

Federal Reserve Bank of Dallas
Globalization and Monetary Policy Institute
Working Paper No. 60

<http://www.dallasfed.org/institute/wpapers/2010/0060.pdf>

International Real Business Cycles with Endogenous Markup Variability*

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September 2010

Abstract

The aggregate impact of decisions made at the level of the individual firm has recently attracted a lot of attention in both the macro and trade literatures. We adapt the benchmark international real business cycle model to a game-theoretic environment to add a channel for the strategic interaction among domestic and foreign firms. We show how the sum of strategic pricing decisions made at the level of the individual firm can have significant effects on the volatility and cross country co-movement of GDP and its components. Specifically we show that the addition of this one channel for strategic interaction leads to a significant increase in the cross-country co-movement of production and investment, as well as a significant decrease in the volatility of investment and the trade balance over the benchmark IRBC model.

JEL codes: L16, E32, F41

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1 Introduction

The recent attention to the role of markup variability in the international trade and finance literature is quite remarkable. Not surprisingly, given that markup variability provides a wedge between change in marginal costs and changes in prices, many (but not all) of these studies incorporate endogenous markup variability into a model with nominal rigidities to explain price or exchange rate dynamics.¹

In addition, some models have employed variable markups to explain either national or international quantity dynamics, but these papers usually focus on the role of markup variability in conjunction with endogenous firm entry and exit.²

This paper constructs a model to evaluate the role of endogenous markup variability in affecting international quantity dynamics. However, unlike most of the papers in this literature, this paper abstracts from the role of endogenous firm entry and exit in order to focus specifically on how strategic decisions made at the level of the individual firm can aggregate up to have a significant effect on the volatility and international co-movement of GDP and its components.

Specifically we will examine how including endogenous markup variability in an international real business cycle model (IRBC) affects the relative volatility of consumption, investment, net exports and employment and the cross-country correlation of GDP, consumption, investment, and employment. These statistics, as presented in Backus et al. (1995, henceforth BKK), Chari et al. (2002, henceforth CKM) and Ambler et al. (2004, henceforth ACZ), are reproduced in table 1. The table also presents the results from the benchmark international real business cycle model in BKK (1995).

These statistics are chosen specifically because these are the key points where the bench-

¹Dotsey and King (2005) show how markup variability can affect inflation dynamics, namely the persistence of inflation and prices. Bouakez (2005) develops a model with markup variability and nominal rigidities that can help explain real exchange rate persistence. Sbordone (2007) shows the effect of variable markups on international inflation dynamics. Atkeson and Burstein (2008) model how markup variability can explain deviations of international relative prices from purchasing power parity. Gust et al. (2010) construct a model where markup variability is responsible for the incomplete pass through of exchange rate changes into import prices.

²Cook (2002) presents a model where endogenous firm entry and markup variability lead to an increase in cross-country business cycle co-movement. Melitz (2003) shows how firm heterogeneity, endogenous firm entry and exit and variable demand elasticities can explain why some firms export and others only supply the domestic market. Ruhl (2005) uses a similar model to show how endogenous firm entry can affect the elasticity of substitution between home and foreign goods. With a closed economy model, Jaimovich and Floetto (2009) show how variable markups and endogenous firm entry leads to a propagation mechanism that can significantly increase the volatility of business cycle fluctuations following productivity shocks. In addition, Arkolakis and Ramanarayanan (2009) show how markup variability and heterogeneous firms may be necessary to explain the link between bilateral trade and the endogenous transfer of measured TFP fluctuations.

mark IRBC model fails to match the data. Namely, the model’s predictions for consumption and employment volatility are too low, while those for investment and net export volatility are too high. Furthermore, the model’s prediction for cross-country consumption co-movement is far too high, and the model predicts negative GDP, investment, and employment correlation.

Table 1: Volatility and cross-country co-movement of GDP and its components in the data and in the benchmark IRBC model.

	Data			Model
	BKK (1995)	CKM (2002)	ACZ (2004)	BKK (1995)
Volatility relative to GDP:				
Consumption	0.75	0.83		0.42
Investment	3.27	2.78		10.99
Net Exports	0.27	0.11		2.51
Employment	0.61	0.67		0.50
Cross-country correlation:				
GDP	0.66	0.60	0.22	-0.21
Consumption	0.51	0.38	0.14	0.88
Investment	0.53	0.33	0.18	-0.94
Employment	0.33	0.39	0.20	-0.78

Many papers have shown how various permutations of the benchmark IRBC model can help resolve these discrepancies between the model and the data.³ Without doubting the validity of these contributions we propose an additional channel that may help resolve these well known discrepancies between the benchmark IRBC model and the data, the channel through which the sum of strategic decisions made at the level of individual firms can have significant aggregate effects.

Our work adapts the international real business cycle model of Backus et al. (1994) to a game-theoretic environment. In the model, intermediate goods from home and foreign firms are aggregated in a discretized Dixit and Stiglitz (1977) aggregator function. Like in the closed economy model in Yang and Heijdra (1993), when deciding its optimal output price, the firm takes into account the effect of its price on the aggregate price level. Thus a strategic game arises where a firm must take into account the impact of its pricing decision

³See Baxter and Crucini (1993) for, among other things, a discussion of the role of capital adjustment costs in increasing the relative volatility of consumption and reducing the relative volatility of investment. Baxter and Crucini (1995), Kehoe and Perri (2002), and Heathcote and Perri (2002 and 2003) show that cross-country consumption correlation falls and cross-country GDP correlation increases when international asset markets are restricted. Kose and Yi (2001), Ambler et al. (2002), and Burstein et al. (2008) show that it is possible to raise the level of international business cycle co-movement by altering the production process to allow for trade in intermediate inputs in the production process.

on the aggregate price level and thus on all other firms' pricing decisions.⁴

The extent to which a firm can influence the aggregate price level and the prices set by other firms is determined by the firm's market share. A firm with a higher market share has greater market power, and this greater market power allows the firm to charge a higher markup over marginal cost. Cyclical changes in the import share that arise in an international real business cycle model as a result of country specific productivity shocks lead to cyclical changes in domestic and foreign firms' market power and thus cyclical changes in domestic or export markups.

The link between an increasing market share of foreign firms and a decreasing domestic markup is well established in the empirical literature. Tybout (2003) provides a survey of ample empirical evidence on how markups generally fall as import competition rises (see, also, Katics and Peterson, 1994; Tribble, 1995; Konings and Vandebussche, 2005; Blonigen et al., 2007; Chen et al., 2009), while Becarelo (1997) especially emphasizes the empirical significance of such markup variability and the impact of import competition at the business cycle frequency.

This markup variability in response to a changing import share reduces the volatility of fluctuations in GDP, investment, and net exports. In addition markup variability leads to greater international co-movement in production, investment and employment. The intuition is as follows. Suppose there is a positive shock to foreign productivity. Foreign marginal costs fall. The relative price of foreign products decreases. Foreign producers will increase production and domestic producers will cut production. Thus the foreign shock leads to volatile business cycle fluctuations and international business cycle divergence.

Foreign producers gain market share at the expense of domestic producers, and thus foreign markups increase while domestic markups fall. These changes in markups cause the relative price of foreign goods to increase. Thus variability in markups causes a change in relative prices that is exactly opposite the change due to the initial productivity shock. Home and foreign business cycles will fluctuate less than they would have without variable markups.

Since home firms find themselves at a comparative disadvantage following the foreign productivity shock they cut production for a period of time. Therefore the productivity shock leads to business cycle divergence and negative bilateral GDP correlation. When there is endogenous markup variability, the changes in the markups of home and foreign firms place home firms at less of a comparative disadvantage and they return to pre-shock levels of production sooner. Thus with endogenous markup variability there is less bilateral

⁴We consider both the cases where this strategic interaction arises from firms engaged in quantity competition (Cournot) or price competition (Bertrand).

business cycle divergence following a country specific productivity shock and higher bilateral cyclical correlation.

When adding a channel for strategic interaction between domestic and foreign firms to an otherwise ordinary IRBC model, we find that strategic interaction leads to a nearly 12 percentage point increase in cross-country GDP correlation. In addition we find that cross-country investment and employment correlation increase by 11 and 15 percentage points, respectively.⁵

Furthermore, a channel for strategic interaction leads to a 5% increase in the relative volatility of consumption, a 5% decrease in the relative volatility of investment, and an 11% decrease in the relative volatility of net exports.

This paper will proceed as follows. In section 2 the model is described and a part of the model is solved in order to find a closed form expression for a firm's elasticity of demand as a function of its market share. The parameterization of the model and the exogenous shock process is described in section 3. The results from the model are presented in section 4. First we log linearize part of the model to provide some intuition for how markup variability should affect volatility and co-movement. Then we simulate the model to show the quantitative effect of endogenous markup variability in an otherwise ordinary IRBC model. Finally, section 5 concludes.

2 The Model

2.1 Production

There are two countries, home and foreign. Foreign variables are written with an asterisk and home variables are not. In the following description of the model, foreign equations are omitted for brevity.

An aggregate good is used by households for consumption and investment, C_t and I_t . This aggregate good, y_t , is formed through the combination of domestic and imported final goods, which are combined in an Armington (1969) aggregator function with an elasticity of substitution ρ .

$$C_t + I_t = y_t = \left[(\omega)^{\frac{1}{\rho}} (y_{D,t})^{\frac{\rho-1}{\rho}} + (1 - \omega)^{\frac{1}{\rho}} (y_{M,t})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \quad (1)$$

where $y_{D,t}$ are domestically produced final goods and $y_{M,t}$ are imported final goods.

⁵To put this in perspective, in their seminal paper, Frankel and Rose (1998) find that the doubling of trade between two countries leads to about an 8 percentage point increase in bilateral GDP correlation and a 10 percentage point increase in bilateral employment correlation (see also Clark and van Wincoop, 2001; Baxter and Kouparitsas, 2005; Kose and Yi, 2006; Calderón et al., 2007)

The demand for domestically produced or imported final goods as a function of aggregate consumption and investment spending is:

$$\begin{aligned} y_{D,t} &= \omega (p_t)^{-\rho} y_t \\ y_{M,t} &= (1 - \omega) \left(q_t \frac{p_t^*}{1 - c} \right)^{-\rho} y_t \end{aligned} \quad (2)$$

where p_t is the price of home final goods relative to the price of the home consumption good, p_t^* is the price of foreign final goods relative to price of the foreign consumption good, q_t is the real exchange rate measured in units of the foreign consumption good per units of the home consumption good, and c is an iceberg trade cost parameter.

Goods shipped internationally are subject to an iceberg trade cost, so when 1 unit of a good is shipped, only $1 - c$ units arrive. Thus $y_{X,t} = \frac{y_{M,t}^*}{(1-c)}$ and $y_{X,t}^* = \frac{y_{M,t}}{(1-c)}$, where $y_{X,t}$ and $y_{X,t}^*$ are home and foreign exports of final goods.

Final goods, which are to be sold domestically or exported, are produced from the combination of value added at the final goods stage and intermediate inputs.

$$y_{D,t} + y_{X,t} = \left[\phi^{\frac{1}{\kappa}} (A_t h_{yt}^{1-\alpha} K_{yt}^\alpha)^{\frac{\kappa-1}{\kappa}} + (1 - \phi)^{\frac{1}{\kappa}} (x_t)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}} \quad (3)$$

where h_{yt} and K_{yt} are labor and capital employed in the production of final goods, A_t is a country specific total factor productivity parameter, x_t are intermediate inputs, and κ is the elasticity of substitution between value added and intermediate inputs in the production of final goods.

The demand for intermediate inputs in the production of final goods is given by:

$$x_t = (1 - \phi) \left(\frac{p_t^x}{p_t} \right)^{-\kappa} (y_{D,t} + y_{X,t}) \quad (4)$$

where p_t^x is the relative price of intermediate inputs.

The intermediate input term x_t in the production of final goods is made up of a combination of intermediate inputs from a continuum of sectors j :

$$x_t = \left(\int_0^1 (x_t^j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \quad (5)$$

where x_t^j represents intermediate inputs from sector j . The demand for inputs from sector j is given by:

$$x_t^j = \left(\frac{p_t^j}{p_t^x} \right)^{-\gamma} x_t \quad (6)$$

where p_t^j is the relative price of intermediate goods from sector j .

The intermediate good from sector j is formed by combining firm specific varieties from N home firms and N^* foreign firms in sector j . We use the convention that firms denoted $i = 1, \dots, N$ are home firms and those denoted $i = N + 1, \dots, N + N^*$ are foreign firms:

$$x_t^j = \left[\sum_{i=1}^{N+N^*} x_t^j(i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (7)$$

where $x_t^j(i)$ is the intermediate good from firm i in sector j .

Firm i can supply to either the home or the foreign market. For notational clarity, suppose that $x_{D,t}^j(i)$ and $x_{X,t}^j(i)$ are the quantities that the firm supplies to the home and foreign markets, respectively. For each of these goods the firm charges a price $p_{D,t}^j(i)$ and $p_{X,t}^j(i)$.

The demand from the home market for the intermediate good from firm i in sector j is:

$$x_{D,t}^j(i) = \left(\frac{p_{D,t}^j(i)}{p_t^j} \right)^{-\sigma} x_t^j \quad (8)$$

and the demand in the foreign market for the intermediate good from the same firm is:

$$x_{X,t}^j(i) = \frac{1}{1-c} \left(\frac{p_{X,t}^j(i)}{q_t(1-c)p_t^{j*}} \right)^{-\sigma} x_t^{j*} \quad (9)$$

Again we see that traded intermediate inputs are subject to an iceberg trade cost. The price of the intermediate good from sector j , p_t^j , is given by:

$$p_t^j = \left[\left(\sum_{i=1}^N p_{D,t}^j(i)^{1-\sigma} + \sum_{i=N+1}^{N+N^*} \left(q_t \frac{p_{X,t}^{j*}(i)}{1-c} \right)^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}} \quad (10)$$

The total quantity sold by firm i in sector j for both the domestic and foreign markets is produced from a combination of domestic capital and labor.

$$x_{D,t}^j(i) + x_{X,t}^j(i) = A_t (h_t^j(i))^{1-\alpha} (K_t^j(i))^\alpha - \psi \quad (11)$$

where $h_t^j(i)$ and $K_t^j(i)$ is labor and capital employed by firm i in sector j , A_t is an exogenous country specific productivity shock, and ψ is a fixed cost which ensures that firms earn zero

profit in the steady state..

2.2 Households

The one representative household per country derives utility from consumption and leisure. The household in the home country maximizes expected lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta_t \frac{1}{1-\zeta} \left[(1-h_t)^\theta (C_t)^{1-\theta} \right]^{1-\zeta} \quad (12)$$

where ζ is the coefficient of relative risk aversion and $h_t = h_{yt} + \int_0^1 \sum_{i=1}^N h_t^j(i) dj$ is aggregate labor supplied by the domestic household to both final goods production, h_{yt} , and to intermediate goods production from all firms i in all sectors j , $\int_0^1 \sum_{i=1}^N h_t^j(i) dj$.

We assume that international asset markets are complete. We can model this by assuming households share one worldwide budget constraint:

$$C_t + I_t + q_t (C_t^* + I_t^*) = w_t h_t + r_t K_t + \int_0^1 \sum_{i=1}^N \Pi_t^j(i) dj + q_t \left(w_t^* h_t^* + r_t^* K_t^* + \int_0^1 \sum_{i=1}^N \Pi_t^{j*}(i) dj \right) \quad (13)$$

where w_t is the home wage rate (in terms of the home consumption good), r_t is the rental rate of home capital, $K_t = K_{yt} + \int_0^1 \sum_{i=1}^N K_t^j(i) dj$, and $\Pi_t^j(i) = p_{D,t}^j(i) x_{D,t}^j(i) + p_{X,t}^j(i) x_{M,t}^{j*}(i) - w_t h_t^j(i) - r_t K_t^j(i)$ is the profit at time t of the home firm i in sector j .

Finally, the home capital stock evolves according to the following:

$$K_{t+1} = (1-\delta) K_t + I_t \quad (14)$$

where δ is the one-period depreciation rate of capital.

2.3 The firm's maximization problem

Firm i in sector j , with $1 \leq i \leq N$, chooses $p_{D,t}^j(i)$, $p_{X,t}^j(i)$, $x_{D,t}^j(i)$, and $x_{M,t}^{j*}(i) = (1-c) x_{X,t}^j(i)$ to solve the following static maximization problem:

$$\max \left(p_{D,t}^j(i) x_{D,t}^j(i) + p_{X,t}^j(i) x_{M,t}^{j*}(i) - w_t h_t^j(i) - r_t K_t^j(i) \right) \quad (15)$$

This profit can be rewritten as:

$$\max \left[(p_{D,t}^j(i) - MC_t) x_{D,t}^j(i) + ((1-c) p_{X,t}^j(i) - MC_t) x_{X,t}^j(i) - MC_t \psi \right] \quad (16)$$

where $MC_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^\alpha$.

We assume a sufficient degree of market segmentation such that firms can price to market, i.e. the firm is able to set a different price for goods sold domestically versus those that are exported.

If firms are engaged in quantity competition then the firm chooses domestic and export quantities according to:

$$\begin{aligned} x_{D,t}^j(i) &= \operatorname{argmax}_{x_{D,t}^j(i)} \{ (p_{D,t}^j(i) - MC_t) x_{D,t}^j(i) \} \\ x_{X,t}^j(i) &= \operatorname{argmax}_{x_{X,t}^j(i)} \{ ((1-c)p_{X,t}^j(i) - MC_t) x_{X,t}^j(i) \} \end{aligned} \quad (17)$$

subject to the inverse demand functions:

$$\begin{aligned} p_{D,t}^j(i) &= \left(\frac{x_{D,t}^j(i)}{x_t^j}\right)^{\frac{-1}{\sigma}} \left(\frac{x_t^j}{x_t}\right)^{\frac{-1}{\gamma}} p_t^x \\ p_{X,t}^j(i) &= q_t(1-c) \left(\frac{(1-c)x_{X,t}^j(i)}{x_t^{j*}}\right)^{\frac{-1}{\sigma}} \left(\frac{x_t^{j*}}{x_t^*}\right)^{\frac{-1}{\gamma}} p_t^{x*} \end{aligned} \quad (18)$$

Thus the domestic and export elasticities of demand faced by the firm are:

$$\begin{aligned} \varepsilon_t^d(i) &= \left(\frac{1}{\sigma} (1 - s_{D,t}^j(i)) + \frac{1}{\gamma} s_{D,t}^j(i)\right)^{-1} \\ \varepsilon_t^x(i) &= \left(\frac{1}{\sigma} (1 - s_{X,t}^j(i)) + \frac{1}{\gamma} s_{X,t}^j(i)\right)^{-1} \end{aligned} \quad (19)$$

where

$$\begin{aligned} s_{D,t}^j(i) &= \frac{p_{D,t}^j(i) x_t^j(i)}{\left(\sum_{i=1}^N p_{D,t}^j(i) x_t^j(i) + \sum_{i=N+1}^{N+N^*} p_{X,t}^{j*}(i) x_t^{j*}(i)\right)} \\ s_{X,t}^j(i) &= \frac{p_{X,t}^j(i) x_t^{j*}(i)}{\left(\sum_{i=1}^N p_{X,t}^j(i) x_t^{j*}(i) + \sum_{i=N+1}^{N+N^*} p_{D,t}^{j*}(i) x_t^j(i)\right)} \end{aligned} \quad (20)$$

are an intermediate good firm's market shares in the home and foreign markets.

If instead firms engaged in price competition the demand elasticities would be the following:

$$\begin{aligned}\varepsilon_t^d(i) &= \sigma(1 - s_{D,t}^j(i)) + \gamma s_{D,t}^j(i) \\ \varepsilon_t^x(i) &= \sigma(1 - s_{X,t}^j(i)) + \gamma s_{X,t}^j(i)\end{aligned}\tag{21}$$

Details of the solution to the firm's problem and the derivation of these elasticities can be found in the appendix.

The impact of strategic interaction among firms and how that can affect firm markups is highlighted by equations (19) and (21). Strategic interaction occurs when a firm takes into account the effect of its pricing decision on the aggregate price level, and thus on the pricing decision of other firms. If the aggregate price level increases after a firm raises its price then other firms will follow suit and raise their prices as well. Thus the firm that is able to significantly influence the aggregate price level and thus the pricing decisions of other firms has greater market power and will maximize profits by charging a higher markup.

In equations (19) and (21), the elasticity of demand for the output from firm i is a convex combination of the technological elasticity of substitution between firms in the same sector and the elasticity of substitution between sectors. As the firm's market share increases, the weight on the elasticity of substitution between sectors increases, and thus the elasticity of demand for the output from firm i decreases. If firm i is small compared to the rest of the market and thus its market share, $s_{D,t}^j(i)$, is close to zero, then when setting its price the firm does not take into account the effect of its decision on other firms. Thus when the market is made up of many small firms there is no channel for strategic interaction and each firm's demand elasticity reduces to σ .

3 Parameterization

The model's parameters and their benchmark values are found in table 2.

σ is the elasticity of substitution across intermediate inputs from different firms. We set this elasticity equal to 10. This is done to match the convention in this literature⁶ and ensure that in the extreme case of atomistic firms ($N + N^* \rightarrow \infty$), firms charge a steady state markup of about 11%, which is common in the literature with imperfect competition and atomistic firms.

⁶Atkeson and Burstein (2008) use an elasticity of 10 in a similar model.

γ is the elasticity of substitution between goods from different sectors. We set this parameter equal to 1.01. Again this is similar to Atkeson and Burstein (2008) and an elasticity of substitution close to one ensures that sectoral expenditure shares are roughly constant.

ρ is the elasticity of substitution between home and foreign final goods. As discussed in Anderson and van Wincoop (2004), in the trade literature, this elasticity is commonly estimated as somewhere between 5 and 10. However the international macro literature finds that this elasticity needs to be low, around 1.5, in order to match the variability of the real exchange rate. Ruhl (2005) attempts to justify these different estimates of the same elasticity in a model with endogenous firm entry. Since we do not consider the role of endogenous firm entry in this model, we follow the convention in the international macro literature and set this elasticity to 1.5.

The next six parameters: θ , the exponent on leisure in the Cobb-Douglas utility function, ζ , the coefficient of relative risk aversion, α , capital's share of income, β , the discount factor, δ , the capital depreciation rate, and κ , the elasticity of substitution between value added and intermediate inputs in the production of final goods, are all set to values commonly found in the real business cycle literature.

The benchmark fixed cost parameter, ψ , from the firm's production function, is set such that firms earn zero profit in the steady state.

The next two parameters determine the market share of foreign firms in either the market for final goods or the market for intermediate inputs. The trade cost parameter c affects both the relative cost of foreign final goods and the cost of foreign intermediate goods, although the purpose of this parameter is to determine the steady state market share of foreign intermediate goods firms. By setting the trade cost parameter equal to 0.159, the steady state market share of foreign firms in the market for intermediate inputs is equal to 25%.

The parameter ω describes the exogenous home bias for domestically produced final goods. This parameter is set to 0.733 to ensure that after accounting for trade costs, the steady state share of foreign goods in the household's consumption basket is equal to 25%.

The steady state domestic markup is equal to 20%. With this steady state domestic markup, the number of firms, N , can be backed out of the elasticity expressions in (19) and (21).⁷

⁷ $\mu^d = \frac{\varepsilon^d}{\varepsilon^d - 1} - 1$, or alternatively $\varepsilon^d = \frac{\mu^d + 1}{\mu^d}$. Under quantity competition, $\varepsilon^d = \left(\frac{1}{\sigma} \left(1 - \frac{1}{N} (1 - m) \right) + \frac{1}{\gamma} \frac{1}{N} (1 - m) \right)^{-1}$, thus $N = \frac{(1-m)(\frac{1}{\gamma} - \frac{1}{\sigma})}{(\frac{1}{\varepsilon^d} - \frac{1}{\sigma})}$. Under price competition, $\varepsilon^d = \sigma \left(1 - \frac{1}{N} (1 - m) \right) + \gamma \frac{1}{N} (1 - m)$, thus $N = \frac{(1-m)(\gamma - \sigma)}{(\varepsilon^d - \sigma)}$ where m is the steady state market share of foreign firms in the intermediate goods market.

Finally, in this real business cycle model, fluctuations in total factor productivity drive business cycle fluctuations. The A_t and A_t^* variables in (11) are exogenous country specific shocks that evolve according to the following VAR(1) process:

$$\begin{bmatrix} A_{t+1} \\ A_{t+1}^* \end{bmatrix} = \begin{bmatrix} 0.91 & 0 \\ 0 & 0.91 \end{bmatrix} \begin{bmatrix} A_t \\ A_t^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}$$

where $\text{var}(\varepsilon_t) = \text{var}(\varepsilon_t^*) = .006^2$ and $\text{corr}(\varepsilon_t, \varepsilon_t^*) = .25$

4 Results

4.1 Qualitative effect of variable markups

To see how variable markups affect the cyclical fluctuations in production and aggregate output, consider the log linearization of the demand function for intermediate goods from domestic firms in (8):

$$\hat{x}_{D,t} = -\sigma \hat{p}_{D,t} + \sigma \hat{p}_t + \hat{x}_t \quad (22)$$

Note that the sectoral superscripts j have been omitted for clarity.

If we log linearize the price index in (10), then the sectoral price level \hat{p}_t can be expressed as:

$$\hat{p}_t = N s_{D,t} (\hat{p}_{D,t}) + N^* s_{X,t}^* (\hat{p}_{X,t}^* + \hat{q}_t) \quad (23)$$

where s_D and s_X , defined in an earlier section, are a home intermediate good firm's market shares in the home and foreign markets, respectively. Since the N home intermediate goods firms and the N^* foreign intermediate goods firms are identical, $N^* s_{X,t}^* = m$ is the market share of foreign firms in the market for intermediate goods, and $N s_{D,t} = 1 - m$ is the market share of domestic firms.

Substituting equation (23) into (22) yields the following:

$$\hat{x}_{D,t} = \sigma m (\hat{q}_t + \hat{p}_{X,t}^* - \hat{p}_{D,t}) + \hat{x}_t \quad (24)$$

The domestic price is simply equal to a markup over the domestic marginal cost of production, $p_{D,t} = (1 + \mu_{D,t}) MC_t$. The foreign export price is equal to a markup multiplied by the foreign marginal cost of production, $p_{X,t}^* = (1 + \mu_{X,t}^*) MC_t^*$. After log-linearizing these pricing formulas, the log-linearized demand function in (24) is:

$$\hat{x}_{D,t} = \sigma m \left(\hat{q}_t + \frac{\mu_X^*}{1 + \mu_X^*} \hat{\mu}_{X,t}^* + M\hat{C}_t^* - \frac{\mu_D}{1 + \mu_D} \hat{\mu}_{D,t} - M\hat{C}_t \right) + \hat{x}_t \quad (25)$$

where μ_D and μ_X^* are the steady state values of the domestic and export markups.

Suppose that there is an exogenous change in the foreign marginal cost of production, $M\hat{C}_t^*$. Equation (25) shows that the elasticity of domestic demand with respect to changes in foreign marginal cost is:

$$\frac{\hat{x}_{D,t}}{M\hat{C}_t^*} = \sigma m \left(\frac{\hat{q}_t}{M\hat{C}_t^*} + \frac{\mu_X^*}{1 + \mu_X^*} \frac{\hat{\mu}_{X,t}^*}{M\hat{C}_t^*} + 1 - \frac{\mu_D}{1 + \mu_D} \frac{\hat{\mu}_{D,t}}{M\hat{C}_t^*} - \frac{M\hat{C}_t}{M\hat{C}_t^*} \right) + \frac{\hat{x}_t}{M\hat{C}_t^*} \quad (26)$$

Suppose markups are constant, then $\frac{\hat{\mu}_{X,t}^*}{M\hat{C}_t^*} = \frac{\hat{\mu}_{D,t}}{M\hat{C}_t^*} = 0$. Then the elasticity in (26) can be rewritten as:

$$\frac{\hat{x}_{D,t}}{M\hat{C}_t^*} = \sigma m \left(\frac{\hat{q}_t}{M\hat{C}_t^*} + 1 - \frac{M\hat{C}_t}{M\hat{C}_t^*} \right) + \frac{\hat{x}_t}{M\hat{C}_t^*} \quad (27)$$

Therefore equations (26) and (27) show how the variability of markups can affect the cyclical fluctuations in output. Define $\Delta \frac{\hat{x}_{D,t}}{M\hat{C}_t^*}$ to be the change in elasticity when the variability of markups is taken into account:⁸

$$\Delta \frac{\hat{x}_{D,t}}{M\hat{C}_t^*} = \sigma m \left(\frac{\mu_X^*}{1 + \mu_X^*} \frac{\hat{\mu}_{X,t}^*}{M\hat{C}_t^*} - \frac{\mu_D}{1 + \mu_D} \frac{\hat{\mu}_{D,t}}{M\hat{C}_t^*} \right) \quad (28)$$

In this model, the markup is an increasing function of market share. Therefore when foreign costs decrease, foreign producers gain market share in the domestic market and thus their markup rises, $\frac{\hat{\mu}_{X,t}^*}{M\hat{C}_t^*} < 0$. Similarly when foreign cost decrease, domestic producers lose market share and thus their markup increases, $\frac{\hat{\mu}_{D,t}}{M\hat{C}_t^*} > 0$. This implies that $\Delta \frac{\hat{x}_{D,t}}{M\hat{C}_t^*} < 0$.

The key result from this paper is summarized in equation (28).

Markup variability should lead to less business cycle volatility and greater international business cycle correlation. Suppose there is a positive shock to foreign productivity. Foreign marginal costs fall, and the relative price of foreign products decreases. Foreign producers will increase production and domestic producers will cut production. Thus the foreign shock leads both business cycle volatility and international business cycle divergence.

Foreign producers will gain market share at the expense of domestic producers, and thus foreign markups increase while domestic markups fall. These changes in markups cause the

⁸This expression for $\Delta \frac{\hat{x}_{D,t}}{M\hat{C}_t^*}$ is an approximation. Other elasticities like the elasticity of the real exchange rate with respect to changes in foreign marginal costs, $\frac{\hat{q}_t}{M\hat{C}_t^*}$, the elasticity of home marginal costs, $\frac{M\hat{C}_t}{M\hat{C}_t^*}$, and the elasticity of home gross output, $\frac{\hat{x}_t}{M\hat{C}_t^*}$, should all be affected by the variability of markups.

relative price of foreign goods to increase. Thus variability in markups causes a change in relative prices that is exactly opposite to the change due to the initial productivity shock. Home and foreign business cycles will be less volatile and will diverge less than they would have without variable markups.

This sequence of events is illustrated in figures 1 and 2. Figure 1 plots the response of home and foreign marginal cost and the response of home domestic markups and foreign export markups to a positive foreign productivity shock. The dotted line in each plot refers to the path of markups and GDP when markups are constant. Figure 2 plots the price of domestically produced intermediate goods in the home market and the price of foreign intermediate goods in the home market, as well as home and foreign GDP. The dotted line in each impulse response diagram refers to the version of the model where markups are held constant over the cycle, and the solid line refers to the version of the model where markups are variable over the cycle and arise from firms engaged in quantity competition, as in (19).⁹

The top row of impulse response diagrams in figure 1 shows how the positive foreign TFP shock causes home marginal costs to increase and foreign marginal costs to decrease. However when this happens, foreign firms gain market share at the expense of domestic firms. The bottom row in the figure shows that when markups vary over the cycle, foreign firms will increase their markup in the home market and domestic firms will cut their markup.

The top row in figure 2 shows how the price of domestically produced intermediate inputs should rise and the price of foreign intermediate inputs should fall following the foreign TFP shock. However the figure shows that the price change is smaller if markups vary over the cycle. Home intermediate goods prices still rise and foreign intermediate goods prices still fall, but not as much under variable markups as under constant markups. Therefore domestic production is higher and foreign production is lower than it would have been without variable markups. Endogenous markup variability means that the initial negative response to home GDP is less negative, and the initial positive response of foreign GDP is less positive, leading to less volatile responses in home and foreign GDP following the foreign TFP shock.

Home firms find themselves at a comparative disadvantage following the foreign productivity shock and thus they cut production for about 16 quarters. Therefore the productivity shock leads to business cycle divergence and negative bilateral *GDP* correlation, which is a feature of many IRBC models with complete international asset markets. When there is endogenous markup variability, the changes in the markups of home and foreign firms place home firms at less of a comparative disadvantage and they return to pre-shock levels of production after about 10 quarters. Thus with endogenous markup variability there is less bilateral business cycle divergence following a country specific productivity shock and

⁹In both cases the steady state markup is 20%.

bilateral correlation should be higher.

4.2 Quantitative impact of variable markups

4.2.1 Business Cycle Volatility

The volatility of GDP and its components as calculated from simulations of the model are presented in table 3. The first two rows of the table present the standard deviation of real GDP. The remaining rows in the table list the relative standard deviation of the components of GDP (and labor input). In the table, the data for each entry is computed twice, once in the model assuming that markups are held constant over the cycle, and once in the model assuming that markups are allowed to vary over the cycle.

The first column of the table presents the results from the benchmark parameterization, as described in section 3. The share of foreign firms in both the intermediate and final goods markets is set to 25%. Intermediate goods firms charge a 20% markup in their home market, firms are engaged in quantity competition, international asset markets are complete, and preferences are non-separable in consumption and leisure.

Under the benchmark parameterization of the model, the standard deviation of real GDP is 3.25% when markups are held constant and 3.06% when markups are allowed to vary. Thus allowing markups to vary over the cycle leads to a nearly 6% reduction in GDP volatility.

The table also shows that allowing markups to vary leads to a nearly 6% increase in the relative volatility of consumption from 0.44 to 0.47, not enough of a change to fully reconcile the IRBC model's predictions about consumption volatility with the volatility observed in the data (see table 1) but definitely a significant move in the right direction.

Another well known feature of the IRBC model is that without investment adjustment costs, the model's predictions for investment and net export volatility is way too high. Table 3 shows that the inclusion of endogenous markup variability leads to a 5% fall in investment volatility and an 11% fall in net export volatility.

However the model cannot help resolve the discrepancy between the IRBC model's prediction about employment volatility and that observed in the data. The model consistently predicts a relative volatility of employment that is below that observed in the data. However the table shows that the introduction of endogenous markup variability actually reduces the relative volatility of employment, and thus markup variability cannot help resolve the well known discrepancy between the model and the data in regards to the relative volatility of employment.

The remaining columns in the table calculate the same statistics from model simulations but under alternative parameterizations. The second column reports the results when firms

are engaged in price competition, the third column reports the results for a 25% steady state markup, the fourth column reports the results for a 15% steady state markup, the fifth column reports the results from a model where the only asset traded internationally is a non-contingent bond, and the sixth column reports the results from the model where preferences are separable in consumption and labor effort. The effect of markup variability on the volatility of GDP and its components remains the same under the alternative parameterizations. In fact, the effect is enhanced when firms are engaged in price competition or the steady state markup is high. When markups are small, markup variability still has an effect on volatility, but the effect is smaller. Similarly the effect of markup variability is slightly reduced when asset markets are restricted. Lastly, the results are largely unaffected by the substitutability of leisure and consumption in the utility function.

The effect of markup variability as the market share of foreign firms increases

The quantitative results in table 3 are calculated assuming that in the steady state foreign firms supply 25% of the intermediate goods used in the home country. The qualitative results earlier in this section, specifically equation (28), suggests that the quantitative effect of markup variability should be increasing in the market share of foreign intermediate goods firms. In this section we will examine the effect of an increasing foreign market share on business cycle volatility.

The standard deviation of GDP as calculated from model simulations for different steady state foreign market shares is presented in figure 3. The figure presents the results from the model assuming markups are held fixed over the cycle and assuming that markups are variable over the cycle. The figure confirms the prediction from equation (28) that the role of variable markups in reducing GDP volatility is increasing in the market share of foreign firms.

The effect of markup variability on the relative volatility of the components of GDP are presented in figure 4. The figure confirms the results that are presented in table 3. Endogenous markup variability has a positive effect on the relative volatility of consumption and a negative effect on the relative volatilities of investment, net exports and employment. Furthermore, this effect of markup variability is increasing in the market share of foreign firms.

4.2.2 International Business Cycle Co-movement

Similarly, markup variability has a sizeable effect on international business cycle co-movement. The results from calculating the cross-country correlation of GDP and its components in the model with and without markup variability are listed in table 4.

As discussed in the introduction, bilateral GDP, investment, and employment correlation is far too low in the benchmark IRBC model with complete international asset markets and no investment adjustment costs. Furthermore, bilateral consumption correlation in the model is too high compared to that observed in the data.

In the benchmark case of this model, the correlation between the two countries' GDP fluctuations is about -8% when markups are held constant over the cycle. When markups are allowed to vary, this correlation coefficient increases to about 4% . Thus endogenous markup variability leads to a 12 percentage point increase in cross-country GDP correlation. Similarly allowing markup variability leads to an 11 percentage point increase in investment co-movement and a 15 percentage point increase in cross-country employment co-movement.

Thus the inclusion of endogenous markup variability partially resolves the well known puzzle that in the benchmark IRBC model, GDP, investment and employment correlation is too low. However the model does not help resolve the puzzle about bilateral consumption correlation being too high. The introduction of endogenous markup variability leads to about a 1 percentage point increase in cross-country consumption correlation. While that is the opposite of what is needed to resolve the discrepancy between the model and the data, the effect of markup variability on cross-country consumption correlation is about one order of magnitude smaller than the effect on GDP, investment, or employment correlation, so it is safe to say that endogenous markup variability doesn't have much of an effect on cross-country consumption correlation.

The remaining columns in the table present the model's simulated correlation coefficients under alternative parameterizations. The results hold under the various parameterizations, and in some cases are enhanced. For instance, when markup variability arises from firms engaged in price competition, cyclical markup variability leads to a 16 percentage point increase in GDP co-movement, a 14 percentage point increase in investment co-movement, and a nearly 20 percentage point increase in the cross-country employment correlation.

The effect of markup variability as the market share of foreign firms increases

The effect of increasing foreign market share on cross-country GDP correlation is presented in figure 5. The figure shows that the gap between correlation in the model with variable markups and that in the model with constant markets is increasing in the foreign market share. As predicted in equation (28), as foreign firms occupy a greater share of the home intermediate goods market, allowing markups to vary over the cycle should have a greater quantitative effect on the cross-country co-movement of business cycles.

The effects of variable markups on cross-country GDP correlation of the components of GDP is presented in figure 6. This figure confirms the results from table 4 and shows that

markup variability has a sizable positive effect on cross-country investment and employment co-movement. The figure also shows that the effect of markup variability is increasing in the market share of foreign intermediate goods firms.

4.2.3 Using command-basis GDP instead of real GDP

As discussed in Kehoe and Ruhl (2008), when the terms of trade are volatile, real GDP may not accurately measure the purchasing power of an economy. In the model, real GDP is measured as:

$$GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}$$

where C_t , I_t , X_t , M_t are the nominal values of home consumption, investment, exports, and imports, and P_t^C , P_t^I , P_t^X , P_t^M are the deflators for consumption, investment, exports and imports. As demonstrated by Kehoe and Ruhl, the actual purchasing power of an economy should instead be measured with as command-basis GDP, also referred to as gross domestic income (GDI):

$$GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{X_t - M_t}{P_t^M}$$

Notice that the only difference between the two measures is that the nominal value of exports is deflated by the import price deflator in the measure of GDI but the export price deflator in the measure of GDP. The reason the command-basis GDP measure is a better measure of purchasing power is that $X_t - M_t$ measures the nominal net income from international trade. By deflating this nominal net income by the import price deflator, you can measure the actual quantity of goods and services from abroad that an economy was able to purchase with this net income from international trade.

Algebraically, the only difference between the measure of real GDP and command-basis GDP is that in the measure of command-basis GDP, the nominal value of exports is divided by the terms of trade. Thus if the terms of trade is relatively stable, fluctuations in real GDP should be similar to fluctuations in command-basis GDP. However if the terms of trade fluctuate, and are positively correlated with fluctuations in exports, then command-basis GDP should be less volatile and have greater international co-movement than fluctuations in real GDP.

The results from simulations of the model where we calculate the moments of command-GDP instead of real-GDP are presented in tables 5 and 6. The table shows that in this model where business cycles are driven by productivity shocks, fluctuations in command-GDP are less volatile and display higher international correlation than fluctuations in real GDP,

however the effect of endogenous markup variability is unchanged. Endogenous markup variability has the same qualitative and quantitative effect on business cycle volatility and international co-movement even when we consider fluctuations in command-GDP.

The volatility and cross-country correlation of command GDP as the market share of foreign intermediate goods firms changes is presented in figures 7 and 8. Again, using command basis GDP instead of real GDP leads to a level shift in the volatility and cross-country correlation, but the net effect of introducing endogenous markup variability is unchanged.

5 Summary and Conclusion

This paper introduces strategic interaction among firms and endogenous markup variability into the international real business cycle model. Specifically this paper shows how this strategic interaction among individual firms can have a significant effect on aggregate quantities at the business cycle frequency.

The intuition here is simple. Due to strategic interactions among firms, a firm's market power is positively related to its market share. Following a productivity shock in an international real business cycle model, there is a change in the relative prices of home and foreign produced goods that leads to a change in firm market shares.

We find that the introduction of strategic interaction and the resulting markup variability into the IRBC model leads to about a 5% decrease in GDP volatility, a 5% increase in the relative volatility of consumption, and a 5% and 10% decrease in the relative volatilities of investment and the trade balance, respectively.

Furthermore this strategic interaction among firms leads to a nearly 12 percentage point increase in bilateral GDP correlation and a 11 and 15 percentage point increase in the cross-country correlation of investment and employment.

This paper fits into the young but growing literature that applies the microeconomic issue of endogenous markup variability to macroeconomic questions. Most, but not all, papers incorporating endogenous markup variability study macroeconomic issues related to prices and exchange rates. This paper shows that the effect of strategic interaction on aggregate quantities is not trivial. The effect of markup variability on other questions pertaining to quantities and production allocation is a promising avenue for further research.

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A Technical Appendix

A.1 The household's maximization problem

The household will maximize the expected present value of lifetime utility, (12), subject to their budget constraint in (13), and capital accumulation equation, (14).

Under complete international financial markets, the home and foreign households' problems are solved as one maximization problem subject to one worldwide budget constraint and two capital accumulation equations:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\lambda_t \left(\begin{aligned} & \frac{1}{1-\zeta} \left[(1-h_t)^\theta (C_t)^{1-\theta} \right]^{1-\zeta} + \frac{1}{1-\zeta} \left[(1-h_t^*)^\theta (C_t^*)^{1-\theta} \right]^{1-\zeta} \\ & C_t + I_t + q_t (C_t^* + I_t^*) \\ & -w_t h_t - r_t K_t - \int_0^1 \sum_{i=1}^N \Pi_t^j(i) dj - q_t \left(w_t^* h_t^* - r_t^* K_t^* - \int_0^1 \sum_{i=1}^N \Pi_t^{j*}(i) dj \right) \\ & -v_t (K_{t+1} - (1-\delta) K_t - I_t) \\ & -v_t^* (K_{t+1}^* - (1-\delta) K_t^* - I_t^*) \end{aligned} \right) \right]$$

The first order conditions with respect to $C_t, C_t^*, h_t, h_t^*, I_t, I_t^*, K_{t+1}$ and K_{t+1}^* are:

$$\begin{aligned} \lambda_t &= (1-\theta) \left[(1-h_t)^\theta (C_t)^{1-\theta} \right]^{1-\zeta} (C_t)^{-\theta} & (29) \\ \lambda_t q_t &= (1-\theta) \left[(1-h_t^*)^\theta (C_t^*)^{1-\theta} \right]^{1-\zeta} (C_t^*)^{-\theta} \\ \lambda_t w_t &= \theta \left[(1-h_t)^\theta (C_t)^{1-\theta} \right]^{1-\zeta} (1-h_t)^{-1} \\ \lambda_t w_t^* q_t &= \theta \left[(1-h_t^*)^\theta (C_t^*)^{1-\theta} \right]^{1-\zeta} (1-h_t^*)^{-1} \\ \lambda_t &= v_t \\ \lambda_t q_t &= v_t^* \\ \frac{v_t}{\beta} &= (1-\delta) v_{t+1} + \lambda_{t+1} r_{t+1} \\ \frac{v_t^*}{\beta} &= (1-\delta) v_{t+1}^* + \lambda_{t+1} r_{t+1}^* q_{t+1} \end{aligned}$$

A.2 The firm's maximization problem

Firm i in sector j will choose a prices and quantities to maximize profit given by:

$$\Pi_t^j(i) = p_{D,t}^j(i) x_{D,t}^j(i) + p_{X,t}^j(i) x_{M,t}^{j*}(i) - w_t h_t^j(i) - r_t K_t^j(i) \quad (30)$$

This profit can be rewritten as:

$$\Pi_t^j(i) = (p_{D,t}^j(i) - MC_t) x_{D,t}^j(i) + ((1-c)p_{X,t}^j(i) - MC_t) x_{X,t}^j(i) - MC_t \psi \quad (31)$$

where $MC_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^\alpha$.

A.2.1 Quantity Competition

The firm engaged in quantity competition will choose quantities $x_{D,t}^j(i)$ and $x_{X,t}^j(i)$ to maximize (17) subject to the inverse demand functions in (18). After substituting the inverse demand function into the profit maximization problem, the firm's univariate maximization problem to choose $x_{D,t}^j(i)$ is:

$$x_{D,t}^j(i) = \operatorname{argmax}_{x_{D,t}^j(i)} \left\{ \left(\left(\frac{x_{D,t}^j(i)}{x_t^j} \right)^{\frac{-1}{\sigma}} \left(\frac{x_t^j}{x_t} \right)^{\frac{-1}{\gamma}} p_t^x - MC_t \right) x_{D,t}^j(i) \right\}$$

The individual firm's choice of $x_{D,t}^j(i)$ can influence total production within sector j , x_t^j , but not the quantity of intermediate inputs from all sectors x_t or marginal cost, MC_t . Using the fact that $\frac{d \ln x_t^j}{d \ln x_{D,t}^j(i)} = (1 - m_t) \frac{1}{N}$ the first order condition of the firm's problem is:

$$\left(\begin{aligned} & \left(1 - \frac{1}{\sigma}\right) \left(x_{D,t}^j(i)\right)^{-\frac{1}{\sigma}} \left(x_t^j\right)^{\frac{1}{\sigma} - \frac{1}{\gamma}} \left(x_t\right)^{\frac{-1}{\gamma}} p_t^x \\ & + \left(\frac{1}{\sigma} - \frac{1}{\gamma}\right) x_{D,t}^j(i)^{-\frac{1}{\sigma}} \left(x_t^j\right)^{\frac{1}{\sigma} - \frac{1}{\gamma}} \left(x_t\right)^{\frac{-1}{\gamma}} p_t^x \frac{\partial x_t^j}{\partial x_{D,t}^j(i)} \frac{x_{D,t}^j(i)}{x_t^j} \end{aligned} \right) = MC_t \quad (32)$$

Substitute the inverse demand function in (18) into this first order condition and note that $\frac{d \ln x_t^j}{d \ln x_{D,t}^j(i)} = (1 - m_t) \frac{1}{N} = s_{D,t}^j$, then the price set by the firm engaging in quantity competition is:

$$p_{D,t}^j(i) = \frac{1}{1 - \left(\frac{1}{\sigma}(1 - s_t^D) + \frac{1}{\gamma}s_t^D\right)} MC_t \quad (33)$$

Thus the domestic elasticity of demand for the firm engaged in quantity competition is:

$$\varepsilon^d = \left(\frac{1}{\sigma}(1 - s_t^D) + \frac{1}{\gamma}s_t^D\right)^{-1}$$

A.2.2 Price Competition

The firm engaged in price competition will choose prices $p_{D,t}^j(i)$ and $p_{X,t}^j(i)$ to maximize:

$$\begin{aligned} p_{D,t}^j(i) &= \operatorname{argmax}_{p_{D,t}^j(i)} \left\{ (p_{D,t}^j(i) - MC_t) x_{D,t}^j(i) \right\} \\ p_{X,t}^j(i) &= \operatorname{argmax}_{p_{X,t}^j(i)} \left\{ ((1 - c)p_{X,t}^j(i) - MC_t) x_{X,t}^j(i) \right\} \end{aligned} \quad (34)$$

subject to the following demand functions:

$$\begin{aligned} x_{D,t}^j(i) &= \left(\frac{p_{D,t}^j(i)}{p_t^j}\right)^{-\sigma} \left(\frac{p_t^j}{p_t^x}\right)^{-\gamma} x_t \\ x_{X,t}^j(i) &= \frac{1}{(1 - c)} \left(\frac{p_{X,t}^j(i)}{q_t(1 - c)p_t^{j*}}\right)^{-\sigma} \left(\frac{p_t^{j*}}{p_t^{x*}}\right)^{-\gamma} x_t^* \end{aligned} \quad (35)$$

After substituting the demand function into the maximization problem, the firm's univariate maximization problem to choose its domestic price, $p_{D,t}^j(i)$, is:

$$p_{D,t}^j(i) = \operatorname{argmax}_{p_{D,t}^j(i)} \left\{ (p_{D,t}^j(i) - MC_t) \left(\frac{p_{D,t}^j(i)}{p_t^j}\right)^{-\sigma} \left(\frac{p_t^j}{p_t^x}\right)^{-\gamma} x_t \right\} \quad (36)$$

The individual firm's choice of $p_{D,t}^j(i)$ can influence sectoral prices, p_t^j , but not the aggregate price of intermediate inputs, p_t^x , or the quantity of intermediate inputs from all sectors, x_t , or marginal cost, MC_t . the first order condition of the firm's problem is, making note of

the fact that $\frac{d \ln p_t^j}{d \ln p_{D,t}^j(i)} = (1 - m_t) \frac{1}{N} = s_{D,t}$:

$$p_{D,t}^j(i) = \frac{\sigma(1 - s_t^D) + \gamma s_t^D}{\sigma(1 - s_t^D) + \gamma s_t^D - 1} MC_t \quad (37)$$

Thus the domestic elasticity of demand for the firm engaged in price competition is:

$$\varepsilon^d = \sigma(1 - s_t^D) + \gamma s_t^D$$

Table 2: Parameter Values

Symbol	Value	Description
σ	10	elasticity of substitution across intermediate goods producing firms
γ	1.01	elasticity of substitution across different sectors
ρ	1.5	elasticity of substitution across home and foreign final goods
θ	0.7	weight on leisure in the utility function
ζ	2	coefficient of relative risk aversion
α	0.36	capital's share of income
β	0.99	discount factor
δ	0.025	capital depreciation rate
κ	0.5	elasticity of substitution between value added and intermediate inputs
ϕ	0.5	coefficient on value added in the production function for final goods
ψ	0.321	fixed cost parameter (benchmark)
c	0.159	trade cost parameter
ω	0.733	exogenous preference for imported goods in the final goods aggregator function
μ^d	20%	steady state domestic markup

Table 3: Standard deviation of GDP and its components as calculated by the model with and without variable markups.

	Variations on the benchmark economy					
	Benchmark economy	Price Competition	High Markups	Low Markups	Incomplete Markets	Separable Preferences
Standard deviation of GDP (%):						
Constant Markups	3.254	3.252	3.301	3.206	2.762	2.762
Variable Markups	3.062	3.000	3.021	3.114	2.705	2.600
Relative standard deviation of:						
Consumption:						
Constant Markups	0.445	0.445	0.444	0.446	0.605	0.321
Variable Markups	0.471	0.479	0.483	0.458	0.609	0.341
Investment:						
Constant Markups	3.603	3.612	3.543	3.668	3.072	3.851
Variable Markups	3.410	3.401	3.275	3.571	2.986	3.672
Net Exports:						
Constant Markups	0.716	0.717	0.709	0.724	0.386	0.776
Variable Markups	0.636	0.629	0.596	0.685	0.347	0.699
Exports:						
Constant Markups	1.192	1.193	1.181	1.204	0.954	1.241
Variable Markups	1.137	1.137	1.108	1.175	0.942	1.182
Imports:						
Constant Markups	1.130	1.131	1.119	1.142	0.941	1.168
Variable Markups	1.074	1.078	1.045	1.113	0.930	1.104
Labor:						
Constant Markups	0.513	0.514	0.499	0.528	0.407	0.480
Variable Markups	0.483	0.476	0.456	0.514	0.399	0.454

Table 4: Cross-country correlation of GDP and its components as calculated by the model with and without variable markups.
 Variations on the benchmark economy

Cross-country correlation of:	Benchmark economy		Price Competition	High Markups	Low Markups	Incomplete Markets	Separable Preferences
	economy	Markups					
GDP:							
Constant Markups	-0.080	-0.079	-0.087	-0.073	0.278	-0.186	
Variable Markups	0.040	0.081	0.091	-0.018	0.332	-0.082	
Consumption:							
Constant Markups	0.885	0.885	0.888	0.882	0.415	0.982	
Variable Markups	0.900	0.912	0.910	0.889	0.459	0.980	
Investment:							
Constant Markups	-0.576	-0.577	-0.567	-0.585	-0.189	-0.557	
Variable Markups	-0.465	-0.440	-0.395	-0.536	-0.106	-0.450	
Net Exports:							
Constant Markups	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	
Variable Markups	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	
Exports:							
Constant Markups	-0.352	-0.353	-0.345	-0.360	0.405	-0.472	
Variable Markups	-0.196	-0.163	-0.111	-0.289	0.502	-0.343	
Imports:							
Constant Markups	-0.279	-0.281	-0.270	-0.290	0.442	-0.403	
Variable Markups	-0.099	-0.070	-0.001	-0.207	0.540	-0.247	
Labor:							
Constant Markups	-0.471	-0.471	-0.477	-0.464	0.170	-0.459	
Variable Markups	-0.327	-0.274	-0.251	-0.401	0.269	-0.319	

Table 5: Standard deviation of GDP and its components as calculated by the model with and without variable markups. Calculated by adjusting exports by the terms of trade as in the measure of command GDP.

	Variations on the benchmark economy					
	Benchmark economy	Price Competition	High Markups	Low Markups	Incomplete Markets	Separable Preferences
Standard deviation of GDP (%):						
Constant Markups	3.054	3.052	3.096	3.011	2.667	2.569
Variable Markups	2.879	2.823	2.841	2.928	2.610	2.418
Relative standard deviation of:						
Consumption:						
Constant Markups	0.474	0.474	0.474	0.474	0.627	0.345
Variable Markups	0.501	0.509	0.513	0.487	0.631	0.367
Investment:						
Constant Markups	3.839	3.848	3.778	3.905	3.181	4.140
Variable Markups	3.626	3.615	3.482	3.799	3.095	3.948
Net Exports:						
Constant Markups	0.697	0.698	0.690	0.706	0.384	0.759
Variable Markups	0.612	0.603	0.568	0.665	0.346	0.674
Exports:						
Constant Markups	1.135	1.136	1.123	1.147	0.967	1.170
Variable Markups	1.084	1.084	1.057	1.121	0.960	1.112
Imports:						
Constant Markups	1.204	1.205	1.193	1.216	0.975	1.255
Variable Markups	1.142	1.146	1.111	1.184	0.964	1.187
Labor:						
Constant Markups	0.547	0.548	0.532	0.562	0.421	0.516
Variable Markups	0.514	0.506	0.485	0.547	0.413	0.489

Table 6: Cross-country correlation of GDP and its components as calculated by the model with and without variable markups. Calculated by adjusting exports by the terms of trade as in the measure of command GDP.

Cross-country correlation of:	Variations on the benchmark economy					
	Benchmark economy	Price Competition	High Markups	Low Markups	Incomplete Markets	Separable Preferences
GDP:						
Constant Markups	0.045	0.045	0.038	0.051	0.371	-0.060
Variable Markups	0.176	0.221	0.233	0.112	0.431	0.061
Consumption:						
Constant Markups	0.885	0.885	0.888	0.882	0.415	0.982
Variable Markups	0.900	0.912	0.910	0.889	0.459	0.980
Investment:						
Constant Markups	-0.576	-0.577	-0.567	-0.585	-0.189	-0.557
Variable Markups	-0.465	-0.440	-0.395	-0.536	-0.106	-0.450
Net Exports:						
Constant Markups	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
Variable Markups	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
Exports:						
Constant Markups	-0.188	-0.190	-0.176	-0.202	0.465	-0.313
Variable Markups	0.001	0.038	0.105	-0.115	0.553	-0.141
Imports:						
Constant Markups	-0.279	-0.281	-0.270	-0.290	0.442	-0.403
Variable Markups	-0.099	-0.070	-0.001	-0.207	0.540	-0.247
Labor:						
Constant Markups	-0.471	-0.471	-0.477	-0.464	0.170	-0.459
Variable Markups	-0.327	-0.274	-0.251	-0.401	0.269	-0.319

Figure 1: Response of home and foreign marginal cost and markups to a positive foreign TFP shock.

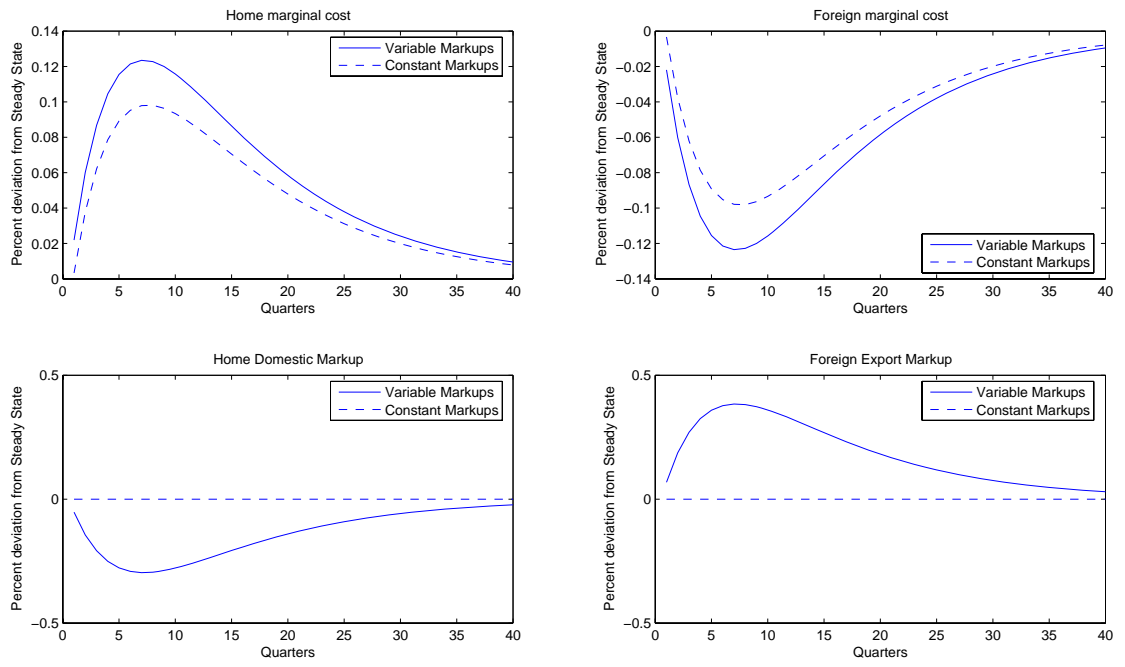


Figure 2: Response of home and foreign prices and GDP to a positive foreign TFP shock.

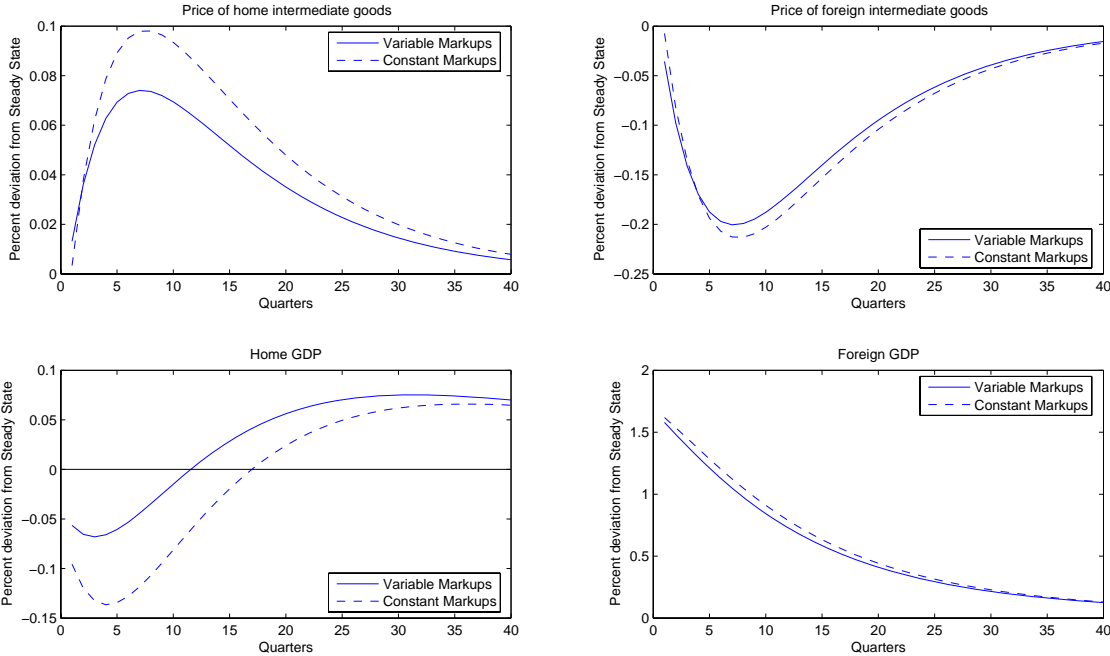


Figure 3: GDP volatility as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

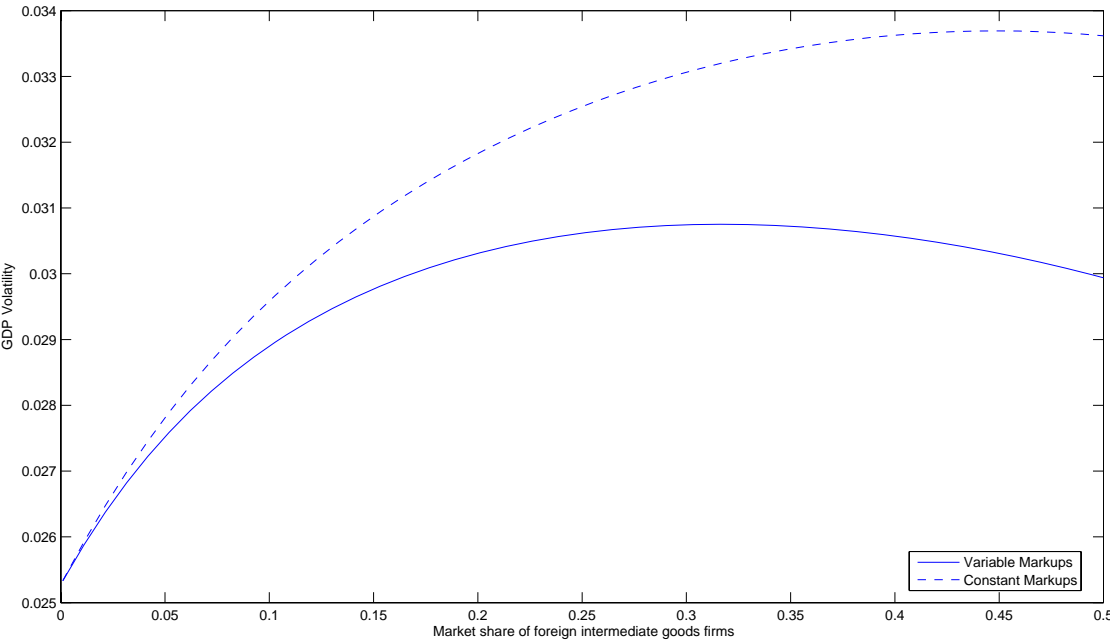


Figure 4: The relative volatility of the components of GDP as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

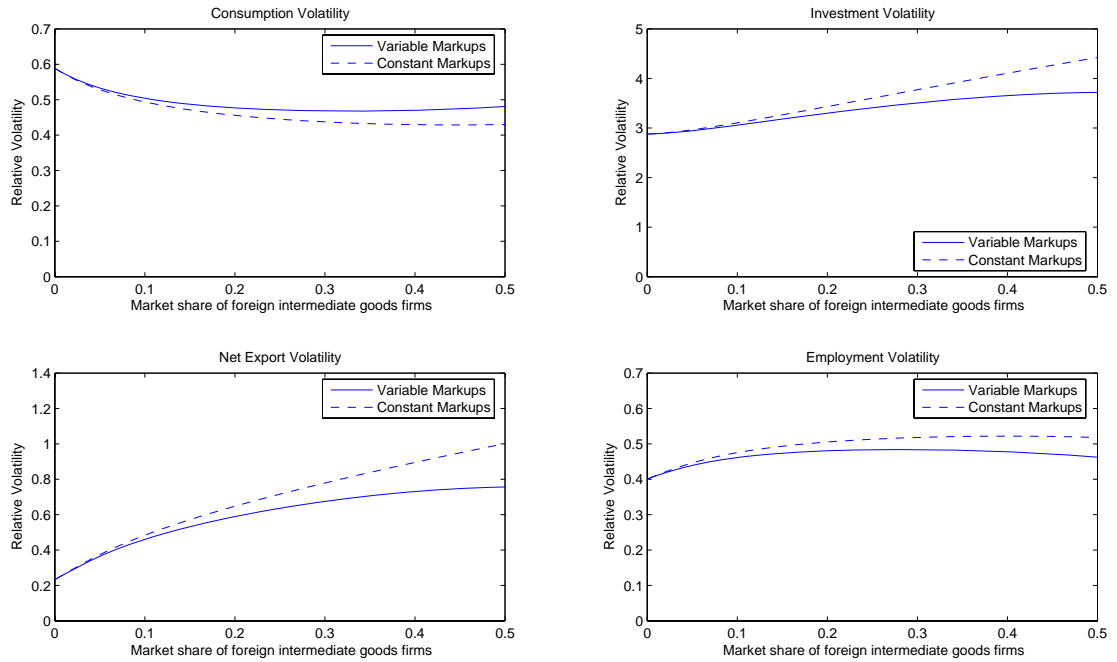


Figure 5: Cross-country GDP correlation as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

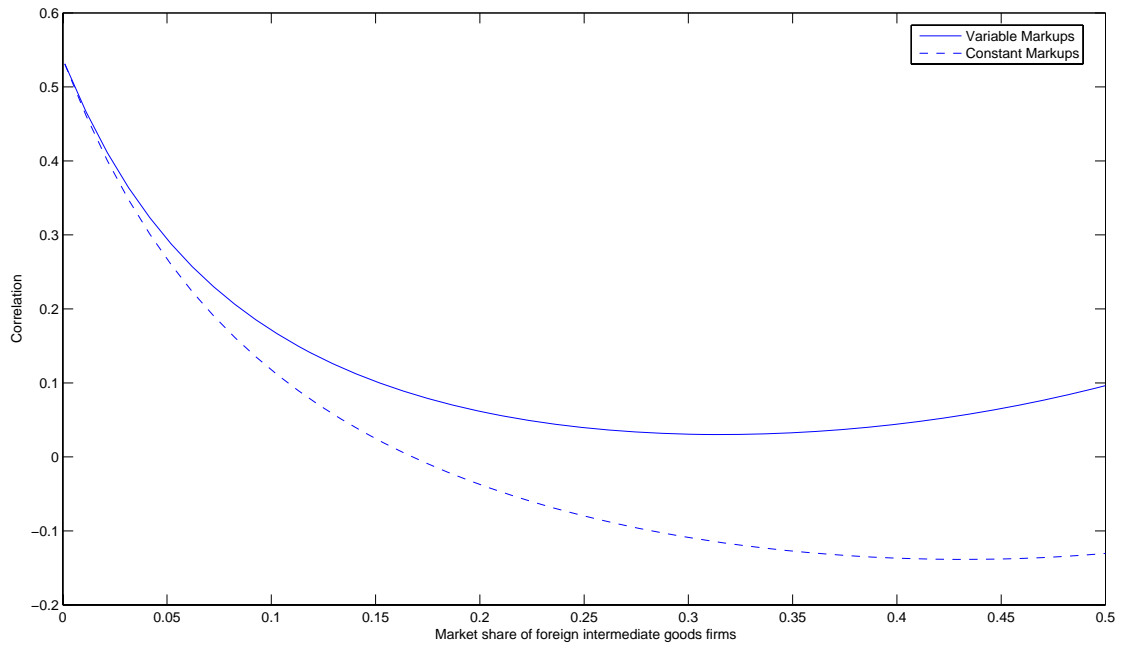


Figure 6: Cross-country correlation of the components of GDP as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

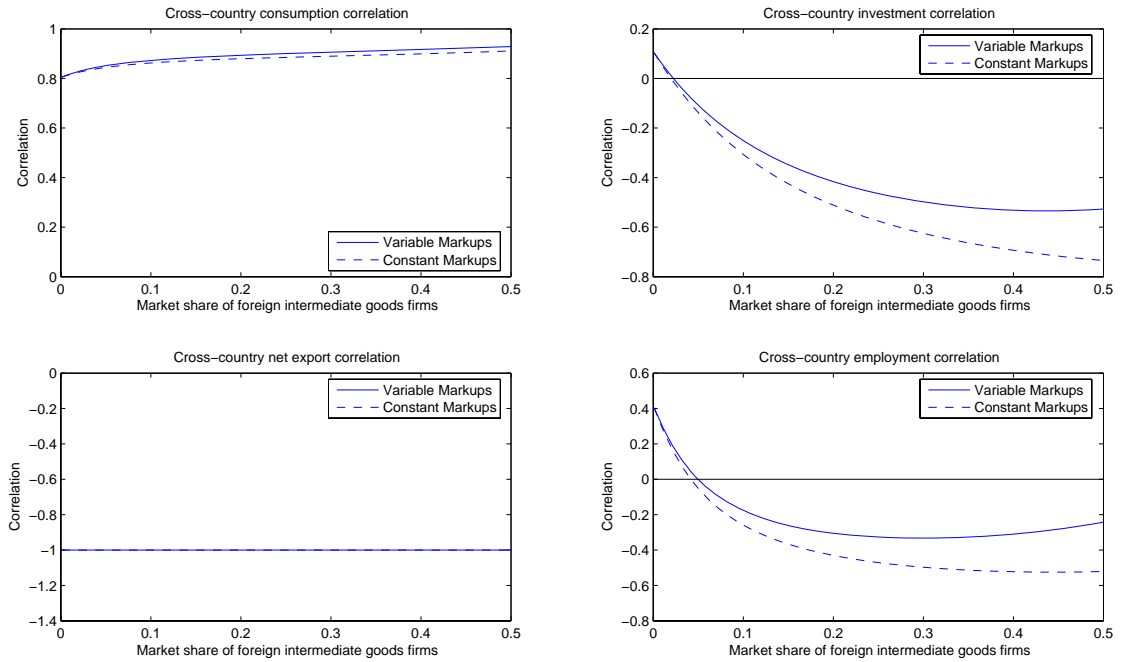


Figure 7: Command GDP volatility as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

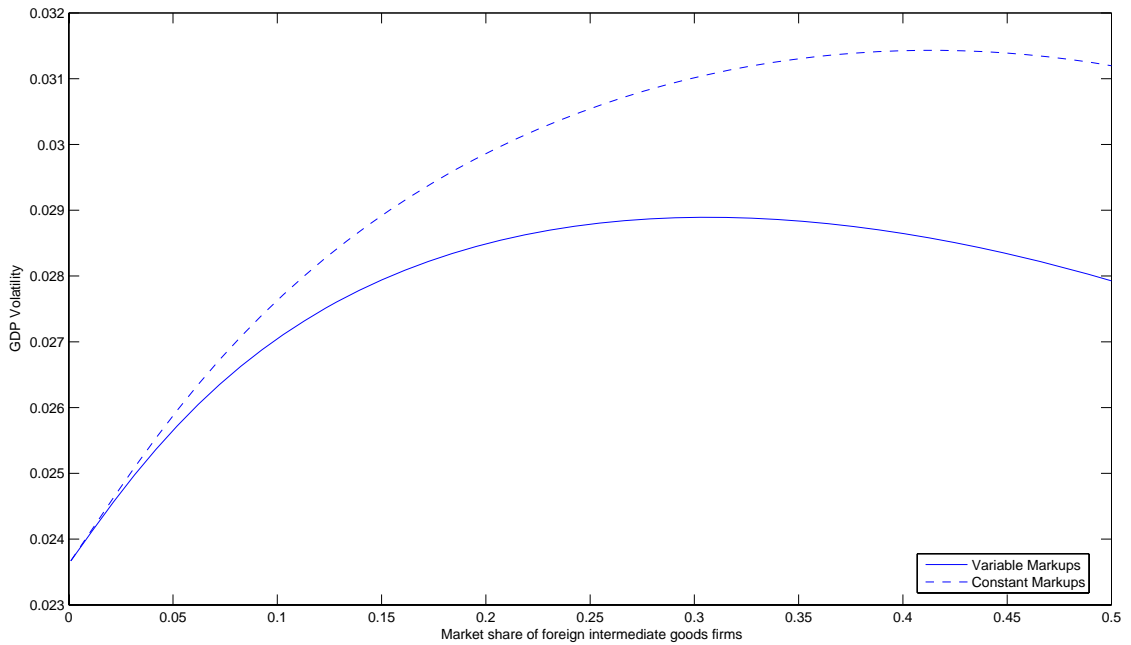


Figure 8: Cross-country command GDP correlation as a function of the steady state market share of foreign intermediate goods firms, calculated with and without variable markups.

