# Firm Risk and Leverage-Based Business Cycles*

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October 25, 2009

**Abstract**

I exploit evidence on cyclical fluctuations in the cross-sectional dispersion of firm-level productivity to quantify how much volatility in borrowers' leverage ratios can be explained by "second-moment shocks." In a standard financial accelerator model, second-moment shocks lead to fluctuations in leverage an order of magnitude larger than due to standard "first-moment" TFP shocks. This result represents substantial improvement over baseline analyses of accelerator models, though it is still five times lower than the volatility of borrowers' (firms') leverage ratios I document using Compustat data. In terms of macroeconomic aggregate quantities, pure dispersion shocks account for a small share of GDP fluctuations in the model, less than five percent. Depending on whether or not second-moment fluctuations are independent from or intertwined with shocks to the mean level of productivity, the model also performs well in explaining either (but not both) the observed acyclicality of borrowers' leverage or the observed countercyclicality of firm-level dispersion. Overall, the mechanism the model articulates is conceptually clear and seems quantitatively promising.

**Keywords:** leverage, second-moment shocks, time-varying volatility, credit frictions, financial accelerator, business cycles

**JEL Classification:** E10, E20, E32, E44

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# Contents

# 1 Introduction

Given recent and ongoing economic events, there is naturally much interest in understanding how financial factors vary over time and how they co-move over the business cycle with standard macroeconomic aggregates. One key to both empirical and theoretical understanding of such phenomena seems to lie in the cyclical behavior of leverage ratios, as recently stressed by Geanakoplos (2009), Adrian and Shin (2008), and others.

In this paper, I modify an existing class of general-equilibrium "financial accelerator" macroeconomic models in a way that leads to large fluctuations in firms' leverage ratios. Specifically, I demonstrate that "second-moment shocks" can fruitfully be employed in an existing class of DSGE models of financial frictions. Such shocks, through their effect on leverage ratios, lead to fluctuations in standard macroeconomic quantities, completely independently from "standard TFP" and other "first-moment shocks" common in the macro literature. However, I do not treat these second-moment shocks as a free parameter. The empirical discipline I bring to bear on the model is from a completely separate recent literature that has studied how time-variation in the *cross-sectional distribution of firm-level outcomes* — "second-moment fluctuations" — are related to and may in and of themselves cause business cycles.

There are three main results, two from the theoretical model and one from complementary empirical work. First, second-moment (dispersion) shocks drive virtually all of the business-cycle volatility of the model's financial-market aggregates, whereas first-moment shocks — I consider only standard TFP shocks — cause virtually no fluctuations in financial-market aggregates. The volatility in the aggregate leverage ratio is an order of magnitude *larger* than in baseline models in this class, although it is still about five times *smaller* than in the empirical evidence I document. Second, pure dispersion shocks, in which average TFP is held constant, lead to fluctuations of standard macro aggregates about two percent as large as those caused by standard TFP shocks alone. While thus only a small fraction of overall aggregate fluctuations, these "leverage-based business cycles" arise through the fluctuations in firms' leverage ratios that are induced by dispersion shocks. Hence, the transmission channel that the model emphasizes is explicitly a financial channel; if there were no financial frictions, these dispersion shocks would have no effects whatsoever in the aggregate. This latter aspect of the model is similar to the qualitative business-cycle model of Williamson (1987).

The third main contribution of the paper is empirical in nature. Using Compustat data, I construct cyclical measures of the aggregate leverage ratio in the U.S. non-financial business sector, which constitutes a large share of the demand side of credit markets. Because basic statistics on the cyclical properties of aggregate leverage — most notably its cyclical volatility — are largely lacking in the macro literature, constructing these statistics provides some metrics against which to assess the performance of the model, and seems to be of interest in its own right.[1] Using non-financial firms selected from Compustat, I find that the average leverage ratio over the period 1987-2009 was about 1.4, that its cyclical volatility was between 20 and 30 percent, and that it displays virtually zero contemporaneous correlation with the cyclical component of GDP — at best, leverage is mildly countercyclical.[2] These nascent stylized facts paint a picture of the evolution of borrowers' (firms') balance sheets over the business cycle and may provide guidance to other business-cycle modeling efforts in which financial frictions play a prominent role.

In the model, all firms have working capital requirements; each firm has private information about its own productivity; and lenders, due to firms' private information and the costs lenders incur to reorganize insolvent firms, charge an interest premium above the risk-free rate on their loans. Loan sizes depend on firms' net worth, thus leverage ratios are an equilibrium outcome. This basic setup descends from the Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999) line of models. To this framework, I introduce time-variation in the cross-sectional dispersion of firms' idiosyncratic productivity, in a way consistent with the direct empirical evidence of Bloom, Floetotto, and Jaimovich (2009), Bachmann and Bayer (2009), and my own analysis of firm-level data constructed by Cooper and Haltiwanger (2006). Time-variation in cross-firm dispersion faces lenders with time-varying risk of their overall loan portfolios, and hence leads them to extend more or less credit to borrowers — i.e., extend more or less leverage.

Two prominent studies in the recent "second-moment shocks" literature are Bloom, Floetotto, and Jaimovich (2009) and Bachmann and Bayer (2009) — henceforth, BFJ and BB, respectively. Regarding theory, the main question I take up in this paper is broadly similar to theirs: to what extent can changes over time in cross-sectional dispersion lead to aggregate fluctuations? However, the focus in this paper is on quantifying the role of financial factors per se in transmitting dispersion shocks to economic activity. In the model I present, the *only* way for second-moment shocks to transmit into fluctuations of GDP and other macro aggregates is through leverage — hence my terminology "leverage-based" business cycles. In contrast, the transmission channels in the models of BFJ and BB are non-financial; their models feature no financial frictions and instead

[1] Some empirical studies that speak to the same sorts of issues I examine in this paper are Levin, Natalucci, and Zakrajsek (2004), Covas and den Haan (2006), Korajczyk and Levy (2003), Hennessy and Whited (2007), and Levy and Hennessy (2007). With the exception of Covas and den Haan (2006), none of these papers presents business-cycle statistics on the aggregate leverage ratio, although in principle they each could given the data they study. In the online Appendix of their paper, Covas and den Haan (2006) present the cyclical correlation of firms' leverage with GDP, although not its cyclical volatility. As described further below, the results I find corroborate their finding regarding correlation with GDP.

[2] I define the leverage ratio as total end-of-quarter debt to total end-of-quarter equity for all non-financial firms in Compustat that report positive revenue and positive debt in a given quarter.

emphasize the role of firm-level factor adjustment costs in transmitting second-moment fluctuations into aggregate quantities.

While the details of BFJ's and BB's models and results of course differ along many dimensions, to me, one of the most important ideas that emerges is that "aggregate shocks" may entail *both* shocks to first moments — the mean of TFP — *and* shocks to second moments — the dispersion of TFP. This notion of a "bundled aggregate shock" is one that representative-agent economies cannot admit, but it seems a potentially promising new concept of an "aggregate shock," as the results of this paper, BFJ, and BB suggest. In Section 6, I treat the case of bundled aggregate shocks. But I start more simply in Sections 3, 4, and 5 with the case that second-moment shocks are completely isolated from first-moment shocks — that they are not bundled. Many of the insights and quantitative results from the baseline model carry over to the complete model with bundled shocks. However, the additional value of the bundled-shock model is that it is consistent with the countercyclicality of firm-level dispersion, which is documented by BFJ and BB and which I also document in Section 5.2 using the Cooper and Haltiwanger (2006) data

Regardless of whether or not aggregate shocks are bundled, it is clear that in order to consider fluctuations in cross-sectional dispersion, the model must have some notion of heterogeneity and cannot be a strict representative-agent economy. In the Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999) class of models on which I build, the heterogeneity is in borrowers' idiosyncratic ability to repay their loans. This feature is central to these models because with no cross-sectional heterogeneity of borrowers' ability to repay, there is no risk at all from the point of view of lenders, and hence no financial friction. In quantitative analysis of these models, parameters for the distribution are typically chosen based on evidence on long-run risk premia or the like, but then the distributional aspect of the model invariably fades into the background. I instead place this feature of the model in the foreground by emphasizing the time-variation in cross-sectional dispersion of firms' productivity, and hence their capacity to repay their debts, using firm-level evidence to guide the calibration.

The idea and mechanism proposed in this paper can thus also be thought of as a way of selecting a class of general-equilibrium financial-accelerator models for applied work. The two major foundations for financial accelerator effects in DSGE settings have been the agency-cost setup of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gerlter, and Gilchrist (1999); or the collateral-constraint setup of Kiyotaki and Moore (1997), which was first fruitfully used in a "medium-scale" DSGE model by Iacoviello (2005). Regarding leverage, the agency-cost setup makes the leverage ratio endogenous and, more to the point at hand, endogenously-time-varying in the presence of business cycles. On the other hand, the Kiyotaki and Moore (1997) setup in its standard form fixes the leverage ratio exogenously at a constant. However, even though the standard agency-cost model admits the possibility of time-varying leverage, standard calibrations of the model lead to miniscule fluctuations in leverage.[3] Comparing the predictions of a baseline model in this class to the empirical evidence I present in Section 2 shows that the baseline model's fluctuations in leverage are *two orders of magnitude smaller than in the data.* What the results of this paper show is that if one needs a model of meaningful cyclical fluctuations in leverage, the agency-cost formulation, when driven by empirically-relevant dispersion shocks, at least increases volatility by *one* order of magnitude.

This paper is also part of an emerging literature studying the joint business-cycle dynamics of real and financial variables. For example, Jermann and Quadrini (2009) also aim to jointly explain some salient facts regarding real and financial fluctuations. In their empirical work, Jermann and Quadrini (2009) document the cyclical properties of flows of firms' equity and debt issuance. However, they do not report the cyclical behavior of the debt-to-equity ratio, which is the focus of this paper.[4] The medium-scale monetary policy model of Christiano, Motto, and Rostagno (2009) also employs the dispersion shock highlighted in this paper — they label it a "risk shock" — but estimates the parameters of the process, rather than using direct firm-level evidence. In terms of main results, while I find that less than five percent of GDP fluctuations can be attributed directly to dispersion shocks, Christiano, Motto, and Rostagno (2009) find that nearly 20 percent of GDP fluctuations stem from dispersion shocks. Evidently, the difference is due to the host of nominal rigidities, real rigidities, and "news shock" events present in their economic model, from which I abstract in order to isolate the role of dispersion shocks.

Finally, a few words regarding terminology are in order. As is hopefully clear from the discussion so far, the idea of "second-moment fluctuations" in this paper is variations over time in the cross-sectional standard deviation of firm-level productivity, holding constant the mean of firm productivity. This is the same notion of "second-moment shocks" that BFJ and BB have in mind. However, it is distinct from another recent conceptualization of "second-moment shocks" emphasized by Justiniano and Primiceri (2008), Fernández-Villaverde and Rubio-Ramirez (2007), and others, in which the standard deviation of the shocks affecting standard macro driving processes such as aggregate TFP, monetary disturbances, etc., vary over time. Crucial in this latter group of studies is that they are all representative-agent economies, so there is no concept of cross-sectional dispersion and hence of course no possibility of changes in cross-sectional dispersion over

[3]For example, Carlstrom, Fuerst, and Paustian (2009, p. 8) recently point this out in their study of monetary policy. Based partly on the apparent inability of the agency-cost framework to predict empirically-meaningful fluctuations in firm leverage, Carlstrom, Fuerst, and Paustian (2009) instead construct a Kiyotaki and Moore (1997) style model with a fixed leverage ratio.

[4]Jermann and Quadrini (2009) use financial data from the Flow of Funds Accounts of the Federal Reserve Board, whereas I use Compustat data.

time. Focusing on the cross section is the main idea in BFJ, BB, and this paper. Finally, Gourio (2008), Christiano, Motto, and Rostagno (2009), and others emphasize the terms "firm-level risk" or "risk shocks" to describe the idea of time-variation in cross-sectional dispersion. I will use the terms "second-moment shocks," "firm-level risk," "firm-level shocks," "risk shocks," and "dispersion shocks" interchangeably: in this paper, these terms all refer to changes over time in the cross-sectional standard deviation of firm-level productivity.

The rest of this paper is organized as follows. Section 2 begins with some new empirical evidence on the cyclical behavior of an aggregate measure of the leverage ratio in the U.S. non-financial business sector. This evidence serves as the metric against which I judge the performance of the model. Section 3 lays out the baseline model, in which first-moment (TFP) shocks and second-moment (dispersion) shocks are completely independent from each other. Section 4 discusses the intuition for why leverage ratios may respond sharply to second-moment shocks, illustrating it with simple calculations from a partial equilibrium core of the full model. Section 5 presents quantitative results for the baseline model. Section 6 presents and studies the modified model with "bundled aggregate shocks." Section 7 concludes.

## 2 Cyclical Properties of Aggregate Leverage Ratios

In this section, I provide quarterly empirical evidence on leverage ratios of U.S. non-financial businesses over the past 20 years. Time-series evidence on leverage of the type I construct is not widely available in the macro literature. The closest available evidence is provided by Levin, Natalucci, and Zakrajsek (2004), who use quarterly Compustat data to construct a time series of non-financial sector leverage over the period 1988-2003; Korajczyk and Levy (2003), who use quarterly Compustat data over the period 1984-1993; and Covas and den Haan (2006), who use Compustat data, although at an annual frequency and with a focus on the behavior of debt and equity separately — that is, on the numerator and denominator of the leverage ratio separately.

With the exception of Covas and den Haan (2006), these studies do not report standard business cycle statistics, such as volatilities and cross-correlations with standard macro aggregates, using filtering procedures common in business-cycle analysis. In the online Appendix to their study, Covas and den Haan (2006) present cyclical correlations of a few measures of leverage with respect to GDP, but not the cyclical volatility of leverage. Relative to Covas and den Haan (2006) and Levin, Natalucci, and Zakrajsek (2004) — henceforth, LNZ — the evidence presented here extends the analysis through 2009 and also documents both business-cycle volatilities and correlations of leverage, providing some metrics against which the predictions of business-cycle models that feature endogenous leverage may be judged, including the model I study below. In more finance-oriented and firm-level applications, Hennesy and Whited (2007) and Levy and Whited (2007) also document some of the types of evidence on which I focus.[5]

Like LNZ and Korajczyk and Levy (2003), I use quarterly Compustat data on publicly-traded non-financial U.S. firms. I examine the period 1987:Q3 — 2009:Q1. My selection criteria are the following: in each quarter, I select only firms that report both strictly positive revenues and strictly positive debt in that quarter.[6] To achieve some comparability with LNZ, I use the book value of a firm's total long-term debt as the debt measure. The measure of equity is total shareholder equity, also available in Compustat. For each firm selected according to these criteria, I compute leverage as this debt-to-equity ratio. For each quarter, I then compute the "aggregate leverage ratio" in the non-financial sector as the revenue-weighted average over the firms selected in that quarter.[7]

Figures 1 and 2 plot the time series of aggregate leverage, its HP-filtered trend component, and its cyclical component. The mean leverage ratio over the sample is 1.41, and Figure 1 shows that there is not much of a trend, if any.[8,9] Three sharp spikes are evident: in the first half of 1995, in the fourth quarter of 2002, and in the third quarter of 2007. The latter spike corresponds to the start of the financial turmoil in August 2007 and associated declines in stock prices that touched off the current recession. The spike in 2002:Q4 is due to the deflation in stock prices throughout the course of 2002, following the bursting of the "tech bubble."

Based on the cyclical component of leverage plotted in Figure 2, the standard deviation of aggregate leverage is 30 percent in the non-financial business sector. While there are no previous estimates against which to compare this volatility, this volatility seems large. Interestingly, Figure 2 reveals that the spike in leverage in 2007:Q3 was not as large as the spikes in 1995:Q1-1995:Q2 and 2002:Q4.

Table 1 presents correlations over the business cycle of aggregate leverage with the standard macro aggregates GDP, consumption, and gross investment. Perhaps counter to conventional wis-

[5]An important distinction between Hennesy and Whited (2007) and Levy and Whited (2007) relative to the type of model-based lens through which LNZ and I view the data is that in the former, external financing can be either in terms of debt or equity, whereas in the latter external financing is only in the form of debt.

[6]The justification for this is that in the model developed below, all firms always have positive debt and positive revenues in every period; the model is silent about firms that have zero revenues and/or zero or negative debt.

[7]The number of firms selected according to this criteria grows through the sample: in 1987:Q3, the selection method picks out 1,596 firms, while in 2009:Q1 the selection method picks out 3,446 firms.

[8]This latter aspect of the leverage ratio I construct differs from LNZ, who show in their Figure 3 that the leverage ratio displays a downward trend during the period 1988-2000, which is not evident here. Some differences may be definitional ones (for example, they use the market value of common equity as their measure of equity, in contrast to my metric of total shareholder equity) and some may be sample selection issues (for example, while they also use sales weighting, it is unclear if they limit their sample to only those firms that report positive debt).

[9]I also note that the mean leverage ratio I compute is substantially larger than that computed by Levy and Whited (2007, Table 1), which may be at least partly, and perhaps almost entirely, attributable to the different sample selection methods employed.

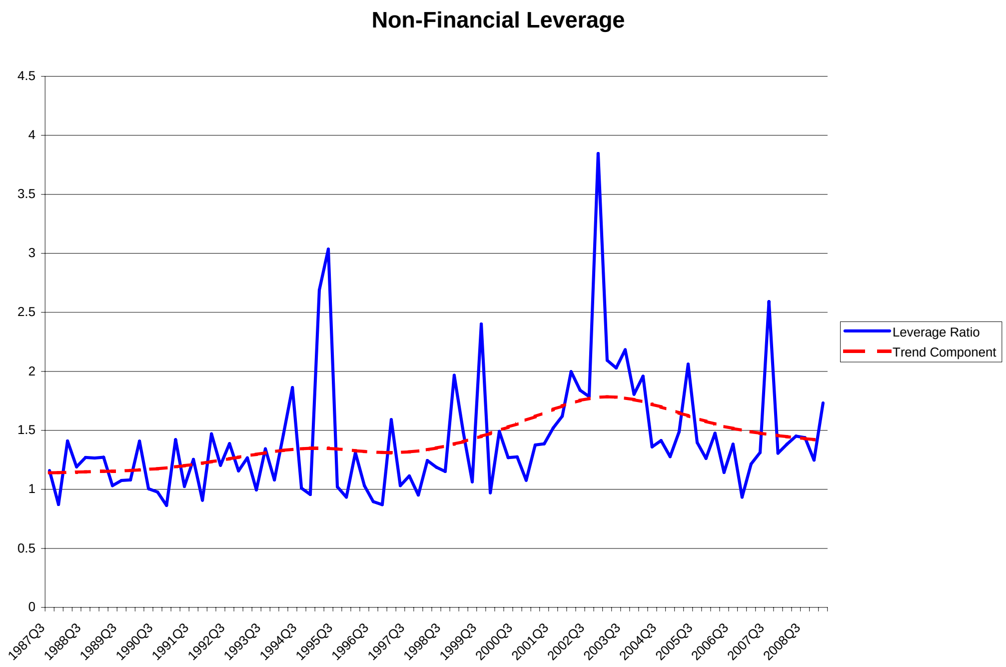


Figure 1: Leverage ratio in U.S. non-financial business sector, 1987Q3-2009Q1. Mean = 1.41. Trend component constructed with HP filter (smoothing parameter 1600).

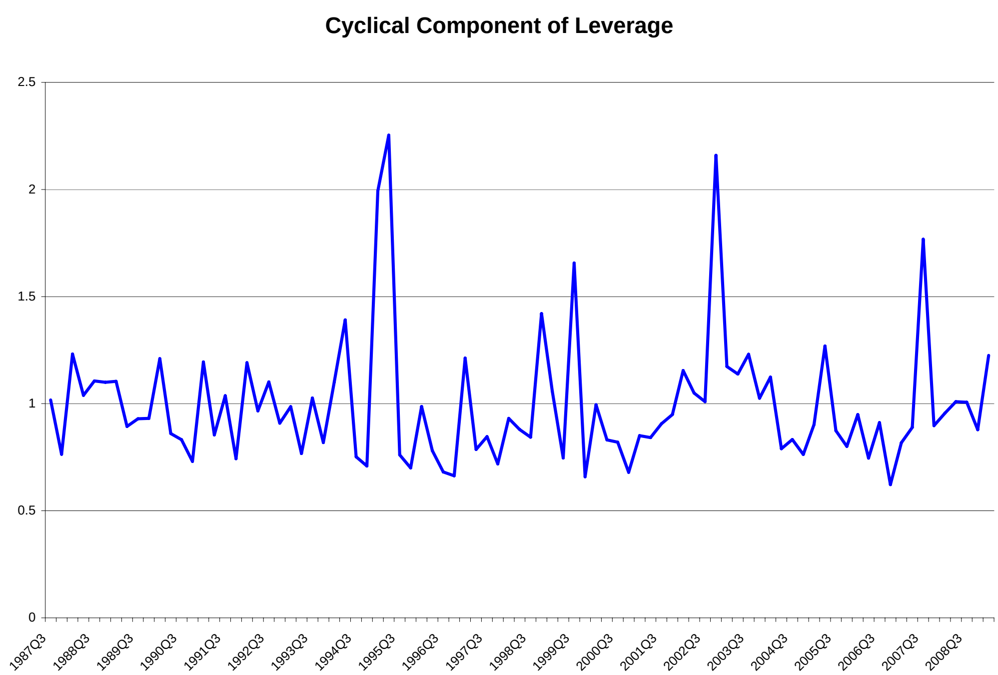


Figure 2: Cyclical component of leverage ratio in U.S. non-financial business sector, 1987Q3-2009Q1. Standard deviation = 30 percent.

| | | GDP | C | I | leverage |
|---|---|---|---|---|---|
| Std. dev. (%) | | 1.04 | 0.89 | 5.17 | 30.15 |
| Auto. corr. | | 0.91 | 0.91 | 0.88 | 0.10 |
| Corr. matrix | GDP | 1 | 0.8662 | 0.9018 | -0.1559 |
| | C | | 1 | 0.7136 | -0.0527 |
| | I | | | 1 | -0.1647 |
| | leverage | | | | 1 |

Table 1: Business cycle statistics for standard macro aggregates (GDP, consumption, and gross investment) and aggregate leverage ratio in U.S. non-financial business sector. Based on HP-filtered cyclical components over the period 1987:Q3-2009:Q1.

dom, the contemporaneous correlation of leverage in the non-financial business sector is slightly countercyclical. Non-financial firms do not seem to load up on leverage during expansions. In fact, quite the opposite; this finding is consistent with those in Levy and Hennessy (2007), who show that leverage ratios in highly-constrained firms are countercyclical, while leverage ratios in less-constrained firms are acyclical.

The magnitudes in Table 1 are small enough that one may even broadly read the aggregative evidence as showing virtual acyclicality of leverage in the non-financial business sector.[10] This finding is also consistent with the conclusion of Covas and den Haan (2006) that leverage is acyclical. A further interesting comparison is to Hennesy and Whited (2007, Table 1), who document essentially zero covariance between non-financial firms' leverage and physical investment over the period 1988-2001. The covariance between aggregate leverage and aggregate investment in the sample used here is -0.0026, virtually identical to the value of -0.0018 they find.

A reasonable alternative way of computing business cycle statistics here may be to remove the three sharp spikes evident in Figures 1 and 2 because there are no corresponding sharp spikes in the real GDP, consumption, and investment series. To the extent that the model constructed below is not meant to or able to capture events such as the bursting of the tech bubble or the onset of the current financial crisis, I present in Table 2 an alternative calculation of business cycle statistics with

[10]Note that the recent evidence of Adrian and Shin (2008), who document procyclicality of leverage amongst the five large U.S. investment banks leading up to the most acute phase of the financial crisis in September 2008, is for the supply side of the credit markets — lenders. The evidence I present is for the demand side of credit markets — (corporate) borrowers. Hence there is no inconsistency between these findings and Adrian and Shin (2008). In fact, my finding of acyclicality, or mild countercyclicality, of non-financial sector leverage is consistent with the one piece of evidence Adrian and Shin (2008) document for non-financial firms: their Figure 2.3 also displays mild countercyclicality of non-financial sector leverage (although note that their notions of cyclicality are with respect to market asset values, rather than with respect to GDP).

| | | GDP | C | I | leverage |
|---|---|---|---|---|---|
| Std. dev. (%) | | 1.04 | 0.89 | 5.17 | 19.96 |
| Auto. corr. | | 0.91 | 0.91 | 0.88 | -0.08 |
| | GDP | 1 | 0.8662 | 0.9018 | -0.0841 |
| Corr. matrix | C | | 1 | 0.7136 | -0.0835 |
| | I | | | 1 | -0.2305 |
| | leverage | | | | 1 |

Table 2: Business cycle statistics for standard macro aggregates (GDP, consumption, and gross investment) and aggregate leverage ratio in U.S. non-financial business sector with the four periods 1995:Q1, 1995:Q2, 2002:Q3, and 2007:Q3 set equal to the mean leverage ratio over the entire sample with these four quarters excluded. Based on HP-filtered cyclical components over the period 1987:Q3-2009:Q1.

the three spikes in leverage over the period 1987:Q3-2009:Q1 removed. To remove these spikes, I compute the average leverage ratio over the period 1987:Q3-2009:Q1 with the four periods 1995:Q1, 1995:Q2, 2002:Q4, and 2007:Q3 excluded. The mean leverage ratio is 1.33, compared to 1.41 for the full sample. I then simply set the leverage ratio during these four quarters to 1.33, and then HP filter the resulting time series. Table 2 shows that the volatility of leverage in this "de-spiked" series is 20 percent, compared to 30 percent for the full sample. Just as for the full sample, the correlation of leverage with macro aggregates is virtually zero to very mildly negative.

Another cut at the data is to allow negative debt observations, unlike the sample selection method up to now, which restricted observations to strictly positive debt levels. Negative debt corresponds to cash on hand for firms — although the model developed below does not allow for cash holdings, it is nonetheless useful to analyze the data allowing for negative debt. Redoing the analysis with this expanded sample, I find that the mean leverage ratio is 1.29, its cyclical volatility is 20.63 percent, and its contemporaneous correlation with GDP is -0.18. The main empirical features conveyed by the baseline sample are thus barely altered when allowing for negative debt.

This evidence amounts to a first step in constructing measures of aggregate leverage in a way familiar to standard business cycle analysis. The aggregates constructed here are broadly in line with more highly disaggregated studies such as Hennessy and Whited (2007) and Levy and Hennessy (2007). Future work will refine these aggregative measures and examine alternative measures.[11] For the purposes of the rest of this paper, I take the following as stylized facts that emerge from this

[11] In addition to, for example, parsing these results into long-term vs. short-term leverage, etc, yet another dimension of analysis would be examining leverage behavior amongst publicly-traded firms (which are what Compustat covers) vs. privately-traded firms. Davis, Haltiwanger, Jarmin, and Miranda (2007) show that firm-level outcomes may be very different for public vs. private firms.

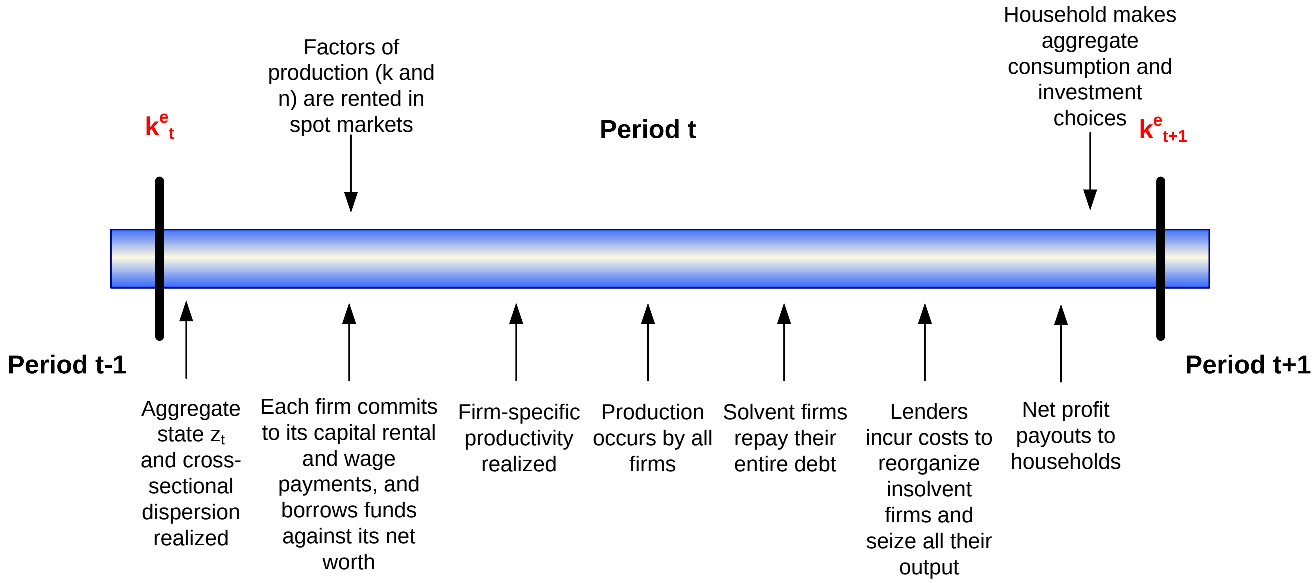


Figure 3: Timing of events in model.

evidence: the cyclical volatility of leverage in the non-financial business sector is large, over an order of magnitude larger than standard macro aggregates; and there is essentially zero correlation, or at most a mildly negative correlation, over the business cycle of leverage with standard macroeconomic quantity aggregates.

## 3 Baseline Model

As described in the introduction, the model is based on the agency-cost formulation of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999). The model I construct is most directly based on the "output model" of Carlstrom and Fuerst (1998), in which all prices are flexible, firms require short-term working capital (formally, intraperiod) to finance their production costs, and there are no other rigidities or frictions whatsoever. This provides me the cleanest starting point to highlight the role of second-moment shocks, so from here on I speak as if I am building "just" on the Carlstrom and Fuerst (1998) — henceforth, CF — analysis, recognizing that it is meant to capture an entire literature of work.

I turn now to describing the economic environment and the equilibrium. As an aid to the ensuing description, Figure 3 illustrates the timing of events in the model.

### 3.1 Households

There is a representative household in the economy that maximizes expected lifetime discounted utility over streams of consumption and leisure,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + v(1 - n_t) \right], \tag{1}$$

subject to the sequence of flow budget constraints

$$c_t + k_{ht+1} = w_t n_t + k_{ht} \left[ 1 + r_t - \delta \right] + \Pi_t. \tag{2}$$

The functions $u(.)$ and $v(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and leisure, respectively. The rest of the notation is as follows. The household's subjective discount factor is $\beta \in (0, 1)$, $c_t$ denotes the household's consumption, $k_{ht}$ denotes the household's capital holdings at the start of period $t$, $w_t$ is the real wage, $\tau_t^n$ is a proportional labor income tax, $r_t$ is the market rental rate on capital, $\tau_t^k$ is a proportional capital income tax, and $\delta$ is the depreciation rate of capital. The household also pays a lump-sum tax $T_t$ to the government and receives dividends $\Pi_t$ from firms as lump-sum income.[12]

Emerging from household optimization is a completely standard labor supply condition

$$\frac{v'(1 - n_t)}{u'(c_t)} = w_t, \tag{3}$$

and a completely standard capital supply condition

$$u'(c_t) = \beta E_t \left\{ u'(c_{t+1}) \left[ 1 + r_{t+1} - \delta \right] \right\}, \tag{4}$$

which follows as usual from the household's period-$t$ first-order conditions with respect to $c_t$ and $k_{ht+1}$. The stochastic discount factor is defined as $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

### 3.2 Firms/Entrepreneurs

There is a continuum of unit mass of firms that produce output by operating a constant-returns technology. Following CF, each firm is assumed to be owned by an "entrepreneur." From here on, I thus refer interchangeably to "a firm" and "an entrepreneur." I emphasize one terminology or another depending on the particular aspect of firm/entrepreneur behavior being analyzed.

[12] I could also introduce shares in order to directly price streams of dividends paid by firms to households; but this extra detail is unnecessary for our main points, so it is omitted.

Firm $i$ produces output using the technology $\omega_{it} z_t F(k_{it}, n_{it})$: $k_{it}$ is the firm's purchase of physical capital on spot markets, $n_{it}$ is the firm's hiring of labor on spot markets, $z_t$ is an aggregate TFP realization that is common across all firms, and $\omega_{it}$ is a firm-specific productivity realization.[13]

Each period, firm $i$'s idiosyncratic technology is a draw from a distribution with cumulative distribution function $\Phi(\omega, z_t)$, with constant mean $\omega_m \equiv 1$, a time-varying standard deviation $\sigma_t^\omega$, and associated density function $\phi(\omega, z_t)$, all of which are identical across firms. The time-varying volatility $\sigma_t^\omega$ is the key — and indeed, only — innovation in the model compared to CF. Note in particular that idiosyncratic productivity for a given firm is completely i.i.d. While obviously an abstraction and at odds with the evidence of Cooper and Haltiwanger (2006) and others, this assumption — in particular, the lack of persistence of the idiosyncratic component of a firm's realized technology — greatly simplifies the computation of the model, yet still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to large fluctuations in aggregate leverage and, in turn, fluctuations in economic activity.[14]

Aggregate TFP is included as the second argument of the CDF and PDF because in the "bundled aggregate shock" version of the model presented in Section 6, variations in $\sigma_t^\omega$ will be induced by variations in TFP, which is why I introduce this general notation. In the model in the current section, however, there is no such bundling, with variations in $\sigma_t^\omega$ completely independent of variations in $z_t$. In any case, from here on, I suppress the $z_t$ argument from $\Phi(.)$ and $\phi(.)$, and it will be understood to be operative in Section 6.

#### 3.2.1 Firm Financing and Contractual Arrangement

In period $t$, total costs of firm $i$, which are the sum of capital rental costs and wage payments, are thus

$$M_{it} = w_t n_{it} + r_t k_{it}. \tag{5}$$

As in CF and as shown in Figure 3, I assume the firm commits to and pays for all of its input costs after observing the aggregate technology realization $z_t$, but before observing the idiosyncratic

[13] I note that $\omega_{it}$ could be also interpreted as a Cooper and Haltiwanger (2006) "profitability shock," which may include idiosyncratic supply and/or demand innovations, rather than strictly a "productivity shock." This interpretation is especially appealing because, as I describe below, I rely on the Cooper and Haltiwanger (2006) evidence in one of my calibration strategies. However, in the rest of the text, I continue to refer to $\omega_{it}$ as a "productivity shock."

[14] In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the CF and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the intraperiod loan contracts of these models could not be improved upon by the contracting parties by, say, multi-period contracts. Sidestepping this issue is yet another reason to assume no persistence in realized idiosyncratic productivity. Note, however, that assuming persistence in shocks to $\sigma_t^\omega$, as I do, does not pose any of these problems; indeed, shocks to $\sigma_t^\omega$ really are aggregate shocks.

component $\omega_{it}$ and thus before any output and revenue is realized.

Part of the financing of the firm's costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each entrepreneur (firm) accumulates is rented on spot markets to (other) firms, just like households rent out their capital on spot markets. Entrepreneur $i$'s capital holdings at the start of period $t$ is $k^e_{it}$. Thus, note that $k^e_{it}$, which reflects the entrepreneur's/firm's capital *supply* decisions, is distinct from $k_{it}$, which reflects the firm's/entrepreneur's capital *demand* decisions for production purposes.

However, internal funds (net worth) are insufficient to cover all input costs. To finance the remainder, a firm borrows short-term — formally, intraperiod — working capital. A firm requires external financing because, in its role as an *accumulator* of assets (that is, in its savings role as an entrepreneur, which is described below), it is assumed to be more impatient than households.[15] By acquiring external funds, the firm is able to leverage its net worth in period $t$,

$$nw_{it} = k^e_{it}\left[1 + (1-\tau^k_t)(r_t - \delta)\right] + e_t, \tag{6}$$

into the project size $M_{it}$. Total borrowing by the firm is thus $M_{it} - nw_{it}$. The component $e_t$ of net worth is a small amount of "endowment income" that each entrepreneur receives to ensure its continued operations in the event that it goes bankrupt in the previous period. In closing the model, this endowment is absorbed into the payout $\Pi_{it}$ the firm pays to its owners, which is the representative household. The payout $\Pi_{it}$ is thus interpreted as net of the endowment $e_t$.[16]

I describe only briefly the outcome of the contracting arrangement between borrowers (firms) and lenders (households) because it is well-known in this class of models.[17] The financial contract is a debt contract, which is fully characterized by a liquidation threshold $\bar{\omega}_t$ and a loan size $M_{it} - nw_{it}$. A firm must be liquidated or "reorganized" if its realized productivity $\omega_{it}$ falls below the contractually-specified threshold $\bar{\omega}_t$. Below this threshold, the firm does not have enough resources to fully repay its loan. In that case, the firm is declared insolvent and receives nothing, while the lender must pay liquidation costs that are proportional to the total output of the firm and receives,

[15] This is a standard assumption in this class of models and avoids the self-financing outcome. See, for example, Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999).

[16] Thus, equivalently, $e_t$ can be interpreted as a lump-sum transfer of "startup funds" provided by households to entrepreneurs, as in Gertler and Karadi (2009). By allowing a "firm's" operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the "costs of bankruptcy" in our model are more properly interpreted as "costs of transferring ownership" from an insolvent owner to a solvent one. An equivalent interpretation is that firm $i$ exits the market upon bankruptcy and a new firm enters the market at zero cost, and is endowed with $e_t$.

[17] In the context of general-equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and Faia and Monacelli (2007). In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).

net of these liquidation costs, all of the output of the firm. Note that all firms, regardless of whether or not they end up requiring reorganization, do produce output up their full (idiosyncratic) capacity.

Define by $f(\bar{\omega}_t)$ the expected share of idiosyncratic output $\omega_{it} z_t F(k_{it}, n_{it})$ the borrower (the firm) keeps after repaying the loan, and by $g(\bar{\omega}_t)$ the expected share received by the lender.[18] These expectations are conditional on the realization of the time-$t$ aggregate state, but before revelation of a firm's idiosyncratic productivity $\omega_{it}$. The contractually-specified loan size is characterized by a zero-profit condition on the part of lenders,

$$M_{it} = \frac{nw_{it}}{1 - p_t g(\bar{\omega}_t)}, \tag{7}$$

and the contractually-specified liquidation threshold is characterized by

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \tag{8}$$

in which $p_t > 1$ is a component of input costs that arises solely from the financing needs of the firm.[19] Thus, for each unit of capital the firm rents, the cost, inclusive of financing costs, is $p_t r_t$, rather than just $r_t$. The same is true for each unit of labor that must be paid. Finally, all contractual outcomes are contingent on the aggregate state $(z_t, \sigma_t^\omega)$ of the economy.

### 3.2.2 Firm Profit Maximization

Can now analyze the firm's profit maximization problem. Taking as given the contractual outcome, firm $i$ maximizes in each period $t$ its expected profits

$$z_t F(k_{it}, n_{it}) - p_t \left[ w_t n_{it} + r_t k_{it} \right]. \tag{9}$$

This expectation is taken before the realization of firm-specific idiosyncratic productivity but after the realization of the aggregate state of the economy. Because the mean of $\omega_{it}$ is unity, ex-ante revenue of the firm is $z_t F(k_{it}, n_{it})$. The idiosyncratic risk $\omega_{it}$ and associated financing costs implied by it are captured by the inclusion of the financing component of input costs $p_t$ in the above expression.[20]

[18]Formally, $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_i - \bar{\omega}_t)\, \phi(\omega_i; .) d\omega_i = \int_{\bar{\omega}_t}^{\infty} \omega_i \phi(\omega_i; .) d\omega_i - [1 - \Phi(\bar{\omega}_t; .)]\bar{\omega}_t$ is the share received by the firm, and $g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} (\omega_i - \mu)\, \phi(\omega_i; .) d\omega_i + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t \phi(\omega_i; .) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i; .) d\omega_i + [1 - \Phi(\bar{\omega}_t; .)]\bar{\omega}_t - \mu\Phi(\omega_i; .)$.

[19]The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into the lending market is costless. Formally, the two conditions characterizing the optimal contract result from maximizing (the firm's share of) the return on the financial contract (because the firm, if it remains solvent, is the residual claimant on output), $p_t f(\bar{\omega}_t) M_{it}$, subject to the zero profit condition of the lender, $p_t g(\bar{\omega}_t) M_{it} = M_{it} - nw_{it}$.

[20]As is common in macro models, writing, for example, $p_t$, is shorthand for the state-contingent equilibrium function $p(z_t, \sigma_t^\omega)$. If the distribution of $\omega$ were degenerate — that is, if there were no idiosyncratic component of technology — then we would have $p_t = 1\ \forall t$, which simply has the interpretation that financing issues are irrelevant as in, say, a baseline RBC model or baseline DSGE search model.

Profit maximization with respect to capital rental and labor demand gives rise to the capital demand condition

$$r_t = \frac{z_t F_k(k_t, n_t)}{p_t} \tag{10}$$

and the labor demand condition

$$w_t = \frac{z_t F_n(k_t, n_t)}{p_t}. \tag{11}$$

Having taken first-order conditions, firm $i$ indexes are now dropped because I analyze an equilibrium symmetric across all firms. In both (10) and (11), the financing cost $p_t$ drives a wedge between marginal product and the associated factor price. That the financing cost drives an endogenous time-varying wedge between prices and marginal returns in neoclassical factor markets is a key feature of the equilibrium of the CF model.

### 3.2.3 Entrepreneurial Capital Accumulation

Now that the characterization of firm profit maximization is complete, all that remains is to characterize optimal behavior by the entrepreneurial side of firms. For the purpose of net worth accumulation, the entrepreneur is assumed to be more impatient than households. Specifically, entrepreneurs have a subjective discount factor for capital accumulation of $\gamma < 1$ over and above any other discount factor they assign to their objective function. The entrepreneur chooses capital accumulation $k^e_{it+1}$ subject to the flow budget constraint

$$\Pi^e_{it} + k^e_{it+1} = p_t f(\bar{\omega}_t) M_{it}, \tag{12}$$

where $\Pi^e_{it}$ is part of the transfer, or payout, of non-retained earnings to the household. Substituting the contractually-specified quantity of borrowing, $M = \frac{nw}{1-pg(\bar{\omega})}$, which is taken as given at the stage of entrepreneurial capital accumulation, this can be re-written as

$$\Pi^e_{it} + k^e_{it+1} = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} nw_{it}. \tag{13}$$

Further substituting the definition of net worth, the budget constraint of the (representative) entrepreneur is

$$\Pi^e_{it} + k^e_{it+1} = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left[k^e_{it} \left[1 + (1 - \tau^k_t)(r_t - \delta)\right] + e_t\right].^{21} \tag{14}$$

Capital accumulation $k^e_{it+1}$ is chosen to maximize the profit function (9); while the "firm's" objective (9) is static, the "entrepreneur's" objective is dynamic because the firm/entrpreneur is

[21] As in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and the ensuing literature, I can analyze a "representative" entrepreneur due to entrepreneurs' risk-neutrality and the fact that their production is linear in their idiosyncratic productivity.

assumed to rebate to households any total profits it earns, and part of these total profits stem from net worth accumulation and decumulation. As stated above, entrepreneurs place an an additional discount factor $\gamma < 1$ above that which the household discounts future flows. Thus, for the purpose of its own capital accumulation $k^e$, the entrepreneur applies the stochastic discount factor $\gamma\Xi_{t|0} < \Xi_{t|0}$. The optimal capital Euler equation for entrepreneurs is thus

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} \left[1 + r_{t+1} - \delta\right] \right\}. \tag{15}$$

Note from above that, in equilibrium, $r_{t+1} = \frac{z_{t+1} F_k(k_{t+1}, n_{t+1})}{p_{t+1}}$. Finally, the end-of-period lump-sum transfer by firms to households is $\Pi_t = \Pi_t^e + \Pi_t^f$, with $\Pi_t^f \equiv z_t F(k_t, n_t) - w_t n_t - r_t k_t$ the usual notion of "profits of the firm."

### 3.3 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes $\{c_t, n_t, k_{ht+1}, k_{t+1}^e, k_{t+1}, \Pi_t^e, w_t, r_t, p_t, \bar{\omega}_t\}$ that satisfy the following conditions: the labor-supply condition

$$\frac{v'(1 - n_t)}{u'(c_t)} = w_t; \tag{16}$$

the labor-demand condition

$$w_t = \frac{z_t F_n(k_t, n_t)}{p_t}; \tag{17}$$

the capital-demand condition

$$r_t = \frac{z_t F_k(k_t, n_t)}{p_t}; \tag{18}$$

the representative household's Euler equation for capital holdings

$$1 = E_t \left\{ \Xi_{t+1|t} \left[1 + r_{t+1} - \delta\right] \right\}; \tag{19}$$

the firm/entrepreneur's Euler equation for capital holdings

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} \left[1 + r_{t+1} - \delta\right] \right\}; \tag{20}$$

aggregate capital market clearing

$$k_t = k_{ht} + k_t^e; \tag{21}$$

the aggregate resource constraint

$$c_t + k_{t+1} - (1 - \delta) k_t = z_t F(k_t, n_t) \left[1 - \mu \Phi(\bar{\omega}_t)\right]; \tag{22}$$

the contractually-specified loan size

$$M_t = \frac{n w_t}{1 - p_t g(\bar{\omega}_t)}, \tag{23}$$

in which expression (6) for $nw_t$ is substituted in; the contractually-specified liquidation threshold

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}; \tag{24}$$

and the budget constraint of the representative entrepreneur

$$\Pi^e_{it} + k^e_{it+1} = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left[ k^e_{it} \left[ 1 + r_t - \delta \right] + e_t \right]. \tag{25}$$

The private sector takes as given the stochastic process $\{z_t, \sigma^\omega_t\}_{t=0}^\infty$.

# 4 Basic Analytics: Firm Productivity Dispersion and Leverage Ratio

Before proceeding to the quantitative analysis of the model, it is useful to consider analytically the intuition behind the model's main mechanism. These analytics do not formally prove the main results, which are quantitative in nature. But they shed light on the transmission mechanism, which is subsequently quantified in Section 5.

To begin this intuitive consideration, note that conditions (23) and (24), which characterize the terms of the financial contract, can be combined to

$$M - nw = -\left( \frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^\omega) g(\bar{\omega}; \sigma^\omega)}{f(\bar{\omega}; \sigma^\omega) g_{\bar{\omega}}(\bar{\omega}; \sigma^\omega)} \right) nw. \tag{26}$$

I drop time indices for ease of notation. The term in parentheses is the leverage ratio because it specifies the total debt obligation of a firm, $M - nw$, as a function of its net worth. Thus, define the leverage ratio as

$$\ell(\bar{\omega}; \sigma^\omega) \equiv -\frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^\omega) g(\bar{\omega}; \sigma^\omega)}{f(\bar{\omega}; \sigma^\omega) g_{\bar{\omega}}(\bar{\omega}; \sigma^\omega)}. \tag{27}$$

The expected share functions $f(.)$ and $g(.)$ and their derivatives depend on the cross-sectional dispersion $\sigma^\omega$ of firm-level productivity, hence the leverage ratio also depends on $\sigma^\omega$. For the purpose at hand, I wish to emphasize this dependence, hence I explicitly highlight it as an argument of these functions.

Figure 4 illustrates why changes in the cross-sectional dispersion of firms' TFP would be expected to cause changes in leverage. Suppose the solid black curve in Figure 4 is the pdf $\phi(\omega, .)$ before a shock occurs. The liquidation threshold $\bar{\omega}$ shown is for this initial distribution. Suppose there is an exogenous reduction in dispersion. If the liquidation threshold $\bar{\omega}$ were to remain unchanged, fewer firms would draw an idiosyncratic $\omega < \bar{\omega}$, which lenders understand because the density $\phi(\omega, .)$ is common knowledge. This in turn means that fewer firms are expected to be able to fully repay their loans, which reduces lenders' risk. Ex-ante, then, lenders would be willing to

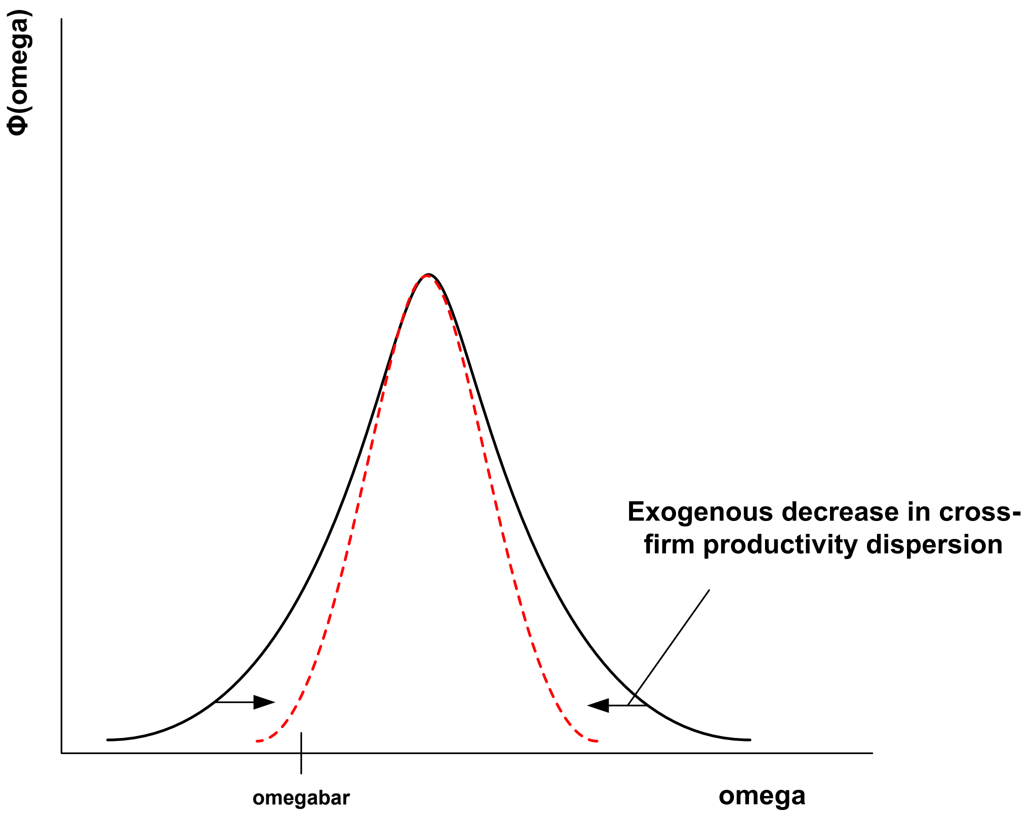


Figure 4: An exogenous decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.

extend more credit, which implies higher leverage ratios for firms (borrowers). In general equilibrium, $\bar{\omega}$ will of course also change. It is thus a quantitative question how much a given-size change in the dispersion $\sigma^{\omega}$ will affect the threshold $\bar{\omega}$ and hence leverage. This question can only be answered in the full general equilibrium model.

However, simple calculations can provide a gauge on the responsiveness of leverage to cross-sectional risk in the full general-equilibrium analysis. For a fixed value of $\bar{\omega}$, Figure 5 plots leverage as a function of cross-sectional risk, and Figure 6 plots the elasticity of the leverage ratio with respect to $\sigma^{\omega}$.[22] As intuition suggests, leverage declines with risk, and Figure 5 gives a sense of levels; in the calibration described below, $\sigma^{\omega} = 0.30$ in the long run, which is why Figures 5 and 6 are centered where they are.

The elasticity of leverage, shown in Figure 6, increases (in magnitude) in cross-sectional risk.

[22]For this illustration, I take $\bar{\omega} = 0.5$, which is roughly the value that emerges endogenously in the full quantitative analysis below.

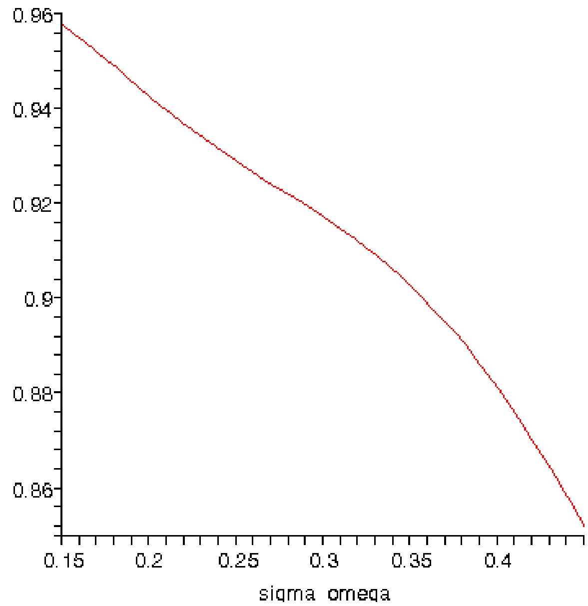


Figure 5: The leverage ratio $\ell$ as a function of the cross-sectional dispersion $\sigma^{\omega}$ of firm-level productivity, for a fixed value of $\bar{\omega}$ ($\bar{\omega} = 0.5$ is taken for illustrative purposes, which is the value that emerges endogenously in the quantitative analysis below).

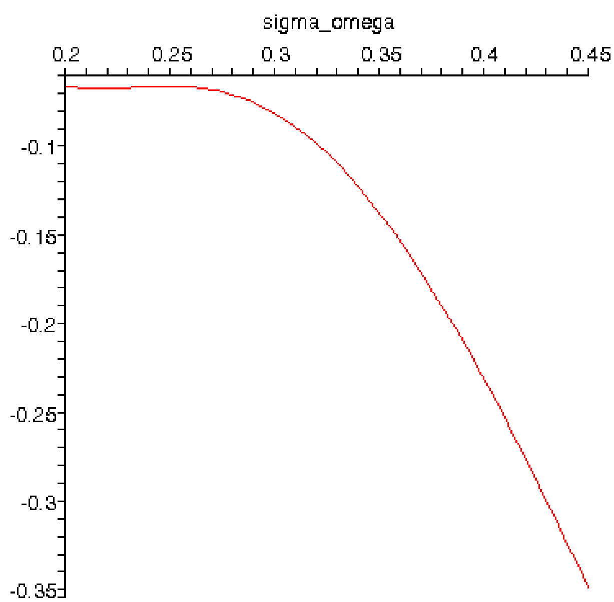


Figure 6: Elasticity of the leverage ratio $\ell$ with respect to the cross-sectional dispersion $\sigma^{\omega}$ of firm-level productivity, for a fixed value of $\bar{\omega}$ ($\bar{\omega} = 0.5$ is taken for illustrative purposes, which is the value that emerges endogenously in the quantitative analysis below).

For the calibration $\sigma^{\omega} = 0.30$ used in the quantitative experiments, Figure 6 shows that a one percent rise in dispersion (from 0.30 to 0.303) leads to a decline in leverage of about one-tenth of one percent. This is for a given value of the liquidation threshold $\bar{\omega}$. Because this threshold is endogenous, the general- equilibrium "effective elasticity" could be larger or smaller than this calculation. The quantitative experiments show that the effective (equilibrium) elasticity of $\ell$ with respect to $\sigma^{\omega}$ is much larger.

# 5 Quantitative Analysis

## 5.1 Computational Strategy

To study the dynamics of the model, I compute a second-order approximation to the equilibrium, using my own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004). Because the main interest is in business cycle fluctuations driven by "small shocks," such methods are likely to portray an accurate picture of the model's dynamic behavior, as the studies by Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) and Caldera, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2009) suggest.

Changes in cross-sectional risk are indeed aggregate, rather than idiosyncratic, shocks in the model economy. Because I thus track only aggregate outcomes and do not track any firm-specific outcomes, there is no reason to think that local approximation methods will misrepresent the model's dynamics. Given a local approximation strategy, I compute a second-order approximation because a first-order approximation would miss the feature that is at the heart of the model — time variation in cross-sectional risk. First-order-accurate decision rules by construction are invariant to risk. Nonetheless, it is useful to note that the results reported below are virtually identical to those I obtain from a linear approximation. This seems to be attributable to the relatively small amount of changes in cross-sectional risk that occur in the data; this is described in detail in the calibration discussion below. The quantitative results reported below are thus fundamentally driven by the model's mechanism — changes in cross-sectional risk leading to changes in aggregate leverage ratios, which then potentially are transmitted to the real economy — rather than issues about the appropriate approximation method.

Before presenting the dynamic results, I describe the calibration of the model and its main long-run predictions.

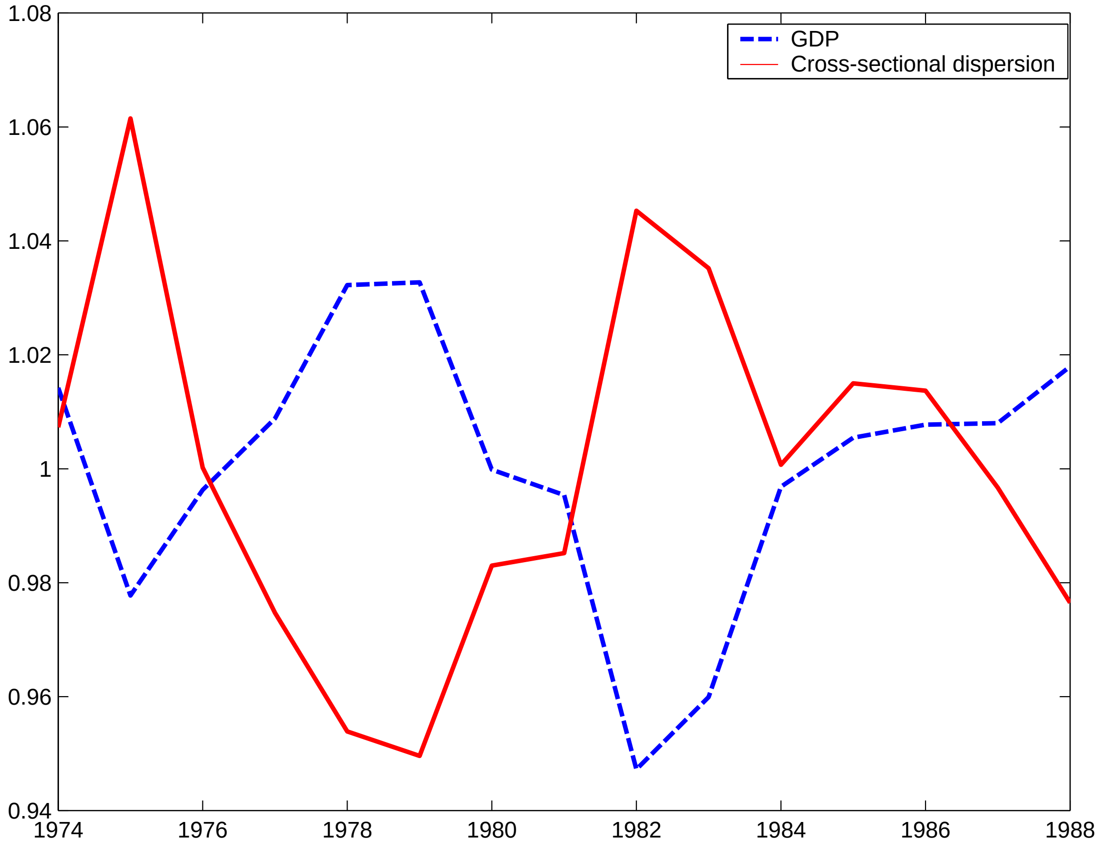


Figure 7: HP-filtered GDP and cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Data are annual. Vertical axis is gross percentage deviation from trend. Profitability series from Cooper and Haltiwanger (2006).

## 5.2 Calibration

The novel aspect of the calibration of the model is in the dispersion process. In the baseline model, I suppose that $\sigma_t^\omega$ follows an AR(1),

$$\ln \sigma_{t+1}^\omega = (1-\rho_{\sigma^\omega})\bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_t^{\sigma^\omega}, \tag{28}$$

with $\epsilon^{\sigma^\omega} \sim N(0, \sigma_{\sigma^\omega})$. To calibrate $\bar{\sigma}^\omega$, $\rho_{\sigma^\omega}$, and $\sigma_{\sigma^\omega}$, I use the series on firm-level profitability constructed by Cooper and Haltiwanger (2006).[23,24]

[23] I thank John Haltiwanger for providing the data.

[24] As Cooper and Haltiwanger (2006) note, given the unavailability of plant-level price deflators in their dataset, it is impossible to distinguish "true productivity shocks" from "revenue shocks;" hence they label their constructed measure a general "profitability shock." In this model, the relative price of all goods is one due to perfect competition, hence I interpret these "profitability shocks" as true productivity shocks.

| Functional Form | Description |
|---|---|
| $\ln \sigma^{\omega}_{t+1} = (1-\rho_{\sigma^{\omega}})\bar{\sigma}^{\omega} + \rho_{\sigma^{\omega}} \ln \sigma^{\omega}_{t} + \epsilon^{\sigma^{\omega}}_{t}$ | Exogenous process for firm productivity dispersion |
| $\ln z_{t+1} = \rho_z \ln z_t + \epsilon^{z}_{t}$ | Exogenous process for TFP |
| $u(c) = \ln c$ | Consumption subutility |
| $v(\ell) = \psi \ln \ell$ | Leisure subutility |
| $F(k, n) = k^{\alpha} n^{1-\alpha}$ | Production technology |

Table 3: Functional forms for quantitative analysis.

Figure 7 displays the HP-filtered components of GDP and the standard deviation of the idiosyncratic component of firms' profitability constructed by Cooper and Haltiwanger (2006) over the period 1974-1988, which is the entire range of their sample. A striking negative cyclical correlation between the two series is apparent — the annual cyclical correlation (the data are annual) between the two series is -0.83, hence good times are associated with a clear decrease in the dispersion of firms' idiosyncratic productivity, and bad times are associated with a clear increase in the dispersion of firms' idiosyncratic productivity. The message conveyed by Figure 7 is the same as that of BB, whose Figure 1 documents strong countercyclicality of firm risk in Germany over roughly the same time period.[25]

I compute using the Cooper and Haltiwanger (2006) data a coefficient of variation of a normalized measure of firm-level dispersion of about three percent over time. More precisely, I first compute the cross-sectional coefficient of variation, which is a normalized measure of the cross-firm productivity dispersion, for each of the 15 years of their sample. I take cross-sectional coefficients of variation because, as described below in Table 4, the mean level of productivity in the model is normalized to unity, which is not the case in their data. The mean of this series is about 30 log points, hence I set the long-run dispersion parameter to $\bar{\sigma}^{\omega} = 0.30$.[26] After HP filtering the time series I construct, I find that the standard deviation over time of this cross-sectional coefficient of variation is 2.92 percent.

I then estimate the AR(1) process (28) to describe the evolution of the dispersion process, and I find $\rho_{\sigma^{\omega}} = 0.48$, with a t-statistic of 1.93. With this AR(1) estimate of $\rho_{\sigma^{\omega}}$ and the standard deviation of the cross-sectional coefficient of variation of 2.92 percent computed above, one can

[25] Figure 1 of BB displays the coefficient of variation of the innovations to firm risk, whereas Figure 7 displays the fluctuations in firm risk itself.

[26] It is interesting to note that CF (1998, p. 590) and Bernanke, Gertler, and Gilchrist (1999, p. 1368) use similar calibrated values, though based on very different calibration targets, not firm-level data: the former set $\bar{\sigma}^{\omega} = 0.37$, and the latter set $\bar{\sigma}^{\omega} = 0.28$.

| Parameter Value | Description/Notes |
| --- | --- |
| | Preferences |
| $\beta = 0.99$ | Households' quarterly subjective discount factor |
| $\gamma = 0.947$ | Entrepreneurs' (additional) subjective discount factor |
| $\psi = 1.8$ | Leisure calibrating parameter (calibrated in baseline model) |
| | Production Technology |
| $\alpha = 0.36$ | Capital's share in production function |
| $\delta = 0.02$ | Depreciation rate of capital |
| | Financial Markets and Agency Costs |
| $\mu = 0.15$ | Per-unit monitoring cost |
| $\omega_m = 1$ | Long-run mean of idiosyncratic productivity |
| $\bar{\sigma}^\omega = 0.30$ | Long-run standard deviation of distribution of $\ln \omega$ |
| $\rho_{\sigma^\omega} = 0.83$ | Quarterly persistence of log firm risk process |
| $\sigma_{\sigma^\omega} = 0.005$ | Standard deviation of log firm risk shock |
| | Exogenous Process |
| $\rho_z = 0.95$ | Quarterly persistence of log TFP process |
| $\sigma_z = 0.0038$ | Standard deviation of log TFP shock, calibrated to deliver GDP volatility of 1.8 percent |

Table 4: Parameter values for baseline model.

compute that the standard deviation of the *innovations* to the annual dispersion process is 0.0216. This implies a coefficient of variation (with respect to the mean dispersion $\bar{\sigma}^{\omega} = 0.30$) of 7.2 percent, which can be directly compared to the empirical evidence reported by BB. Computed in a variety of ways, BB find a coefficient of variation of innovations to firm-level productivity for their entire sample of German firms between two and three percent. However, because the Cooper and Haltiwanger (2006) analysis is of large manufacturing plants, the most comparable result in BB is their finding for the largest (ranked by employment) five percent of firms in their sample. For this sample, BB find a coefficient of variation of firm-level innovations of 5.5 percent (see their Table 8). The 7.2 percent coefficient of variation of plant-level innovations in the Cooper and Haltiwanger (2006) sample is thus slightly higher than, though in line with, the largest firms in BB's sample.[27]

However, rather than an annual calibration, I pursue a quarterly calibration of the model because the leverage evidence documented in Section 2 is quarterly. Because the Cooper and Haltiwanger (2006) data are annual, I use a persistence of $\rho_{\sigma^{\omega}} = 0.48^{0.25} = 0.83$. I then set $\sigma_{\sigma^{\omega}} = 0.005$, which enables the model to match a time-series standard deviation of $\sigma^{\omega}_t$ of three percent.[28]

In addition to the exogenous law of motion for the dispersion process, Table 3 lists the functional forms used in the quantitative experiments, and Table 4 lists the baseline parameter settings. The preference and production parameters are standard in business cycle models. The value for entrepreneurs' discount factor, $\gamma = 0.947$, is taken from CF, as is the agency cost parameter $\mu$.[29] Besides the dispersion process, the other exogenous driving force of the model economy is standard shocks to aggregate (log) TFP. I set the conventional value $\rho_z = 0.95$ for the persistence of TFP shocks, and choose the standard deviation of shocks to TFP such that in the model economy driven *only* by TFP innovations, the standard deviation of GDP is 1.8 percent, which is the historical volatility of GDP fluctuations in the U.S.; this procedure leads to $\sigma_z = 0.0038$. The somewhat low calibrated value of shocks to TFP itself indicates that, given the rest of the parameterization, the model delivers more amplification of TFP shocks than a baseline RBC model, which can be attributed to the financial accelerator mechanism.

[27] I thank Rudi Bachmann for pointing these issues out.

[28] Note that $\sigma_{\sigma^{\omega}}$ is the standard deviation of the innovation to the dispersion process, not the standard deviation of the dispersion process itself; the latter depends on the former along with the persistence $\rho_{\sigma^{\omega}}$ of the dispersion process. Also note that the calibrated quarterly $\sigma_{\sigma^{\omega}} = 0.005$ follows simply as the standard deviation of the annual innovations computed above (0.0216) divided by four.

[29] The parameterized value for the agency cost parameter $\mu$ is the same as the calibrated value in Covas and den Haan (2006) and is also very close to that estimated by Levin, Natalucci, and Zakrajsek (2004).

## 5.3 Long-Run Dispersion and Long-Run Equilibrium

I compute the long-run deterministic (steady-state) equilibrium numerically using a standard nonlinear equation solver. The main comparative static exercise I conduct is presented in Figure 8, which plots the long-run (steady-state) equilibria as a function of the long-run cross-sectional dispersion $\bar{\sigma}^{\omega}$. All other parameters are held fixed at those presented in Table 4.

Figure 8 shows that the long-run response of the economy to changes in $\bar{\sigma}^{\omega}$ is non-monotonic. For low dispersion of idiosyncratic productivity, GDP falls as dispersion rises, but for high dispersion, the comparative static result reverses. The nonmonotonicity is also evident in the long-run behavior of the markup (lower right panel) as well as other standard aggregate quantities such as consumption and gross investment (for brevity, the latter two are not shown in Figure 8). This effect is not due to any nonmonotonicity of the contract terms, as the loan amount (upper middle panel) is strictly decreasing in $\bar{\sigma}^{\omega}$, and the bankruptcy threshold $\bar{\omega}$ (not shown) and hence bankruptcies (lower left panel) are strictly increasing in $\bar{\sigma}^{\omega}$. The calibrated long-run dispersion is $\bar{\sigma}^{\omega} = 0.30$, and, when I allow $\sigma_t^{\omega}$ to fluctuate, dispersion never reaches as high as 0.50 in the simulations, hence the model's fluctuations do not cover the inflection point Figure 8 reveals.[30] I leave to future investigation further study of the nonmonotonicity.

It is useful to highlight the long-run values implied by the model of the three key financial variables of the model: the leverage ratio, the bankruptcy rate, and the risk premium. These are collected in Table 5 for the baseline calibration of the model. The long-run bankruptcy rate is in line with the Dun & Bradstreet evidence cited by CF (1998, p. 590), and the risk-premium is in line with, although a bit higher than, most of the measures of the risk premium presented in DeGraeve (2008).[31]

The long-run leverage ratio, at 0.97, is smaller than the 1.4 long-run leverage ratio documented in Section 2. In the model, the only determinant of long-run leverage is long-run dispersion, $\bar{\sigma}^{\omega}$. As dispersion shrinks to zero, which means that lenders face no risk whatsoever on their loans, the leverage ratio grows unboundedly, as shown in Figure 8.[32] For $\bar{\sigma}^{\omega} > 0$, there indeed is a value

[30] As Table 4 shows, our calibrated value of the standard error of the shocks to the dispersion process is $\sigma_{\sigma^{\omega}} = 0.005$, which is sufficiently small that during simulations, $\sigma_t^{\omega} = 0.50$ was never reached.

[31] As discussed extensively by DeGraeve (2008), it is not clear what is the most relevant empirical counterpart to model's external finance premium. Many natural alternatives suggest themselves, such as the difference between the prime borrowing rate and the short-term T-bill rate, the interest spread between AAA-rated commercial paper and T-bills, the spread between BBB-commercial paper and T-bills, and so on. DeGraeve (2008) documents that these various empirical measures of "the external finance premium" behave differently enough over the business cycle that it remains, in my interpretation, a wide-open question what the "right" empirical counterpart of the model's external finance premium is.

[32] That is, as $\bar{\sigma}^{\omega} \rightarrow 0$, lenders are willing to lend ever larger quantities. Alternatively, one could say that leverage is undefined because financial frictions do not matter and the model technically pins down neither loan amounts nor

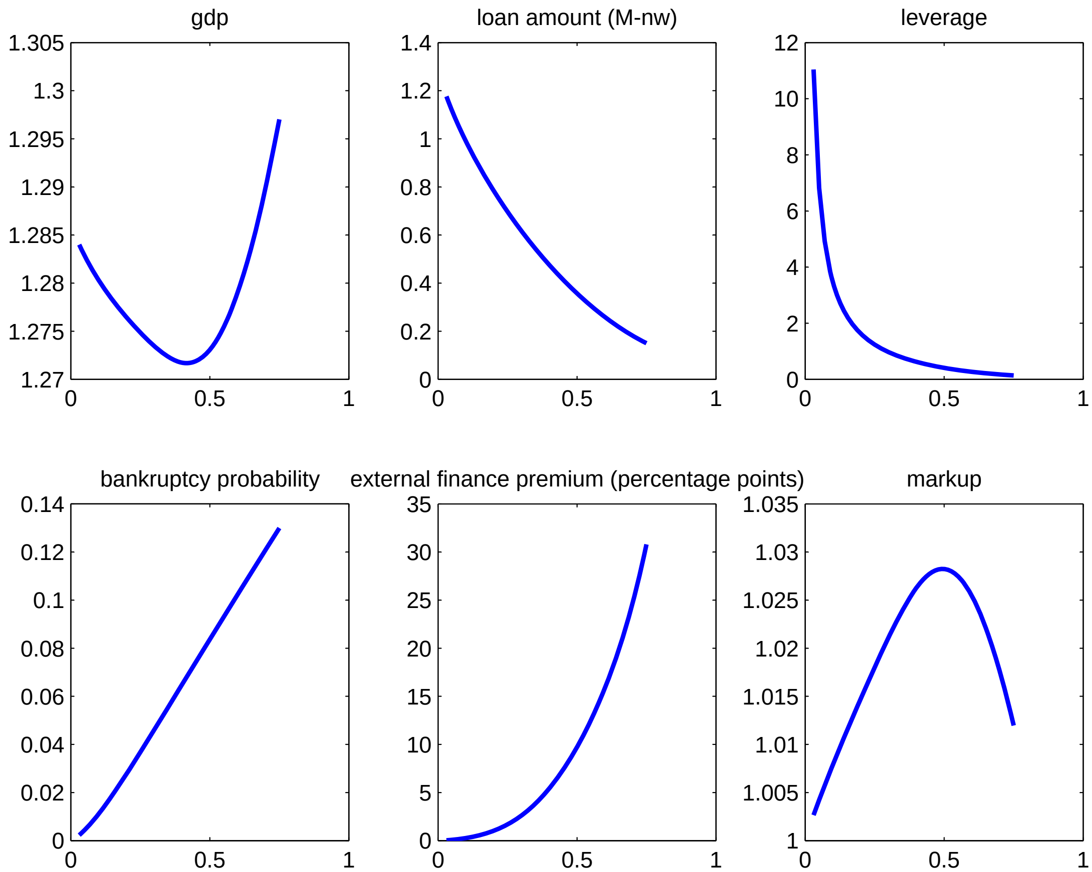


Figure 8: Long-run equilibrium as long-run standard deviation of idiosyncratic productivity distribution, $\bar{\sigma}^{\omega}$, varies; $\bar{\sigma}^{\omega}$ plotted on horizontal axis.

| Financial Measure | Long-Run Value |
|---|---|
| Leverage ratio, $\ell(\bar{\omega})$ | 0.97 |
| Risk premium, $100\,(\bar{\omega}/g(\bar{\omega})-1)$ | 2.61 basis points |
| Bankruptcy rate, $100\Phi(\bar{\omega})$ | 4.61 percent |

Table 5: Long-run financial variables for the baseline calibration of the model.

that implies a long-run leverage ratio of 1.4 — that value is $\bar{\sigma}^{\omega} = 0.23$, which is not too far from the calibrated value of $\bar{\sigma}^{\omega} = 0.30$ implied by the firm-level evidence. Given a $\bar{\sigma}^{\omega} > 0$, in reality there are many other factors not present in the model that affect long-run leverage besides just the cross-section of borrowers' risk. I thus view the model's predicted long-run leverage ratio as broadly in line with the evidence.

### 5.4 Business Cycle Dynamics

I divide the presentation of the baseline model's cyclical dynamics into three parts. First, I document how macro as well as financial aggregates respond to standard TFP shocks, with cross-sectional dispersion of firm productivity held constant at $\bar{\sigma}^{\omega}$. Then, I document how the model behaves in response to only cross-sectional dispersion shocks, with TFP held constant at $\bar{z} = 1$. Finally, I allow both shocks to simultaneously drive the economy. In the language of the recent literature, I am parsing the model's dynamics into behavior following pure first-moment shocks, pure second-moment shocks, and an independent combination of first- and second-moment shocks. The latter is not necessarily what I consider to be, using my terminology from above, "bundled aggregate shocks;" I defer consideration of this until the extended model in Section 6.

#### 5.4.1 First-Moment Shocks

To establish a baseline, I first demonstrate that the model's predictions nest the results obtained by CF when $\sigma_t^{\omega} = \bar{\sigma}^{\omega}\ \forall t$ and it is only standard TFP (first-moment) shocks that drive fluctuations. Figure 9 displays impulse responses to a one-time, one-standard deviation positive shock to TFP, holding constant cross-sectional dispersion of firm productivity. The results are in line with those documented in CF (1998, Figure 1) for their "output model" in terms of both the qualitative responses of variables and their magnitudes. Note that CF (1998, Figure 1) do not report the impulse responses of the leverage ratio, the bankruptcy rate, or the risk premium, so I cannot compare their responses. These measures are included in Figure 9 because these are key dimensions

leverage.

along which I assess the model; furthermore, Figure 9 shows previously-unreported aspects of how the basic CF model works. In particular, the leverage ratio rises by about one percent, the bankruptcy rate rises modestly from 4.6 percent to 4.65 percent, and the risk premium rises a miniscule two basis points, from 2.61 percent to 2.63 percent.

Table 6 presents business cycle statistics from model simulations when the only exogenous process is fluctuations in TFP.[33] While CF do not report simulation-based moments, the model reproduces basic business cycle stylized facts: for example, gross investment is roughly three times as volatile as GDP, consumption is about two-thirds as volatile, and so on. Also reported in Table 6 are the model's business cycle statistics for key financial variables. The first row of the table shows that the leverage ratio $\ell(\bar{\omega})$, the bankruptcy rate $\Phi(\bar{\omega})$, and the risk premium hardly fluctuate, confirming the results from Figure 9.[34] Compared to the empirical evidence presented in Section 2, *the volatility of the leverage ratio is smaller by a factor of roughly 40 compared to the data.* This prediction of the model is the baseline against which I compare the experiments presented next.[35]

### 5.4.2 Second-Moment Shocks

With the baseline dynamics of the model established, I now present the first set of novel experiments conducted in the model, namely dynamics in the face of pure second-moment shocks. Figure 10 presents impulse responses to a one-time, one-standard deviation positive shock to the cross-sectional dispersion of firm productivity, holding constant the level of TFP. Complementing this impulse-response analysis are the simulated business cycle statistics reported in Table 7, in which it is only exogenous fluctuations in cross-sectional dispersion of productivity that generate business cycles.

Comparing Figure 10 with Figure 9 shows that a pure shock to cross-sectional dispersion induces a GDP response only about two percent as large as that induced by a pure shock to the mean level of TFP. This suggests some role for empirically-relevant dispersion shocks as a driver of aggregate fluctuations, but a minor one. It is interesting to note that this is one of the main messages of the theoretical model of BB, as well, even though their focus is not on financial frictions as a transmission channel. Taking into account the role of financial frictions in the transmission

[33] These business cycle statistics are generated by simulating the model 1000 times around the deterministic steady state equilibrium, with each simulation 1000 periods in length, and then computing the medians across simulations of standard deviations, correlations, etc.

[34] Moreover, $\ell(\bar{\omega})$, $\Phi(\bar{\omega})$, and the risk premium are all perfectly correlated with each other. This is easy to understand because, for a time-invariant distribution of $\omega_{it}$, fluctuations in each depend only on fluctuations in the contractually-specified bankruptcy threshold $\bar{\omega}_t$.

[35] As mentioned above, the virtually constant leverage ratio in the basic CF model is noted by Carlstrom, Fuerst, and Paustian (2009, p. 8), although not documented in CF (1998) or Carlstrom and Fuerst (1997) (nor, once again, in Bernanke, Gertler, and Gilchrist (1999)).

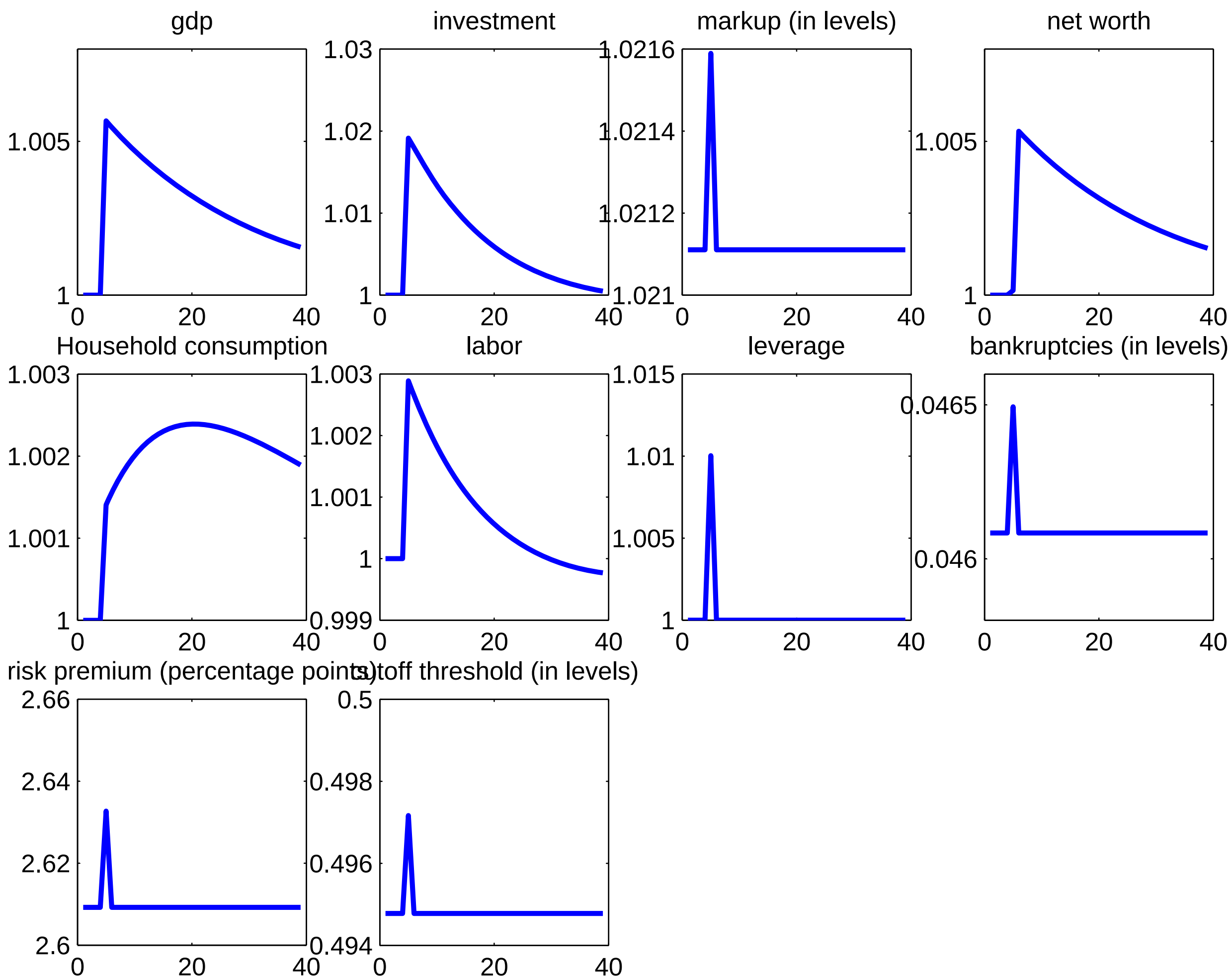


Figure 9: Impulse response to a one-standard-deviation exogenous increase in the level of TFP, holding constant the dispersion $\sigma^{\omega}$ of firm productivity. Except where noted, scale is gross percentage point deviation from steady state.

| | | Financial aggregates | | | Macro aggregates | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\ell(\bar{\omega})$ | $\Phi(\bar{\omega})$ | $R^p$ | $gdp$ | $c$ | $i$ | $n$ |
| Std. dev. (%) | | 0.0056 | 0.0067 | 0.0002 | 0.0189 | 0.0125 | 0.0490 | 0.0069 |
| Auto. corr. | | 0.0263 | 0.0157 | 0.0151 | 0.9504 | 0.9919 | 0.9172 | 0.9042 |
| | $\ell(\bar{\omega})$ | 1 | 1 | 1 | 0.3282 | 0.1252 | 0.4228 | 0.4499 |
| | $\Phi(\bar{\omega})$ | | 1 | 1 | 0.3449 | 0.1438 | 0.4349 | 0.4583 |
| | $R^p$ | | | 1 | 0.3415 | 0.1396 | 0.4328 | 0.4572 |
| | $gdp$ | | | | 1 | 0.8628 | 0.9138 | 0.7926 |
| Corr. matrix | $c$ | | | | | 1 | 0.5866 | 0.3824 |
| | $i$ | | | | | | 1 | 0.9715 |
| | $n$ | | | | | | | 1 |

Table 6: Simulation-based business cycle statistics from first-order approximation, only TFP (first-moment) shocks.

| | | Financial aggregates | | | Macro aggregates | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\ell(\bar{\omega})$ | $\Phi(\bar{\omega})$ | $R^p$ | $gdp$ | $c$ | $i$ | $n$ |
| Std. dev. (%) | | 0.0362 | 0.0236 | 0.0005 | 0.0002 | 0.0003 | 0.0014 | 0.0003 |
| Auto. corr. | | 0.8139 | 0.0535 | -0.0787 | 0.8513 | 0.8021 | 0.8962 | 0.7910 |
| | $\ell(\bar{\omega})$ | 1 | -0.0548 | 0.3143 | -0.8147 | 0.5573 | -0.9356 | -0.7456 |
| | $\Phi(\bar{\omega})$ | | 1 | 0.9298 | 0.5757 | -0.6323 | 0.3360 | 0.6672 |
| | $R^p$ | | | 1 | 0.2475 | -0.3969 | -0.0240 | 0.3601 |
| | $gdp$ | | | | 1 | -0.6520 | 0.8721 | 0.8956 |
| Corr. matrix | $c$ | | | | | 1 | -0.8096 | -0.9202 |
| | $i$ | | | | | | 1 | 0.9230 |
| | $n$ | | | | | | | 1 |

Table 7: Simulation-based business cycle statistics, only cross-sectional dispersion (second-moment) shocks.

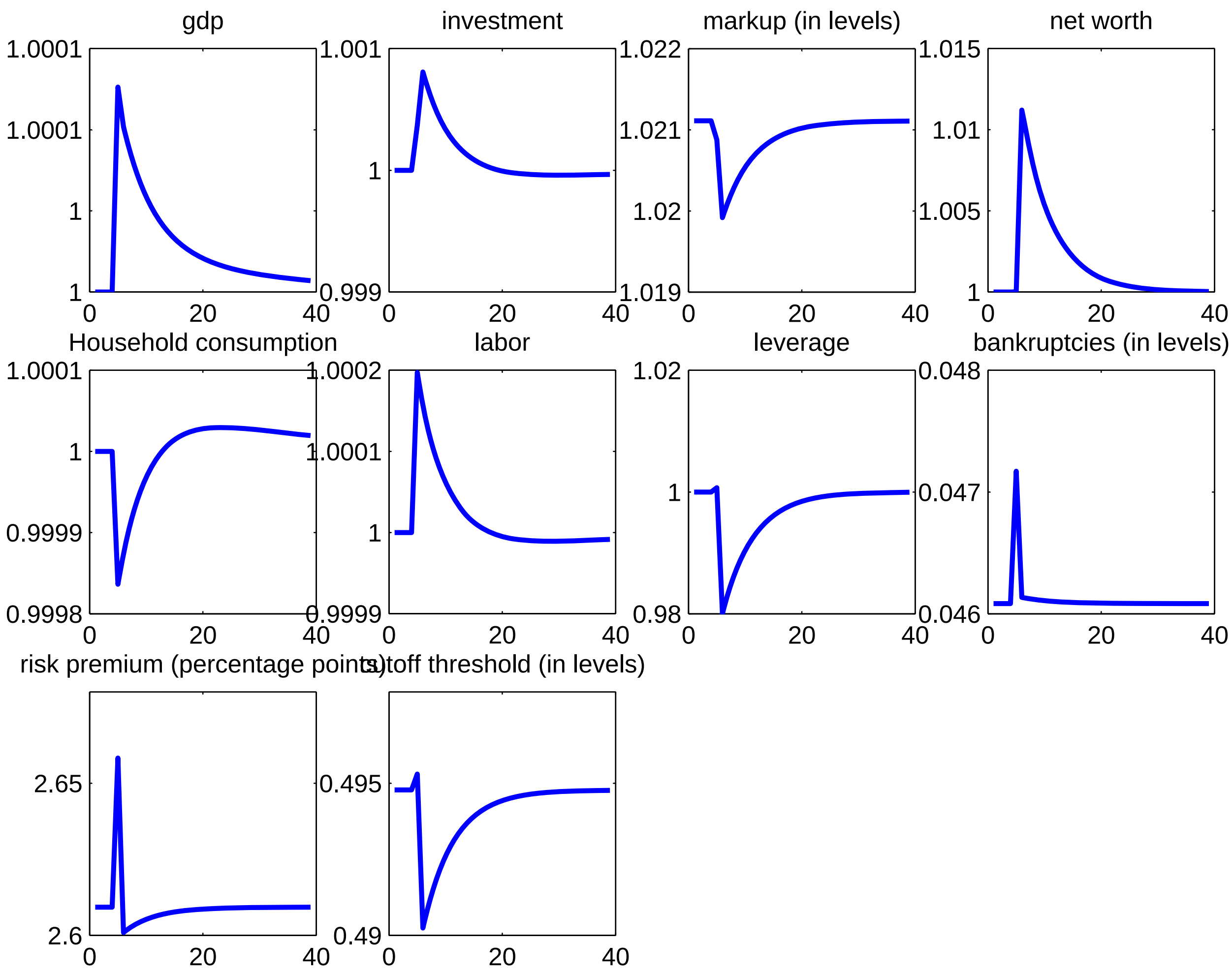


Figure 10: Impulse response to a one-standard-deviation exogenous increase in the dispersion $\sigma^{\omega}$ of firm productivity, holding constant the level of TFP. Except where noted, scale is gross percentage point deviation from steady state.

mechanism leads to a broadly similar conclusion as BB.

Comparing Figure 10 with Figure 9 also reveals that financial variables react much more strongly to a dispersion shock than to a TFP shock. This is the most important result of the baseline model, one that carries over to the modified model in Section 6. Ignoring the direction of the responses for a moment, the leverage ratio moves by two percent following a dispersion shock, whereas it moves only one percent following a TFP shock; and the bankruptcy rate and risk premium move by roughly the same magnitudes following a dispersion shock as they do following a TFP shock, even though the GDP response is only two percent as large. In terms of relative volatilities of fluctuations in financial variables compared to aggregate macro quantities, then, dispersion shocks can be quite powerful quantitatively.

The quantitative power of pure dispersion shocks is more clearly revealed by the simulation-based results reported in Table 7. The cyclical volatility of leverage, bankruptcies, and the risk premium are all an order of magnitude larger in the face of pure dispersion shocks than in the face of only TFP shocks, even though fluctuations of standard macro aggregate quantities are only two percent as large. Moreover, dispersion shocks break the unit correlation amongst $\ell(\bar{\omega})$, $\Phi(\bar{\omega})$, and the risk premium because these quantities now depend not just on endogenous movements in the contractually specified bankruptcy threshold $\bar{\omega}_t$, but also directly on the exogenous movements in the cross-sectional distribution due to fluctuations in $\sigma_t^{\omega}$.[36] In terms of cyclicality of the leverage ratio, it is strongly countercyclical (-0.8147) with respect to GDP, at odds with the virtual acyclicality documented in Section 2. Note, however, that this result arises in the face of only second-moment shocks — below, I show that if the economy is hit by both first-moment shocks and second-moment shocks, the implications for the cyclicality of leverage are consistent with the data.

The basic intuition behind why the leverage ratio fluctuates in response to second-moment shocks was discussed in Section 4. There, a back-of-the-envelope calculation that held constant the liquidation threshold $\bar{\omega}$ led me to conclude that the volatility of the leverage ratio should be about 0.3, given the calibrated standard deviation of the cross-sectional dispersion of productivity of about 3 percent (i.e., an elasticity of -0.1, as shown in Figure 6). The first row of Table 7 shows that the coefficient of variation of the leverage ratio is nearly 4 percent, quite a bit larger than the simple analytics presented in Section 4. The effective equilibrium elasticity of the leverage ratio with respect to firm-level dispersion that emerges from the simulations is thus roughly unity, which seems attributable to the endogenous variations in $\bar{\omega}_t$ that I was abstracting away from in Section 4.

Somewhat counterintuitively, and clearly counterfactually, an increase in cross-sectional disper-

[36]That is, even for a given $\bar{\omega}$, variations in $\sigma^{\omega}$ affect the share functions $f(\bar{\omega})$ and $g(\bar{\omega})$ and hence leverage, bankruptcies, and the risk premium. Although this cannot be shown analytically, once endogenous changes in $\bar{\omega}$ are considered, too, the unit correlations are broken.

sion induces an *increase* in GDP. As shown in Figure 7 and as also documented by BB and BFJ, firm-level dispersion is clearly countercyclical. The reason this result seems to arise in the model is due to the "Hartman-Abel effect," which also arises in the simplest version of the BFJ model that features a minimum of adjustment costs for capital and labor. The idea, as described by BFJ (p. 20), is that absent sufficient adjustment costs, a higher variance of productivity increases output because marginal revenue products are convex in productivity. While I do not model "adjustment costs" in the way the firm-level literature typically does, the entire agency cost/financial friction of the CF model can be viewed broadly as a type of "adjustment cost." However, it apparently is not strong enough to overturn the Hartman-Abel effect. In Section 6, I modify the model in a simple way to deliver countercyclical firm risk. The leverage volatility result in this baseline model, though, will carry over to the modified model, hence it is useful to understand how the baseline model works, both its successes and shortcomings.

#### 5.4.3 Both First-Moment Shocks and Second-Moment Shocks

Although dispersion shocks cause great amplification of financial variables compared to TFP shocks, the correlation of leverage with macro quantities does not line up with its empirical counterpart. In particular, the strong negative contemporaneous correlation of leverage with GDP is not in line with the empirical evidence presented in Section 2. However, business cycle fluctuations are likely driven by both first-moment shocks and second-moment shocks. In Table 8, I report business cycle statistics when the model economy is hit by independent shocks to both TFP and dispersion.

In terms of cross-correlations, the model's dynamics improve substantially relative to the model driven by either first-moment or second-moment shocks alone. Focusing on the leverage ratio, leverage is essentially *acyclical* when the model economy is hit by both TFP and dispersion shocks, broadly in line with the evidence presented in Section 2. I view this as a positive result of the model.

The volatility results for standard macro quantities reported in Table 6 and the volatility results for financial variables reported in Table 7 carry through to the model hit by both first-moment and second-moment shocks. For example, volatility of GDP is still 1.8 percent, and the volatilities of leverage, bankruptcies, and the risk premium are all high. Thus, in terms of volatilities, dispersion shocks are the predominant force affecting financial variables, while TFP shocks are the predominant force affecting standard macro aggregates. The volatilities of macro vs. financial variables are thus in a sense driven by independent forces — first-moment shocks drive macro aggregates, while second-moment shocks drive financial aggregates. This is an interesting property of the model, and seems a useful feature to note for future development of this type of model.

Finally, the standard view in macroeconomics is that bankruptcies and risk premia are countercyclical. The model here does not deliver this result, instead predicting that they are essentially acyclical or mildly procyclical. However, the procyclicality of the risk premium in the CF model is a well-known (counterfactual) prediction, and there are known "fixes" to this problem.[37] In the interest of not cluttering the model and intuition too much, I do not apply a "fix" to this prediction of the model.

However, no matter which configuration of shocks is considered — first-moment shocks alone, second-moment shocks alone, or both in tandem — all the experiments conducted in the baseline model lead to procyclicality of firm-level dispersion. That is, an exogenous rise in the cross-section of firm-level risk leads to an increase in GDP. As discussed above, this is opposite the empirical evidence presented in Figure 7, in BB, and in BFJ, which all portray strong *countercyclicality* of firm-level dispersion. In Section 6, I modify the model to accommodate this.

# 6 Bundled Aggregate Shocks: TFP-Induced Dispersion Shocks

I model countercylicality of firm-level dispersion by linking time-variation in aggregate TFP directly to fluctuations in firm-level risk. Specifically, the cross-sectional dispersion of productivity across firms is now assumed to decline when aggregate TFP improves. First-moment shocks are thus assumed to be bundled with second-moment shocks, and I refer to the entire bundle as an "aggregate shock." The two processes are assumed to be linked according to

$$\sigma_t^{\omega} = \bar{\sigma}^{\omega} + \varphi \ln z_t. \tag{29}$$

This condition replaces the exogenous law of motion (28) for $\sigma_t^{\omega}$, and the evolution of $z_t$ is still described by the standard log-linear law of motion presented in Table 4. The rest of the model is exactly the same as above. The parameter $\varphi$ is obviously the key parameter of this version of the model, with $\varphi < 0$ implying countercyclicality of firm-level risk.[38]

Figure 11 illustrates why $\varphi < 0$ leads to countercyclical firm risk. A positive shock to TFP will, all else equal, increase GDP. If at the same time cross-sectional dispersion is declining due to $\varphi < 0$, and supposing initially that the bankruptcy threshold $\bar{\omega}$ were fixed, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios. Indeed, the second part of the intuitive argument is exactly the same as that underlying Figure 4. What is different from the baseline model is the event that now induces the change in dispersion;

[37] See Gomes, Yaron, and Zhang (2003) for analysis of why the standard CF model generates a procyclical finance premium. Faia and Monacelli (2007) propose a mechanism that allows the CF model to generate a countercyclical finance premium, a mechanism that is also used by Chugh (2009).

[38] Clearly, $\varphi > 0$ would deliver procyclical firm-level risk, and $\varphi = 0$ would recover the baseline CF model in which there are no changes in cross-sectional dispersion.

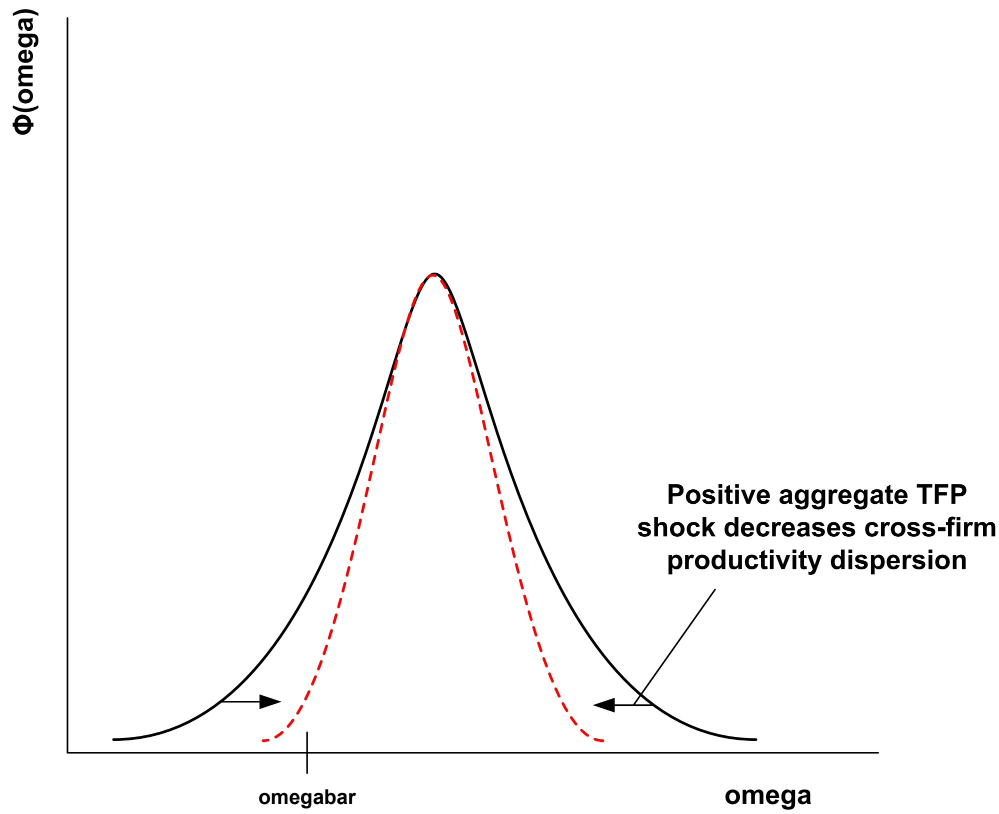


Figure 11: A positive shock to the mean of aggregate TFP causes a decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.

in the baseline model, the change in dispersion itself was the exogenous event, whereas here it is a positive TFP shock that will, again all else equal, cause GDP to rise.

This bundled aggregate shock is of course a reduced-form construct. However, I bring the same empirical evidence presented in Section 5.2 to bear on the calibration of the crucial elasticity parameter $\varphi$. The calibration strategy here is a simulated method of moments (SMM) approach to choose $\varphi$ so that the model matches the observed time-series variation in cross-sectional dispersion. In Section 5.2, I documented based on the Cooper and Haltiwanger (2006) data that the time-series volatility in cross-sectional dispersion is three percent. Given this calibration target and holding fixed all the parameters in Table 4, the SMM procedure (with TFP fluctuations now as the sole truly exogenous driving process) leads to $\varphi = -1.9$.

Figure 12 presents impulse responses to a positive bundled aggregate shock. The most salient point of comparison for these impulse responses are those presented in Figure 9, in which the

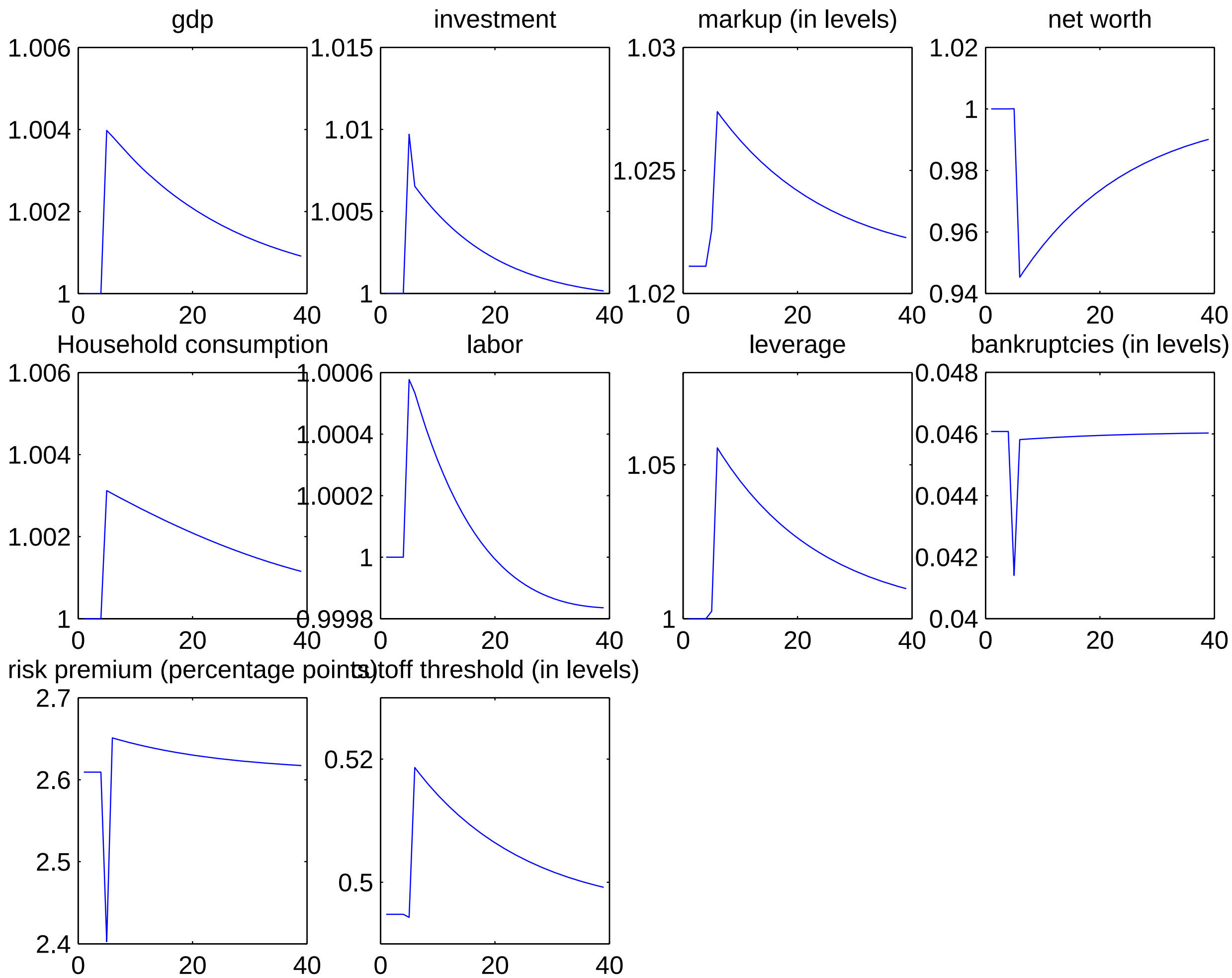


Figure 12: Impulse response to a positive "bundled aggregate shock," in which a one-standard-deviation exogenous increase in TFP induces a decrease in cross-sectional dispersion. Except where noted, scale is gross percentage point deviation from steady state.

same size first-moment shock is also the exogenous impulse except with no change in cross-firm dispersion. Comparing Figure 12 with Figure 9, we see that the bundled aggregate shock leads to a slightly smaller response of GDP than the unbundled first-moment shock alone: the impact effect on GDP is about 90 percent as large. This assumes the parameter $\sigma_z = 0.0038$ shown in Table 4, which itself, as noted in Section 5.2, is a somewhat low value for the standard deviation of shocks to TFP. In the interest of isolating the model's mechanisms, however, $\sigma_z$ is kept fixed.

In terms of bankruptcies and the risk premium, the bundled-shock model predicts responses that are more in line with conventional wisdom. The risk premium falls, although only modestly, on impact and is thus countercyclical on impact with respect to GDP, which accords with conventional wisdom and the evidence presented in Gomes, Yaron, and Zhang (2003) and most of the evidence presented in DeGraeve (2008).[39] Bankruptcies (middle right panel) are countercyclical with respect to GDP in the bundled-shock model, which is also in line with the conventional view and is opposite the effect seen in Figure 9 and even opposite that seen in Figure 10 for a pure second-moment shock. The leverage ratio rises by nearly one percent, but is procyclical with respect to GDP, which is opposite the evidence presented in Section 2.

Finally, Table 9 presents simulation-based business cycle statistics. As the first row of the table shows, the high volatility of leverage carries over from the pure-second-moment shock experiments presented in Table 7. However, a shortcoming of the bundled-shock model is that the leverage ratio is extremely procyclical, at odds with the evidence presented in Section 2.

To summarize, the bundled-shock model by construction is consistent with the empirically-observed countercyclicality of cross-sectional firm risk, and it improves substantially on the CF model in its predictions about the volatility of leverage. However, it fails to capture empirically-relevant (a)cyclicality of leverage. On the other hand, the baseline model driven by a complete set of independent, "unbundled," shocks performed well on both the volatility dimension and the cyclicality dimension, but failed to capture the countercyclicality of firm-level risk. Although I do not take up this model extension here, a conjecture is that a combination of bundled shocks along with independent, exogenous, shocks to dispersion may help in capturing all three of these dimensions of the data.[40]

[39] However, note that the risk premium then shoots up, before declining back towards its long-run value. When measuring the cross-correlation between the risk premium and GDP over the entire time path, this feature actually leads to a mildly *procyclical* measure between the risk premium and GDP. The simulation statistics presented in Table 9 also reveal this.

[40] Another conjecture is that one could introduce non-convex factor adjustment costs like BFJ. Indeed, as noted above, their model absent adjustment costs also features the "Hartman-Abel" effect of procyclical firm risk. It is only in the presence of adjustment costs that countercyclical firm risk arises in the BFJ model.

| | | Financial aggregates | | | Macro aggregates | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\ell(\bar{\omega})$ | $\Phi(\bar{\omega})$ | $R^p$ | $gdp$ | $c$ | $i$ | $n$ |
| Std. dev. (%) | | 0.0375 | 0.0252 | 0.0005 | 0.0189 | 0.0125 | 0.0489 | 0.0069 |
| Auto. corr. | | 0.7557 | 0.0454 | -0.0672 | 0.9504 | 0.9918 | 0.9173 | 0.9038 |
| | $\ell(\bar{\omega})$ | 1 | 0.0428 | 0.3858 | 0.0657 | 0.0330 | 0.0792 | 0.0816 |
| | $\Phi(\bar{\omega})$ | | 1 | 0.9374 | 0.1054 | 0.0165 | 0.1408 | 0.1725 |
| | $R^p$ | | | 1 | 0.1203 | 0.0267 | 0.1578 | 0.1879 |
| | $gdp$ | | | | 1 | 0.8632 | 0.9143 | 0.7914 |
| Corr. matrix | $c$ | | | | | 1 | 0.5883 | 0.3814 |
| | $i$ | | | | | | 1 | 0.9705 |
| | $n$ | | | | | | | 1 |

Table 8: Simulation-based business cycle statistics, with both TFP (first-moment) and cross-sectional dispersion (second-moment) shocks.

| | | Financial aggregates | | | Macro aggregates | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\ell(\bar{\omega})$ | $\Phi(\bar{\omega})$ | $R^p$ | $gdp$ | $c$ | $i$ | $n$ |
| Std. dev. (%) | | 0.0417 | 0.0073 | 0.0002 | 0.0156 | 0.0099 | 0.0408 | 0.0056 |
| Auto. corr. | | 0.9618 | 0.1639 | 0.5357 | 0.9377 | 0.9830 | 0.9048 | 0.8969 |
| | $\ell(\bar{\omega})$ | 1 | -0.4625 | 0.7330 | 0.9843 | 0.8302 | 0.9428 | 0.8574 |
| | $\Phi(\bar{\omega})$ | | 1 | 0.2521 | -0.5662 | -0.3511 | -0.6428 | -0.6396 |
| | $R^p$ | | | 1 | 0.6372 | 0.6362 | 0.5340 | 0.4417 |
| | $gdp$ | | | | 1 | 0.8793 | 0.9300 | 0.8276 |
| Corr. matrix | $c$ | | | | | 1 | 0.6469 | 0.4690 |
| | $i$ | | | | | | 1 | 0.9755 |
| | $n$ | | | | | | | 1 |

Table 9: Simulation-based business cycle statistics for bundled aggregate shocks, in which TFP (first-moment) shocks induce changes in cross-sectional dispersion. Parameter $\varphi = -1.9$.

# 7 Conclusion

This paper documented the business cycle properties of the leverage (debt-to-equity) ratio in the publicly-traded U.S. non-financial business sector. The most important facts to emerge are that leverage is very volatile over the business cycle (an order of magnitude more volatile than GDP), and is essentially acyclical to mildly countercyclical. I then demonstrated that a baseline financial accelerator DSGE model commonly used in the business-cycle literature, based on the agency-cost framework of Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999), does a poor job explaining the volatility of leverage when parameterized in the standard way and driven by standard TFP shocks. In a baseline agency-cost model, the volatility of leverage is two orders of magnitude smaller than in the data. This discrepancy between model volatility and empirical volatility is more dramatic than the one-order-of-magnitude discrepancy Shimer (2005) documented for the key variables of a baseline labor search and matching model.

To try to address this shortcoming of the model, I incorporated recent empirical evidence on business-cycle fluctuations in the cross-sectional dispersion of firm productivity into a baseline accelerator model. The most robust result from the theoretical model is that second-moment fluctuations cause firms' (borrowers') leverage ratios to be an order of magnitude more volatile than in baseline models driven by only standard (first-moment) TFP shocks. Nonetheless, leverage is still five times less volatile than in the empirical evidence I construct. Depending on whether or not second-moment fluctuations are independent from or intertwined with shocks to the mean level of productivity, the model also does a reasonable job of explaining either (but not both) the observed acyclicality of leverage or the observed countercyclicality of firm-level dispersion.

I point out that the inability of the model to generate fluctuations as large as observed in the data is not a shortcoming of the model's mechanism per se. If the observed degree of second-moment fluctuations were roughly five times larger than documented by BB and my analysis of the Cooper and Haltiwanger (2006) data, the model's volatility of leverage would be in line with the evidence. It is simply that empirically-relevant second-moment fluctuations *alone* cannot predict *all* of the observed volatility in leverage; but it nonetheless takes us an order of magnitude closer than baseline agency-cost models.

A broader idea that emerges, then, is that understanding changes directly in the distribution of firm-level risk may be important for guiding the further development of business-cycle models featuring financial frictions. This paper has exploited second-moment disturbances. As noted by Levin, Natalucci, and Zakrajsek (2004, p. 33), fluctuations in third- or higher-order moments may also need to be considered for understanding some aspects of the financial data. This requires moving away from the symmetry of a log-normally productivity distribution. Given the recurring evidence on phenomena like firm sizes being distributed non-normally, there may be reason to think that firm productivity is also distributed non-normally and, moreover, that skewness and higher moments may be time-varying. Such fluctuations would also be expected to affect leverage.

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