Optimal Monetary Policy Under Financial Sector Risk[*]

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Abstract

We consider whether or not a central bank should respond directly to financial market conditions when setting monetary policy. Specifically, should a central bank put weight on interbank lending spreads in its Taylor rule policy function? Using a model with risk and balance sheet effects in both the real and financial sectors (Davis, "The Adverse Feedback Loop and the Effects of Risk in the both the Real and Financial Sectors" Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper No. 66, November 2010) we find that when the conventional parameters in the Taylor rule (the coefficients on the lagged interest rate, inflation, and the output gap) are optimally chosen, the central bank should not put any weight on endogenous fluctuations in the interbank lending spread. However, the central bank should adjust the risk free rate in response to fluctuations in the spread that occur because of exogenous financial shocks, but we find that the central bank should not be too aggressive in its easing policy. Optimal policy calls for a two-thirds of a percentage point cut in the risk free rate in response to a financial shock that causes a one percentage point increase in interbank lending spreads.

JEL codes: E32; E44; E50; F40; G01

1 Introduction

The past few years have witnessed how financial sector risks could be spread across countries' borders to cause a financial and economic calamity worldwide. This overwhelming episode has stimulated a surged interest in investigating whether and how monetary policy should respond to variations in financial market conditions. A popular view is that the central bank should include some interbank credit spreads in a Taylor type monetary policy rule. In his testimony on February 26, 2008 before the Committee on Financial Services of the U.S. House of Representatives, John B. Taylor argued that the intercept term in a Taylor type rule for monetary policy, that is, the natural rate of interest, should be adjusted downward in proportion to observed increase in the spread between the term Libor rate at three month maturity and an index of overnight federal funds rates expected for the same period. Similar views have been expressed by others, including Goodfriend and McCallum (2007), De Fiore and Tristani (2007), McCulley and Toloui (2008), Meyer and Sack (2008), Curdia and Woodford (2009 and 2010), Woodford (2010), and Mishkin (2010a and 2010b).

In this paper, we examine whether and how a Taylor type monetary policy rule should be modified in an environment featuring financial frictions and financial sector risks. A defining feature of our model is a careful distinction between exogenous and endogenous changes in the financial market conditions, which we capture by an interbank lending spread. A robust finding in the paper is that both the natural rate of interest and the risk free policy rate should be adjusted directly to an exogenous variation in the spread, which we call a financial sector shock, but neither should respond directly to any endogenous movement in the spread.

Our paper is a study of optimal simple rules for central banks in an open-economy monetary model that features multiple sources of frictions and risks in both real and financial sectors. Estimated simple monetary policy rules typically take the form of a policy rate as a function of inflation and the output gap, as well as the lagged policy rate. Whereas stabilizing the variability in inflation and the output gap are the basic characteristics of the celebrated Taylor rule (e.g., Taylor, 1993), subsequent studies reveal substantial evidence of interest rate smoothing in monetary policy practice (e.g., Rudebusch, 1995; Clarida, Gali, and Gertler, 1998 and 2000; Orphanides, 2001). Arguments in favor of such policy rules have been made using structural models where the cen-

tral bank's loss function consists of variations in inflation, the output gap, and interest rate (e.g., Woodford, 2003a). Desirability of such policy rules or their variants have been shown in models with sticky prices in one sector (e.g., Clarida, Gali, and Gertler, 1999; Goodfriend and King, 2001; Aoki , 2001), in multiple sectors (e.g., Mankiw and Reis, 2003; Huang and Liu, 2005), in multiple countries (e.g., Benigno, 2004; Clarida, Gali, and Gertler, 2002), and with both sticky prices and sticky wages (e.g., Erceg, Henderson, and Levin, 2000; Amato and Laubach, 2003). The importance of interest rate smoothing is especially emphasized by Woodford (2003b). The robustness of such simple policy rules have been shown in various modeling environments (e.g., Levin and Williams, 2003; Levin, Wieland, and Williams, 1999 and 2003; Levin, Onatski, Williams, and Williams, 2005; see Taylor and Williams, 2009b for a survey).

These models all feature a frictionless financial world. In this paper, we examine how such simple monetary policy rules might need to be modified in an environment featuring financial frictions and financial sector risks. Our model takes its root in the classic financial accelerator literature pioneered by Gertler (1988), Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999), among others. These papers incorporate frictions in finance for manufacturing firms, but they do not model financial intermediaries. Some recent studies incorporate financial intermediaries in modeling, but they abstract from frictions or risks in the financial sector. For instance, Christiano, Motto, and Rostagno (2008) model banks, but they do not allow for frictions or risks within the banking sector. A few recent papers like Meh and Moran (2010), Gertler and Karadi (2009), Gertler and Kiyotaki (2010), and Dib (2010) model financial frictions within the financial sector, in the form of collateral constraints, but they do not allow for financial sector risks. All of these studies focus on the mechanism of financial frictions in transmitting real or monetary shocks.

Our model builds on Davis (2010) but is concerned about the design of optimal simple monetary policy rules in both a closed and an open-economy monetary environment that features multiple sources of frictions and risks, in both real and financial sectors. The model has four important features. First, it incorporates financial intermediaries, and frictions and risks in both the manufacturing industries and the financial sector. Thus, the model features balance sheet effects on both the demand and supply sides of the credit market. Second, it incorporates sticky wages, in addition to sticky prices, so monetary policy faces a nontrivial trade-off between different components of the

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central bank's objective even when the financial frictions and risks are muted, while in the baseline case the policy trade-off is multi-dimensional. Third, we consider both a closed and open-economy setting to take into account frictions and risks in not only domestic but international interbank lending markets for short-term unsecured loans. Last, it distinguishes between endogenous and exogenous fluctuations in interbank lending spreads, the spread between the bank's cost of capital and the risk free rate. As we show below, exogenous and endogenous fluctuations in the interbank spreads play fundamentally different roles in determining an optimal simple monetary policy rule.

Our results are easy to summarize. Starting with the standard case that abstracts from financial frictions and financial sector risks, we derive a simple optimal rule in terms of the respective responsiveness of policy rate to the variability in inflation and the output gap as well as lagged policy rate, much in line with the standard literature on optimal monetary policy. We then find that the presence of financial frictions calls for a greater degree of gradualism in the interest rate rule. This finding supports the classic view of Goodfriend (1987) and Cukierman (1991) in support of interest rate smoothing from the perspective of financial stability. In addition, policy also shifts weight from output gap stabilization to inflation stabilization in the presence of financial frictions. This is consistent with the finding by Lee (2010) who models financial frictions on the households' side, whereas we model financial frictions and risks on the sides of firms and banks.[1]

But, in contrast to the standard models that abstracts from financial frictions and financial sector risks, our model features more than one interest rate. Time varying spreads in our model have an allocative role. Issing (2006) and Goodhart (2007) suggest that such spreads may have implications for monetary policy practice. It is on this end we find it important to distinguish between what we term endogenous fluctuations in the spread from exogenous fluctuations.

Endogenous fluctuations in the spread are the essence of a financial accelerator model. In this model with financial frictions in the banking sector, the interbank lending spread fluctuates countercyclically in response to endogenous fluctuations in a bank's capital structure and loan loss ratios. In this paper's most interesting finding, we find that when the conventional parameters of the Taylor rule, the coefficients on the lagged interest rate, the inflation rate, and the output gap, are chosen optimally, the central bank should ignore these endogenous fluctuations in the

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[1] Recent welfare-based monetary policy evaluations in models with financial frictions on the households' side also include Iacoviello (2005) and Monacelli (2009).

spread. Since these fluctuations are the endogenous reactions to other macroeconomic variables in the model, these fluctuations in the spread provide no new information that is not already contained in measures of the output gap and inflation. Since the conventional Taylor rule parameters were chosen optimally, the central bank has already found the optimal weighting of the information contained in the inflation rate and the output gap. Therefore putting any weight on a new term that contains no new information would be sub-optimal.

However this may not be true for exogenous fluctuations in the spread. We define exogenous fluctuations in the spread as occurring because of some exogenous financial sector shock, for instance, a sudden tightening of the credit market that occurs because of a sudden increase in financial sector risk or uncertainty. This type of shock is documented by Taylor and Williams (2009a) who describe the sudden increase in interbank lending spreads at the beginning of the financial crisis in August 2007. Bordo and Haubrich (2010) document historical instances of these credit market shocks going back to 1875. Helbling et al. (2010) and Gilchrist, Yankov and Zakrajsek (2009) use econometric techniques the single out these credit shocks and demonstrate their importance in explaining the fluctuations in broader macro aggregates. Within the framework of a financial accelerator model, a number of recent papers, like Attah-Mensah and Dib (2008), Christiano et al. (2003 and 2008), Nolan and Thoenissen (2009), Jermann and Quadrini (2009), and Gilchrist, Ortiz, and Zakrajsek (2009) have introduced credit shocks into a DSGE model.

We find that fluctuations in the spread that are caused by these exogenous credit shocks may contain new information that is not already found in measures of the output gap and the inflation rate. Thus the central bank may want to reduce both the natural rate of interest and the risk free policy rate in response to an exogenous increase in the interbank spread that is caused by a credit market shock. Furthermore, in the open economy versions of the model, the central bank will want to react to exogenous fluctuations in both home and foreign interbank lending spreads. While the central bank will want to employ an accommodative monetary policy in response to a credit market shock, we find that the central bank will not want to fully accommodate the shock. To fully accommodate the shock would imply that the central bank would lower the risk free rate by 1% in response to a 1% exogenous increase in the spread. We find that this degree of accommodation is too extreme. Optimal policy is for the central bank to reduce the risk free rate by about 0.7% in response to a 1% exogenous increase in the spread.

Our paper is related to an emerging literature of welfare-based monetary policy evaluations using models with financial frictions, including Moessner (2006), Faia and Monacelli (2007), De Fiore and Tristani (2007), Teranishi (2008), Sudo and Teranishi (2008), Curdia (2008), Curdia and Woodford (2009 and 2010), Faia and Iliopulos (2010), Merola (2010), and Kolasa and Lombardo (2011), among others. Besides our modeling details, such as the open-economy setup with both sticky prices and sticky wages, what distances our study from this literature is the incorporation of financial sector shocks, frictions in finance in both real and financial sectors, their distinguished roles, and the distinction between exogenous changes of domestic and foreign interbank spreads and endogenous movements in these spreads in determining simple optimal monetary policy rules.

This paper will proceed as follows. Section 2 presents the model that is used to assess the desirability of including interbank spreads in the central bank's policy function. The model is a multi-country new Keynesian model, with financial frictions introduced in both real and financial sectors that enable the model to move away from the irrelevance of balance sheets implied by the Modigliani and Miller (1958) theorem. The model is very similar to that presented in Davis (2010), but unlike the model in Davis (2010), the central bank's policy function is modified to give the central bank the option of responding to financial market conditions. In addition, in this paper we introduce a new type of shock that is a direct shock to risk and uncertainty in the financial sector. This new type of shock provides the basis for the exogenous fluctuations in the interbank spread that become so important when discussing optimal simple policy rules. Then the calibration of the model is discussed in section 3. The optimal parameters in the Taylor rule as derived from simulations of the model are presented in section 4. First we discuss how the presence of financial frictions in the model induces the central bank to put more weight on interest rate smoothing in their Taylor rule function. Then we discuss whether or not the central bank will want to directly target interbank lending spreads. Finally, section 5 concludes and offers some suggestions for further research.

2   Model

In the model there are five types of agents: firms, entrepreneurs, capital builders, banks, and households. There is also a central bank that sets the risk free nominal rate of interest.

Firms use capital and labor inputs to produce tradeable output that is used for consumption and investment. Each firm produces a differentiated good and sets prices according to a Calvo (1983) style price setting framework, thus giving rise to nominal price rigidity.

Entrepreneurs own physical capital and rent it to firms. This physical capital is financed partially through debt and partially through equity. In every period, an individual entrepreneur faces an idiosyncratic shock to the value of their physical capital assets. While these shocks have no direct aggregate effects, they introduce heterogeneity among entrepreneurs. The shock is uninsurable, and a fraction of entrepreneurs may experience an abnormally large shock to the value of their physical capital stock and be pushed into bankruptcy, while most will not. The uncertainty over which entrepreneurs will be pushed into bankruptcy and which will not is a type of financial friction in the real sector. The ratio of debt to equity on an entrepreneur's balance sheet determines their ability to withstand a shock to the value of their capital stock. Creditors use the entrepreneur's debt-equity ratio to determine the riskiness of lending to the entrepreneurial sector, giving rise to a default risk interest premium that depends on the debt-equity ratio.[2]

Capital builders purchase final goods from firms for physical capital investment. There are diminishing marginal returns to physical capital investment. In periods when investment is high, the marginal return of that investment in producing new physical capital is low, and vice versa. This gives rise to a procyclical relative value of physical capital.

Banks channel savings from households to firms in the form of working capital loans and to entrepreneurs in the form of physical capital loans. A bank finances its asset portfolio partially through equity and partially through debt, which is made up of deposits from domestic and foreign households.

Due to bankruptcies in the real sector, a portion of a bank's portfolio of physical capital loans will go into default in any given period. While these loan losses are not great enough to push the entire banking sector into insolvency, there is heterogeneity among banks with regards to their exposure to the set of non-performing loans. A few banks may be over-exposed to the set of bad loans, and they themselves may be pushed into insolvency. The uncertainty about which banks are over-exposed to the set of non-performing loans and which are not is a type of financial friction in

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[2]The fact that this idiosyncratic shock is uninsurable provides the necessary violation of the complete markets assumption necessary to overcome the implications of the Miller and Modigliani (1958) theorem.

the banking sector. The ratio of debt to equity on a bank's balance sheet determines their ability to absorb loan losses, so the debt-equity ratio determines the ex-ante riskiness of a particular bank. This gives rise to an environment where the spread between interbank lending rates and the risk free rate is increasing in the leverage ratio of the banking sector.

Households supply labor to firms and consume final output. Furthermore they supply a differentiated type of labor and set wages according to a Calvo-style wage setting process, giving rise to nominal wage rigidity.

Finally, the central bank tries to stabilize output and prices by controlling the risk free nominal rate of interest. The central bank sets policy using a Taylor rule function combining the current period's inflation rate, output gap, and the lagged risk free nominal interest rate. We will also consider the case where the home and foreign interbank lending spreads are also part of the Taylor rule. When considering the central bank's optimal reaction to financial sector developments, specifically we are trying to find the optimal coefficients on the spreads in the Taylor rule.

The remainder of this section presents the actual details of the model. In section 4 we will examine two versions of the model, the closed economy, and two open economies. In what follows, the key model equations are written for the open economy case. In the model's notation, the relative size of the home country is n and the relative size of the foreign country is $1 - n$. In the case of two large economies, $n = \frac{1}{2}$. In the case of the closed economy, imagine that $n = 1$.

In what follows, all variables are written in per capita terms and foreign variables are distinguished by an asterisk (*). In the open economy version of the model, the two countries are symmetric, so foreign equations have been omitted for brevity except where absolutely necessary.

2.1 Firms

In the home country, intermediate goods producing firms, indexed $i \in [0 \ n]$, combine capital and labor, $k_t(i)$ and $h_t(i)$ to produce a unique intermediate good $Y_t(i)$. The firm's production function is:

$$Y_t(i) = A_t h_t(i)^{1-\alpha} k_t(i)^{\alpha} - \phi \tag{1}$$

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where A_t is an exogenous country specific stochastic TFP parameter that is common to all firms and ϕ is a fixed cost parameter that is calibrated to ensure that firms earn zero profit in the steady state.

The output from firm i can be sold to the domestic market or sold as imports in the foreign market:

$$Y_t(i) = y_t^d(i) + y_t^{m*}(i)$$

where $y_t^d(i)$ is output from firm i that is sold domestically and $y_t^{m*}(i)$ is the output that is imported into the foreign country.

Intermediate goods from domestic and foreign firms are then combined into one aggregate final good. As in Chari, Kehoe, and McGrattan (2002), domestically supplied and imported intermediate goods are aggregated by the following:

$$y_t = \left[(\gamma)^{\frac{1}{\rho}} \left[\left(\int_0^n y_t^d(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\rho-1}{\rho}} + \left(\gamma^f \right)^{\frac{1}{\rho}} \left[\left(\int_n^1 y_t^m(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{2}$$

where σ is the elasticity of substitution between domestic varieties and ρ is the elasticity of substitution between home and foreign varieties.

From this aggregator function the demand in the home country for the intermediate good from domestic firm i, where $i \in [0\ n]$, as a function of aggregate demand is:

$$y_t^d(i) = \gamma (n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{P_t^d(i)}{P_t^d} \right)^{-\sigma} \left(\frac{P_t^d}{P_t} \right)^{-\rho} y_t \tag{3}$$

Similarly, the demand in the home country for the intermediate good from foreign firm i, where $i \in (n\ 1]$, as a function of aggregate demand is:

$$y_t^m(i) = \gamma^f (1-n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{P_t^m(i)}{P_t^m} \right)^{-\sigma} \left(\frac{P_t^m}{P_t} \right)^{-\rho} y_t \tag{4}$$

where $P_t^d(i)$ is the price in the domestic market for the intermediate good from firm i, $P_t^d = \left(\frac{1}{n} \int_0^n \left(P_t^d(i) \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ is a price index of domestically produced intermediate goods, $P_t^m = \left(\frac{1}{1-n} \int_n^1 \left(P_t^m(i) \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ is a price index of imported intermediate goods, and the aggregate

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price level is given by $P_t = \left[\gamma (n)^{\frac{1-\rho}{1-\sigma}} \left(P_t^d \right)^{1-\rho} + \gamma^f (1-n)^{\frac{1-\rho}{1-\sigma}} \left(P_t^m \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$.

Firm i can discriminate when setting prices for the domestic or foreign market. Thus they can set separate prices for the domestic and export markets. In period t, the firm will be able to change its price in the domestic market with probability $1 - \xi_p$. If the firm cannot change prices then they are reset automatically according to $P_t^d (i) = \pi_{t-1} P_{t-1}^d (i)$, where $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$.

Thus if allowed to change their domestic price in period t, the firm will set a price to maximize:

$$\max_{P_t^d(i)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} \left\{ \Pi_{t,t+\tau} P_t^d (i) y_{t+\tau}^d (i) - MC_{t+\tau} y_{t+\tau}^d (i) \right\}$$

where λ_t is the marginal utility of income in period t. As discussed in this paper's technical appendix, the firm that is able to change its domestic price in period t will set its price to:

$$P_t^d (i) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} MC_{t+\tau} \left(\frac{\Pi_{t,t+\tau}}{P_{t+\tau}^d} \right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}} \right)^{-\rho} y_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} \Pi_{t,t+\tau} \left(\frac{\Pi_{t,t+\tau}}{P_{t+\tau}^d} \right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}} \right)^{-\rho} y_{t+\tau}}$$

If prices are flexible, and thus $\xi_p = 0$, then this expression reduces to:

$$P_t^d (i) = \frac{\sigma}{\sigma - 1} MC_t$$

which says that the firm will set a price equal to a constant mark-up over marginal cost.

Write the domestic price set by the firm that can reset prices in period t as $\tilde{P}_t^d (i)$ to denote that it is an optimal price. Firms that can reset prices in period t will all reset to the same level, so $\tilde{P}_t^d (i) = \tilde{P}_t^d$. Substitute this optimal price into the price index $P_t^d = \left(\frac{1}{n} \int_0^n \left(P_t^d (i) \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$. Since a firm has a probability of $1 - \xi_p$ of being able to change their price, then by the law of large numbers in any period $1 - \xi_p$ percent of firms will reoptimize prices, and the prices of ξ_p percent of firms will be automatically reset using the previous periods inflation rate. Thus the domestic price index, P_t^d, can be written as:

$$P_t^d = \left(\xi_p \left(\Pi_{t-1,t} P_{t-1}^d \right)^{1-\sigma} + (1 - \xi_p) \left(\tilde{P}_t^d \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

The full details of this derivation as well as the derivation for prices set for the foreign market

is located in the appendix.

The firm hires labor and capital inputs, where W_t is the wage rate paid for labor input and R_t is the capital rental rate, both of which the firm takes as given. Furthermore the firm must pay their wage bill at the beginning of the period, prior to production. To do so they borrow $b_t^{wc}(i) = W_t h_t(i)$. The firm's income after paying for capital and labor inputs is:

$$d_t^f(i) = P_t^d(i) y_t^d(i) + P_t^x(i) y_t^x(i) - W_t h_t(i) - R_t k_t(i) - r_t^{wc} b_t^{wc}(i) \tag{5}$$

where $P_t^x(i)$ is the export price for the intermediate good from firm i, and r_t^{wc} is the interest rate on working capital loans. Since there is no default risk from lending working capital to firms, competition in the banking sector forces the rate on working capital loans down to the bank's own cost of capital, $r_t^{wc} = r_t^b$.

The aggregate income from all firms is returned to households as a lump sum payment, $d_t^f = \int_0^n d_t^f(i) \, di$.

The firm will choose $h_t(i)$ and $k_t(i)$ to maximize profit in (5) subject to the production function in (1). The working capital requirement implies that the cost of the labor input is $W_t(1 + r_t^{wc})$ and the cost of the capital input is R_t. Given these prices, the firm's demand for labor and capital inputs are:

$$
\begin{aligned}
h_t(i) &= (1 - \alpha) \frac{MC_t}{W_t(1 + r_t^{wc})} Y_t(i) \\
k_t(i) &= \alpha \frac{MC_t}{R_t} Y_t(i)
\end{aligned}
\tag{6}
$$

where $MC_t = \frac{1}{A_t} \left(\frac{W_t(1 + r_t^{wc})}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha$.

2.2 Entrepreneurs

Entrepreneurs, indexed $j \in [0 \ n]$, buy capital from capital builders and rent it to firms. At the beginning of period t, entrepreneur j has a stock of capital, $K_t(j)$, that he will rent to firms in period t at a rental rate R_t. In equilibrium, the aggregate stock of capital supplied by all domestic entrepreneurs j is equal to the aggregate stock of capital demanded by all domestic firms

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i, $\int_0^n K_t(j)\,dj = \int_0^n k_t(i)\,di$.

Entrepreneurs finance this stock of capital partially through debt. The entrepreneur borrows $b_t^e(j)$ from domestic banks to finance their capital stock $K_t(j)$. Thus the market value of the assets and liabilities for entrepreneur j at the beginning of period t are:

$$\text{Assets:} \quad P_t^K K_t(j)$$
$$\text{Liabilities:} \quad b_t^e(j) \tag{7}$$

where P_t^K is the price of existing capital.

The end of the period the value of the non-depreciated capital stock for the average entrepreneur is $P_t^K(1-\delta)K_t$. However during the period, the individual entrepreneur j receives an idiosyncratic draw that affects the relative price of their existing capital, so for entrepreneur j the end of period value of their non-depreciated capital stock is:

$$\omega_t^e(j)\,P_t^K(1-\delta)K_t(j)$$

where $\omega_t^e(j)$ is a i.i.d. draw from a lognormal distribution on the interval $[0\ \infty)$ with mean 1 and variance σ_e^2.

Since this draw has a mean 1, it has no effect on the aggregate capital stock. It simply introduces heterogeneity among entrepreneurs, and in any given period a fraction of entrepreneurs receive a draw that has a large adverse effect on the value of their existing capital (a small $\omega_t^e(j)$) and thus at the end of the period, the value of their liabilities exceeds the value of their assets.

During the period the entrepreneur rents his capital stock to firms for a rental rate of R_t. The entrepreneur finances this capital stock with a loan from the bank with an interest rate r_t^e. Thus at the end of the period, after the realization of $\omega_t^e(j)$, the nominal market value of entrepreneur j's assets is $\omega_t^e(j)\,P_t^K(1-\delta)K_t(j) + R_t K_t(j)$. At the end of the period the nominal value of the entrepreneur's liabilities is $(1+r_t^e)\,b_t^e(j)$.

Thus, after the realization of $\omega_t^e(j)$, entrepreneur j is bankrupt if:

$$\omega_t^e(j)\,P_t^K(1-\delta)K_t(j) + R_t K_t(j) < (1+r_t^e)\,b_t^e(j) \tag{8}$$

Thus the threshold value of $\omega_t^e(j)$ below which the entrepreneur goes bankrupt in period t and above which they continue operations is:

$$\bar{\omega}_t^e = \frac{(1 + r_t^e)\frac{b_t^e(j)}{K_t(j)} - R_t}{P_t^K(1-\delta)} \tag{9}$$

where $DA_t^e(j) = \frac{b_t^e(j)}{K_t(j)}$ is the ratio of the book value of debt to the book value of assets on an entrepreneur's balance sheet. The history of individual entrepreneur j will influence the level of $b_t^e(j)$ and $K_t(j)$, but the ratio $DA_t^e(j) = \frac{b_t^e(j)}{K_t(j)}$ is equal across all entrepreneurs. This is a key result for aggregation, for it implies that the bankruptcy cutoff value $\bar{\omega}_t^e$ does not depend on an entrepreneur's history. More intuition behind this result is presented at the end of this section and a formal proof is presented in the appendix.

When deciding how much to lend to entrepreneurs going into next period and at what rate, banks factor in the fact that if entrepreneur j does not default in period $t + 1$, creditors receive a return of r_{t+1}^e. If the entrepreneur defaults, creditors receive a share of the entrepreneur's remaining assets, less the bankruptcy cost μ^e. The threshold value $\bar{\omega}_{t+1}^e$ in equation (9) determines whether or not an entrepreneur goes into default next period. Thus the payoff to creditors conditional of the realization of the shock $\omega_{t+1}^e(j)$ is:

$$\begin{array}{ll}
(1 + r_{t+1}^e)(b_{t+1}^e(j)) & \text{if } \omega_{t+1}^e(j) \geq \bar{\omega}_{t+1}^e \\
(1 - \mu^e)[\omega_{t+1}^e(j)(1-\delta)P_{t+1}^K K_{t+1}(j) + R_{t+1}K_{t+1}(j)] & \text{if } \omega_{t+1}^e(j) < \bar{\omega}_{t+1}^e
\end{array} \tag{10}$$

Perfect competition in the banking sector implies that the bank's expected profit is zero. So the interest rate the bank charges on physical capital loans, r_{t+1}^e, is set such that the expected return, after factoring in the cost of bankruptcy, is equal to the bank's cost of capital, r_{t+1}^b:

$$\begin{aligned}
\left(1 + r_{t+1}^b\right)b_{t+1}^e(j) =\ & \int_0^{\bar{\omega}_{t+1}^e} (1-\mu^e)\left(\omega_{t+1}^e(j)(1-\delta)P_{t+1}^K K_{t+1}(j) + R_{t+1}K_{t+1}(j)\right) dF\left(\omega_{t+1}^e\right) \\
& + \int_{\bar{\omega}_{t+1}^e}^{\infty} \left(1 + r_{t+1}^e\right)b_{t+1}^e(j)\, dF\left(\omega_{t+1}^e\right)
\end{aligned}$$

where $F\left(\omega_{t+1}^e\right)$ is the c.d.f. of the lognormal distribution of ω_{t+1}^e.

Thus the interest rate charged by banks for physical capital loans is:

$$1 + r_{t+1}^e = \frac{\left(1 + r_{t+1}^b\right)}{1 - F\left(\bar{\omega}_{t+1}^e\right)} - \frac{(1 - \mu^e)\left[R_{t+1}F\left(\bar{\omega}_{t+1}^e\right) + (1 - \delta)P_{t+1}^K \int_0^{\bar{\omega}_{t+1}^e} \omega_{t+1}^e dF\left(\omega_{t+1}^e\right)\right]}{\left(1 - F\left(\bar{\omega}_{t+1}^e\right)\right)\frac{b_{t+1}^e(j)}{K_{t+1}(j)}} \tag{11}$$

where $F\left(\bar{\omega}_{t+1}^e\right)$ is the percent of manufacturing firms that declare bankruptcy.

Holding all else equal, this interest rate, r_{t+1}^e, is increasing in $F\left(\bar{\omega}_{t+1}^e\right)$. If there are financial frictions in the entrepreneurial sector, $F\left(\bar{\omega}_{t+1}^e\right)$ is increasing in $\bar{\omega}_{t+1}^e$. $\bar{\omega}_{t+1}^e$ is increasing in the manufacturing firm's debt-asset ratio. Thus when there are financial frictions in the entrepreneurial sector, the interest rate on physical capital loans is increasing in the level of debt on an entrepreneur's balance sheet.

The cutoff value of $\omega_{t+1}^e(j)$ in equation (9) combined with the interest rate expression in (11) demonstrates the feedback loop associated with financial frictions in the entrepreneurial sector. When the price of existing capital, P_{t+1}^K falls, the cutoff value $\bar{\omega}_{t+1}^e$ rises. This implies that more firms will receive draws of $\omega_{t+1}^e(j)$ below this cutoff value and be forced into bankruptcy. When more firms go into bankruptcy, $F\left(\bar{\omega}_{t+1}^e\right)$ increases, and r_{t+1}^e increases as banks now demand a higher interest rate to compensate for the increased bankruptcy risk. This higher r_{t+1}^e means higher interest expenses and lower profit for the entrepreneur, which leads to a further increase in the cutoff value $\bar{\omega}_{t+1}^e$.

The end of period net worth for the entrepreneur that survives is the entrepreneur's profit in time t plus the value of their non-depreciated capital stock:

$$\tilde{N}_t^e(j) = R_t K_t(j) - (1 + r_t^e) b_t^e(j) + \omega_t^e(j) P_t^K (1 - \delta) K_t(j)$$

The entrepreneur will pay a dividend to shareholders of $d_t^e(j)$ and begin the next period with net worth $N_{t+1}^e(j) = \tilde{N}_t^e(j) - d_t^e(j)$. Entrepreneurs that declare bankruptcy in period t pay no dividend and drop out of the market, they are replaced with new entrepreneurs, which are endowed with start up capital of $\bar{N}^e$. Thus the net worth of the entrepreneurial sector at the beginning of next period is:

$$N_{t+1}^e = \int_0^{\bar\omega_t^e} N_{t+1}^e(j) \, dF(\bar\omega_t^e) + \int_{\bar\omega_t^e}^\infty N_{t+1}^e(i) \, dF(\bar\omega_t^e) \tag{12}$$

$$= \bar N^e F(\bar\omega_t^e) + \left(r_t^k K_t - (1+r_t^e) b_t^e - d_t^e \right)(1 - F(\bar\omega_t^e)) + P_t^K(1-\delta) K_t \int_{\bar\omega_t^e}^\infty \omega_t^e \, dF(\bar\omega_t^e)$$

At the beginning of any period, entrepreneurs have different levels of net worth $N_{t+1}(j)$ that will depend on the entrepreneur's history of idiosyncratic shocks $\omega_t^e(j)$.

The entrepreneur will acquire capital up to the point where the interest rate on bank loans is equal to the expected return to holding a unit of capital:

$$r_{t+1}^e = E_t \left(\frac{R_{t+1} + \omega_{t+1}^e(j)(1-\delta) P_{t+1}^K}{P_t^K} \right)$$

Since $\omega_{t+1}^e(j)$ is i.i.d. and $E_t\left(\omega_{t+1}^e(j)\right) = 1$, the left hand side of the above expression is the same across all entrepreneurs j, which implies that r_{t+1}^e is the same across all entrepreneurs.

2.3   Capital Builders

The representative capital builder converts final goods, given by equation (2), into the physical capital purchased by entrepreneurs. At the end of period t, the non depreciated physical capital stock is $(1-\delta) K_t$, and the physical capital stock at the beginning of the next period is K_{t+1}. The evolution of the physical capital stock is given by:

$$K_{t+1} - (1-\delta) K_t = \phi \left(\frac{I_t}{K_t} \right) K_t$$

where $\phi' > 0$ and $\phi'' < 0$ implying that there are diminishing marginal returns to physical capital investment. Capital builders purchase final goods for investment at a price P_t and sell existing capital to entrepreneurs at a price P_t^K. Thus the profits of the representative capital builder are given by:

$$d_t^c = P_t^K \left(K_{t+1} - (1-\delta) K_t \right) - P_t I_t$$

In a competitive capital building sector, profit maximization implies that the relative price of

existing capital is:

$$\frac{P_t^K}{P_t} = \left[\phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}$$

Since $\phi'' < 0$, when $\frac{I_t}{K_t}$ is high, $\phi'\left(\frac{I_t}{K_t}\right)$ is low, so $\frac{P_t^K}{P_t}$ is high. This implies that during times of high physical capital investment, when the ratio of investment to the existing capital stock is high, the relative price of existing capital is high. Since investment is highly procyclical, capital adjustment costs imply that the relative price of capital is highly procyclical as well.

2.4 Banks

Banks, indexed $k \in [0\ n]$ make physical capital loans to domestic entrepreneurs. They finance this loan portfolio partially with equity and partially with borrowing from domestic and foreign households.

At the beginning of period t, the value of the bank's assets is $B_t^e(k)$, which is the bank's stock of loans to entrepreneurs. The value of the bank's liabilities is $b_t^s(k) + b_t^{sf}(k)$, where $b_t^s(k)$ are the deposits of domestic households and $b_t^{sf}(k)$ are the deposits of foreign households.[3]

The bank also makes working capital loans to firms in order to finance the firm's wage bill. This however is not listed as a beginning of period asset for the bank. By assumption this loan is made after the beginning of the period and repaid before the end of the period. If the stock of working capital loans were to appear as a asset for the bank at the beginning of period t, that would imply that the loan was made in period $t-1$, which implies that the firm made a decision about period t's labor input in period $t-1$.

Bankruptcy in the entrepreneurial sector in period t means the bank's assets are worth less at the end of the period. The value of the average bank's assets at the end of the period is $(1 - \zeta_t^e)(1 + r_t^e) B_t^e$, where ζ_t^e is the share of the average bank's physical capital loan portfolio that is lost to bankruptcy and liquidation costs.

ζ_t^e represents the share of the *average* bank's physical capital loan portfolio that is lost to

---

[3]The same stock of bonds that is a liability to one party is an asset to another. Throughout this paper, when a stock of bonds is an asset, it is written with a capital B, when the stock of bonds is a liability it is written with a lower case b.

Thus market clearing in the bond market requires that the sum of physical capital loans across all banks equals the sum of borrowing by entrepreneurs, $\int_0^n B_t^e(k)\,dk = \int_0^n b_t^e(j)\,dj$.

bankruptcy and liquidation costs, however banks don't hold fully diversified loan portfolios. Some banks may be overexposed to the set of non-performing loans to the entrepreneurial sector. This overexposure may be due to a regional bias in the bank's portfolio, or it may be because a bank has a certain core competency and is therefore overexposed to a certain sector of the economy.[4]

The percent of the bank k's loan portfolio that is lost to bankruptcy or liquidation costs is $\omega_t^b(k)\zeta_t^e$, where $\omega_t^b(k)$ is an i.i.d. draw from a lognormal distribution on the interval $\left[0 \ \frac{1}{\zeta_t^e}\right]$ with mean 1 and standard deviation σ_t^b.

If bank k receives a large draw $\omega_t^b(k)$, it implies that the bank is overexposed to the set of non-performing loans and may itself face insolvency. The bank is insolvent if the end of period value of its assets is less than the end of period value of its liabilities:

$$\left(1 - \omega_t^b(k)\zeta_t^e\right)(1 + r_t^e)B_t^e(k) < \left(1 + r_t^b(k)\right)\left(b_t^s(k) + b_t^{sf}(k)\right)$$

The threshold value of $\omega_t^b(k)$ above which bank k is forced to declare bankruptcy and below which the bank will continue operations is:

$$\bar{\omega}_t^b = \frac{(1 + r_t^e) - \left(1 + r_t^b(k)\right)\frac{b_t^s(k) + b_t^{sf}(k)}{B_t^e(k)}}{\zeta_t^e(1 + r_t^e)} \tag{13}$$

Bank k's history of idiosyncratic draws, $\omega_t^b(k)$, thus its history of exposure to non-preforming sectors of the economy, will determine the levels of $B_t^e(k)$, $b_t^s(k)$, and $b_t^{sf}(k)$. However, at the beginning of the period, all banks will have the same ratio of total debt to total assets, $DA_t^b(k) = \frac{b_t^s(k) + b_t^{sf}(k)}{B_t^e(k)}$ and will have the same cost of capital, $r_t^b(k)$. This result is key for the aggregation of balance sheet variables across a continuum of individual banks, for this implies that the cutoff value $\bar{\omega}_t^b$ is common across all banks. The formal proof of this claim is presented in the appendix.

When deciding how much to lend to bank k in the next period and at what rate, the bank's creditors factor in the fact that if the bank does not default, they receive a gross interest rate $1 + r_{t+1}^b(k)$. If bank k defaults, creditors receive nothing.[5] Thus the expected payoff to a bank's

---

[4]Like the banks, many of which are now bankrupt or were acquired by healthier rivals, who were overexposed to the subprime sector of the mortgage market during the recent financial crisis.

[5]The assumption that creditors receive nothing in the case of bank default is because the model is later calibrated such that the spread between the interbank rate, r^b, and the risk free rate, i, in the steady state of the model is equal to the historical average of the spread between the 3-month Libor and the 3-month T-bill. The Libor is an interbank index rate that is based on the interest rate for unsecured lending to banks.

creditors conditional on the bank's exposure to the set of non-preforming loans is:

$$
\left(1 + r_{t+1}^b(k)\right)\left(b_{t+1}^s(k) + b_{t+1}^{sf}(k)\right) \qquad \text{if } \omega_{t+1}^b(k) \leq \bar{\omega}_{t+1}^b
$$
$$
0 \qquad \text{if } \omega_{t+1}^b(k) > \bar{\omega}_{t+1}^b
$$

(14)

Domestic and foreign depositors will extend bank k credit up to the point where the expected return, after factoring in the probability of default is equal to the risk free rate:

$$
\left(1 + i_{t+1}\right)\left(b_{t+1}^b(k) + b_{t+1}^{bf}(k)\right) = \int_0^{\bar{\omega}_{t+1}^b} \left(1 + r_{t+1}^b(k)\right)\left(b_{t+1}^b(k) + b_{t+1}^{bf}(k)\right) dG\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)
$$

This condition can be used to solve for the interest rate on interbank lending to bank k:

$$
1 + r_{t+1}^b(k) = \frac{1 + i_{t+1}}{G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)}
$$

(15)

where $G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)$ is the c.d.f. of the lognormal distribution of $\bar{\omega}_{t+1}^b$, and thus measures the proportion of banks that do not go bankrupt in period $t+1$. Since $DA_{t+1}^b(k) = \frac{b_{t+1}^s(k) + b_{t+1}^{sf}(k)}{B_{t+1}^e(k)}$ is constant across all banks, the interbank lending rate, and thus banks' cost of capital, is constant across all banks.

A first order Taylor series expansion of the expression in (15) highlights the two factors, one endogenous and one exogenous, that can cause fluctuations in the interbank lending spread, $rp_t^b = \frac{1 + r_{t+1}^b}{1 + i_{t+1}}$:

$$
rp_t^b \approx rp_{ss}^b \left[1 + g_1\left(\frac{\bar{\omega}_{t+1}^b - \bar{\omega}_{ss}^b}{\bar{\omega}_{ss}^b}\right) + g_2\left(\frac{\sigma_{t+1}^b - \sigma_{ss}^b}{\sigma_{ss}^b}\right)\right]
$$

(16)

where $g_1 = -\frac{\partial G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)}{\partial \bar{\omega}_{t+1}^b} \frac{\bar{\omega}_{t+1}^b}{G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)} < 0$ is the elasticity of the spread with respect to changes in the endogenous cutoff value $\bar{\omega}_{t+1}^b$, and $g_2 = -\frac{\partial G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)}{\partial \sigma_{t+1}^b} \frac{\sigma_{t+1}^b}{G\left(\bar{\omega}_{t+1}^b; \sigma_{t+1}^b\right)} > 0$ is the elasticity of the spread with respect to changes in the exogenous variable describing uncertainty in the interbank mark, σ_{t+1}^b. Thus fluctuations in the spread, $r\hat{p}_t^b = \frac{rp_t^b}{rp_{ss}^b} - 1$, can be dividend into two components, an endogenous component due to fluctuations in the endogenous cutoff value $\bar{\omega}_{t+1}^b$, and an exogenous component due to change in the exogenous uncertainty variable σ_{t+1}^b.

Define $r\hat{p}_t^{b,endo} = g_1 \left(\frac{\bar{\omega}_{t+1}^b - \bar{\omega}_{ss}^b}{\bar{\omega}_{ss}^b} \right)$ as the endogenous component of fluctuations in the spread and $r\hat{p}_t^{b,exo} = g_2 \left(\frac{\sigma_{t+1}^b - \sigma_{ss}^b}{\sigma_{ss}^b} \right)$ as the exogenous component.

The end of period t net worth of the bank that is not over-exposed to the set of non-preforming loans and is able to continue operations is:

$$\tilde{N}_t^b = \left(1 - \omega_t^b (k) \zeta_t^e \right) (1 + r_t^e) B_t^e (k) - \left(1 + r_t^b \right) \left(b_t^s (k) + b_t^{sf} (k) \right)$$

The bank will pay a dividend to shareholders and begin the next period with a net worth $N_{t+1}^b (k) = \tilde{N}_t^b (k) - d_t^b (k)$. Banks that were overexposed to the set of non-preforming loans and thus were forced into bankruptcy end the period with no net worth and drop out of the market. They are replaced with new banks that are endowed with start up capital $\bar{N}^b$. Thus the net worth of the entire banking sector at the beginning of next period is:

$$
\begin{aligned}
N_{t+1}^b &= \int_{\bar{\omega}_t^b}^{\infty} N_{t+1}^b (i) \, dG \left(\omega_t^b; \sigma_t^b \right) + \int_0^{\bar{\omega}_t^b} N_{t+1}^b (k) \, dG \left(\omega_t^b; \sigma_t^b \right) \qquad (17) \\
&= \bar{N}^b \left(1 - G \left(\omega_t^b; \sigma_t^b \right) \right) + \left((1 + r_t^e) B_t^e - \left(1 + r_t^b \right) \left(b_t^s + b_t^{sf} \right) - d_t^b \right) G \left(\omega_t^b; \sigma_t^b \right) \\
&\quad - (1 + r_t^e) B_t^e \zeta_t^e \int_0^{\bar{\omega}_t^b} \omega_t^b dG \left(\omega_t^b; \sigma_t^b \right)
\end{aligned}
$$

2.5 Households

Households, indexed $l \in [0 \; n]$, supply heterogeneous labor to firms and consume from their labor income, interest on savings, and profit income from firms, entrepreneurs, capital builders, and banks.

The household maximizes their utility function:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t (l) \right) - \psi \left(H_t (l) \right)^{\frac{1+\sigma_H}{\sigma_H}} \right] \qquad (18)$$

subject to their budget constraint:

$$P_t C_t(l) + B_{t+1}^s(l) + S_t B_{t+1}^{sf*}(l) + F(\bar{\omega}_t^e) \bar{N}^e + \left(1 - G\left(\omega_t^b; \sigma_t^b\right)\right) \bar{N}^b \qquad (19)$$

$$= W_t(l) H_t(l) + d_t^f(l) + d_t^e(l) + d_t^c(l) + d_t^b(l) + \left(1 - \zeta_t^b\right)\left(1 + r_t^b\right) B_t^s(l)$$

$$+ \left(1 - \zeta_t^{b*}\right)\left(1 + r_t^{b*}\right) S_t B_t^{sf*}(l) + \zeta_t^e + \zeta_t^b - \frac{\chi^b}{2}\left(S_t B_t^{sf*}(l) - S_t \bar{B}^{sf*}\right)^2$$

where $C_t(l)$ is consumption by household l in period t, $H_t(l)$ is the household's labor effort in the period, $B_t^s(l)$ is the household's stock of deposits with domestic banks at the beginning of the period, $B_t^{sf*}(l)$ is the stock of deposits with foreign banks, $W_t(l)$ is the wage paid for the household's heterogenous labor supply, ζ_t^b (ζ_t^{b*}) represents the small share of deposits to the home (foreign) banking sector that are lost to bankruptcy and liquidation costs, and $d_t^f(l)$, $d_t^e(l)$, $d_t^c(l)$ and $d_t^b(l)$ are the household's share of period t profits from firms, entrepreneurs, capital builders and banks, respectively.[6]

The household pays a small quadratic transactions cost to holding other than the steady state level of deposits with foreign banks, $\frac{\chi^b}{2}\left(B_t^{sf*}(l) - \bar{B}^{sf*}\right)^2$.

Each household supplies a differentiated type of labor. The function to aggregate the labor supplied by each household into the aggregate stock of labor employed by domestic firms is:

$$H_t = \left(\int_0^n H_t(l)^{\frac{\theta-1}{\theta}} dl\right)^{\frac{\theta}{\theta-1}} \qquad (20)$$

where $H_t = \int_0^n h_t(i) \, di$. Since the household supplies a differentiated type of labor, it faces a downward sloping labor demand function:

$$H_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\theta} H_t$$

In any given period, household j faces a probability of $1 - \xi_w$ of being able to reset their wage, otherwise it is reset automatically according to $W_t(l) = \pi_{t-1} W_{t-1}(l)$.

If household j is allowed to reset their wages in period t they will set a wage to maximize the

---

[6] Market clearing in the market for deposits requires that the sum of deposits with domestic banks across all domestic households equals the sum of borrowing from domestic households across all domestic banks, $\int_0^n B_t^s(l) \, dl = \int_0^n b_t^s(k) \, dk$, and that the sum of deposits with foreign banks across all domestic households equals the sum of borrowing from domestic households across all foreign banks, $\int_0^n B_t^{sf*}(l) \, dl = \int_n^1 b_t^{sf*}(k) \, dk$.

expected present value of utility from consumption minus the disutility of labor.

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left\{ \lambda_{t+\tau} \Pi_{t,t+\tau} W_t(l) H_{t+\tau}(l) - \psi (H_{t+\tau}(l))^{\frac{1+\sigma_H}{\sigma_H}} \right\}$$

Thus after technical details which are located in the appendix, the household that can reset wages in period t will choose a wage:

$$W_t(l)^{\frac{\theta}{\sigma_H}+1} = \frac{\theta}{\theta-1} \frac{1+\sigma_H}{\sigma_H} \psi (W_t)^{\frac{\theta}{\sigma_H}} \frac{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left(\frac{W_{t+\tau}}{\Pi_{t,t+\tau} W_t} \right)^{\frac{\theta}{\sigma_H}+\theta} (H_{t+\tau})^{\frac{1+\sigma_H}{\sigma_H}}}{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \lambda_{t+\tau} \Pi_{t,t+\tau} \left(\frac{W_{t+\tau}}{\Pi_{t,t+\tau} W_t} \right)^{\theta} H_{t+\tau}}$$

If wages are flexible, and thus $\xi_w = 0$, this expression reduces to:

$$W_t(l) = \frac{\theta}{\theta-1} \frac{\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}}{\lambda_t}$$

Thus when wages are flexible the wage rate is equal to a mark-up, $\frac{\theta}{(\theta-1)}$, multiplied by the marginal disutility of labor, $\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}$, divided by the marginal utility of consumption, λ_t.

Write the wage rate for the household that can reset wages in period t, $W_t(l)$, as $\tilde{W}_t(l)$ to denote it as an optimal wage. Also note that all households that can reset wages in period t will reset to the same wage rate, so $\tilde{W}_t(l) = \tilde{W}_t$.

All households face a probability of $(1 - \xi_w)$ of being able to reset their wages in a given period, so by the law of large numbers $(1 - \xi_w)$ of households can reset their wages in a given period. The wages of the other ξ_w will automatically reset by the previous periods inflation rate.

Substitute $\tilde{W}_t$ into the expression for the average wage rate $W_t = \left(\int_0^n W_t(l)^{1-\theta} dl \right)^{\frac{1}{1-\theta}}$, to derive an expression for the evolution of the average wage:

$$W_t = \left(\xi_w (\Pi_{t-1,t} W_{t-1})^{1-\theta} + (1 - \xi_w) \left(\tilde{W}_t \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

2.6 Monetary Policy

The monetary policy instrument is the short term risk free rate, i_t, which is determined by the central bank's Taylor rule function:

$$i_{t+1} = i_{ss} + \theta_i \left(i_t - i_{ss} \right) + (1 - \theta_i) \left(\theta_p \pi_t + \theta_y \hat{y}_t + \theta_r r\hat{p}_t^b + \theta_{rf} r\hat{p}_t^{b*} \right) \tag{21}$$

where $\pi_t = \frac{P_t}{P_{t-4}} - 1$, and $\hat{y}_t = \frac{GDP_t}{G\tilde{D}P_t} - 1$, where $G\tilde{D}P_t$ is the level of GDP at time t in an economy with the same structure as the one just described and subject to the same shocks, only there are no price or wage frictions, $\xi_p = \xi_w = 0$, and there are no financial frictions, $\sigma^e = \sigma^b = 0$.

When $\theta_r = \theta_{rf} = 0$, the central bank does not place any weight on conditions in the interbank markets and the Taylor rule is simply the conventional Taylor rule with smoothing.

Recall from equation (16) that fluctuations in the interbank lending spread $r\hat{p}_t^b$ have an endogenous and exogenous component, $r\hat{p}_t^b = r\hat{p}_t^{b,endo} + r\hat{p}_t^{b,exo}$. In another version of the Taylor rule, we assume that the central bank can distinguish between these two components and potentially respond differently to fluctuations in the spread depending on whether it is due to endogenous or exogenous factors. In this case the Taylor rule would take the form:

$$i_{t+1} = i_{ss} + \theta_i \left(i_t - i_{ss} \right) + (1 - \theta_i) \left(\theta_p \pi_t + \theta_y \hat{y}_t + \theta_r^{endo} r\hat{p}_t^{b,endo} + \theta_r^{exo} r\hat{p}_t^{b,exo} + \theta_{rf}^{endo} r\hat{p}_t^{b,endo*} + \theta_{rf}^{exo} r\hat{p}_t^{b,exo*} \right) \tag{22}$$

3 Parameter Values

The model in the previous section is solved with a first-order approximation and the results are found from simulations of the calibrated model. This section will begin by presenting the basic parameter values used in this calibration. Then we will describe the various types of exogenous shocks that will drive the simulations of the model and the estimation of these different shock processes.

The full list of the model's parameters and their values is found in table 1.

The first eight parameters: the discount factor, the capital depreciation rate, capital's share of income, the elasticity of substitution between home and foreign goods, the bond adjustment cost parameter, the labor supply elasticity, the elasticity of substitution between goods from different firms, and the elasticity of substitution between labor from different households are all set to values that are commonly found in the literature.

The capital adjustment cost parameter, κ, describes the curvature of the capital adjustment function $\phi\left(\frac{I_t}{K_t}\right)$. It is the elasticity of the relative price of capital with respect to changes in the investment-capital ratio. This parameter preforms the important functions of lowering the relative volatility of investment and ensuring the procyclicality of the price of capital. Empirical estimates of this parameter vary, but the value of 0.125 is in the middle of the range of empirical estimates and ensures that the relative volatility of investment in the model is near what we see in the data.

The next two parameters in the table are the Calvo price and wage stickiness parameters. The wage stickiness parameter is chosen such that on average a household adjusts their wages once a year. The price stickiness parameter implies that prices are a little more flexible than wages and is taken from the DSGE estimation literature (see e.g. Christiano et al. 2005).

The next four parameters are all determined so that the steady state of the model is able to match certain features of the data. The γ and γ^f parameters from the function that aggregates home and foreign goods in (2) are set such that the home country in the open economy version of the model have a steady state import share of 25%.[7] The next two parameters, ϕ and ψ are the fixed cost in the production of intermediate goods and the weight on the disutility from labor in the household's utility function, respectively. These are set to ensure that in the steady state, intermediate goods firms earn zero economic profit and the household's labor supply is unity.

Finally the last three parameters in the table relate to the risk of bankruptcy and liquidation costs in either the banking or entrepreneurial sectors. The steady state value of σ_t^b measures the steady state level of uncertainty in the financial sector. This parameter is determined to ensure that in the steady state of the model, when banks have a debt-asset ratio of about 0.9, there is a 13 basis point spread between interbank rates and the risk free rate, the average quarterly spread between the 3-month Libor and the 3-month T-bill from 1984 to 2007.

The cost of liquidation and the idiosyncratic bankruptcy risk in the entrepreneurial sector, μ^e and σ_e are jointly determined. These parameters ensure that in the steady state of the model, when firms in the entrepreneurial sector have a debt-asset ratio of 0.5, an entrepreneur faces a 2% probability of bankruptcy and the steady state spread between the interest rate on physical capital

---

[7]From the demand functions for domestically supplied intermediate inputs and imports, equations (3) and (4), the steady state import share is: $m = \dfrac{\displaystyle\int_n^1 P_t^m(i) y_t^m(i)}{\displaystyle\int_0^n P_t^d(i) y_t^d(i) + \int_n^1 P_t^m(i) y_t^m(i)} = \dfrac{\gamma^f (1-n)^{\frac{1-\rho}{1-\sigma}}}{\gamma(n)^{\frac{1-\rho}{1-\sigma}} + \gamma^f (1-n)^{\frac{1-\rho}{1-\sigma}}}$

loans and the bank's cost of capital is approximately 70 basis points.[8]

3.1 Exogenous Shock Processes

In this model there are two types of shocks. The first shock is simply a country specific shock to total factor productivity (TFP) in (1). The second shock is a shock to the uncertainty about the health of a bank's assets, σ_t^b, and is unique to a model with financial frictions in the banking sector.

Since TFP shocks are not the primary focus of the study, we set the exogenous process that governs TFP shocks to a simple process that is familiar in the real business cycle literature. Shocks to TFP, $\hat{A}_t$ follow an AR(1) process with an autoregressive coefficient of 0.9. Since the model is solved using a first order approximation, we simply normalize the variance of the shocks to TFP to one. To ensure comparability across the closed and open economy versions of the model, we assume that TFP follows this same process regardless of country type, this ensures that any differences in optimal policy between the two versions of the model are due to the internal dynamics of the model and not the exogenous shock process.

Alternatively we can consider shocks to the financial sector uncertainty variable, σ_{t+1}^b. Equation (16) describes how fluctuations in the interbank lending spread can be broken down into two components, one due to fluctuations in the endogenous cutoff value $\bar{\omega}_{t+1}^b$, and one due to exogenous fluctuations in financial sector risk, σ_{t+1}^b.

$$\hat{rp}_t^b \approx g_1 \left(\frac{\bar{\omega}_{t+1}^b - \bar{\omega}_{ss}^b}{\bar{\omega}_{ss}^b} \right) + g_2 \left(\frac{\sigma_{t+1}^b - \sigma_{ss}^b}{\sigma_{ss}^b} \right)$$

We construct a time series of the interbank spread rp_t^b by taking the difference between the 3-month USD Libor and the 3-month T-bill from 1985:1-2010:3 (alternatively we consider the period 1985:1-2007:2). A time series of the cutoff value $\bar{\omega}_{t+1}^b$ is constructed using data on U.S. commercial bank debt asset ratios, loan delinquency rates, the prime rate, and the 3-month USD Libor over the same period. In an OLS regression where $\hat{rp}_t^b$, is the dependent variable, $\frac{\bar{\omega}_{t+1}^b - \bar{\omega}_{ss}^b}{\bar{\omega}_{ss}^b}$ is the independent variable, and $g_2 \left(\frac{\sigma_{t+1}^b - \sigma_{ss}^b}{\sigma_{ss}^b} \right)$ is the residual, the estimated coefficient $\hat{g}_1 = -0.274$ with a standard error of 0.079 and an $R^2 = 0.108$ (for the 1985:1-2007:2 period $\hat{g}_1 = -0.258$ with $SE = 0.053$ and

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[8]The calibration that entrepreneurs have a steady state debt-asset ratio of about 0.5 and banks have a steady state debt-asset ratio of about 0.9 is based on the historical average debt-asset ratios for U.S. non-financial and financial firms as reported in the Federal Reserve's Flow of Funds Accounts.

$R^2 = 0.211$).

We use the residual term $g_2 \left(\frac{\sigma^b_{t+1} - \sigma^b_{ss}}{\sigma^b_{ss}} \right)$ to estimate an AR(1) process for σ^b_{t+1} and find that the financial sector uncertainty variable has an autoregressive coefficient of 0.74. When presenting the results in the next section, we will parameterize the exogenous shock process for σ^b_{t+1} such that the autoregressive coefficient is 0.74, but we will experiment with the variance of the process and find that when identified correctly, the optimal monetary policy is largely invariant to the variance of the exogenous financial sector uncertainty shocks, σ^b_{t+1}.

4 Results

To find the optimal coefficients in the central bank's Taylor rule and access how these optimal parameters change as the degree of financial sector risk in the economy changes, we first have to define the loss function that the central bank will attempt to minimize. In order to ensure that the changes in optimal policy are due to changes in the endogenous structure of the economy and the transmission mechanism and not due to changes in the central bank's preferences, this loss function should remain the same regardless of the degree of financial sector risk in the economy.

When finding the optimal coefficients in the Taylor rule, the central bank will attempt to minimize:

$$\mathcal{L} = var\left(\pi_t\right) + 0.5 * var\left(\hat{y}_t\right) + 0.1 * var\left(i_t - i_{t-1}\right)$$

where π_t is the inflation rate, $\hat{y}_t$ is the output gap, and $i_t - i_{t-1}$ is the quarter-over-quarter difference in the nominal risk free interest rate.

4.1 The parameters in the conventional Taylor rule

To evaluate how monetary policy should respond to financial sector risk, we first identify the optimal weights on inflation, the output gap, and the lagged interest rate in the Taylor rule function in the model where business cycles are driven by productivity shocks. We find these optimal parameters three times, in a model without a financial accelerator, $\sigma^e = \sigma^b = 0$, in a model with a financial accelerator only in the entrepreneurial sector, $\sigma^e > 0$ and $\sigma^b = 0$, and in a model with a financial accelerator in both the entrepreneurial and banking sectors, $\sigma^e > 0$ and $\sigma^b > 0$.

We find these parameters through a grid search. We vary θ_p, θ_y, and θ_i until we find the combination of the three terms in the central bank's Taylor rule function that minimizes the central bank's loss function. The results from this search are presented in table 2. The table presents optimal parameters for both the closed economy and the open economy versions of the model.

In the benchmark parameterization of the model, regardless of country type, the presence of a financial accelerator in the model means that the central bank will want to put more weight on the lagged interest rate. In the closed economy version of the model, the optimal coefficient on the lagged interest rate increases from 0.74 in the version of the model with no financial frictions to 0.76 in the model with financial frictions only in the real sector, to 0.78 in the model with frictions in both the real and financial sectors. The results for the open economy version of the model show a similar trend.

Similarly, as the level of financial frictions in the model increases, the central bank will put more weight on inflation and less weight on the output gap. In the closed economy, adding a financial accelerator in both the entrepreneurial and banking sectors means that the central bank will want to increase the weight on inflation by 0.05 from 1.62 to 1.67, and they will decrease the weight on the output gap by the same (absolute) amount from 0.4 to 0.35.

Thus as the level of financial frictions in the model increases, the central bank will want to put more weight on the lagged interest rate as opposed to contemporaneous variables, and more weight on inflation as opposed to output.

In table 2, financial frictions lead to endogenous changes in lending spreads (either the entrepreneurial lending spread, $r_t^e - r_t^b$, or the interbank spread, $r_t^b - i_t$) in response to fluctuations in balance sheets and asset prices that are ultimately caused by an exogenous productivity shocks. Many recent papers have also considered the role of exogenous financial shocks. In this model, exogenous financial shocks take the form of exogenous shocks to the interbank credit supply schedule. As described in equation (16), fluctuations in the interbank lending rate are driven by endogenous fluctuations in $\bar{\omega}_t^b$, the cutoff value for insolvency in the banking sector, which is determined by endogenous variables like debt-asset ratios and loan loss ratios, and exogenous fluctuations in σ_t^b, the stochastic variable that measures the degree of ex-ante uncertainty about the health of a particular bank's assets.

Define Σ as the standard deviation of the exogenous fluctuations in the interbank lending spread.

The standard deviation of exogenous TFP fluctuations is normalized to one, so Σ measures the ratio of the standard deviations of the two shocks in the model, the exogenous shocks to the interbank spread and the exogenous shocks to TFP.

$$\Sigma^2 = \frac{var\left[g_2\left(\frac{\sigma^b_{t+1}-\sigma^b_{ss}}{\sigma^b_{ss}}\right)\right]}{var\left[A_t\right]}$$

The optimal weights on inflation, the output gap, and the lagged interest rate under different values of Σ are presented in table 3. It should first be noted that in this model, exogenous financial shocks only make sense in the version of the model with a financial accelerator in both the entrepreneurial and banking sectors. The case where $\Sigma = 0$, and thus TFP shocks are the only active shocks, is the same as the "Bank FA" rows of table 2.

An interesting result comes out of the results in table 3. As Σ, the strength of exogenous credit shocks, increases, the central bank will want to put slightly more weight on the lagged interest rate. Quantitatively, the shift in θ_i is small, it is only interesting because it seems to fit well with the results from table 2, as the strength of the fluctuations in interbank borrowing spreads increases, the central bank will want to put more weight on the lagged interest rate.

However, this connection between table 2 and 3 does not hold when considering the effect of an increasing Σ on θ_p. In table 2, when added layers of financial frictions lead to increased variability in the interbank lending spread, the central bank would want to respond by placing more weight on inflation and less on the output gap. In table 3, when the increasing severity of financial sector shocks is leading to increased variability in the interbank lending spread, the central bank will want to respond by placing less weight on inflation and more weight on the output gap. This fact that the central bank's optimal response to variability in the spread depends on whether that variability is the endogenous reaction to real variables in the model like balance sheets and default rates or whether it is the reaction to exogenous credit shocks is a theme that will appear in the next subsection where we consider the desirability of including spreads in the Taylor rule.

Also, notice the last column in table 3. This column reports the distance between the value of the loss function when the Taylor rule parameters are optimally chosen and the value of the loss function under the Ramsey true optimal policy.[9] Thus when there are only TFP shocks in the

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[9]In the open economy case, the true optimal is defined as the cooperative Ramsey equilibrium.

model, the value of the loss function under the optimal Taylor rule is about 15% higher than the value in the true optimum. As Σ increases, the Taylor rule function of inflation, the output gap, and the lagged interest rate becomes as worse approximation of optimal policy. When $\Sigma = 0.2$, the minimum value of the loss function under the Taylor rule is 26% higher than the true optimum.

4.2 The optimal weight on the interbank lending spread

The previous section discusses how a central bank would shift the relative weights in their Taylor rule function towards interest rate stability and (maybe) price level stability when there is variability in the interbank lending spread. This section will consider if in addition to doing this, is there any benefit to putting a measure of financial risk, like interbank lending spreads, directly in the Taylor rule.

Recall from equation (21), θ_r and θ_{rf} are the weights on the home and foreign interbank lending spreads, respectively, in the central bank's Taylor rule. If $\theta_r = 0$, then the central bank does not react to changes in home interbank lending spreads, but if $\theta_r < 0$, the central bank responds to an increase in interbank lending spreads by lowering the nominal risk free rate.

We use the same procedure that was used to calculate the results in tables 2 and 3, only now instead of finding the optimal coefficients θ_p, θ_y, and θ_i, we vary θ_p, θ_y, θ_i, and θ_r (and θ_{rf} for the open economy case). These optimal coefficients are presented in table 4. The table presents the optimal Taylor rule coefficients as a function of the standard deviation of the exogenous financial sector shocks, Σ, for both the closed economy and the open economy versions of the model.

The first and most important finding in the table is that the optimal coefficient on the interbank spread is zero when Σ is zero. In other words, when the conventional Taylor rule parameters are chosen optimally, there is no need to also include interbank spreads in the central bank's policy function provided that any fluctuations in the spread are endogenous reactions to real shocks in the model. If fluctuations in the spread are the endogenous reaction to fluctuations in real and nominal variables in the model, then interbank spreads provide no new information that is not already provided by measures of the output gap and inflation. If the central bank chooses the optimal weights on the output gap and inflation, then there is no need to also pay attention to changes in spreads. If the central bank assigns a non-zero coefficient on the interbank spread, $\theta_r < 0$ or $\theta_{rf} < 0$, the weighting of the available information is no longer optimal.

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This result changes when the model is also driven by exogenous financial sector shocks. When there are exogenous financial sector shocks, there is information in the interbank spread that may not be contained in readings of current inflation and the current output gap. Thus the central bank may find it worthwhile to also pay attention to interbank spreads, $\theta_r < 0$ and $\theta_{rf} < 0$, given that they potentially contain new information that is not already factored into the optimal weighting of the output gap and inflation.

The table shows that as the standard deviation of the exogenous financial sector shocks increases, the amount of new information contained in the interbank spread increases and thus the coefficient on the interbank spread increases (in absolute value). This highlights the fact that when setting a non-zero coefficient to the interbank spread, the central bank faces a trade-off. When there are exogenous financial sector shocks in the model, fluctuations in the spread are driven by both endogenous and exogenous factors. Since the endogenous fluctuations contain no new information that is not already contained in the optimally chosen weighting of inflation and the output gap, assigning any weight to these endogenous components is suboptimal. However the exogenous component does contain new information, and thus ignoring this component is suboptimal. When increasing the weight on the interbank spread, the central bank is balancing the marginal benefit of increasing the coefficient on the exogenous component against the marginal cost of increasing the weight on the endogenous component. Thus when $\Sigma = 0.05$, the exogenous financial sector shocks are weak enough that the cost still outweighs the benefit and thus the optimal coefficient is zero, $\theta_r = 0$ or $\theta_{rf} = 0$. When $\Sigma = 0.10$ the exogenous financial sector shock is stronger and now the marginal benefit of including spreads in the Taylor rule equals the marginal cost at a non-zero coefficient. When $\Sigma = 0.20$ the exogenous financial shock is stronger still and thus the marginal benefit of targeting the exogenous component of the spread is equal to the marginal cost of targeting the endogenous component at an even higher coefficient (in absolute value).

Recall from table 3 that when Σ increases, the conventional Taylor rule quickly becomes a poor approximation for the true optimal policy. When $\Sigma = 0$, the value of the loss function under the Taylor rule is about 15% higher than the value under the Ramsey solution but when $\Sigma = 0.2$, the conventional Taylor rule now misses the true optimum by 26%. This trend is largely halted when the Taylor rule is also a function of the interbank lending spread. Table 4 shows that when $\Sigma = 0$, the Taylor rule with interbank lending spreads as a potential argument misses the true optimum by

15%, and when $\Sigma = 0.2$ the modified Taylor rule only misses the true optimum by about 17%. Thus when credit shocks are a potential source of economic fluctuations, including interbank spreads in the Taylor rule leads to a significant improvement over conventional monetary policy that is simply a function of inflation and the output gap.

4.2.1 The optimal weights when endogenous fluctuations are separated from exogenous fluctuations

Given that when setting the optimal coefficients on the home and foreign interbank lending spreads the central bank must balance the benefit of targeting the exogenous component of the spread against the cost of targeting the endogenous component of the spread, the natural question arises, what are the optimal coefficients on the home and foreign interbank lending spreads when the central bank can distinguish between endogenous and exogenous fluctuations?

The optimal coefficients on interbank lending spreads assuming that the central bank can distinguish between endogenous and exogenous components of the spread are presented in table 5. The results in the table confirm the earlier intuition. The optimal policy is to assign a coefficient of zero to endogenous fluctuations in the spread while assign a negative coefficient to exogenous fluctuations. Thus the optimal policy is to ignore endogenous fluctuations since they contain no new information that is not already contained in the optimal weighting of the output gap and the inflation rate, but at the same time accommodate exogenous fluctuations in the spread, and thus lower the nominal risk free rate in response to an exogenous increase in the interbank spread.

In the closed economy version of the model, the central bank should ignore endogenous fluctuations in the spread and lower the risk free rate in the long run by about 1 percentage point for every 1 percentage point increase in the interbank lending spread that can be attributed to an exogenous credit shock. The results are fairly stable, the central bank will want to pursue a similar policy regardless of the relative strength of the exogenous financial shocks.

Furthermore, the results for the open economy are similar, but now instead of only focusing on exogenous fluctuations in the domestic interbank spread, the central bank also responds to exogenous fluctuations in the foreign spread. In the closed economy case, the central bank cut the risk free rate by about 100 basis points in response to a 100 basis point increase in the exogenous portion of the spread, in the open economy case, the central bank cuts the risk free rate by about

70 basis points in response to a 100 basis point increase in the exogenous portion of the domestic spread and by about 30 basis points in response to a similar increase in the foreign spread.

The last column of table 5 shows again that when the Taylor rule is also a function of interbank lending spreads, the Taylor rule continues to be a good approximation for optimal policy even as the severity of the exogenous financial shocks increases. Furthermore a comparison of the last column in table 4 with that in table 5 shows that there is a slight welfare improvement when the central bank is able to distinguish between the endogenous and exogenous portions of the spread. In the closed economy, when $\Sigma = 0.2$, the modified Taylor rule where the central bank cannot identify the the source of the fluctuations in the spread yields a loss function about 17.7% above the optimum, when the central bank can distinguish between different types of fluctuations, the value of the loss function is only 16.7% above the optimum.

4.2.2 The optimal use of interbank spreads as an intermediate target (or instrument)

In congressional testimony, Taylor (2008) describes the experience of the Swiss National Bank in the early days of the 2007-2009 financial crisis. Unlike the Federal Reserve and most other central banks which use a short term nominal risk free rate as their intermediate target, the intermediate target for the Swiss National Bank is the 3-month interbank rate. Thus if there is an exogenous increase in financial sector risk, like that which occurred in August 2007, Swiss monetary policy is automatically accommodative. In terms of the Taylor rule function in (22), it was as if $\theta_r^{exo} = -1$ in the special case where $\theta_i = 0$.

Given that there is a role for interest rate smoothing in the model, the results presented thus far as not directly comparable to the case of using interbank rates as an intermediate target. The results presented in table 5 show that following an exogenous one percentage point increase in the interbank spread, the optimal policy response by the closed economy central bank is to reduce the nominal risk free rate by about 1 percentage point *in the long run*.

If instead we rearrange the central bank's Taylor rule function and put the risk spread outside of the smoothing parameter then the Taylor rule function that is more comparable to changes in the natural rate is given by:

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$$i_{t+1} = i_{ss} + \theta_i \left(i_t - i_{ss} - \theta_r r\hat{p}^b_{t-1} - \theta_{rf} r\hat{p}^{b*}_{t-1} \right) + (1 - \theta_i)\left(\theta_p \pi_t + \theta_y \hat{y}_t\right) + \theta_r r\hat{p}^b_t + \theta_{rf} r\hat{p}^{b*}_t \quad (23)$$

Recall that $r\hat{p}^b_t \approx \left(r^b_t - i_t\right) - \left(r^b_{ss} - i_{ss}\right)$, if $\theta_{rf} = 0$, this revised Taylor rule becomes:

$$r_{t+1} = r_{ss} + \theta_i \left(r_t\right) + (1 - \theta_i)\left(\theta_p \pi_t + \theta_y \hat{y}_t\right)$$

where $r_t = -\theta_r r^b_t + (1 + \theta_r) i_t$. Since $\theta_r \leq 0$, the revised Taylor rule simply says that the central bank's intermediate target (or instrument)[10] is determined in a conventional Taylor rule with smoothing function, and the intermediate target is a convex combination of the interbank rate and the risk free rate. If $\theta_r = -1$, the central bank uses the interbank rate as their intermediate target.

The values of θ_r and θ_{rf} in the alternative version of the Taylor rule in (23) that minimize the central bank's loss function are presented in table 6. The table presents the optimal combination of parameters both for the case where the central bank cannot distinguish between endogenous and exogenous fluctuations in the interbank lending spread and thus they are grouped in the policy rule function (Grouped), and also the case where the central bank can distinguish between the two and thus they enter as separate terms in the Taylor rule (Separate).

The key results from the earlier analysis where interbank lending spreads were simply another term in the Taylor rule still hold here. Mainly, the central bank will want to ignore endogenous fluctuations in the spread, which contain no new information, and actively adjust the risk free rate to accommodate exogenous fluctuations in the spread.

Furthermore, the central bank will want to adjust the natural rate in response to exogenous fluctuations in the home and foreign interbank lending spread, but they will not be fully accommodative. This would be the case where $\theta_r = -1$, and thus the central bank is using the interbank borrowing rate as their intermediate target. In the closed economy case, the central bank will want to lower the risk free rate by about two-thirds of a percentage point in response to an exogenous one percentage point increase in the spread. In the open economy case, the central bank will want

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[10]Since the policy rate like the Fed Funds rate is determined in a market, it is technically not an instrument but an intermediate target. The central bank's instrument is the monetary base or the quantity of reserves. However since most new Keynesian models simply assume that the central bank sets the policy rate according to a Taylor rule function, is is treated as the central bank's monetary policy instrument. The distinction is simply one of semantics.

to reduce the risk free rate by about half of a percentage point in response to an exogenous one percentage point increase in the home interbank spread and they will want to reduce the risk free rate by about a sixth of a percentage point in response to a similar exogenous increase in the foreign interbank spread.

The last column of the table shows that when the central bank adjusts the natural rate in response to fluctuations in the spread and can distinguish between endogenous and exogenous fluctuations in the spread, the relative size of exogenous financial shocks has almost no impact on the quality of monetary policy under the Taylor rule. In the closed economy version of the model, when $\Sigma = 0$, the value of the loss function under the modified Taylor rule is 15.3% above the true optimum, when $\Sigma = 0.2$, the modified Taylor rule yields results which are only 15.8% above the true optimum.

4.3 Impulse responses under Ramsey optimal policy and Taylor rules with and without spreads

In the previous section we show how including spreads in the Taylor rule can numerically bring us closer to true optimal policy. In this section we will instead consider impulse responses to see the path of the output gap, inflation and other macro variables following a shock and show how including spreads in the Taylor rule can make the path of these variables following a shock closer to the true optimal path.

Figure 1 presents the responses of the output gap, inflation, investment, the nominal risk free rate, the entrepreneurial risk spread $(r_t^e - r_t^b)$, and the interbank lending spread $(r_t^b - i_t)$ in the closed economy following an exogenous shock to financial sector uncertainty. The responses are plotted under three assumptions for monetary policy. The solid line represents Ramsey optimal monetary policy, the dashed line is the path when policy is determined by a Taylor rule function of the output gap, inflation, and the lagged interest rate, and the line with stars is the path when monetary policy is determined by a Taylor rule function of the output gap, inflation, the lagged interest rate, and the interbank lending spread.[11]

The entire process is driven by an exogenous 30 basis point increase in the interbank lending

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[11]In the case of the optimally chosen conventional Taylor rule without spreads, we use the coefficients from the $\Sigma = 0.2$ line of table 3. The parameters for the optimally chosen modified Taylor rule are taken from the $\Sigma = 0.2$ line of table 4.

spread, as shown in the lower right-hand diagram. When monetary policy is determined by a conventional Taylor rule without interbank lending spreads, this exogenous increase in the spread leads to a 4% fall in investment and a 60 basis point increase in the gap between potential and actual output. If however monetary policy is the true optimal, there is a sudden cut in the risk free rate. This ensures that there is only a 2% fall in investment and a slight improvement in the output gap.

When spreads are included in the Taylor rule, the path of the risk free rate following the shock is closer to the path determined by Ramsey optimal policy. As a result the path of investment and the output gap when policy is determined by the Taylor rule with spreads is very similar to the optimal policy under Ramsey policy.

It should be noted however that the policy of including spreads in the Taylor rule, while closer to true optimal policy than when spreads are ignored, is not costless. The exogenous financial sector uncertainty shock is a shock to the efficiency of financial intermediation. Specifically it represents a shift in the supply curve in the interbank lending market. The central bank can cut the risk free rate to accommodate the shock, but it cannot reverse the shock. The cost of accommodation is higher inflation, as shown in the top right-hand diagram in the figure. Specifically, when monetary policy is determined by a Taylor rule with spreads, accommodating the exogenous 30 basis point increase in the interbank lending spread results in a $10 - 15$ basis point increase in inflation.

Figure 2 presents the responses of the same variables in the closed economy to a TFP shock. Here any movement in the interbank lending spread in the lower right-hand diagram is an endogenous reaction to changes in real variables. In response to a TFP shock, there is not much of a difference between policy under the conventional Taylor rule and under the Taylor rule with spreads. The figure also shows that the responses under the Taylor rules are pretty similar to those under Ramsey policy, implying that the Taylor rule is a close approximation to optimal policy in the model in response to TFP shocks. The difference is that the optimal response seems to allow a little more inflation and thus there is less of a fall in the output gap and investment following the shock.

Figure 3 presents the responses of the same variables, but this time in the open economy model in response to a *foreign* exogenous financial sector uncertainty shock.[12] Again, the dashed line in

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[12]The responses in the open economy to a home shock looks very similar to the responses in the closed economy, and thus have been omitted for brevity

the figure represents the optimal Taylor rule function of the output gap, inflation, and the lagged interest rate in the open economy version of the model, and the line with stars plots the optimal responses when the Taylor rule is also a function of both home and foreign interbank lending spreads. The solid line in the figure plots the responses where monetary policy in both countries is determined in a cooperative Ramsey equilibrium.

When spreads are included in the Taylor rule, monetary policy reacts to the endogenous fluctuations in the home interbank spread, which are plotted in the lower right-hand diagram, but policy also reacts to (largely exogenous) movements in the foreign interbank spread (not pictured). As a result, monetary policy is much more accommodative when spreads are included in the policy rule, and investment and the output gap are higher when spreads are included in the rule, and there is an endogenous fall in both the entrepreneurial risk spread and the interbank lending spread.

What is interesting from the figure is how closely the responses under the modified Taylor rule track the true optimal policy. For all six variables in the figure, the line with stars is nearly indistinguishable from the solid line, implying that the modified Taylor rule is very close to the true optimal policy.

It should be noted that in this open economy version of the model where the shock is to foreign financial sector uncertainty, the modified Taylor rule does not produce the same sharp increase in inflation as in the closed economy version of the model. Recall that in figure 3 the two countries are symmetric. The home country central bank with a modified Taylor rule is cutting interest rates in response to an increase in the foreign interbank lending spread, but at the same time, the foreign country central bank is cutting interest rates in response to an increase in domestic interbank lending spreads. Recall also from table 4 that the foreign central bank will react more strongly to this increase in spreads in the foreign country. With the foreign central bank cutting rates by more in response to the same shock, the home country currency will appreciate. This will hold down import price inflation and thus overall consumer inflation even through under the modified Taylor rule, the home country central bank is following an accommodative monetary policy in response to the foreign financial sector shock.

Finally, figure 4 presents the responses of the same variables to a foreign TFP shock under the three different monetary policy scenarios. As before in the closed economy version of the model, the figure shows that including interbank lending spreads in the Taylor rule are of little consequence

following a TFP shock.

5   Conclusion

This paper presents a framework to think about how monetary policy should react to periods of stress in the financial markets. Specifically, should the central bank incorporate interbank lending spreads into their Taylor rule function.

The answer is a resounding maybe! More specifically, it should depend on whether or not the spread contains any new information that isn't already contained in measures of the output gap and inflation. If the coefficients in the Taylor rule policy function are chosen optimally, then the central bank has already chosen the optimal weighting to assign to information contained in the output gap and information contained in the inflation rate. Thus assigning any weight to a new term like the interbank lending spread that contains no new information is suboptimal.

However when fluctuations in the interbank rate are driven by exogenous financial sector shocks, the central bank may want to reduce the risk free rate in response to an exogenous increase in the interbank lending spread. However full accommodation is too extreme. Calculations from the model instead show that the central bank will want to reduce the natural rate by about two-thirds of a percentage point in response to a one percentage point increase in the spread, and thus interestingly, while accommodation is warranted, full accommodation of the exogenous financial shock is too much.

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A Technical Appendix

This appendix will present some of the more technical derivations in the paper related to the nominal rigidities and financial frictions present in the model. The first part of the appendix, section A.1 presents the derivations involved with the Calvo style wage and price equations. The second part of this appendix, section A.2 presents the proofs necessary for aggregation in the presence of financial frictions.

A.1 Nominal Rigidities

A.1.1 Sticky Wages

In any given period, household j faces a probability of $1 - \xi_w$ of being able to reset their wage, otherwise it is reset automatically according to $W_t(l) = \pi_{t-1}W_{t-1}(l)$, where $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$.

If household j is allowed to reset their wages in period t they will set a wage to maximize the expected present value of utility from consumption minus the disutility of labor.

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau (\xi_w)^\tau \left\{ \lambda_{t+\tau}\Pi_{t,t+\tau}W_t(l)H_{t+\tau}(l) - \psi(H_{t+\tau}(l))^{\frac{1+\sigma_H}{\sigma_H}} \right\} \tag{24}$$

where $\lambda_{t+\tau}$ is the marginal utility of consumption in period $t + \tau$.[13]

$$\Pi_{t,t+\tau} = \begin{cases} 1 & \text{if } \tau = 0 \\ \pi_{t+\tau-1}\Pi_{t,t+\tau-1} & \text{if } \tau > 0 \end{cases}$$

The imperfect combination of labor from different households is described in (20). Use this function to derive the demand function for labor from a specific household:

$$H_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\theta} H_t \tag{25}$$

where $W_t = \left(\int_0^n W_t(l)^{1-\theta} dl\right)^{\frac{1}{1-\theta}}$ is the average wage across households, and H_t is aggregate labor supplied by all households.

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[13] We assume complete contingent claims markets among households within a country. This implies that the marginal utility of consumption is the same across all households within a country, regardless of their income. Therefore the total utility from the consumption of labor income in any period is simply the country specific marginal utility of comsumption, λ_t, multiplied by the household's labor income, $W_t(l)N_t(l)$.

Substitute the labor demand function into the maximization problem to express the maximization problem as a function of one choice variable, the wage rate, $W_t(l)$:

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left\{ \lambda_{t+\tau} \Pi_{t,t+\tau} W_t(l) \left(\frac{\Pi_{t,t+\tau} W_t(l)}{W_{t+\tau}} \right)^{-\theta} H_{t+\tau} - \psi \left(\left(\frac{\Pi_{t,t+\tau} W_t(l)}{W_{t+\tau}} \right)^{-\theta} H_{t+\tau} \right)^{\frac{1+\sigma_H}{\sigma_H}} \right\}$$

After some rearranging, the first order condition of this problem is:

$$W_t(l)^{\frac{\theta}{\sigma_N}+1} = \frac{\theta}{\theta-1} \frac{1+\sigma_H}{\sigma_H} \psi (W_t)^{\frac{\theta}{\sigma_H}} \frac{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left(\frac{W_{t+\tau}}{\Pi_{t,t+\tau} W_t} \right)^{\frac{\theta}{\sigma_H}+\theta} (H_{t+\tau})^{\frac{1+\sigma_H}{\sigma_H}}}{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \lambda_{t+\tau} \Pi_{t,t+\tau} \left(\frac{W_{t+\tau}}{\Pi_{t,t+\tau} W_t} \right)^{\theta} H_{t+\tau}}$$

If wages are flexible, and thus $\xi_w = 0$, this expression reduces to:

$$W_t(l) = \frac{\theta}{\theta-1} \frac{\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}}{\lambda_t}$$

Thus when wages are flexible the wage rate is equal to a mark-up, $\frac{\theta}{(\theta-1)}$, multiplied by the marginal disutility of labor, $\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}$, divided by the marginal utility of consumption, λ_t.

Write the wage rate for the household that can reset wages in period t, $W_t(l)$, as $\tilde{W}_t(l)$ to denote it as an optimal wage. Also note that all households that can reset wages in period t will reset to the same wage rate, so $\tilde{W}_t(l) = \tilde{W}_t$.

All households face a probability of $(1 - \xi_w)$ of being able to reset their wages in a given period, so by the law of large numbers $(1 - \xi_w)$ of households can reset their wages in a given period. The wages of the other ξ_w will automatically reset by the previous periods inflation rate.

So substitute $\tilde{W}_t$ into the expression for the average wage rate $W_t = \left(\int_0^n W_t(l)^{1-\theta} dl \right)^{\frac{1}{1-\theta}}$, to derive an expression for the evolution of the average wage:

$$W_t = \left(\xi_w (\Pi_{t-1,t} W_{t-1})^{1-\theta} + (1 - \xi_w) \left(\tilde{W}_t \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

A.1.2 Sticky Output Prices

Domestic Prices In the model, intermediate goods prices are sticky. Intermediate goods firms can set separate domestic and export prices.

In period t, the firm will be able to change it's price in the domestic market with probability $1 - \xi_p$. If the firm cannot change prices then they are reset automatically according to $P_t^d(i) = \pi_{t-1} P_{t-1}^d(i)$.

The firm that can reset prices in period t will choose $P_t^d(i)$ to maximize discounted future profits:

$$\max_{P_t^d(i)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \lambda_{t+\tau} \left\{ \Pi_{t,t+\tau} P_t^d(i) y_{t+\tau}^d(i) - MC_{t+\tau} y_{t+\tau}^d(i) \right\}$$

where $MC_{t+\tau}$ is marginal cost of production in period $t + \tau$.

The firm's domestic demand is given in (3). Substitute this demand function into the maximization problem to express this problem as a function of one choice variable, $P_t(i)$:

$$\max_{P_t^d(i)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \lambda_{t+\tau} \left\{ \begin{array}{l} \Pi_{t,t+\tau} P_t^d(i) \gamma(n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{\Pi_{t,t+\tau} P_t^d(i)}{P_{t+\tau}^d}\right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}}\right)^{-\rho} y_{t+\tau} \\ -MC_{t+\tau} \gamma(n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{\Pi_{t,t+\tau} P_t^d(i)}{P_{t+\tau}^d}\right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}}\right)^{-\rho} y_{t+\tau} \end{array} \right\}$$

After some rearranging, the first order condition with respect to $P_t^d(i)$ is:

$$P_t^d(i) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \lambda_{t+\tau} MC_{t+\tau} \left(\frac{\Pi_{t,t+\tau}}{P_{t+\tau}^d}\right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}}\right)^{-\rho} y_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \lambda_{t+\tau} \Pi_{t,t+\tau} \left(\frac{\Pi_{t,t+\tau}}{P_{t+\tau}^d}\right)^{-\sigma} \left(\frac{P_{t+\tau}^d}{P_{t+\tau}}\right)^{-\rho} y_{t+\tau}}$$

If prices are flexible, and thus $\xi_p = 0$, then this expression reduces to:

$$P_t^d(i) = \frac{\sigma}{\sigma - 1} MC_t$$

which says that the firm will set a price equal to a constant mark-up over marginal cost.

Write the domestic price set by the firm that can reset prices in period t as $\tilde{P}_t^d(i)$ to denote that it is an optimal price. Firms that can reset prices in period t will all reset to the same level, so $\tilde{P}_t^d(i) = \tilde{P}_t^d$. Substitute this optimal price into the price index $P_t^d = \left(\frac{1}{n} \int_0^n \left(P_t^d(i)\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$ and use the fact that in any period $1 - \xi_p$ percent of firms will reoptimize prices, and the prices of ξ_p percent of firms will be automatically reset using the previous periods inflation rate, to derive an expression for the domestic price index, P_t^d:

$$P_t^d = \left(\xi_p \left(\Pi_{t-1,t} P_{t-1}^d \right)^{1-\sigma} + (1 - \xi_p) \left(\tilde{P}_t^d \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Export Prices   Domestic firm i, where $i \in [0, n]$, will set a price $P_t^{m*}(i)$ for its intermediate input in the foreign market.

The demand for the intermediate good from domestic firm i in the rest of the world is given by:

$$y_t^{m*}(i) = (n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{P_t^{m*}(i)}{P_t^{m*}} \right)^{-\sigma} \left(\frac{P_t^{m*}}{P_t^*} \right)^{-\rho} y_t^*$$

In period t, the firm will be able to change it's export price with probability $1 - \xi_p$. If the firm cannot change its price in the foreign market then it is reset automatically according to $P_t^{m*}(i) = \pi_{t-1}^* P_{t-1}^{m*}(i)$, where $\pi_{t-1}^* = \frac{P_{t-1}^*}{P_{t-2}^*}$.

If domestic firm i was last able to change their export price in period t, the demand for the intermediate good from firm i in the rest of the world in period $t + \tau$ is:

$$y_{t+\tau}^{m*}(i) = \gamma^{f*} (n)^{\frac{1-\rho}{1-\sigma}-1} \left(\frac{\Pi_{t,t+\tau}^* P_t^{m*}(i)}{P_{t+\tau}^{m*}} \right)^{-\sigma} \left(\frac{P_{t+\tau}^{m*}}{P_{t+\tau}^*} \right)^{-\rho} y_{t+\tau}^*$$

The firm that can reset prices in period t will choose $P_t^{m*}(i)$ to maximize discounted future profits:

$$\max_{P_t^{m*}(i)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} \left\{ \Pi_{t,t+\tau}^* \frac{P_t^{m*}(i)}{S_{t+\tau}} y_{t+\tau}^{m*}(i) - MC_{t+\tau} y_{t+\tau}^{m*}(i) \right\}$$

where S_t is the nominal exchange rate denoted in units of the foreign currency per units of the home currency.

After some rearranging, the first order condition with respect to $P_t^{m*}(i)$ is:

$$P_t^{m*}(i) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} MC_{t+\tau} \left(\frac{\Pi_{t,t+\tau}^*}{P_{t+\tau}^{m*}} \right)^{-\sigma} \left(\frac{P_{t+\tau}^{m*}}{P_{t+\tau}^*} \right)^{-\rho} y_{t+\tau}^*}{E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p \right)^\tau \lambda_{t+\tau} \frac{\Pi_{t,t+\tau}^*}{S_{t+\tau}} \left(\frac{\Pi_{t,t+\tau}^*}{P_{t+\tau}^{m*}} \right)^{-\sigma} \left(\frac{P_{t+\tau}^{m*}}{P_{t+\tau}^*} \right)^{-\rho} y_{t+\tau}^*}$$

If prices are flexible, and thus $\xi_p = 0$, then this expression reduces to:

$$P_t^{m*}(i) = \frac{\sigma}{\sigma - 1} S_t MC_t$$

Denote $\tilde{P}_t^{m*}(i)$ as the optimal price for the foreign market set by a firm that was able to change their prices in period t. Firms that can reset prices in period t will all reset to the same level, so $\tilde{P}_t^{m*}(i) = \tilde{P}_t^{m*}$. Substitute this optimal price into the price index $P_t^{m*} = \left(\frac{1}{n}\int_0^n (P_t^{m*}(i))^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$ and use the fact that in any period $1 - \xi_p$ percent of firms will reoptimize prices, and the prices of ξ_p percent of firms will be automatically reset using the previous periods inflation rate, to derive an expression for the import price index, P_t^{m*}:

$$P_t^{m*} = \left(\xi_p \left(\Pi_{t-1,t}^* P_{t-1}^{m*}\right)^{1-\sigma} + \left(1 - \xi_p\right)\left(\tilde{P}_t^{m*}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

A.2 Financial Frictions

The derivation of the various interest rates in the model, r_t^e, r_t^b, r_t^{wc} is presented in the text. However in the text, aggregation was only possible because at the beginning of the period, entrepreneur j's debt-asset ratio, $DA_t^e(j) = \frac{b_t^e(j)}{K_t(j)}$, was equal across all entrepreneurs, and bank k's debt-asset ratio, $DA_t^b(k) = \frac{b_t^s(k) + b_t^{sf}(k)}{B_t^e(k)}$, was equal across all banks. This section of the appendix will present the formal proof to both of these claims.

A.2.1 Entrepreneurial sector

Prove: $DA_{t+1}^e(i) = DA_{t+1}^e(j)$:

Entrepreneur i will purchase capital up to the point where:

$$1 + r_{t+1}^e(i) = E_t\left(\frac{R_{t+1} + \omega_{t+1}^e(i) P_{t+1}^K (1-\delta) K_{t+1}}{P_t^K}\right)$$

Since $E_t\left(\omega_{t+1}^e(i)\right) = 1$ and $cov\left(\omega_{t+1}^e(i), P_{t+1}^K(1-\delta)K_{t+1}\right) = 0$, $E_t\left(\frac{R_{t+1} + \omega_{t+1}^e(i) P_{t+1}^K (1-\delta) K_{t+1}}{P_t^K}\right) = E_t\left(\frac{R_{t+1} + P_{t+1}^K (1-\delta) K_{t+1}}{P_t^K}\right)$

Since $E_t\left(\frac{R_{t+1} + P_{t+1}^K (1-\delta) K_{t+1}}{P_t^K}\right)$ does not depend on any characteristics that are specific to entrepreneur i, in equilibrium $r_{t+1}^e(i) = r_{t+1}^e(j)$ for any two entrepreneurs i and j.

Proof by contradiction:

Suppose $DA_{t+1}^e(i) < DA_{t+1}^e(j)$

From the bank's loan supply schedule:

$$1 + r^e_{t+1}(j) = \frac{\left(1 + r^b_{t+1}\right)}{1 - F\left(\bar{\omega}^e_{t+1}(j)\right)} - \frac{(1 - \mu^e)\left[R_{t+1}F\left(\bar{\omega}^e_{t+1}(j)\right) + (1 - \delta)P^K_t \int_0^{\bar{\omega}^e_{t+1}(j)} \omega^e_{t+1} dF\left(\omega^e_{t+1}\right)\right]}{\left(1 - F\left(\bar{\omega}^e_{t+1}(j)\right)\right) \frac{b^e_{t+1}(j)}{K_{t+1}(j)}}$$

where

$$\bar{\omega}^e_{t+1}(j) = \frac{\left(1 + r^e_{t+1}\right)\frac{b^e_{t+1}(j)}{K_{t+1}(j)} - R_{t+1}}{P^K_{t+1}(1 - \delta)}$$

If $DA^e_{t+1}(i) < DA^e_{t+1}(j)$, then $\frac{b^e_t(i)}{K_t(i)} < \frac{b^e_t(j)}{K_t(j)}$, so $\bar{\omega}^e_t(i) < \bar{\omega}^e_t(j)$ and $r^e_t(i) < r^e_t(j)$.

This contradicts with the earlier equilibrium condition that $r^e_{t+1}(i) = r^e_{t+1}(j)$, thus $DA^e_{t+1}(i) \not< DA^e_{t+1}(j)$ and since the choice of i and j where arbitrary the only possible equilibrium is one where $DA^e_{t+1}(i) = DA^e_{t+1}(j)$.

A.2.2 Banking sector

Prove $DA^b_{t+1}(i) = DA^b_{t+1}(j)$:

Bank i will make loans up to the point where:

$$1 + r^b_{t+1}(i) = E_t\left(\left(1 - \omega^b_{t+1}(i)\zeta^e_{t+1}\right)\left(1 + r^e_{t+1}\right)\right)$$

Since $\omega^b_{t+1}(i)$ is i.i.d. and $E_t\left(\omega^b_{t+1}(i)\right) = 1$, $E_t\left(\left(1 - \omega^b_{t+1}(i)\zeta^e_{t+1}\right)\left(1 + r^e_{t+1}\right)\right) = E_t\left(\left(1 - \zeta^e_{t+1}\right)\left(1 + r^e_{t+1}\right)\right)$
Thus $r^b_{t+1}(i) = r^b_{t+1}(j)$ for any two banks i and j.

Proof by contradiction:

Suppose $DA^b_{t+1}(i) < DA^b_{t+1}(j)$

From the equilibrium condition that determines how much credit is extended to a bank:

$$1 + r^b_{t+1}(i) = \frac{1 + i_{t+1}}{G\left(\omega^b_{t+1}; \sigma^b_{t+1}\right)}$$

where

$$\bar{\omega}_{t+1}^b (i) = \frac{\left(1 + r_{t+1}^e\right) - \left(1 + r_{t+1}^b (i)\right) \frac{b_{t+1}^s(i) + b_{t+1}^{sf}(i)}{B_{t+1}^e(i)}}{\zeta_{t+1}^e \left(1 + r_{t+1}^e\right)}$$

If $DA_{t+1}^b (i) < DA_{t+1}^b (j)$ then $\frac{b_{t+1}^s(i) + b_{t+1}^{sf}(i)}{B_{t+1}^e(i)} < \frac{b_{t+1}^s(j) + b_{t+1}^{sf}(j)}{B_{t+1}^e(j)}$, so $\bar{\omega}_{t+1}^b (i) > \bar{\omega}_{t+1}^b (j)$, so $r_{t+1}^b (i) < r_{t+1}^b (j)$.

This contradicts with the earlier equilibrium condition that $r_{t+1}^b (i) = r_{t+1}^b (j)$, thus $DA_{t+1}^b (i) \not< DA_{t+1}^b (j)$ and since the choice of i and j where arbitrary the only possible equilibrium is one where $DA_{t+1}^b (i) = DA_{t+1}^b (j)$.

Table 1: Benchmark Parameter Values

Symbol	Value	Description
β	0.99	discount factor
δ	0.025	depreciation rate
α	0.36	capital's share of income
ρ	1.5	substitution elasticity between home and foreign goods
χ^b	0.01	cost of adjusting foreign bond holdings
σ_n	1	labor supply elasticity
σ	10	substitution elasticity across goods from domestic firms
θ	21	substitution elasticity across differentiated labor inputs
κ	0.125	capital adjustment cost parameter
ξ_p	0.62	probability that a firm cannot change prices in the current period
ξ_w	0.75	probability that a worker cannot change wages in the current period
γ	0.78	weight on domestic goods (open economy)
γ^f	0.26	weight on imported goods (open economy)
ϕ	0.271	fixed cost in production
ψ	0.021	coefficient on labor effort in the utility function
σ^b	1.133	standard deviation of idiosyncratic bank shocks
μ^e	0.134	cost of liquidation in the entrepreneurial sector
σ^e	0.370	standard deviation of idiosyncratic entrepreneur shocks

Table 2: The optimal coefficients on inflation, the output gap, and the lagged interest rate in the central bank's Taylor rule in the model with and the model without a financial accelerator.

	θ_p	θ_y	θ_i
Closed			
No FA	1.621	0.404	0.740
Ent FA	1.658	0.366	0.764
Bank FA	1.671	0.350	0.783
Open			
No FA	1.640	0.388	0.719
Ent FA	1.672	0.354	0.751
Bank FA	1.684	0.340	0.772

Table 3: The optimal coefficients on home and foreign interbank lending spreads.

	θ_p	θ_y	θ_i	Rel. Loss
Closed				
$\Sigma = 0$	1.671	0.350	0.783	15.3%
$\Sigma = 0.05$	1.670	0.353	0.783	16.0%
$\Sigma = 0.10$	1.667	0.359	0.783	18.1%
$\Sigma = 0.15$	1.662	0.369	0.783	21.5%
$\Sigma = 0.20$	1.657	0.379	0.785	25.8%
Open				
$\Sigma = 0$	1.684	0.340	0.772	15.3%
$\Sigma = 0.05$	1.682	0.343	0.772	16.1%
$\Sigma = 0.10$	1.677	0.351	0.772	18.3%
$\Sigma = 0.15$	1.671	0.362	0.773	21.9%
$\Sigma = 0.20$	1.664	0.374	0.776	26.2%

Table 4: The optimal coefficients on home and foreign interbank lending spreads.

	θ_p	θ_y	θ_i	θ_r	θ_{rf}	Rel. Loss
Closed						
$\Sigma = 0$	1.670	0.350	0.783	0.000	na	15.3%
$\Sigma = 0.05$	1.669	0.352	0.783	0.000	na	16.0%
$\Sigma = 0.10$	1.669	0.353	0.789	−0.631	na	16.9%
$\Sigma = 0.15$	1.671	0.349	0.791	−0.799	na	17.3%
$\Sigma = 0.20$	1.674	0.345	0.791	−0.846	na	17.7%
Open						
$\Sigma = 0$	1.684	0.340	0.772	0.000	0.000	15.3%
$\Sigma = 0.05$	1.682	0.342	0.774	−0.170	0.000	16.1%
$\Sigma = 0.10$	1.681	0.341	0.779	−0.555	−0.128	16.7%
$\Sigma = 0.15$	1.682	0.338	0.780	−0.632	−0.217	17.1%
$\Sigma = 0.20$	1.683	0.333	0.780	−0.663	−0.249	17.5%

Table 5: The optimal coefficients on the endogenous and exogenous parts of the home and foreign interbank lending spreads.

	θ_p	θ_y	θ_i	θ_r^{endo}	θ_{rf}^{endo}	θ_r^{exo}	θ_{rf}^{exo}	Rel. Loss
Closed								
$\Sigma = 0$	1.670	0.349	0.783	0.000	na	na	na	15.3%
$\Sigma = 0.05$	1.670	0.353	0.783	0.000	na	0.000	na	16.0%
$\Sigma = 0.10$	1.667	0.359	0.783	0.000	na	-1.024	na	15.1%
$\Sigma = 0.15$	1.662	0.369	0.783	0.000	na	-0.953	na	15.9%
$\Sigma = 0.20$	1.657	0.379	0.785	0.000	na	-0.940	na	16.7%
Open								
$\Sigma = 0$	1.684	0.340	0.772	0.000	0.000	na	na	15.3%
$\Sigma = 0.05$	1.682	0.343	0.772	0.000	0.000	-1.198	-0.490	16.1%
$\Sigma = 0.10$	1.677	0.351	0.772	0.000	0.000	-0.752	-0.309	15.2%
$\Sigma = 0.15$	1.671	0.362	0.773	0.000	0.000	-0.705	-0.293	15.9%
$\Sigma = 0.20$	1.664	0.374	0.776	0.000	0.000	-0.691	-0.292	16.8%

Table 6: The optimal coefficients on the interbank lending spread in the version of the Taylor rule where fluctuations in the spread directly affect the natural rate.

Grouped

	θ_p	θ_y	θ_i	θ_r	θ_{rf}	Rel. Loss
Closed						
$\Sigma = 0$	1.669	0.350	0.782	0.000	na	15.3%
$\Sigma = 0.05$	1.669	0.352	0.783	0.000	na	16.0%
$\Sigma = 0.10$	1.670	0.350	0.794	−0.538	na	16.3%
$\Sigma = 0.15$	1.672	0.349	0.797	−0.612	na	16.3%
$\Sigma = 0.20$	1.673	0.346	0.799	−0.636	na	16.4%
Open						
$\Sigma = 0$	1.684	0.340	0.772	0.000	na	15.3%
$\Sigma = 0.05$	1.682	0.341	0.779	−0.330	0.000	15.9%
$\Sigma = 0.10$	1.681	0.340	0.784	−0.446	−0.132	16.2%
$\Sigma = 0.15$	1.682	0.338	0.786	−0.479	−0.176	16.2%
$\Sigma = 0.20$	1.682	0.336	0.788	−0.492	−0.192	16.2%

Separate

	θ_p	θ_y	θ_i	θ_r^{endo}	θ_{rf}^{endo}	θ_r^{exo}	θ_{rf}^{exo}	Rel. Loss
Closed								
$\Sigma = 0$	1.670	0.349	0.783	0.000	na	na	na	15.3%
$\Sigma = 0.05$	1.670	0.353	0.783	0.000	na	−0.901	na	14.5%
$\Sigma = 0.10$	1.667	0.359	0.783	0.000	na	−0.693	na	15.2%
$\Sigma = 0.15$	1.662	0.369	0.783	0.000	na	−0.676	na	15.5%
$\Sigma = 0.20$	1.657	0.379	0.785	−0.002	na	−0.667	na	15.8%
Open								
$\Sigma = 0$	1.684	0.340	0.772	0.000	0.000	na	na	15.3%
$\Sigma = 0.05$	1.682	0.343	0.772	0.000	0.000	−0.715	−0.293	14.2%
$\Sigma = 0.10$	1.677	0.351	0.772	0.000	0.000	−0.528	−0.217	15.1%
$\Sigma = 0.15$	1.671	0.362	0.773	0.000	0.000	−0.513	−0.212	15.4%
$\Sigma = 0.20$	1.664	0.374	0.776	0.000	0.000	−0.557	−0.121	15.5%

Figure 1: Responses in the closed economy to a shock to financial sector uncertainty. Calculated under three assumptions for monetary policy, Ramsey optimal policy (solid line), the conventional Taylor rule (dashed line), and the modified Taylor rule that is a function of interbank lending spreads (line with stars).
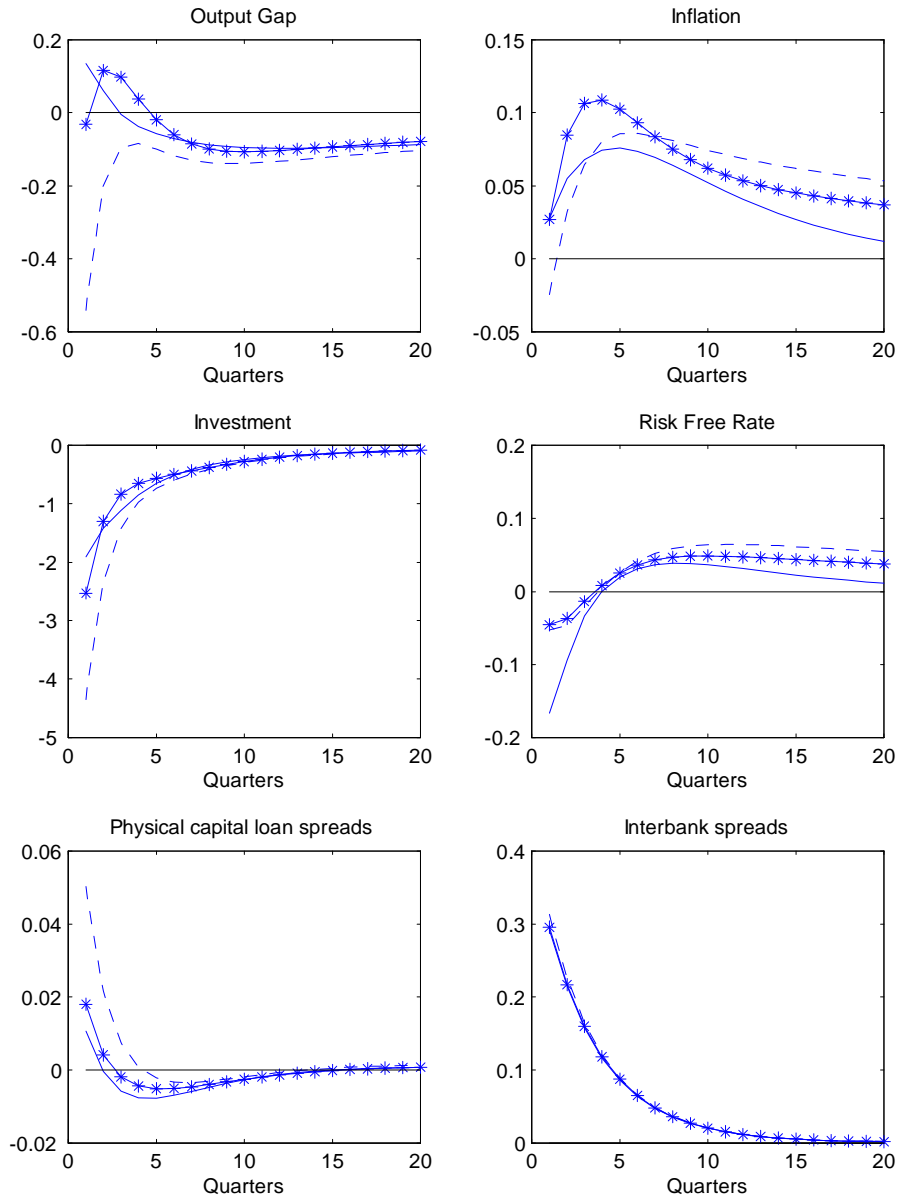
Figure 2: Responses in the closed economy to a one percent TFP shock. Calculated under three assumptions for monetary policy, Ramsey optimal policy (solid line), the conventional Taylor rule (dashed line), and the modified Taylor rule that is a function of interbank lending spreads (line with stars).
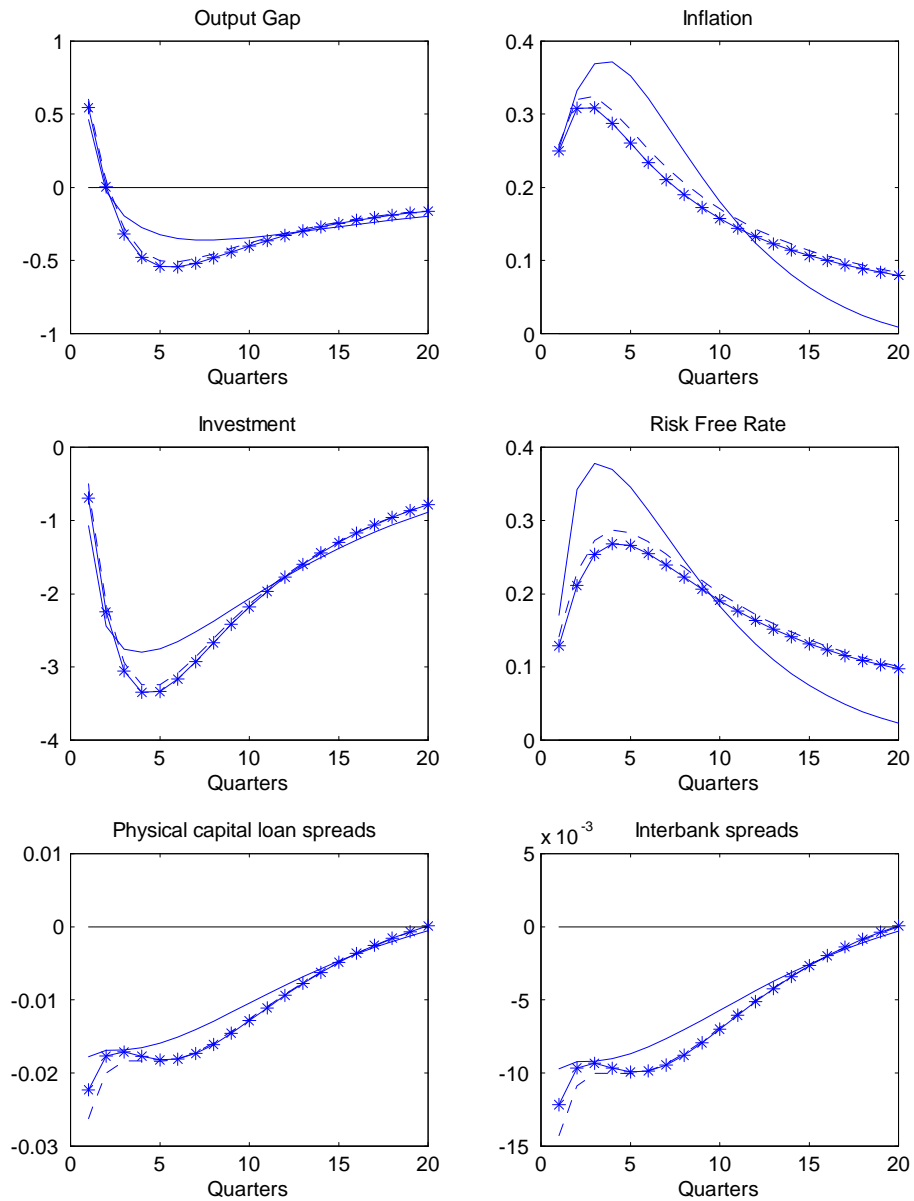
Figure 3: Responses in the open economy to a shock to foreign financial sector uncertainty. Calculated under three assumptions for monetary policy, Ramsey optimal policy (solid line), the conventional Taylor rule (dashed line), and the modified Taylor rule that is a function of interbank lending spreads (line with stars).
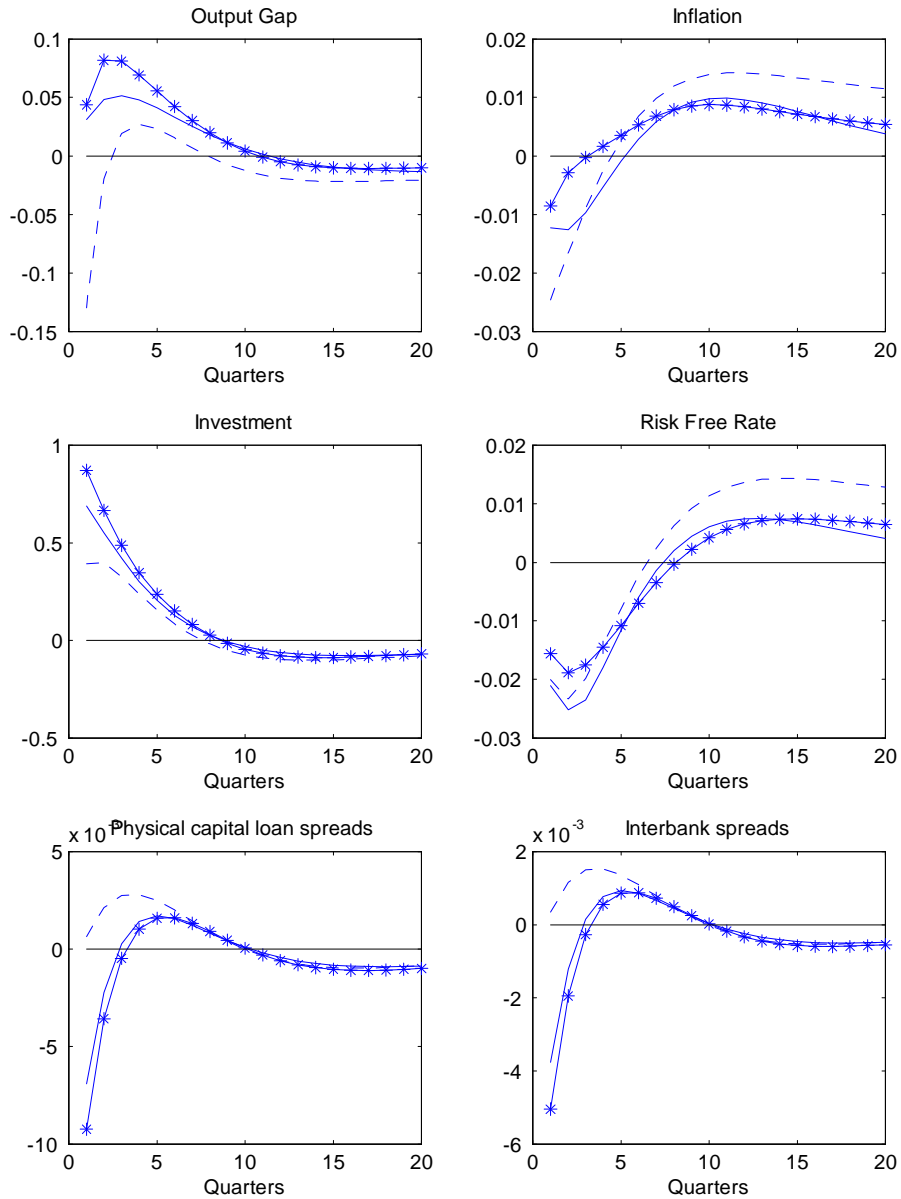
Figure 4: Responses in the open economy to a one percent foreign TFP shock. Calculated under three assumptions for monetary policy, Ramsey optimal policy (solid line), the conventional Taylor rule (dashed line), and the modified Taylor rule that is a function of interbank lending spreads (line with stars).