International and Intra-national Real Exchange Rates: Theory and Evidence[*]

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Abstract

In this paper we study two long-standing puzzles in the International Finance literature: the fact that the real exchange rate (RER) is very volatile (RER volatility puzzle) and that it covaries negatively with domestic consumption relative to foreign consumption (Backus-Smith puzzle). To understand the two puzzles we depart from the existing literature by focusing on a disaggregated analysis of consumption and RER. First, using region-level data for a large number of developed and developing economies we document the characteristics of the two puzzles in cross-country and within-country data. We then develop a combined model of inter-regional and international trade. The model exhibits variations in inter-regional as well as in inter-national consumption and real exchange rates. We show that with a combination of within country and across country shocks, and endogenous tradability the model can rationalize the two puzzles and does so in both international and intra-national dimensions.

1 Introduction

Despite dramatic changes affecting international asset markets, there exists ample evidence that international risk-sharing is far from perfect. One of the well-known corroborations of this fact is the Backus-Smith puzzle, due to Backus and Smith (1993) and Kollmann (1995). The puzzle is the result of a striking disagreement between theory and data. In particular, existing theoretical models with complete asset markets imply a high positive correlation between the cross-country consumption growth rates and the real exchange rate (RER). In the data, however, such positive correlation is forcefully rejected. A related though distinct puzzle in the international finance literature is known as the real exchange rate volatility puzzle. This puzzle says that the volatility of the real exchange rate is very high and significantly exceeds the volatility of consumption and GDP, while the corresponding statistic predicted by the theory is small. Both these puzzles pose significant challenge to the standard open-economy models with complete markets and representative agents.

In this paper we propose a novel explanation for the Backus-Smith and volatility puzzles. Our explanation relies on two central elements: within country (regional) heterogeneity and endogenous non-tradability. We show that each feature, when introduced in an incomplete markets setting, implies a lower international consumption-RER correlation, but is not sufficient by itself. Only a combination of regional heterogeneity and endogenous tradability resolves the Backus-Smith puzzle and generates realistic volatility of RER relative to consumption.

Our theoretical analysis is supplemented by a novel empirical study of the Backus-Smith correlation and real exchange rate volatility. In particular, we depart from the existing studies of the consumption-RER relationship that focus on cross-country analysis and instead develop a more disaggregated analysis using region-level data for a large sample of economies. This analysis helps us to establish the key properties of the real exchange rate and consumption in regional data, a line of research not explored in the existing literature. We then show that our theoretical model can rationalize these regional facts as well.

For this purpose we construct a new dataset of historical series for consumption and prices for a large number of regions in a number of developed and developing countries. To date we have collected regional data for the US, Canada, Japan, Spain, Germany, India and China. Two sets of our empirical findings are particularly interesting. First, the data indicates that the Backus-Smith puzzle is not a purely cross-country phenomenon. In fact, we find that the departures from perfect risk-sharing are significant within countries as well. For within country data, we normally find a positive correlation between relative consumption

and real exchange rate. But this correlation is very small, much less than predicted by the theory. For instance, the average correlation between relative consumption and the regional real exchange rate across Canadian provinces is equal to 0.12, a significant departure from a coefficient of 1 predicted by the theory. Similar results characterize the degree of inter-regional risk-sharing in the other countries in our sample. We also find that the degree of risk-sharing varies substantially across regions and over time in a given country. For instance, in Canada, the correlation ranges from -0.3 (between Alberta and Ontario) to 0.55 (between Quebec and Newfoundland). Figure 1 illustrates these findings. For each country in our sample it presents a scatterplot of all unique bilateral regional average CPI growth and consumption growth pairs over a corresponding sample period. Japan and China exhibit practically no correlation between the two variables, while the correlation is slightly positive for Canada, Spain, Germany, US and India.

Figure **1**. Backus-Smith correlation across regions
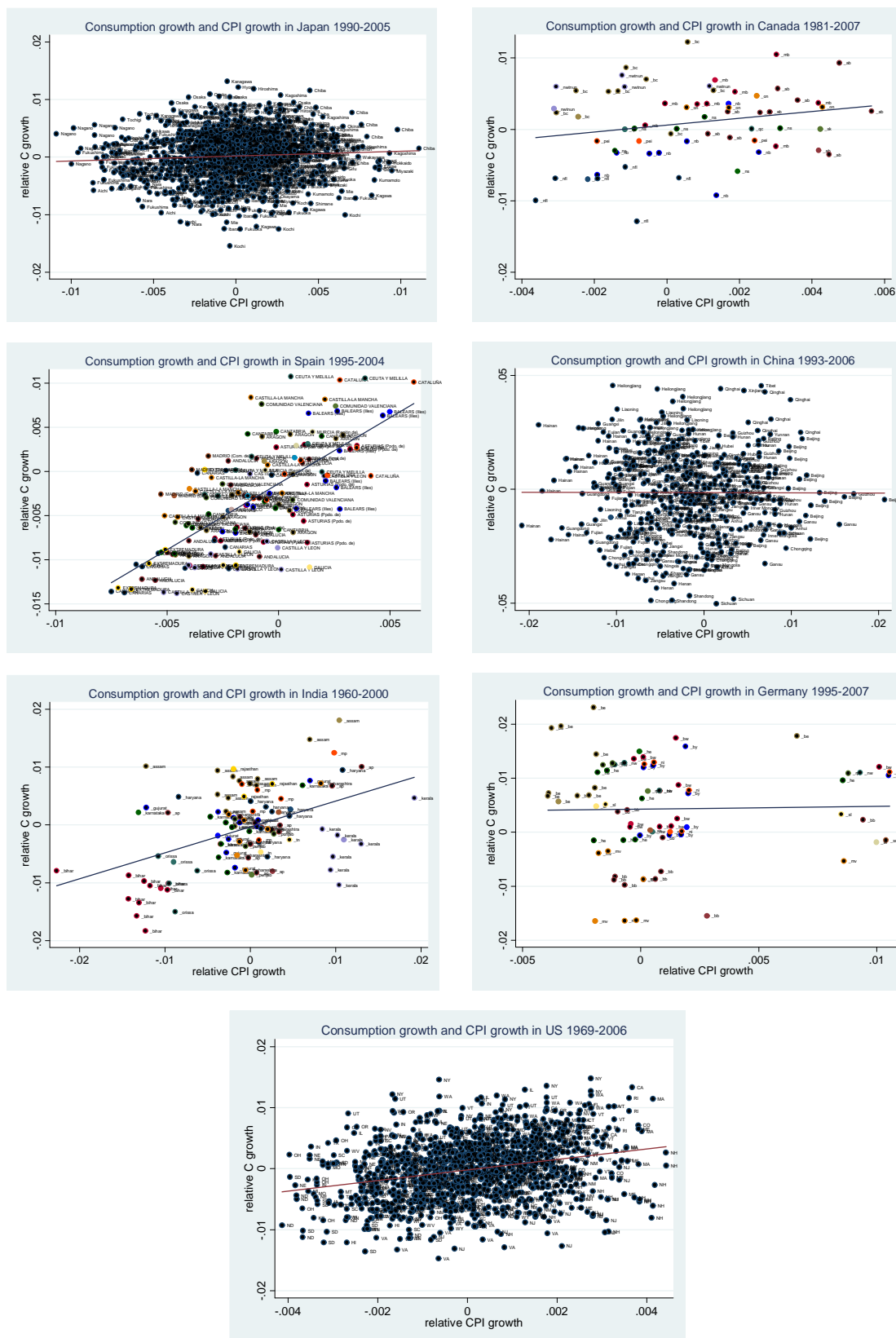
Notes: Each point in the scatterplots above represents the sample average of the RER growth versus the sample average of the relative consumption growth for a pair of regions within a given country over a corresponding sample period (excluding repeated pairs). Details on our data sources and coverage are provided in Appendix A.1.

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In terms of volatilities we find that real exchange rate is significantly less volatile than consumption in the regional data and significantly more volatile than consumption in the international data. These empirical findings discipline our theoretical work as they provide us with the additional testable implication that our model should comply with. In particular, any explanation for the Backus-Smith puzzle and the real exchange rate volatility puzzle in the international data must also comply with the corresponding properties of the RER and consumption in the regional data. Providing such an explanation is the main objective of our paper.

Our model is a relatively canonical model of international business cycles with incomplete asset markets. We consider a world economy composed of two symmetric countries. Each country consists of two regions. Each region is represented by a continuum of households and a continuum of monopolistically competitive firms who choose whether to sell their goods in the domestic regional market, the other regional market within the same country, or to export to the foreign country. The split is endogenously determined by a combination of regional and international transportation costs and fixed costs of exporting. Each firm is subject to a sector-specific total factor productivity (TFP) shock and a firm-specific productivity shock. As TFP varies, the mass of firms producing in each sector fluctuates over time, inducing changes in the supply of goods and relative prices. We find that allowing for regional heterogeneity and endogenous international and regional tradability generates a negative correlation between RER and relative consumption across countries and a positive correlation across regions within a country, thus allowing us to account for the Backus-Smith puzzle in both international and regional dimensions. The two features also generate volatility of RER relative to consumption that is qualitatively comparable to the corresponding statistic in the data.

TFP shocks in our model originate in the internationally and regionally traded and regionally nontraded sectors. All shocks trigger wealth effects and a Balassa-Samuelson effect, often in the opposite directions. Internationally traded goods shocks imply a negative correlation between international RER and relative consumption. Internationally nontraded shocks have the opposite effect. By introducing within country heterogeneity and endogenous tradability, one at a time, we show that the international Backus-Smith correlation can be reduced.

Regional heterogeneity is important because it allows households to trade goods with their countrymen living in the other regions, thus effectively reducing the demand elasticity for nontraded goods and the implied magnitude of nontraded price movements.[1] As a result,

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[1] Such regionally-traded goods constitute a large fraction of the goods that are commonly classified as

the effects of nontraded shocks on RER and relative consumption are moderated and the implied positive correlation between RER and relative consumption following these shocks is lower. Endogenous tradability is important because it makes the size of each sector cyclical, as new varieties become traded across regions and countries. This amplifies the effects of internationally traded shocks and suppresses the effects of internationally-nontraded shocks. A combination of the two features generates the Backus-Smith correlation consistent with both the intra-national and international data.

This paper contributes to the existing literature along several dimensions. First, it contributes to the literature studying Backus-Smith and real exchange rate volatility puzzles in the context of open economy models. A number of papers have tried to explain the Backus-Smith puzzle using models with incomplete financial market structure. Benigno and Thoenissen (2008) introduce a production economy with non-traded and traded goods sectors. Under the assumption that productivity shocks to the traded goods sector are more persistent and more volatile than those in the non-traded goods sector, they can obtain a negative correlation between relative consumption and the real exchange rate. Corsetti, Dedola, and Leduc (2008) show that with a very low elasticity of substitution between domestic and foreign traded goods, they can account for the Backus-Smith puzzle.[2] By contrast, our model can account for the Backus-Smith puzzle while using more symmetric sectoral productivity processes and standard values for substitution elasticities.

Further, the existing work focuses only on international risk-sharing and does not study the intra-national risk-sharing. Bodenstein (2006) introduces limited contract enforcement into a two-country complete markets model with the production structure of Corsetti, Dedola, and Leduc (2008) to rationalize both the Backus-Smith and RER volatility puzzles. However, he too does not study intra-national risk-sharing. Engel and Wang (2008) introduce durable goods into an otherwise standard business cycle model and show that they can replicate the Backus-Smith correlation in the data under the parametrization similar to that in Benigno and Thoenissen (2008). Our contribution is to extend the existing work by departing from a representative agent framework and incorporating regional heterogeneity within each country. Furthermore, our approach allows sectoral tradability to be endogenously determined as a result of profit maximizing decisions by heterogenous firms.

nontraded. In particular, for OECD economies, the average share of regionally-traded sectors, to which we allocate some of the services (i.e. finance, insurance, real estate and business services), as well as utilities (electricity, gas and water supply) constituted 50% of the real value added produced by all nontraded sectors in 2007.

[2]When elasticity of substitution between domestic and imported tradable goods is high, the model requires very persistent productivity shocks (approaching unit root) to generate negative Backus-Smith correlation.

Studies that do allow for inter-regional risk-sharing are primarily empirical and focus entirely on the comovement between consumption and output growth rates (Sorensen and Yosha, 1998; Crucini and Hess, 1999; Labhard and Sawicki, 2006). Our contribution to this empirical literature is to focus on a measure of risk-sharing due to Backus and Smith (1993) and provide a set of stylized facts about the intra-national Backus-Smith correlation and real exchange rate volatility for a relatively large sample of developed and developing countries. Furthermore, we show that these regional and international facts can be rationalized jointly.

Finally, several recent papers have studied the role of endogenous non-tradability for the international business cycles. Ghironi and Melitz (2005) incorporate Melitz (2003) model of trade into a dynamic two-country model and show that it improves the model performance in terms of several business cycle moments. Alessandria and Choi (2007) find that endogenous firms export decisions have negligible effects on the properties of international business cycles. In a similar framework, Naknoi (2008) studies the dynamics of RER under different exchange rate regimes. Corsetti, Martin, and Pesenti (2008) use endogenous tradability model to study the dynamics of current account adjustment. Bergin and Glick (2005) and Bergin, Glick, and Taylor (2006) use a model of endogenous tradability to understand price dispersion and the Balassa-Samuelson effect. Our approach embeds the sectoral tradability decisions into a dynamic international business cycles framework and allows for both international and regional export decisions in the firm's problem.

The rest of the paper is structured as follows. The next section describes the Backus-Smith and real exchange rate volatility puzzles. Section 3 presents our empirical findings on the two puzzles in the international and intra-national data. Section 4 develops the model that we propose to rationalize our empirical results. Section 5 describes how the model is calibrated and solved. The results are presented in Section 6. The last section concludes.

2   Stating the Puzzles

To illustrate the main idea behind the Backus-Smith puzzle, consider a multi-country international business cycle model. A problem facing a representative household in country $j = 1, .., J$ is as follows:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \delta^i U(C_{t+i}^j, X_{t+i}^j),$$

where δ is the subjective discount factor, C_t denotes a composite consumption good in period t, X_t denotes factors other than consumption that can affect utility (e.g. preference shocks, labor choice, etc.). C_t is defined as a CES aggregator over the consumption of traded, C_t^{T},

and nontraded, C_t^N, goods. Let P_t^j denote consumption price index in country j.

Every period households receive stochastic endowments of traded and nontraded goods: Y_t^T and Y_t^N, respectively. Further, suppose that in this world a complete set of state-contingent securities is available to households in all countries. In this case, the key optimality condition is

$$U_c(C_{i,t}, X_{i,t}) = \frac{P_{i,t}}{P_{j,t}} U_c(C_{j,t}, X_{j,t}),$$

where $U_c(C_{i,t}, X_{i,t})$ denotes the marginal utility of aggregate consumption in country i. Further, let $e_t^{i,j} = P_{i,t}/P_{j,t}$ denote the real exchange rate between country i and j. Then the condition above can be written as

$$U_c(C_{i,t}, X_{i,t}) = e_t^{i,j} U_c(C_{j,t}, X_{j,t}). \tag{1}$$

This equation must hold in every date and state of the world, between any two countries i and j. It says that in equilibrium, consumption between households in countries i and j must be allocated in a way that its marginal utility (converted into the same units using the real exchange rate) is equalized across countries. If utility is of a constant relative risk aversion (CRRA) form, with the coefficient of relative risk aversion ω, equation (1) becomes

$$\left(\frac{C_{i,t}}{C_{j,t}}\right)^{-\omega} = e_t^{i,j},$$

or equivalently in logs

$$-\omega \left(\ln C_{i,t} - \ln C_{j,t}\right) = \ln e_t^{i,j}.$$

These expressions establish the close relationship between the real exchange rate and relative consumption in countries i and j. They imply that during the times when the real exchange rate in country i appreciates, the consumption in that country should fall relative to country j's consumption. The expression above must also hold in growth rates:

$$-\omega \left(\Delta \ln C_{i,t} - \Delta \ln C_{j,t}\right) = \Delta \ln e_t^{i,j},$$

where $\Delta \ln X_{i,t} = \ln X_{i,t} - \ln X_{i,t-1}$. Therefore, if markets are complete, the correlation, $\rho_{e,c_j/c_i} = corr(\Delta \ln e_t^{i,j}, \omega \Delta \ln \frac{C_{j,t}}{C_{i,t}})$, should be equal to 1, as pointed out by Backus and Smith (1993). In the data, a number of authors have found this correlation to be very low and often negative (see Backus and Smith, 1993; Kollmann, 1995; Chari, Kehoe, and McGrattan, 2002; Corsetti, Dedola, and Leduc, 2008). Ravn (2001) and Lewis (1996) study the role of preference nonseparabilities in explaining Backus-Smith puzzle and find that the puzzle is

robust to them.

A related puzzle is the real exchange rate volatility puzzle. It relates the volatility of the real exchange rate to the volatility of macroeconomic aggregates, such as consumption or output. In the context of the complete markets model presented above, the relative volatility of the real exchange rate and consumption can be written as

$$
\begin{aligned}
\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}} &= \frac{\mathbb{V}(\Delta \ln e_t^{i,j})}{\sqrt{\mathbb{V}(\Delta \ln C_{i,t})\mathbb{V}(\Delta \ln C_{j,t})}} \\
&= \omega^2 \frac{\mathbb{V}(\Delta \ln C_{i,t}) + \mathbb{V}(\Delta \ln C_{j,t}) - 2\mathbb{CV}(\Delta \ln C_{i,t}, \Delta \ln C_{j,t})}{\sqrt{\mathbb{V}(\Delta \ln C_{i,t})\mathbb{V}(\Delta \ln C_{j,t})}} \\
&= \omega^2 \left(\sqrt{\frac{\mathbb{V}(\Delta \ln C_{i,t})}{\mathbb{V}(\Delta \ln C_{j,t})}} + \sqrt{\frac{\mathbb{V}(\Delta \ln C_{j,t})}{\mathbb{V}(\Delta \ln C_{i,t})}} - 2\rho_{c_i,c_j} \right),
\end{aligned}
$$

where $\mathbb{V}(.)$ and $\mathbb{CV}(.)$ denote the unconditional variance and covariance; and $\rho_{c_i,c_j} = corr(\Delta \ln C_{i,t}, \Delta \ln C_{j,t})$. The complete markets model above predicts $\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}$ to be very small, unless ω is very large. In the data, this relative volatility is large (see Chari, Kehoe, and McGrattan, 2002; Bodenstein, 2006 and our estimates in the next section).

3   Empirical analysis

In this section we study the properties of real exchange rate and relative consumption empirically. We start by documenting these properties in the cross-country data, and then develop a more disaggregated analysis that relies on the region-level data for a large sample of economies. To conduct this analysis we construct a novel dataset of historical series for consumption and prices for a large number of regions in a number of developed and developing countries. Our dataset contains regional data for US, Canada, Japan, Spain, Germany, India and China and covers at least ten years of annual data. A detailed description of the data and its sources is provided in Appendix A.1.

Following Backus and Smith (1993), a number of papers have studied the correlation between consumption and real exchange rates in the international data. Chari, Kehoe, and McGrattan (2002) report the median value for the correlation between real exchange rate and relative consumption in quarterly data between 1973-1994 for a sample of OECD countries to be -0.07. Corsetti, Dedola, and Leduc (2008) in a sample of OECD economies during 1970-2001 find that this correlation is well below one and often is negative when measured in levels or growth rates. Benigno and Thoenissen (2008) re-estimate the correlations between the two variables (both logged and Hodrick-Prescott filtered) in levels and first-differences

using the US as a reference country. In their sample of annual data during 1970-2000 they find that the median correlation is -0.288 in levels and -0.168 in the first-differences.

We confirm these findings for our sample of countries in annual data during 1970-2004. Table 1 summarizes the results. Columns (i)-(iii) report the average, maximum and minimum of the bilateral correlations between the growth rates of real exchange rate and relative consumption for 7 different countries measured relative to OECD economies.[3] For all countries, on average, the correlation is negative.

	$\rho_{e,c_j/c_i}$			$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$
	mean	max	min	mean
	(i)	(ii)	(iii)	(iv)
Canada	-0.104	0.185	-0.568	3.086
Japan	-0.042	0.323	-0.656	3.280
Spain	-0.159	0.435	-0.570	2.345
India	-0.150	0.567	-0.608	2.883
China	-0.127	0.345	-0.617	4.582
Germany	-0.105	0.486	-0.674	2.033
US	-0.205	0.138	-0.654	3.634

Table 1. International C-RER correlation and relative volatility

Notes: Columns under $\rho_{e,c_j/c_i}$ heading report summary statistics on the cross-sectional distribution of the correlation coefficient between the growth rate of RER and the growth rate of relative consumption for a set of all bilateral pairs of a given country with other OECD countries calculated over the sample period. Mean, max, min refer to the average, maximum and minimum of that distribution. In the same spirit, column under $\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$ heading reports the sample mean of the cross-sectional distribution of standard deviation of RER growth between countries i and j relative to the square root of the product of standard deviations of consumption growth in those countries. Details on the sample coverage and data sources are provided in Appendix A.1.

Column (iv) reports the volatility of the growth rate in the real exchange rate relative to the volatility of consumption growth in country i and OECD economies (j). For all countries in our sample the volatility of real exchange rate exceeds the volatility of consumption. The real exchange rate in China exhibits the most volatility relative to consumption among the countries in our sample. These numbers are consistent with the available estimates in the literature. For instance, in the quarterly data for US relative to 9 European economies during 1972:1-1993:3, Chari, Kehoe, and McGrattan (2002) find that the volatility of real exchange rate is 4.4 times that of US GDP. For the period of 1978:4-2003:4 Bodenstein (2006) computes that real exchange rate in US relative to the other G7 countries is 3.4 times as volatile as US consumption. Similarly, Johri and Lahiri (2008) report a number for this

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[3]For each OECD economy in our sample, we used the remaining OECD countries as a reference sample.

volatility equal to 5.5. Our findings are for the annual data and are consistent with these numbers.

Next, we re-estimate the correlation and volatility properties of real exchange rate in the intra-national data. Such analysis also allows us to control for the movements in the nominal exchange rate. Table 2 presents the results.

Table 2. Regional C-RER correlation and relative volatility				
	$\rho_{e,c_j/c_i}$			$\sqrt{\dfrac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$
	mean	max	min	mean
	(i)	(ii)	(iii)	(iv)
Canada	0.123	0.55	-0.35	0.47
Japan	0.121	0.85	-0.69	0.47
Spain	0.196	0.80	-0.70	0.25
India	0.656	0.85	0.34	0.73
China	0.117	0.88	-0.84	0.36
Germany	0.108	0.89	-0.63	0.23
US	0.085	0.59	-0.43	0.26

Notes: Columns under $\rho_{e,c_j/c_i}$ heading report summary statistics on the cross-sectional distribution of the correlation coefficient between the growth rate of regional RER and the growth rate of relative consumption for a set of all bilateral regional pairs in a given country calculated over the sample period. Mean, max, min refer to the average, maximum and minimum of that distribution. In the same spirit, column under $\sqrt{\dfrac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$ heading reports the sample mean of the cross-sectional distribution of standard deviation of RER growth between regions i and j relative to the square root of the product of standard deviations of consumption growth in those regions. Details on the sample coverage and data sources are provided in Appendix A.1.

As in the international data, columns (i)-(iii) report the summary statistics on the bilateral correlation of the growth rate in real exchange rate and relative consumption between two regions within a given country. All of the average correlations are well below 1, with the minimum values well into the negative range. The numbers clearly indicate a failure of full RER-adjusted risk sharing within countries, but provide some evidence of more risk sharing within countries than across countries. Column (iv) reports the relative volatility of real exchange rate growth between two regions within a given country to their consumption growth. All numbers are well below one, suggesting that regional real exchange rate is significantly less volatile than regional consumption.

Overall, two results stand out from our empirical study. First, the Backus-Smith puzzle is not a purely cross-country phenomenon. The puzzle remains in the region/state-level data. Further, the Backus-Smith correlation varies substantially across regions/states in a given country. Second, the RER is more volatile than consumption in the international data and

is less volatile than consumption in the intra-national data. We next develop a model to rationalize these findings.

4 Model economy

We work with a conventional two-country model with incomplete asset markets. This type of model has been used extensively in the literature to study the properties of international business cycles, including the Backus-Smith correlation and the volatility puzzle. First, we extend this model to allow for regional heterogeneity within each country and show that this modified model can not account for the properties of international or intra-national data. We then introduce endogenous non-tradability into this model and show that it too can not produce a realistic Backus-Smith correlation and RER volatility. Only when both regional heterogeneity and endogenous tradability are added simultaneously, can our model generate negative correlation between real exchange rate and relative consumption (in both levels and growth rates) in the international dimension, while predicting a positive correlation between the two variables in the intra-national dimension. Furthermore, the model's predictions for the volatility of real exchange rate and relative consumption across regions and across countries are in accord with the data. We next present our full model that allows for both regional heterogeneity and endogenous non-tradability.

4.1 Households

We consider a world economy consisting of two symmetric economies, home (H) and foreign (F). Each country is populated by a continuum of firms and households who reside in two regions, A and B, within each country. Households residing in country H, region $j = \{A,B\}$ supply L_j units of labor inelastically to regional firms in return for the wage rate $w_{j,t}$. We assume that labor is perfectly mobile across sectors within a region, but not across regions or countries. Households also derive utility from consuming two goods: a composite tradable, C_j^T and a composite domestic nontradable, C_j^N. In particular, preferences of H households in region j are given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \delta^s U(C_{j,t+s}^T, C_{j,t+s}^N), \tag{2}$$

where $0 < \delta < 1$ is the discount factor, and $U(.)$ is a concave sub-utility function. Let all goods available to H households in period t be normalized to a [0,1] interval. We denote each individual good on this interval by index i. Further, let $i_{j,t}^*$ denote the endogenous time-t share of goods that are non-traded. Then, at time t household consumes a nontraded goods

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basket defined over a continuum of goods $I_t^{\mathrm{N}} = [0, i_{j,t}^*]$ and a traded goods basket defined over a continuum of goods $I_t^{\mathrm{T}} = [i_{j,t}^*, 1]$. In what follows we show that the measures of I_t^{T} and I_t^{N} are determined from the firms' profit maximizing decisions. The utility function of H household from region j can be written as:

$$U(C_{j,t}^{\mathrm{T}}, C_{j,t}^{\mathrm{N}}) = \frac{1}{1-\omega} \left(\left[\lambda_{\mathrm{T}}(i_{j,t}^*)^{1-\phi} \left(C_{j,t}^{\mathrm{T}} \right)^{\phi} + \lambda_{\mathrm{N}}(i_{j,t}^*)^{1-\phi} \left(C_{j,t}^{\mathrm{N}} \right)^{\phi} \right]^{\frac{1}{\phi}} \right)^{1-\omega},$$

where ω is the inverse of the intertemporal elasticity. Here $\lambda_{\mathrm{T}}(i_{j,t}^*)$ and $\lambda_{\mathrm{N}}(i_{j,t}^*) = 1 - \lambda_{\mathrm{T}}(i_{j,t}^*)$ are the weights of tradable and nontradable consumption in the aggregate consumption basket; and are endogenous functions $i_{j,t}^*$. In the Appendix A.2 we show that $\lambda_{\mathrm{N}}(i_{j,t}^*) = i_{j,t}^*$ and $\lambda_{\mathrm{T}}(i_{j,t}^*) = 1 - i_{j,t}^*$. The elasticity of substitution between tradable and nontradable consumption is $(1-\phi)^{-1} > 0$.

A composite tradable good, $C_{j,t}^{\mathrm{T}}$, in turn is given by a CES aggregator over tradables produced in the H and F countries:

$$C_{j,t}^{\mathrm{T}} = \left[\lambda_{\mathrm{H}}(i_{j,t}^*)^{1-\rho} \left(C_{j,t}^{\mathrm{H}} \right)^{\rho} + \lambda_{\mathrm{F}}(i_{j,t}^*)^{1-\rho} \left(C_{j,t}^{\mathrm{F}} \right)^{\rho} \right]^{\frac{1}{\rho}},$$

where $(1-\rho)^{-1} > 0$ is the elasticity of substitution between tradable goods. Here $\lambda_{\mathrm{H}}(i_{j,t}^*)$ and $\lambda_{\mathrm{F}}(i_{j,t}^*) = 1 - \lambda_{\mathrm{H}}(i_{j,t}^*)$ denote the weights that households in country H assign to the consumption of H and F-produced tradable goods. Again, these weights are endogenously linked to the shares of non-traded goods in the two countries as:

$$\lambda_{\mathrm{H}}(i_{j,t}^*) = \frac{1 - i_{j,t}^*}{2 - i_{j,t}^* - \hat{i}_t^*} \qquad \text{and} \qquad \lambda_{\mathrm{F}}(i_{j,t}^*) = \frac{1 - \hat{i}_t^*}{2 - i_{j,t}^* - \hat{i}_t^*},$$

where $\hat{i}_t^* = 0.5(\hat{i}_{\mathrm{A},t}^* + \hat{i}_{\mathrm{B},t}^*)$ is the average nontraded share in country F in period t. Hereafter we will use a hat, "^", over a variable to denote F country variables.

The key feature that distinguishes the preference structure outlined above from the one used commonly in the open-economy macro models is the endogenous nature of consumption expenditure weights. This is the outcome of endogenous tradability feature of our model.

By introducing regions we can decompose the nontraded consumption basket of households living in region j into two categories: regionally-traded goods, $Z_{j,t}^{\mathrm{T}}$ and regionally-nontraded goods, $Z_{j,t}^{\mathrm{N}}$. Regionally-traded basket is defined over a measure of goods I_t^{RT}, while regionally-nontraded basket is defined over a measure of goods I_t^{RN}, where $I_t^{\mathrm{RT}} + I_t^{\mathrm{RN}} = I_t^{\mathrm{N}}$. Regionally-tradable consumption is given by a CES aggregator over consumption of tradables produced in regions A and B, $Z_{j,t}^{\mathrm{A}}$ and $Z_{j,t}^{\mathrm{B}}$, respectively, so that the aggregate nontraded

consumption becomes:

$$C_{j,t}^{\text{N}} = \left[\gamma_{\text{T}}^{1-\alpha} \left[\gamma_{\text{A}}(i_{j,t}^*)^{1-\beta} \left(Z_{j,t}^{\text{A}} \right)^{\beta} + \gamma_{\text{B}}(i_{j,t}^*)^{1-\beta} \left(Z_{j,t}^{\text{B}} \right)^{\beta} \right]^{\frac{\alpha}{\beta}} + \gamma_{\text{N}}^{1-\alpha} \left(Z_{j,t}^{\text{N}} \right)^{\alpha} \right]^{\frac{1}{\alpha}},$$

where γ_{T} and $\gamma_{\text{N}} = 1 - \gamma_{\text{T}}$ represent the weights that households in region j assign to the consumption of regionally-traded and regionally-nontraded goods. The elasticity of substitution between regionally tradable and nontradable goods is given by $(1 - \alpha)^{-1} > 0$. For tractability, we assume that I_t^{RN} measure is exogenously given and fixed. The measure of regionally traded goods, I_t^{RT}, will then vary with changes to I_t^{N}. Further, $\gamma_{\text{A}}(i_{j,t}^*)$ and $\gamma_{\text{B}}(i_{j,t}^*) = 1 - \gamma_{\text{A}}(i_{j,t}^*)$ denote the weights that households in region j assign to the consumption of A- and B−produced tradable goods. These weights are also functions of $i_{j,t}^*$ as

$$\gamma_{\text{A}}(i_{j,t}^*) = \frac{i_{\text{A},t}^*}{i_{\text{A},t}^* + i_{\text{B},t}^*} \qquad \text{and} \qquad \gamma_{\text{B}}(i_{j,t}^*) = \frac{i_{\text{B},t}^*}{i_{\text{A},t}^* + i_{\text{B},t}^*}.$$

The elasticity of substitution between two regionally tradable goods is given by $(1 - \beta)^{-1} > 0$.

Each consumption basket is a CES aggregate of individual goods. For instance, period-t consumption aggregates in region j, country H are given by:

$$\left(Z_{j,t}^{\text{N}} \right)^{\beta} = \int_0^{\gamma_{\text{N}} i_{j,t}^*} \left(\frac{1}{\gamma_{\text{N}} i_{j,t}^*} \right)^{1-\beta} z_{j,t}(i)^{\beta} di, \qquad\qquad i \in I^{\text{RN}}$$

$$\left(Z_{j,t}^{\text{J}} \right)^{\beta} = \int_{\gamma_{\text{N}} i_{j,t}^*}^{i_{j,t}^*} \left(\frac{1}{(1-\gamma_{\text{N}}) i_{j,t}^*} \right)^{1-\beta} z_{j,t}(i)^{\beta} di, \qquad\qquad i \in I^{\text{RT}}$$

$$\left(Z_{j,t}^{\text{-J}} \right)^{\beta} = \int_{i_{j,t}^*}^{i_{j,t}^* + (1-\gamma_{\text{N}}) i_{-j,t}^*} \left(\frac{1}{(1-\gamma_{\text{N}}) i_{-j,t}^*} \right)^{1-\beta} z_{-j,t}^*(i)^{\beta} di, \qquad i \in I^{\text{RT}}$$

$$\left(C_{j,t}^{\text{H}} \right)^{\rho} = \int_{i_{j,t}^*}^{1} \left(\frac{1}{1-i_{j,t}^*} \right)^{1-\rho} c_{j,t}(i)^{\rho} di, \qquad\qquad i \in I^{\text{T}}$$

$$\left(C_{j,t}^{\text{F}} \right)^{\rho} = \int_{1}^{2-\hat{i}_t^*} \left(\frac{1}{1-\hat{i}_t^*} \right)^{1-\rho} \left(c_{j,t}^*(i) \right)^{\rho} di \qquad\qquad i \in I^{\text{T}},$$

where $\{-j\}$ is used to denote region other than j within the same country. See Appendix A.2 for derivations. For simplicity, we assume that households treat internationally-traded goods originated in the two regions of the foreign country as perfect substitutes. Note also that by defining consumption aggregates in this way we rule our "love for variety" effects, which are characteristic of Dixit-Stiglitz aggregators. In doing so we follow the tradition of the standard international business cycle models, in which consumption aggregates are defined over a constant measure of varieties.

Let $p_{j,t}(i)$ denote the price of good $i \in I_t^{\text{T}}$, while $r_{j,t}(i)$ denote the price of good $i \in I_t^{\text{N}}$. Then the consumption-based price indices for different consumption baskets in country H, region j are given by:

$$\left(R_{j,t}^{\mathrm{N}}\right)^{\beta/(\beta-1)} = \int_0^{\gamma_{\mathrm{N}} i_{j,t}^*} \left(\frac{1}{\gamma_{\mathrm{N}} i_{j,t}^*}\right) r_{j,t}(i)^{\beta/(\beta-1)} di, \qquad\qquad i \in I^{\mathrm{RN}}$$

$$\left(R_{j,t}^{\mathrm{J}}\right)^{\beta/(\beta-1)} = \int_{\gamma_{\mathrm{N}} i_{j,t}^*}^{i_{j,t}^*} \left(\frac{1}{(1-\gamma_{\mathrm{N}}) i_{j,t}^*}\right) r_{j,t}(i)^{\beta/(\beta-1)} di, \qquad\qquad i \in I^{\mathrm{RT}}$$

$$\left(R_{j,t}^{\mathrm{J}}\right)^{\beta/(\beta-1)} = \int_{i_{j,t}^*}^{i_{j,t}^* + (1-\gamma_{\mathrm{N}}) i_{-j,t}^*} \left(\frac{1}{(1-\gamma_{\mathrm{N}}) i_{-j,t}^*}\right) r_{-j,t}^*(i)^{\beta/(\beta-1)} di, \qquad\qquad i \in I^{\mathrm{RT}}$$

$$\left(P_{j,t}^{\mathrm{H}}\right)^{\rho/(\rho-1)} = \int_{i_{j,t}^*}^{1} \left(\frac{1}{1-i_{j,t}^*}\right) p_{j,t}(i)^{\rho/(\rho-1)} di, \qquad\qquad i \in I^{\mathrm{T}}$$

$$\left(P_{j,t}^{\mathrm{F}}\right)^{\rho/(\rho-1)} = \int_{1}^{2-\hat{\imath}_t^*} \left(\frac{1}{1-\hat{\imath}_t^*}\right) \left(p_{j,t}^*(i)\right)^{\rho/(\rho-1)} di, \qquad\qquad i \in I^{\mathrm{T}}.$$

Households in each country finance their consumption expenditures with wage income and profits, $\Pi_{j,t}$, received from domestic firms. Households also have access to international borrowing and lending at interest rate R_t. We assume that bonds are denominated in units of internationally tradable goods produced by country H.[4] As a result, asset markets in our model are incomplete both across regions within a country, and across countries. We use this simple asset structure in our model to preserve comparability with the existing models of international business cycles, and to maintain the symmetry across regions and across countries in the asset structure dimension.

As noted before, we assume that internationally-traded goods produced in both regions within the same country are perfect substitutes. In combination with no barriers to regional trade in these goods, we get for H country $P_{\mathrm{A},t}^\chi = P_{\mathrm{B},t}^\chi \equiv P_t^\chi$, with $\chi = \{\mathrm{H,F}\}$; and for F country $\hat{P}_{\mathrm{A},t}^\chi = \hat{P}_{\mathrm{B},t}^\chi \equiv \hat{P}_t^\chi$, with $\chi = \{\mathrm{H,F}\}$.

Therefore, the period$-t$ budget constraint of households living in country H, region j is given by

$$P_t^{\mathrm{H}} C_{j,t}^{\mathrm{H}} + P_t^{\mathrm{F}} C_{j,t}^{\mathrm{F}} + R_{j,t}^{\mathrm{A}} Z_{j,t}^{\mathrm{A}} + R_{j,t}^{\mathrm{B}} Z_{j,t}^{\mathrm{B}} + R_{j,t}^{\mathrm{N}} Z_{j,t}^{\mathrm{N}} + \frac{1}{R_t} P_t^{\mathrm{H}} B_{j,t} \leq P_t^{\mathrm{H}} B_{j,t-1}$$
$$+ \left(L_{j,t}^{\mathrm{H}} + L_{j,t}^{\mathrm{J}} + L_{j,t}^{\mathrm{N}} + f_x(1 - i_{j,t}^*)\right) w_{j,t} + \Pi_{j,t}, \tag{3}$$

where $B_{j,t}$ denotes period$-t$ holdings of international bond, P_t^χ is the H country price of internationally traded goods produced in country χ, with $\chi = \{\mathrm{H,F}\}$, and $R_t^\varkappa$ is the price of regional good $\varkappa = \{\mathrm{A,B,N}\}$ as defined in (6)-(7). $L_{j,t}^{\mathrm{H}}, L_{j,t}^{\mathrm{J}}, L_{j,t}^{\mathrm{N}}$ denote aggregate labor employed in the production of internationally tradable, regionally tradable and non-tradable goods, respectively, in region j. The term $f_x(1-i_{j,t}^*)w_{j,t}$ captures the wage payments received by regional labor that was hired by the domestic regional firms to cover the fixed cost of exporting, f_x. We discuss this aspect in details below.

Using household's first-order conditions, we can now define household's demand for each individual goods i belonging to the different consumption baskets as

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[4]Denomination of the bond does not influence our results.

$$z_{j,t}(i) = \tfrac{1}{\gamma_{\text{N}}i^*_{j,t}} \left(r_{j,t}(i)/R^{\text{N}}_{j,t}\right)^{1/(\beta-1)} Z^{\text{N}}_{j,t}, \qquad\qquad i \in I^{\text{RN}}$$

$$z_{j,t}(i) = \tfrac{1}{(1-\gamma_{\text{N}})i^*_{j,t}} \left(r_{j,t}(i)/R^{\text{J}}_{j,t}\right)^{1/(\beta-1)} Z^{\text{J}}_{j,t}, \qquad\qquad i \in I^{\text{RT}}$$

$$z^*_{-j,t}(i) = \tfrac{1}{(1-\gamma_{\text{N}})i^*_{-j,t}} \left(r^*_{-j,t}(i)/R^{\text{J}}_{j,t}\right)^{1/(\beta-1)} Z^{\text{J}}_{j,t}, \qquad i \in I^{\text{RT}} \qquad (4)$$

$$c_{j,t}(i) = \tfrac{1}{1-i^*_{j,t}} \left(p_{j,t}(i)/P^{\text{H}}_{j,t}\right)^{1/(\rho-1)} C^{\text{H}}_{j,t}, \qquad\qquad i \in I^{\text{T}}$$

$$c^*_{j,t}(i) = \tfrac{1}{1-\hat{\imath}^*_t} \left(p^*_{j,t}(i)/P^{\text{F}}_{j,t}\right)^{1/(\rho-1)} C^{\text{F}}_{j,t}, \qquad\qquad i \in I^{\text{T}}.$$

Note here, that the terms with $i^*_{j,t}$ appear in the expressions above as we adjust for the fact that the set of varieties over which aggregates are defined can expand or contract.

Preferences of F households living in region $j = \{\text{A,B}\}$ are similarly defined in terms of tradable consumption basket, $\hat{C}^{\text{T}}_{j,t}$, and a nontradable consumption basket, $\hat{C}^{\text{N}}_{j,t}$. The tradable consumption bundle in F country is defined symmetrically in terms of H tradables, $\hat{C}^{\text{H}}_{j,t}$, and F tradables, $\hat{C}^{\text{F}}_{j,t}$, as

$$\hat{C}^{\text{T}}_{j,t} = \left[\hat{\lambda}_{\text{H}}(\hat{\imath}^*_{j,t})^{1-\rho}(\hat{C}^{\text{H}}_{j,t})^\rho + \hat{\lambda}_{\text{F}}(\hat{\imath}^*_{j,t})^{1-\rho}(\hat{C}^{\text{F}}_{j,t})^\rho\right]^{\frac{1}{\rho}}.$$

Here $\hat{\imath}^*_{j,t}$ denotes the endogenous time-t share of goods that are non-traded in region j, country F. Nontradable consumption bundle of F households living in region j is defined in terms of tradables produced in the regions A and B, $\hat{Z}^{\text{A}}_{j,t}$ and $\hat{Z}^{\text{B}}_{j,t}$, respectively, and consumption of region$-j$ nontradables, $\hat{Z}^{\text{N}}_{j,t}$:

$$\hat{C}^{\text{N}}_{j,t} = \left[\gamma_{\text{T}}^{1-\alpha}\left[\gamma_{\text{A}}(\hat{\imath}^*_{j,t})^{1-\beta}\left(\hat{Z}^{\text{A}}_{j,t}\right)^\beta + \gamma_{\text{B}}(\hat{\imath}^*_{j,t})^{1-\beta}\left(\hat{Z}^{\text{B}}_{j,t}\right)^\beta\right]^{\frac{\alpha}{\beta}} + \gamma_{\text{N}}^{1-\alpha}\left(\hat{Z}^{\text{N}}_{j,t}\right)^\alpha\right]^{\frac{1}{\alpha}}.$$

Households in the F country, living in region j face the budget constraint:

$$\hat{P}^{\text{H}}_t \hat{C}^{\text{H}}_{j,t} + \hat{P}^{\text{F}}_t \hat{C}^{\text{F}}_{j,t} + \hat{R}^{\text{A}}_{j,t} \hat{Z}^{\text{A}}_{j,t} + \hat{R}^{\text{B}}_{j,t} \hat{Z}^{\text{B}}_{j,t} + \hat{R}^{\text{N}}_{j,t} \hat{Z}^{\text{N}}_{j,t} + \frac{1}{R_t} P^{\text{H}}_t \hat{B}_{j,t} \le P^{\text{H}}_t \hat{B}_{j,t-1}$$

$$+ \left(\hat{L}^{\text{F}}_{j,t} + \hat{L}^{\text{J}}_{j,t} + \hat{L}^{\text{N}}_{j,t} + f_x(1-\hat{\imath}^*_{j,t})\right) \hat{w}_{j,t} + \hat{\Pi}_{j,t}, \qquad (5)$$

where $\hat{B}_{j,t}$ denotes the bond holdings of F households in region j. We choose internationally traded goods produced in country H to be a numeraire in our economy.

4.2 Firms

Each region in our economy specializes in the production of a continuum of goods, indexed by $i \in [0,1]$. Each differentiated good i is produced using constant returns to scale technology in just one input, labor, $l_t(i)$:

$$y_t(i) = X_t A(i) l_t(i).$$

Here X_t is the total factor productivity (TFP), and $A(i)$ is the good/firm-specific productivity. Productivity differences across firms give rise to firm heterogeneity. Firms can sell their output in three markets: in the local region (*regional market*), in another region within the same country (*national market*), and abroad (*international market*).[5]

We define 'regionally-nontraded' (RN) sector as a sector comprising of firms that sell their goods only on the regional market; similarly, we define 'regionally-traded' (RT) sector as a sector consisting of firms that sell on the national market; finally, all firms that sell on the international market, we define to comprise 'internationally-traded' (IT) sector. These are the goods that form the corresponding consumption baskets of the households. We assume that TFP is sector-specific and affects all firms who choose to locate in that sector equally.

Exporting to a foreign country is costly. There is a fixed cost of beginning to export, denoted by f_x; and iceberg transportation costs, τ_I. In order to sell goods in another region within the same country producers have to incur iceberg transportation costs τ_R, where $\tau_R < \tau_I$.[6] As in Ghironi and Melitz (2005) and Bergin and Glick (2005), we assume that firms hire domestic labor to cover the fixed costs of exporting. Transportation costs are common to all producers.

Differences in productivities also imply different unit costs of production across firms. In particular, if, as before, we let $w_{j,t}$ denote the wage rate in region j of country H measured in units of a numeraire good, then $w_{j,t}/X_{j,t} A_j(i)$ represents such unit costs in region j. Further, in each destination market, a firm faces a constant elasticity of substitution (CES) demand function, which we derived in equations (4). For instance, when selling in the regional market in country H region j, firm i faces demand function given by $z_{j,t}(i)$; similarly, the demand coming from the other region within country H is given by $z^*_{-j,t}(i)$; while $\hat{c}_{j,t}(i)$ denotes the country F's demand for good i. When making a decision of which market to service, the firm decomposes its profits into parts earned from regional sales, potential national sales and potential international sales. In particular, these components for a firm i operating in region

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[5]In our setup, it will be the case that a firm that decides to service the other regional market or export internationally, will also sell its products in the local regional market.

[6]Thus, two key features distinguish regions from countries in our setup: smaller regional transportation costs and, as we will see later, factor price equalization across regions.

j in country H can be written as:

(i) profits from regional sales:

$$\pi_{j,t}(i) = r_{j,t}(i)z_{j,t}(i) - \frac{w_{j,t}}{X_{j,t}A_j(i)}z_{j,t}(i), \qquad i \in I^{\mathrm{RN}}, I^{\mathrm{RT}}$$

$$\pi_{j,t}(i) = p_{j,t}(i)c_{j,t}(i) - \frac{w_{j,t}}{X_{j,t}A_j(i)}c_{j,t}(i), \qquad i \in I^{\mathrm{T}}$$

(ii) profits from national sales:

$$\pi^*_{j,t}(i) = \begin{cases} r^*_{j,t}(i)z^*_{j,t}(i) - \frac{w_{j,t}}{X_{j,t}A_j(i)}\frac{1}{1-\tau_R}z^*_{j,t}(i), & \text{if firm sells in the other region} \\ 0, & \text{otherwise} \end{cases}$$

(iii) profits from international sales:

$$\hat{\pi}_{j,t}(i) = \begin{cases} \hat{p}_{j,t}(i)\hat{c}_{j,t}(i) - \frac{w_{j,t}}{X_{j,t}A_j(i)}\frac{1}{1-\tau_I}\hat{c}_{j,t}(i) - f_x w_{j,t}, & \text{if firm exports} \\ 0, & \text{otherwise} \end{cases}$$

In this setup, the maximization problem of a firm i operating in region $j = \{\mathrm{A,B}\}$ in country H yields a mark-up pricing rule. In particular, for goods sold on the domestic markets, prices are

$$r_{j,t}(i) = \frac{1}{\beta}\frac{w_{j,t}}{X_{j,t}A_j(i)} \qquad i \in I^{\mathrm{RN}}, I^{\mathrm{RT}}$$

$$p_{j,t}(i) = \frac{1}{\rho}\frac{w_{j,t}}{X_{j,t}A_j(i)} \qquad i \in I^{\mathrm{T}}.$$

Prices for goods sold in the other regional and international markets are

$$r^*_{j,t} = \frac{1}{\beta}\frac{w_{j,t}}{X_{j,t}A_j(i)}\frac{1}{1-\tau_R} \qquad i \in I^{\mathrm{RT}},$$

$$\hat{p}_{j,t} = \frac{1}{\rho}\frac{w_{j,t}}{X_{j,t}A_j(i)}\frac{1}{1-\tau_I} \qquad i \in I^{\mathrm{T}},$$

respectively. Throughout the paper we will use letter r to denote regional individual goods prices, and letter p to denote international individual goods prices. Here $\frac{1}{\beta}$ and $\frac{1}{\rho}$ are constant markups, linked to the elasticities of substitution across different varieties of traded and nontraded goods. Markups vary with the market in which goods are sold. Due to the fixed costs of exporting, firms with lower productivity levels will choose to sell in the domestic market. When making this decision, a firm computes potential profits from export sales after

accounting for the fixed costs of exporting. A firm will export if and only if these profits are non-negative, that is

$$\hat{p}_{j,t}(i)\hat{c}_{j,t}(i) - \frac{w_{j,t}}{X_{j,t}A_j(i)}\frac{1}{1-\tau_I}\hat{c}_{j,t}(i) - f_x w_{j,t} \geq 0 \qquad i \in I^{\mathrm{T}},$$

where $\hat{c}_{j,t}(i)$, $i \in I^{\mathrm{T}}$ is consumption demand by F-country households for the internationally-traded good produced by firm i residing in country H, region j. A firm for which $\hat{\pi}_{j,t}(i) = 0$ will pin down the threshold index $i^*_{j,t}$ of the marginal firm that will export. In particular, let $A_j(i^*) \equiv \inf\{A : \hat{\pi}_{j,t}(i) > 0\}$ be a productivity cut-off level. Then all firms with productivities below with cutoff, $A_j(i) < A_j(i^*)$, will only sell in country H, while the firms with $A_j(i) > A_j(i^*)$ will also be able to sell in country F. Within a group of firms selling in the home country, a share γ_{N} of firms with the lowest productivity levels will sell goods in the domestic regional market. The remaining share of firms will be able to export goods to the other region within the same country. We assume that the share of firms servicing domestic region, γ_{N}, is fixed to preserve tractability and to focus on the role played by international non-tradability. Figure **2** plots an arbitrary productivity function for firms within region j to illustrate their decision as to which market to service. As noted before, these decisions also endogenously determine the sectoral composition of each region's economy. Firms operating in country F face a similar problem.

Figure **2**. Firm productivity function within a region
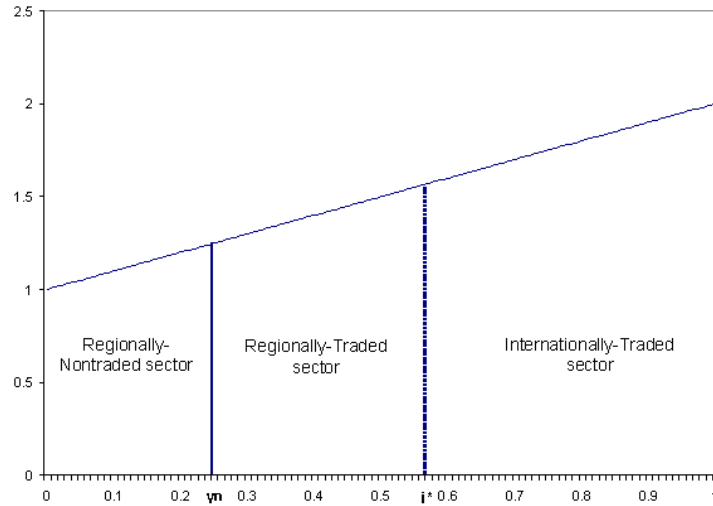
Following Melitz (2003) and Bergin and Glick (2005), we define "average" productivity

levels – average A – for firms producing different categories of goods in region j:

$$\tilde{A}_{\text{RN},j}^{\frac{\beta}{1-\beta}} = \frac{1}{\gamma_{\text{N}} i_j^*} \int_0^{\gamma_{\text{N}} i_j^*} A_{ij}^{\frac{\beta}{1-\beta}} \, di,$$

$$\tilde{A}_{\text{RT},j}^{\frac{\beta}{1-\beta}} = \frac{1}{(1-\gamma_{\text{N}}) i_j^*} \int_{\gamma_{\text{N}} i_j^*}^{i_j^*} A_{ij}^{\frac{\beta}{1-\beta}} \, di,$$

$$\tilde{A}_{\text{IT},j}^{\frac{\rho}{1-\rho}} = \frac{1}{1 - i_j^*} \int_{i_j^*}^1 A_{ij}^{\frac{\rho}{1-\rho}} \, di,$$

$$\tilde{A}_j^{\frac{\phi}{1-\phi}} = \int_0^1 A_{ij}^{\frac{\phi}{1-\phi}} \, di.$$

Our focus is on aggregate dynamics, and as shown in Melitz (2003), the average productivities are sufficient to characterize these dynamics. We can now define average goods prices in terms of these productivity averages. In particular, prices of goods sold on the domestic market in region j, country H are given by:

$$R_{j,t}^{\text{N}} = \frac{1}{\beta} \frac{w_{j,t}}{X_{j,t} \tilde{A}_{\text{RN},j}}, \qquad R_{j,t}^{\text{J}} = \frac{1}{\beta} \frac{w_{j,t}}{X_{j,t} \tilde{A}_{\text{RT},j}}, \qquad P_{j,t}^{\text{H}} = \frac{1}{\rho} \frac{w_{j,t}}{X_{j,t} \tilde{A}_{\text{IT},j}}. \tag{6}$$

Here $R_{j,t}^{\text{N}}$, $R_{j,t}^{\text{J}}$, and $P_{j,t}^{\text{H}}$ denote prices of regionally-nontraded, regionally-traded, and internationally-traded goods, respectively. Prices of goods that originated in region j, country H and are sold in the other regional and international markets are given, respectively, by

$$R_{-j,t}^{\text{J}} = \frac{1}{\beta} \frac{w_{j,t}}{X_{j,t} \tilde{A}_{\text{RT},j}} \frac{1}{1 - \tau_R}, \qquad \hat{P}_{j,t}^{\text{H}} = \frac{1}{\rho} \frac{w_{j,t}}{X_{j,t} \tilde{A}_{\text{IT},j}} \frac{1}{1 - \tau_I}, \tag{7}$$

As before we use letter R to denote regional price indices, and letter P to denote international price indices. An analogous set of prices applies to the F country.

Profits received by domestic firms in country H are given as the sum of profits received by domestic firms selling in three different markets:

$$\Pi_{j,t} = \int_0^{\gamma_{\text{N}} i_{j,t}^*} \pi_{j,t}(i) di \mid_{i \in I^{\text{RN}}} + \int_{\gamma_{\text{N}} i_{j,t}^*}^{i_{j,t}^*} \pi_{j,t}(i) di \mid_{i \in I^{\text{RT}}} + \int_{\gamma_{\text{N}} i_{j,t}^*}^{i_{j,t}^*} \pi_{j,t}^*(i) di \mid_{i \in \text{RT}} + \int_{i_{j,t}^*}^1 \pi_{j,t}(i) di \mid_{i \in \text{IT}} + \int_{i_{j,t}^*}^1 \hat{\pi}_{j,t}(i) di \mid_{i \in \text{IT}}.$$

Profits received by firms in country F are defined analogously.

4.3 Equilibrium

The first-order conditions for H households living in region j are given by

$$\frac{\partial U_t/\partial C_{j,t}^{\text{F}}}{\partial U_t/\partial C_{j,t}^{\text{H}}} = \frac{P_t^{\text{F}}}{P_t^{\text{H}}}, \tag{8a}$$

$$\frac{\partial U_t/\partial Z_{j,t}^{\text{A}}}{\partial U_t/\partial C_{j,t}^{\text{H}}} = \frac{R_{j,t}^{\text{A}}}{P_t^{\text{H}}} \tag{8b}$$

$$\frac{\partial U_t/\partial Z_{j,t}^{\text{B}}}{\partial U_t/\partial C_{j,t}^{\text{H}}} = \frac{R_{j,t}^{\text{B}}}{P_t^{\text{H}}} \tag{8c}$$

$$\frac{\partial U_t/\partial Z_{j,t}^{\text{N}}}{\partial U_t/\partial C_{j,t}^{\text{H}}} = \frac{R_{j,t}^{\text{N}}}{P_t^{\text{H}}}. \tag{8d}$$

These equations define the relative prices of F traded goods, region A and B-originated traded goods, and region$-j$ nontraded goods in terms of H international tradables as ratios of their respective marginal utilities to the marginal utility of H tradables. In the bond economy, the first-order conditions also include

$$P_t^{\text{H}}\partial U_t/\partial C_{j,t}^{\text{H}} = \beta R_t \mathbb{E}_t P_{t+1}^{\text{H}}\partial U_t/\partial C_{j,t+1}^{\text{H}}, \tag{9}$$

which is the standard pricing equations for the bond. The first-order conditions for F households are symmetric.

In equilibrium, households and firms decisions must also be consistent with the market clearing conditions. The market clearing conditions in the nontraded goods sector in region $j = \{\text{A,B}\}$ are

$$z_{j,t}(i) = y_{j,t}(i) \qquad \text{and} \qquad \hat{z}_{j,t}(i) = \hat{y}_{j,t}(i), \qquad i \in I^{\text{RN}}.$$

For the regional traded goods originated within country H, the market clearing requires

$$z_{\text{A}t}(i) + z_{\text{A}t}^{*}(i)\frac{1}{1 - \tau_R} = y_{\text{A}t}(i) \qquad \text{and} \qquad z_{\text{B}t}(i) + z_{\text{B}t}^{*}(i)\frac{1}{1 - \tau_R} = y_{\text{B}t}(i), \qquad i \in I^{\text{RT}}.$$

while for those originated within country F, the market clearing is

$$\hat{z}_{\text{A}t}(i) + \hat{z}_{\text{A}t}^{*}(i)\frac{1}{1 - \tau_R} = \hat{y}_{\text{A}t}(i) \qquad \text{and} \qquad \hat{z}_{\text{B}t}(i) + \hat{z}_{\text{B}t}^{*}(i)\frac{1}{1 - \tau_R} = \hat{y}_{\text{B}t}(i), \qquad i \in I^{\text{RT}}.$$

Here $z_j^{*}(i)$ $[\hat{z}_j^{*}(i)]$, $i \in I^{\text{RT}}$ is consumption demand for H [F] country, region $j-$produced regionally-traded goods sold in the other region within H [F] country, as before.

Finally, in equilibrium, the world demand for each internationally-traded good must

be equal to its corresponding supply. The internationally traded goods in each country originate in the two regions within the country. Therefore, for the internationally-tradable good produced in the H country, market clearing requires that

$$c_{\text{A}t}(i) + c_{\text{B}t}(i) + (\hat{c}_{\text{A}t}(i) + \hat{c}_{\text{B}t}(i)) \frac{1}{1 - \tau_I} = y_{\text{A}t}(i) + y_{\text{B}t}(i), \qquad i \in I^{\text{T}}$$

while for the international tradables originated in the F country, market clearing requires

$$\hat{c}_{\text{A}t}^*(i) + \hat{c}_{\text{B}t}^*(i) + (c_{\text{A}t}^*(i) + c_{\text{B}t}^*(i)) \frac{1}{1 - \tau_I} = \hat{y}_{\text{A}t}(i) + \hat{y}_{\text{B}t}(i), \qquad i \in I^{\text{T}}.$$

Here $\hat{c}_{ij} [c_{ij}^*], i \in \text{IT}$ is consumption demand for H [F] country, region $j-$produced internationally-traded goods sold in the F [H] country, as defined before.

Labor market clearing in each region within country H requires

$$\int_0^{\gamma_{\text{N}} i_{j,t}^*} l_{j,t}(i)di + \int_{\gamma_{\text{N}} i_{j,t}^*}^{i_{j,t}^*} l_{j,t}(i)di + \int_{i_{j,t}^*}^1 l_{j,t}(i)di + f_x(1 - i_{j,t}^*) = L_j, \qquad j = \{\text{A,B}\},$$

where L_j is exogenously given labor supply in region j. Similarly, for each region in country F, labor market clearing implies

$$\int_0^{\gamma_{\text{N}} \hat{i}_{j,t}^*} \hat{l}_{j,t}(i)di + \int_{\gamma_{\text{N}} \hat{i}_{j,t}^*}^{\hat{i}_{j,t}^*} \hat{l}_{j,t}(i)di + \int_{\hat{i}_{j,t}^*}^1 \hat{l}_{j,t}(i)di + f_x(1 - \hat{i}_{j,t}^*) = \hat{L}_j, \qquad j = \{\text{A,B}\},$$

where $\hat{L}_j$ is labor supply in region j in country F.

We also require an asset market clearing condition. We assume that bonds are in zero net supply, so that bond market clearing condition is $B_{\text{A},t} + B_{\text{B},t} + \hat{B}_{\text{A},t} + \hat{B}_{\text{B},t} = 0$.

An equilibrium in this economy consists of a sequence of goods prices $\{P_t^{\text{H}}, P_t^{\text{F}}, \hat{P}_t^{\text{H}}, \hat{P}_t^{\text{F}}, R_{j,t}^{\text{A}}, R_{j,t}^{\text{B}}, \hat{R}_{j,t}^{\text{A}}, \hat{R}_{j,t}^{\text{B}}, R_{j,t}^{\text{N}}, \hat{R}_{j,t}^{\text{N}}\}$ and an interest rate R_t, such that households in both countries make their consumption and bond allocation decisions optimally, taking prices as given; firms in both countries make their profit maximizing decisions; and all markets clear.

4.4 Variables of interest

In our economy there is a sequence of price indices that comprise regional and international real exchange rates. To simplify the notation, we omit explicit references to i_j^* in the consumption weights, $\gamma_{\text{A}}, \gamma_{\text{B}}, \lambda_{\text{H}}, \lambda_{\text{F}}, \lambda_{\text{N}}, \lambda_{\text{T}}$.

Recall that $R_{j,t}^{\text{T}}$ denotes the price of regionally traded basket in region $j = \{\text{A, B}\}$. This

price index in H country is given by

$$R_{\mathrm{A},t}^{\mathrm{T}} = \left[\gamma_{\mathrm{A}} \left(R_{\mathrm{A},t}^{\mathrm{A}} \right)^{\frac{\beta}{\beta-1}} + \gamma_{\mathrm{B}} \left(R_{\mathrm{A},t}^{\mathrm{B}} \right)^{\frac{\beta}{\beta-1}} \right]^{\frac{\beta-1}{\beta}} \qquad \text{and} \qquad R_{\mathrm{B},t}^{\mathrm{T}} = \left[\hat{\gamma}_{\mathrm{A}} \left(R_{\mathrm{B},t}^{\mathrm{A}} \right)^{\frac{\beta}{\beta-1}} + \hat{\gamma}_{\mathrm{B}} \left(R_{\mathrm{B},t}^{\mathrm{B}} \right)^{\frac{\beta}{\beta-1}} \right]^{\frac{\beta-1}{\beta}}. \tag{10}$$

Next, recall that $P_{j,t}^{\mathrm{N}}$ denotes the price of aggregate internationally nontraded consumption basket in region j. It is a composite of prices of regionally traded and regionally nontraded goods:

$$P_{\mathrm{A},t}^{\mathrm{N}} = \left[\gamma_{\mathrm{T}} \left(R_{\mathrm{A},t}^{\mathrm{T}} \right)^{\frac{\alpha}{\alpha-1}} + \gamma_{\mathrm{N}} \left(R_{\mathrm{A},t}^{\mathrm{N}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{\alpha-1}{\alpha}} \qquad \text{and} \qquad P_{\mathrm{B},t}^{\mathrm{N}} = \left[\hat{\gamma}_{\mathrm{T}} \left(R_{\mathrm{B},t}^{\mathrm{T}} \right)^{\frac{\alpha}{\alpha-1}} + \hat{\gamma}_{\mathrm{N}} \left(R_{\mathrm{B},t}^{\mathrm{N}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{\alpha-1}{\alpha}}. \tag{11}$$

Finally, recall that P_t^{T} denotes the price of aggregate internationally traded consumption basket in regions A and B in country H. It is composed of prices of internationally traded goods in country H:

$$P_t^{\mathrm{T}} = \left[\lambda_{\mathrm{H}} \left(P_t^{\mathrm{H}} \right)^{\frac{\rho}{\rho-1}} + \lambda_{\mathrm{F}} \left(P_t^{\mathrm{F}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}. \tag{12}$$

The aggregate price index in region j in country H, therefore, is given by

$$P_{j,t} = \left[\lambda_{\mathrm{T}} \left(P_t^{\mathrm{T}} \right)^{\frac{\phi}{\phi-1}} + \lambda_{\mathrm{N}} \left(P_{j,t}^{\mathrm{N}} \right)^{\frac{\phi}{\phi-1}} \right]^{\frac{\phi-1}{\phi}}. \tag{13}$$

Notice that only the price of internationally nontraded goods, $P_{j,t}^{\mathrm{N}}$, is region-specific. The price of internationally traded goods, P_t^{T}, is common across the regions within the same country.

The price indices in the foreign country are symmetrically defined. Thus, the prices of regional traded baskets in regions A and B in country F are given by

$$\hat{R}_{\mathrm{A},t}^{\mathrm{T}} = \left[\gamma_{\mathrm{A}} \left(\hat{R}_{\mathrm{A},t}^{\mathrm{A}} \right)^{\frac{\beta}{\beta-1}} + \gamma_{\mathrm{B}} \left(\hat{R}_{\mathrm{A},t}^{\mathrm{B}} \right)^{\frac{\beta}{\beta-1}} \right]^{\frac{\beta-1}{\beta}} \qquad \text{and} \qquad \hat{R}_{\mathrm{B},t}^{\mathrm{T}} = \left[\hat{\gamma}_{\mathrm{A}} \left(\hat{R}_{\mathrm{B},t}^{\mathrm{A}} \right)^{\frac{\beta}{\beta-1}} + \hat{\gamma}_{\mathrm{B}} \left(\hat{R}_{\mathrm{B},t}^{\mathrm{B}} \right)^{\frac{\beta}{\beta-1}} \right]^{\frac{\beta-1}{\beta}}. \tag{14}$$

Prices of aggregate internationally nontraded consumption baskets in regions A and B in country F are:

$$\hat{P}_{\mathrm{A},t}^{\mathrm{N}} = \left[\gamma_{\mathrm{T}} \left(\hat{R}_{\mathrm{A},t}^{\mathrm{T}} \right)^{\frac{\alpha}{\alpha-1}} + \gamma_{\mathrm{N}} \left(\hat{R}_{\mathrm{A},t}^{\mathrm{N}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{\alpha-1}{\alpha}} \qquad \text{and} \qquad \hat{P}_{\mathrm{B},t}^{\mathrm{N}} = \left[\hat{\gamma}_{\mathrm{T}} \left(\hat{R}_{\mathrm{B},t}^{\mathrm{T}} \right)^{\frac{\alpha}{\alpha-1}} + \hat{\gamma}_{\mathrm{N}} \left(\hat{R}_{\mathrm{B},t}^{\mathrm{N}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{\alpha-1}{\alpha}}. \tag{15}$$

Price of aggregate internationally traded consumption basket in regions A and B in country

F is given by

$$\hat{P}_t^{\mathrm{T}} = \left[\hat{\lambda}_{\mathrm{H}} \left(\hat{P}_t^{\mathrm{H}} \right)^{\frac{\rho}{\rho-1}} + \hat{\lambda}_{\mathrm{F}} \left(\hat{P}_t^{\mathrm{F}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}, \tag{16}$$

while the aggregate price index in region j in country F is

$$\hat{P}_{j,t} = \left[\hat{\lambda}_{\mathrm{T}} \left(\hat{P}_t^{\mathrm{T}} \right)^{\frac{\phi}{\phi-1}} + \hat{\lambda}_{\mathrm{N}} \left(\hat{P}_{j,t}^{\mathrm{N}} \right)^{\frac{\phi}{\phi-1}} \right]^{\frac{\phi-1}{\phi}}. \tag{17}$$

Now we are ready to define RERs. The regional real exchange rate is defined as the price of consumption basket in region B relative to the price of consumption basket in region A. Thus, the regional real exchange rates in country H, RER_t^{H}, and country F, RER_t^{F}, are given by

$$RER_t^{\mathrm{H}} = \frac{P_{\mathrm{B},t}}{P_{\mathrm{A},t}} \qquad \text{and} \qquad RER_t^{\mathrm{F}} = \frac{\hat{P}_{\mathrm{B},t}}{\hat{P}_{\mathrm{A},t}}. \tag{18}$$

International real exchange rate in our model, RER_t^{I}, is given by the ratio of F to H aggregate price indices:

$$RER_t^{\mathrm{I}} = \frac{\frac{1}{2}\hat{P}_{\mathrm{A},t} + \frac{1}{2}\hat{P}_{\mathrm{B},t}}{\frac{1}{2}P_{\mathrm{A},t} + \frac{1}{2}P_{\mathrm{B},t}}, \tag{19}$$

where we exploited the symmetry of the regions within a given country to construct the country-wide aggregate price index as the average of price indices across regions within that country.

The terms-of-trade in the model are defined as a relative price of foreign to domestic internationally-traded goods and are given by $TOT_t = \hat{P}_t^{\mathrm{F}} / P_t^{\mathrm{H}}$.

5   Parameter values and computations

Parameter values for the calibration of our benchmark model are summarized in Table **3**. We consider the world economy as consisting of two symmetric countries, matching the properties of US economy in annual data. Most of the preference parameter values are standard in the literature and, in particular, follow closely those adopted by Stockman and Tesar (1995). In particular, δ is set to 0.96 to obtain the steady-state real interest rate of 4% per annum. Coefficient of relative risk aversion, ω, is set to 2. The values for substitution elasticities are chosen as follows. First, the value for ϕ is set, following Mendoza (1995), to obtain the elasticity of substitution between tradable and nontradable consumption equal to 0.74. Second, the elasticity of substitution between H and F traded goods is set to equal 6 to obtain a 20% mark-up of price over marginal costs, a value commonly used in the literature

(Obstfeld and Rogoff, 2000).

Table 3. Benchmark Model Parameters

PREFERENCES				
Subjective discount factor	δ	0.96		
Risk-aversion	ω	2		
	International goods:		Regional goods:	
share of nontraded goods			$\gamma_{\mathrm{N}} = \hat{\gamma}_{\mathrm{N}}$	0.55
elasticity of substitution b/n				
traded and nontraded goods	$1/(1-\phi)$	0.74	$1/(1-\alpha)$	0.74
H and F traded goods	$1/(1-\rho)$	6	$1/(1-\beta)$	6
PRODUCTIVITY				
	International goods:		Regional goods:	
persistence of traded shocks	$a_{ii}^{\mathrm{H}} = a_{ii}^{\mathrm{F}}$	0.9	$a_{ii}^{\mathrm{A}} = a_{ii}^{\mathrm{B}}$	0.9
persistence of nontraded shocks			a_{ii}^{N}	0.9
volatility of				
traded innovations (std.dev.)	$\Omega_e^{\mathrm{H}} = \Omega_e^{\mathrm{F}}$	0.01	$\Omega_e^{\mathrm{A}} = \Omega_e^{\mathrm{B}}$	0.01
nontraded innovations (std.dev.)		0.005	Ω_e^{N}	0.01

We parameterize the fixed cost of exporting parameter, f_x, in both countries to obtain the share of nontradables in aggregate consumption expenditure, λ_{N} and $\hat{\lambda}_{\mathrm{N}}$, equal to 0.55 in the steady state. This number is calculated using OECD STructural ANalysis (STAN) database.[7] We set the shares of home goods in the internationally-traded consumption basket in both countries, λ_{H} and $\hat{\lambda}_{\mathrm{F}}$, to 0.5 in the steady state, so that there is no consumption home bias built in exogenously in the model. Instead we calibrate the international iceberg transportation costs to match the share of international imports to be equal to 10% of output in the steady state.

The regional dimension of the model is more difficult to parametrize as the estimates are not readily available in the literature. Under our benchmark calibration, we assume that preference parameters over regional consumption are symmetric to the preference parameters in the international dimension. That is, we assume that the value of α is chosen to set the elasticity of substitution between regional tradable and nontradable goods to 0.74, while the share of nontraded regional goods, $\gamma_{\mathrm{N}} = \hat{\gamma}_{\mathrm{N}}$, is set to 0.55. Regional iceberg transportation costs, τ_R, are calibrated to obtain regional trade volume to be 1.25 times the international trade volume in the steady state. The latter number was obtained as an average using interprovincial and international trade data for Canada during 1984-2005. State-level trade

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[7]These numbers are similar to the estimates in the literature. For instance, Corsetti, Dedola, and Leduc (2008) and Dotsey and Duarte (2006) use $\lambda_{\mathrm{N}} = 0.55$, Stockman and Tesar (1995) report λ_{N} close to 0.5; Pesenti and van Wincoop (2002) also argue that 0.5 of consumers budget is allocated to nontradables; Benigno and Thoenissen (2008) assume $\lambda_{\mathrm{N}} = 0.45$.

data for US is not available, to the best of our knowledge, so we use the trade numbers for Canada to calibrate τ_R parameter in our model. The elasticity of substitution between two regional goods is set to 6 in both countries, which pins down the value for β.

The available estimates for sectoral productivity processes in the literature are very dissimilar (see, for instance, Corsetti, Dedola, and Leduc, 2008; Benigno and Thoenissen, 2008; Tesar, 1993; Stockman and Tesar, 1995), thus we assume independent productivity processes across sectors, across regions and across countries. Each of the productivity processes follows an AR(1) process. The AR(1) coefficients are all set to 0.9. Innovations to internationally traded productivity have standard deviation of 0.01, while innovations to internationally nontraded productivity are half that size. These numbers are consistent with the empirical findings that traded productivity exhibits more volatility than nontraded productivity. We also assume that international shocks are common across the regions within the same country, while all other shocks are region-specific. We parameterize good/firm-specific productivity following Bergin and Glick (2005) as $A_i = \psi(1 + i)$, with $\psi = 1$.

When regions are present, internationally nontraded TFP shocks have two components: regionally-traded and regionally-nontraded. We parameterize the two components in such a way that the variance of the internationally nontraded shocks remains unchanged. Because we assume that regionally-traded and regionally-nontraded shocks are independent, we increase the standard deviation of these shocks to 0.01.

The model is solved by linearizing the system of equilibrium conditions and solving the resulting system of linear difference equations. To make our bond economy stationary, we introduce small quadratic costs on bond holdings. We study the properties of the model's equilibrium by simulating it over 100 periods. The statistics reported in the next section are derived from 200 simulations.

6   Results

This section presents the main findings from the numerical solution to the model. We start by characterizing the properties of RER in a production economy with no firm heterogeneity due to firm-specific productivity. Such simplification eliminates the endogenous non-tradability feature in our model and reduces the setting to a standard international business cycle economy with a representative firm in each region. Then we allow for firm heterogeneity and consider a production economy with endogenous tradability. Under each scenario we also evaluate the role of within country heterogeneity by studying a version of the model with and without regions. We use an economy without endogenous tradability and with no

regional heterogeneity as a benchmark.

6.1   No firm heterogeneity

The properties of real exchange rate and consumption generated by the production bond economy with homogeneous and perfectly competitive firms are summarized in Table **4**. Column (i) reports the volatility of the two variables and the correlation between them across regions, while column (ii) presents the corresponding statistics across countries. We study two versions of this model: without regional heterogeneity, the results for which are reported in the top panel; and with regional heterogeneity, the results for which are summarized in the bottom panel of Table **4**. The latter exercise allows us to isolate the contribution of each new feature that we introduce.

Without the regional dimension, our model has been used extensively in the literature to study international business cycles, terms of trade and real exchange rate movements (see Tesar, 1993; Corsetti, Dedola, and Leduc, 2008; Benigno and Thoenissen, 2008). The international business cycle properties of our model under the benchmark parametrization, therefore, are standard. As expected, it predicts international RER that is positively correlated with consumption across countries, even though asset markets are incomplete. This correlation is positive and high when we consider both levels and growth rates of the two variables (0.3 in levels and in growth rates). When regional heterogeneity is introduced, the international correlation between RER and relative consumption declines slightly, but remains positive. Across regions, the correlation between RER and consumption is also positive, consistent with the data, but is significantly above its empirical counterpart. These results re-confirm the consumption-RER puzzle in the international data, and show that within country heterogeneity is not sufficient to resolve the Backus-Smith puzzle.

The model also fails to account for the volatility of RER relative to that of consumption. The implied relative volatility of international RER is well above one, but below its value in the data. At the same time, the model predicts too much volatility for the regional RER.

Overall, our benchmark model with heterogenous regions and a single non-state-contingent international bond is not able to account for the differences in consumption-RER relationship across regions and across countries. We next explore the role of endogenous tradability in capturing the link between the two variables.

Table 4. RER and relative consumption: representative firm		
	Across regions	Across countries
No regional heterogeneity	(i)	(ii)
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$		1.57
Correlation b/n		
RER and $C - \hat{C}$		0.30
growth rate of RER and $C - \hat{C}$		0.30
Regional heterogeneity		
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$	0.98	1.57
Correlation b/n		
RER and $C - \hat{C}$	0.99	0.29
growth rate of RER and $C - \hat{C}$	0.99	0.29

Notes: Statistics reported in the table are based on 200 simulations of annual series, each 100 periods long.

6.2 Endogenous tradability

Next, we present the results obtained in the production bond economy with heterogeneous firms. As before, we consider two versions of the model. The top panel of Table **5** reports the statistics obtained in the version of our model without regional heterogeneity, while the bottom panel summarizes the corresponding statistics in the version of the model with regional heterogeneity.

Table **5**. RER and relative consumption: endogenous tradability		
	Across regions	Across countries
No regional heterogeneity	(i)	(ii)
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$		1.67
Correlation b/n		
RER and $C - \hat{C}$		0.03
growth rate of RER and $C - \hat{C}$		0.04
Regional heterogeneity		
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$	0.67	1.59
Correlation b/n		
RER and $C - \hat{C}$	0.88	-0.01
growth rate of RER and $C - \hat{C}$	0.88	-0.02

Notes: Statistics reported in the table are based on 200 simulations of annual series, each 100 periods long.

The contribution of endogenous non-tradability to the properties of RER and relative

consumption can be isolated by comparing the statistics in the top panels of Tables **4** and **5**. Introducing endogenous tradability into a generic international business cycle model (with no regions) increases the relative volatility of RER, and reduces consumption-RER correlation both in levels and growth rates. This correlation, however, remains positive (0.03 in levels, 0.04 in growth rates). Therefore, endogenous non-tradability per se is not sufficient to overturn the Backus-Smith puzzle.

The results from our full model that incorporates both regional heterogeneity and endogenous non-tradability are presented in the bottom panel of Table **5**. This model provides the best match to the data. It is able to generate a positive correlation between RER and relative consumption across regions and a corresponding negative correlation across countries. It also implies relative volatility of RER to consumption that is well above one across countries and well below one across regions, consistent with the data. We next develop the intuition for these results.

6.3 Discussion

To understand the results above it is useful to decompose the international and regional real exchange rates into their components. In particular, in Appendix A.3 we show that the log international real exchange rate (in deviation from the steady state), can be expressed as

$$rer^{\mathrm{I}}_{\mathrm{A},t} = \left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}\left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right)\kappa_1(\hat{p}^{\mathrm{F}}_t - p^{\mathrm{H}}_t) + \hat{\lambda}_{\mathrm{N}}\kappa_2\left(\hat{p}^{\mathrm{N}}_{\mathrm{A},t} - \hat{p}^{\mathrm{T}}_{\mathrm{A},t}\right) - \lambda_{\mathrm{N}}\kappa_2\left(p^{\mathrm{N}}_{\mathrm{A},t} - p^{\mathrm{T}}_{\mathrm{A},t}\right), \quad (20)$$

where lowercase letters denote the log transformations for all variables in deviations from their steady state values (e.g., $p^{\mathrm{H}}_t \equiv \ln P^{\mathrm{H}}_t - \ln P^{\mathrm{H}}$, etc.); and $\kappa_1^{\frac{\rho-1}{\rho}} = \left(\hat{P}^{\mathrm{F}}/P^{\mathrm{T}}\right)$, $\kappa_2^{\frac{\phi-1}{\phi}} = \left(\hat{P}^{\mathrm{N}}_{\mathrm{A}}/\hat{P}_{\mathrm{A}}\right) = (P^{\mathrm{N}}_{\mathrm{A}}/P_{\mathrm{A}})$ are coefficients that depend on the steady state values of relative prices. For analytical tractability, in this decomposition we focused on the aggregate price indices between regions A in countries H and F. Clearly, the international real exchange rate will depend on the weighted average of the aggregate price indices in regions A and B in the two countries.

The expression in (20) decomposes international real exchange rate into two components: (i) a component associated with the international terms of trade movements, $\left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}\left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right)\kappa_1(\hat{p}^{\mathrm{F}}_t - p^{\mathrm{H}}_t)$; (ii) a component arising due to variations in the relative prices of internationally-nontraded goods (the last two terms in the expression above).[8]

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[8] A similar decomposition is also derived in Benigno and Thoenissen (2008) in the context of a two-country two-sector model with no iceberg trade costs.

If there is consumption home bias in households' preferences, $\left(\lambda_{\text{H}} - \lambda_{\text{F}} \left(\frac{1}{1-\tau_I} \right)^{\frac{\rho}{\rho-1}} \right) > 0$, then the improvements in the terms of trade will be associated with the real exchange rate appreciation. Furthermore, any variations in the relative price of nontradable goods in the two countries will also contribute to real exchange rate movements to the extent of the weight of nontradable consumption in the aggregate consumption basket of the two countries, $\lambda_{\text{N}} \kappa_2$ and $\hat{\lambda}_{\text{N}} \kappa_2$.

6.3.1 No regions and no endogenous tradability

The decomposition above allows us to develop the intuition for the mechanism underlying the model without regions and no endogenous tradability. In that model the productivity shocks can originate in two sectors: internationally tradable (T) and internationally nontradable (N). In response to a positive shock in the T sector of country H, its terms of trade deteriorates, which will tend to depreciate the RER. This effect, however, may be counterbalanced by the movements in the relative prices of nontraded goods. The latter will arise from two sources. The first is due to a standard Balassa-Samuelson effect, where positive productivity shock in the T sector will trigger an increase in the real wage, thus driving up relative prices of nontraded goods; the latter adjustment is necessary to prevent all labor from reallocating into the traded sector and out of the nontraded sector. This is a supply side effect and we refer to it as the *resource-shifting* channel.

The second effect arises from the CES structure of preferences and the desire of households to consume a balanced basket of traded and nontraded different good. Thus, following a productivity improvement in the T sector, domestic households experience a positive wealth effect, which leads them to increase their demand for nontraded goods, which in turn will drive up their prices. This is the demand-side effect and we refer to it as the *demand-composition* channel. In our model these two effects dominate the fall in the terms of trade and, as a result, positive productivity shocks in the T sector are associated with RER appreciation and an increase in relative consumption. This gives rise to the negative Backus-Smith correlation.

Positive shocks that originate in the N sector have the opposite effect on the RER. Such shocks lead to a fall in the relative prices of nontraded goods, thus depreciating the RER. Given the low elasticity of nontraded demand, this price decline can be large enough to generate negative wealth effect for domestic households. As a result, they decrease their demand for traded goods. If there is home bias in consumption, this demand-composition effect leads to a deterioration in the terms of trade, thus further depreciating the RER. The

resource-shirting channel moderates this adjustment through reallocation of labor towards the traded sector. This effect is, however, weak. Thus, shocks to the N productivity lead to RER depreciation and an increase in relative consumption, implying a positive Backus-Smith correlation and working against the resolution of the Backus-Smith puzzle. As the top panel of Table **4** illustrates, a model with no regions and no endogenous non-tradability implies that productivity shocks to N sector dominate the consumption-RER correlation, predicting large positive numbers for that correlation. The reason is that in the presence of an international bond, households can smooth out the effects of T shocks much better than the effects of N shocks. The effects of the former, therefore, are moderated through bond trade.

Several mechanisms have been proposed in the literature in order to moderate the effects of N shocks on the Backus-Smith correlation. Benigno and Thoenissen (2008) reduced the volatility and persistence of N shocks relative to T shocks in their parameterization. Corsetti, Dedola, and Leduc (2008) by assuming a very low implied elasticity of substitution between traded goods, in effect induced large wealth effects following T shocks. Both of these approaches required strong assumptions on preferences and technology parameters. Our approach, instead, moderates the effects of N shocks endogenously, by allowing for regional heterogeneity and goods trade across regions. This feature is also empirically relevant as a large fraction of goods that are usually classified as internationally nontradable, are in fact tradable across regions within a country.

6.3.2 Role of regional heterogeneity

When heterogenous regions are introduced, TFP shocks can originate in three sectors: traded (T), regionally traded (RT) and regionally nontraded (RN). We start by discussing the effects of heterogenous regions for *international* Backus-Smith correlation. We find that when regional heterogeneity is introduced the positive association between relative consumption and RER becomes slightly weaker. This decline is due to attenuated effects of the shocks in the N sector. In fact, by introducing heterogeneous regions and recognizing that a large fraction of nontraded goods is actually traded, across regions within the same country, we in effect are raising the demand elasticity for nontraded goods. With higher elasticity, the fall in the price of nontraded goods following a productivity improvement in that sector (when originated in the RT sector) becomes smaller. As a result, the wealth effects of such shocks can turn positive, leading to higher demand for traded goods, and thus generating terms of trade improvements. In addition, wages in the nontraded sector can increase following N shocks, leading to labor reallocation towards the nontraded sector and out of the traded sector.

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With fewer traded goods produced, terms of trade can improve. If these two effects are strong enough, terms of trade improvement can offset the drop in nontraded prices triggered by the positive N productivity shock. In this case, the RER appreciation, accompanied by an increase in relative consumption will produce a negative Backus-Smith correlation. If the two effects are not sufficiently strong, they will still be of help for us by reducing the adjustment in the price of nontraded goods and thus weakening the positive Backus-Smith correlation arising from N shocks. The strength of such adjustments becomes a quantitative matter.

Next, we study the *regional* RER-relative consumption correlation. Due to the presence of regional heterogeneity in our model, we can decompose the price of nontraded goods in region A in country H as

$$p_{A,t}^{N} = r_{A,t}^{A} + \gamma_{B} \left(\frac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(r_{A,t}^{B} - r_{A,t}^{A} \right) + \gamma_{N} \kappa_4 \left(r_{A,t}^{N} - r_{A,t}^{T} \right), \tag{21}$$

where $\kappa_3^{\frac{\beta-1}{\beta}} = (R_B^B/R_A^T) = \left(\hat{R}_B^B/\hat{R}_A^T \right) = (R_B^B/R_B^T) = \left(\hat{R}_B^B/\hat{R}_B^T \right)$ and $\kappa_4^{\frac{\alpha-1}{\alpha}} = (R_A^N/P_A^N) = \left(\hat{R}_A^N/\hat{P}_A^N \right) = (R_B^N/P_B^N) = \left(\hat{R}_B^N/\hat{P}_B^N \right)$. The relative price of nontraded goods contains two components: (i) a component associated with the movements in the relative prices of regionally-produced goods, r_t^A, and regional terms of trade movements, $\left(r_{A,t}^{B} - r_t^{A} \right)$; (ii) a component arising due to variations in the relative prices of regionally-nontraded goods (the last term).

In region B in country H, the price of nontraded goods is given by the same two components as in (21):

$$p_{B,t}^{N} = r_{B,t}^{A} + \hat{\gamma}_{B} \kappa_3 \left(r_{B,t}^{B} - r_{B,t}^{A} \right) + \hat{\gamma}_{N} \kappa_4 \left(r_{B,t}^{N} - r_{B,t}^{T} \right).$$

Using these decompositions, the regional real exchange rate in country H can be written as[9]

$$rer_t^{H} = \lambda_{N} \kappa_2 \left(p_{B,t}^{N} - p_{A,t}^{N} \right).$$

From the results in (6)-(7), the prices of regional goods in country H in deviations from the steady state are related as $r_{A,t}^{A} = r_{B,t}^{A} \equiv r_t^{A}$ and $r_{A,t}^{B} = r_{B,t}^{B} \equiv r_t^{B}$. Further, since regions are symmetric, we get $\hat{\gamma}_{B} = \gamma_{A}$, which allows to write the regional real exchange rate in country

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[9]Note that the international terms of trade component of the RER is common to the two regions within the same country and thus cancels out.

H as:

$$rer_t^{\text{H}} = \lambda_{\text{N}} \kappa_2 \left[\left(\gamma_{\text{A}} - \gamma_{\text{B}} \left(\frac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \right) \kappa_3 \left(r_t^{\text{B}} - r_t^{\text{A}} \right) + \hat{\gamma}_{\text{N}} \kappa_4 \left(r_{\text{B},t}^{\text{N}} - r_{\text{B},t}^{\text{T}} \right) - \gamma_{\text{N}} \kappa_4 \left(r_{\text{A},t}^{\text{N}} - r_{\text{A},t}^{\text{T}} \right) \right].$$

$$(22)$$

These derivations show that the regional real exchange rate, like its international counterpart, is a composite of two components: (i) regional terms of trade, $\left(\gamma_{\text{A}} - \gamma_{\text{B}} \left(\frac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \right) \kappa_3 \left(r_t^{\text{B}} - r_t^{\text{A}} \right),$ which depends on the degree of regional consumption home bias; and (ii) relative price of regionally-nontraded goods in regions A and B, given by $\hat{\gamma}_{\text{N}} \kappa_4 \left(r_{\text{B},t}^{\text{N}} - r_{\text{B},t}^{\text{T}} \right) - \gamma_{\text{N}} \kappa_4 \left(r_{\text{A},t}^{\text{N}} - r_{\text{A},t}^{\text{T}} \right).$

To understand the adjustments in regional RER and regional relative consumption, we again consider three sectoral shocks: T, RT and RN. Under our parameterization, however, we assume that T shocks are perfectly correlated across regions within a country. As a result, the effects on this shocks are felt symmetrically in both regions, and cancel out in terms of their role for regional RER.

Two other productivity shocks, RT and RN, act very much like the T and N shocks on the international RER. For instance, positive RN shocks will tend to depreciate regional RER and increase regional consumption, implying a positive correlation between the two. Positive RT shocks will worsen regional terms of trade, an effect that will be partially offset by increasing prices of regionally nontraded goods. Turns out that this increase is not strong enough to appreciate the regional RER. The reason is the factor price equalization across regions within the same country that takes place in our model.

Recall that in our setup factor prices are equalized across sectors due to the assumption of perfect labor mobility across sectors within the same region. Further, due to the presence of an internationally-traded good that is homogeneous across regions in the same country, we find that factor prices are also equalized across regions within the same country. With this outcome labor does not have to flow across regions to bring about the factor price equalization, goods flows are enough. This result also implies that RT shocks can be smoothed out across regions through factor flows, leading to almost perfect risk-sharing within each country. Therefore, regional RER and relative regional consumption are always positively correlated in our model, consistent with the data. This correlation, obviously, is too high relative to the data.

Next, we illustrate these effects on regional real exchange rate, rer_t^{H}, and international real exchange rate, $rer_{\text{A},t}^{\text{I}}$ in more details using impulse response analysis. We start by analyzing the bond economy with a representative firm. The responses of international and regional real exchange rates and relative consumption, as well as their decomposition into the

components derived in (20) and (22) are presented in Figures $\mathbf{2a} - \mathbf{2c}$. Figure $\mathbf{2a}$ presents the effects of a 1% positive productivity shock in the T sector at H.[10]

Figure $\mathbf{2a}$. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Int'l shock
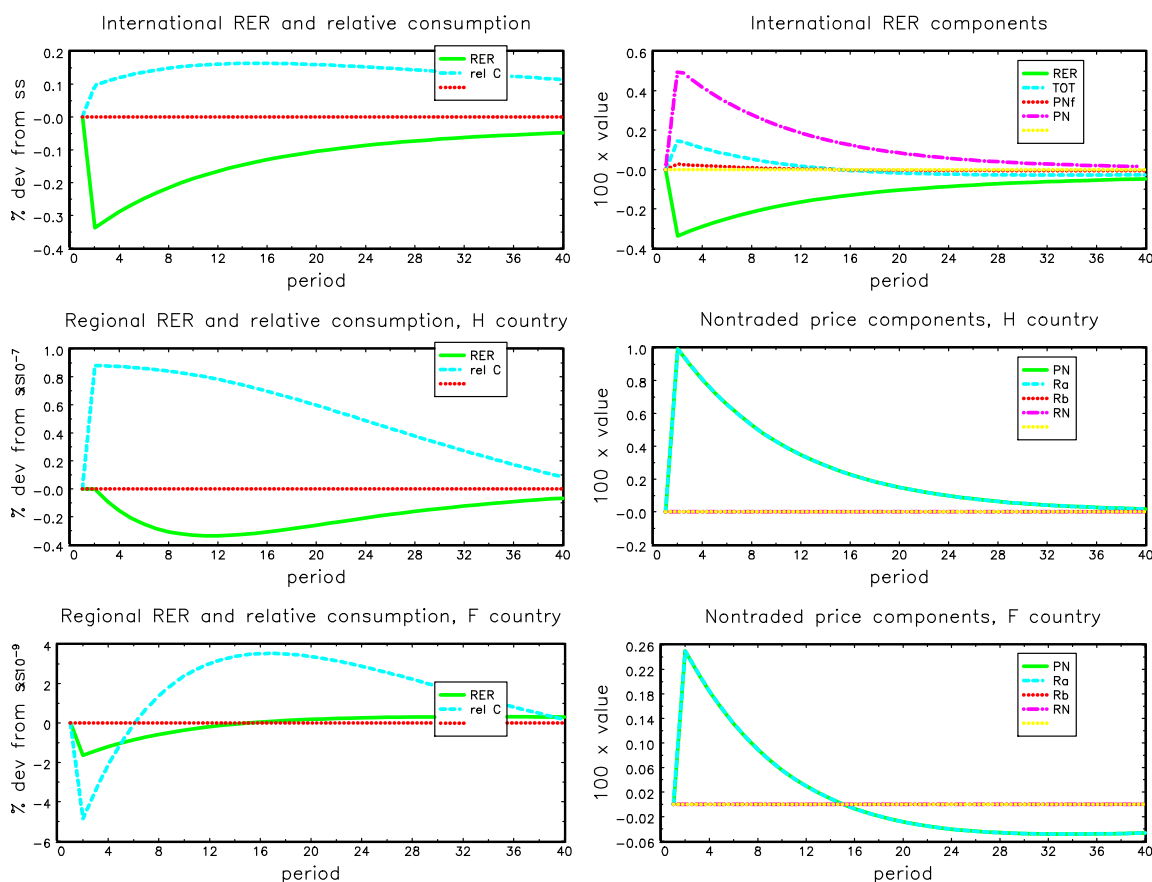

The left-hand-side panel of Figure $\mathbf{2a}$ presents the impulse responses of RER and relative consumption across countries and across regions within each country, H and F. The correlation between the two variables is clearly negative across countries. Regional RER and relative consumption are unaffected by T shocks, because both regions within the country are subject to that shock. The reason for the negative international correlation, as discussed in section 6.3.1 is twofold: First, there is a strong wealth effect that is triggered by the supply shocks. Second, there is a Balassa-Samuelson effect that results from higher wages in the sector experiencing the shock. The right-hand-side panel presents the decomposition of the international and regional RERs. In response to a positive shock to productivity of T goods, terms of trade depreciate. Because households' preferences are such that consumption of internationally-traded goods is close to complementary with the consumption

---

[10] As before, in the model calibration we assume that shocks to T goods productivity experienced by the two regions within each country are perfectly positively correlated.

of internationally-nontraded goods, the demand for regional goods, both traded and non-traded, goes up. As a result, their relative prices increase, which drives P^{N} (pictured in the top right chart of Figure **2a**) up. Productivity changes in country H also spill over to country F through their effects on the international terms of trade. As the relative price of foreign internationally-traded goods goes up, households residing in the F country increase their consumption of regional goods. This drives up the relative price of these goods ($\hat{P}^{\text{N}}$ increases). This increase, however, is smaller than that in H country. While the depreciation of terms-of-trade and an increase in $\hat{P}^{\text{N}}$ tend to depreciate the international real exchange rate, and increase in P^{N} tends to appreciate it. The latter effect dominates, leading to international RER appreciation. At the same time, H country relative to F country consumption will increase, implying a negative correlation between the two variables.

Next, we look at the effects of regional shocks. Figure **2b** presents the responses of RERs and relative consumption, as well as the components of RERs to a 1% positive shock to productivity of RT goods in region A of country H.

Figure **2b**. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Reg. T shock
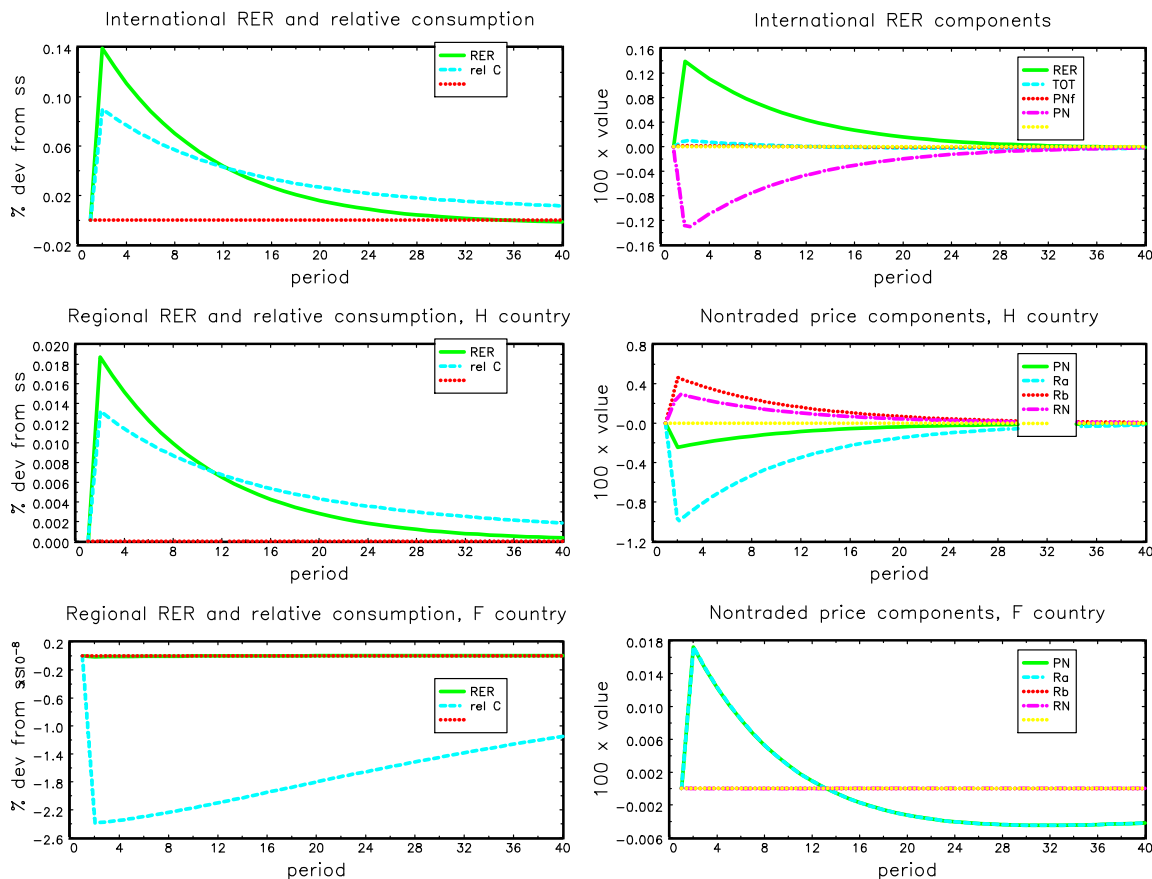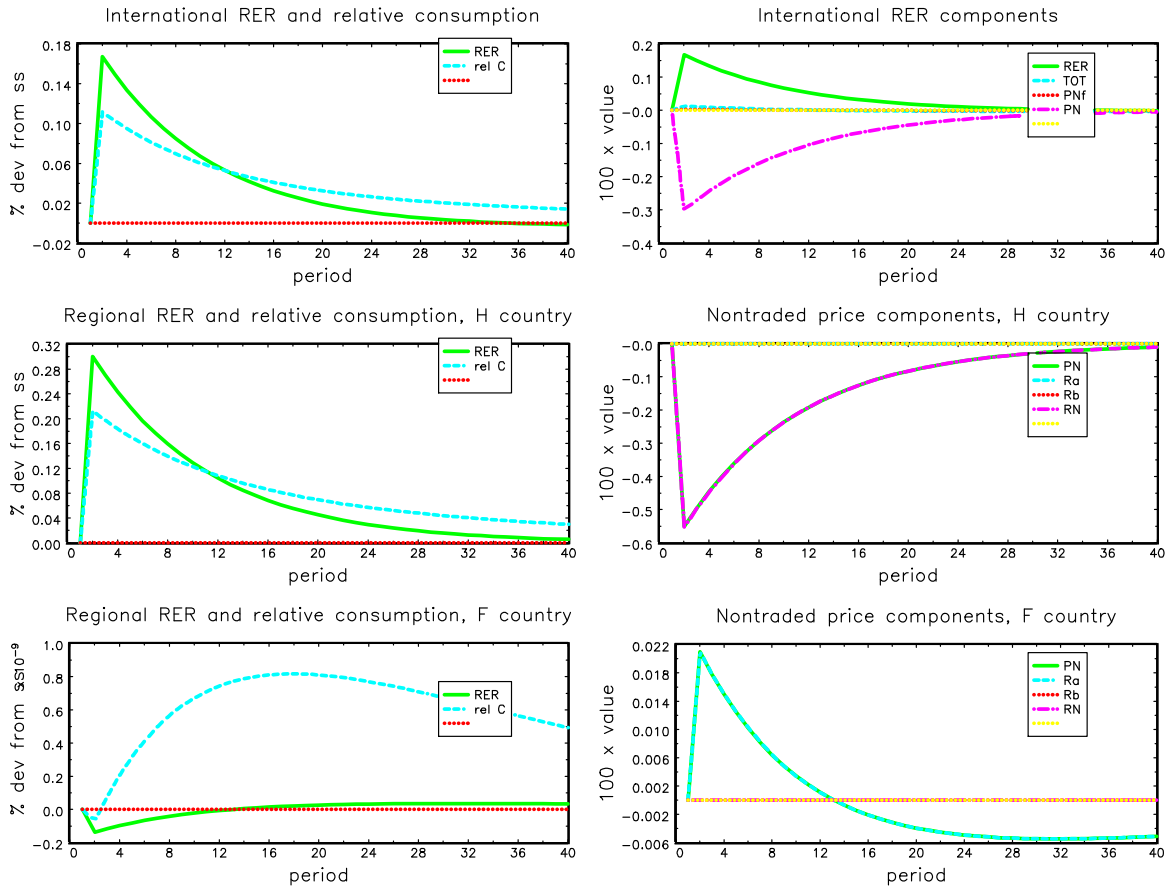

Here both international RER and relative consumption increase in response to the shock, implying a positive correlation between the two variables. The reason for such dynamics can

be seen from the right-hand-side panel. In response to the shock, the international terms-of-trade and the relative price of foreign internationally-nontraded goods do not change much. The dynamics of the international RER are therefore dominated by the drop in the relative price of domestic internationally-nontraded goods, P^{N}, which leads to RER depreciation.

The regional RER follows similar dynamics following the shock. When the supply of regionally-traded goods in region A of country H increases, the price of these goods, R^{A}, drops, while the relative price of regionally-traded goods originated in region B, R^{B}, goes up. This depreciates regional terms of trade in region A. Households also demand more of regionally-nontraded goods, driving their price, R^{N} up. Overall, the fall in R^{A} dominates the other effects, and causes P^{N} to fall. In region B of country H, the effects of the shock are similar, but less pronounced, also leading to a fall in the relative price P^{N} in that region. The regional RER, which is the sum of the regional terms of trade and the log-difference of the internationally-nontraded prices in regions B and A, therefore, increases (depreciates).

Finally, Figure **2c** presents the impulse responses of international and regional RERs and relative consumption following a 1% positive shock to productivity of the RN good in region A, country H.

Figure **2c**. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Reg. N shock

Both international and regional RERs are positively correlated with relative consumptions. An increase in the supply of regionally-nontraded goods, depresses their prices in ʜ country, but does so more in region ᴀ than in region ʙ. The regional RER, therefore, depreciates. The drop in the relative price of internationally-nontraded goods also leads to a depreciation of the international RER. At the same time, relative consumption rises both across countries and across regions.

To sum up, ᴛ shocks imply a negative correlation between international RER and relative consumption and have no effect on the corresponding correlation across regions. The shocks to productivity of ɴ goods (both ʀᴛ and ʀɴ) generate a positive Backus-Smith correlation in both international dimension and regional dimension. As a result, regional consumption-RER correlation is always positive in our model. The net international consumption-RER correlation depends on which shocks dominate. Under our benchmark calibration the effects of regional shocks dominate, resulting in a positive Backus-Smith correlation. This is because regions have access to an international bond, so they are able to smooth out shocks to productivity of internationally-traded goods with the other country reasonably well. The effects of those shocks, therefore, are moderated. The shocks to regional productivity are harder to

insure using international asset markets, implying that those shocks have profound effects on real exchange rates and relative consumption. However, when regions are heterogenous, factor mobility allows to smooth out some of the RT shocks across regions within the same country. This limits the effects of N shocks. The effect, however, is not strong enough to overturn the Backus-Smith puzzle. Next we study the role played by endogenous non-tradability in the model.

6.3.3 Role of endogenous non-tradability

Next, consider the impulse responses arising in the model with endogenous tradability, but no regional heterogeneity. Shutting down regional heterogeneity allows us to isolate the contribution of firm heterogeneity and their export entry-exit decisions on the risk-sharing. Figures **3a** − **3b** summarize the responses.

Figure **3a**. International RER and $C - \hat{C}$ with endog. tradability: Positive 1% Int'l T shock
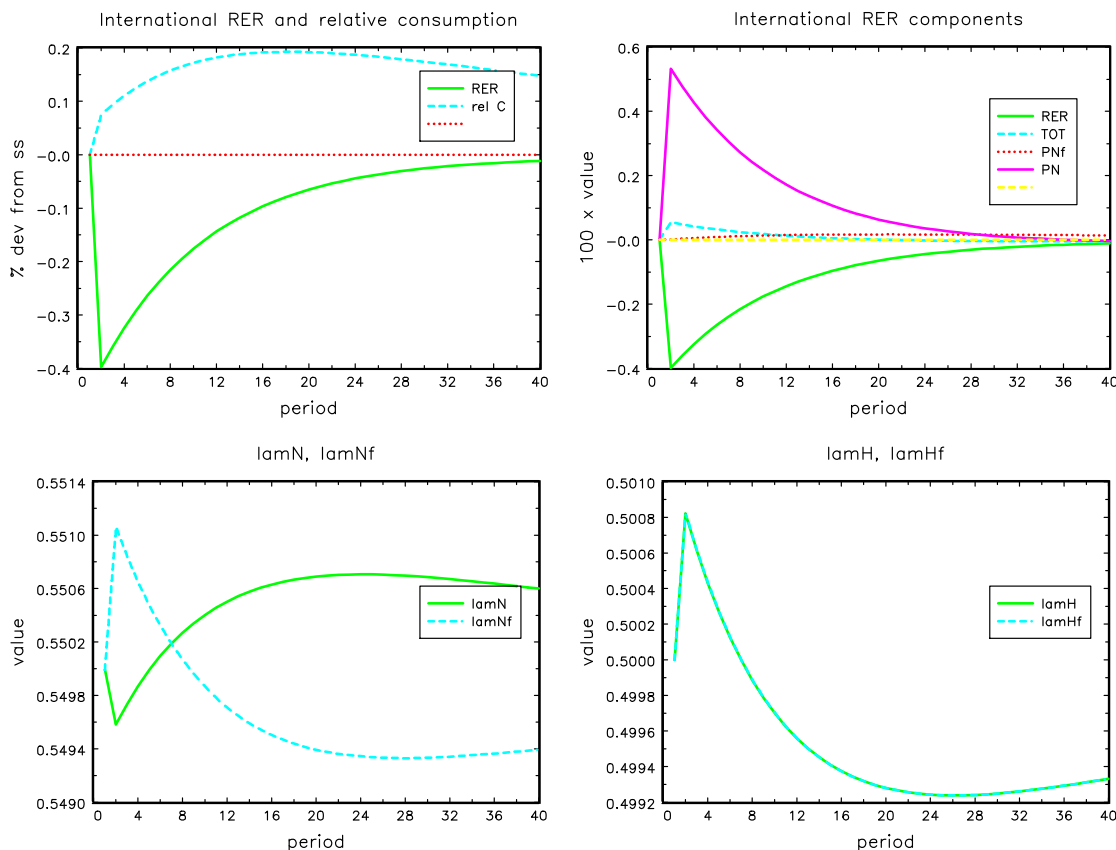

The top panel of Figure **3a** shows the responses of relative consumption, international RER and its components to a 1% positive productivity innovation in the T sector. These responses are qualitatively similar to those in the model with no endogenous tradability

presented in Figure **2a**, however, they are quantitatively larger. The amplifications is due to an endogenous change in the size of the internationally traded sector as new varieties become exported after a positive TFP shock. The bottom panel of Figure **3a** confirms this intuition. It show the impulse responses of nontraded, λ_N and $\hat{\lambda}_N$, consumption shares as well as domestic and foreign traded consumption shares in country H. Recall that λ_N is also equal to the threshold index i^* of the marginal firm that will export, thus controlling the size of nontraded sector in country H.

In the model with no endogenous tradability studied earlier, following a positive shock, labor was reallocating from the nontraded sector into the traded sector. That is, the adjustment on the supply side was taking place at the intensive margin, through the labor employment per firm. With endogenous tradability, the intensive margin is still present. However, it gets amplified by the extensive margin as more firms enter the export market in country H. The reverse happens in country F. The reason is that the extra profit that firm can make by exporting goes up (higher demand elasticity for traded goods in comparison to demand elasticity for nontraded goods is key for this result). As a result, more firms become exporters. The expansion in the size of the H internationally traded sector and a corresponding decline in the size of the internationally nontraded sector leads to a larger increase in the relative price of domestic nontraded goods, P^N, implying a greater appreciation of the RER, as a result.

Figure **3b** present impulse responses following a 1% positive productivity shock in the N sector.

Figure **3b**. International RER and $C - \hat{C}$ with endog. tradability: Positive 1% Int'l N shock
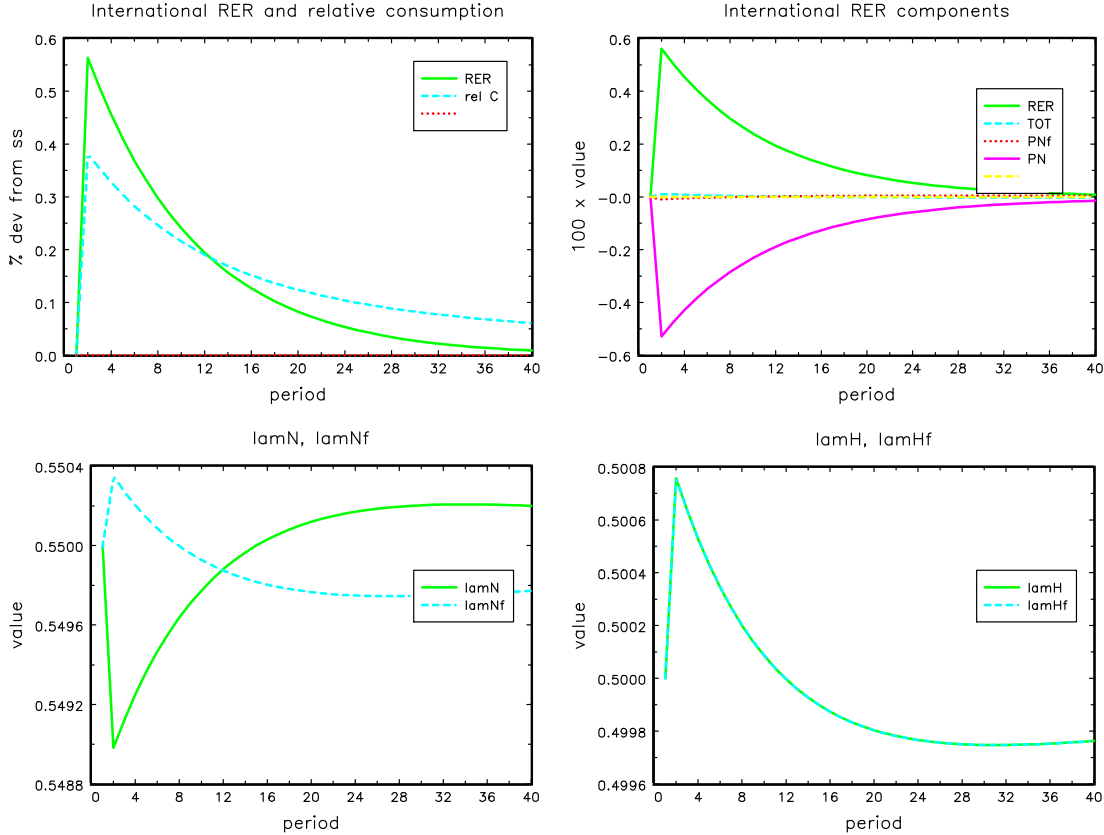
As in Figure **2c**, relative consumption in country H goes up, and RER depreciates following the shock. Much of the RER depreciation is driven by the fall in relative prices of nontraded goods in country H. As was the case with T productivity changes, the extensive margin in our model fortifies the adjustment that occurs at the intensive margin. In particular, following a N productivity improvement, labor was reallocating into the traded sector. The reason was a large drop in the price of nontraded goods, which put downward pressure on wages in the nontraded sector. In the economy with endogenous tradability, the fall in nontraded price also reduces firms profits in the nontraded sector and increases potential profits in the traded sector. As a result, initially nontraded firms enter the export market. This leads to a contraction in the size of internationally non-traded sector. This effect partially offsets the drop in prices of N goods in country H. As a result, RER depreciates by less, weakening the positive correlation between RER and relative consumption.

Overall, endogenous tradability amplifies the adjustment along the intensive margin (firms follow labor) and thus reinforces the effects of T shocks and counteracts the effects of N shocks on the RER. This lowers the international Backus-Smith correlation in the model with endogenous tradability.

6.3.4 Full model

Finally, we explore the properties of the full model that allows for both within-country heterogeneity and endogenous tradability. We find that the dynamics of this model in response to T shocks and RN shocks are very similar to those in the two versions of the model presented above. The responses of consumption and RER following RT shocks are also qualitatively similar, however, are smaller quantitatively. We present these responses in Figure **4** below.

Figure 4. Int'l and Regional RER and $C - \hat{C}$ with endog. tradability: Positive 1% Reg. T shock
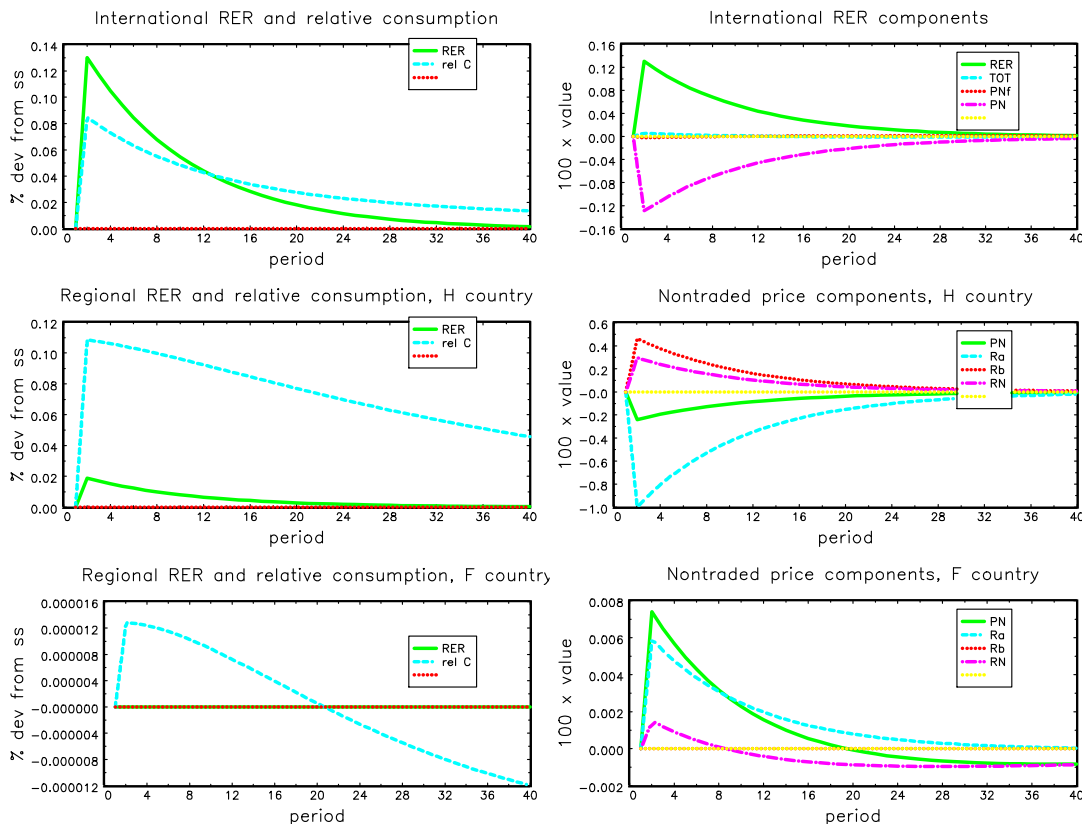
Consider first the adjustment along the intensive margin, that is in the employment per firm, in the RT sector. Following a positive RT productivity shock in region A, and given that consumption demand for RT goods is relatively elastic, there is a pressure for the wage in that sector in region A to go up. Therefore, labor reallocates from the internationally-traded sector into the RT sector. This reduces the size of internationally-traded sector and can lead to terms of trade appreciation. However, in our model with regional factor price equalization, the effects arising in the RT sector in region A are more than offset by the adjustment taking place in the RT sector in region B. In particular, RT sector employment in region B falls. This is due to two effects: first, there is an increase in the relative price of RT good produced by region B firms, inducing them to produce more; second, there is a

decrease in the demand for their good, due to higher prices, inducing them to produce less. The latter effect dominates. Due to factor price equalization across regions, all adjustment in the production must happen through factor adjustment. As a result, firms in the RT sector in region B reduce employment, and do so by a larger amount than an increase in employment by RT firms in region A. The overall employment in RT sector in country H falls, as a result.[11]

With endogenous tradability, extensive margin reinforces the intensive margin. As a result, following RT shock firms choose to exit RT sector and become exporters. This reduces the size of the nontraded sector in the economy and moderates the resulting adjustment in the price of nontraded goods. This effect leads to a lower RER appreciation in the model with endogenous tradability.

Overall, the model predicts that productivity improvements in the sector that produces internationally-tradable goods are accompanied by an entry of new firms/varieties into that sector as firms try to take advantage of favorable conditions. These productivity improvements can then be shared with households in other country. Productivity shocks to regionally-traded and regionally-nontraded sectors are associated with an exit of firms from the nontraded sector. A combination of regionally-traded shocks and endogenous tradability strengthen the negative correlation between international RER and relative consumption, thus helping us to rationalize the Backus-Smith puzzle in the international data. At the same time, the regional consumption-RER correlation remains positive, consistent with the empirical evidence.

7   Conclusion

This paper studies two well-known puzzles in the International Finance: Backus-Smith puzzle and RER volatility puzzle. First, we document the two puzzles in the international data. We complement this empirical analysis with a set of stylized facts on RER and relative consumption using regional/state level data for a large sample of countries. Second, we rationalize the two puzzles in a two-country real business cycles model, and do so in both international and intra-national dimensions. Our explanation relies on two key elements: within country heterogeneity and endogenous non-tradability. Each feature individually allows us to reduce the Backus-Smith correlation, but is not sufficient by itself. Only a

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[11]A way to dampen the offsetting adjustment in the region B is to reduce the degree of substitutability between regionally-traded goods within the same country. In the robustness checks we explored this avenue and found that the international Backus-Smith correlation becomes more negative as the elasticity of substitution between RT goods is reduced.

combination of regional heterogeneity and endogenous tradability resolves the Backus-Smith puzzle in the international and regional dimensions, and qualitatively can account for the differences in volatility of RER relative to consumption across countries and regions.

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A   Appendix

A.1   Data description and sources

A.1.1   International data

Our sample includes 29 OECD countries (no data is available for Korea) plus China and India over 1970-2004 period. The aggregate series for consumption are taken from Penn World Table 6.2 (Heston, Summers, and Aten, 2006). In particular, we used "Real gross domestic product per capita: CGDP" series in combination with "Consumption Share of CGDP: CC" series to obtain our measure of real per capita consumption for each country. The components of the RER are obtained from the following sources: (i) nominal exchange rate series are period averages calculated as local currency units per US \$, and are from IMF's International Financial Statistics (IFS) database; (ii) prices for all OECD countries in our sample and India are obtained from SourceOECD database and are consumer price index (CPI) series with base year of 2005. CPI for China is from China Statistical Yearbooks published by the Department of National Accounts, National Bureau of Statistics of China.

A.1.2   Regional data

To be completed.

A.2   Derivation of consumption aggregates

In our setup households have access to three types of consumption goods: regionally-nontraded goods, goods that can be traded between regions but not internationally, and internationally-traded goods. Let's consider first the final consumption aggregate. It consists of internationally nontraded goods and a basket of internationally traded goods. The latter can be produced by local firms or imported from the foreign country. Recall that all local firms are located on $[0, 1]$ interval. Firms that produce internationally nontraded goods occupy $[1, i^*]$ interval, where i^* is the threshold export index in a given region. Domestic internationally-traded goods are produced by firms located on $(i^*, 1]$ interval. We append this continuum of local firms by a $[1, 2 - \hat{\imath}^*]$ measure of foreign firms that can export their goods to the local market. As a result, the consumption basket of households residing in

region A in country H becomes

$$
\begin{aligned}
C^\phi &= (i^*)^{1-\phi} \left[\int_0^{i^*} \left(\tfrac{1}{i^*}\right)^{1-\rho} c_i^\rho \, di \right]^{\frac{\phi}{\rho}} + (1-i^*)^{1-\phi} \left[\left(\tfrac{1}{2-\hat{i}^*-i^*}\right)^{1-\rho} \left(\int_{i^*}^1 c_i^\rho \, di + \int_1^{2-\hat{i}^*} \hat{c}_i^\rho \, di \right) \right]^{\frac{\phi}{\rho}} \\
&= (i^*)^{1-\phi} \left[\int_0^{i^*} \left(\tfrac{1}{i^*}\right)^{1-\rho} c_i^\rho \, di \right]^{\frac{\phi}{\rho}} + (1-i^*)^{1-\phi} \, * \\
&\quad * \left[\left(\tfrac{1}{2-\hat{i}^*-i^*}\right)^{1-\rho} \left((1-i^*)^{1-\rho} \int_{i^*}^1 \left(\tfrac{1}{1-i^*}\right)^{1-\rho} c_i^\rho \, di + (1-\hat{i}^*)^{1-\rho} \int_1^{2-\hat{i}^*} \left(\tfrac{1}{1-\hat{i}^*}\right)^{1-\rho} \hat{c}_i^\rho \, di \right) \right]^{\frac{\phi}{\rho}} \\
&= (i^*)^{1-\phi} (C^{\mathrm{N}})^\phi + (1-i^*)^{1-\phi} \left[\left(\tfrac{1-i^*}{2-\hat{i}^*-i^*}\right)^{1-\rho} (C^{\mathrm{H}})^\rho + \left(\tfrac{1-\hat{i}^*}{2-\hat{i}^*-i^*}\right)^{1-\rho} (C^{\mathrm{F}})^\rho \right]^{\frac{\phi}{\rho}} \\
&= (i^*)^{1-\phi} (C^{\mathrm{N}})^\phi + (1-i^*)^{1-\phi} (C^{\mathrm{T}})^\phi ,
\end{aligned}
$$

where

$$
\begin{aligned}
(C^{\mathrm{N}})^\rho &= \int_0^{i^*} \left(\tfrac{1}{i^*}\right)^{1-\rho} c_i^\rho \, di, \\
(C^{\mathrm{H}})^\rho &= \int_{i^*}^1 \left(\tfrac{1}{1-i^*}\right)^{1-\rho} c_i^\rho \, di, \\
(C^{\mathrm{F}})^\rho &= \int_1^{2-\hat{i}^*} \left(\tfrac{1}{1-\hat{i}^*}\right)^{1-\rho} \hat{c}_i^\rho \, di.
\end{aligned}
$$

From above, consumption shares are endogenously linked to the threshold export index of firms:

$$
\lambda_{\mathrm{N}}(i^*) = i^* \qquad \text{and} \qquad \lambda_{\mathrm{T}}(i^*) = 1 - i^*,
$$
$$
\lambda_{\mathrm{H}}(i^*) = \tfrac{1-i^*}{2-\hat{i}^*-i^*} \qquad \text{and} \qquad \lambda_{\mathrm{F}}(i^*) = \tfrac{1-\hat{i}^*}{2-\hat{i}^*-i^*}.
$$

Next, consider the internationally nontraded consumption aggregate. It consists of regionally-nontraded goods (i.e. haircuts, real estate services, etc.); regionally traded goods that can be produced by the local firms or by the firms residing in another region within the same country (i.e. utility and government services, agricultural produce, many retail services, like those provided by Future Shop in Canada - a retail chain that does not operate in the US, etc.). Firms residing in region A that do not export to region B and thus produce regionally nontraded goods are located on $[1, \gamma_{\mathrm{N}}]$ interval. Firms operating in region A that produce goods that are regionally tradable occupy $[\gamma_{\mathrm{N}}, i_{\mathrm{A}}^*]$ interval. Appending this measure of firms with the $[i_{\mathrm{A}}^* + i_{\mathrm{B}}^* (1 - \gamma_{\mathrm{N}})]$ interval of firms residing in region B who supply goods to the local market. Therefore, internationally nontraded consumption aggregate in region A

can be written as

$$
\begin{aligned}
(i_{\mathrm{A}}^*)^{1-\alpha}(C^{\mathrm{N}})^\alpha &= (\gamma_{\mathrm{N}}i_{\mathrm{A}}^*)^{1-\alpha}\left[\int_0^{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\left(\frac{1}{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\right)^{1-\beta}z_i^\beta di\right]^{\frac{\alpha}{\beta}} + (i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}}))^{1-\alpha}\, * \\
&\quad * \left[\left(\frac{1}{i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}})+i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}\right)^{1-\beta}\left(\int_{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*}z_i^\beta di + \int_{i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}z_{i\mathrm{B}}^\beta di\right)\right]^{\frac{\alpha}{\beta}} \\
&= (\gamma_{\mathrm{N}}i_{\mathrm{A}}^*)^{1-\alpha}\left[\int_0^{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\left(\frac{1}{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\right)^{1-\beta}z_i^\beta di\right]^{\frac{\alpha}{\beta}} + (i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}}))^{1-\alpha}\, * \\
&\quad * \left[\left(\left(\frac{i_{\mathrm{A}}^*}{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*}\right)^{1-\beta}\int_{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*}\left(\frac{1}{i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}})}\right)^{1-\beta}z_i^\beta di + \left(\frac{i_{\mathrm{B}}^*}{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*}\right)^{1-\beta}\int_{i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}\left(\frac{1}{i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}\right)^{1-\beta}z_{i\mathrm{B}}^\beta di\right)\right]^{\frac{\alpha}{\beta}} \\
&= (\gamma_{\mathrm{N}}i_{\mathrm{A}}^*)^{1-\alpha}(Z^{\mathrm{N}})^\alpha + (i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}}))^{1-\alpha}\left[\left(\frac{i_{\mathrm{A}}^*}{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*}\right)^{1-\beta}(Z^{\mathrm{A}})^\beta + \left(\frac{i_{\mathrm{B}}^*}{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*}\right)^{1-\beta}(Z^{\mathrm{B}})^\beta\right]^{\frac{\alpha}{\beta}} \\
&= (i_{\mathrm{A}}^*)^{1-\alpha}\left[\gamma_{\mathrm{N}}^{1-\alpha}(Z^{\mathrm{N}})^\alpha + (1-\gamma_{\mathrm{N}})^{1-\alpha}(Z^{\mathrm{T}})^\alpha\right],
\end{aligned}
$$

where

$$
\begin{aligned}
(Z^{\mathrm{N}})^\beta &= \int_0^{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\left(\frac{1}{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}\right)^{1-\beta}z_i^\beta di, \\
(Z^{\mathrm{A}})^\beta &= \int_{\gamma_{\mathrm{N}}i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*}\left(\frac{1}{i_{\mathrm{A}}^*(1-\gamma_{\mathrm{N}})}\right)^{1-\beta}z_i^\beta di, \\
(Z^{\mathrm{B}})^\beta &= \int_{i_{\mathrm{A}}^*}^{i_{\mathrm{A}}^*+i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}\left(\frac{1}{i_{\mathrm{B}}^*(1-\gamma_{\mathrm{N}})}\right)^{1-\beta}z_{i\mathrm{B}}^\beta di.
\end{aligned}
$$

A.3 International and intra-national RER

In order to derive the decomposition for international RER, we log-linearize aggregate consumption price indices as follows. The log-pice of aggregate price index in region j in country H can be written as

$$
\begin{aligned}
p_{j,t} &= \lambda_{\mathrm{T}}\left(\frac{P_j^{\mathrm{T}}}{P_j}\right)^{\frac{\phi}{\phi-1}}p_t^{\mathrm{T}} + \lambda_{\mathrm{N}}\left(\frac{P_j^{\mathrm{N}}}{P_j}\right)^{\frac{\phi}{\phi-1}}p_{j,t}^{\mathrm{N}} \\
&= p_t^{\mathrm{T}} + \lambda_{\mathrm{N}}\left(\frac{P_j^{\mathrm{N}}}{P_j}\right)^{\frac{\phi}{\phi-1}}\left(p_{j,t}^{\mathrm{N}} - p_t^{\mathrm{T}}\right),
\end{aligned} \tag{A1}
$$

with

$$
\begin{aligned}
p_t^{\mathrm{T}} &= \lambda_{\mathrm{H}}\left(\frac{P_j^{\mathrm{H}}}{P_j^{\mathrm{T}}}\right)^{\frac{\rho}{\rho-1}}p_t^{\mathrm{H}} + \lambda_{\mathrm{F}}\left(\frac{P_j^{\mathrm{F}}}{P_j^{\mathrm{T}}}\right)^{\frac{\rho}{\rho-1}}p_t^{\mathrm{F}} \\
&= p_t^{\mathrm{H}} + \lambda_{\mathrm{F}}\left(\frac{P_j^{\mathrm{F}}}{P_j^{\mathrm{T}}}\right)^{\frac{\rho}{\rho-1}}\left(p_t^{\mathrm{F}} - p_t^{\mathrm{H}}\right)
\end{aligned} \tag{A2}
$$

and

$$p_{j,t}^{\text{N}} = \gamma_{\text{T}} \left(\frac{R_j^{\text{T}}}{P_j^{\text{N}}} \right)^{\frac{\alpha}{\alpha-1}} r_{j,t}^{\text{T}} + \gamma_{\text{N}} \left(\frac{R_j^{\text{N}}}{P_j^{\text{N}}} \right)^{\frac{\alpha}{\alpha-1}} r_{j,t}^{\text{N}}. \tag{A3}$$

Finally, the aggregate log-price of regionally traded goods in region j is

$$r_{j,t}^{\text{T}} = \gamma_{\text{A}} \left(\frac{R_j^{\text{A}}}{R_j^{\text{T}}} \right)^{\frac{\beta}{\beta-1}} r_{j,t}^{\text{A}} + \gamma_{\text{B}} \left(\frac{R_j^{\text{A}}}{R_j^{\text{T}}} \right)^{\frac{\beta}{\beta-1}} r_{j,t}^{\text{B}}. \tag{A4}$$

A similar set of conditions applies to the foreign country. For analytical tractability, we assume that international RER is a relative price of foreign to domestic consumption basket in regions A in countries H and F. In the numerical analysis of the model we use the definition of RER that relies on the average price in regions A and B as defined in (19). Now RER can be written as

$$rer_{\text{A},t}^{\text{I}} = \hat{p}_{\text{A},t} - p_{\text{A},t}.$$

Substituting in the definitions above, we get

$$rer_{\text{A},t}^{\text{I}} = \left(\lambda_{\text{H}} - \lambda_{\text{F}} \left(\frac{1}{1-\tau_I} \right)^{\frac{\rho}{\rho-1}} \right) \kappa_1 (\hat{p}_t^{\text{F}} - p_t^{\text{H}}) + \hat{\lambda}_{\text{N}} \kappa_2 \left(\hat{p}_{\text{A},t}^{\text{N}} - \hat{p}_{\text{A},t}^{\text{T}} \right) - \lambda_{\text{N}} \kappa_2 \left(p_{\text{A},t}^{\text{N}} - p_{\text{A},t}^{\text{T}} \right),$$

where lowercase letters, as before, denote the log transformations for all variables in deviations from their steady state values (e.g., $p_t^{\text{H}} \equiv \ln P_t^{\text{H}} - \ln P^{\text{H}}$, etc.); and $\kappa_1^{\frac{\rho-1}{\rho}} = \left(\hat{P}^{\text{F}} / P^{\text{T}} \right), \kappa_2^{\frac{\phi-1}{\phi}} = \left(\hat{P}_{\text{A}}^{\text{N}} / \hat{P}_{\text{A}} \right) = (P_{\text{A}}^{\text{N}} / P_{\text{A}})$. This is expression (20) in the text.

Having regional dimension in the model allows us to better understand the dynamics of nontradable goods prices. In particular, substituting equation (A4) into equation (A3), regionally nontraded log-price index can be written as:

$$p_{\text{A},t}^{\text{N}} = r_{\text{A},t}^{\text{A}} + \gamma_{\text{B}} \left(\frac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(r_{\text{A},t}^{\text{B}} - r_{\text{A},t}^{\text{A}} \right) + \gamma_{\text{N}} \kappa_4 \left(r_{\text{A},t}^{\text{N}} - r_{\text{A},t}^{\text{T}} \right),$$

where $\kappa_3^{\frac{\beta-1}{\beta}} = (R_{\text{B}}^{\text{B}} / R_{\text{A}}^{\text{T}}) = \left(\hat{R}_{\text{B}}^{\text{B}} / \hat{R}_{\text{A}}^{\text{T}} \right) = (R_{\text{B}}^{\text{B}} / R_{\text{B}}^{\text{T}}) = \left(\hat{R}_{\text{B}}^{\text{B}} / \hat{R}_{\text{B}}^{\text{T}} \right)$ and $\kappa_4^{\frac{\alpha-1}{\alpha}} = (R_{\text{A}}^{\text{N}} / P_{\text{A}}^{\text{N}}) = \left(\hat{R}_{\text{A}}^{\text{N}} / \hat{P}_{\text{A}}^{\text{N}} \right) = (R_{\text{B}}^{\text{N}} / P_{\text{B}}^{\text{N}}) = \left(\hat{R}_{\text{B}}^{\text{N}} / \hat{P}_{\text{B}}^{\text{N}} \right)$. This is equation (21) in the text. In region B, country H, the corresponding expression is

$$p_{\text{B},t}^{\text{N}} = r_{\text{B},t}^{\text{A}} + \hat{\gamma}_{\text{B}} \kappa_3 \left(r_{\text{B},t}^{\text{B}} - r_{\text{B},t}^{\text{A}} \right) + \hat{\gamma}_{\text{N}} \kappa_4 \left(r_{\text{B},t}^{\text{N}} - r_{\text{B},t}^{\text{T}} \right).$$

In country F the prices of internationally-nontraded goods can be approximated in an

analogous way. For instance, in region A it is given by

$$\hat{p}^{\text{N}}_{\text{A},t} = \hat{r}^{\text{A}}_{\text{A},t} + \gamma_{\text{B}} \left(\tfrac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(\hat{r}^{\text{B}}_{\text{A},t} - \hat{r}^{\text{A}}_{\text{A},t} \right) + \gamma_{\text{N}} \kappa_4 \left(\hat{r}^{\text{N}}_{\text{A},t} - \hat{r}^{\text{T}}_{\text{A},t} \right),$$

while in region B it is given by

$$p^{\text{N}}_{\text{B},t} = \hat{r}^{\text{A}}_{\text{B},t} + \hat{\gamma}_{\text{B}} \kappa_3 \left(\hat{r}^{\text{B}}_{\text{B},t} - \hat{r}^{\text{A}}_{\text{B},t} \right) + \hat{\gamma}_{\text{N}} \kappa_4 \left(\hat{r}^{\text{N}}_{\text{B},t} - \hat{r}^{\text{T}}_{\text{B},t} \right).$$

Using these decompositions, the regional real exchange rate in country H can be written as

$$rer^{\text{H}}_t = \lambda_{\text{N}} \kappa_2 \left(p^{\text{N}}_{\text{B},t} - p^{\text{N}}_{\text{A},t} \right).$$

We can simplify the derivations by recognizing that in our model $r^{\text{A}}_{\text{A},t} = r^{\text{A}}_{\text{B},t} \equiv r^{\text{A}}_t$ and $r^{\text{B}}_{\text{A},t} = r^{\text{B}}_{\text{B},t} \equiv r^{\text{B}}_t$. Furthermore, because regions are symmetric, we get $\hat{\gamma}_{\text{B}} = \gamma_{\text{A}}$. Substituting for $p^{\text{N}}_{\text{B},t}$ and $p^{\text{N}}_{\text{A},t}$ in the expression for rer^{H}_t and combining with the simplifications above allows to write rer^{H}_t as

$$rer^{\text{H}}_t = \lambda_{\text{N}} \kappa_2 \left[\left(\gamma_{\text{A}} - \gamma_{\text{B}} \left(\tfrac{1}{1-\tau_R} \right)^{\frac{\beta}{\beta-1}} \right) \kappa_3 \left(r^{\text{B}}_t - r^{\text{A}}_t \right) + \hat{\gamma}_{\text{N}} \kappa_4 \left(r^{\text{N}}_{\text{B},t} - r^{\text{T}}_{\text{B},t} \right) - \gamma_{\text{N}} \kappa_4 \left(r^{\text{N}}_{\text{A},t} - r^{\text{T}}_{\text{A},t} \right) \right],$$

which is equation (22) in the text.