Does the US Government Hedge against Expenditure Risk?

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ABSTRACT

The US federal government is partially hedged against wars and other increases in defense expenditures. 14 percent of the total cost of defense spending shocks in the post-war era was absorbed by lower real returns on the federal government's outstanding liabilities. More than half of this is due to reductions in expected future, rather than contemporaneous holding returns on government debt. One corollary of this is that fiscal shocks predict future returns on government debt.

I. Introduction

This paper examines the response of US fiscal policy to government spending shocks and measures of fiscal imbalance. The government budget constraint dictates that spending shocks must be financed through either an increase in the endogenous component of current and future surpluses or a reduction in current and future expected returns on the government's portfolio of liabilities. Applying this logic, we isolate the return and surplus response to news about defense spending growth and find that adjustments to returns have absorbed 14% of the cost of defense spending shocks in the postwar era. In this sense, the US government has made significant use of its debt portfolio to hedge fiscal shocks and damp variation in surplus growth. Normative theories of fiscal

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policy emphasize a narrower form of hedging, namely that achieved through return variations that are contemporaneous with fiscal news. We find that this sort of ex ante hedging makes a much smaller contribution to the financing of defense shocks. Only 4% of their cost is absorbed by adjustments in contemporaneous returns. One immediate implication of these results is that fiscal shocks predict future debt returns. We introduce a new measure of defense shocks constructed from the abnormal return on defense stocks and show that it predicts increases in yields and reductions in debt returns.

Following the predictability literature in finance (e.g. Lettau and Ludvigson (2001)), Campbell's (1993) asset pricing analysis and Gourinchas and Rey's work on international financial adjustment, we organize our thinking around a log-linearized budget constraint; in our case, the government's budget constraint. The log-linearized budget constraint implies that innovations to the present discounted value (PDV) of expected defense spending growth, where the discount factor is one minus the mean surplus/debt ratio, provide the appropriate measure of defense shocks. The linearized budget constraint permits a tractable decomposition of the policy response to shocks into news about the PDV of expected future surplus growth and news about the PDV of expected current and future debt returns. We obtain empirical measures for these news variables and use them to construct hedging betas that describe the response of return news to defense shocks. In postwar US data, the PDV of expected current and future real returns on government debt decreases by *twenty-six* basis points when the PDV of expected future defense spending increases by one percent.

We define hedging to be the fraction of the variation in the cost of fiscal shocks absorbed by variation in debt returns. This fraction depends upon both the hedging betas and the average defense/total spending ratio. Over the post war period, this has been 41% percent, implying that about 14% percent of total defense spending risk is hedged. Evaluated at the lower current defense/total spending ratio, the fraction is closer to 24%. There are two components to this. The first is standard, ex ante hedging, the focus of much of the normative literature on optimal fiscal policy. News about an increase in the PDV of expected defense spending growth coincides with an unexpected drop in the contemporaneous value of government liabilities. The ex ante hedging beta (of current debt returns) is *-.075* in postwar US data. Second, the PDV of expected *future* debt returns decreases when news about higher defense spending growth is released. The ex post hedging beta (of news about future debt returns) is *-.18.* When expected future defense spending growth increases by one percent, the average real return investors expect to earn in the future on government debt decreases by eighteen basis points. This second part is a, perhaps, surprising, but robust feature of the data. The ex ante and ex post beta add up to the total beta of *-.26*.

In the data, nominal yields rise persistently after the news about an increase in future defense spending growth reaches financial markets. Additionally, our results indicate that bonds with longer maturities experience larger increases in their yields. Thus the maturity structure of the government's debt can affect the degree of hedging. We find that if the government were to increase the average maturity of its outstanding liabilities to an average of 15 years, the ex post beta would increase to *-.24* but the ex ante beta would double to *-.17*. The total average current and future return decreases by forty-one basis points when the average defense spending growth rate increases by one percent.

II. Literature

Budget constraints and predictability Our log-linearization of the budget constraint follows Campbell (1993) and, especially, Gourinchas and Rey (2003). Campbell's focus is asset pricing, Gourinchas and Rey's is international adjustment to large trade or asset imbalances. The issue of hedging exogenous spending shocks is absent from both of these papers.

Our results on the ability of fiscal shocks to predict government portfolio returns and yields corroborate those found in Dai and Philippon (2005).

Optimal tax literature The normative theory of fiscal policy provides perspective. Standard models in this literature feature a benevolent government that minimizes the welfare losses arising from variation in marginal tax rates over time and states. If the tax system is sufficiently constrained, then the government will wish to smooth inter-state marginal tax rates and the excess burden of taxation by varying the return it pays on its debt.[1,2] The extent to which it can do this is determined by the asset market structure it faces. In complete market models, there are no restrictions on the government's ability to hedge shocks through return variations. In the simplest versions of these models, fiscal variables such as taxes are functions of shocks only and inherit their

[1] If the government has access to lump sum taxation, then Ricardian Equivalence implies that it need make no recourse to bond markets. If it can tax private assets without inducing any contemporaneous distortion, then asset taxation can substitute for variations in debt returns. Finally, if it can flexibly adjust both consumption and income tax rates in response to shocks, then again debt is redundant as a fiscal hedge (see Correia and Teles (2003)). On the other hand, if the tax system is sticky or if the government is constrained to adjust income tax rates in the aftermath of shocks, then debt's essential role as a fiscal hedge is reinstated.

[2] Scott (2007) shows that when markets are complete, the government maintains the excess burden of taxation (the shadow value of the future primary surplus stream) at a constant level. Labor tax rates still vary to the extent that the compensated labor supply elasticity varies. However, these variations are typically damped relative to an incomplete markets setting.

statistical properties from these shocks.[3] At the other extreme, if the government can trade only one period real non-contingent debt, then interstate hedging is proscribed and optimal policy entails intertemporal rather than interstate smoothing of taxes and the excess burden. Tax rates and debt values now evolve according to (risk-adjusted) martingales; they exhibit a unit root component and are more persistent than underlying shocks.[4] Intermediate cases in which fiscal hedging is possible, but costly, deliver intermediate results. In these, the government optimally responds to shocks with a mixture of interstate and intertemporal smoothing of taxes and the excess burden.[5]

Several contributors beginning with Barro (1979) have used normative models of the sort described above to assess empirical fiscal policy. Early analysis found evidence of persistence in tax rates consistent with incomplete market models.[6] More recent work by Marcet and Scott (2001) and Scott (2007) has obtained and empirically assessed the implications of complete and incomplete market optimal policy models. This work provides further evidence of persistence in debt levels and tax rates relative to allocations that is suggestive of incomplete markets models.

Relative to these papers, we are, to the best of our knowledge, the first to use the budget constraint to quantify directly the degree of hedging. Our work complements these existing contributions by suggesting that contemporaneous hedging of shocks is limited - although we make no attempt to ascribe this to market incompleteness per se. That is, we do not distinguish between an inability or an unwillingness to engage in ex ante hedging. On the other hand, our work indicates the relative importance of ex post hedging - variations in expected future returns play a significant role in financing shocks - that is ignored in the optimal tax literature.

III. Government Budget Constraint and Hedging

To quantify the extent to which the government is hedged against defense shocks, we use the government's budget constraint. The dynamic period-by-period version of the government's budget constraint is given by:

$$B_{t+1} = R^b_{t+1} \left(B_t - S_t \right),$$

[3] In more elaborate versions with capital or habit formation, they depend on other real state variables, but they are no more persistent than these variables.

[4] See Barro (1979) and Aiyagari, Marcet, Sargent and Seppala (2002).

[5] One example is Lustig, Sleet and Yeltekin (2006). There a government trades non-contingent nominal debt of various maturities. Costly contemporaneous or expected future inflations allow it to hedge fiscal shocks. See also Siu (2004). Another is Sleet (2004) who requires fiscal policy to satisfy incentive compatibility restrictions.

[6] See, for example, Bizer and Durlauf (1990), Hess (1993) and Sahaskul (1986). However, as Bohn (1998) and Scott (2007) point out the unit root tests used in this literature have low power against the alternative of optimal policy in a complete markets/ persistent shock environment.

where B_t denotes the market value of outstanding government debt[7] at the start of period t, $S_t = T_t - G_t$ denotes the federal government's primary surplus -receipts T_t less expenditures G_t - and R^b_{t+1} denotes the simple gross return paid on the government's portfolio between t and $t+1$. This equation can be re-arranged to yield the following expression for the growth rate of government debt as a function of the return on this debt and the primary surplus-debt ratio:

$$\frac{B_{t+1}}{B_t} = R^b_{t+1}\left(1 - \frac{S_t}{B_t}\right). \tag{1}$$

Our goal is to measure the impact of news about future defense spending on the budget constraint, and the extent to which this impact is offset by contemporaneous and subsequent declines in the market value of outstanding debt. This is obviously much easier in a linear setting and so we follow Campbell (1993) and Gourinchas and Rey (2005) and linearize the budget constraint (1).

A. The linearized budget constraint

Campbell's linearization of the the household budget constraint treats labor income as the return on human capital and, hence, part of the return on the household's overall portfolio. The constraint is then re-expressed as a function of household wealth (inclusive of human capital) and consumption, both of which are taken to be positive. In contrast, we treat government income from taxation as a part of the flow surplus rather than as a return on a government asset. The fact that the surplus may be either positive or negative creates additional difficulties for the log-linearization. We circumvent these difficulties by expanding around both the log receipts/debt and log spending/debt ratios and then constructing a weighted log primary surplus.

We assume that for all t, $B_t > 0$ and $1 - S_t/B_t > 0$. Additionally, we assume that the receipts/debt and spending/debt ratios, T_t/B_t and G_t/B_t, are stationary around their average values which we denote $\overline{TB}$ and $\overline{GB}$ respectively. Finally, we assume that average surplus/debt ratio, $\overline{SB} := \overline{TB} - \overline{GB}$ is between 0 and 1.[8] Throughout, our notational convention is to use lower cases to denote log variables and Δ's to denote log differences, so that $b_t = \log B_t$, $\Delta b_{t+1} = \log b_{t+1} - \log b_t$ and so on. In the appendix, we show that under the above assumptions and ignoring unimportant constants, the law of motion for debt can be approximated as:

$$\Delta b_{t+1} = r^b_{t+1} + \left(1 - \frac{1}{\rho}\right)(ns_t - b_t), \tag{2}$$

[7] Inclusive of cash.

[8] For example, one might assume that because of some initial amount of debt, surpluses are positive on average.

where $\rho = 1 - (\overline{S/B}) \in (0, 1)$ and ns_t is the *weighted log primary surplus*:

$$ns_t = \mu_\tau \tau_t - \mu_g g_t. \tag{3}$$

The weights in (3) are given by $\mu_\tau = \frac{\mu_{\tau,b}}{\mu_{\tau,b} - \mu_{g,b}}$ and $\mu_g = \frac{\mu_{g,b}}{\mu_{\tau,b} - \mu_{g,b}}$, where $\mu_{\tau,b} = \overline{T/B}$ and $\mu_{g,b} = \overline{G/B}$.

Equation (2) implies the first order difference equation:

$$b_t - ns_t = \rho r^b_{t+1} + \rho \Delta ns_{t+1} + \rho(b_{t+1} - ns_{t+1}). \tag{4}$$

Solving (4) forward and imposing the tail condition $\lim_{j \to \infty} \rho^{t+j}(ns_{t+j} - b_{t+j}) = 0$, we obtain the following expression for $ns_t - b_t$, the *log surplus/debt ratio* :

$$ns_t - b_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j} \right). \tag{5}$$

As noted, this derivation hinges on the assumption that the receipts/debt and spending/debt ratios are stationary, so that their average values, along with the average surplus/debt ratio, exist. The quality of the approximation relies, additionally, on these variables remaining close to their average values. We assume throughout variations in these ratios is relatively small.

Measuring the the log surplus/debt ratio. The measurement of the log surplus to debt ratio relies on the computation of the weights μ_g and μ_τ and the market value of outstanding government debt. We compute the market value of outstanding debt by aggregating the price of all future coupon and principal payments promised by the government. The price of these coupon payments is computed off the zero-coupon yield curve we constructed from CRSP data (see the appendix for a detailed description). In what follows, we abstract from other federal government assets and liabilities that may change in value when exogenous shocks arise.

We use two different methods to compute the weights μ_g and μ_τ. In the first, we calculate sample averages for the receipts/debt and spending/debt ratios. We substitute these into the previously given formulas for the weights and obtain the weighted log surplus/ debt ratios:

$$ns_t - b_t = \mu_\tau \tau_t - \mu_g g_t - b_t.$$

The result is plotted in figure (1).

If $\{\tau_t\}$, $\{g_t\}$ and $\{b_t\}$ are co-integrated, the residuals from the co-integrating vector between $\{\tau_t\}$, $\{g_t\}$ and $\{b_t\}$ give the deviations from the long-relation between weighted log surplus and the market value of debt. This provides a second measure of the weighted log surplus/debt ratio. We estimate the co-integrating vector by simple OLS regression of g_t on a constant, τ_t and b_t. The residual (inclusive of constant) from this co-integrating relation is labeled $\widetilde{nsb}_t$ and is obtained as:

$$\widetilde{nsb}_t = \widetilde{\mu}_\tau \tau_t - \widetilde{\mu}_g g_t - b_t,$$

where $\widetilde{\mu}_\tau$ and $\widetilde{\mu}_g$ are the estimated weights. Figure (2) plots the results. It's clear from the plots that the two series are strongly correlated.

Figure 1. The surplus debt ratio nsb: The figure plots $ns_t - b_t$. The weights are computed using the sample averages. The sample is 1946.I-2003.IV.

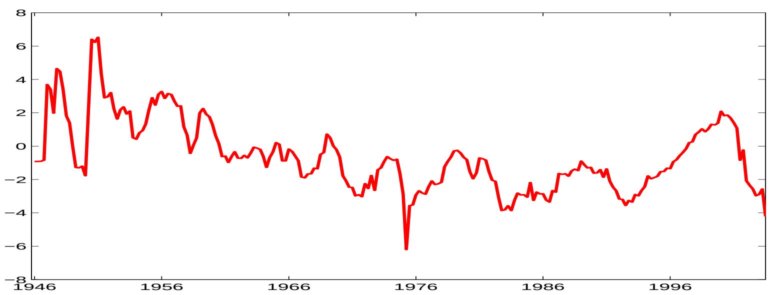
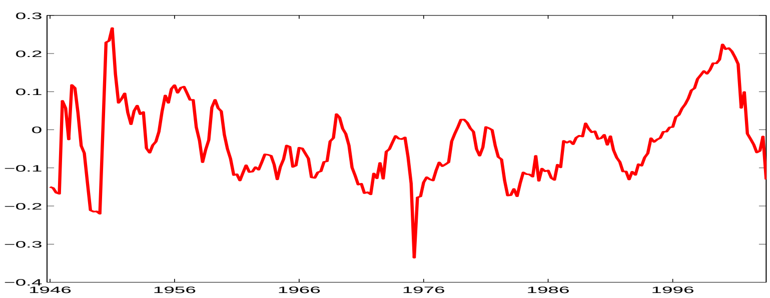


Figure 2. The surplus debt ratio in deviations from co-integrating relation $\widetilde{nsb}$: The figure plots the deviations from co-integrating relation for τ_t, g_t and b_t: $\widetilde{nsb}_t$. The co-integrating vector is $[1, -.779, -.072]$, estimated over the sample 1946.I-2003.IV.



Implications of the linearized budget The log surplus/debt ratio expression in (5) looks like the Campbell-Shiller expression for the price-dividend ratio. It implies that, if the surplus/debt ratio fluctuates, it has to be due to a change in future expected returns on outstanding debt or a change in future expected deficit growth. The log surplus/debt ratio reveals deviations from the long-run relationship between surpluses and debt. If it is negative, the surplus is small relative to the market value of debt; we expect low future real returns on the government debt or high future surplus growth. If it is positive, we expect high future real returns on the debt or low future surplus growth. Analogously, in the case of the dividend/price ratio, a high ratio reveals high expected returns or low expected future dividend growth:

$$d_t - p_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r^s_{t+j} - \Delta d_{t+j} \right). \tag{6}$$

In short, the government budget constraint dictates that the log surplus/debt ratio predicts either future returns on the government's portfolio of outstanding debt or future surpluses. If the sum of expected, discounted future returns on government debt are constant over time, e.g. if the returns are i.i.d., then equation (5) implies that all of the adjustment occurs through an increase in the growth rate of future log surpluses $\{\Delta ns_{t+j}\}$. If not, at least part of the adjustment in to a decrease in the surplus/debt ratio is brought about by lower expected future returns on government debt holdings. In the latter case, $ns_t - b_t$ ought to predict future returns $\{r^b_{t+j}\}$. Lettau and Ludvigson (2001) apply the same reasoning to the household budget constraint and show that the consumption/wealth ratio ought to predict returns, which it does.

IV. Government Hedging

Through further manipulation the linearized government budget constraint, we gain a better understanding of the different ways in which the government can hedge against shocks to its expenditures.

News about future surplus growth and future returns The expression for the log surplus/debt ratio, equation (5) can be re-arranged to decompose the news about the log surplus into two parts:

$$ns_{t+1} - E_t ns_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^b_{t+j+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta ns_{t+j+1}. \tag{7}$$

A surprise increase in the (weighted log) surplus today must correspond to either surprise lower surplus growth in the future or to surprise higher expected returns on government debt in current and future periods. As a corollary, we can infer news about current and future surplus growth from news about the returns on government debt.

Exogenous Shocks to Government Expenditures We decompose the news about future surplus growth into two components: news about *exogenous* spending increases and news about endogenous future (weighted) surplus growth. In much of this paper, we identify exogenous spending with defense spending and anticipating this denote growth in exogenous spending by $\{\Delta g_t^{def}\}$. We denote endogenous surplus growth by $\{\Delta ns_t^{endo}\}$. If defense spending excludes exogenous and uncorrelated components of the government's surplus, then the precision of our subsequent empirical estimates of fiscal hedging will be reduced. If defense spending contains an endogenous component that increases in response to shocks to the non-defense surplus, then our later hedging estimates will be biased downwards.

We use μ_{def} to denote the average fraction of government spending that is exogenous (defense) spending. Re-arranging the previous equation produces the following relation between news about expenditures, returns on government debt and future surplus growth:

$$\begin{aligned}(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta g_{t+j+1}^{def} \quad &= \quad -\frac{1}{\mu_g} \frac{1}{\mu_{def}} \left((E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+j+1}^b \right) \\ &\quad + \frac{1}{\mu_g} \frac{1}{\mu_{def}} \left((E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta ns_{t+j+1}^{endo} \right). \qquad (8)\end{aligned}$$

The above equation implies that an increase in the expected PDV of exogenous expenditure growth has to coincide with one of two things: a decrease in the expected PDV of real returns on debt, or an increase in the expected PDV of future endogenous surplus growth. We will refer to the first of these adjustments as government hedging, broadly defined. Notice that the (linearized) budget constraint implies that the relevant measure of a spending shock is the innovation to the expenditure growth PDV: $(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta g_{t+j+1}^{def}$. When the government is fully hedged, the decline in expected future real returns completely offsets the increase in exogenous (i.e. defense) spending growth. The second effect absorbs the slack: if changes in expected returns on government debt do not offset the effect of news about an increase in exogenous expenditure growth, the government will have to run larger surpluses in the future.

A. Quantifying Government Hedging

Ex ante versus ex post hedging Government hedging broadly defined occurs either through a contemporaneous decline in the returns on the government's debt portfolio when the news about higher future spending growth is revealed, or a decline in expected future returns on the government's debt portfolio. We distinguish between these two channels and label them *ex ante hedging* and *ex post hedging* respectively. Normative theory emphasizes the value of ex ante hedging as a device for smoothing the excess burden of taxation.

To simplify matters, we introduce some additional notation. Denote news about current and future (exogenous) defense spending growth by:

$$\widetilde{h}^{g,def}_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta g^{def}_{t+j+1},$$

news about current returns on government debt by:

$$\widetilde{r}^b_{t+1} = r^b_{t+1} - E_t r^b_{t+1},$$

and news about future returns on government debt by:

$$\widetilde{h}^{r^b}_{t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^b_{t+j+1}.$$

The linearized budget constraint (7) then implies that news about log weighted surplus growth $\widetilde{h}^{ns}_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta ns_{t+j+1}$ is given by $\widetilde{r}^b_{t+1} + \widetilde{h}^{r^b}_{t+1}$.

With these pieces of notation in place, we may formally define ex ante hedging to be a negative covariance between innovations to current returns and news about exogenous defense spending growth in the current period and in the future:

$$cov\left(\widetilde{h}^{g,def}_{t+1}, \widetilde{r}^b_{t+1}\right) < 0.$$

Analogously, ex post hedging is a negative covariance between news about future expected returns on government debt and news about expenditure growth in the current period and in the future:

$$cov\left(\widetilde{h}^{g,def}_{t+1}, \widetilde{h}^{r^b}_{t+1}\right) < 0.$$

Measuring the government portfolio's g-beta To asses how well the government is hedged, we compute the government portfolio betas of news about current and future returns, one for ex post hedging, β^p, and one for ex ante hedging, β^a, and a last one for the total amount of hedging by the government β^f:

$$\begin{aligned} \widetilde{h}^{r^b}_{t+1} &= \beta^p_0 + \beta^p_1 \widetilde{h}^{g,def}_{t+1} + \varepsilon_{t+1} \\ \widetilde{r}^b_{t+1} &= \beta^a_0 + \beta^a_1 \widetilde{h}^{g,def}_{t+1} + \varepsilon_{t+1} \\ \widetilde{r}^b_{t+1} + \widetilde{h}^{r^b}_{t+1} &= \beta^f_0 + \beta^f_1 \widetilde{h}^{g,def}_{t+1} + \varepsilon_{t+1} \end{aligned} \tag{9}$$

If the total g-beta is one, the news about the sum of current and future returns changes by 1 basis point when the news about current and future defense expenditure growth changes by 1 basis point.

These betas map directly into fractions of total exogenous expenditure risk hedged by the government. If $\beta^p_1 \frac{1}{\mu_g \mu_{def}}$ is minus one, the government is obviously fully hedged in the ex post sense. The government budget constraint does not require any additional decline in future deficit growth. Similarly, if $\beta^a_1 \frac{1}{\mu_g \mu_{def}}$ is minus one, the government is obviously fully hedged in the ex ante sense. No additional adjustment is required.

V. Empirical results

A. Estimating the news variables

A benchmark VAR We use unrestricted VARs to forecast future government debt returns and defense spending growth; from these forecasts we construct estimates of return and defense spending news. We do not impose identifying restrictions on the VARs and we make no attempt to isolate deep structural shocks. To begin with we define the state vector z_t to include r^b_t, the real log returns on government debt, $\widetilde{nsb}_t$, i.e. $ns_t - b_t$, with the weights computed from the co-integrating vector, and Δg^{def}_t, the HP-filtered growth rate of defense spending. Note that the inclusion of $\widetilde{nsb}_t$ is motivated by our linearized budget constraint (5). By this condition, $\widetilde{nsb}_t$ is likely to contain useful information about future return and spending growth. The vector z_t also includes inflation and the slope of the yield curve, as additional forecasting variables. The slope of the yield curve is defined as the difference between the 10-year and the one-year yield on zero-coupon bonds. Thus,

$$z_t = \begin{pmatrix} r^b_t & \pi_t & \widetilde{nsb}_t & sl_t & \Delta g^{def}_t \end{pmatrix}.$$

All variables (except inflation and the slope of the yield curve) are deflated using the Consumer Price Index (BLS). We use quarterly data. We demean all the variables and impose a first-order structure on the VAR:

$$z_t = Az_{t-1} + \varepsilon_t.$$

Table I reports the GMM estimates with the t-statistics. The table shows that this simple specification does reasonably well in explaining the returns on government debt. Additionally, $\widetilde{nsb}_t$ helps to predict the returns on government debt: the coefficient on the log surplus/debt ratio is negative and statistically significant. Not surprisingly, defense expenditure growth is hard to predict. Later, we include additional lags, which we stack into the state vector, and forecasting variables and show that our results are robust to these extensions.

Table I
VAR Estimates

The VAR includes a total of 5 variables. We use $ns_t - b_t$ for the log surplus/debt ratio with the weights obtained from the co-integrating vector. The VAR includes 1 lag (quarterly data). The sample is 1945.I-2003.IV. T-stats are reported in brackets. GMM estimates. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The last column reports the R-squared. The zero coupon prices are calculated from CRSP price data.

Variables	r^b_{t-1}	π_{t-1}	$\widetilde{nsb}_{t-1}$	sl_{t-1}	Δg^{def}_{t-1}	R^2
r^b_t	0.049	−0.479	−0.231	−0.178	0.005	0.133
	[0.359]	[−2.569]	[−3.195]	[−1.160]	[1.142]	
π_t	−0.024	0.509	0.001	−0.147	−0.003	0.373
	[−0.675]	[7.464]	[0.021]	[−2.811]	[−1.260]	
$\widetilde{nsb}_t$	−0.097	−0.203	0.899	−0.023	0.000	0.862
	[−2.709]	[−2.086]	[39.100]	[−0.448]	[0.409]	
sl_t	−0.008	−0.038	−0.010	0.848	−0.000	0.734
	[−0.134]	[−0.503]	[−0.496]	[15.534]	[−1.005]	
Δg^{def}_t	0.060	1.286	0.704	0.386	−0.041	0.061
	[0.224]	[2.212]	[1.665]	[1.039]	[−1.916]	

Calculating the news variables We set $\rho = 1 - (\overline{S/B})$ equal to its post-war sample value of *.977*. We can then easily back out news about future defense spending growth from the VAR estimates:

$$\widetilde{h}^{g,def}_{t+1} = e_5(I - \rho A)^{-1}\varepsilon_{t+1}.$$

Similarly, we may obtain news about current and future government debt returns as:

$$\widetilde{r}^b_{t+1} = e_1\varepsilon_{t+1},$$

and

$$\widetilde{h}^{r^b}_{t+1} = e_1 \rho A (I - \rho A)^{-1} \varepsilon_{t+1}.$$

B. Hedging results

Empirical correlations Table II reports the raw correlations between the news variables on its off diagonals; the diagonals contain the standard deviations of these variables. The main results are as follows. First, news about current and future debt returns are positively correlated; there is no multi-variate mean-reversion in debt returns. Second, news about current returns on government debt and news about future defense spending are only weakly negatively correlated (-*.29*), but this news is strongly negatively correlated with news about future returns on government debt (-*.68*). The defense innovations are three times more volatile than the government debt return innovations.

Figure 3 plots $\widetilde{h}^{g}_{t+1}$ and $\widetilde{h}^{r^b}_{t+1}$. The two largest positive shocks to expected future defense spending growth occur in 1947 and 1950 at the start of the Korean war. As is apparent from Figure 3, these are accompanied by negative shocks to the *PDV* of expected future returns on government debt of roughly 1/4th the size of the two g-shocks -both objects are in the same units. The defense shocks in the remainder of the sample are much smaller, but the negative correlation between shocks to future expected returns and defense spending growth is still apparent. Figure 4 plots the current innovations to returns (i.e. in the same quarter) against the shocks to current and future defense spending growth. The contemporaneous response of government debt returns is much smaller than the shocks to defense spending growth.

Table II
Correlation between innovations

The diagonals report the standard the deviations and the off-diagonals report the correlations. The VAR contains 5 variables; it is first-order on quarterly data. We use $ns_t - b_t$ for the log surplus/debt ratio with the weights obtained from the co-integrating vector. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The last column reports the R-squared. The zero coupon prices are constructed from CRSP price data.

	$\widetilde{r}^b_t$	$\widetilde{h}^{r^b}_t$	$\widetilde{h}^{g,def}_t$	$\widetilde{h}^{nd}_t$	$\widetilde{\pi}_t$	$\widetilde{h}^{\pi}_t$
$\widetilde{r}^b_t$	**0.02**					
$\widetilde{h}^{r^b}_t$	**0.39**	**0.02**				
$\widetilde{h}^{g,def}_t$	**-0.29**	**-0.68**	**0.07**			
$\widetilde{h}^{nd}_t$	0.82	0.85	−0.59	0.03		
$\widetilde{\pi}_t$	−0.62	**-0.19**	0.10	−0.48	**0.01**	
$\widetilde{h}^{\pi}_t$	−0.71	**-0.26**	0.10	0.17	0.74	**0.02**

Figure 3. News about Defense Spending Growth and Future Debt Returns: Plots the innovations to future government debt returns $\widetilde{h}_t^{r^b}$ and innovations to current and future defense spending growth $\widetilde{h}_t^{g,def}$. The scale on the right hand side is for $\widetilde{h}_t^{g,def}$. Innovations were computed from the 5-dimensional VAR. The sample is 1946.I-2003.IV.

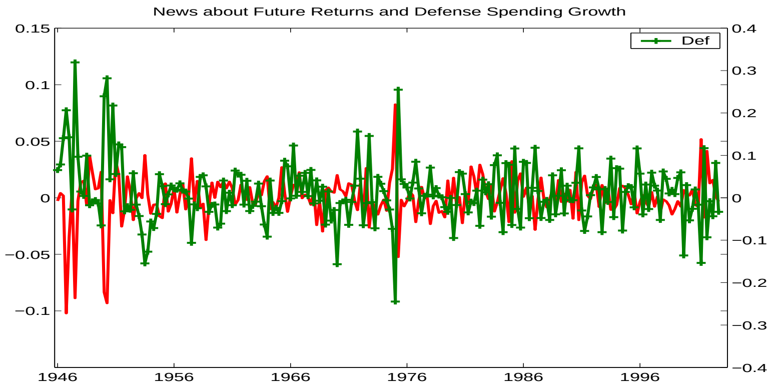
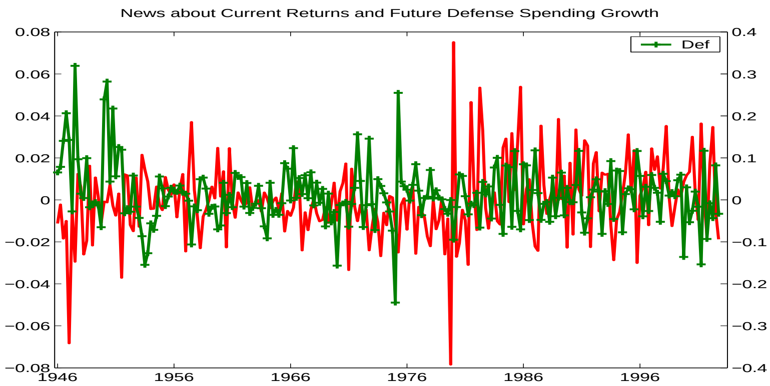


Figure 4. News about Defense Spending Growth and Current Debt Returns: Plots the innovations to current government debt returns $\widetilde{r}_t^b$ and innovations to current and future defense spending growth $\widetilde{h}_t^{g,def}$. The scale on the right hand side is for $\widetilde{h}_t^{g,def}$. Innovations were computed from the 5-dimensional VAR. The sample is 1946.I-2003.IV.



***g*-Betas** Table III reports the g-betas for the government portfolio. The ex post g-beta is *-.186*, its ex ante counterpart is *-.075*. Both are significantly different from zero, but the ex post beta is much more precisely estimated. This adds up to a total beta of *-.26*: a one percent increase in the *PDV* of defense spending growth on average induces a *twenty-six* basis points drop in the *PDV* of returns on outstanding debt. Over the post-war sample, variation in the *PDV* of defense spending growth can account for *thirty-five percent* of the total variation in current and future returns on the federal government's outstanding debt.

Table III
Hedging Betas

We regress $\widetilde{h}^{r^b}$, $\widetilde{r}^b$ etc. on $\widetilde{h}^{g,def}$. The VAR used to compute the innovations contains 5 variables; it is first-order on quarterly data. We use $ns_t - b_t$ for the log surplus/debt ratio with the weights obtained from the co-integrating vector. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The last but one column reports the R-squared. The final column is the hedging fraction. The zero coupon prices are computed from CRSP price data.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.002	**-0.186**	0.062	0.102
	[2.214]	[−6.798]		
$\widetilde{r}^b$	0.001	**-0.075**	0.085	0.041
	[0.677]	[−4.222]		
$\widetilde{r}^b + \widetilde{h}^{r^b}$	0.003	**-0.260**	0.347	0.143
	[1.723]	[−8.593]		
$\widetilde{r}^\pi + \widetilde{h}^\pi$	−0.001	**0.034**	0.012	0.018
	[−0.357]	[1.433]		

Quantifying fiscal hedging To compute the fraction of defense expenditures that is hedged by the government, we need values for the share of defense expenditure in total (federal government) expenditures μ_{def} and μ_g, the weight on expenditures in the log surplus ratio. Over the 1946.I-2003.IV sample, the defense spending/total spending ratio is *.41* and the weight on expenditures is *4.5*.

What fraction of defense shocks is hedged? Because of the response of the term structure to news about defense spending, the government hedges on average about ten percent of its total defense expenditure risk through ex post hedging, and a much smaller fraction, about four percent through ex ante hedging. This adds up to a total of *14 percent* (last column of Table III) for the post-war US sample.

Obviously, these fractions are very sensitive to the weights we use. The sample average of forty percent for defense spending seems high; at the end of our sample, in 2003.IV, the defense/total spending ratio was only twenty-four percent. Evaluated at that end-of-sample ratio, the government is hedged against *twenty-four percent* of defense spending risk, rather than fourteen percent.

C. Additional results

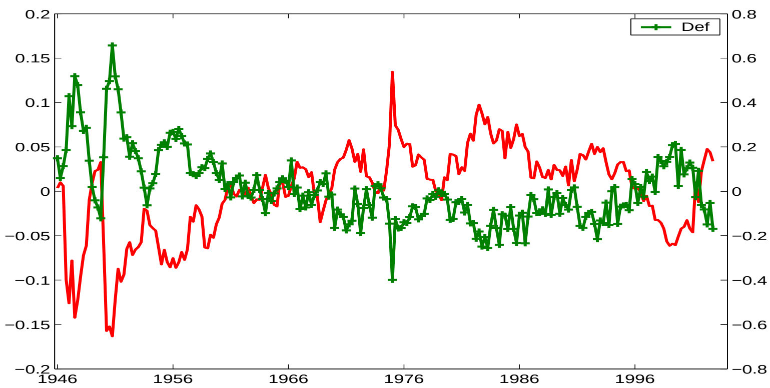
Expected Returns The correlation between innovations to spending and returns adds up to a striking pattern between the expected returns and the expected defense spending. Instead of considering the innovations, we can also look at the expected *PDV* of future defense spending growth $h_{t+1}^{g,def} = E_t \sum_{j=1}^{\infty} \rho^j \Delta g_{t+j}^{def}$; Figure 5 plots it (in deviations from the sample mean) against the expected PDV of future returns on government debt $h^{r^b}_{t+1} = E_t \sum_{j=1}^{\infty} \rho^j r^b_{t+j}$, where $h^{g,def}_{t+1}$ and $h^{r^b}_{t+1}$ are calculated as:

$$h^{r^b}_{t+1} = e_1 \rho A (I - \rho A)^{-1} z_{t+1}$$
$$h^{g,def}_{t+1} = e_5 \rho A (I - \rho A)^{-1} z_{t+1}$$

Over the entire sample, the correlation between these two objects is *-.93*. For example, at the start of Korean war, expected defense spending growth increases in PDV to roughly *eighty percent* above its sample mean, while the expected returns on government debt decrease to *fifteen percent* below its sample mean. Similarly, in the late nineties, expected defense spending was five percent above its mean and expected returns on bonds were five percent below the mean.

Obviously, this strong negative correlation merely reflects the fact that the loadings on the state vector for both objects have opposite signs, but this finding is remarkably robust to changes in the specification (including additional lags and forecasting variables).

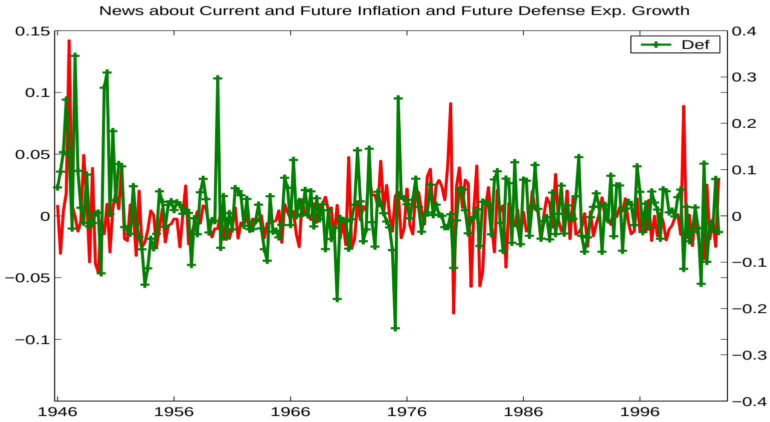
Figure 5. Expected Defense Spending Growth and Expected Future Debt Returns: Plots expected future government debt returns $h^{r^b}_t$ and expected current and future defense spending growth $h^{g,def}_t$, both in deviations from the sample mean. The scale on the right hand side is for $h^{g,def}_t$. Expected values were computed from the 5-dimensional VAR. The sample is 1946.I-2003.IV.



The role of inflation At quarterly frequencies, inflation only seems to account for a small part of the decrease in real returns on government debt after a surprise shock to defense spending. The correlation between news about future inflation and news about total defense spending growth is *.10*; the same correlation is *.106* for news about current inflation.

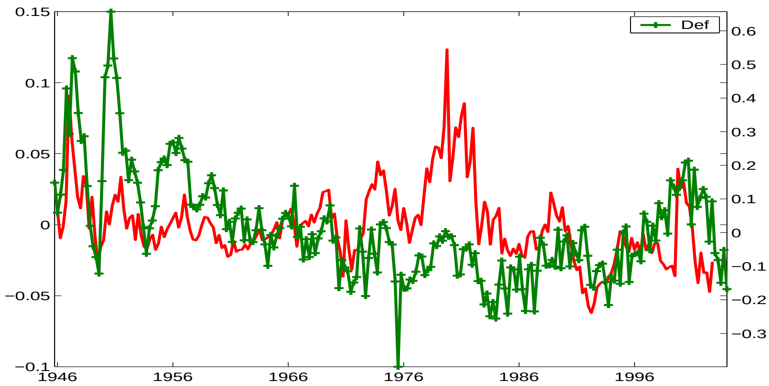
However, there are two very large surprises for defense spending in the early part of the sample:

Figure 6. News about Defense Spending Growth and Inflation: Plots the innovations to current and future inflation $\widetilde{r}_t^{\pi} + \widetilde{h}_t^{\pi}$ and innovations to future defense spending growth $\widetilde{h}_t^{g,def}$. The scale on the right hand side is for $\widetilde{h}_t^{g,def}$. Innovations were computed from the 5-dimensional VAR. The sample is 1946.I-2003.IV. ρ was set equal to .977.



one in 1946/1947 and one in 1950. These are accompanied by substantial surprises about current and future inflation. Figure 6 plots the news about current and future inflation against news about current and future defense spending growth. Immediately after the second world war expected inflation rises *ten percent* above its mean in *PDV*, and it increases to *four percent* above its mean at the outset of the Korean war (see Figure 7). This pattern breaks down in the seventies and the eighties. On average, over the entire sample, inflation accounts for only one percent or a tenth of the total amount of hedging (see last row in Table III).

Figure 7. Expected Defense Spending Growth and Expected Inflation: Plots expected current and future defense spending growth h_t^{def} and expected current and future inflation $r_t^{\pi} + h_t^{\pi}$, both in deviations from the sample mean. The scale on the right hand side is for $h_t^{g,def}$. Expected values were computed from the 5-dimensional VAR. The sample is 1946.I-2003.IV.



VI. Implications for debt management: Why issue long term debt?

Why does the government issue long-term debt at all? If the Treasury's objective is simply to minimize the cost of outstanding debt, issuing long-term debt seems hard to rationalize. Its average return is higher than that of short-term debt and it is more volatile.

A. Average Cost of Funds

We use $hpr_t = p_{t+12}^{N-1} - p_t^N$ to denote the log 12-month holding return on a zero coupon bond of maturity N (in years), while $y_t^N = -\frac{1}{N} p_t^N$ denotes the log yield on a zero coupon bond of maturity N (in years).

On average, the one-year excess return on a 10- year zero coupon bond was sixty basis points higher over the postwar sample (compared to a one-year zero coupon), while the average yield spread was about forty-five basis points higher. Our second sample (the Fama-Bliss sample) confirms these results (see Table IV). Of course, the data are not definitive because the standard deviation of the excess returns (and, to a lesser extent the spreads) is very high. Still, cost considerations alone suggest that the government ought to be exploring the short end of the maturity structure: longer debt is more expensive and its returns are more volatile, causing investors to bear more risk.

Table IV
Yield Spreads and Excess Returns

We compute the 12-month log holding period return $hpr_{t+12}^N = p_{t+12}^{N-1} - p_t^N$ (in excess of the return on a 3-month T-bill) and the yield spread $y_t^n - r_t^{tb}$(relative to a 3-month T-bill) using monthly data. The standard deviations are in parentheses. Zero coupon yield curve constructed from CRSP data (see appendix).

Maturity	1	2	5	10
		1946.1-2003.12		
Spread	0.95	1.04	1.20	1.37
	(0.77)	(0.85)	(1.04)	(1.31)
Excess Return	0.95	1.11	1.24	1.66
	(0.76)	(1.90)	(5.21)	(9.47)
		1952.1-2003.12		
Spread	0.98	1.08	1.25	1.42
	(0.80)	(0.88)	(1.09)	(1.37)
Excess Return	0.97	1.15	1.31	1.77
	(0.80)	(2.00)	(5.49)	(9.98)

B. Active Maturity Management

The expectations hypothesis fails in US data: on average, a high forward spread predicts mainly high holding returns on bonds, not just higher interest rates in the future. Table V reports the excess returns on a managed portfolio that goes long in high forward spread maturities and shorts low forward spread maturities. As a result, the government should be issuing more long term debt only when the forward spread (the marginal increase in the yield as the maturity increases) is low, because that predicts, on average, low excess returns on this maturity. When the forward spread is high, it should buy back long term debt instead.[9]

Is this what the US government actually does? Not quite. Figure 8 plots the forward spread against the average maturity. The government did reduce the average maturity starting in the mid-sixties, when forward spreads increased dramatically and it did increase the average maturity when the spreads started to drop in the early eighties, but it continued to issue more long-term debt in the nineties. The forward spread reached four percent in 1991.

Table V
Excess Returns on Managed Portfolios

We compute the 12-month log holding period return on a portfolio that goes long when the forward spread is high: $(hpr^N_{t+12} - y^1_t)(f^{N-1\rightarrow N}_t - y^1_t)$, using monthly data. The standard deviations are in parentheses. Zero coupon yield curve constructed from CRSP price data.

Maturity	2	5	10
	1946.1-2003.12		
Excess Return	0.23	1.43	10.52
	(1.12)	(8.42)	(43.37)
	1952.1-2003.12		
Excess Return	0.26	1.64	11.91
	(1.18)	(8.91)	(45.82)

These arguments ignore the potential hedging benefits of longer-term debt.

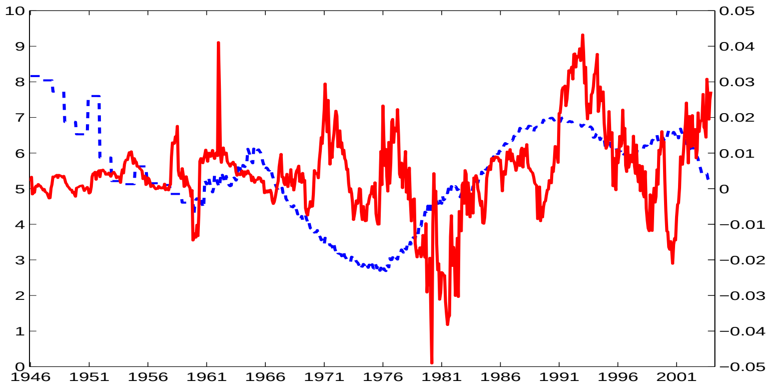
C. Maturity Structure of Government Debt

If we take the response of the term structure to defense expenditure shocks as given, how should the Treasury structure its debt in order to increase the state-contingency built in to its liabilities?[10]

[9] Such a strategy was advocated by Campbell (1995).

[10] This exercise is in the (partial equilibrium) spirit of Campbell (1995), but not of the (general equilibrium) optimal tax literature. Still it is a useful starting point.

Figure 8. Maturity Structure of Publicly Held Debt: Plots the average maturity of publicly held debt against the forward spread on a 5-year zero coupon (full line). The debt level is in market values (excludes the monetary base). The right axis is the forward spread. The sample is 1946.I-2003.IV.



Government issues only one Maturity To begin to answer this question, we assume the federal government only has one of the following securities outstanding: 1-year, 5-year, 10-year, 15-year and 20-year zero-coupon bonds. We include the real returns $r_t^{b,k}$ on each of these securities (in addition to all the previous variables) in a separate VAR. The state now includes six variables. Next, we re-compute the news about current and future government returns r^b and h^{r^b} assuming the government only has bonds of one maturity outstanding:

$$z_t = \begin{pmatrix} r_t^b & \pi_t & \widetilde{nsb}_t & sl_t & \Delta g_t^{def} & r_t^{b,k} \end{pmatrix}$$

where the maturity of the zero coupon bond ranges from one year to 20 years: $k = 1, 5, 10, 15, 20$. We re-estimate the VAR in each case.

Hedging The resulting beta estimates are reported in Table VI[11]. All of the estimated betas are significantly negative at the 5 percent level. The total g-beta of government debt returns increases from -*.21* for one-year debt to -*.41* for 15-year debt, almost double. Correspondingly, the fraction of total risk hedged increases from *eleven to twenty percent*, evaluated at the sample average weights. When evaluated at the current defense spending ratio of twenty-four percent, the fraction increases from *nineteen percent* to *thirty-six percent.*

There are two parts to this. First, the ex ante beta increases from -*.02* for the one-year bond to -*.16* for the 15-year bond. Second, the ex post beta increases from -*.18* to -*.25*. The higher ex

[11]The VAR estimates are not included.

ante beta just reflects the fact that the price of longer maturity bonds experience a much larger (in fact eight times larger) price drop when the news about higher future defense spending growth is revealed, and this is the main the source of the superior hedging performance of longer maturities.

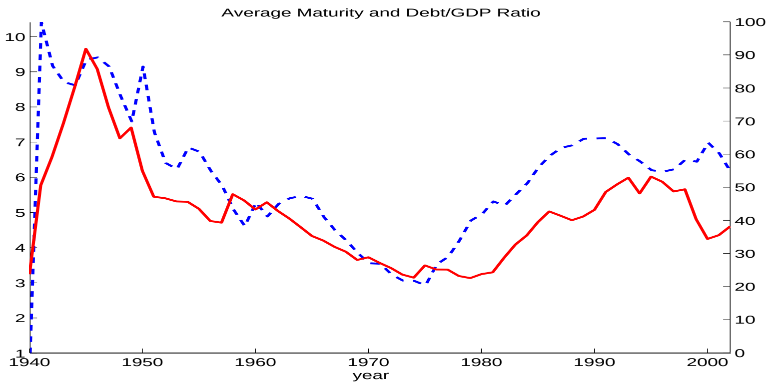
Table VI
g-Betas

This table reports the regression coefficients in the g-beta regressions, maturity by maturity. The VAR contains 6 variables; it is first-order on quarterly data. We use $ns_t - b_t$ for the log surplus/debt ratio with the weights obtained from the co-integrating vector. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The fourth column reports the R-squared; the final column reports the hedging fractions. The zero coupon prices are constructed from CRSP data. In the third panel, we regress $\widetilde{h}^{r^b}$ and $\widetilde{r}^b$ on $\widetilde{h}^{g,def}$.

	β_0	β_1	R^2	*fraction*
Maturity		*Hedging Betas* $\widetilde{h}^{r^b}$		
1	0.002	**-0.187**	0.391	0.102
	1.911	-7.148		
5	0.003	**-0.217**	0.477	0.119
	[2.322]	[-6.897]		
10	0.002	**-0.172**	0.219	0.094
	[1.447]	[-4.778]		
15	0.003	**-0.245**	0.293	0.134
	[1.717]	[-5.186]		
20	0.004	**-0.250**	0.242	0.137
	[1.444]	[-4.725]		
		Hedging Betas $\widetilde{r}^b$		
1	0.000	$-$**0.023**	0.017	0.013
	[0.299]	[-1.929]		
5	0.001	**-0.096**	0.040	0.052
	[0.485]	[-3.314]		
10	0.002	**-0.138**	0.035	0.076
	[0.409]	[-3.014]		
15	0.002	**-0.165**	0.030	0.090
	[0.376]	[-2.959]		
20	0.001	**-0.109**	0.060	0.036
	[0.179]	[-1.512]		
		Hedging Betas $\widetilde{h}^{r^b} + \widetilde{r}^b$		
1	0.002	$-$**0.210**	0.387	0.115
	[1.823]	[-7.257]		
5	0.004	**-0.313**	0.256	0.172
	[1.356]	[-8.653]		
10	0.004	**-0.310**	0.112	0.173
	[0.866]	[-6.213]		
15	0.005	**-0.410**	0.131	0.225
	[0.900]	[-6.898]		
20	0.005	**-0.359**	0.059	0.197
	[0.612]	[-4.230]		

Actual Maturity Structure of Publicly Held Debt The actual maturity structure of US government debt does fluctuate substantially at low frequencies, and it seems to track the debt/GDP ratio: when the government has to issue lots of new debt, it does so at the long end of the maturity structure. At the end of the Second World War, the average maturity was around 10 years, while the average maturity was around three years in the mid-seventies. This explains the dramatic decline in the *PDV* of returns on government debt at the start of the Korean war. The average maturity starts to increase again in the eighties, and this clearly shows in Figure 9.

Figure 9. Maturity Structure of Publicly Held Debt: Plots the average maturity of publicly held debt against the debt/gdp ratio (full line). The debt level is in market values (excludes the monetary base). The right axis is the debt/GDP ratio scale. The sample is 1946.I-2003.IV.



VII. Robustness

To check the robustness of our results, we add one more lag to the VAR; we add the excess returns on defense stocks relative to the CRSP-VW return $r_t^{def} - r_t^m$ and the log price/dividend ratio of defense stock (relative to the market) as additional forecasting variables. The state space now includes 12 variables (including the lags):

$$z_t = \begin{pmatrix} r_t^b & \widetilde{nsb}_t & \pi_t & \Delta g_t^{def} & pd_t^{def-m} & r_t^{def-m} \\ r_{t-1}^b & \widetilde{nsb}_{t-1} & \pi_{t-1} & \Delta g_{t-1}^{def} & pd_{t-1}^{def-m} & r_{t-1}^{def-m} \end{pmatrix}$$

Defense Shocks and Defense Stocks By fiscal hedging, we mean the use of government debt returns to absorb variations in the expected *PDV* of (exogenous) defense spending growth. To assess the extent of fiscal hedging, it is clearly essential to forecast future defense spending. Two factors potentially complicate the extraction of such forecasts from macro-data. First, agents may learn about the political/military events driving future defense spending growth in advance of this growth occurring or affecting other macro-variables. Thus, VARs relying on such data may fail to identify the true date of the shock. Second, innovations to the expected *PDV* of defense spending growth may not result in realized increases in defense spending. An international dispute may raise expectations of future defense spending, but if the dispute is resolved through negotiation, this spending may not occur. In this case, a VAR relying on defense spending and other macro-data may fail to detect the shock at all.

In short, our exercise relies on a rich enough of model of the agent's information set that encompasses relevant political and military events. To partially address these issues, we extend our VARs to include information embedded in the returns on defense stocks. Defense stock return variables should respond contemporaneously to news about perceived future defense spending growth and, hence, their inclusion partially addresses the above concerns. As noted above, we include the returns on defense stocks and the the defense stock dividends/price ratio (both relative to market). It follows from standard linear expansions of return equations that both of these variables contain information on the PDV of future defense dividend growth. Insofar as dividends and defense firm profits are tied to defense spending, these variables are revealing of shocks to the PDV of defense spending growth.

Results Naturally, this specification does much better in explaining the variation in defense spending growth: the R^2 more than doubles to fourteen percent (see Table VII), and we also do much better in explaining the variation in quarterly returns on government debt (the R^2 is eighteen percent).

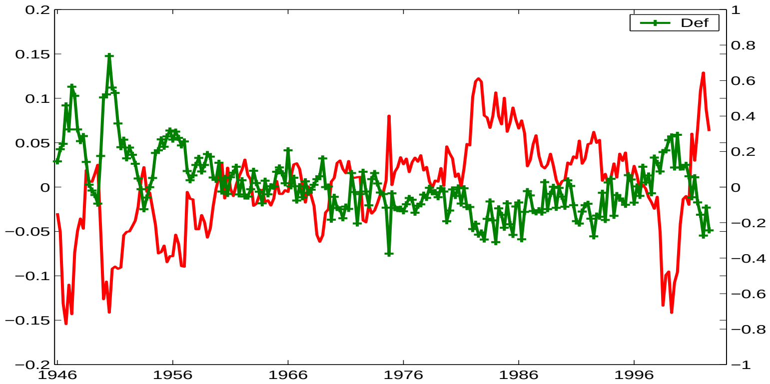
The excess returns on defense stocks not only help to predict future defense spending growth, but also future returns on the government's liabilities. This is exactly what we would expect to find: if the defense stock returns contain new information about future defense spending growth and the government manipulates interest rates to hedge, then the return on government debt will immediately respond.

Hedging Our hedging results are essentially unchanged. A total of *thirteen percent* of total defense spending shocks are hedged; ten percent is hedged ex post and three percent is hedged ex ante. The betas are estimated even more precisely than in the original specification: all of the estimated coefficients a re significant at the one percent level. The ex ante beta is *-.18*, the ex post beta is *-.6*; so, in total, the PDV of returns drops by twenty-four basis points when the PDV of defense spending growth increases by one percent.

In this specification, we also get a more precise estimate of the inflation hedge: the expected

PDV of future inflation increases by five basis points when the *PDV* of defense spending growth increases by one percent.

Figure 10. Expected Defense Spending Growth and Expected Future Debt Returns:
Plots expected future government debt returns $h_t^{r^b}$ and expected current and future defense spending growth $h_t^{g,def}$, both in deviations from the sample mean. The scale on the right hand side is for $h_t^{g,def}$. Expected values were computed from the 12-dimensional VAR. The sample is 1946.I-2003.IV.



A. Maturity Composition

Finally, we check our earlier results about the optimal maturity composition by running the 2-lag VAR with defense stock returns and the defense price/dividend ratio, but after replacing the return on government debt with the 1-year, 2-year, 5-year, 15-year and 20-year zero coupon bond[12]. The ex post beta increases from -*.16* at the one year maturity to -*.32* at the 20-year maturity; the ex ante beta increases from -*.03* to -*.13* at the 15-year maturity. This adds up to a total beta of *.35*, in line with our earlier results. The fraction of defense expenditure risk that is hedged varies from *eleven* percent to *twenty-two* percent.

[12]The VAR estimates are not reported.

Table VII: VAR Estimates

The VAR includes 2 lags (quarterly data). The sample is 1945.I-2003.IV. T-stats are reported in brackets. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The last column reports the R-squared. The zero coupon prices are computed from CRSP price data. $\widetilde{nsb}$ is constructed from weights obtained from the co-integrating vector.

Maturity	r^b_{t-1}	$\widetilde{nsb}_{t-1}$	π_{t-1}	Δg^{def}_{t-1}	dp^{def-m}_{t-1}	r^{def-m}_{t-1}	r^b_{t-2}	$\widetilde{nsb}_{t-2}$	π_{t-2}	Δg^{def}_{t-2}	dp^{def-m}_{t-2}	r^{def-m}_{t-2}	R^2
r^b_t	0.04	−0.19	−0.00	−0.04	0.02	−0.01	−0.01	−0.31	0.00	−0.00	−0.01	0.01	0.18
	[0.24]	[−0.79]	[−1.02]	[−1.94]	[1.52]	[−0.75]	[−0.12]	[−1.96]	[0.45]	[−0.02]	[−1.17]	[0.46]	
$\widetilde{nsb}_t$	−0.01	0.50	0.00	0.01	−0.00	−0.00	0.08	0.22	−0.00	0.01	−0.00	0.00	0.42
	[−0.19]	[4.76]	[0.75]	[1.09]	[−0.23]	[−0.35]	[2.22]	[2.30]	[−0.80]	[3.64]	[−0.11]	[0.50]	
π_t	−7.73	−13.58	0.98	0.57	−0.79	0.92	−6.93	−15.45	−0.12	0.00	0.58	−0.54	0.87
	[−2.35]	[−1.50]	[7.07]	[0.44]	[−1.43]	[0.97]	[−1.64]	[−0.87]	[−0.91]	[0.02]	[1.07]	[−0.68]	
Δg^{def}_t	−0.01	1.15	0.01	0.24	0.05	0.06	0.18	−0.08	−0.00	−0.01	−0.04	−0.04	0.14
	[−0.03]	[2.01]	[0.99]	[2.25]	[1.33]	[1.09]	[0.64]	[−0.12]	[−0.06]	[−0.51]	[−1.19]	[−0.85]	
dp^{def-m}_t	0.30	0.15	0.00	0.23	1.08	−0.00	0.37	2.26	−0.00	−0.03	−0.18	0.16	0.87
	[0.40]	[0.11]	[0.05]	[1.20]	[9.02]	[−0.02]	[0.37]	[1.11]	[−0.09]	[−1.38]	[−1.56]	[1.33]	
r^{def-m}_t	0.10	0.46	−0.01	−0.05	0.11	−0.08	−0.09	0.22	0.01	−0.01	−0.14	0.17	0.06
	[0.17]	[0.41]	[−0.77]	[−0.37]	[1.40]	[−0.62]	[−0.16]	[0.17]	[0.60]	[−0.53]	[−1.83]	[1.60]	

Table VIII
Correlation between innovations

The diagonals report the standard the deviations and the off-diagonals report the correlations. The VAR contains 6 variables and 2 lags on quarterly data. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix.

	$\widetilde{r}^b_t$	$\widetilde{h}^{r^b}_t$	$\widetilde{h}^{g,def}_t$	$\widetilde{h}^{ns}_t$	$\widetilde{\pi}_t$	$\widetilde{h}^{\pi}_t$
$\widetilde{r}^b_t$	**0.02**					
$\widetilde{h}^{r^b}_t$	0.51	**0.02**				
$\widetilde{h}^{g,def}_t$	**-0.29**	**-0.66**	**0.09**			
$\widetilde{h}^{ns}_t$	0.83	0.90	−0.57	0.04		
$\widetilde{\pi}_t$	−0.59	−0.20	0.04	−0.42	0.01	
$\widetilde{h}^{\pi}_t$	−0.50	−0.45	0.25	−0.54	0.89	0.02

Table IX
Hedging Betas

We regress h^{r^b},r^b etc. on $h^{g,def}$. The VAR used to compute the innovations contains 6 variables and 2 lags on quarterly data. We use $ns_t - b_t$ for the log surplus/debt ratio with the weights obtained from the co-integrating vector. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The fourth column reports the R-squared and the final column reports the hedging fraction.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.002	**-0.179**	0.435	0.098
	[1.587]	[−10.315]		
$\widetilde{r}^b$	0.001	**-0.059**	0.083	0.032
	[0.979]	[−4.379]		
$\widetilde{h}^{r^b} + \widetilde{r}^b$	0.003	**-0.238**	0.330	0.130
	[1.501]	[−10.006]		
$\widetilde{h}^{\pi} + \widetilde{r}^{\pi}$	−0.001	**0.051**	0.034	0.028
	[−0.367]	[3.214]		

Table X
***g*-Betas**

This table reports the regression coefficients in the g-beta regressions, maturity by maturity. The VAR contains 6 variables; it is second-order on quarterly data. The sample is 1945.I-2003.IV. We use the Newey-West variance-covariance matrix with 4 lags as the weighting matrix. The fourth column reports the R-squared and the last column reports the hedging fraction.

	β_0	β_1	R^2	*fraction*
	Hedging Betas $\widetilde{h}^{r^b}$			
1	0.002	−0.168	0.412	0.092
	[1.763]	[−8.700]		
5	0.002	−0.204	0.466	0.112
	[1.707]	[−8.932]		
10	0.002	−0.195	0.369	0.107
	[1.447]	[−7.620]		
15	0.003	−0.255	0.140	0.084
	[1.688]	[−8.238]		
20	0.004	−0.322	0.177	0.107
	[1.563]	[−8.930]		
	Hedging Betas $\widetilde{r}^b$			
1	0.001	−0.035	0.060	0.019
	0.964	−2.933		
5	0.001	−0.068	0.030	0.037
	[0.547]	[−3.258]		
10	0.002	−0.130	0.045	0.071
	[0.471]	[−3.549]		
15	0.002	−0.133	0.029	0.073
	[0.373]	[−3.006]		
20	0.001	−0.034	0.001	0.019
	[0.176]	[−0.551]		
	Hedging Betas $\widetilde{r}^b + \widetilde{h}^{r^b}$			
1	0.003	−0.203	0.317	0.111
	[1.594]	[−7.023]		
5	0.003	−0.272	0.232	0.149
	[1.129]	[−8.456]		
10	0.004	−0.325	0.154	0.178
	[0.829]	[−7.128]		
15	0.005	−0.388	0.147	0.213
	[0.823]	[−6.677]		
20	0.005	−0.356	0.077	0.195
	[0.627]	[−4.333]		

VIII. Defense Shocks and predictability

Measuring defense shocks Until now we have treated $\widetilde{h}_{t+1}^{g,def}$ as a measure of a defense spending shock. Implicitly, we have treated defense spending as exogenous. Additionally, in computing this and other news variables, we have adopted the stance of an econometrician with access to the entire sample. The econometrician uses the estimated VAR coefficients over the entire sample. Obviously, agents cannot run this VAR, extract the residuals and compute the news variable $\widetilde{h}_t^{def}$. This is a source of concern in any forecasting exercise that uses information that is not measurable w.r.t. agent's information set. In this section, we propose a new measure of defense spending shocks that partially addresses these concerns. Specifically, we estimate abnormal returns on defense stocks using rolling regression windows. We then construct dummies from statistically significant abnormal returns.

Innovations to defense stock returns reflect news about future dividend growth and future risk premia. By focussing on *abnormal* returns, we seek to isolate that component due to innovations in current and future dividend growth. Insofar as dividend growth and profits are tied to US government defense spending, we obtain an estimate of news about future defense spending growth.[13] By using rolling regressions on moving windows, we avoid using information that is not in the agent's information set.

The literature has proposed two other methods for isolating fiscal shocks. First, a series of contributors has made various identifying assumptions on VAR's to isolate shocks to overall (rather than defense) spending by the government, see for example Blanchard and Perotti (1999). However, the VAR's used may not adequately describe agent's information sets. In particular, agents may receive news about future spending increases sometime before these increases occur and sometime before the spending shock is detected by existing VAR analyses. Ramey (2006) cogently argues how this failure to identify the true timing of shocks can seriously bias estimates of the effect of shocks.[14] The alternative narrative approach (see Ramey and Shapiro (1998)) identifies major political/military events that led to subsequent increases in government spending. While this approach may resolve timing issues, it suffers from the fact that there are few of these events (four in the post war period). Additionally, while each Ramey-Shapiro date precedes a major build up of defense spending, it is less clear how these events were perceived at the time.

[13]Other factors may drive these abnormal returns. Insofar as they are uncorrelated with US defense spending, they will reduce the precision of our estimates.

[14]Ramey's focus is on real variables, but the bias is likely to be more severe with respect to asset prices.

Estimating abnormal returns To construct the abnormal returns on defense shocks, we project the excess returns $R_t^{e,def}$ on the market excess return $R_t^{e,m}$.[15] Our model for the returns on the defense industry is:

$$R_t^{e,def} = \alpha_{def} + \beta_{def} R_t^{e,m} + \varepsilon_t$$

We estimate the market model on $T = 24$-month windows. The residuals $\{\varepsilon_t\}$ are the abnormal returns. These variables are orthogonal to the market return and are not (easily) predictable on the basis of past information available to investors. Define X_t^{def} to be the $T \times 2$ matrix of regressors. The variance covariance matrix of the residuals is given by:

$$V_i = I\sigma_\varepsilon^2 + X^{def}(X^{def\prime}X^{def})^{-1}X^{def}\sigma_\varepsilon^2$$

The first part captures the variance due to actual disturbances, while the second part captures the additional variance due to sampling uncertainty in the estimated coefficients. Next, we compute the t-statistic on the T-th abnormal return ε_T, the last observation in the estimation window. If this abnormal return is positive and significantly different from zero (at the 5 percent level), we classify this observation as a defense shock. We define a dummy variable $\{d_t^{def}\}$ that is one when the abnormal return on defense stocks is positive.

Figure 11 plots the defense shock dummies against a one-year moving average of Δg_t^{def}. While the dummies do precede large and persistent increases in the growth rate of defense spending, the timing varies somewhat.

A. Impulse responses

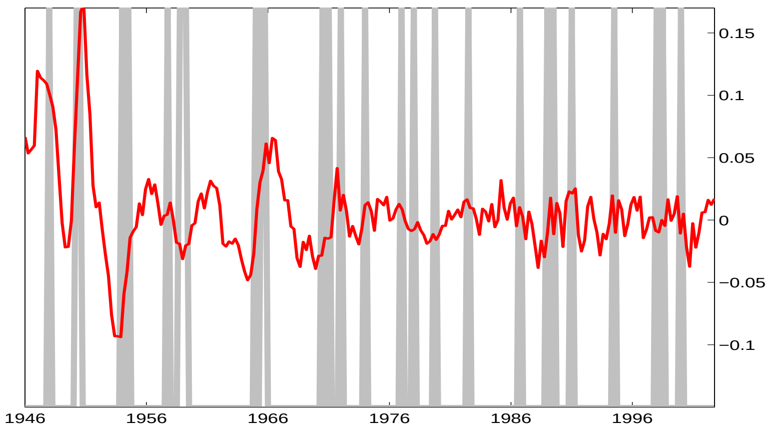
The defense dummies are taken to be completely exogenous and there is no need for VAR's to trace out the response of a variable y_t to a defense shock. Instead, we simply run a univariate regression of the variable of interest y_t on a constant, the exogenous defense dummy and six lags of the defense dummies and y_t over the 1946-I-2003.IV sample:

$$y_t = \alpha_0 + \Phi(L)d_t^{def} + \Theta(L)y_{t-1} + \varepsilon_t$$

We use quarterly data over the 1946.I-2003.IV sample. To compute the impulse-responses, we set $d_0^{def} = 1$ and simulate the effect on $\{y_t\}_{t=0}^{\infty}$. To construct the confidence intervals, we use

[15] We have also constructed abnormal returns by regressing defense returns onto Fama-French factors. The results are essentially unaltered.

Figure 11. Defense Expenditure Growth and Defense Shocks 1950-2003: The figure plots a one-year MA of the HP-filtered growth rate of defense expenditures against the defense shock dummies



a bootstrap Monte Carlo procedure. We draw a new time series of residuals from the estimated residuals (drawing randomly with replacement). Using the newly constructed time series on $\{y_t^s\}$ and defense shocks, we re-estimate the regression and construct the new impulse responses. For each lag we compute the 80th and the 420 impulse response coefficient (after sorting on size at each lag).

Defense Shocks Are our dummy variables really shocks to the *PDV* of defense expenditures? On average, a one-unit shocks raises the defense spending/GDP ratio by sixty basis points after 10 quarters (Figure 12). Looking at the growth rate of defense expenditures, a one-unit shock raises the HP-filtered growth rate of defense expenditures by about 200 basis points after 5 quarters. The estimated response of the growth rate is much less persistent.

Figure 12. Defense Exp. and Defense Shocks 1945-2003: The figure plots the impulse response of the Defense Spending/GDP (in percentage points) for a one-unit defense shock against the time in quarters. The impulse responses are constructed from a univariate regression of the defense/GDP ratio on 6 lagged values of itself and on the current and 6 lagged values of the defense dummies. We construct the 68 percent confidence interval by bootstrapping from the residuals with replacement.

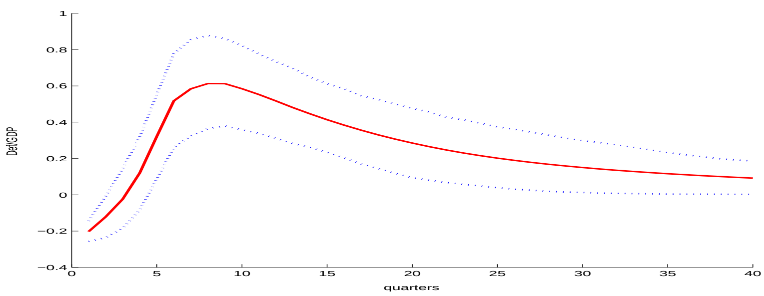
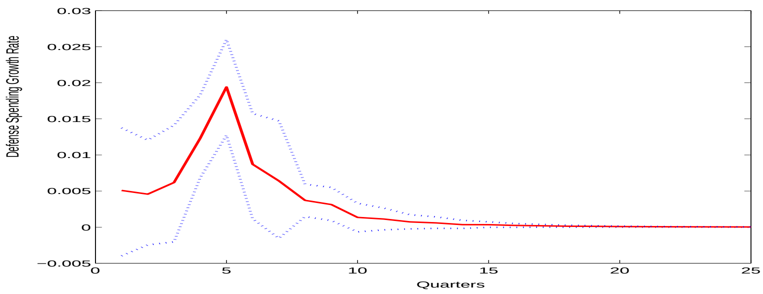


Figure 13. Defense Exp. Growth and Defense Shocks 1945-2003: The figure plots the impulse response of the defense spending growth rate for a one-unit defense shocks against the time in quarters. The impulse responses are constructed from a univariate regression of the Δg_t^{def} on 6 lagged values of itself and on the current and 6 lagged values of the defense dummies. The growth rate of defense spending was HP-filtered. We construct the 68 percent confidence interval by bootstrapping from the residuals with replacement.



Our dummies pick out all of the Ramey-Shapiro dates, although their profusion in the mid 1960's and early 1970's suggest that information about the spending implications of the Vietnam War was released slowly over this period, rather than at a single (Ramey-Shapiro) date. Our dummies also pick up events early in the Cold War such as the adoption of the Truman Doctrine and the Berlin air lift, defense growth spikes in the mid 1950's, early 1960's, the first Gulf War and 9/11.

Inflation, Interest Rates and Yields In response to a one-unit shock, the inflation rate increases by a total sixty basis points after 5 quarters and then reverts to normal after 10 quarters. The effect on zero coupon yields peaks at about fifty basis points after 5 quarters, but is more persistent than the inflation response. After five to 10 quarters, it seems like real yields decrease in response to a defense shock.

B. Predictability of Changes in Yields and Ex Ante Hedging

Lagged values of the defense dummies do predict future increases in the yields, across the board (see Table XI). Most of the estimated coefficients are statistically significant. The magnitudes of the coefficients are quite large.

All of this is consistent with the government slowly increasing the nominal short rate after the initial shock. This accounts for the ex ante hedging: bond markets anticipate the increase in future short run yields and there is an instantaneous drop in the market value of, especially, longer term outstanding bonds. This accounts for the greater degree of ex ante hedging obtained from longer term debt. (Recall Table X). These observations are consistent with the results of Lustig, Sleet and

Yeltekin (2006) who suggest that it is optimal for governments to hedge fiscal shocks by providing larger contemporaneous capital losses on longer term nominal debt.

Table XI
Predictability of Changes in Yields.

Regression of yield changes $y_{t+4}^{n-1} - y_t^n$ on a zero coupon bond with maturity n years between t and $t+4$ on lagged defense expenditure shocks: Regression of yield changes $y_{t+4}^{n-1} - y_t^n = b_0 + B(L)d_{t-1}^{def} + e_t$. Results shown for 6 lags. The sample is 1945.1-2003.4. T-stats are reported in brackets. The first row uses the truncated kernel of Hansen (1982) and Hansen and Hodrick (1980) as the weighting matrix with 4 lags. The second row uses the Newey-West variance-covariance matrix with 6 lags as the weighting matrix. The last column reports the R-squared and the χ^2-statistic. The zero coupon yields are constructed from CRSP.

Maturity	b_0	b_1	b_2	b_3	b_4	R^2	χ^2
1	−0.18	0.31	0.59	0.78	0.43	0.04	
	[−0.77]	[0.74]	[1.65]	[1.90]	[1.38]		4.55
	[−0.85]	[0.78]	[1.65]	[1.98]	[1.38]		5.11
5	−0.19	1.05	0.48	0.66	0.28	0.04	
	[−0.84]	[1.13]	[1.77]	[2.82]	[0.88]		22.27
	[−0.88]	[1.21]	[1.55]	[2.36]	[0.91]		5.79
10	−0.18	1.49	0.43	0.56	0.02	0.04	
	[−0.76]	[1.27]	[1.98]	[5.47]	[0.07]		
	[−0.77]	[1.37]	[1.53]	[2.44]	[0.07]		7.32
15	−0.12	1.25	0.38	0.40	0.01	0.04	
	[−0.59]	[1.72]	[1.77]	[3.20]	[0.02]		2.27
	[−0.59]	[6.24]	[1.81]	[1.46]	[0.02]		6.24

C. Predictability of Holding Returns and Ex Post Hedging

Defense shocks predict much lower subsequent (real) holding returns on bonds, especially for longer maturities.

We plot the slope coefficients in a regression of one-year holding returns on lagged defense shock dummies against the maturity of the zero coupon bond in Figure 14. The slope coefficients increase (in absolute value) almost monotonically in the maturity of the zero coupon bonds. The longer the maturity, the more (ex post) hedging this security provides against defense spending shocks. This confirms the VAR results. Table XII and report the regression results. Even though the defense shocks explain only 2-3 percent of the variation in returns, most of the coefficient estimates are significantly different from zero. A defense shock predicts a 360 basis points drop in the dollar return on 15-year bond over the next year, but only a seventy basis points drop in the dollar return on a one-year bond.

Figure 14. Holding Returns and Defense Shocks 1945-2002: The figure plots the slope coefficients in the regression of one-year holding returns on defense shocks against the lags, maturity by maturity. Regression of log holding returns $\log(p_{t+4}^{n-1}) - \log(p_t^n) = b_0 + B(L)d_{t-1}^{def} + e_t$. Results shown for 6 lags, on quarterly data. The zero coupon prices are the Fama-Bliss series from CRSP.

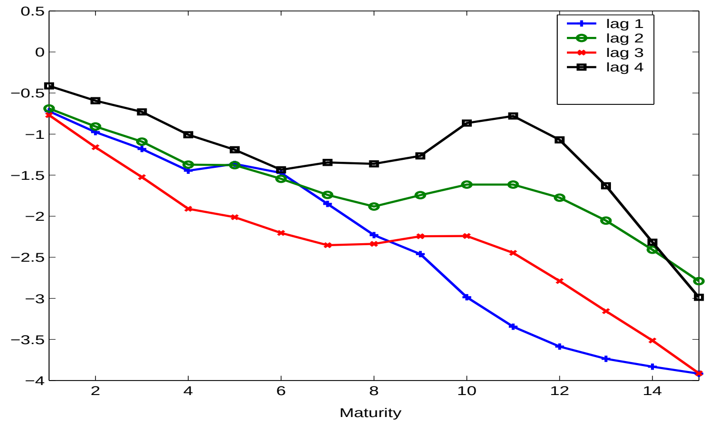


Table XII
Predictability of Holding Returns.

Regression of log holding returns $\log(p_{t+4}^{n-1}) - \log(p_t^n)$ on a zero coupon bond with maturity n years between t and $t+4$ on lagged defense expenditure shocks: Regression of log holding returns $\log(p_{t+4}^{n-1}) - \log(p_t^n) = b_0 + B(L)d_{t-1}^{def} + e_t$. Results shown for 4 lags on quarterly data. The sample is 1945.1-2003.4. T-stats are reported in brackets. The first row uses the truncated kernel of Hansen (1982) and Hansen and Hodrick (1980) as the weighting matrix with 4 lags. The second row uses the Newey-West variance-covariance matrix with 6 lags as the weighting matrix. The last column reports the R-squared and the χ^2-statistic. The zero coupon prices are the Fama-Bliss series from CRSP.

Maturity	b_0	b_1	b_2	b_3	b_4	R^2	χ^2
2	6.06	−0.72	−0.73	−0.92	−0.60	0.02	
	[7.68]	[−1.42]	[−1.47]	[−1.57]	[−1.14]		2.73
	[8.68]	[−1.38]	[−1.47]	[−1.61]	[−1.16]		3.15
5	6.61	−1.34	−1.26	−1.99	−1.25	0.04	
	[5.93]	[−1.66]	[−1.52]	[−1.93]	[−1.39]		4.50
	[6.63]	[−1.62]	[−1.56]	[−1.94]	[−1.45]		5.05
10	7.20	−2.40	−1.61	−2.16	−1.48	0.02	
	[4.11]	[−1.69]	[−1.33]	[−1.84]	[−1.10]		10.00
	[4.55]	[−1.69]	[−1.35]	[−1.62]	[−1.10]		4.07
15	6.92	−3.62	−2.03	−3.19	−2.50	0.03	
	[3.06]	[−1.85]	[−1.27]	[−2.00]	−[1.44]		4.28
	[3.39]	[−1.93]	[−1.30]	[−1.83]	[−1.46]		4.79

IX. Conclusion

In this paper we empirically explore how the US Federal Government has responded to fiscal shocks. We report preliminary evidence that some 10-20% of the cost of fiscal shocks is absorbed by returns on government debt, with the remainder absorbed by lower surplus growth. However, a much smaller fraction of this cost is absorbed by movements in contemporaneous returns. The ex ante hedging emphasized in the optimal tax literature plays a relatively smaller role in US fiscal

Table XIII
Predictability of Real Holding Returns.

Regression of log holding returns $\log(p_{t+4}^{n-1}) - \log(p_t^n)$ on a zero coupon bond with maturity n years between t and $t+4$ on lagged defense expenditure shocks: Regression of real log holding returns $\log(p_{t+4}^{n-1}) - \log(p_t^n) - (\log(CPI_{t+4}) - \log(CPI_t)) = b_0 + B(L)d_{t-1}^{def} + e_t$. Results shown for 4 lags on quarterly data. The sample is 1945.1-2003.4. T-stats are reported in brackets. The first row uses the truncated kernel of Hansen (1982) and Hansen and Hodrick (1980) as the weighting matrix with 4 lags. The second row uses the Newey-West variance-covariance matrix with 6 lags as the weighting matrix. The last column reports the R-squared and the χ^2-statistic. The zero coupon prices are the Fama-Bliss series from CRSP.

$Maturity$	b_0	b_1	b_2	b_3	b_4	R^2	χ^2
1	2.26	−0.97	−0.53	−0.69	−0.16	0.01	
	[2.87]	[−1.73]	[−1.06]	[−1.10]	[−0.27]		9.36
	[3.23]	[−1.54]	[−0.96]	[−1.02]	[−0.26]		3.55
5	2.84	−1.63	−1.15	−1.86	−0.89	0.02	
	[2.29]	[−1.93]	[−1.28]	[−1.59]	[−0.82]		8.54
	[2.56]	[−1.77]	[−1.21]	[−1.47]	[−0.81]		4.61
10	3.45	−2.72	−1.58	−2.12	−1.15	0.02	
	[1.77]	[−1.83]	[−1.22]	[−1.55]	[−0.71]		2.20
	[1.96]	[−1.77]	[−1.16]	[−1.28]	[−0.68]		3.92
15	3.23	−4.03	−2.28	−3.47	−2.49	0.03	
	[1.30]	[−1.96]	[−1.30]	[−1.85]	[−1.19]		5.34
	[1.44]	[−1.99]	[−1.29]	[−1.63]	[−1.15]		4.58

policy. Under the assumption that the price of risk is exogenous to the conduct of fiscal policy, we show that government could achieve a greater degree of hedging by substituting long for short term debt. Finally, we provide some evidence that fiscal shocks predict movements in yields and expected debt returns.

Following much of the recent predictability literature in finance, we use a linearized version of the budget constraint to organize our thinking. This constraint suggests useful measures of fiscal shocks and fiscal hedging. The latter are informative of the conduct of policy and may be viewed through the lens of normative fiscal policy theory. They supplement other attempts to test directly specific versions of this theory (e.g. Marcet and Scott (2003), Scott (2007)). One implication of our paper is that movements in expected future returns are important for the financing of shocks. This aspect of fiscal policy is not much emphasized in the normative literature, which does not usually utilize models suitable for serious analyses of asset pricing. Similarly, the literature on the positive effects of fiscal policy places more emphasis on the evolution of real quantities than asset prices and often relegates the government budget constraint to the background. We leave the modeling and analysis of this channel of fiscal policy to later work.

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A. Appendix

A. Linearization of the Government Budget Constraint

We start with the dynamic budget constraint of the federal government. B_{t+1} denotes the market value of outstanding (federal) government liabilities at the start of period $t+1$. The government budget constraint is given by:

$$B_{t+1} = R^b_{t+1}\left(B_t - S_t\right).$$

where R^b_{t+1} denotes the gross return on government debt and S_t denotes the government surplus $S_t = T_t - G_t$, government receipts less expenditures. The growth rate of debt can be stated simply as the gross return times one minus the surplus to debt ratio:

$$\frac{B_{t+1}}{B_t} = R^b_{t+1}\left(1 - \frac{S_t}{B_t}\right) \tag{10}$$

We assume that for all t, $B_t > 0$ and $1 - S_t/B_t > 0$. Additionally, we assume that the receipts/debt and spending/debt ratios, T_t/B_t and G_t/B_t, are stationary around their average values which we denote $\overline{TB}$ and $\overline{GB}$ respectively. Finally, we assume that average surplus/debt ratio, $\overline{SB} := \overline{TB} - \overline{GB}$ is between 0 and 1. Using lower case letters to denote logs, (10) may be rewritten as:

$$\begin{aligned}\Delta b_{t+1} &= r^b_{t+1} + \log(1 - \exp(s_t - b_t)) \text{ if } S_t > 0\\ &= r^b_{t+1} + \log(1 + \exp(d_t - b_t)) \text{ if } D_t = -S_t > 0,\end{aligned}$$

where we distinguish between the case in which the government is running deficits and the case in which it is running surpluses. If the government only ran surpluses, then we could expand the right hand side of the log budget constraint as a function of $s_t - b_t$ around $\overline{sb} := \log \overline{SB}$:

$$\log(1 - \exp(s_t - b_t)) = \log(1 - \exp(\overline{sb})) - \frac{\exp(\overline{sb})}{1 - \exp(\overline{sb})}\left[(s_t - b_t) - \overline{sb}\right].$$

First-order Expansion Since, government's run deficits an alternative expansion is required. We rewrite $\log(1 - S_t/B_t)$ as $\log(1 - \exp(\tau_t - b_t) + \exp(g_t - b_t))$ and expand around $(\overline{\tau b}, \overline{gb})) :=$

$(\log(\overline{TB}), \log(\overline{GB}))$. We obtain:

$$\begin{aligned}\log\left(1-\frac{S_t}{B_t}\right) &\approx \log(1-\overline{TB}+\overline{GB}) - \frac{\mu_{sb}}{1-\mu_{sb}}\left(\frac{\mu_{\tau b}\left(\tau_t - b_t - \overline{\tau b}\right) - \mu_{gb}\left(g_t - b_t - \overline{gb}\right)}{\mu_{sb}}\right)\\ &= K - \frac{\mu_{sb}}{1-\mu_{sb}}\left(\frac{\mu_{\tau b}}{\mu_{\tau b}-\mu_{gb}}\tau_t - \frac{\mu_{gb}}{\mu_{\tau b}-\mu_{gb}}g_t - b_t\right),\end{aligned}$$

where K absorbs unimportant constants and the shares are defined as: $\mu_{sb} = \overline{\left(\frac{T-G}{B}\right)}$, $\mu_{\tau b} = \overline{\left(\frac{T}{B}\right)}$ and $\mu_{gb} = \overline{\left(\frac{G}{B}\right)}$.

Law of Motion for Debt This approximation implies the following law of motion for debt:

$$\Delta \log b_{t+1} = r^b_{t+1} + \left(1 - \frac{1}{\rho}\right)(ns_t - b_t)$$

where $\rho = 1 - \mu_{sb}$. Subtracting and adding $ns_{t+1} - ns_t$ produces:

$$b_{t+1} - ns_{t+1} + \Delta ns_{t+1} = r_{t+1} - (\frac{1}{\rho})(ns_t - b_t).$$

This is a first-order difference equation that can be solved by repeated substitution for the weighted log surplus/debt ratio. Imposing the tail condition $\lim_{j\to\infty}\rho^j(ns_{t+j} - b_{t+j}) = 0$ and taking expectations, we obtain:

$$(ns_t - b_t) = E_t\sum_{j=1}^{\infty}\rho^j r^b_{t+j} - E_t\sum_{j=1}^{\infty}\rho^j \Delta ns_{t+j}. \tag{11}$$

Equation (11) also implies that the news about current future returns on government debt equals the news about current and future surplus growth:

$$(E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j r^b_{t+j+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta ns_{t+j+1}.$$

This follows because:

$$(ns_{t+1} - b_{t+1}) - E_t(ns_{t+1} - b_{t+1}) = (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r^b_{t+j+1} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \Delta ns_{t+j+1}.$$

And so using $b_{t+1} - E_t b_{t+1} = r^b_{t+1} - E_t r^b_{t+1}$, we have:

$$(E_{t+1} - E_t)\, ns_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^b_{t+j+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta ns_{t+j+1}$$

B. Computing zero-coupon bond prices, value of government's total liabilities and interest cost

The Treasury reports the interest cost of total government debt, calculated by summing up all the principal and coupon payments the government has promised to deliver at $t+j$ as of time t. The Treasury methodology makes no distinction between coupon payments and the principal, hence mismeasures its cost of funds. We follow Hall and Sargent's (1997) accounting method for computing government's cost of funds, capital gains and losses from holding long-term government debt and the change in the value of government total liabilities. To accomplish this task, we first convert nominal yields to maturity on government debt into prices of claims on future dollars in terms of current prices. In other words, we unbundle a coupon bond into its constituent pure discount bonds and value these components. We then add up the values of the components to attain the value of the bundle.

Let $s_{j,t}$ be the number of $t+j$ dollars the government has promised to deliver (promised at time t). Let $a_{j,t}$ be the number of time t goods it takes to buy a dollar delivered at time $t+j$. Hence $a_{j,t}$ is the real price (inflation-adjusted) at time t of a zero-coupon bond maturing j periods ahead. Let def_t be the government's real net-of-interest budget deficit, measured in units of time t goods. Finally, let m_t be the nominal value of the monetary base. The government's budget constraint can be written as:

$$\underbrace{a_{o,t}m_t}_{\substack{\text{value of mon. base}\\ \text{at end of } t}} + \underbrace{\sum_{j=1}^{n} a_{j,t}s_{j,t}}_{\substack{\text{real value of}\\ \text{debt at end of } t}} = \underbrace{a_{0.t}m_{t-1}}_{\substack{\text{value of mon. base}\\ \text{at beginning of } t}} + \underbrace{\sum_{j=1}^{n} a_{j-1,t}s_{j,t-1}}_{\substack{\text{real value of}\\ \text{debt at beginning of } t}} \underbrace{+def_t}_{\text{real primary deficit}} \quad (12)$$

where $a_{0,t}$ is the inverse of price level at time t. Monetary base is viewed as matured government bonds, hence bond-holders and money-holders are treated symmetrically. We can re-arrange equation (12) to get

$$a_{o,t}m_t + \underbrace{\sum_{j=1}^{n} a_{j,t}s_{j,t}}_{\substack{\text{real value of}\\ \text{debt at end of } t}} = \underbrace{(a_{0.t} - a_{0,t-1})m_{t-1}}_{\substack{\text{- seignorage}\\ \text{borrowing cost of "money"}}} + \underbrace{\sum_{j=1}^{n}(a_{j-1,t} - a_{j,t-1})s_{j,t-1}}_{\substack{\text{borrowing cost of debt}\\ \text{"cost of funds"}}}$$

$$+ \underbrace{a_{0,t-1}m_{t-1}}_{\substack{\text{mon. base value}\\ \text{at } t-1}} + \underbrace{\sum_{j=1} a_{j,t-1}s_{j,t-1}}_{\substack{\text{value of debt}\\ \text{at } t-1}} + \ def_t \quad (13)$$

B.1. The Price Data

To decompose the government's budget constraint in this manner, we need to calculate the quantities $s_{j,t}$ and the prices $a_{j,t}$. We compute $s_{j,t}$ from CRSP Government Bonds Files, Monthly Treasuries. These files contain monthly maturity, publicly held face value outstanding, coupon rate data on virtually all negotiable direct obligations of the United States Treasury for the period December 31, 1925, to the present. We construct the series $s_{j,t}$ by adding up all the dollar principal plus coupon payments that the government has promised to deliver to the public at date $t + j$ as of date t. CRSP does not report the face value of Treasury bills held by the public, so these data are obtained from table FD-5 of the monthly Treasury Bulletin. The value of the currency v_t is computed by,

$$v_t = \frac{100}{p_t}$$

where p_t is the price level. The price level is the monthly series CPI-all items from the Bureau of Labor Statistics.

B.2. Computation of the Zero-Coupon Bond Prices

To compute the series $a_{j,t}$,[16] we extract the nominal implicit discount function (or the forward rate curve) from the bill and coupon price data and then convert it to real terms by using the CPI. This is accomplished in several steps. We first eliminate from the CRSP sample any flower, callable bonds and notes, as well as bonds and notes with less than 1 year maturity and bills less than 30 days maturity. We also eliminate from the sample, bonds with 1.5% coupon rates as they have been documented to contain large spurious errors. Then we estimate the discount function and/or the forward rate curve. Let $\{B_i\}_{1\leq i\leq N}$ be a set of bonds and $\tau_1 < \tau_2 < \cdots \tau_K$ the set of dates on which principal and interest payments of $c_{i,j}$ are made. In an environment with no taxes or transaction costs, no arbitrage implies the pricing equation:

$$P_i = \sum_{j=1}^{K} c_{i,j}\delta(t_j) = \sum_{j=1}^{K} c_{i,j}\ exp\left(-\int_0^{t_j} f(s)ds\right), \tag{14}$$

where $\delta(t)$ is the discount function and $f(t)$ is the forward rate curve. In the real world of taxes and transaction costs, the price of a coupon bond can only be estimated by 14 and the pricing equation is

$$P_i = \hat{P}_i + \epsilon_i, \tag{15}$$

where

$$\hat{P}_i = \sum_{j=1}^{K} c_{i,j}\delta(t_j) = \sum_{j=1}^{K} c_{i,j}\ exp\left(-\int_0^{t_j} f(s)ds\right). \tag{16}$$

To estimate either the discount function or the forward rate curve, we follow Waggoner(1997) and use smooth cubic splines. We sort all outstanding bonds not eliminated by the first step of our estimation scheme by their settlement dates. The set of nodes for the cubic spline are the set of

[16]The value of the currency $a_{0,t}$ is the inverse of the end of month observations of the consumer price index.

cash flow dates $\{\tau_j\}$. The estimated forward rate curve minimizes the expression:

$$\sum_{i=1}^{N}\left(P_i-\sum_{j=1}^{K}c_{i,j}\ exp\left(-\int_0^{t_j}f(s)ds\right)\right)^2+\int_0^{\tau_K}\lambda(s)\left[f''(s)\right]^2 ds, \tag{17}$$

where $\lambda(t)$ is the penalty function used to ensure a smooth forward curve. $\lambda(t)$ is set to:

$$\lambda(t)=\begin{cases}0.1 & 0\le t<1\\ 100 & 1\le t\le 10\\ 100,000 & 10\le t\end{cases} \tag{18}$$

for (t) measured in years. This penalty function allows for the spline to be more flexible (hence less smooth) on the short end and more smooth on the long end of the maturity range.[17] The zero coupon bond prices are then computed using the estimated instantaneous forward rate curve.

C. Cointegrating relation between Spending, Receipts and Market Value of Debt

We deflate the market value of debt, government expenditures and government receipts using the CPI (availabale from the BLS BLS). We test for a cointegrating relationship between rec_t, exp_t and b_t[18] using the Johansen procedure and we find evidence in favor a cointegrating relation between these three variables on the 1932.I-2003.IV sample. The vector we estimate is : $[-10.86650.0280]$ for $[rec_t exp_t b_t]$. We can back out the implied weights from these estimates.

D. Defense Shocks

For the government receipts T and expenditures G, we used NIPA Table 3.2. Federal Government Current Receipts and Expenditures [Billions of dollars] Seasonally adjusted at annual rates: current receipts (line 1) less current asset income received (line 12) and current expenditures (line 19) less current interest paid (line 28). These data can be downloaded from $http://www.bea.gov/bea/dn/nipaweb$. For the defense expenditures g^{def}, we used Table 3.9.5. Government Consumption Expenditures and Gross Investment [Billions of dollars], seasonally adjusted at annual rates (line 11).

[17]See Waggoner(1997) for the choice of the penalty function. We have explored, in our own calculations, other penalty functions and have found the Waggoner method to be successful in producing smooth forward rates and as well as pricing bonds well.

[18]All lowercase variables are in logs.

Returns on the government's debt portfolio We compute the returns on the government's outstanding debt as $\sum_{j=1} s_{j,t} \frac{(a_{j-k,t+k} - a_{j,t})}{aj,t}$.

Defense Industry Returns The defense industry returns were computed using CRSP cum-dividend returns for all firms with SIC codes between 3760-3769 (Guided missiles and space vehicles), 3795-3795 (Tanks and tank components) and 3480-3489 (Ordnance & accessories). This is the definition used by Kenneth French in the 48 industry definition.

Fama French factors Fama and French update the benchmark returns approximately two weeks after the end of each month. The benchmark factors summarize (1) the overall market return (R^m), (2) the performance of small stocks relative to big stocks (R^{SMB}, Small Minus Big), and (3) the performance of value stocks relative to growth stocks (R^{HML}, High Minus Low). The Fama/French benchmark portfolios are rebalanced quarterly using independent sorts on size (market equity) and the ratio of book equity to market equity. The book-to-market ratio is high for value stocks and low for growth stocks. These data were downloaded from Kenneth French's web site at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

Returns The market return R^m_{t+1} is the cum-dividend return on the value-weighted CRSP index (downloaded from CRSP). The risk-free rate is the average one-month T-bill rate (Fama risk-free rate) made available by CRSP.

Figure 15. Holding Returns and Defense Shocks 1945-2002: The figure plots the slope coefficients in the regression of one-year holding returns on defense shocks against the lags, maturity by maturity. Regression of log holding returns $\log(p^{n-1}_{t+4}) - \log(p^n_t) = b_0 + B(L)d^{def}_{t-1} + e_t$. Results shown for 6 lags. The zero coupon prices are the Fama-Bliss series from CRSP.