

Federal Reserve Bank of Minneapolis
November 2008

Trade Liberalization, Growth, and Productivity*

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ABSTRACT

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. This paper investigates the theoretical relation between trade and growth. We show that standard models — including Ricardian models, Heckscher-Ohlin models, monopolistic competition models with homogeneous firms, and monopolistic competition models with heterogeneous firms — predict that opening to trade increases welfare, not necessarily real GDP. In a dynamic model where trade changes the incentives to accumulate factors of production, trade liberalization may lower growth rates even as it increases welfare. To the extent that trade liberalization leads to higher rates of growth in real GDP, it must do so primarily through mechanisms outside of those analyzed in standard models.

*This paper was prepared for the conference “New Directions in International Trade Theory” at the University of Nottingham. We thank the participants and the discussant, Doug Nelson, for comments. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

How does trade liberalization affect a country's growth and productivity? How does it affect a country's social welfare? As Rodriguez and Rodrik (2001) point out, "growth and welfare are not the same thing. Trade policies can have positive effects on welfare without affecting the rate of economic growth."

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. The findings are mixed. Many studies find a connection between trade, or some other measure of openness, and growth. But Rodriguez and Rodrik (2001), among others, are skeptical that these studies find a connection between *trade policy* and growth. (We provide an overview of these literatures below.) A further criticism of the empirical literature, posed by Slaughter (2001), is that it largely does not address the specific mechanisms through which trade may affect growth.

We investigate the theoretical relation between trade policy and growth. We do so using simple versions of some of the most common international trade models, including a Heckscher-Ohlin model, a Ricardian model with a continuum of goods, a monopolistic competition model with homogeneous firms, a monopolistic competition model with heterogeneous firms, and a dynamic Heckscher-Ohlin model. These models allow us to investigate a number of specific mechanisms by which trade liberalization is commonly thought to enhance growth or productivity: improvements in the terms of trade, increases in product variety, reallocation toward more productive firms, and increased incentive to accumulate capital.

For each model we provide an analytical solution for the autarky equilibrium and for the free trade equilibrium. We then look at the extreme case of trade liberalization by comparing autarky and free trade. To be consistent with empirical work, we measure real GDP in each of these models as real GDP is typically measured in the data, as GDP at constant prices. In each model the supply of labor is fixed, so changes in real GDP are also changes in measured labor productivity. We then contrast real GDP with a theoretical measure of real income, or social welfare.

In each model, trade liberalization improves social welfare. This is to be expected, but our results on real GDP may come as a surprise to many economists. In the static models, there is no general connection between trade liberalization and increases in real GDP per capita — the relationship may even be negative. Moreover, in a dynamic model with capital accumulation, some countries will have slower rates of growth under free trade than under autarky. Opening to trade improves welfare, but does not necessarily increase real GDP per capita or speed up growth. If openness does in fact lead to large increases in real GDP, these increases do not come from the standard mechanisms of international trade.

There is a vast empirical literature on the relationship between trade and growth. This literature typically studies the correlation between some measure of openness — for example, trade relative to GDP — and the growth of real GDP or real GDP per capita. Early papers in this line of research include Michaely (1977) and Balassa (1978). Lewer and Van den Berg (2003) present an extensive survey of this literature. They argue that most studies in this literature find a positive relationship between trade volume and growth and that they are fairly consistent on the size of this relationship. Other studies that find a positive relationship between trade openness and growth (using different techniques and openness measures) include World Bank (1987), Dollar (1992), Sachs and Warner (1995), Frankel and Romer (1999), Hall and Jones (1999), and Dollar and Kraay (2004).

Rodriguez and Rodrik (2001) question the findings of these studies. They argue that the indicators of openness used in these studies are either bad measures of trade barriers or are highly correlated with variables that also affect the growth rate of income. In the latter case, the studies may be attributing to trade the negative effects on growth of those other variables. Following this argument, Rodrik, Subramanian, and Trebbi (2002) find that openness has no significant effect on growth once institution-related variables are added in the regression analysis. Several studies using tariff rates as their specific measures of openness have found the relationship between trade policy and growth to depend on a country's level of development. In particular, Yanikkaya (2003) and DeJong and Ripoll (2006) find a negative relationship between trade openness and growth for developing countries.

Wacziarg (2001) and Hall and Jones (1999) find that trade affects growth mainly through capital investment and productivity. A smaller set of papers study the relationship between openness to trade and productivity. Examples are Alcalá and Ciccone (2004) and Hall and Jones (1999), both of which find a significant positive relationship between trade and productivity.

Theoretical studies on the relationship between trade and growth do not offer a clear view on whether there should be a relationship between trade openness (measured as lower trade barriers) and growth in income.

Models following the endogenous growth literature with increasing returns, learning-by-doing, or knowledge spillovers predict that opening to trade increases growth in the world as a whole, but may decrease growth in developing countries if they specialize in the production of goods with less potential for learning. Young (1991), Grossman and Helpman (1991), and Lucas (1988) are examples of examples of papers in this area. By contrast, Rivera-Batiz and Romer (1991) find that trade leads to higher growth for all countries by promoting investment in research and development.

Models of trade using the Dixit-Stiglitz theory of industrial organization have typically focused on welfare. Krugman () shows, for instance, that trade liberalization leads to welfare increases because of increases in product variety.

Melitz (2003) incorporates heterogeneous firms into a Krugman model and finds that trade liberalization increases a theoretical measure of productivity. Chaney (2006) also considers a simple model of heterogeneous firms, similar to the one we study here. When productivity is measured in the model as in the data, Gibson (2006) shows that trade liberalization does not, in general, increase productivity in these sorts of models. The increase is, rather, in welfare. Gibson (2006) finds that adding mechanisms to allow for technology adoption generate increases in measured productivity from trade liberalization.

Standard growth models also do not have a clear prediction for the relationship between trade and growth. In particular, in dynamic Heckscher-Ohlin models — models that integrate a neoclassical growth model with a Heckscher-Ohlin model of trade — opening to trade may increase or decrease a country's growth rate of income depending on parameter values. Trade may slow down growth in the capital-scarce country even

while it raises welfare. Papers in this literature are Ventura (1997), Cuñat and Maffezzoli (2004), and Bajona and Kehoe (2006).

2. General approach and measurement

In this paper we consider five commonly used models of trade. In each model we choose standard functional forms and, as needed, make assumptions so as to obtain analytical solutions for both the autarky equilibrium and the free trade equilibrium. (Throughout the paper we denote autarky equilibrium objects by a superscript A and free trade equilibrium objects by a superscript T). This allows us to examine the extreme case of trade liberalization. In most of the models, however, it is straightforward to add *ad valorem* tariffs or iceberg transportation costs.

In each model we measure real GDP as it is measured in the data. We then contrast this with a theoretical measure of real income, or social welfare. Using this approach, Gibson (2007) and Kehoe and Ruhl (2007) show that the difference between the data-based measure and the theoretical measure can be surprising.

In each model there is a perfect real income index with which we measure a country's social welfare in each period (throughout the paper we denote this by v). For simplicity, in each of the models the period utility function takes the form $u(c) = \log f(c)$, where $f(c)$ is homogeneous of degree one in c . The real income index is simply given by $f(c)$.

We strive to measure statistics in our models the same way they are measured in the data. This allows us to directly compare the model with the data. The issue here is the measurement of real GDP. Empirical studies use real GDP as reported in the national income and product accounts. This is either GDP at constant prices or GDP at current prices deflated by a chain-weighted price index. To be consistent with this empirical work, we measure real GDP in each of our models as GDP at constant prices. (Throughout the paper we denote GDP at current prices by gdp and GDP at constant prices by GDP .) For instance, in each of the static models we measure real GDP as GDP at autarky prices. In the dynamic model, we measure real GDP as GDP at period-0 prices.

Finally, in each model the supply of labor is fixed. Thus changes in real GDP are also changes in measured labor productivity, value added per worker. The terms real GDP, real GDP per capita, and labor productivity are all equivalent here.

Add total factor productivity for the Heckscher-Ohlin models?

3. Do improvements in the terms of trade increase real GDP?

In traditional trade theory, Ricardian and Heckscher-Ohlin frameworks, trade affects income through changes in relative prices. In particular, improvements in the terms of trade — the price of imports relative to the price of exports — lead to reallocation of resources towards goods in which a country has comparative advantage. Comparative advantage is driven by differences in technology, as in Ricardian models, or in factor endowments, as in Heckscher-Ohlin models. In this section we consider how changes in the terms of trade affect real GDP in both a Heckscher-Ohlin model and a Ricardian model with a continuum of goods. Kehoe and Ruhl (2007) consider the same issue in a small open economy model. In particular, they show that in standard models income effects due to changes in the terms of trade are not reflected in data-based measures of real GDP. Similar issues are addressed by Diewert and Morrison (1986) and Kohli (1983, 2004). In each of these models, trade liberalization leads to an improvement in the terms of trade. This increases welfare. The effects on real GDP and productivity, though, differ. In the Ricardian model, real GDP and productivity do not change after trade liberalization. In the static Heckscher-Ohlin model, when real GDP is measured at autarky prices, real GDP and productivity decrease.

3.1. A static Heckscher-Ohlin model

Consider a world with n countries, where each country i , $i = 1, 2, \dots, n$, has measure L_i of consumers. Each consumer in country i is endowed with one unit of labor and \bar{k}_i units of capital. There are two tradable goods, $j = 1, 2$, which are produced using capital and labor. The technology to produce the two tradable goods is the same across countries.

A consumer in country i derives utility from the consumption of both traded goods and chooses c_{ij} , $j = 1, 2$, to maximize

$$u(c_{i1}, c_{i2}) = a_1 \log c_{i1} + a_2 \log c_{i2}, \quad (1)$$

where $a_1 + a_2 = 1$, subject to the budget constraint

$$p_{i1}c_{i1} + p_{i2}c_{i2} = w_i + r_i\bar{k}_i. \quad (2)$$

Here p_{ij} is the price of good j , r_i is the rental rate of capital, and w_i is the wage rate.

Good j , $j = 1, 2$ is produced by combining capital and labor according to the Cobb-Douglas production function (identical in all countries)

$$y_j = \theta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}, \quad (3)$$

where we assume that $\alpha_1 > \alpha_2$ (that is, good 1 is capital-intensive and good 2 is labor-intensive). The markets for the traded goods are perfectly competitive and producers are price takers.

The autarkic and free trade versions of this model differ in the conditions that determine feasibility in the traded goods' markets. Under autarky, both markets have to clear in each country:

$$c_{ij} = y_{ij}, \quad (4)$$

whereas under free trade, the markets have to clear at the world level:

$$\sum_{i=1}^n L_i c_{ij} = \sum_{i=1}^n L_i y_{ij}. \quad (5)$$

Given our choice of functional forms for preferences and technologies, the model can be solved analytically. In both the autarkic and free trade equilibrium, prices and allocations can be expressed as functions of the allocation of capital per person. In what follows we list the expressions for the relevant variables for our analysis. The complete solution can be found in the appendix. To simplify the notation, let

$$A_1 = a_1\alpha_1 + a_2\alpha_2 \quad (6)$$

$$A_2 = 1 - A_1 = a_1(1 - \alpha_1) + a_2(1 - \alpha_2) \quad (7)$$

$$D_j = \frac{\theta_j a_j \alpha_j^{\alpha_j} (1 - \alpha_j)^{1 - \alpha_j}}{A_1^{\alpha_j} A_2^{1 - \alpha_j}} \quad (8)$$

$$D = D_1^{a_1} D_2^{a_2} . \quad (9)$$

Autarky

The autarky prices for the traded goods and the consumption and production allocations for country i , $i = 1 \dots n$, are

$$p_{ij}^A = \frac{a_j D}{D_j} \bar{k}_i^{-A_1 - \alpha_j} , \quad (10)$$

$$c_{ij}^A = y_{ij}^A = D_j \bar{k}_i^{\alpha_j} \quad (11)$$

Our variables of interest are nominal and real GDP, productivity and welfare. Since we take autarky as the base year in computing real GDP, nominal and real GDP coincide in autarky:

$$\begin{aligned} gdp_i^A = GDP_i^A &= p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A \\ &= D \bar{k}_i^{A_1} , \end{aligned} \quad (12)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^A = \frac{GDP_i^A}{\bar{k}_i^{A_1}} = D . \quad (13)$$

Using the monotonic transformation of the utility function $v = e^u$ to measure welfare, we obtain:

$$v_i^A = (c_{i1}^A)^{a_1} (c_{i2}^A)^{a_2} = D \bar{k}_i^{A_1} . \quad (14)$$

Free trade

In the free trade equilibrium we focus on the case where countries have similar enough factor endowments so that all countries are in the cone of diversification. That is, letting

$$\bar{k} = \frac{\sum_{i=1}^n L_i \bar{k}_i}{\sum_{i=1}^n L_i} \quad (15)$$

$$\gamma_i = \frac{\bar{k}_i}{\bar{k}} \quad (16)$$

$$\kappa_j = \left(\frac{\alpha_j}{1 - \alpha_j} \right) \frac{A_2}{A_1}, \quad (17)$$

we examine the case where $\kappa_2 \leq \gamma_i \leq \kappa_1$, $i = 1, \dots, n$. In this case, the equilibrium prices and aggregate variables of the free trade equilibrium can be obtained by solving for the equilibrium of the integrated economy (a closed economy with factor endowments equal to world factor endowments) and then splitting the aggregate allocations across countries in a way that is consistent with their factor endowments. Let \bar{k} be as in equation (15). Then the world prices and each country's production and consumption patterns are given by:

$$p_j^T = \frac{a_j D}{D_j} \bar{k}^{A_1 - \alpha_j} \quad (18)$$

$$c_{ij}^T = (A_1 \gamma_i + A_2) D_j \bar{k}^{\alpha_j} \quad (19)$$

$$y_{ij}^T = \mu_{ij} D_j \bar{k}^{\alpha_j} \quad (20)$$

where γ_i is defined in equation (16), and μ_{ij} are:

$$\mu_{i1} = \frac{A_1 \gamma_i (1 - \alpha_2) - A_2 \alpha_2}{a_1 (\alpha_1 - \alpha_2)} \quad (21)$$

$$\mu_{i2} = \frac{A_2 \alpha_1 - A_1 \gamma_i (1 - \alpha_1)}{a_2 (\alpha_1 - \alpha_2)}. \quad (22)$$

Notice that setting $\gamma_i = 1$ we obtain the same values as in autarky.

In this version of the model, nominal and real GDP do not longer coincide.

Nominal GDP in country i , is GDP measured at current prices:

$$\begin{aligned} gdp_i^T &= p_1^T y_{i1}^T + p_2^T y_{i2}^T \\ gdp_i^T &= (A_1 \gamma_i + A_2) D \bar{k}^{A_1}, \end{aligned} \quad (23)$$

whereas real GDP is measured at autarky prices:

$$\begin{aligned} GDP_i^T &= p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T \\ GDP_i^T &= \frac{\gamma_i^{-\alpha_1} (A_1 \gamma_i (1 - \alpha_2) - A_2 \alpha_2) + \gamma_i^{-\alpha_2} (A_2 \alpha_1 - A_1 \gamma_i (1 - \alpha_1))}{\alpha_1 - \alpha_2} D \bar{k}_i^{A_1}. \end{aligned} \quad (24)$$

Using the same measure as in the autarkic model, welfare under free trade becomes:

$$v_i^T = (c_{i1}^T)^{a_1} (c_{i2}^T)^{a_2} = (A_1 \gamma_i + A_2) D \bar{k}^{A_1}. \quad (25)$$

Effect of trade liberalization

Trade liberalization in this model increases the prices of the exported goods and decreases the prices of the imported goods, improving the terms of trade. This improvement in the terms of trade increases welfare, but decreases measured real GDP and productivity. The intuition for the latter is simple: given factor endowments, the autarkic production pattern in country i is the optimal production pattern for country i at the autarkic prices. Any deviation from that production pattern will lower the value of production at those prices. Since productivity is measured using real GDP, a decrease in real GDP also implies a decrease in measured productivity.

Proposition 1. In the static Heckscher-Ohlin model described above, if $\gamma_i \neq 1$, following trade liberalization:

- (i) welfare strictly increases
- (ii) real GDP and productivity decrease.

For $\gamma^i = 1$ all measures stay the same.

Proof. (i) Comparing (14) and (29) we need to show that to show that

$$(A_1 \gamma_i + A_2) D \bar{k}^{A_1} > D \bar{k}_i^{A_1}, \quad (26)$$

or equivalently, using the definition of γ_i , that

$$A_1 \gamma_i + 1 - A_1 > \gamma_i^{A_1}. \quad (27)$$

Define $f(\gamma) = A_1 \gamma + 1 - A_1$ and $g(\gamma) = \gamma^{A_1}$. The result comes from the fact that,

$$f(1) = g(1), \quad f'(1) = g'(1), \quad f'(\gamma) > g'(\gamma) \text{ if } \gamma < 1 \text{ and } f'(\gamma) < g'(\gamma) \text{ if } \gamma > 1. \quad \blacksquare$$

(ii) We need to show that

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T. \quad (28)$$

Define the function

$$\begin{aligned}
\pi(p_1, p_2, k_i) &= \max \left[p_1 \theta_1 k_{i1}^{\alpha_1} \ell_{i1}^{1-\alpha_1} + p_2 \theta_2 k_{i2}^{\alpha_2} \ell_{i2}^{1-\alpha_2} \right] \\
&\text{s.t. } k_{i1} + k_{i2} \leq k_i \\
&\quad \ell_{i1} + \ell_{i2} \leq 1 \\
&\quad k_{ij} \geq 0, \ell_{ij} \geq 0
\end{aligned} \tag{29}$$

Since $\alpha_1 > \alpha_2$, this function is strictly concave. Notice that

$$\pi(p_{i1}^A, p_{i2}^A, \bar{k}_i) = p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A. \tag{30}$$

The free trade allocation also satisfies the feasibility constraints in (29), so

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T, \tag{31}$$

where the strict inequality follows from the strict concavity of π . Figure 1 illustrates the proof.

The decrease in productivity follows immediately from the decrease in real GDP. ■

3.2. A Ricardian model with a continuum of goods

Consider a world with two symmetric countries. In each country i , $i = 1, 2$, the representative consumer is endowed with $\bar{\ell}$ units of labor. There is a continuum of tradable goods, $z \in [0, 1]$.

The representative consumer derives utility from the consumption of all tradable goods and chooses $c_i(z)$, $z \in [0, 1]$, to maximize

$$u(c_i) = \int_0^1 \log c_i(z) dz \tag{32}$$

subject to the budget constraint

$$\int_0^1 p_i(z) c_i(z) dz = w_i \bar{\ell}. \tag{33}$$

Here $p_i(z)$ is the price of good z and w_i is the wage rate.

Good z is produced using only labor. The production technology to produce good z differs across countries and it is given by:

$$y_i(z) = \ell_i(z) / a_i(z), \tag{34}$$

where $a_i(z)$ is the quantity of labor required to produce one unit of good z in country i . Let us assume that goods are ordered from lowest to highest unit labor requirements in country 1, and that countries are symmetric in terms of productivities in the sense that good z in country 1 uses the same technology in production as good $1-z$ in country 2. That is, let

$$a_1(z) = e^{\alpha z} \quad (35)$$

$$a_2(z) = e^{\alpha(1-z)}, \quad (36)$$

where $\alpha > 0$. The markets for the traded goods are perfectly competitive and producers are price takers.

The autarkic and free trade versions of this model differ in the conditions that determine feasibility in the traded goods' markets. Under autarky, the market for good all goods $z \in [0,1]$ has to clear in each country, whereas under free trade, only the world market for each good has to clear:

$$c_1(z) + c_2(z) = y_1(z) + y_2(z). \quad (37)$$

We have chosen functional forms for which there is an analytical solution of the model. In what follows, we list the values of the relevant variables, as well as the values for nominal and real GDP, productivity and welfare for the model under both, autarky and free trade.

Autarky

Let us normalize $w_i = 1$. The autarkic prices for good $z \in [0,1]$ in each country and the consumption and production levels are:

$$p_1^A(z) = e^{\alpha z} \quad (38)$$

$$p_2^A(z) = e^{\alpha(1-z)}. \quad (39)$$

$$c_i^A(z) = y_i^A(z) = \frac{\bar{\ell}}{p_i^A(z)}. \quad (40)$$

In measuring real GDP, we take the autarkic prices as the base prices. Therefore, in the autarkic equilibrium, nominal and real GDP coincide, and take the value:

$$gdp_i^A = GDP_i^A = \int_0^1 p_i^A(z) y_i^A(z) dz = \bar{\ell}. \quad (41)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^A = \frac{GDP_i^A}{\bar{\ell}} = 1. \quad (42)$$

Using the monotonic transformation of the utility function $v = e^u$ to measure welfare, we

obtain:
$$v_i^A = \exp \int_0^1 \log c_i^A(z) dz = \bar{\ell} e^{-\alpha^2/2}. \quad (43)$$

Free trade

Given the symmetry imposed in the model, we normalize $w_1 = w_2 = 1$. Country 1 produces and exports goods $z \in [0, 0.5]$ and country 2 produces and exports goods $z \in (0.5, 1]$. The prices of the goods and the consumption patterns in each country are

$$p^T(z) = \begin{cases} e^{\alpha z} & z \in [0, 0.5] \\ e^{\alpha(1-z)} & z \in (0.5, 1] \end{cases}. \quad (44)$$

$$c_1^T(z) = c_2^T(z) = \frac{\bar{\ell}}{p^T(z)}. \quad (45)$$

The production patterns are as follows. For goods $z \in [0, 0.5]$, the production plans are

$$y_1^T(z) = \frac{2\bar{\ell}}{p^T(z)}, \quad \ell_1^T(z) = 2\bar{\ell} \quad (46)$$

$$y_2^T(z) = \ell_2^T(z) = 0. \quad (47)$$

For goods $z \in (0.5, 1]$, the production plans are

$$y_1^T(z) = \ell_1^T(z) = 0 \quad (48)$$

$$y_2^T(z) = \frac{2\bar{\ell}}{p^T(z)}, \quad \ell_2^T(z) = 2\bar{\ell} \quad (49)$$

Regarding the variables of interest in our paper, notice that, in a given country, autarky and trade prices differ for the goods that the country is importing, but they are the same for the goods that the country is exporting. Overall, though, the changes in production

completely offset the changes in prices, and nominal and real GDP coincide in the free trade model. In particular, GDP at current prices is:

$$gdp_i^T = \int_0^1 p^T(z) y_i^T(z) dz = \bar{\ell}, \quad (50)$$

and GDP at autarky prices is

$$GDP_i^T = \int_0^1 p_i^A(z) y_i^T(z) dz = \bar{\ell}. \quad (51)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^T = \frac{GDP_i^T}{\bar{\ell}} = 1 \quad (52)$$

Using the same measure of welfare as in the autarky model, we obtain that welfare in the free trade economy equals:

$$v_i^T = \exp \int_0^1 \log c_i^T(z) dz = \bar{\ell}. \quad (53)$$

Effect of trade liberalization

After trade liberalization, the prices of each country's imports decrease, resulting in an improvement in the terms of trade. As a result, real income increases in both countries, but real GDP and productivity stay the same. The next proposition summarizes the results:

Proposition 2: In the Ricardian model described above, following trade liberalization:

- (i) welfare increases
- (ii) real GDP and measured productivity do not change.

Proof: (i) We need to show $\bar{\ell} e^{-\alpha^2/2} < \bar{\ell}$, which follows directly from the fact that $e^{-\alpha^2/2} < 1$.

(ii) results from direct comparison of (41) and (51). Since GDP does not change, productivity stays the same. ■

4. Do increases in product variety from trade liberalization increase real GDP?

It is well known that, in standard monopolistic competition models with homogeneous firms, trade liberalization leads to an increase in the number of product varieties available to the consumer. This increase in product variety leads to an increase in real income, but does it lead to an increase in real GDP? We find that this depends on the nature of competition in the product market. If there is a continuum of product varieties, then real GDP does not change. If there is a finite number of product varieties, then real GDP increases. The reason is that, with Cournot (or Bertrand) competition among firms, markups over marginal cost decrease when the number of firms supplying goods to a market increases. We make this point using a monopolistic competition model with a finite number of product varieties.

A monopolistic competition model with homogeneous firms

In each country i , $i = 1, 2, \dots, n$, the representative consumer is endowed with $\bar{\ell}_i$ units of labor. Let J_i be the number of goods available to the consumer in country i .

Consumer i chooses c_{ij} , $j = 1, 2, \dots, J_i$, to maximize

$$(1/\rho) \log \sum_{j=1}^{J_i} c_{ij}^\rho \quad (54)$$

subject to the budget constraint

$$\sum_{j=1}^{J_i} p_{ij} c_{ij} = w_i \bar{\ell}_i. \quad (55)$$

Here p_{ij} is the price of good j and w_i is the wage rate.

A firm producing good j in country i has the increasing-returns-to-scale technology

$$y_{ij} = (1/b) \max[\ell_{ij} - f, 0], \quad (56)$$

where f is the fixed cost, in units of labor, of operating.

There is Cournot competition among firms. Taking as given the consumer's demand function and the decisions of all other firms, a firm's problem is to choose the

quantity of output that maximizes its profits. There is free entry of firms, so there are no aggregate profits.

Clearing in the labor market requires that

$$\sum_{j=1}^{J_i} \ell_{ij} = \bar{\ell}_i. \quad (57)$$

Under autarky,

$$c_{ij} = y_{ij}. \quad (58)$$

Under free trade, if good j is produced in country i ,

$$y_{ij} = \sum_{i=1}^n c_{ij}. \quad (59)$$

Autarky

Normalize $w_i = 1$. Each firm takes the consumer's indirect demand function as given. Consumer i 's indirect demand function for good j is

$$p_{ij} = \frac{c_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} c_{im}^{\rho}} \bar{\ell}_i. \quad (60)$$

The firm in country i producing good j chooses y_{ij} to maximize profits,

$$p_{ij} y_{ij} - b y_{ij} - f. \quad (61)$$

Plugging (60) into (61), the expression for profits becomes

$$\frac{y_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} y_{im}^{\rho}} \bar{\ell}_i y_{ij} - b y_{ij} - f. \quad (62)$$

Profit maximization implies that marginal revenue is equal to marginal cost, so

$$\left(\frac{y_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} y_{im}^{\rho}} \right) \left(\frac{\left(\sum_{m=1}^{J_i} y_{im}^{\rho} \right) \rho y_{ij}^{\rho-1} - y_{ij}^{\rho} \rho y_{ij}^{\rho-1}}{\left(\sum_{m=1}^{J_i} y_{im}^{\rho} \right)^2} \right) \bar{\ell}_i = b. \quad (63)$$

Imposing symmetry across firms (the j subscripts are omitted), we obtain

$$c_i^A = y_i^A = \frac{\rho (J_i^A - 1) \bar{\ell}_i}{(J_i^A)^2 b} \quad (64)$$

$$p_i^A = \frac{bJ_i^A}{\rho(J_i^A - 1)}. \quad (65)$$

The profits of a firm are

$$p_i^A y_i^A - b y_i^A - f = \frac{\bar{\ell}_i}{J_i^A} - \frac{\rho(J_i^A - 1)\bar{\ell}_i}{(J_i^A)^2} - f. \quad (66)$$

Since there is free entry, firm profits must be zero in equilibrium:

$$f(J_i^A)^2 - (1 - \rho)\bar{\ell}_i J_i^A - \rho\bar{\ell}_i = 0 \quad (67)$$

Let N_i be the number of firms in country i . Using the quadratic formula, we solve for the number of varieties and firms:

$$J_i^A = N_i^A = \frac{(1 - \rho)\bar{\ell}_i + \sqrt{(1 - \rho)^2 \bar{\ell}_i^2 + 4f\rho\bar{\ell}_i}}{f^2}. \quad (68)$$

Notice that the number of goods is not necessarily an integer. Alternatively, we could allow for aggregate profits and calculate N_i as the integer such that there are nonnegative profits but that, if one more firm entered, profits would be negative.

GDP at current prices is

$$\begin{aligned} GDP_i^A &= N_i^A p_i^A y_i^A \\ &= \bar{\ell}_i \end{aligned} \quad (69)$$

Real income is

$$\begin{aligned} v_i^A &= \left(J_i^A (c_i^A)^\rho \right)^{1/\rho} \\ &= \left(J_i^A \right)^{\frac{1-\rho}{\rho}} \frac{\rho(J_i^A - 1)}{J_i^A b} \bar{\ell}_i \end{aligned} \quad (70)$$

Free trade

We can use the above approach to solve for the integrated equilibrium of the world economy, in which the supply of labor is $\bar{\ell} = \sum_{i=1}^n \bar{\ell}_i$. We again normalize $w = 1$ and obtain

$$J^T = \frac{(1-\rho)\bar{\ell} + \sqrt{(1-\rho)^2\bar{\ell}^2 + 4f\rho\bar{\ell}}}{f^2} \quad (71)$$

$$y^T = \frac{\rho(J^T - 1)\bar{\ell}}{(J^T)^2 b} \quad (72)$$

$$p^T = \frac{bJ^T}{\rho(J^T - 1)}. \quad (73)$$

Disaggregating proportionally,

$$c_i^T = \frac{\bar{\ell}_i}{\bar{\ell}} \bar{y}^T. \quad (74)$$

$$N_i^T = \frac{\bar{\ell}_i}{\bar{\ell}} J^T. \quad (75)$$

Notice that the equilibrium values for free trade are the same as those for autarky if $\bar{\ell}_i = \bar{\ell}$.

GDP at current prices is

$$\begin{aligned} gdp_i^T &= N_i^T p^T y^T \\ &= \bar{\ell}_i \end{aligned} \quad (76)$$

GDP at autarky prices is

$$\begin{aligned} GDP_i^T &= N_i^T p_i^A y^T \\ &= \frac{J_i^A}{(J_i^A - 1)} \frac{(J^T - 1)}{J^T} \bar{\ell}_i \end{aligned} \quad (77)$$

Real income is

$$\begin{aligned} v_i^T &= \left(J^T (c_i^T)^\rho \right)^{1/\rho} \\ &= \left(J^T \right)^{\frac{1-\rho}{\rho}} \frac{\rho (J^T - 1)}{J^T b} \bar{\ell}_i \end{aligned} \quad (78)$$

Effect of trade liberalization

Proposition 3. If $\bar{\ell}_i < \bar{\ell}$, then real income in country i strictly increases following trade liberalization.

Proof. We want to show that

$$\left(J^T\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J^T-1\right)}{J^T b} \bar{\ell}_i > \left(J_i^A\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J_i^A-1\right)}{J_i^A b} \bar{\ell}_i. \quad (79)$$

It suffices to show that $J^T > J_i^A$, which is evident from comparing (68) and (71). ■

Proposition 4. If $\bar{\ell}_i < \bar{\ell}$, then GDP at autarky prices in country i strictly increases following trade liberalization.

Proof. We want to show that

$$\frac{J_i^A}{\left(J_i^A-1\right)} \frac{\left(J^T-1\right)}{J^T} \bar{\ell}_i > \bar{\ell}_i. \quad (80)$$

Again, this follows from the fact that $J^T > J_i^A$. ■

Real GDP increases because markups decrease. Since $J^T > J_i^A$, there are more firms competing in each market. With Cournot competition, this lowers the markup over marginal cost:

$$\frac{J^T}{\rho\left(J^T-1\right)} < \frac{J_i^A}{\rho\left(J_i^A-1\right)}. \quad (81)$$

If there is a continuum, rather than a finite number, of product varieties, then the markup over marginal cost is constant at $1/\rho$, regardless of trade policy. In this case, GDP at autarky prices remains constant following trade liberalization.

5. Does reallocation across heterogeneous firms following trade liberalization increase measured productivity?

With heterogeneous firms and fixed costs of exporting, trade liberalization can lead to a reallocation of resources across firms. In a simple model, trade liberalization

causes the least productive firms to exit and the most productive firms to become exporters. Intuitively, this reallocation of resources toward more productive firms should increase aggregate productivity. But we find that it does not. The finding here is explored further in Gibson (2007), where a positive mechanism is also provided.

A monopolistic competition model with heterogeneous firms

There are two symmetric countries. In each country i , $i = 1, 2$, the representative consumer is endowed with $\bar{\ell}$ units of labor and measure μ of potential firms (potential firms may choose not to operate). Each firm produces a differentiated good.

Let Z_i be the set of goods available to consumer i . The consumer chooses $c_i(z)$, $z \in Z_i$, to maximize

$$(1/\rho) \log \int_{Z_i} c_i(z)^\rho dz \quad (82)$$

subject to the budget constraint

$$\int_{Z_i} p_i(z) c_i(z) dz = w_i \bar{\ell} + \pi_i. \quad (83)$$

Here $p_i(z)$ is the price of good z , w_i is the wage rate, and π_i is the profits of firms.

Firms differ in their productivity levels. Let $x(z)$ be the productivity level of the firm that produces good z . The firm producing good z in country i has the increasing-returns-to-scale technology

$$y_i(z) = \max \left[x(z) (\ell_i(z) - f_d), 0 \right], \quad (84)$$

where f_d is the fixed cost, in units of labor, of operating. If the economies are open to trade, then a firm can choose to export by paying an additional fixed cost of f_e units of labor.

Potential firms draw their productivities from a Pareto distribution

$$F(x) = 1 - x^{-\gamma}, \quad (85)$$

$x \geq 1$. The choice of one as the lower bound on the Pareto distribution can be thought of as a normalization. For reasons that will be clear later, we impose the restriction that $\gamma > \max \left[2, \rho/(1-\rho) \right]$.

Taking the consumer's demand functions as given, the firm's problem is to choose the profit-maximizing price. Each firm decides whether to operate. If there is free trade, each firm decides whether to export.

Clearing in the labor market requires that

$$\int_{Z_i} \ell_i(z) dz = \bar{\ell}. \quad (86)$$

Autarky

There are two possibilities: Either all potential firms choose to produce or not. We examine the latter case. In this case, there is a cutoff \bar{x}_d , $\bar{x}_d > 1$, such that a firm with productivity x produces if $x \geq \bar{x}_d$.

Since the countries are symmetric, country subscripts are omitted. Set $w = 1$.

The profit-maximizing prices are

$$p^A(x) = \frac{1}{\rho x}. \quad (87)$$

The aggregate price index is

$$\begin{aligned} P^A &= \left(\mu \int_{\bar{x}_d}^{\infty} p^A(x)^{\frac{-\rho}{1-\rho}} dF(x) \right)^{\frac{-(1-\rho)}{\rho}} \\ &= \left(\frac{\gamma(1-\rho) - \rho}{\rho^{\frac{\rho}{1-\rho}} (1-\rho) \gamma \mu (\bar{x}_d^A)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}} \right)^{\frac{1-\rho}{\rho}}. \end{aligned} \quad (88)$$

The demand for a good produced by a firm with productivity $x \geq \bar{x}_d^A$ is

$$\begin{aligned} c^A(x) &= y^A(x) = p^A(x)^{\frac{-1}{1-\rho}} (P^A)^{\frac{\rho}{1-\rho}} (\bar{\ell} + \pi^A) \\ &= \frac{\rho(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^A) x^{\frac{1}{1-\rho}}}{(1-\rho) \gamma \mu (\bar{x}_d^A)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}}. \end{aligned} \quad (89)$$

A firm with productivity \bar{x}_d^A must make zero profits in equilibrium, so

$$p^A(\bar{x}_d^A) c^A(\bar{x}_d^A) - \frac{c^A(\bar{x}_d^A)}{\bar{x}_d^A} - f_d = 0. \quad (90)$$

Plugging (87) and (89) into (90), we obtain

$$\bar{x}_d^A = \left(\frac{\mu \gamma f_d}{(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^A)} \right)^{1/\gamma}, \quad (91)$$

where

$$\begin{aligned} \pi^A &= \mu \int_{\bar{x}_d^A}^{\infty} \left(p^A(x) c^A(x) - \frac{c^A(x)}{x} - f_d \right) dF(x) \\ &= \frac{\rho \bar{\ell}}{\gamma - \rho} \end{aligned} \quad (92)$$

Plugging (92) into (91), the cutoff for operating is

$$\bar{x}_d^A = \left(\frac{\mu(\gamma - \rho) f_d}{(\gamma(1-\rho) - \rho) \bar{\ell}} \right)^{1/\gamma}. \quad (93)$$

GDP at current prices is

$$\begin{aligned} GDP^A &= \mu \int_{\bar{x}_d^A}^{\infty} p^A(x) y^A(x) dF(x) \\ &= \frac{\gamma}{\gamma - \rho} \bar{\ell} \end{aligned} \quad (94)$$

Real income is

$$\begin{aligned} v^A &= \left(\mu \int_{\bar{x}_d^A}^{\infty} c^A(x)^\rho dF(x) \right)^{1/\rho} \\ &= \frac{\gamma}{(\gamma - \rho) P^A} \bar{\ell} \end{aligned} \quad (95)$$

Free trade

We again examine the case in which not all firms choose to produce. That is, firm z produces if $x(z) \geq \bar{x}_d$, $\bar{x}_d > 1$. With free trade, each firm faces an additional decision: whether to pay the fixed cost f_e to export. There is a cutoff \bar{x}_e , $\bar{x}_e > \bar{x}_d$, such that firm z exports if $x(z) \geq \bar{x}_e$.

Since the countries are symmetric, we set $w_1 = w_2 = 1$. The profit-maximizing prices are

$$p^T(x) = \frac{1}{\rho x}. \quad (96)$$

The aggregate price index is

$$\begin{aligned} P^T &= \left(\mu \int_{\bar{x}_d^T}^{\infty} p^T(x)^{\frac{-\rho}{1-\rho}} dF(x) + \mu \int_{\bar{x}_e^T}^{\infty} p^T(x)^{\frac{-\rho}{1-\rho}} dF(x) \right)^{\frac{-(1-\rho)}{\rho}} \\ &= \left(\frac{\gamma(1-\rho) - \rho}{\rho^{\frac{\rho}{1-\rho}} (1-\rho) \gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} \right)^{\frac{1-\rho}{\rho}}. \end{aligned} \quad (97)$$

The demand in a country for a good produced by a firm with productivity $x \geq \bar{x}_d^T$ is

$$\begin{aligned} c^T(x) &= p^T(x)^{\frac{-1}{1-\rho}} (P^T)^{\frac{\rho}{1-\rho}} (\bar{\ell} + \pi^T) \\ &= \frac{\rho(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^T) x^{\frac{1}{1-\rho}}}{(1-\rho) \gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)}. \end{aligned} \quad (98)$$

Then

$$y^T(x) = \begin{cases} c^T(x) & \bar{x}_d^T \leq x < \bar{x}_e^T \\ 2c^T(x) & x \geq \bar{x}_e^T \end{cases}. \quad (99)$$

The cutoff for operating, \bar{x}_d^T , must satisfy

$$p^T(\bar{x}_d^T) c^T(\bar{x}_d^T) - \frac{c^T(\bar{x}_d^T)}{\bar{x}_d^T} - f_d = 0, \quad (100)$$

so

$$\frac{(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^T)(\bar{x}_d^T)^{\frac{\rho}{1-\rho}}}{\gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} - f_d = 0. \quad (101)$$

Similarly, the cutoff for exporting, \bar{x}_e^T , must satisfy

$$p^T(\bar{x}_e^T) c^T(\bar{x}_e^T) - \frac{c^T(\bar{x}_e^T)}{\bar{x}_e^T} - f_e = 0, \quad (102)$$

so

$$\frac{(\gamma(1-\rho)-\rho)(\bar{\ell}+\pi^T)(\bar{x}_e^T)^{\frac{\rho}{1-\rho}}}{\gamma\mu\left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}+(\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}\right)}-f_e=0. \quad (103)$$

Here

$$\begin{aligned} \pi^T &= \mu \int_{\bar{x}_d^T}^{\infty} \left(p^T(x) c^T(x) - \frac{c^T(x)}{x} - f_d \right) dF(x) \\ &+ \mu \int_{\bar{x}_e^T}^{\infty} \left(p^T(x) c^T(x) - \frac{c^T(x)}{x} - f_e \right) dF(x) \quad . \\ &= (1-\rho)(\bar{\ell}+\pi^T) - \mu \left((\bar{x}_d^T)^{-\gamma} f_d + (\bar{x}_e^T)^{-\gamma} f_e \right) \end{aligned} \quad (104)$$

Notice that (101), (103), and (104) give us a system of 3 equations in 3 unknowns to be solved for \bar{x}_d^T , \bar{x}_e^T , and π^T . The solution is

$$\bar{x}_d^T = \left(\frac{\mu(\gamma-\rho)f_d \left(1 + (f_e/f_d)^{\frac{\rho-\gamma(1-\rho)}{\rho}} \right)}{(\gamma(1-\rho)-\rho)\bar{\ell}} \right)^{1/\gamma} \quad (105)$$

$$\bar{x}_e^T = \left(\frac{f_e}{f_d} \right)^{\frac{1-\rho}{\rho}} \left(\frac{\mu(\gamma-\rho)f_d \left(1 + (f_e/f_d)^{\frac{\rho-\gamma(1-\rho)}{\rho}} \right)}{(\gamma(1-\rho)-\rho)\bar{\ell}} \right)^{1/\gamma} \quad (106)$$

$$\pi^T = \frac{\rho\bar{\ell}}{\gamma-\rho}. \quad (107)$$

GDP at current prices is

$$\begin{aligned} gdp^T &= \mu \int_{\bar{x}_d^T}^{\infty} p^T(x) y^T(x) dF(x) \\ &= \frac{\gamma}{\gamma-\rho} \bar{\ell} \end{aligned} \quad (108)$$

GDP at autarky prices is

$$\begin{aligned} GDP^T &= \mu \int_{\bar{x}_d^T}^{\infty} p^A(x) y^T(x) dF(x) \\ &= \frac{\gamma}{\gamma-\rho} \bar{\ell} \end{aligned} \quad (109)$$

Real income is

$$\begin{aligned}
 v^T &= \left(\mu \int_{\bar{x}_d^T}^{\infty} c^T(x)^\rho dF(x) + \mu \int_{\bar{x}_e^T}^{\infty} c^T(x)^\rho dF(x) \right)^{1/\rho} \\
 &= \frac{\gamma}{(\gamma - \rho) P^T} \bar{\ell}
 \end{aligned} \tag{110}$$

Effect of trade liberalization

Proposition 5. The cutoff for operating strictly increases following trade liberalization.

Proof. Compare (93) and (105). ■

Proposition 6. GDP at autarky prices does not change following trade liberalization.

Proof. Compare (94) and (109). ■

Proposition 7. Real income increases following trade liberalization.

Proof. Comparing (95) and (110), it suffices to show that $P^A > P^T$. Comparing (88) and (97), we see that this follows from Proposition 5. ■

The effect of reallocation across firms — the exit of the least productive firms and the movement of resources toward the most productive firms which start exporting — increases welfare, not real GDP.

6. How does trade liberalization affect growth rates?

Trade liberalization can change the incentives to accumulate capital, which in turn affects growth. In particular, capital scarce countries may concentrate in the production of labor intensive goods under free trade and, therefore, accumulate capital at a slower rate than in autarky. In this section we analyze this effect under the framework of a

dynamic Heckscher-Ohlin model with endogenous capital accumulation like the one studied in Bajona and Kehoe (2006).

A dynamic Heckscher-Ohlin model

Consider a world with n countries, where in each country i , $i = 1, 2, \dots, n$, there is measure L_i of infinitely-lived consumers. Each consumer in country i is endowed with one unit of labor and \bar{k}_{i0} units of capital. There are two tradable goods, $j = 1, 2$ which are produced using capital and labor. The technology to produce the two tradable goods is the same across countries.

A consumer in country i derives utility from the consumption of both traded goods in each period of his life, and chooses consumption and investment allocations $\{c_{ijt}, x_{ijt}, k_{it}\}$, $j = 1, 2$, $t = 0, 1, \dots$, to maximize lifetime utility.

$$\sum_{t=0}^{\infty} \beta^t (a_1 \log c_{i1t} + a_2 \log c_{i2t}), \quad (111)$$

where $a_1 + a_2 = 1$, subject to the budget constraints

$$p_{i1t}(c_{i1t} + x_{i1t}) + p_{i2t}(c_{i2t} + x_{i2t}) = w_{it} + r_{it}k_{it} \quad (112)$$

and the laws of motion of capital

$$k_{it+1} = (1 - \delta)k_{it} + x_{i1t}^a x_{i2t}^{1-a}, \quad (113)$$

for $t = 0, 1, 2, \dots$, given $k_{i0} = \bar{k}_{i0}$. Here p_{ijt} is the price of good j , w_{it} is the wage rate, and r_{it} is the rental rate of capital.

The production of the traded goods, as well as the feasibility conditions under autarky and free trade follow exactly the description of the static model in section 2.1 and we do not repeat it here.

The autarkic and free trade versions of this model differ in the conditions that determine feasibility in the traded goods' markets.

The model described above has an analytical solution under the assumption of complete depreciation, $\delta = 1$. In what follows we use the same notational conventions used in the static Heckscher-Ohlin model. Notice that in our specification of the model, the traded goods are combined in the same way in consumption and investment. This

assumption greatly simplifies the solution of the dynamic model. In particular, given k_{it} , the equilibrium prices and production patterns of the dynamic model for period t can be solved by solving a static Heckscher-Ohlin model with initial capital per person k_{it} in each country i . Values for consumption and investment in each period are solved by using the intertemporal consumer's problem. See Bajona and Kehoe (2006) for details.

Autarky

Let us normalize prices so that the price of a unit of investment is equal to one in each period. The autarky prices for the traded goods and the consumption, investment, and production allocations for country i , $i = 1 \dots n$, are

$$p_{ijt}^A = \frac{a_j D}{D_j} (k_{it}^A)^{A_1 - \alpha_j} \quad (114)$$

$$y_{ijt}^A = D_j (k_{it}^A)^{\alpha_j} \quad (115)$$

$$c_{ijt}^A = (1 - \beta A_1) D_j (k_{it}^A)^{\alpha_j} \quad (116)$$

$$x_{ijt}^A = \beta A_1 D_j (k_{it}^A)^{\alpha_j} \quad (117)$$

where

$$k_{it}^A = \beta A_1 D (k_{i,t-1}^A)^{A_1} = (\beta A_1 D)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_{i0}^{A_1^t}. \quad (118)$$

Our variables of interest are nominal and real GDP, productivity and welfare. GDP measured at current prices is equal to:

$$\begin{aligned} gdp_{it}^A &= p_{i1t}^A y_{i1t}^A + p_{i2t}^A y_{i2t}^A \\ &= D (k_{it}^A)^{A_1} \\ &= (\beta A_1 D)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_{i0}^{A_1^{t+1}}, \end{aligned} \quad (119)$$

where the last expression, derived using (118), includes only parameters and initial conditions. Notice that GDP at current prices is equal to $(y_{i1t}^A)^{a_1} (y_{i2t}^A)^{a_2}$. This is a direct result of our assumption that the traded goods are combined using the same technology in consumption and investment.

We measure real GDP by using the period-0 as the base year. Its value is

$$\begin{aligned}
GDP_{it}^A &= p_{i10}^A y_{i1t}^A + p_{i20}^A y_{i2t}^A \\
&= \left(a_1 \left(\frac{k_{it}^A}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \left(\frac{k_{it}^A}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{-A_1} \\
&= \left(a_1 \left((\beta A_1 D)^{\frac{1-A_1'}{1-A_1}} \bar{k}_{i0}^{A_1'-1} \right)^{\alpha_1} + a_2 \left((\beta A_1 D)^{\frac{1-A_1'}{1-A_1}} \bar{k}_{i0}^{A_1'-1} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{-A_1}
\end{aligned} \tag{120}$$

where the last equation expresses real GDP as a function of initial capital per worker and parameters.

In what follows we are interested in two measures of welfare: period welfare and lifetime welfare. We measure period welfare as a homogeneous transformation of the period utility:

$$\begin{aligned}
v_{it}^A &= (c_{i1t}^A)^{a_1} (c_{i2t}^A)^{a_2} \\
&= (1 - \beta A_1) D (k_{it}^A)^{A_1} \\
&= (1 - \beta A_1) (\beta A_1 D)^{\frac{A_1 - A_1'^{+1}}{1 - A_1}} D \bar{k}_{i0}^{-A_1'^{+1}}
\end{aligned} \tag{121}$$

Lifetime welfare is just the discounted sum of period welfares. Its analytical expression is:

$$\begin{aligned}
W_i^A &= \sum_{t=0}^{\infty} \beta^t \log v_{it}^A \\
&= \frac{\log[(1 - \beta A_1) D]}{1 - \beta} + \frac{\beta A_1 \log(\beta A_1 D)}{(1 - \beta)(1 - \beta A_1)} + \frac{A_1 \log \bar{k}_{i0}}{1 - \beta A_1}
\end{aligned} \tag{122}$$

Free trade

Following the same strategy as in the static version of the model, we assume that the initial factor endowments are such that factor prices are equalized in the first period. Bajona and Kehoe (2006) show that, in this case, factor price equalization occurs along the entire equilibrium path for the Cobb-Douglas model. This implies that the model can

be solved by calculating the equilibrium of the integrated economy — the economy with initial endowments equal to the world endowments — and then splitting production, consumption, and investment across countries in each period. If all countries are in the cone of diversification in all periods, it can be shown that for all $t = 0, 1, 2, \dots$

$$k_{it} = \gamma_i k_t, \quad (123)$$

where $\gamma_i = \bar{k}_{i0} / \bar{k}_0$ and $\bar{k}_0 = \sum_{i=1}^n L_i \bar{k}_{i0} / \sum_{i=1}^n L_i$.

The expressions for the relevant variables for our analysis, prices, consumption, production, and investment patterns, are (a complete solution is described in the appendix)

$$p_{jt}^T = \frac{a_j D}{D_j} (k_t^T)^{A_1 - \alpha_j} \quad (124)$$

$$c_{ijt}^T = ((1 - \beta) A \gamma_i + A_2) D_j (k_t^T)^{\alpha_j} \quad (125)$$

$$x_{ijt}^T = \gamma_i \beta A_1 D_j (k_t^T)^{\alpha_j} \quad (126)$$

$$y_{ijt}^T = \mu_{ij} D_j (k_t^T)^{\alpha_j}, \quad (127)$$

where k_t^T can be expressed as a function of the world's initial level of capital per person:

$$k_t^T = \beta A_1 D (k_{t-1}^T)^{A_1} = (\beta A_1 D)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t}. \quad (128)$$

Our variables of interest are nominal and real GDP, productivity and welfare. GDP at current prices is:

$$\begin{aligned} gdp_{it}^T &= p_{1t}^T y_{1it}^T + p_{2t}^T y_{2it}^T \\ &= (A_1 \gamma_i + A_2) D (k_t^T)^{A_1} \\ &= (A_1 \gamma_i + A_2) (\beta A_1 D)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_0^{A_1^{t+1}} \end{aligned} \quad (129)$$

We compute real GDP using two different base year prices. First, GDP at period-0 prices is

$$\begin{aligned}
GDP_{it}^T &= p_{10}^T y_{i1t}^T + p_{20}^T y_{i2t}^T \\
&= \left(a_1 \mu_{i1} \left(\frac{k_t^T}{k_0} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{k_t^T}{k_0} \right)^{\alpha_2} \right) D \bar{k}_0^{\bar{A}_i} \\
&= \left(a_1 \mu_{i1} \left((\beta A_1 D)^{\frac{1-A_i'}{1-A_1}} \bar{k}_0^{\bar{A}_i' - 1} \right)^{\alpha_1} + a_2 \mu_{i2} \left((\beta A_1 D)^{\frac{1-A_i'}{1-A_1}} \bar{k}_0^{\bar{A}_i' - 1} \right)^{\alpha_2} \right) D \bar{k}_0^{\bar{A}_i}.
\end{aligned} \tag{130}$$

Second, if the countries are initially in autarky, we may be interested in measuring real GDP as GDP at period-0 autarky prices. In this case, the expression becomes:

$$\begin{aligned}
GDP_{it}^T &= p_{10}^A y_{i1t}^T + p_{20}^A y_{i2t}^T \\
&= \left(a_1 \mu_{i1} \left(\frac{k_t^T}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{k_t^T}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{\bar{A}_i} \\
&= \left(a_1 \mu_{i1} \left(\frac{(\beta A_1 D)^{\frac{1-A_i'}{1-A_1}} \bar{k}_0^{\bar{A}_i'}}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{(\beta A_1 D)^{\frac{1-A_i'}{1-A_1}} \bar{k}_0^{\bar{A}_i'}}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{\bar{A}_i}.
\end{aligned} \tag{131}$$

We again consider two measures of welfare: period welfare and lifetime welfare. The value of period welfare is:

$$\begin{aligned}
v_i^T &= (c_{i1t}^T)^{\alpha_1} (c_{i2t}^T)^{\alpha_2} \\
&= ((1-\beta)A\gamma_i + A_2) D (k_t^T)^{A_i} \\
&= ((1-\beta)A\gamma_i + A_2) (\beta A_1 D)^{\frac{A_i - A_i^{t+1}}{1-A_1}} D \bar{k}_0^{\bar{A}_i^{t+1}}.
\end{aligned} \tag{132}$$

Effect of trade liberalization

We begin with the analysis of real income and discuss real GDP later. First we analyze rates of growth of real income under both autarky and free trade.

Proposition 8. Under autarky, if $\bar{k}_{i0} < \bar{k}_{j0}$, then the growth rate of real income is higher in country i than in country j in every period.

Proof. Under autarky, the growth rate of real income in country i is

$$\begin{aligned}\frac{v_{i,t+1}^A}{v_{it}^A} - 1 &= \left(\frac{k_{i,t+1}^A}{k_{it}^A} \right)^{A_i} - 1 \\ &= (\beta A_1 D)^{A_i} (k_{it}^A)^{A_i(A_i-1)} - 1. \\ &= (\beta A_1 D)^{A_i+1} \bar{k}_{i0}^{A_i+1(A_i-1)} - 1\end{aligned}\tag{133}$$

This is decreasing in \bar{k}_{i0} . ■

Proposition 9. Under free trade, real income grows at the same rate in every country.

Proof. With free trade, the growth rate of real income is

$$\frac{v_{i,t+1}^T}{v_{it}^T} - 1 = (\beta A_1 D)^{A_i+1} \bar{k}_0^{A_i+1(A_i-1)} - 1.\tag{134}$$

This is independent of i . ■

Notice that, under free trade, income in country i relative to income in the world is constant over time.

Proposition 10. If $\bar{k}_{i0} > \bar{k}_0$, then real income in country i grows at a faster rate under free trade than under autarky in every period. If $\bar{k}_{i0} < \bar{k}_0$, then real income in country i grows at a slower rate under free trade than under autarky in every period.

Proof. This follows directly from the previous two propositions. ■

Despite the fact that trade liberalization leads to slower growth of real income in some countries, trade liberalization increases welfare in every country.

Proposition 11. If $\gamma_i \neq 1$, welfare is strictly higher under free trade than under autarky.

Proof. Welfare in country i under autarky is

$$\begin{aligned}
W_i^A &= \sum_{t=0}^{\infty} \beta^t \log v_{it}^A \\
&= \sum_{t=0}^{\infty} \beta^t \log \left[(1-\beta A_1) (\beta A_1 D)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_{i0}^{A_1^{t+1}} \right] \\
&= \frac{\log[(1-\beta A_1) D]}{1-\beta} + \frac{\beta A_1 \log(\beta A_1 D)}{(1-\beta)(1-\beta A_1)} + \frac{A_1 \log \bar{k}_{i0}}{1-\beta A_1}
\end{aligned} \tag{135}$$

Welfare in country i under free trade is

$$\begin{aligned}
W_i^T &= \sum_{t=0}^{\infty} \beta^t \log v_{it}^T \\
&= \sum_{t=0}^{\infty} \beta^t \log \left[(A_1 \gamma_i + A_2 - \beta A_1) (\beta A_1 D)^{\frac{A_1 - A_1^{t+1}}{1-A_1}} D \bar{k}_0^{A_1^{t+1}} \right] \\
&= \frac{\log[(A_1 \gamma_i + A_2 - \beta A_1) D]}{1-\beta} + \frac{\beta A_1 \log(\beta A_1 D)}{(1-\beta)(1-\beta A_1)} + \frac{A_1 \log \bar{k}_0}{1-\beta A_1}
\end{aligned} \tag{136}$$

We want to show that $W_i^T > W_i^A$, or equivalently that

$$\frac{A_1(1-\beta)}{1-\beta A_1} \gamma_i + 1 - \frac{A_1(1-\beta)}{1-\beta A_1} > \gamma_i^{\frac{A_1(1-\beta)}{1-\beta A_1}}. \tag{137}$$

From here, the proof is the same as that for Proposition 1. ■

What happens to real GDP following trade liberalization? We can infer from the static model that, if a country is initially in autarky, then trade liberalization initially causes a decrease, or at least a decrease in the growth rate of, real GDP in that country.

Proposition 12. If $\gamma_i \neq 1$, GDP at period-0 autarky prices is strictly lower under free trade than under autarky in period 0.

Proof. This follows from Proposition 2. ■

At this point we would like to analyze the growth rates of GDP at period-0 prices under both autarky and free trade. The expressions for these growth rates are not analytically comparable, however. We instead provide an illustrative numerical example.

There are two countries, and country 1 is relatively capital-rich. We set $L_1 = L_2 = 1$, $\beta = 0.96$, $a_1 = a_2 = 0.5$, $\theta_1 = \theta_2 = 1$, $\alpha_1 = 0.6$, $\alpha_2 = 0.4$, $\bar{k}_{10} = 0.05$, and $\bar{k}_{20} = 0.03$. The results on growth rates of real GDP are similar to those on growth rates of real income. As Figure 2 shows, under autarky the capital-poor country grows much faster than the capital-rich country, just as we would expect from a standard growth model. This completely changes under free trade. Figure 3 shows that the capital-rich country grows faster than the capital-poor country. Figures 4 and 5 reiterate this finding from the perspective of each individual country.

7. Conclusion

To the extent that trade liberalization leads to higher productivity or higher rates of growth in real GDP, it does so through mechanisms that are, for the most part, outside of those analyzed in standard models. Determining the relation between trade liberalization and growth is not just a challenge for empirical research but also for theoretical research.

References

- Alcalá, F., and A. Ciccone (2004), "Trade and Productivity," *Quarterly Journal of Economics*, 119, 613-46.
- Atkeson, A., and P. J. Kehoe (2000), "Paths of Development for Early and Late Bloomers in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Staff Report 256.
- Bajona, C., and T. J. Kehoe (2006), "Demographics in Dynamic Heckscher-Ohlin Models: Overlapping Generations versus Infinitely Lived Consumers," Federal Reserve Bank of Minneapolis Staff Report.
- Bajona, C., and T. J. Kehoe (2006), "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Staff Report.
- Balassa, B. (1978), "Exports and Economic Growth: Further Evidence," *Journal of Development Economics*, 5, 181-189.
- Baldwin, L. E. (2003), "Openness and growth: what is the empirical relationship?" In L. E. Baldwin and L. A. Winters (eds) *Challenges to Globalization*. University of Chicago Press. Chicago. (NBER 9578)
- Chaney, T. (2006), "Distorted Gravity: Heterogeneous Firms, Market Structure, and the Geography of International Trade," University of Chicago.
- Cuñat A., and M. Maffezzoli (2004), "Neoclassical Growth and Commodity Trade," *Review of Economic Dynamics* 7, 3: 707-736.
- DeJong, D. N. and M. Ripoll (2006) "Tariffs and growth: comparing relationships among the rich and poor." *Review of Economics and Statistics* 88: 625-40.
- Diewert, W. E. and C. J. Morrison (1986) "Adjusting output and productivity indexes for changes in the terms of trade." *Economic Journal* 96: 659-79.
- Dollar, D. (1992) "Outward-oriented developing economies really do grow more rapidly: evidence from 95 LDCs, 1976-1985." *Economic Development and Cultural Change* 40: 523-544.
- Dollar, D. and A. Kraay (2004) "Trade, growth, and poverty." *The Economic Journal* 114: 22-49.

- Feenstra, R. C. (1994), "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157-177.
- Ferreira and Trejos (2006) "On the output effects of barriers to trade." *International Economic Review*, 47(4): 1319-1340.
- Frankel, J. A., and D. Romer (1999), "Does Trade Cause Growth?" *American Economic Review*, 89, 379-399.
- Gibson, M. J. (2007), "Trade Liberalization, Reallocation, and Productivity," University of Minnesota.
- Grossman, G. and E. Helpman (1991) *Innovation and growth in the global economy*. MIT Press, Cambridge, Massachusetts.
- Hall, R. and C. Jones (1999) "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics* 114: 83-116.
- Holmes, T. and J. Schmitz Jr. (1995) "Resistance to new technology and trade between areas." *FRB of Minneapolis Quarterly Review* Winter: 2-18.
- Kehoe, T. J., and K. J. Ruhl (2006), "Are Shocks to the Terms of Trade Shocks to Productivity?" Federal Reserve Bank of Minneapolis Staff Report.
- Kohli, U. (1983) "technology and the demand for imports," *Southern Economic Journal* 50: 137-150.
- Kohli, U. (2004) "Real GDP, real domestic income, and terms of trade changes." *Journal of International Economics* 62: 83-106.
- Levine, R. and D. Renelt (1992) "A sensitivity analysis of cross-country growth." *American Economic Review* 82: 942-963.
- Lewer, Joshua J. and Hendrik Van den Berg (2003). "How large is international trade's effect on economic growth?" *Journal of Economic Surveys* 17(3): 363-396
- Lewer, Joshua J. and Hendrik Van den Berg (2007). *International trade and economic growth*. M.E. Sharpe. Armonk. New York.
- Lucas, R. (1988) "On the mechanics of economic development."
- McGrattan, E. R., and E. C. Prescott (2007), "Openness, Technology Capital, and Development," Federal Reserve Bank of Minneapolis.
- Melitz, M. J. (2003), "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695-1725.

- Michaely M. (1977) "Exports and growth: an empirical investigation." *Journal of Development Economics* : 49-53.
- Rivera-Batiz, L. A. and P. Romer (1991) "Economic integration and endogenous growth." *Quarterly Journal of Economics* 106: 531-56.
- Rodriguez, F. and D. Rodrik (2001) "Trade policy and economic growth: a skeptics guide to the cross-national evidence." In Bernanke, B., and K. S. Rogoffs (eds) *NBER Macroeconomics Annual 2000*, MIT Press, Cambridge, Massachusetts.
- Rodrik D., A. Subramanian and F. Trebbi (2002) "Institutions rule: the primacy of institutions over geography and integration in economic development." *NBER Working Paper* 9305.
- Sachs, J., and A. Warner (1995), "Economic Reform and the Process of Global Integration," *Brookings Papers on Economic Activity*, 1, 1-118.
- Sala-i-Martin, X. (1997) "I just run two million regressions." *American Economic Review* 87: 178-183.
- Slaughter (2001)
- Ventura (1997) "Growth and interdependence." *Quarterly Journal of Economics* 112: 57-84.
- Wacziarg, R. (2001), "Measuring the dynamic gains from trade." *World Bank Economic Review* 15: 393-429.
- World Bank (1987), *The World Bank World Development Report*. The World Bank, New York.
- Yanikkaya, H. (2003), "Trade openness and economic growth: a cross-country empirical investigation." *Journal of Development Economics* 72: 57-89.
- Young, A. (1991), "Leaning by doing and the dynamic effects of international trade." *Quarterly Journal of Economics* 106: 369-405.

Figure 1

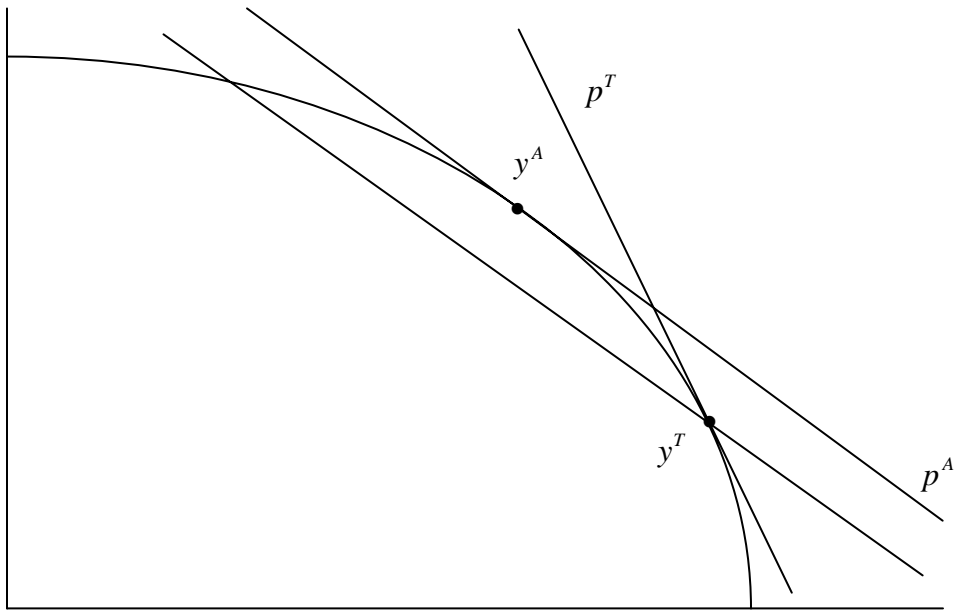


Figure 2

Autarky: GDP at period-0 prices

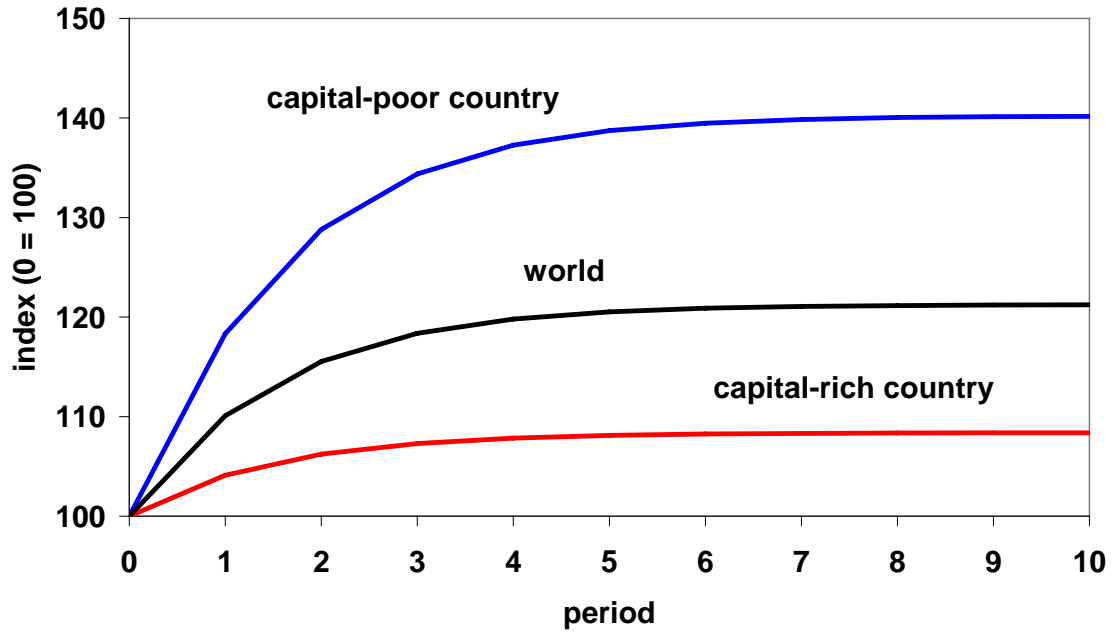


Figure 3

Free trade: GDP at period-0 prices

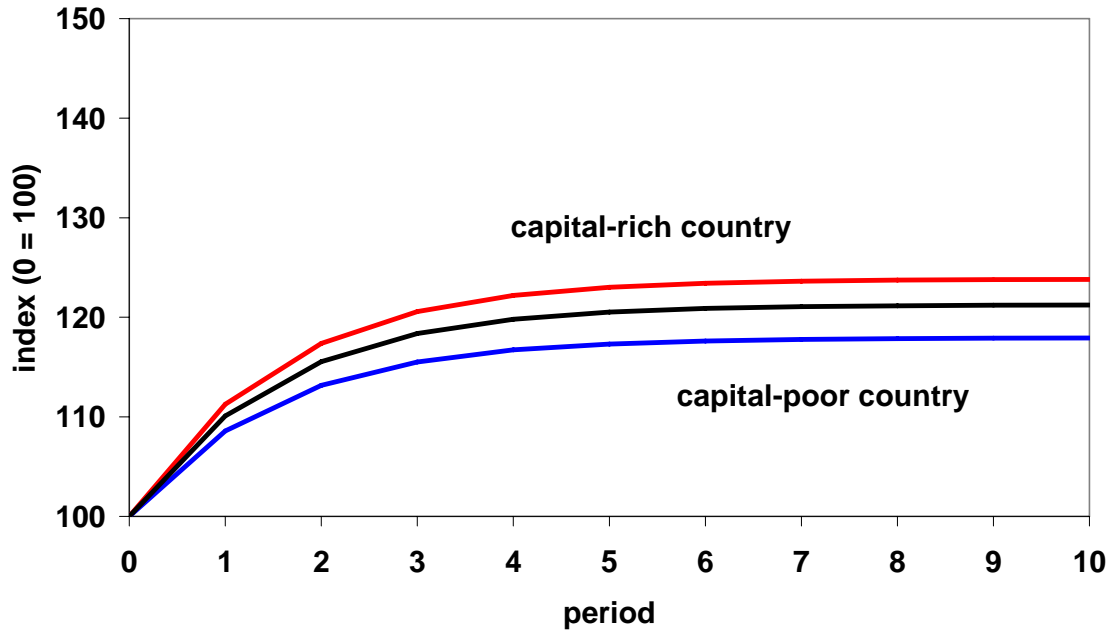


Figure 4

Capital-rich country: GDP at period-0 prices

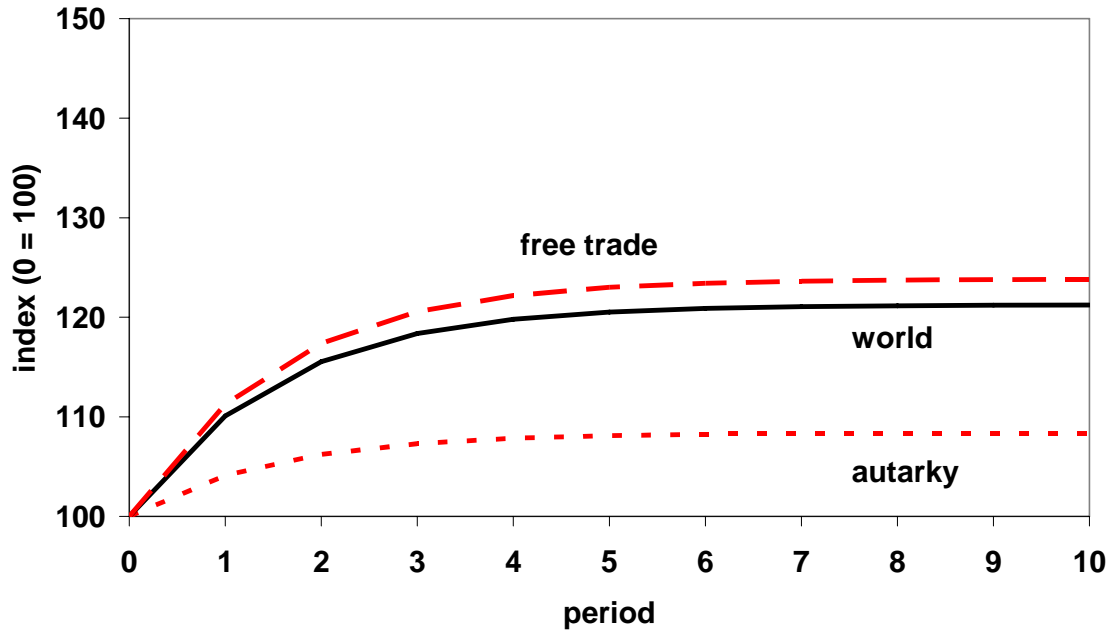


Figure 5

Capital-poor country: GDP at period-0 prices

