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Is Openness Inflationary? Imperfect Competition and Monetary Market Power*

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Abstract

Much empirical work has documented a negative correlation between different measures of globalization or openness and inflation levels across countries and across time. However, here is much less work exploring this relationship through structural international models based on explicit microeconomic foundations. This paper asks the question of how the degree of openness of an economy affects the equilibrium inflation level in a simple two-country OLG model with imperfect competition in which the monetary authority in each country chooses the money growth rate to maximize the welfare of its citizens. I find that a higher degree of openness in a country is associated with a higher equilibrium inflation rate. This result is driven by the fact that the monetary authority enjoys a degree of monopoly power in international markets as Foreign consumers have some degree of inelasticity in their demand for goods produced in the Home country. The decision of the monetary authority is then to balance the benefits of increased money growth that come from the open economy setting with the well-known consumption tax costs of inflation. In addition, I find that the level of imperfect competition among producers within a country is a perfect substitute for the international market power of the monetary authority in extracting the monopoly rents available in this international structure.

JEL codes: E52, E61, F41, F42

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1 Introduction

The goal of this paper is to try to answer the question of whether or not increased openness to international markets is inflationary using a structural international general equilibrium model derived from microeconomic foundations. This question has been the subject of a large body of research beginning as early as 1962 and continuing to the present. Most of these papers have been empirical in focus and provide strong evidence of a negative relationship between openness and inflation. However, much less work exists that structurally models this relationship beginning with the behavior of individual agents. This paper is intended as an attempt at furthering the theoretical understanding of some specific channels through which economic openness may influence a country's inflation rate.

A major focus of this work and one of its main innovations is how the level of imperfect competition, both within a country and between countries, affects the relationship between openness and inflation. Some of the literature has begun to assess the relationship between imperfect competition and inflation in open economies, but this is the first paper that specifically models how imperfect competition affects the relationship between openness and inflation.

To address this question, I use a two-country overlapping generations (OLG) model in which agents are born in each country in each period and live for only two periods. The agents use their labor to produce a differentiated good in the first period of their lives for which they enjoy a degree of market power, and they sell the good to consumers from both countries in exchange for the producers' country's currency. A monetary authority in each country chooses and commits to a money growth rate at the beginning of time and implements that policy through non-proportional transfers to the consumers of its own country in each period so as to maximize the welfare of its citizens.

The results derived from this model run counter to most of the findings from the literature addressing the question of the effect of openness on inflation. I find that an increased level of openness actually increases the steady-state equilibrium inflation rate in a country. In a closed economy and in environments in which money is not neutral, increased money growth generates inflation which provides a leisure subsidy and levies a consumption tax. However, in the environment laid out in this paper, increased openness to international trade opens up two new channels through which a country's inflation rate benefits its citizens.

First, increased openness reduces the burden of the inflation tax borne by the citizens of the inflating country in that they spend a larger portion of their currency holdings on Foreign goods. Second, inflation causes the terms of trade to appreciate in favor of the Home country. That is, the price of exports increase in relation to the price of imports. These two benefits working together result in a country's real wage increasing in response to higher Home inflation levels. These benefits are generated by a degree of market power enjoyed by each monetary authority in the international markets due to the assumption that consumers in each country prefer some consumption combination of its own country's production, the assumption that a consumer's expenditure share on the other country's goods is inelastic to some degree, and the institutional assumption that consumers must hold both countries' currencies in order to consume both of their goods. The problem of the monetary authority then becomes choosing the money growth rate and the associated rate of inflation so as to balance the resulting consumption tax with the real-wage benefit (consumption tax burden shift plus terms of trade appreciation).

In addition, I find that the level of imperfect competition among the producers within a country acts as a perfect substitute for the market power enjoyed by a country's monetary authority. That is, an increased level of imperfect competition among producers within a country reduces the benefits that result from inflation generated by that country's monetary authority. Put differently, a fixed amount of international monopoly rents are available to the citizens of each country given the structure of the model, and whatever percentage of those rents are not obtained through the pricing behavior of each country's producers is obtained by that country's monetary authority changing the inflation rate through the money growth rate. So this model predicts a negative relation between a country's inflation rate and the level of imperfect competition, given the degree of openness to international markets. Thus, the channel through which openness affects inflation is the international market power that a country enjoys.

The structure of the paper is as follows. In Section 2, I survey the literature that has addressed the question of openness and inflation. Section 3 presents the model and its equilibrium properties. Section 4 presents the key results from the model, and Section 5 concludes.

2 Literature

This paper's place in the international monetary literature is to provide a simple attempt at a micro-founded structural model of openness, inflation, and imperfect competition in order to try to match the relationship between openness and inflation documented in the empirical literature. The oldest branch of the theoretical literature uses a structural model that is an international version of Barro and Gordon (1983) which predicts that, other things equal, openness leads to a lower inflation rate. But a newer branch of the literature can be loosely grouped under the rubric of "new open economy macroeconomics" (NOEM) models, and predicts that, other things equal, more openness leads to a higher inflation rate. The modeling approach I use in this paper will follow the NOEM style for reasons that I will detail below.

One of the earliest empirical papers addressing the question of the relationship between openness and inflation, although somewhat indirectly, is Triffin and Grudel (1962). Using data from six European countries during the 1950s, they provide evidence that inflationary pressures are more correlated, and thus less independent, across countries that are more integrated. They propose that, among countries that are more open and integrated, inflation generated by a monetary authority can have more of an effect on the balance of payments, than on inflation. However, they only mention in passing that this balance of payments effect can only be short-term, and they assume no optimizing behavior by the government, consumers, or firms.

In his famous AEA Presidential address, Friedman (1968) proposed that monetary policy should target inflation or money growth rates. But he also added indirectly that exchange rate targeting could be more desirable if imports were a bigger share of GDP, thus implying a potential connection between openness and inflation.[1]

The first structural model directly addressing the question of openness and inflation is Rogoff (1985). His approach is to extend the Barro and Gordon (1983) framework to a two-country Mundell-Fleming model. As in Barro and Gordon, a labor market friction causes the optimal time-consistent policy of the monetary authority to be increased inflation in order to raise the level of employment. However, in Rogoff's international version, the increased inflation has an extra cost in that optimal employment is a function of the real exchange rate and that the real exchange rate depreciates with higher inflation. Thus the optimal time-consistent inflation rate chosen by a monetary authority is lower as the deteriorating effect on the exchange rate increases. More openness leads to a lower equilibrium inflation rate in this time consistent environment.

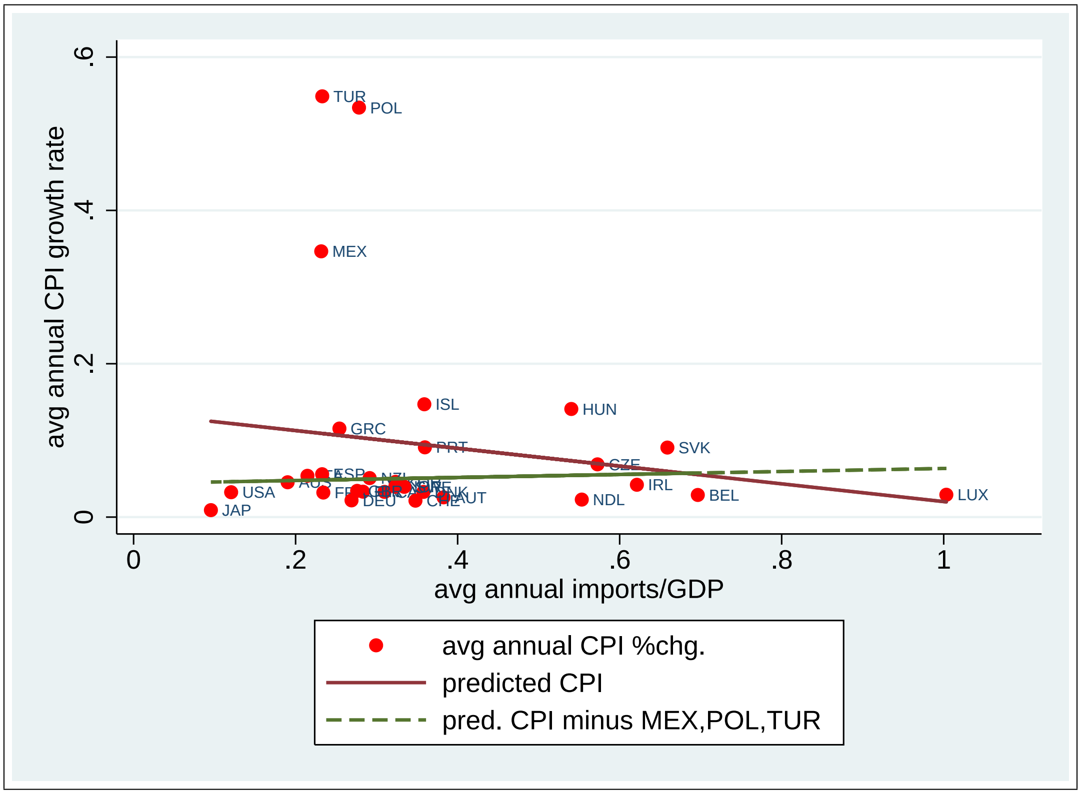
The empirical literature testing the effect of openness on inflation primarily cites the model and conclusions of Rogoff (1985). The most important empirical paper that addresses this question is Romer (1993). He cites the Rogoff prediction that, in his time-consistent environment, more openness should lead to lower inflation. In his regressions, Romer controls for endogeneity, includes political controls, development level controls, regional controls, and uses many different samples of countries over the post-Bretton Woods period from 1973 to the early 1990s. Romer's empirical findings lend support to the theoretical results of Rogoff (1985) in that he finds robust evidence of a negative relationship between openness and inflation and that the negative relationship becomes weaker in countries with less independent central banks and more political instability.[2]

Figure 1 shows a scatterplot of the average annual import share and average

[1]On page 15, Friedman makes the contrapositive statement that, with only 5 percent of U.S. resources devoted to international trade in 1967, "it would be better to let the market, through floating exchange rates, adjust to world conditions."

[2]A number of empirical papers follow up on Romer (1993), and most of them either confirm his finding of a negative relationship or find that the relationship is not statistically significant. Wynne and Kersting (2007) provide a good survey of the empirical literature as well as some of their own analyses.

Figure 1: Import share vs. CPI for 30 OECD countries: annual avg. for 1982 to 2005



annual CPI growth rate for the 30 OECD countries over the period from 1982 to 2005. This picture is similar to figures in Romer (1993) and Wynne and Kersting (2007) and is common to this empirical literature. However, the conclusions to take from Figure 1 are not obvious. A slight negative correlation exists between import share and inflation over the sample period (solid line), but that negative relationship becomes positive when I drop the three high-inflation outliers of Mexico, Poland, and Turkey (dotted line). Restricting the sample to the G7 countries produces a positive correlation nearly identical to that of the whole sample minus Mexico, Poland, and Turkey. When the sample period is shortened to more recent periods, the negative relationship with all the countries and the positive relationship without the high-inflation countries both diminish to the point where the two predicted value lines for the year 2005 are nearly indistinguishable and are both slightly positive. However, none of the slopes in any specification is significantly different from zero.[3]

[3]Other authors obtain statistically significant correlations by expanding the sample of countries

The "natural rate" approach of the model used in Rogoff (1985) has been criticized on a number of dimensions. Azariadis (1981) questions the Phillips curve assumption of dropping all but the first two terms of a Taylor series expansion of the aggregate supply equation around the expected logarithm of price. Also, the natural rate models on which so much of empirical monetary policy today is based, assume that the welfare of a representative agent is a quadratic loss function in the deviation of output from its natural rate and in the deviation of inflation from expected inflation. This type of disutility function is a step removed from maximization of individual's utility functions that is standard in most micro-founded macroeconomics.

Another key characteristic implicit in the Rogoff model is that the labor market friction that causes the optimal employment level to be higher than the level desired by the suppliers of labor could be caused by some form of monopoly power on the part of these suppliers such as a labor union. Thus, the monetary authority uses the inflationary money injection to induce higher demand which causes the owners of labor to supply more. Intuitively, the more open an economy is, the less market power the monopolistic labor suppliers enjoy and the less incentive a monetary authority has to inflate.

An alternative to the natural rate international models mentioned above for addressing the relationship between openness and inflation are some more recent works related to the NOEM models. A number of optimal monetary policy papers have come out in recently in this vein of the literature that address optimal inflation levels generated by a monetary authority in general equilibrium multi-country environments in which firms and consumers are acting optimally and the monetary authority is maximizing the utility of its citizens.

Cooley and Quadrini (2003) and Cooper and Kempf (2003) both use models in which the production market is perfectly competitive to answer the questions of whether and when countries gain from cooperating in currency unions. An attempt to categorize them might place them close to the "new open economy macroeconomic" (NOEM) models literature, except that they both feature perfectly competitive mar-

and by controlling for other variables to isolate the effect of openness on inflation.

kets. Cooley and Quadrini (2003) employ a model in which Home final goods producers use inputs from both Home and Foreign intermediate goods producers, and then consumers in each country only consume the final goods produced in their own country. Monetary policy in Cooley and Quadrini is a country's monetary authority choosing a nominal interest rate on a bond that final goods producers in both countries purchase to finance the intermediate inputs from both countries.

Cooper and Kempf (2003) use a technique that is conceptually different but structurally similar in which consumers only care about final goods consumption and that the final goods consumption is an aggregation of a Home produced good and a Foreign produced good in an OLG setting. Monetary policy in Cooper and Kempf is a country's monetary authority choosing a currency growth rate. They impose two cash-in-advance constraints such that a Home consumer must pay for Home produced goods in his own currency and he must pay for Foreign produced goods in the Foreign currency.

In both papers, the standard consumption tax of inflation results. But, in the two-country setting with international trade, both papers find that the a degree of monetary market power—derived in Cooley and Quadrini (2003) from some degree of inelasticity in the demand for both Home and Foreign intermediate goods and derived in Cooper and Kempf (2003) from a degree of inelasticity in the demand for Home and Foreign final goods—generates an added benefit to inflation of being able to appreciate the terms of trade in favor of the inflating country. Cooley and Quadrini find that this inflationary bias in open economies is actually larger if the monetary authority cannot commit to a policy.

In a more traditional NOEM paper, Arseneau (2007) uses a model very similar to Corsetti and Pesenti (2001) that adds imperfectly competitive firms in each country. In an environment in which the monetary authority can commit to policy, Arseneau confirms the inflationary bias of monetary policy result from Cooley and Quadrini (2003) and Cooper and Kempf (2003). In addition, Arseneau shows that the degree of imperfect competition can dampen the inflationary bias and can even fully offset it such that the equilibrium inflation rate is zero or negative. However, none of the four

NOEM papers discussed in the previous paragraphs attempts to answer the question of how the degree of openness in a country affects its equilibrium inflation level when monetary policy is set optimally.

Analogous to the interpretation of the mechanism of the "natural rate" models but with an opposite result, the following interpretation applies to these NOEM models with imperfectly competitive firms. In a closed economy, the monetary authority has an incentive to deflate in order to offset the inefficiently high price and low output level caused by the market power held by firms. However, this degree of market power is eroded as the country becomes more open and the elasticity of substitution between Home and Foreign consumption is less than the elasticity of substitution among the goods of a given country.

The goal of this paper is to use the micro-founded two-country model with optimal monetary policy in this paper that borrows heavily from the NOEM literature, instead of following the Mundell-Fleming "natural rate" approach, to try and match the relationship borne out in the data that openness is negatively correlated with inflation levels.

3 Model

Following Cooper and Kempf (2003), I use a two-country OLG general equilibrium framework with an independent monetary authority in each country that maximizes the welfare of its own citizens. In addition, similar to Arseneau (2007), the model includes imperfectly competitive producers in each country. The model includes no stochastic shocks and agents enjoy perfect foresight.

I will call the two countries Home and Foreign, which are not relative terms but are the names of the actual countries. Most of the exposition in this section will focus on the problem of Home agents and the Home monetary authority, but the Foreign problem is symmetric in almost every dimension. However, I will allow Home and Foreign countries to differ in their respective levels of openness to international trade in a way that I will specify. Within a country, I assume the equilibrium is symmetric, so I will drop any subscripting of individuals.

This stylized economy is made up of two countries, each of which has a monetary authority, producers, and consumers. The overlapping generations of agents live for two periods. In the first period of their lives, they produce differentiated goods in a monopolistically competitive environment and sell the goods to both Home and Foreign consumers in exchange for the producer's Home-currency. The producers then choose how much of their Home currency to hold and how much of the Foreign currency to hold given that they will use a portfolio of each respective currency to consume Home and Foreign goods in the second period of their lives.

The role of each country's monetary authority is to maximize the lifetime welfare of the representative agent in the Home country by giving a non-proportional transfer of Home currency to the consumers of its own country in each period. Money is held in this economy because it is the only store of value, and changes in the money supply are not neutral due to the transfers being non-proportional.[4] The two cash-in-advance constraints and consumer preferences generate demand for both currencies by a given consumer.

3.1 Money

The objective of the monetary authority in each country, which will be made more explicit in Section 3.4, is to choose a fixed (gross) money growth rate $x_t = x$ or $x_t^* = x^*$ at the beginning of time in such a way as to maximize the welfare of its own citizens. I assume here that the monetary authority is committed to its money growth rate and cannot deviate once it has chosen its money growth path.[5]

Let M_t and M_t^* be the aggregate supply of Home currency and Foreign currency, respectively, in period t. I normalize the initial supply of Home and Foreign currency

[4]See Azariadis (1981) for a proof that non-proportional monetary transfers are not neutral, even in a perfect foresight economy.

[5]The reason to avoid discretionary monetary policy in this paper is due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps. King and Wolman (2004) is a good reference on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). Also, see Chatterjee, Cooper, and Ravikumar (1993).

to 1 and divide it equally among the period-1 consumers at the beginning of the period.

$$M_0 = M_0^* = 1 \quad \text{and} \quad m_0^h = m_0^f = m_0^{h*} = m_0^{f*} = \frac{1}{2} \tag{1}$$

where m_0^h and m_0^{f*} are the individual holdings of Home currency by Home consumers and Foreign currency by Foreign consumers, respectively, at the beginning of period 1. Each country's monetary authority makes non-proportional transfers of $(x-1)M_{t-1}^h$ to each Home consumer in period t and $(x^*-1)M_{t-1}^f$ to each Foreign consumer where x and x^* represent the respective constant gross money growth rates of each country. So aggregate supply of currency in each country obeys the following laws of motion.

$$M_{t+1} = xM_t \tag{2}$$

$$M_{t+1}^* = x^* M_t^* \tag{3}$$

This implies that the following relationships for τ_{t+1} and τ_{t+1}^* represent the non-proportional transfer to each Home consumer and to each Foreign consumer by their respective monetary authorities.

$$\tau_{t+1} = (x-1)\, M_t \tag{4}$$

$$\tau_{t+1}^* = (x^*-1)\, M_{t^*} \tag{5}$$

At the end of the first period of their lives, producers make a portfolio decision of how much of each type of currency to hold. They have just received either $p_t(z)n_t(z)$ in Home currency or $p_t(z^*)n_t(z^*)$ in Foreign currency from the sale of their differentiated goods. Now, before the end of the first period of life, producers in each country exchange some of their Home currency balances from sales revenues for Foreign currency balances at the exchange rate e_t as shown in the budget constraint equation (27). Let m_t^h and m_t^f represent each Home producer's portfolio choice between Home and Foreign currency, respectively, in period t. Because the monetary authority of each country only transfers currency to its own consumers, the laws of motion for individual currency balances are the following:

$$m_{t+1}^{h} = m_{t}^{h} + \tau_{t+1} \tag{6}$$

$$m_{t+1}^{f} = m_{t}^{f} \tag{7}$$

$$m_{t+1}^{f*} = m_{t}^{f*} + \tau_{t+1}^{*} \tag{8}$$

$$m_{t+1}^{h*} = m_{t}^{h*} \tag{9}$$

Because the equilibrium currency holdings within each country are symmetric, then $m_t^h, m_t^f, m_t^{f*}, m_t^{h*}$ represent the aggregate amounts of each currency $(M_t^h, M_t^f, M_t^{h*}, M_t^{f*})$ held in each country in each period.

3.2 Individuals

A unit measure of agents are born in each period in both the Home country (indexed by z) and the Foreign country (indexed by z^*). In the first period of their lives, individuals can either enjoy leisure l_t or provide labor $n_t(z)$ subject to their endowment of one unit of time.

$$l_t + n_t(z) = 1 \quad \forall t, z \tag{10}$$

Each individual also has access to a technology through which she can convert labor hours into a differentiated good indexed by the individual z for each Home producer and z^* for each Foreign producer.

$$y_t(z) = f\left(n_t(z)\right) \quad \forall t, z \quad \text{where} \quad f\left(n_t(z)\right) = n_t(z) \tag{11}$$

I follow an international variation of the model of monopolistic competition of Dixit and Stiglitz (1977).[6] I assume that individuals only care about aggregate consumption, where each Home consumer's aggregate consumption of Home produced

[6] Good example of this type of international monetary model with monopolistic competition are Corsetti and Pesenti (2001) and Arseneau (2007). The Technical Appendix T-2 has a full derivation of the demand and price functions shown below that result from this monopolistic competition structure and is available upon request.

goods C^h_{t+1} and aggregate Home consumption of Foreign produced goods C^f_{t+1} are defined as:

$$C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{12}$$

$$C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{13}$$

where $\varepsilon \geq 0$ represents the elasticity of substitution among all the differentiated goods in country either the Home country or the Foreign country. Symmetric to the Home consumer, each Foreign consumer's aggregate consumption of Foreign produced goods C^{f*}_{t+1} and aggregate Foreign consumption of Home produced goods C^{h*}_{t+1} is defined as:

$$C^{f*}_{t+1} \equiv \left(\int_0^1 c^*_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{14}$$

$$C^{h*}_{t+1} \equiv \left(\int_0^1 c^*_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{15}$$

Total consumption by each Home and Foreign consumer is further aggregated over her aggregate consumption of Home produced goods and aggregate consumption of Foreign produced goods using an analogous CES aggregator of total consumption:

$$C_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}} \left(C^h_{t+1} \right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(C^f_{t+1} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_h \in \left[0, \frac{1}{2} \right] \tag{16}$$

$$C^*_{t+1} \equiv \left[(1-\theta_f)^{\frac{1}{\rho}} \left(C^{f*}_{t+1} \right)^{\frac{\rho-1}{\rho}} + \theta_f^{\frac{1}{\rho}} \left(C^{h*}_{t+1} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_f \in \left[0, \frac{1}{2} \right] \tag{17}$$

where θ_h and θ_f are the Home bias parameters for the Home country and the Foreign country, respectively, and $\rho \geq 0$ is the elasticity of substitution between a unit of Home consumption and a unit of Foreign consumption.[7] For the analysis in this paper, I will assume that the elasticity of substitution between a unit of Home aggregate consumption and a unit of Foreign aggregate consumption is equal to 1 ($\rho = 1$) which

[7] To be more specific, the parameter space of the two Home bias parameters is $\{\theta_h, \theta_f\} = \{0, 0\}$ or $\{\theta_h, \theta_f\} = \{(0, 0.5], (0, 0.5]\}$.

results in the following Cobb-Douglas aggregation at this level.

$$C_{t+1} \equiv \left(C^h_{t+1}\right)^{1-\theta_h} \left(C^f_{t+1}\right)^{\theta_h} \quad \text{for} \quad \theta_h \in \left[0, \frac{1}{2}\right] \tag{18}$$

$$C^*_{t+1} \equiv \left(C^{f*}_{t+1}\right)^{1-\theta_f} \left(C^{h*}_{t+1}\right)^{\theta_f} \quad \text{for} \quad \theta_f \in \left[0, \frac{1}{2}\right] \tag{19}$$

The Home and Foreign countries are symmetric in every dimension except for the Home bias parameters. This assumption seems to fit the empirical evidence that import shares differ across countries, as shown in Figure 1.

This functional form assumption is strong because it forces individuals to spend a fixed portion of their earnings on consumption of Home-produced goods. However, the general case results in a degenerate equilibrium. The Cobb-Douglas aggregator has the desirable pedagogical property of allowing for analytical solutions.[8] Whatever the specification, a key intuitive relationship is that the elasticity of substitution among differentiated goods within a country is different from and greater than the elasticity of substitution between aggregate and Foreign consumption $\varepsilon > \rho$. This is the main source of the international market power a monetary authority enjoys when a country becomes more open.

The following individual demand and price relationships result from the problem of an agent minimizing her expenditure given a certain level of aggregate consumption.[9]

$$\begin{aligned} c_{t+1}(z) &= \left(\frac{p_{t+1}(z)}{P^h_{t+1}}\right)^{-\varepsilon} C^h_{t+1} \quad \forall\, t, z \\ c_{t+1}(z^*) &= \left(\frac{p_{t+1}(z^*)}{P^f_{t+1}}\right)^{-\varepsilon} C^f_{t+1} \quad \forall\, t, z^* \end{aligned} \tag{20}$$

[8]Technical Appendix T-4 provides some of the results of what happens when using the general form of the CES aggregator and is available upon request.

[9]The expenditure minimization problem is preferred to the utility maximization problem because the multiplier on the aggregate consumption constraint in the minimization problem has the interpretation of the aggregate price.

$$
\begin{aligned}
P^h_{t+1} &= \left(\int_0^1 p_{t+1}(z)^{1-\varepsilon}\, dz\right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t \\
P^f_{t+1} &= \left(\int_0^1 p_{t+1}(z^*)^{1-\varepsilon}\, dz^*\right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t
\end{aligned} \tag{21}
$$

$$
P_{t+1} = \frac{1}{(1-\theta_h)^{1-\theta_h}\theta_h^{\theta_h}} \left(P^h_{t+1}\right)^{1-\theta_h} \left(e_t P^f_{t+1}\right)^{\theta_h} \tag{22}
$$

where $p_{t+1}(z)$, P^h_{t+1}, and P_{t+1} are prices of individual consumption, aggregate country-specific consumption, and aggregate total consumption, respectively. Analogous to the derivation for the demand for individual differentiated goods z and z^* in (20), each Home consumer's demand for aggregate Home consumption and aggregate Foreign consumption, respectively, are the following:

$$
C^h_{t+1} = (1-\theta_h)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-1} C_{t+1} \tag{23}
$$

$$
C^f_{t+1} = \theta_h \left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-1} C_{t+1} \tag{24}
$$

These two equations simply imply that the expenditure on Home aggregate consumption and the expenditure on Foreign aggregate consumption are constant shares of total expenditure. Another way of putting this is to divide (23) by (24), which gives the following relationship that describes the relationship between total expenditures on Home consumption to total expenditure on Foreign consumption.

$$
\frac{P^h_{t+1} C^h_{t+1}}{e_t P^f_{t+1} C^f_{t+1}} = \frac{1-\theta_h}{\theta_h} \tag{25}
$$

The ratio of total Home consumption to total Foreign consumption is a constant. That is, θ_h represents the Home expenditure share on Foreign consumption or the import share. Equations (20) through (25) result directly from the Dixit-Stiglitz monopolistic competition structure and from the CES aggregation.[10]

Individuals seek to maximize lifetime utility derived from disutility of work in the

[10]The CES consumption aggregation in (12), (13), and (18) can also be interpreted as CES utility over Home and Foreign differentiated goods. Technical Appendix T-3 details the properties of the CES aggregator and is available upon request.

first period of life in order to sell a differentiated production good for own-country currency balances that are carried over to the second period of life in which the individual can spend those balances on consumption of both Home and Foreign goods. Because the monopolistically competitive producers can set the quantity demanded by choosing price in order to clear their goods, the consumer's problem is characterized by choosing how much to charge for her differentiated good $p_t(z)$ and then the portfolio decision of how much of her sales to keep in the form of Home currency m_t^h and how much to exchange for Foreign currency m_t^f.[11]

$$\begin{aligned} &\max_{m_t^h, m_t^f, p_t(z)} \quad u\left(C_{t+1}\right) - g\left(n_t(z)\right) \\ &\text{where} \quad u\left(C_{t+1}\right) = \frac{\left(C_{t+1}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{for} \quad \sigma > 0 \\ &\text{and} \quad g\left(n_t(z)\right) = \chi\left(n_t(z)\right)^{\xi} \quad \text{for} \quad \chi > 0 \quad \text{and} \quad \xi \geq 1 \end{aligned} \tag{26}$$

$$\text{s.t.} \quad p_t(z) n_t(z) = m_t^h + e_t m_t^f \tag{27}$$

$$P_{t+1}^h C_{t+1}^h = m_t^h + \tau_{t+1} \tag{28}$$

$$P_{t+1}^f C_{t+1}^f = m_t^f \tag{29}$$

where (27) is the budget constraint reflecting the portfolio decision and (28) and (29) are cash-in-advance constraints.

The two cash-in-advance constraints can be thought of as a simplification of one equilibrium outcome of a richer environment in which governments or monetary authorities strategically choose what currencies to accept for exchange that takes place within their borders. Matsuyama, Kiyotaki, and Matsui (1993) present a random matching search model of money after the flavor of Kiyotaki and Wright (1989) in which blocks of agents (countries) choose which currencies to accept for local and international transactions based on the likelihood of that currency being accepted in

[11] An implicit assumption in this setup is that the producer will meet demand, whatever it is. Thus the producer sets price $p_t(z)$ and then produces $n_t(z)$ to meet the resulting demand. Some other interesting cases arise in a model with shocks when producers are not required to meet demand.

future transactions. In one equilibrium, corresponding to the two cash-in-advance constraint environment of this paper, each block of agents (country) only accepts local currency for all local and international transactions.

Another equilibrium in the Matsuyama, Kiyotaki, and Matsui (1993) is the case in which vendors in both countries accept currency of both countries. This is analogous to the more standard approach in the NOEM literature as exemplified by Corsetti and Pesenti (2001). Their environment is one characterized by a single cash-in-advance constraint in which producers sell their goods in both countries and charge a price in terms of Home currency and a price in terms of Foreign currency. The exchange rate is then pinned down by an assumption of the law of one price.

The reason for choosing the two cash-in-advance constraints approach as shown in equations (28) and (29) instead of the more standard Corsetti and Pesenti (2001) method of one cash-in-advance constraint and the law of one price is that the method employed here gives rise to a portfolio decision. The law of one price is implicit in the two cash-in-advance constraint assumption because, by definition, vendors only accept one currency and therefore only charge one price. As will be in Section 3.3, the exchange rate here serves as a price that clears the currency exchange market rather than a mechanism for enforcing the law of one price. Furthermore, the currency portfolio decision is an interesting one that has not received much attention.[12] However, both the single CIA constraint with the law of one price method and the dual CIA constraints with currency exchange market clearing method deliver the same results for optimal monetary policy.

Using the individual demand equations represented by (20), I define the total demand $d_t(z)$ for differentiated Home good z as the sum of the individual Home and Foreign demands:[13]

$$d_t(z) \equiv c_t(z) + c_t^*(z) = \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h} \tag{30}$$

[12] Add some references here to the currency portfolio choice literature, such as Engel and Matsumoto (2006) and Evans and Lyons (2005).

[13] The derivation is given in Derivation 1 in Technical Appendix T-1 and is available upon request.

I assume that producers always choose price to maximize utility given their knowledge of total demand $d_t(z)$ and then meet the demand.

$$n_t(z) = d_t(z) = \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h} \tag{31}$$

Using the cash-in-advance constraints (28) and (29), the money laws of motion (6) and (7), and the expressions for the non-proportional transfer in terms of the Home money growth rate (4), country-specific aggregate consumptions can be expressed in the following way:

$$C_{t+1}^h = \frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h} \tag{32}$$

$$C_{t+1}^f = \frac{m_t^f}{P_{t+1}^f} \tag{33}$$

The expression for Home aggregate total consumption is then:

$$C_{t+1} = \left(\frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h} \tag{34}$$

Using the portfolio constraint in (27) to substitute out either $m_{i,t}^h$ or $m_{i,t}^f$ and substituting in the expression for labor supply from (31), the maximization problem then becomes

$$\max_{m_t^f, p_t(z)} \frac{\left[\left(\left[\frac{p_t(z)}{P_t^h}\right]^{1-\varepsilon} \frac{xM_{t-1}}{P_{t+1}^h} - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} \ldots \\ -\chi \left[\left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h}\right]^{\xi} \tag{35}$$

The first order conditions with respect to m_t^f and $p_t(z)$, respectively, are:

$$\frac{P_{t+1}^h C_{t+1}^h}{e_t P_{t+1}^f C_{t+1}^f} = \frac{1-\theta_h}{\theta_h} \tag{36}$$

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P^h_{t+1}}\left(C^h_{t+1}\right)^{(1-\theta_h)(1-\sigma)-1}\left(C^f_{t+1}\right)^{\theta_h(1-\sigma)}=\chi\xi\left(n_t(z)\right)^{\xi-1} \tag{37}$$

where equation (36) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (37) equates the marginal benefit of raising price to its marginal cost in terms of reduced demand, increased utility of leisure, and the change in income in the next period of life. Because each agent within a country is identical, other than for a differentiated production good, the resulting individual equilibrium price $p_t(z)$ and the amount of total revenues held in Foreign currency m^f_t will be symmetric across individuals in a given country.

Notice that the first order condition for m^f_t in (36) is equivalent to the condition (25) that results from the two first order conditions in the imperfect competition expenditure minimization problem. This is because the optimal choice of m^f_t in period t is equivalent to the optimal choice of C^h_{t+1} and C^f_{t+1} in period $t+1$. These two decisions are equivalent and take the labor or pricing decision as given.

3.3 Market clearing conditions

This economy has three markets that must clear—the goods market, the money market, and the exchange market. The following paragraphs describe each market and the respective market clearing condition.

Goods Market. Both Home and Foreign consumers demand goods from both countries. Producers meet that demand by construction in this model. Let $n_t(z)$ represent the amount of production by each Home producer of differentiated good z. Goods market clearing requires that production equal the sum of all the Home demands $c_t(z)$ and Foreign demands $c^*_t(z)$ for differentiated good z.

$$n_t(z)=d_t(z)=c_t(z)+c^*_t(z)\quad\forall\, t,z \tag{38}$$

$$n_t(z^*)=d_t(z^*)=c_t(z^*)+c^*_t(z^*)\quad\forall\, t,z^* \tag{39}$$

where the the right-hand side of each equation is characterized by equation (30) and its Foreign country analogue. This market clearing condition is actually assumed in the individual maximization stage as shown in (31).

Money Market. Money market clearing simply requires that money supply equal money demand at the time that goods are purchased.

$$M_t = m_t^h + m_t^{h*} \quad \forall\, t \tag{40}$$

$$M_t^* = m_t^f + m_t^{f*} \quad \forall\, t \tag{41}$$

where M_t and M_t^* are the Home and Foreign aggregate money supplies, respectively, at time t.

Currency Exchange Market. After trade has taken place in the goods market, period-t producers go to the currency market and make a portfolio decision of how much of each currency to hold. The exchange rate e_t is the price that equates the amount of Foreign currency purchased with Home currency by Home producers with the amount of Home currency purchased by Foreign producers with Foreign currency.

$$e_t m_t^f = m_t^{h*} \quad \forall t \tag{42}$$

It is important to note that the exchange rate here is not pinned down by the assumption of the law of one price as in models with a single cash-in-advance constraint, such as Corsetti and Pesenti (2001) and Arseneau (2007). Here, the exchange rate is a price that clears the currency exchange market in period-t. Because of the two cash-in-advance constraints, the law of one price holds by definition. Using the cash-in-advance constraint (29) and its Foreign country analogue, it can be shown that exchange rate market clearing implies that the nominal value of imports equals the nominal value of exports.

$$e_t P_{t+1}^f C_{t+1}^f = P_{t+1}^h C_{t+1}^{h*} \quad \forall t \tag{43}$$

3.4 Equilibrium

This perfect foresight overlapping generations model has one unique nonautarkic steady state equilibrium. As noted in Section 3.1, I avoid discretionary monetary policy in this paper due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps.[14] Table 1 shows the conditions that must hold in a perfect foresight equilibrium. I define the steady state international equilibrium given both Home and Foreign monetary policy (x, x^*) as follows:

Definition 1 (**Steady State International Equilibrium given** x **and** x^*)**.** A steady state international equilibrium, given Home and Foreign monetary policy (x, x^*) is the set of Home consumption of both Home and Foreign aggregate goods C^h and C^f, Home production n, Home portfolio holdings of both Home and Foreign currency m^h and m^f (or rather, as a percentage of initial Home holdings, ϕ and $1-\phi$), the Foreign counterparts $(C^{h*}, C^{f*}, n^*, m^{h*}, m^{f*})$, individual Home and Foreign prices $p_t(z)$ and $p_t(z^*)$, and exchange rate e_t such that:

- **Individual optimization**: Home and Foreign agents choose the price level of their differentiated good as well as their currency portfolio holdings in order to maximize lifetime utility in (26) and its Foreign counterpart subject to a budget constraint (27) and two cash-in-advance constraints (28) and (29).

- **Market Clearing** The goods markets (38) and (39), money markets (40) and (41), and currency exchange market (42) all clear.

Following Cooper and Kempf (2003), let ϕ_t represent the share of revenues $p_t(z)n_t(z)$ kept in the form of Home currency in period t, and let $1-\phi_t$ be the share of revenues exchanged for Foreign currency as characterized in the portfolio budget constraint

[14] King and Wolman (2004) is a good current reference on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). See also Chatterjee, Cooper, and Ravikumar (1993).

Table 1: **Equilibrium conditions given x and x^***

	Home country	Foreign country
(36)	$\frac{P^h_{t+1}C^h_{t+1}}{e_t P^f_{t+1}C^f_{t+1}} = \frac{1-\theta_h}{\theta_h}$	$\frac{e_t P^f_{t+1}C^{f*}_{t+1}}{P^h_{t+1}C^{h*}_{t+1}} = \frac{1-\theta_f}{\theta_f}$
(37)	$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P^h_{t+1}}\frac{\left(C^h_{t+1}\right)^{(1-\theta_h)(1-\sigma)-1}}{\left(C^f_{t+1}\right)^{-\theta_h(1-\sigma)}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z^*)}{P^f_{t+1}}\frac{\left(C^{f*}_{t+1}\right)^{(1-\theta_f)(1-\sigma)-1}}{\left(C^{h*}_{t+1}\right)^{-\theta_f(1-\sigma)}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(31)	$n_t(z) = \left(\frac{p_t(z)}{P^h_t}\right)^{-\varepsilon}\frac{xM_{t-1}}{P^h_t}$	$n_t(z^*) = \left(\frac{p_t(z^*)}{P^f_t}\right)^{-\varepsilon}\frac{x^*M^*_{t-1}}{P^f_t}$
(27)	$p_t(z)n_t(z) = m^h_t + e_t m^f_t$	$p_t(z^*)n_t(z^*) = m^{f*}_t + \frac{m^{h*}_t}{e_t}$
(32)	$C^h_{t+1} = \frac{m^h_t + (x-1)xM_{t-1}}{P^h_{t+1}}$	$C^{f*}_{t+1} = \frac{m^{f*}_t + (x^*-1)x^*M^*_{t-1}}{P^f_{t+1}}$
(33)	$C^f_{t+1} = \frac{m^f_t}{P^f_{t+1}}$	$C^{h*}_{t+1} = \frac{m^{h*}_t}{P^h_{t+1}}$
(18)	$C_{t+1} = \left(C^h_{t+1}\right)^{1-\theta_h}\left(C^f_{t+1}\right)^{\theta_h}$	$C^*_{t+1} = \left(C^{f*}_{t+1}\right)^{1-\theta_f}\left(C^{h*}_{t+1}\right)^{\theta_f}$
	Market clearing conditions	
(38)	$n_t(z) = c_t(z) + c^*_t(z)$	
(39)	$n_t(z^*) = c_t(z^*) + c^*_t(z^*)$	
(40)	$M_t = m^h_t + m^{h*}_t$	
(41)	$M^*_t = m^f_t + m^{f*}_t$	
(42)	$e_t m^f_t = m^{h*}_t$	

(27). Then the following expressions hold.

$$p_t(z)n_t(z) - m^h_t = e_t m^f_t = (1-\phi_t)\,M_t \tag{44}$$

$$p_t(z^*)n_t(z^*) - m^{f*}_t = \frac{m^{h*}_t}{e_t} = (1-\phi^*_t)\,M^*_t \tag{45}$$

$$m^h_t + \tau_{t+1} = (\phi_t + x - 1)\,M_t \tag{46}$$

$$m^{f*}_t + \tau^*_{t+1} = (\phi^*_t + x^* - 1)\,M^*_t \tag{47}$$

Plugging (44), (45), (46), and (47) into the first order condition (36) and its Foreign country analogue, the unique nonautarkic steady state equilibrium share of currency from sales held for own-country consumption is given by:

$$\phi = 1 - \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{48}$$

$$1 - \phi = \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{49}$$

$$\phi^* = 1 - \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{50}$$

$$1 - \phi^* = \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{51}$$

From the aggregate money laws of motion in (2) and (3) and from the money market clearing conditions in (40) and (41), it is clear that the non-autarkic steady state equilibrium country-specific consumption inflation rates are:

$$\frac{P^h_{t+1}}{P^h_t} = x \tag{52}$$

$$\frac{P^f_{t+1}}{P^f_t} = x^* \tag{53}$$

Furthermore, using the definition of the Home country CPI level P_{t+1} from (22) and its Foreign country analogue, the expressions for the share Home country revenues traded for Foreign currency balances (49) and the share of Foreign country revenues traded for Home currency balances (51), and the currency exchange market clearing condition (42), the Home country CPI growth rate and the Foreign country CPI growth rates can be shown to be equal to their respective countries' money growth rates.[15]

$$\frac{P_{t+1}}{P_t} = x \tag{54}$$

$$\frac{P^*_{t+1}}{P^*_t} = x^* \tag{55}$$

Using (48), (49), (50), and (51), as well as the equilibrium inflation rates from (52) and (53), equilibrium consumption can be derived in terms of steady state em-

[15]The derivation is given in Derivation 2 in Technical Appendix T-1 and is available upon request.

ployment from the cash-in-advance constraints as:

$$C^h = (1-\theta_h)n \tag{56}$$

$$C^f = \theta_f n^* \tag{57}$$

$$C^{f*} = (1-\theta_f)n^* \tag{58}$$

$$C^{h*} = \theta_h n \tag{59}$$

where the steady state employment levels n and n^* are characterized below in equations (62) and (63).

The expressions for the steady state international equilibrium employment is then found by solving the two equilibrium forms of the Home first order condition (37) and its Foreign analogue.

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x}\left[(1-\theta_h)n\right]^{(1-\theta_h)(1-\sigma)-1}\left[\theta_f n^*\right]^{\theta_h(1-\sigma)} = \chi\xi(n)^{\xi-1} \tag{60}$$

$$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x^*}\left[(1-\theta_f)n^*\right]^{(1-\theta_f)(1-\sigma)-1}\left[\theta_h n\right]^{\theta_f(1-\sigma)} = \chi\xi(n^*)^{\xi-1} \tag{61}$$

Solving (61) for n^* and plugging it into (60), and doing the symmetric operation for the Foreign country gives the expressions for Home and Foreign equilibrium labor supply:

$$n\left(x,x^*\right) = \Omega_H\left(x\right)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x^*\right)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{62}$$

$$n^*\left(x^*,x\right) = \Omega_F\left(x^*\right)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x\right)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{63}$$

where the symbols in (62) and (63) summarize the parameters of the model in the

Table 2: Properties of representative parameters

Symbol	Sign	$\frac{\partial(\cdot)}{\partial\theta_h}$	$\frac{\partial(\cdot)}{\partial\theta_f}$
Δ_h	$(-)$ always	$(+)$ when $\sigma > 1$	
Δ_f	$(-)$ always		$(+)$ when $\sigma > 1$
Σ_h	$(-)$ when $\sigma > 1$ and $\theta_h > 0$	$(-)$ when $\sigma > 1$	
Σ_f	$(-)$ when $\sigma > 1$ and $\theta_f > 0$		$(-)$ when $\sigma > 1$
Ω_h	$(+)$ when $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_f > 0$	$(+)$ when $\sigma > 1$ and $\theta_h > 0$
Ω_f	$(+)$ when $\theta_h > 0$	$(+)$ when $\sigma > 1$ and $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_h > 0$
$\Delta_h\Delta_f - \Sigma_h\Sigma_f$	$(+)$ always	$(-)$ when $\sigma > 1$	$(-)$ when $\sigma > 1$

Note: The results from this table are derived in Derivation 3 in Technical Appendix T-1 and are available upon request.

following way:

$$\Delta_h = (1-\theta_h)(1-\sigma) - \xi \tag{64}$$

$$\Delta_f = (1-\theta_f)(1-\sigma) - \xi \tag{65}$$

$$\Sigma_h = \theta_h(1-\sigma) \tag{66}$$

$$\Sigma_f = \theta_f(1-\sigma) \tag{67}$$

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \tag{68}$$

$$\Omega_f = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \tag{69}$$

$$\Omega_H = (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{70}$$

$$\Omega_F = (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{71}$$

The signs of these expressions and their derivatives with respect to the openness parameters θ_h and θ_f are given in Table 2. From the signs of the representative parameters, it is clear that steady state equilibrium Home employment n decreases in x always and increases in x^* when $\sigma > 1$.

Looking at the equation for Home labor supply in (62), the sign of Σ_h determines how Foreign monetary policy affects the real economy in the Home country.

$$\Sigma_h = \begin{cases} > 0 \quad \text{if} \quad \theta_h \in (0, 0.5] \quad \text{and} \quad \sigma \in (0,1) \\ = 0 \quad \text{if} \quad \theta_h = 0 \quad \text{or} \quad \sigma = 1 \\ < 0 \quad \text{if} \quad \theta \in (0, 0.5] \quad \text{and} \quad \sigma > 1 \end{cases} \tag{72}$$

The third case is the most common in which $\Sigma_h < 0$, implying that Foreign inflation causes an increase in the equilibrium level of Home production and, therefore, an increase in equilibrium consumption of the Home good by both Home and Foreign consumers.

If one were to make the strong assumption that the coefficient of relative risk aversion σ were less than one, the first case in (72) occurs in which Foreign inflation causes a decrease in the equilibrium level of Home production. Lastly, it is interesting to notice the cases in which Foreign monetary policy has no real effect on the Home country ($\Sigma = 0$). Obviously, when the economies do not trade with each other, $\theta_h = 0$, Foreign monetary policy will be neutral. But it is interesting to note that the case of log utility ($\sigma = 1$) also induces the real neutrality of Foreign monetary policy.

The monetary authority in each country seeks to maximize the lifetime utility of a representative agent in this economy by choosing Home monetary policy x given Foreign monetary policy x^*. Define $V(x, x^*)$ as the lifetime utility of a representative agent. The objective of the Home monetary authority is then:

$$\max_x V(x, x^*) = \max_x \frac{\left([(1-\theta_h) n(x, x^*)]^{1-\theta_h} [\theta_f n^*(x^*, x)]^{\theta_h} \right)^{1-\sigma} - 1}{1-\sigma} - \chi n(x, x^*)^{\xi} \tag{73}$$

Definition 2 (**Home Country Steady State Monetary Equilibrium**)**.** A Home country steady state monetary equilibrium is a function for the optimal Home money growth rate $\hat{x}(x^*)$ given the Foreign money growth rate such that:

- the individual steady state equilibrium conditions from Definition 1 hold for each country,
- the Home monetary authority chooses x to maximize the lifetime utility of the

representative agent of its country as in equation (73).

Definition 2 can be thought of as a monetary partial equilibrium in a world monetary environment because it implies a best response function for Home monetary policy that is a function of any level of Foreign monetary policy. Taking the derivative of (73), the resulting solution for optimal Home monetary policy is:[16]

$$\begin{aligned} \hat{x} &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h \Sigma_f} \\ &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_f)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_h)\xi} \end{aligned} \tag{74}$$

The analogous solution for the Foreign monetary authority is:

$$\begin{aligned} \hat{x}^* &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f \Sigma_h} \\ &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_h)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_f)\xi} \end{aligned} \tag{75}$$

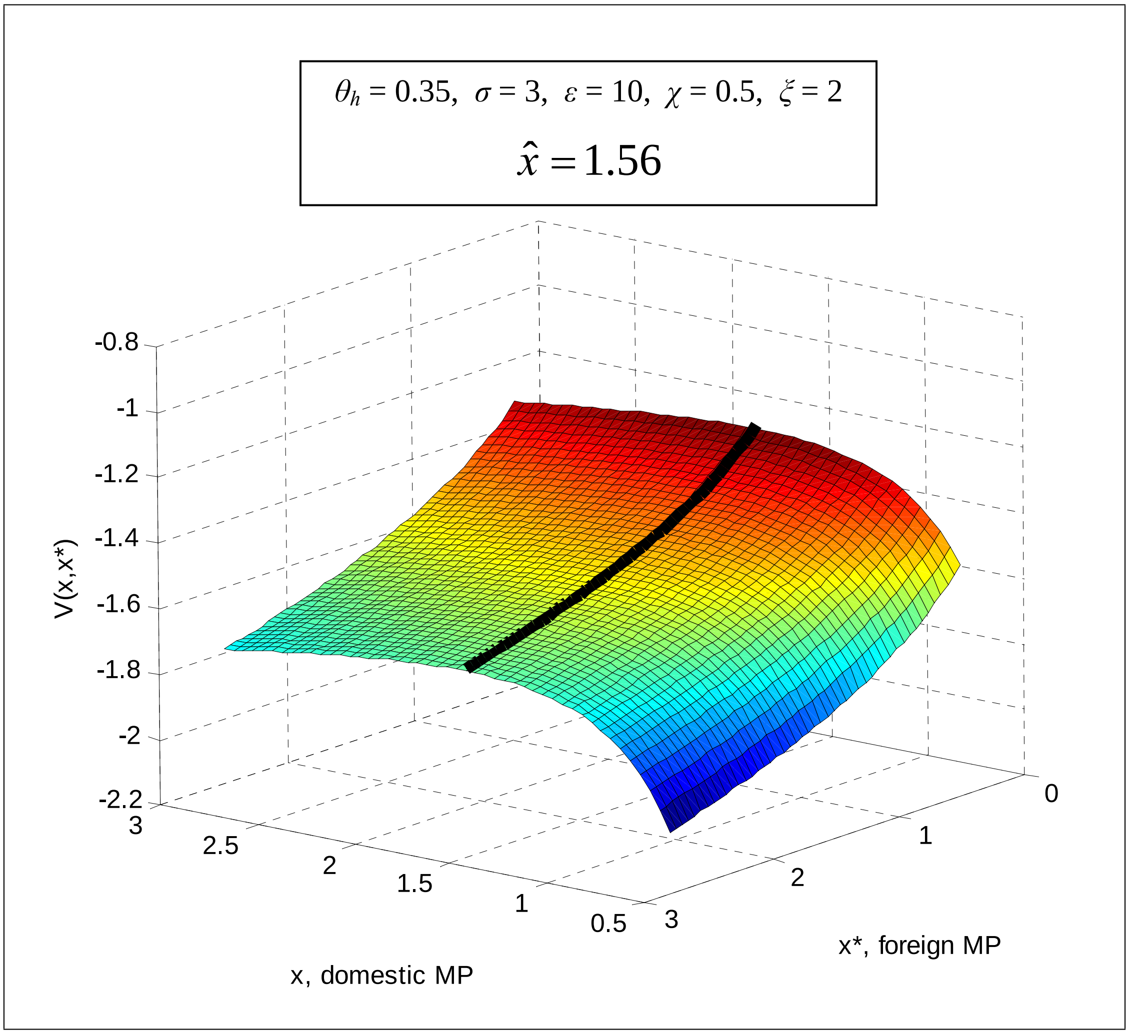
The first characteristic to note about the optimal Home monetary policy function in (74) is that it is independent of Foreign monetary policy x^*. That is, the optimal level of the Home money growth rate does not change with changes in the Foreign money growth rate and is a dominant strategy equilibrium.[17]

This dominant strategy equilibrium is shown in Figure 2 which plots the lifetime utility of a representative Home agent from (73) as a function of Home inflation x and Foreign inflation x^*. The parameters $(\theta, \sigma, \varepsilon, \chi, \xi)$ are simply chosen to reflect values estimated in the empirical literature in order to make a simple example. The dark line running across the top of Figure 2 represents the Home monetary policy best response function from (74). The optimal Home inflation level at the selected parameter values is a constant $\hat{x} = 1.56$, which is not overly high given that the duration of a period is a generation. Because each country's best response function for monetary policy is a dominant strategy equilibrium, the world Nash monetary

[16]See Derivation 4 in Technical Appendix T-1 which is available upon request.

[17]Derivation 4 in Technical Appendix T-1 details why $\hat{x}$ is independent of x^* and is available upon request.

Figure 2: Home lifetime utility V as a function of x and x^*



equilibrium is the same as the country partial monetary equilibrium.

3.5 Frictions

Before moving on to the results from Section 3.4, it is instructive to highlight the two frictions present in this model—money and imperfect competition—and their interplay with the level of openness. The two frictions are most easily isolated in a closed economy when the other friction is shut down. The inefficiencies caused by these two frictions are manifested in this setting as the "labor wedge" outlined in

Chari, Kehoe, and McGrattan (2007).[18] The efficient allocation is found by solving the planner's problem of maximizing the utility of the period-t old from consumption minus the disutility of labor of the period-t young in the closed economy case $\theta_h = 0$, subject to the resource constraint.

$$\begin{aligned} &\max_{C_t^h, n_t} u\left(C_t^h\right) - g\left(n_t\right) \\ &\text{s.t.} \quad C_t^h = n_t \end{aligned} \tag{76}$$

The planner's solution steady state equilibrium is the following:

$$(C_{ps}, n_{ps}): \quad u'(C^h) = g'(n) \tag{77}$$

The deviation from the planner's solution created by the presence of imperfect competition is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which the money growth rate is fixed at $x = 1$. The first order condition in (37) can be written as:

$$(C_{ic}, n_{ic}): \quad u'(C) = \Phi g'(n) \tag{78}$$

where $\Phi = \frac{\varepsilon}{\varepsilon - 1} \geq 1$ and (78) represents that marginal utility of consumption equals a markup over marginal cost. The monopoly power enjoyed by firms resulting from the imperfect substitutability ε of their goods allows producers to raise prices above the efficient level and lower output in order to maximize profits. Thus, $(C_{ic}, n_{ic}) \ll (C_{ps}, n_{ps})$, and (C_{ic}, n_{ic}) decreases as the degree imperfect competition Φ increases (as ε decreases).

In like manner, the deviation from the planner's solution created by the money growth rate is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which producers are perfectly competitive $\varepsilon = \infty$ $(\Phi = 1)$.

[18]However, a key point on which this paper differs from Chari, Kehoe, and McGrattan (2007) is that money is set optimally in this paper and not stochastic. But Chari, Kehoe, and McGrattan (2007) do conclude that the labor-wedge channel does explain much of the observed variation in business cycles.

The first order condition in (37) can now be written as:

$$(C_{mp}, n_{mp}): \quad \frac{1}{x}u'(C) = g'(n) \tag{79}$$

Equation (79) highlights the reason why expansionary monetary policy is thought of as an inflation tax. For higher money growth rates, the marginal benefit of an extra unit of labor decreases. Another way of looking at this problem is that the marginal productivity of labor is equal to 1, given the linear production technology. But the real wage in the closed economy is $\frac{1}{x}$. So for any money growth rate greater than 1, the real wage is less than the marginal productivity of labor. The result is that labor supplied is inefficiently low and $(C_{mp}, n_{mp}) \ll (C_{ps}, n_{ps})$ for all $x > 1$. Conversely, $(C_{mp}, n_{mp}) \gg (C_{ps}, n_{ps})$ for all $x < 1$. If the money growth rate is set optimally, the first best policy is $x = 1$ in this closed economy setting.

The interplay between openness, monetary policy and imperfect competition is seen when the closed economy frictions described preceding paragraphs are compared to their open economy counterparts. In the closed economy above, any money growth rate greater than the inverse of the markup gives a leisure subsidy that is dominated by an inflation tax, both of which are borne entirely by the agents of the closed country. However, when the two countries are open ($\theta_h, \theta_f > 0$), the inflation tax imposed by increasing the money growth rate is no longer borne solely by Home agents. Furthermore, increased money growth by the Home monetary authority actually increases the real wage through the terms of trade appreciation and increased preference weight on Foreign consumption. This added benefit of Home money growth is due to the international monopoly power of the Home monetary authority derived from the degree of inelastic demand fo imports by Foreign consumers.[19]

From the expressions for Home and Foreign employment in (62) and (63), the Home leisure subsidy results from the negative effect of an increase in x and the Foreign leisure tax results from the positive effect of an increase in x. The consumption tax of inflation can be seen by taking the derivative of equilibrium Home aggregate

[19]Recall that the constant expenditure share principle derives from the first order condition of the utility with the Cobb-Douglas aggregate consumption.

consumption C and Foreign aggregate consumption with respect to x.

$$C = [(1-\theta_h)n]^{1-\theta_h}\,[\theta_f n^*]^{\theta_h}$$
$$= (1-\theta_h)^{1-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x)^{\frac{(1-\theta_h)\Delta_f-\theta_h\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x^*)^{\frac{\theta_h\Delta_h-(1-\theta_h)\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{80}$$

$$C^* = [(1-\theta_f)n^*]^{1-\theta_f}\,[\theta_h n]^{\theta_f}$$
$$= (1-\theta_f)^{1-\theta_f}\theta_h^{\theta_f}\left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}\right]^{\frac{\theta_f(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x^*)^{\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x)^{\frac{\theta_f\Delta_f-(1-\theta_f)\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{81}$$

The exponents on x in both (80) and (81) are both negative, but the exponent on x in (80) is larger in absolute value. That is, an increase in the Home money growth rate will cause a decrease in both the Home aggregate consumption C and Foreign aggregate consumption C^*, but the decrease in C is greater than the decrease in C^*. This latter fact is seen more clearly when steady state equilibrium relative aggregate consumption is expressed as follows:

$$\frac{C}{C^*} = \frac{(1-\theta_h)^{(1-\theta_h)}\theta_f^{\theta_h}}{(1-\theta_f)^{(1-\theta_f)}\theta_h^{\theta_f}}\left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}\right]^{\frac{(1-\theta_h-\theta_f)(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{82}$$

The exponent on the bracketed term is negative, so an increase in x makes C decrease more than C^*. Thus, the inflation tax in the open economy is not just a decrease in equilibrium Home aggregate consumption C as in the closed economy case, but also a decrease in Foreign aggregate consumption C^* and an increase in Foreign employment n^*.

As was mentioned earlier, the Home leisure subsidy is the only benefit of inflation in the open economy that also exists in the closed economy. However, in contrast to the decrease in the real wage in a closed economy, an increase in the Home money growth rate x increases the real wage in the open economy setting. The real wage in the open economy is the extra aggregate consumption from an extra unit of labor. Thus, the Home real wage is the derivative of Home aggregate consumption C with respect to n.

$$\begin{aligned}\frac{\partial C}{\partial n} &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left(\frac{n^*}{n}\right)^{\theta_h} \\ &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\end{aligned} \tag{83}$$

Because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the real wage. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the real wage due to the positive effect of x^* on n.

This real-wage benefit of Home inflation is driven by two components. First, as has been documented by Corsetti and Pesenti (2001), Cooley and Quadrini (2003), Cooper and Kempf (2003), and Arseneau (2007), an increase in the Home money growth rate x causes the terms of trade to appreciate in favor of the Home country. The terms of trade for a given country is defined as the price of its exports in terms of its imports. In the steady state equilibrium, the terms of trade for the Home country can be expressed as follows:

$$ToT \equiv \frac{P_{t+1}^h}{e_t P_{t+1}^f} = \frac{\theta_f}{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{1-\sigma-\xi}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{84}$$

Again, because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the cost of Home exports in terms of Home imports. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the terms of trade. The second component of the real-wage benefit of Home inflation is simply that increased openness means that more weight is placed on Foreign consumption which is amplified by the terms-of-trade appreciation.

So the objective of the Home monetary authority is to set its money growth rate such that the benefits of the inflation caused by x (leisure subsidy and real-wage benefit) equal the costs (consumption tax). The real-wage benefit is a direct result of the monopoly power that the monetary authority enjoys in international markets. And this monopoly power derives from the degree of inelastic demand for Foreign goods, as shown in the first order condition for Foreign currency balances (36).

Lastly, looking at the expression for the optimal Home money growth rate x in (74), it is no surprise that as the degree of imperfect competition increases in the Home country, the country-specific welfare benefits that the monetary authority can obtain from increasing the money growth rate decrease. Intuitively, the monopoly rents from the imperfect competition structure replace the monopoly rents obtained by the monetary authority through increasing the money growth rate.

4 Results

The main question of this paper is whether openness is inflationary. The following proposition answers this question with regard to both absolute inflation rate (Home country CPI growth rate) and what I will define as relative inflation rate (Home country CPI growth rate over Foreign country CPI growth rate) or a real exchange rate.

Proposition 1 (Monetary response to changes in openness). The equilibrium optimal Home money growth rate $\hat{x}$ in (74) increases with more Home openness in the form of a higher level of θ_h and in response to more Foreign openness in the form of a higher level of θ_f. The argument for the Foreign country is symmetric. However, when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$. Conversely, when θ_f increases, the increase in $\hat{x}^*$ is greater than the increase in $\hat{x}$.

Proof. See Appendix A-1. □

Because the Home country CPI growth rate (P_{t+1}/P_t) is equal to the Home money growth rate x, an increase in θ_h increases Home country inflation as well as Foreign country inflation. From the perspective of the Home monetary authority, if the Home marginal utility of Home consumption decreases relative to the Home marginal utility of Foreign consumption as is the case when θ_h increases while θ_f remains constant (see first order condition (37)), Home country agents bear a smaller proportion of the inflation tax. In effect, higher θ_h increases the welfare benefits from higher money growth rates to the Home country and lowers the costs. Consequently, the optimal response by the Home monetary authority is to raise the Home money growth rate or the CPI inflation rate in response to a higher degree of openness.

The next two propositions further explain how the level of imperfect competition among producers in a country, as parameterized by the elasticity of substitution among a country's differentiated goods ε, influences the optimal money growth rate x and the real outcomes of the economy in equilibrium.

Proposition 2 (Deflationary bias of imperfect competition). Both the optimal Home money growth rate $\hat{x}$ and the optimal Foreign money growth rate $\hat{x}^*$ decrease as the level of imperfect competition increases (as ε decreases). Furthermore, there exist two critical within-country elasticities of substitution for the Home country and Foreign country $(\bar{\varepsilon}, \bar{\varepsilon}^*)$ such that $\hat{x} = 1$ when $\varepsilon = \bar{\varepsilon}$ and $\hat{x}^* = 1$ when $\varepsilon = \bar{\varepsilon}^*$. That is, these two critical levels of the imperfect competition parameter implement the Friedman Rule in the Home and Foreign country, respectively.

$$\bar{\varepsilon} = \frac{(1-\theta_f)(1-\sigma)-\xi}{\theta_h(1-\sigma-\xi)} \tag{85}$$

$$\bar{\varepsilon}^* = \frac{(1-\theta_h)(1-\sigma)-\xi}{\theta_f(1-\sigma-\xi)} \tag{86}$$

Proof. See Appendix A-1. □

This result that the level imperfect competition induces a deflationary bias in monetary policy has been shown recently by Arseneau (2007).

Lastly, Proposition 3 highlights the relationship between the level of market power held by producers within a country and the monopoly power held by the each monetary authority in international markets.

Proposition 3 (Market power neutrality). In the case of symmetric countries $\theta_h = \theta_f$, the steady state equilibrium levels of employment n and n^* are not affected by the level of imperfect competition ε within both countries.

Proof. See Appendix A-1. □

Proposition 3 says that the real outcomes in each country $(n, n^*, C^h, C^f, C^{h*}, C^{f*})$ are the same regardless of whether the countries are characterized by perfect competition $\varepsilon = \infty$ or whether any degree of monopoly power is enjoyed by producers $\varepsilon < \infty$. The implication of this result is that if any monopoly rents available to Home or Foreign agents are not collected through producer price setting, the remainder will be collected by the monetary authority raising prices. As stated in Proposition 2, a level of imperfect competition exists at which all the monopoly rents are collected through producer price setting along. That is, inflation generated by the monetary authority increasing the money growth rate is not needed.

These results provide an interesting interpretation of the empirical findings summarized in Figure 1. If one is looking at the negative relationship between openness and inflation from the entire sample predicted values, Propositions 1 through 3 suggest that the inflationary bias of openness is dominated by the deflationary bias of imperfect competition. That is, the level of imperfect competition is greater than the critical value at which optimal monetary policy causes zero inflation $(\varepsilon < \bar{\varepsilon}, \bar{\varepsilon}^*)$. On the other hand, if one is looking at the positive relationship between openness and inflation that results when looking at low-inflation countries, the conclusion is that the inflationary bias of openness slightly dominates the deflationary bias of imperfect competition.

5 Conclusion

The main result of this work is that increased openness, as defined by the import share of GDP, is associated with a higher level of steady state equilibrium inflation. In a closed economy, the leisure subsidy of inflation is strictly dominated by the consumption tax, so the only role for the optimal money growth rate is to offset the inefficiencies of imperfect competition. However, as a country becomes more open, more of the burden of the consumption tax of inflation is borne by Foreign consumers, and the terms of trade and the real wage appreciate with increased inflation. These extra benefits from higher money growth rates cause an inflationary effect of openness in equilibrium.

However, another important finding of this paper is that, not only does the level of imperfect competition among producers in a given country dampen the incentive for a monetary authority to increase the money growth rate, but is a perfect substitute. That is, any monopoly rents that are available to the agents of a country that are not collected through price setting behavior of producers derived from the level of imperfect competition within the country are extracted by the monetary authority.

The result that openness is inflationary runs contrary to much previous work that has documented a negative correlation between various measures of the level of globalization or openness and inflation. However, much less work exists that explores this relationship through structural international models based on microeconomic foundations. This work is a first pass at studying, specifically, the imperfect competition and monetary market power channel.

Further work includes relaxing the strong assumption that the elasticity of substitution between aggregate Home-produced consumption and aggregate Foreign-produced consumption is unity $\rho = 1$, which results in the Cobb-Douglas form of the final consumption aggregator. Relaxing this assumption would break the constant expenditure share result and allow consumers to substitute away from expenditures on a country's production when the monetary authority raises the money growth rate. This may also break the dominant strategy equilibrium result in which the optimal monetary policy of each country is independent of the policy of the other country. Other extensions that may break the dominant strategy equilibrium result are to add pricing or exchange rate frictions such as time- or state-dependent pricing or pricing-to-market.

Also, this paper assumes that the two countries are asymmetric with respect to the level of openness θ. However, another dimension of asymmetry that might be interesting is the elasticity of substitution ε that parameterizes the level of imperfect competition. Furthermore, a vein of the literature exists that studies environments with endogenous markups in which the elasticity of substitution changes as firms enter and exit.[20]

And lastly, if the degree of openness has such important effects on the ability of the monetary authority to extract monopoly rents for its citizens, then how would an entity like a congressional body set openness policy optimally if it could? That is, what would be the equilibrium outcomes with endogenous openness θ?

[20] See Ferreira and Lloyd-Braga (2005), Ferreira and Dufourt (2006), and D'Aspremont, Ferreira, and Gérard-Varet (1996).

APPENDIX

A-1 Proofs

Proof of Proposition 1: Monetary response to changes in openness. Taking the derivative of the expression for $\hat{x}$ in (74) with respect to θ_h and θ_f gives the following results:

$$\hat{x} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h \Sigma_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_f)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_h)\xi}$$

$$\frac{\partial \hat{x}}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f - \theta_h\Sigma_f]^2} > 0$$
$$\frac{\partial \hat{x}}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_h(1-\sigma)(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f - \theta_h\Sigma_f]^2} > 0$$

Taking the derivative of the expression for $\hat{x}^*$ in (75) with respect to θ_f and θ_h gives the following results:

$$\hat{x}^* = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f \Sigma_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_h)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_f)\xi}$$

$$\frac{\partial \hat{x}^*}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h - \theta_f\Sigma_h]^2} > 0$$
$$\frac{\partial \hat{x}^*}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_f(1-\sigma)(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h - \theta_f\Sigma_h]^2} > 0$$

Now the proposition that when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$, simply means that $\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} > 0$.

$$\begin{aligned}
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \frac{\partial\hat{x}}{\partial\theta_h}\left[\frac{1}{\hat{x}^*}\right] - \frac{\partial\hat{x}^*}{\partial\theta_h}\left[\frac{\hat{x}}{(\hat{x}^*)^2}\right] \\
&= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h}\ldots \\
&\quad -\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Sigma_f(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\Delta_f\left[(1-\theta_f)\Delta_h-\theta_f\Sigma_h\right]^2}{\Delta_h^2\left[(1-\theta_h)\Delta_f-\theta_h\Sigma_f\right]}\right) \\
&= \frac{\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]}{\Delta_h[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} - \frac{\Delta_f\Sigma_f(1-\sigma-\xi)}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]} \\
&= \frac{\Delta_h\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Delta_f\Sigma_f(1-\sigma-\xi)[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Sigma_f[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi]-\Sigma_f[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \Delta_f(1-\sigma-\xi)\left(\frac{(\Delta_h-\Sigma_f)(1-\theta_h-\theta_f)(1-\sigma)+\xi\left[\Sigma_f(1-\theta_h)-\Delta_h(1-\theta_f)\right]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) > 0
\end{aligned}$$

The last line is true because $\Delta_h - \Sigma_f < 0$ and $\Sigma_f(1-\theta_h) - \Delta_h(1-\theta_f) > 0$. □

Proof of Proposition 2: Deflationary bias of imperfect competition. From (74) and (75):

$$\begin{aligned}
\frac{\partial\hat{x}}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f} > 0 \\
\frac{\partial\hat{x}^*}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h} > 0
\end{aligned}$$

Then, to find the respective levels of ε that induce the Home and Foreign monetary authorities, respectively, to set their money growth rates equal to 1 is found by solving (74) and (75) for ε when $\hat{x} = 1$ and when $\hat{x}^* = 1$.

$$\begin{aligned}
\bar{\varepsilon}: \quad 1 &= \left(\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi} \\
\bar{\varepsilon}^*: \quad 1 &= \left(\frac{\bar{\varepsilon}^*-1}{\bar{\varepsilon}^*}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}
\end{aligned}$$

Solving these two equations for $\bar{\varepsilon}$ and $\bar{\varepsilon}^*$, respectively, gives the results in (85) and

(86).

$$\bar{\varepsilon} = \frac{\Delta_f}{\Sigma_h - \theta_h \xi} = \frac{(1-\theta_f)(1-\sigma) - \xi}{\theta_h(1-\sigma-\xi)}$$
$$\bar{\varepsilon}^* = \frac{\Delta_h}{\Sigma_f - \theta_f \xi} = \frac{(1-\theta_h)(1-\sigma) - \xi}{\theta_f(1-\sigma-\xi)}$$

□

Proof of Proposition 3: Market power neutrality. When the Home and Foreign Country are symmetric $\theta_h = \theta_f = \theta$, the equilibrium employment level is given by:

$$n = \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\left(\frac{\varepsilon}{\varepsilon-1}\right)\right]^{\frac{\Delta-\Sigma}{\Delta^2-\Sigma^2}} (\hat{x})^{\frac{\Delta}{\Delta^2-\Sigma^2}} (\hat{x}^*)^{\frac{-\Sigma}{\Delta^2-\Sigma^2}}$$

where $\Delta = (1-\theta)(1-\sigma) - \xi$ and $\Sigma = \theta(1-\sigma)$. The expressions for the optimal money growth rates in this symmetric case are given by:

$$\hat{x} = \hat{x}^* = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta}{(1-\theta)\Delta - \theta\Sigma}$$

Now the equilibrium employment level can be written as:

$$\begin{aligned} n = n^* &= \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right]^{\frac{1}{1-\sigma-\xi}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\sigma-\xi}}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1-\sigma-\xi}}\left[\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right]^{\frac{1}{1-\sigma-\xi}} \\ &= \left[\left(\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right)\left(\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right)\right]^{\frac{1}{1-\sigma-\xi}} \end{aligned}$$

It is clear that neither n nor n^* is a function of the level of imperfect competition ε. And because the equilibrium consumption levels are simply constant fractions of the output level, consumption is also not affected by changes in the level of imperfect competition. □

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