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Model Documentation Report: System for the Analysis of Global Energy Markets (SAGE)

Volume 1

Model Documentation

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Table of contents

1 Introduction

1.1 Purpose of this Report

Volume 1 of the SAGE documentation report documents the objectives, analytical approach and development of the System for the Analysis of Global Energy markets (SAGE). It lists and describes the model's data assumptions, computational methodology, parameter definitions and source code. Volume 2 of the SAGE documentation report details the technology, process, and technology naming conventions used in SAGE.

This document serves three purposes. First, it is a reference document providing a detailed description for model analysts, users, and the public. Second, this report meets the legal requirements of the Energy Information Administration (EIA) to provide adequate documentation in support of its models *(Public Law 93-275, section 57.b.1).* Third, it facilitates continuity in model development by providing documentation from which energy analysts can undertake model enhancements, data updates, and parameter refinements as future projects.

1.2 Model Summary

SAGE is an integrated set of regional models that provides a technology-rich basis for estimating regional energy supply and demand. For each region, reference case estimates of end-use energy service demands (e.g., car, commercial truck, and heavy truck road travel; residential lighting; steam heat requirements in the paper industry) are developed on the basis of economic and demographic projections. Projections of energy consumption to meet energy demands are estimated on the basis of each region's existing energy use patterns, the existing stock of energy-using equipment, and the characteristics of available new technologies, as well as new sources of primary energy supply.

Period-by-period, market simulations aim to provide each region's energy services at least cost by simultaneously making equipment investment and operating decisions and primary energy supply decisions. For example, in SAGE, if there is an increase in residential lighting energy service (perhaps due to a decline in the cost of residential lighting), either existing generation equipment must be used more intensively or new equipment must be installed. The choice of generation equipment (type and fuel) incorporates analysis of both the characteristics of alternative generation technologies and the economics of primary energy supply.

1.3 Archival Media

Archived on a CD-R, July, 2003.

1.4 Software Requirements

SAGE is a PC-based application requiring the Microsoft Windows 2000 Professional (or later) operating system as well as Microsoft Office 2000 (or later). While the SAGE source code, written in the General Algebraic Modeling System (GAMS), is publicly available at EIA's web site www.eia.doe.gov/oiaf/sage/sage.zip three other commercial software packages are required to use this source code. These consist of GAMS along with a powerful commercial linear program solver (e.g., XPRESS/CPLEX), and VEDA-SAGE the data handling and results analysis "shell" overseeing all aspects of working with SAGE. The interested reader should contact the vendor links listed below.

For information on GAMS and the CPLEX solver used to optimize the SAGE model, contact garygoldstein@gams.com. For information on VEDA-SAGE, contact www.KanORS.com.

In addition, the reader may wish to review the wealth of material available covering the family of MARKAL models developed under the auspice of the International Energy Agency's Energy Technology Systems Analysis Program (ETSAP) that served as the stable platform upon which SAGE development took place (www.etsap.org).

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1.9 Organization of this Report
Chapters 2 and 3 are designed for the analyst needing a clear description of the model's purpose, structure and data assumptions as well as the proper interpretation of model results. Chapter 2 provides a general overview of SAGE that summarizes the model's paradigm, its equations, and the most salient features of its data structure. Chapter 3 offers a description of the economic rationale of SAGE. It starts with a brief review of alternative energy model designs and then goes to describe the economic and mathematical properties of the SAGE equilibrium. That chapter also contains more technical descriptions of the key features of SAGE that distinguish it from other partial equilibrium models. The chapter ends with a brief primer on Linear Programming and Duality Theory, and their application to the SAGE model.

Chapter 4, a comprehensive reference manual, is intended for the programmer/modeler looking for an in-depth understanding of the relationship between the input data and the model mathematics, or contemplating augmenting the capabilities of SAGE. It includes a full description of the attributes, sets, variables, and equations of the system, accompanied by text explaining their purpose and function. It also includes descriptions of how the market sharing mechanism functions in SAGE, and of how inter-regional trade is handled by the model. A full listing of the SAGE source code (written in the Generalized Algebraic Modeling System (GAMS) language) is available as well.

Volume 2 provides a detailed and graphical description of all energy and emissions flows in SAGE. This description starts with resources and technologies needed to produce primary energy and ends with satisfaction of consumers' equilibrium demand for energy services. This section also provides a full description of the naming conventions used in the current implementation of SAGE.

2 Overview of SAGE

2.1 Introduction

The objectives of this chapter are to provide enough description to enable users of SAGE to understand the:

- Reference Energy System (RES) modeling approach;
- Nature of the parameters and other assumptions the user must provide to develop a SAGE reference case projection,
- Simplified mathematical formulation of the SAGE equilibrium, and
- Proper interpretation of SAGE results.

It is useful to distinguish between a model's *structure* and a particular instance of its' implementation. A model's structure exemplifies its' fundamental approach for representing and analyzing a problem—it does not change from one implementation to the next. SAGE models for different regions have identical structures, however, because SAGE is data[1] driven, each regional model will vary as data inputs vary. For example, one region may, as a matter of user data input, have undiscovered domestic oil reserves. Accordingly, SAGE generates technologies and processes that account for the cost of discovery and field development. If, alternatively, user supplied data indicate that a region does not have undiscovered oil reserves no such technologies and processes would be included in the representation of that region's reference energy system (RES, see section 2.4).

The structure of SAGE is ultimately defined by variables and equations determined by data input provided by the user. This information collectively defines a SAGE regional model database, and therefore the resulting mathematical representation of the RES for the region. The database itself contains both qualitative and quantitative data. The qualitative data includes, for example, a list of future technologies that the modeler feels are applicable to a certain region over a specified time horizon. Quantitative data, in contrast, contains the technological and economic parameter assumptions specific to each technology, region, and time period. A technology may be available for use in two distinct regions; however, cost and performance assumptions may be assumed to be quite different (i.e., consider a residential heat pump in Canada versus the same piece of equipment in the United States). This chapter discusses both qualitative and quantitative assumptions in the SAGE modeling system.

[1] Data in this context refers to parameter assumptions, technology characteristics, projections of energy service demands, etc. It does not refer to historical data series.

2.2 Summary of Key Elements of SAGE

The SAGE energy economy is made up of producers and consumers of energy carriers.[1] SAGE, like most equilibrium models, assumes perfectly competitive markets for energy carriers—producers maximize profits and consumers maximize their collective utility. The result is a supply-demand equilibrium that maximizes the net total surplus (i.e. the sum of producers' and consumers' surpluses) as will be fully discussed in chapter 3 of this documentation. SAGE may, however, depart from perfectly competitive market assumptions by the introduction of user-defined, explicit, special constraints.

Operationally, a SAGE run configures the *energy system* of a *set of regions* over a certain *time horizon* in such a way as to *minimize the net total cost (or* equivalently *maximize the net total surplus*) of the system, while satisfying a number of *constraints*. SAGE is generally run in a *time-stepped* manner, meaning investment decisions are made in each period without knowledge of future events.[2] As a consequence, the efficiency of the stock of energy-using equipment in period ***t*** can only be influenced by investment decisions actually made in period ***t***. In addition to time-periods that are usually 5 years, SAGE recognizes three seasons (Winter, Summer, Intermediate), and two diurnal divisions (Day, Night). These time divisions result in six *time-slices*. All six time-slices are recognized only for technologies producing electricity (seasonal and diurnal) or low-temperature heat (seasonal), both of which may not be easily stored and thus require a finer time disaggregation than other energy carriers. SAGE may also be run as a dynamic optimization problem where investment decisions are made with full knowledge of the future.[3]

2.2.1 Reference Energy System Overview

Configuring a region's energy system means defining a schedule of future:

- Investments in technologies (e.g., gasoline fueled light trucks, fuel cell powered light trucks, etc.);
- Operating levels for each type of technology (e.g., billion vehicle kilometers per year), in each time period;[4]
- Levels of primary resource availability, and
- Levels of energy carrier purchased from and/or sold to other regions[5].

[1] An *energy carrier*, also denoted in this report as an *energy source* or *energy form*, is anything in the energy system containing usable energy and utilized as such either to produce another energy carrier (e.g. coal or gas used to produce electricity) or to produce usable heat or mechanical movement via certain technologies (e.g. gasoline, electricity, wood).

[2] This is often referred to as myopic foresight. Note that SAGE has perfect foresight within the current period (usually five years long).

[3] This requires access to the MARKAL modeling system, which can be arranged by contacting ETSAP.

[4] Note that in some circumstances it may be cost effective to purchase new equipment rather than operate existing capacity.

[5] Multi-region SAGE considers both bilateral and global endogenous trade, however, trade in many energy carriers is properly disallowed. For example, noncontiguous regions are not allowed to trade electricity and regions are not allowed to trade end-use energy services such as auto travel.

All of these decisions are made within the context of each region’s Reference Energy System (RES). The configuration of a regional RES is defined by the set of technologies available in each period; the lists of energy service demand projections and of energy sources (including imports), linked by commodities (Figure 2.1). Further details on the RES concept are addressed in section 2.4.

2.2.2 SAGE Regions

Each region of SAGE is either a country or a group of countries. Currently there are 15 SAGE regions which, when combined, cover the entire world.

- Africa
- Australia-New Zealand
- Canada
- Central and South America
- China
- Eastern Europe
- Former Soviet Union
- India
- Japan
- Mexico
- Middle-East
- Rest of Asia
- South Korea
- United States
- Western Europe

The regions are interconnected by special technologies that transport energy carriers (transmission lines, pipelines, shipping, etc.) and by mechanisms to trade emission permits.

2.2.3 Time Horizon

The time horizon may extend for up to eleven 5-year periods, however, to date only projections through 2025 have been developed. The first period extends from 1998 to 2002 and is called 2000 by reference to its middle year. The last potential period is centered at year 2050. In SAGE, the 5 years belonging to the same period are considered identical. In other words, any model input or output related to period ***t*** applies to each of the 5 years in that period. Similarly, the energy flows and emissions levels reported in the results represent annual flows in each of the 5 years embodied in a period. The important exception is that all investments are assumed to occur at the beginning of each modeling period with the resulting installed capacity available throughout that period.

The initial period calibrates the model to the latest comprehensive historical data available. No new investments are allowed in this period. This calibration is one of the important tasks required for setting up a SAGE model. The main variables to be calibrated are: the capacities and operating levels of all technologies, as well as the extracted, exported and imported quantities for all energy carriers. The associated emission levels are also validated. Note carefully that the initial period's calibration also influences the model's decisions over several future periods, since the profile of residual capacities is provided over the remaining life of the technologies existing in the initial period.

2.2.4 System Costs

The SAGE objective is to minimize the total cost of the system, which includes the following elements at each time period:

- *Annualized investments* in various technologies*;*
- *Fixed* and *variable annual Operation and Maintenance (O&M)* costs of technologies;
- Cost of exogenous energy *imports* and domestic resource *production* (e.g., mining);
- Revenue from exogenous energy *exports;*
- *Delivery* costs;
- *Losses* incurred from reduced end-use demands (as will be discussed in section 3.3), and
- *Taxes* and *subsidies* associated with energy sources, technologies, and emissions.

The SAGE objection function: minimization of total cost

In SAGE, run in time-stepped mode, the investment costs are first annualized, and then added to the other costs, which are all annual costs. This annualized cost replaces a fixed lump sum investment by a stream of equal annual payments over the life of the equipment, in such a way that the present value of the stream is exactly equal to the lump sum investment. The discount rate used for annualizing costs may be technology, sector and/or region specific, so as to reflect the financial characteristics the analyst believes appropriate for each investment decision. For instance, if the initial capital cost of a car is $20,000, and if its technical life is 10 years, the annualized value of the capital cost assuming an 8% discount rate must be such that a stream of 10 such annualized payments adds up, after discounting, to exactly $20,000. The equivalent annualized value is $3,255, as computed using expression below.

$$\boldsymbol{INVCOST} \leq \boldsymbol{ANNUALIZED\_INVCOST} \bullet \sum_{\boldsymbol{j}=1}^{\boldsymbol{LIFE}} (1+\boldsymbol{d})^{-\boldsymbol{j}} \qquad (2.2-1)$$

Where:
INVCOST is the investment cost of a technology
ANNUALIZED_INVCOST is the annualized equivalent of *INVCOST*
LIFE is the physical life of the technology
d is the discount rate used

SAGE (in the default time-stepped mode) first minimizes the total cost for the first period alone, and then freezes the resulting investment decisions for that period. It then proceeds to minimize the total cost for the second period alone, and then freezes the second period's decisions as well. This is done sequentially for all periods. By so doing, SAGE assumes that, when making their investment and operating decisions at period *t*, producers and consumers have no information on later periods (or do not wish to act on such information). Such behavior is sometimes referred to as *myopic*[1].

The maximization of total surplus

As will be seen in detail in section , the minimization of total cost desribed above is fully equivalent to the maximization of the total surplus defined as the sum of producers and consumers surpluses. This equivalence is important as it confers a valid economic rationale to the supply-demand equilibrium computed by SAGE.

2.2.5 Constraints

The constraints of the SAGE model express the logical relationships that must be satisfied by the model. If any constraint is not satisfied the model is said to be infeasible. A summary list of the main constraints are listed below:

2.2.5.1 Satisfaction of Energy Service Demands:

Reference (or base) energy service demand projections are developed by the modeler for the entire forecast horizon based on exogenous regional economic and demographic projections (drivers), assumptions regarding each service demands sensitivity to changes in the assumed driver, and calibration factors to account for planned or assumed structural changes in the energy system. Note that service demand elasticities to drivers and calibration factors are defined for each model period. These demands are set for the reference case, but are endogenously determined in alternate scenarios where the price of energy services vary from the reference case prices (section 3.3). For example, a scenario causing the price of oil to rise would increase the cost of auto travel relative to the reference case and, *ceteris paribus* auto service demand would decline relative to the reference case. An increase in the price of oil relative to the reference case would also affect investment decisions. Since past investment decisions are frozen, the initial response to higher oil prices may largely be felt in a reduction of auto energy service. Over time, as the stock of equipment turns over, more efficient autos may be chosen, tending to lower the cost of auto travel service, thereby increasing auto service demand.

2.2.5.2 Capacity Transfer (conservation of investments):

Investment in a piece of equipment increases capacity for the duration of the physical life of the equipment. At the end of that life, the total capacity for this equipment type decreases by the same amount (unless some other investment is decided by the model at that time). While computing the available capacity at some time period, the model takes into account the capacity resulting from all investments up to that period, some of which may have been made prior to the initial period but are still in operating condition (embodied by the residual

[1] Alternatively, a *clairvoyant* (perfect foresight) optimization mode is possible where SAGE would optimize the net present value of costs for all periods.

capacity of the equipment), and others have been decided by the model at, or after, the initial period, but prior to the period in question.

2.2.5.3 Use of capacity:

In each period, the model may use some or all of the installed capacity in that period according to a maximum (or fixed) region and technology specific Availability Factor (AF). Note that the model may decide to use *less* than the available capacity during certain time-slices, or even throughout the whole period. This will of course occur only if such a decision contributes to minimizing the overall cost. Optionally, there is a provision for the modeler to force specific technologies to use their capacity fully.

2.2.5.4 Balance for Commodities (except electricity and low-temperature heat):

At each time period, the production plus import (from other regions) of an energy carrier (or material) must be at least (equal to) as much as that consumed, plus exports (to other regions), For example, this constraint requires that the level of gasoline produced be greater than or equal to the level consumed as part of delivering auto vehicle miles traveled to satisfy said demanded.

2.2.5.5 Electricity & Heat Balance:

These two commodities are defined in each time slice and therefore: at each time period, each region, each season and time-of-day, electricity produced plus electricity imported (from other regions) must be at least as much as electricity consumed, plus electricity exported (to other regions), plus grid losses. A similar balance exists for low temperature heat, although only tracked by season.

2.2.5.6 Electricity Peaking Reserve Constraint:

This constraint requires that in each time period and for each region, total available capacity exceed the average load of the peaking time-slice by a certain percentage. This percentage is the Peak Reserve Factor (ERESERV) and is chosen to insure against possible electricity shortfalls due to uncertainties regarding electricity supply that may decrease capacity in an unpredictable way (e.g. water availability in a reservoir, or unplanned equipment down time). The peaking time-slice is defined as the time-slice when load is heaviest (it may be Winter Day in cold countries, Summer Day in warm countries, etc.).

2.2.5.7 Base Load (electricity generation only):

The user may identify which technologies should be considered as base load technologies by SAGE; i.e. those whose operation must not fluctuate from day to night in a given season. The user may also specify what is the maximum fraction of night production that may be supplied from all base load technologies.

2.2.5.8 Seasonal availability factors (electricity sector only):

The user may specify seasonal and even day-night limitations on the use of the installed capacity of some equipment. This is especially needed when the operation of the equipment depends on the availability of a resource that cannot be stored, such as Wind and Sun, or that can be partially stored, such as water in a reservoir.

2.2.5.9 Market share constraints:

For end-use technologies, and for power sector technologies SAGE possesses a mechanism that allows the user to bound the market share that any single technology or group of technologies is allowed to capture at each period (e.g., natural gas heating technologies as a share of all residential heating technologies). Without such a mechanism, it would be possible for a single technology to capture the entire market (e.g. the gas furnace might provide 100 percent of space heating for new residences in an entire country or region) even if its cost were only slightly lower than the cost of competing technologies. It is more generally observed that end-users' technological choices result in a market split between several technologies, for a variety of reasons, including individual preferences other than pure financial costs. In addition to direct technology market share constraints, SAGE can also assign technology market shares based on relative technology costs (via a logit function formulation) rather than on simply a least cost basis.

2.2.5.10 Emission constraint(s):

The user may impose on the whole system upper limits on emissions of one or more pollutants. The limits may be set for each time period separately, so as to simulate a particular emission profile (also called emission target).

The complete list of constraints along with the full mathematical details may be found in chapter 4.

2.3 SAGE Linear Program (Simplified)

The description of the objective function and constraints of the previous subsections may be translated into a formal set of mathematical expressions. In this section, we present a simplified formulation of the equations, which ignores exceptions and some complexities that are not essential to a basic understanding of the model. The formal mathematical description is contained in section 4. The notation used in section 4 differs from the simplified one used here.

An optimization problem formulation consists of three sets of entities: *decision variables* (unknowns), *objective function* (expressing the quantity to minimize or maximize), and *constraints* (equations or inequalities that must be satisfied)

2.3.1 Decision Variables

The decision variables represent individual choices made by the model

INV(k,t,r)**:** new capacity addition for technology ***k***, at period ***t***, in region ***r***. Units: PJ/year, Million metric tons per year (for steel, aluminum, and paper industries), Billion vehicle kilometers per year (B-vkms/year) for road vehicles and GW for electricity equipment, where 1GW=31.536 PJ/year. Note that is investment in new capacity is assumed to occur at beginning of period ***t***, as a lump investment.

CAP(k,t,r): installed capacity of technology ***k***, at period ***t***, in region ***r.*** Unit: Same as above.

ACT(k,t,r,s): activity level of technology ***k***, time period ***t***, in region ***r***, during time-slice ***s*** Unit: PJ per year. ***ACT*** variables are not defined for end-use technologies, for which it is assumed that activity is always directly proportional to capacity.

TRADE(c,t,r,r'): quantity of commodity ***c*** (PJ per year) sold by region ***r*** to region ***r'*** at time period ***t***. A negative value indicates a purchase. This variable represents endogenous trade between regions. Trade of any given commodity must balance at each period, i.e. the sum of trade variables over all pairs of regions is equal to 0.

IMPORT(c,t,r,l), EXPORT(c,t,r,l,): quantity of commodity ***c***, price level ***l,*** (PJ per year) imported or exported by region ***r*** at time period ***t***. These variables play a role similar to the trade variables described above, but here the origin of imports (or the destination of exports) is not specified. Therefore, these variables will not be automatically balanced by SAGE, and hence must be balanced by the user. These variables are convenient whenever endogenous trade is not being considered. For example, a SAGE model run including only the 3 regions comprising North America would have to treat the imports and exports of oil and oil products to North America as exogenous; the coefficient of the import or export variable in the objective function is the unit price of importing or exporting the commodity.. However, these variables are entirely unnecessary in a global model where all commodities are traded.[1]

MINING(c,t,r,l): quantity of commodity ***c*** (PJ per year) mined in region ***r*** at price level ***l*** at time period ***t***; the coefficient to the objective function is the unit cost of extracting the commodity.

D(t,r): demand for end-use ***D*** in region ***r***, at time period ***t. D(r,t)*** may differ from the reference case demand ***d***, based on each service demand's own-price elasticity.

ENV(p,t,r): Emission of pollutant ***p*** at period ***t*** in region ***r.***

[1] In the actually implementation of SAGE regional import/export variables are used to balance the global trade, but with prices determined endogenously.

2.3.2 Objective function

For each time period t, ***minimize the Sum*** *over all regions* **r,** *all technologies* **k,** *all demand segments* **d,** *all pollutants* **p**, *and all input fuels* **f** *of the various costs incurred, namely: annualized investments, annual operating costs (including fixed and variable technology costs, fuel delivery costs, costs of extracting and importing energy carriers), minus revenue from exported energy carriers, plus taxes on emissions, plus cost of demand losses.*

Mathematically, the expression to be minimized is as follows:

$$\begin{aligned}
&\mathbf{AnnualizedInvcost(k,t,r)\ {*}INV(k,t,r)} \qquad (2.3\text{-}1)\\
&\mathbf{+Fixom(k,t,r)\ {*}CAP(k,t,r)}\\
&+ \boldsymbol{Varom(k,t,r)\ {*}\Sigma_s\, ACT(k,t,r,s)}\\
&+ \boldsymbol{Delivcost(f,k,t,r){*}Input(f,k,t,r){*}\ \Sigma_s\, ACT(k,t,r,s)}\\
&+ \boldsymbol{Miningcost(c,r,l,t){*}Mining(c,r,l,t)}\\
&+ \boldsymbol{Tradecost(c,r,t){*}\ TRADE(c,r,r',t){+}{+}\ Importprice(c,r,l,t){*}Import(c,r,l,t)}\\
&- \boldsymbol{Exportprice(c,r,l,t){*}Export(c,r,l,t)}\\
&+ \boldsymbol{Tax\ (p,t,r)\ {*}\ ENV(p,t,r)}\\
&+ \boldsymbol{\{DemandLoss(d,r,t)\}}
\end{aligned}$$

Where:

AnnualizedInvcost(k,t,r), Fixom(k,t,r), Varom(k,t,r), are annual unit costs of investment, fixed maintenance, and operation, of technology *k*, in region *r*, in time period *t*;
Delivcost(f,k,t,r) is the delivery cost per unit of fuel *f* to technology *k*;
Input(f,k,t,r) is the amount of fuel *f* required to operate one unit of technology *k*;
Miningcost(c,r,l,t) is the cost of mining commodity *c* in the price level *l*;
Tradecost(c,r,t) is the trade transport or transaction cost for commodity *c* in region r and time period t;
Importprice(c,r,l,t) is the (exogenous) import price of commodity c;
Exportprice(c,r,l,t) is the (exogenous) export price of commodity c;
Tax(p,t,r) is the tax on pollutant p; and
DemandLoss (in non reference scenarios) represents the loss in utility incurred by consumers when service demand is less than its value in the reference case, which will be explained further in section 3.3.

Note that the ***TRADE(c,r,r',t)*** variables have no objective function coefficients, other than possibly transport or transaction charges, since the revenue from export by the exporting region ***r*** is exactly cancelled by the cost of import by the importing region ***r'***. Note that the quantities as well as the unit cost (corresponding to the marginal price) of an endogenously traded commodity are model results.

2.3.3 Constraints

The constraints translate the rules governing the nature of the energy system and the operation of the individual technologies. The most important groups of constraints are described here.

2.3.3.1 Satisfaction of Demands

At each time period *t*, region *r*, demand *D*, the total activity of end-use technologies servicing that demand must be at least equal to the specified demand (See Table 2.1 for a list of the service demands modeled in SAGE). Hence:

Sum *{over all end-use technologies k} of* ***ACT(k,t,r) ≥ D(t,r)*** **(2.3-2)**

For example, car travel expressed in billion vehicle kilometers per year may be satisfied by a combination of several types of cars (gasoline, diesel, etc.).

2.3.3.2 Capacity transfer

For each technology *k*, region *r*, period *t*, the available capacity at period t is equal to the sum of investments made at past and current periods, and whose physical life has not ended.

CAP(k,t,r) = ***Sum*** *{over t and all periods t' preceding t and such that t-t'<LIFE(k)} of* ***INV(k,t',r)*** **+** *RESID(k,t)* **(2.3-3)**

Where *RESID(k,t)* is the capacity of technology ***k*** due to investments that were made prior to the initial model period and still existing at time ***t***. Note that since SAGE is generally run in a time-stepped simulation mode, all past investments are taken as fixed, and only the current period's investment is subject to optimization.

Example: *gasoline cars in period t have a total capacity limited by the investments in periods t-1 and t, if the assumed life duration of a car is equal to 2 periods (10 years). If the assumed life is 3 periods, then the capacity in t is limited by the investments in t-2, t-1 and t. Furthermore, if the life were 12 years, then 40% of the capacity would be available in third period after that in which the investment was made.*

2.3.3.3 Use of capacity

For each technology, each period, each region, each time-slice, the activity of the technology may not exceed its available capacity, as specified by a user defined availability factor

ACT (k,t,r,s) ≤ *AF(k,s,t,r)* CAPUNIT*****CAP(k,t,r)*** **(2.3-4)**

Example**:** *a coal fired power plant's activity in any time-slice is bounded above by 80% of its capacity, i.e.* ***ACT (k,t,r,s) ≤*** *.8*31.536*****CAP(k,t,r), where 31.536 is the unit conversion between units of capacity (Gw) and activity (PJ/year).***

Note that this constraint is not written for end-use technologies, because an activity variable is not defined. Activity for end-use technologies is always assumed to be directly proportional to their installed capacities.

2.3.3.4 Energy Balance

For each commodity ***c,*** time period ***t***, region ***r***, (and time-slice ***s*** in the case of electricity and low-temperature heat), this constraint demands that the disposition of each commodity may not exceed its supply. The disposition includes consumption in the region and/or exports; the supply includes production in the region plus imports.

Sum {*over all k} of: output(c,k,r,t)*****ACT(c,k,r,t,s) +*** **(2.3-5)**
Sum {*over all l} of:* ***MINING(c,r,l,t) +***
Sum {*over all l} of:* ***IMP(c,r,l,t) +***
Sum {*over all r'} of:* ***TRADE(c,t,r',r)****FR(s)* ***≥***
Sum {*over all r'} of:* ***TRADE(c,t,r,r')****FR(s)* ***+***
Sum {*over all l} of:* ***EXP(c,r,l,t) +***
Sum *{over all k} of: input(c,r,t)****ACT(c,k,r,t,s)***

Where:

Input(c,k,t,r) is the amount of fuel *c* required to operate one unit of technology *k;*
Output(f,k,t,r) is the amount of fuel *c* produced per unit of technology *k, andFR(s)* is the fraction of the year covered by time-slice *s* (equal to 1 for non-seasonal commodities).

<u>Example</u>*: Gasoline consumed by vehicles plus gasoline exported to other regions must not exceed gasoline produced from refineries plus gasoline imported from other regions.*

2.3.3.5 Electricity (and heat) Peak Reserve Constraint

For each electricity commodity c, at each time period *t* and for each region *r* in each season *s* (daytime), there must be enough installed capacity to exceed the required capacity in the season with largest electricity (and heat) demand by a safety factor called the peak reserve factor.

Sum *{over all k} of CAPUNIT*****CAP(k,t,r)****peak(k) +* **(2.3-6)**
Sum *{over all regions r'} of* ***TRADE(c,t,r',r)****FR(s)* ***≥***
[1+ERESERVE(r)][****D(t,r,s) + Sum****{over all r'} of:*
TRADE(c,t,r,r')*FR(s)]*

Where:

ERESERVE is the region specific reserve coefficient, which allows for unexpected down time of equipment, for unexpectedly large demand at peak, and for uncertain hydroelectric availability.

The *peak(k)* (always <=1) specifies the fraction of technology k's capacity that is allowed to contribute to the peak load. Many types of generating equipments are predictably available during peak load and thus have a peak coefficient equal to unity, whereas others such as wind turbines are attributed a peak coefficient less than 1 since they are on average only fractionally available at peak.

Example*: a wind turbine typically has a peak coefficient of .25 or .3, whereas a hydroelectric plant, a gas plant, or a nuclear plant typically have peak coefficients equal to 1.*

2.3.3.6 Emission constraint[1]

For each time period ***t*** this constraint tracks emissions and ensures that the total emission of pollutant ***p*** will not be greater than a user-selected upper bound, if provided. In SAGE, pollutants may be emitted when a technology is active, but also when it is inactive (for example a hydro reservoir may emit methane even if no electricity is being produced). Emissions may also occur at the time of construction of the technology, if desired. This flexibility allows the accurate representation of various kinds of emissions.

Sum *{over all technologies k, regions r, slices s} of* **(2.3-7)**
*{eminv(p,k)****INV(k,t,r)**** + emcap(p,k,t)****CAP(k,t,r)**** + emact(p,k,t)****ACT(k,t,r,s)****} – ENV(p,t,r) = 0*

Where:

eminv, emcap, emact are emission coefficients linked respectively to the construction, the capacity, and the operation of a technology, and

ENV(p,t,r) ≤ Emlimit(p,t), the upper limit set by the user on the total emission of pollutant *p* in the region at period *t.*
Note that if *etax(p,t,r)* is specified by the modeler then *ENV(p,t,r) * tax(p,t,r)* enters the objective function encouraging the model to generate less of said emissions.

Alternatively, emission caps may be set for all regions together, or by sector, etc., via special additional constraints. It is also possible to set a cumulative emission cap (for a group of time periods) for a region.

2.3.3.7 Electricity Base Load constraint

Electricity generating technologies that are labeled as Base load must produce the same amount of electricity at night as in the day. They may, however, vary their production from season to season. Therefore, for Base load technologies, there are only three *ACT* variables (one per season) instead of 6 for other electric generation technologies. In addition, a base load constraint ensures that only a maximum percentage of the total highest nighttime demand for electricity is met by such plants.

[1] Environmental constraints do not have to be specified to specify a SAGE model.

2.3.3.8 Market share constraints

These constraints allow the user to limit the penetration of each end-use technology; so as to avoid it's capturing the entire energy service demand market. Without such constraints, it might happen that a single technology (whose cost is lowest among all technologies satisfying a given demand segment) would capture 100% of that demand in an entire region. This effect, due to optimization, is sometimes called the 'winner-takes-all', 'penny-switching', or 'knife edge' effect of Linear Programming. For practical purposes, there is more diversity in the end-users preferences and there is more variety within a region, both of which make it unlikely that all end-users would select the same technology. To mimic the more diversified end-user choices, the SAGE analyst may introduce the built-in market share constraints that avoid knife-edge behavior. While used primarily for the demand sector, this option may also be applied to the supply technologies (e.g., a set of competing renewable technologies).

2.3.3.9 User-defined constraints

In addition to the standard SAGE constraints discussed above, the user interested in developing reference case projections of energy market behavior typically introduces many additional linear constraints to express special conditions.

User defined constraints may serve many functions in SAGE. Their general purpose is to constrain the optimization problem in some way to account for either policy- or market behavior-based factors that affect investment decisions but that are not endogenous to the model. For example, there may a user defined constraint limiting investment in new nuclear capacity (regardless of the type of reactor), or dictating that a certain percentage of new electricity generation capacity must be powered by renewable energy sources.

2.3.4 Dual variables (Shadow prices)

Decision variables are referred to as the *primal variables*. They all have the dimension of a physical flow (of energy or other commodities) or of the capacity or activity of some technology. In Linear Programming, there exists another set of variables, each of which corresponds to a constraint of the linear program. These variables are called *dual variables*. The dual variable of a constraint has a value (at optimum) that is equal to the marginal value of the constraint's right-hand-side, i.e. *the change in the optimal Objective function value induced by one additional unit of the constraint in question.* Each optimal dual variable is also called the shadow price of the corresponding constraint. Three examples of dual variables illustrate this concept.

2.3.4.1 Commodity price

The dual variable of balance constraint is the additional dollar value, if one additional unit of the commodity is injected in the system. This is simply the *price* of commodity ***c*** in the model, which is therefore equal to the marginal value of the commodity for the system as a whole. In the case of electricity, there are six prices, one for each time-slice; in addition, the shadow price of the peak constraint (which is the shadow price of the peak reserve constraint) indicates the additional value to the system of another unit of peak reserve capacity. This extra charge should be added to electricity guaranteed during the peak. It is thus possible to construct a composite electricity price which is the weighted average of the 7 electricity shadow prices, and which represents the price of a kWh produced throughout the 6 time-slices and the peak period of the year.

2.3.4.2 Price of energy services (demands)

The dual variable of the demand constraint (Equation 2.3-1) is the additional cost of increasing that demand by one unit. It is therefore the price of the energy service**.**

2.3.4.3 Price of pollutant

The dual variable of the emission constraint (Equation in section 2.3.4.6) is the marginal cost of abating one additional unit of pollutant. Maintaining the Linear Programming paradigm of perfectly competitive markets, it is also the price of that pollutant.

Dual variables (shadow prices) play a very important role in SAGE (as well as in any optimization based equilibrium model), since they represent the price side of the equilibrium. Chapter 3 contains a more complete discussion of dual variables and other Linear Programming concepts and applications.

2.4 The Reference Energy System and its components

2.4.1 The RES concept

Summarizing elements of section 2.3 of this report, the SAGE energy economy consists of:

- *Demands*, that represent the energy services that must be satisfied by the system;
- Energy *sources* (mining or imports) that represent methods of acquiring various energy carriers;
- Energy *sinks*, which represent exports;
- *Technologies* (also called processes), that either transform an energy carrier to another form or into a useful energy service, and
- *Commodities* consisting of energy carriers, energy services, and emissions that are either produced or consumed by the energy sources, sinks, technologies and demands.

It is helpful to summarize the relationships among these various entities using a network diagram referred to as a Reference Energy System (RES). In the complete SAGE RES a node depicts each region, source, sink, technology, and demand, and each commodity (energy carrier, energy service, emission) is depicted by a link.

Figure 2.1 depicts a small portion of a hypothetical RES containing one energy service demand, residential space heating. There are three end-use space heating technologies using the energy carriers gas, electricity, and heating oil, respectively. These energy carriers in turn are produced by other technologies, represented in the diagram by three electricity-generating plants (coal fired, gas fired, oil fired), an oil refinery, and a gas plant. To complete the production chain on the primary energy side, the diagram also represents two sources of crude oil (one extracted domestically and one imported, each using a pipeline), an extraction source for natural gas, and an extraction source for coal. Note that in the RES every time a commodity enters/leaves a process it's name is changed (e.g., wet gas becomes dry gas, crude becomes pipeline crude). This simple rule enables the inter-connectively between the processes to be properly maintained throughout the network.

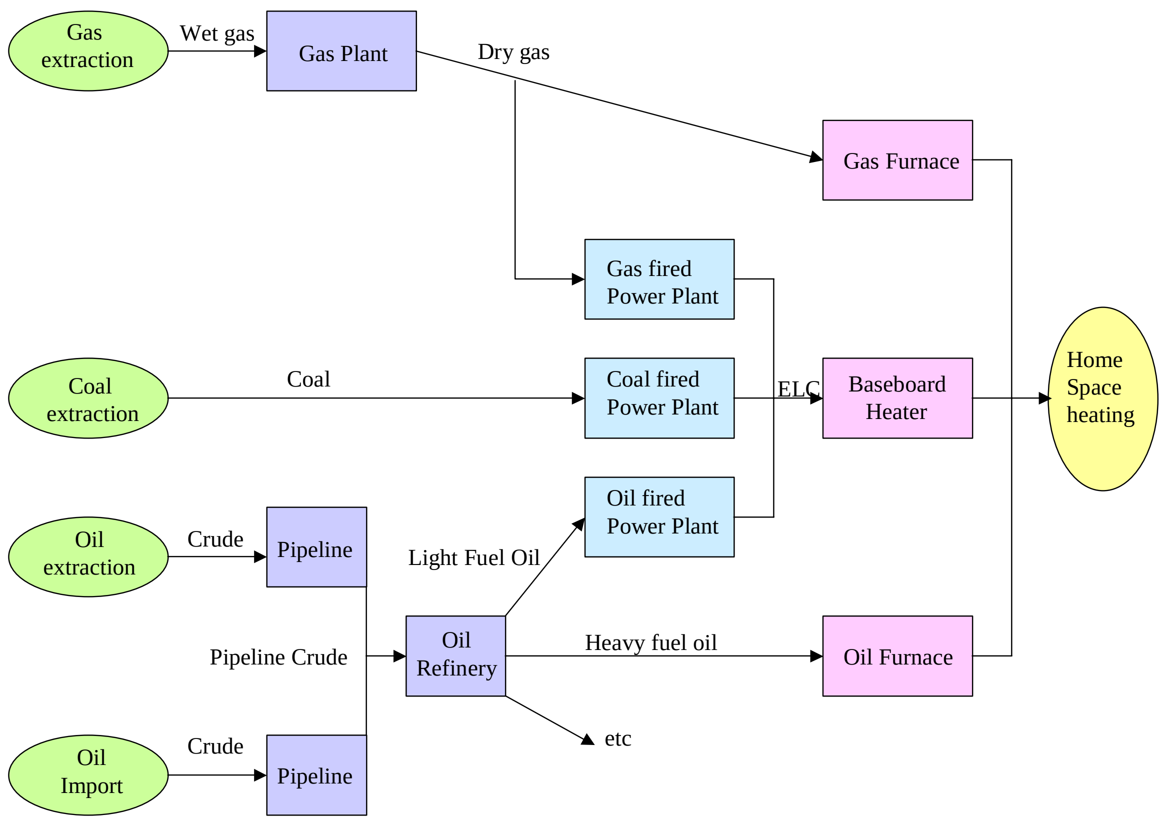

Figure 2.1. Partial view of a Reference Energy System

Nodes and commodities are classified into *sets*. Each set of SAGE regroups components of a similar nature. In this section, we briefly review the main sets of components and we informally describe the properties and features of the components in each set. The complete description of all sets and of the attributes of each set member, is covered in chapter 4. Volume 2: Data Implementation Guide, contains a very detailed description of all technologies, demands, sources and sinks, as well as the energy carriers and emissions of SAGE.

In SAGE, technology and process naming conventions play a critical role in allowing the easy identification of RES element set memberships. This is a very important feature that allows the user to quickly explore the structure of SAGE. For instance, each energy carrier for use in the transportation sector has a six-letter name starting with TRA and ending with the energy form (thus gasoline for vehicles is called TRAGSL). Technology names have nine letters, the first three indicating the subsector, the next three the fuel/technology combination (standard, medium and high efficiency heat pumps), and the last three reserved to distinguish between vintages of technologies. The characters '000' indicate that the technology is an existing technology; '005' refers to a technology first available in 2005. For instance, the standard residential electric water heater available for purchase in 2005 is named RWHELC005. We refer the reader to Volume 2 for a comprehensive description of the components of the SAGE RES.

2.4.2 Overview of SAGE SET Definitions

2.4.2.1 SRCENCP: the set of sources and sinks

SRCENCP contains all source nodes: Mining, Imports, Exports, and physical Renewables such as biomass (not wind, etc.). Each source node deals with a single commodity such as mined natural gas. However, it is possible to distinguish between several steps in the mining of the same resource (gas), each step having its own cost and other parameters. For example in the case of natural gas there may be several steps, for production from located reserves, from enhanced recovery and from undiscovered reserves. Each step has its own production cost and volume available at that cost. This allows the modeling of supply curves. Each source has attributes that specify its cumulative resource availability, cost of procurement (or revenue from export), and bounds on its annual production activity at each period. Sources may also require the consumption of other commodities in order to operate (just like a technology).

Note again that in SAGE, imports into, and exports from, a region may be exogenously specified by the user or endogenously determined. In the former case, a commodity is purchased from (or sold to) an unspecified region, at an exogenously specified price, whereas, in the latter case, trade is between specific regions and the unit value (shadow price) of the traded energy carrier is calculated endogenously.

Renewables for which there are physical (e.g., biomass, MSW), as opposed to "free" (the sun, wind) commodities are also modeled as source nodes in the same manner as the mining.

2.4.2.2 PRC: the set of Process technologies

The set **PRC** contains all technologies that transform energy forms into other energy forms, *except those producing electricity or low temperature heat.* Each PRC technology may also have material inputs and outputs as well as emissions. The set PRC is usually quite large, having typically hundreds of elements. Apart from the inputs and outputs, a PRC member has up to four types of cost attributes attached to it: investment cost (per unit of new capacity), fixed annual maintenance (per unit of existing capacity), variable operating cost (per unit of activity), and delivery costs (per unit of each commodity required to operate the technology). Each PRC technology is further characterized by its life duration, the period when it first becomes available, and its discount rate (used to compute the annualized investment cost). Its operation may also be altered by bounds expressed in absolute terms or as maximum growth rate per period. Finally, each PRC technology may have emission coefficients attached to each of its three variables: investment, capacity, and activity.

2.4.2.3 CON: the set of conversion technologies

The set **CON** contains technologies that produce either electricity or low temperature heat. **CON** consists of three primary subsets: **ELE**, **HPL,** and **CPD**. **ELE** contains the electricity conversion technologies, **HPL** the set of technologies producing low temperature heat, and **CPD** the coupled production technologies (producing heat and power in combination). The two energy carriers *heat* and *electricity* are special, since they are not easily stored. They have the dedicated sets *ELC* and *LTH*. The members of these sets are defined for each of the relevant time-slices (six for electricity, three for heat), whereas all other commodities are defined only as annual flows. The **ELE** set is further composed of subsets, whose members may have somewhat different roles in SAGE. For instance, power plants designated as base load plants (**BAS)** are constrained to operate at the same rate day and night in the same season. Another example: all elements of the subset of centralized plants (**CEN),** incur transmission losses on the grid; whereas those in the subset of decentralized plants (**DCN)** do not.

Each **CON** technology has the same attributes mentioned earlier for **PRC** technologies, plus a few more describing the seasonal and diurnal availability of the capacity. For example, solar technologies (without battery storage) are not available during the night. And wind power may vary by season.

In addition, each **CON** technology has a peaking attribute defining its ability to produce during the peaking demand time-slice.

2.4.2.4 DMD: the set of end-use technologies

The set **DMD** contains all *end-use technologies*; defined as technologies whose output provides an energy service to a final consumer (table 2.1 lists the energy services modeled in SAGE). The first three letters of a technology's name must be adhered to rigidly since the model relies on these letters to assign each technology to a particular energy service demand. For instance, technology RK1KER000 is an existing kerosene cooker. The first three letters RK1 signal to SAGE that this device services cooking demand in sub region 1.

Note that **DMD** technologies do not have an activity variable. It is implicitly assumed that the technology operates at a level proportional to its capacity[1].

2.4.2.5 DM: the set of demands

The set **DM** contains the energy services that must be satisfied. Each energy service is expressed in SAGE in units of useful energy. Table 2.1 contains the list of the 42 energy service demand categories in SAGE, grouped by economic sector. Note that the first

[1] When developing technology cost estimates for SAGE the modeler must explicitly make assumptions regarding capacity. For example, the number of cars needed to provide a billion vehicle kilometers per year depends on assumptions regarding kilometers per year.

letter of each energy service designates the sector (C for commercial, R for residential, A for agriculture, I for industry, and T for transport).

Each energy service has several attributes that describe (a) the amounts of service to be satisfied at each time period, (b) the seasonal/time-of-day nature of these requirements (or electricity and heat), and (c) the price-elasticity of the demand and the allowed interval of demand variation.

Table 2.1. Energy services in SAGE

Transportation segments (15)	Codes
Autos	TRT
Buses	TRB
Light trucks	TRL
Commercial trucks	TRC
Medium trucks	TRM
Heavy trucks	TRH
Two wheelers	TRW
Three wheelers	TRE
International aviation	TAI
Domestic aviation	TAD
Freight rail transportation	TTF
Passengers rail transportation	TTP
Internal navigation	TWD
International navigation (bunkers)	TWI
Non-energy uses in transport	NEU
Residential segments (11)	**Codes**
Space heating	RH1, RH2, RH3, RH4
Space cooling	RC1, RC2, RC3, RC4
Hot water heating	RWH
Lighting	RL1, RL2, RL3, RL4
Cooking	RK1, RK2, RK3, RK4
Refrigerators and freezers	RRF
Cloth washers	RCW
Cloth dryers	RCD
Dish washers	RDW
Miscellaneous electric energy	REA
Other energy uses	ROT
Commercial segments (8)	**Codes**
Space heating	CH1, CH2. CH3, CH4
Space cooling	CC1, CC2. CC3. CC4
Hot water heating	CHW
Lighting	CLA
Cooking	CCK
Refrigerators and freezers	CRF
Electric equipments	COE
Other energy uses	COT
Agriculture segments (1)	**Codes**
Agriculture	AGR
Industrial segments (6)	**Codes**
Iron and steel	IIS
Non ferrous metals	INF
Chemicals	ICH
Pulp and paper	ILP

Non metal minerals	INM
Other industries	IOI
Other segment (1)	**Codes**
Other non specified energy consumption	ONO

Industrial energy services such as IIS (iron and steel) are made up of a 'recipe' of more detailed services—steam, process heat, machine drive, electrolytic service, other, and feedstock. For example ISIS, IPIS, and IMIS refer, respectively, to the steam, process heat, and machine drive needed in the iron and steel industry.

Investment in iron and steel capacity only includes the energy cost [process heat, steam, feedstock, etc] of a million metric tons of steel capacity. The unit of output and capacity for iron and steel, non-ferrous metals and pulp and paper is million metric tons. The output of other industries is measured in units of Petajoules of energy service at base year technology efficiencies.

2.4.2.6 ENC: the set of energy carriers

Energy carriers are classified as fossil, nuclear, renewable, or synthetic, plus electricity and heat. ENC is a primary subset of all energy carriers that excludes electricity and heat.

In SAGE, a deliberate choice is made to strongly identify the fuels to the sector or subsector that produces or consumes it, thus allowing distribution costs, price markups, taxes, and carbon constraints to be sector and/or subsector specific. Thus a tax may be levied on industrial distillate use but not on agricultural distillate use. As before, this is accomplished by strict adherence to energy carrier naming conventions. A full list of energy carrier names is provided in Volume 2, Data Implementation Guide. In Table 2.2, we show as an example the subset of fuels used in the transportation sector. Note that all fuel names for that sector start with the three letters TRA, thus clearly identifying the sector. The same rule is applied to energy carriers in all sectors.

Note also that the renaming of fuels in each sector requires the creation of technologies whose only role is to rename a fuel. For instance, technology TRAELC000 transforms one unit of electricity (ELC) into one unit of electricity dedicated to the transportation sector (TRAELC). On the other hand, most technologies producing a transportation sector fuel are bona fide technologies, as for instance technology TRAMET100 that processes natural gas (GASMET) into methanol (TRAMET).

Table 2.2. List of fuel names in the transportation sector

Energy Carrier's code	Description
TRANGA	Natural Gas
TRADST	Diesel fuel
TRAGSL	Gasoline
TRAHFO	Heavy fuel oil
TRACOA	Coal
TRALPG	Light Petroleum Gas
TRAAVG	Aviation Gasoline

TRAJTK	Let Fuel
TRAMET	Methanol
TRAETH	Ethanol
TRAELC	Electricity

Despite the fact that SAGE does not attribute a specific variable to each energy carrier, there are nevertheless certain attributes that are attached to the electricity and low temperature heat energy carrier, describing the transmission/distribution costs and efficiency of the “grid” to which these energy carriers are attached.

2.4.2.7 ENV: the set of environmental emissions

ENV is the set of environmental emissions. Contrary to energy carriers, a SAGE emission variable is created for each emission commodity. This allows the user to model certain environmental policy instruments such as a cap or a tax on the total emission of a particular substance. Caps may be applied at each period or cumulatively.
There is also the possibility to transform an emission into another emission (for instance methane may be converted into CO2-equivalent).

Finally, negative emission coefficients are perfectly acceptable in SAGE, thus permitting technologies to absorb an emitted substance (for instance, a geological CO2 sequestering technology would have a negative CO2 emission coefficient).

2.4.3 System-Wide attributes

Several attributes are not attached to any particular technology or commodity, but are defined for a region as a whole. Examples are: discount rate[1], number of years per period, and the fraction of the year of the six time-slices used for electricity (and heat) production. Also in this category are a few switches that allow the user to activate or deactivate the elastic demands and technology market sharing mechanism.

[1] Technology specific discount rates override the general regional discount rate if the modeler specifies them.

3 Evolution of the SAGE modeling approach

This chapter provides an economic interpretation of SAGE and other partial equilibrium models. These models have at least one thing in common--they consider the production and consumption of energy services simultaneously. That is, the price of producing an energy service affects energy service demand, while, at the same time, energy service demand affects energy service price. A market is said to have reached an equilibrium price when no consumers wish to purchase any more and no producers wish to produce any more. As described further below, when all markets are in equilibrium the economies 'total surplus' is maximized.

The concept of total surplus maximization extends the cost minimization approach upon which earlier Bottom-up models were based. These former models had fixed energy service demands, and thus were content to minimize the cost of supplying these demands. In SAGE the demands for energy services are elastic to their own prices, thus allowing the model to compute bona fide supply-demand equilibrium.[1] Section 3.1, provides a brief review of different types of energy models. Section 3.2, discusses the economic rationale of the SAGE model with emphasis given to the features that distinguish SAGE from other partial equilibrium models such as the standard MARKAL[2] model (Fishbone and Abilock, 1981, Berger et al., 1992). Section 3.3 describes how price elastic demands are modeled in SAGE, and section 3.4 provides additional information concerning the economic properties of SAGE.

3.1 A brief discussion of energy models

Many energy models are in current use around the world, each designed to emphasize a particular facet of interest. Differences include economic rationale, level of disaggregation of decision variables, time horizon over which decisions are made, and geographic scope. One of the most significant differentiating features among energy models is the degree of detail with which commodities and technologies are represented.

3.1.1 Top Down Models

At one end of the spectrum are aggregated general equilibrium models. In these, each sector is represented by a single production function designed to simulate the possible substitutions between the main factors of production [also highly aggregated]: energy, capital, and labor in the production of goods and energy services. In this model category are found a number of *General equilibrium* (G.E.) models of national or global energy systems. These models are usually called "Top Down", because they represent an entire economy by a relatively small number of aggregate variables and equations. In these models, production function parameters are calculated for each sector such that inputs

[1] It is assumed that cross price elasticities are zero.

[2] MARket ALlocation model, a technology rich model developed by the Energy Technology Systems Analysis Programme, an implementing agreement of the International Energy Agency. The reader is referred to the ETSAP web site, www.etsap.org, for a thorough treatment of the MARKAL model and its many variations. The SAGE model includes most features of the MARKAL model, plus new ones described in this chapter.

and outputs reproduce a single base historical year.[1] In policy runs, the mix of inputs[2] required to produce one unit of a sector's output is allowed to vary according to user-selected elasticities of substitution. Sectoral production functions most typically have the following general form:

$$X_s = A_0\left(B_K \cdot K_s^{\rho} + B_L \cdot L_S^{\rho} + B_E \cdot E_S^{\rho}\right)^{1/\rho} \quad (3.1\text{-}1)$$

Where X_S is the output of sector *S*,
K_S, L_S, *and* E_S are the inputs of capital, labor and energy needed to produce one unit of output in sector S
ρ is the elasticity of substitution parameter
A_0 and the *B*'s are scaling coefficients.

Note that the choice of ρ determines the ease or difficulty with which one production factor may be substituted for another (the smaller ρ is –[but still greater than or equal to 1], the easier it is to substitute the factors to produce the same amount of output from sector *S*). And that the degree of factor substitutability does not vary among the factors of production—the ease with which capital can be substituted for labor is equal to the ease with which capital can be substituted for energy, while maintaining the same level of output.

3.1.2 Bottom Up Models

At the other end of the spectrum are the very detailed, *technology explicit* models that focus only on the energy sector of an economy. In these models each important[3] energy-using technology is identified by a detailed description of its inputs, outputs, unit costs, and several other technical and economic characteristics. In these so-called "Bottom-up" models, a sector is constituted by a (usually large) number of logically arranged technologies, linked together by their inputs and outputs (which may be energy forms or *carriers*, materials, and/or emissions). Some bottom-up models compute a partial equilibrium via maximization of the total net [consumer and producer] surplus; while others simulate other types of behavior by economic agents, as will be discussed further down. In bottom-up models, one unit of sectoral output (e.g., a billion vehicle kilometers of heavy truck service or a Petajoule of residential cooling service) is produced using a mix of individual technologies' outputs. Thus the production function of a sector is *implicitly* constructed, rather than explicitly specified as in more aggregated models. Such implicit production functions may be quite complex, depending on the complexity of the Reference Energy System (RES) of each sector.

3.1.3 Recent Modeling Advances

While the above dichotomy applied fairly well to earlier models, these distinctions now tend to be somewhat blurred by recent advances in both categories of models. In the case

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[1] These models assume that the relationships (as defined by the form of the production functions as well as the calculated parameters) between sector level inputs and outputs are in equilibrium in the base year.
[2] Most models use inputs such as labor, energy, and capital, but other input factors may conceivably be added, such as arable land, water, or even technical know-how)
[3] The technologies that are deemed important reflect the purpose for which the model is being developed.

of aggregate top down models, several general equilibrium models now include a fair amount of fuel and technology disaggregation in the key energy producing sectors (for instance: electricity production, oil and gas supply). This is the case of MERGE[1] and SGM[2], for instance. In the other direction, the more advanced bottom-up models are 'reaching up' to capture some of the effects of the entire economy on the energy system. For instance, the MARKAL and SAGE models have end-use demands [including demands for industrial output] that are sensitive to their own prices, and thus capture the impact of rising energy prices on economic output and *vice versa*. Recent incarnations of technology-rich models are multi-regional, and thus are able to consider the impacts of energy-related decisions on trade. It is worth noting that while the multiregional top-down models have always represented trade, they have done so with a very limited set of traded commodities --typically one or two, whereas there may be quite a number of traded energy forms and materials in multi-regional bottom-up models). MARKAL-MACRO (see [9]) is a hybrid model combining the technological detail of MARKAL with a succinct representation of the macro-economy consisting of a single producing sector. Because of its succinct single-sector production function, MARKAL-MACRO is able to compute equilibrium in a single optimization step. The NEMS[3] model is another example of a full linkage between several technology rich modules of the various sectors and macro-economic equations, although the linkage here is not as tight as in MARKAL-MACRO, and thus requires iterative resolution methods.

In spite of these advances, there remain important differences. Specifically:

- Top-down models encompass macroeconomic variables beyond the energy sector proper, such as wages, consumption, and interest rates, and
- Bottom-up models have a rich representation of the variety of technologies (existing and/or future) available to meet energy needs, and, they often have the capability to track a wide variety of traded commodities.

The Top-down vs. Bottom-up strategy is not the only relevant difference among energy models. Among Top-down models, the so-called Computable General Equilibrium models (CGE) differ markedly from the macro econometric models. The latter do not compute equilibrium solutions, but rather simulate the flows of capital and other monetary quantities between sectors. They use econometrically derived input-output coefficients to compute the impacts of these flows on the main sectoral indicators, including economic output (GDP) and other variables (labor, investments). The sectoral variables are then aggregated into national indicators of consumption, interest rate, GDP, labor, and wages.

Among technology explicit models, two main classes are usually distinguished: the first class is that of the partial equilibrium models such as SAGE that use optimization techniques to compute a least cost (or maximum profit) path for the energy system. The second class is that of simulation models, where the emphasis is on representing a system

[1] Model for Evaluating Regional and Global Effects (Manne et al., 1995)
[2] Second Generation Model (Edmonds et al., 1991)
[3] National Energy Modeling System, a detailed integrated equilibrium model of an energy system linked to the economy at large, US Dept of Energy, Energy Information Administration (2000)

not governed purely by profit or utility maximizing behavior. In pure simulation models (e.g. ISTUM) investment decisions taken by a representative firm or consumer are only partially based on profit maximization, and technologies may capture a share of the market even though their life-cycle cost may be higher than that of other technologies. Simulation models use market-sharing algorithms that preclude the easy computation of equilibrium--at least in a single pass. The SAGE model, although mainly an optimizing partial equilibrium model, nevertheless possesses a market sharing mechanism that allows it to reproduce certain behavioral characteristics of observed markets.

Section 3.2 focuses on the class of partial equilibrium, technology explicit models. These models are also known as Optimization based equilibrium models, since the equilibrium is obtained by maximizing the net total consumer and producer surplus. The SAGE model and its precursor, MARKAL, are both based on this paradigm, but SAGE incorporates some additional features that enable it to better approximate the behavior of observed markets. For instance, when run in time-stepped mode, SAGE takes a myopic view of the evolution of the energy system over the planning horizon; while MARKAL, a dynamic linear programming system, optimizes over the entire modeling horizon at once. The appropriate modeling tactic, as usual, depends on the purpose of the study at hand. SAGE only solves for one period at a time making it much easier to solve than the equivalent dynamic optimization problem. In addition, the results obtained for the current (and previous) period(s) can be examined and adjustments made to “guide” the model. This is the case with the SAGE market share feature (Section 3.2). This time-stepped approach is appropriate if the goal is to develop reference case and policy analysis scenarios describing how energy markets are likely to evolve over time given consumer and producer behaviors, and given no knowledge of future events beyond the current period.

3.2 The SAGE paradigm

This documentation is as self-contained as possible; however, certain portions of this and the next sections require an understanding of the theory and terminology of Linear Programming. The reader requiring a brush-up on this topic may first read section 3.4, and then, if needed, some standard textbook on LP, such as Hillier and Lieberman (1990) or Chvatal (1983). The application of Linear Programming to microeconomic theory is covered in Gale (1960), and in Dorfman, Samuelson, and Solow (1958, and subsequent editions).

A brief description of SAGE would indicate that it is a:

- *Technology explicit*,
- *Multi-regional*,
- *Partial equilibrium* model with
- *Elastic demands* and
- *Limited foresight*, employing a
- *Behavioral approach* for determining *market shares*.

While the first four of these characteristics are inherited from its MARKAL predecessor, the last two (limited foresight and a behavior-based market share mechanism) are features that are unique to SAGE, and make the model a hybrid between optimizing and behavioral models. These two features have been added to help depict the evolutionary nature of energy and technology markets.

3.2.1 A technology explicit model

As already presented in chapter 2 (and described in much more detail in chapter 4), each technology in SAGE is described by a number of technical and economic parameters. Thus, each technology is explicitly identified and distinguished from all others in the model. The current version of SAGE already includes several thousand technologies in all sectors of the energy system of a given region (energy procurement, conversion, processing, transmission, and end-uses). Thus, SAGE is not only technology explicit; it is *technology rich* as well. Furthermore, the number of technologies may be increased without the user ever having to modify the model's equations. Volume 2 provides a sector-by-sector description of the technological variety existing in the current SAGE model.

3.2.2 Multi-regional feature

SAGE currently includes 15 regional modules, linked by energy trading variables--and by emission permit trading variables if desired. The trade variables transform the 15 regional modules into a single global energy model where actions taken in one region may affect all other regions. This feature is of course essential when global as well as regional energy and emission policies are being simulated via SAGE. SAGE also tracks OPEC versus non-OPEC oil and petroleum product trading. This allows the analyst to consider scenarios in which OPEC oil production is curtailed or in which a limit is set on the share of oil imported from OPEC.

3.2.3 Partial equilibrium properties

SAGE computes a partial equilibrium for energy markets. This means that the model computes both the flows of energy forms and materials as well as their prices, in such a way that, at the prices computed by the model, the suppliers of energy produce exactly the amounts that the consumers are willing to buy. This equilibrium feature is present at every stage of the energy system: primary energy forms, secondary energy forms, and energy services. Any supply-demand equilibrium model must have a rationale that drives the computation of the equilibrium. The underlying principles central to the SAGE equilibrium are that:

- Output of a technology is a linear function of its inputs;
- Total net surplus is maximized over the current period (alternatively: over all periods if a dynamic version of SAGE is used);
- Energy markets are competitive, and that
- Economic agents have limited foresight (over one five-year period only, unless the dynamic version of SAGE is used).

As a result of these assumptions the:

- Market price of each commodity is exactly equal to its marginal value in the overall system, and
- Each economic agent maximizes profit (or utility).

3.2.3.1 Linearity

A linear input-to-output relationship means that each technology may be implemented at any capacity, from zero to some upper limit, without economies or diseconomies of scale. In a real economy, a given technology is usually available in discrete sizes, rather than on a continuum. In particular, for some real life technologies, there may be a minimum size below which the technology cannot be implemented (or else at a prohibitive cost), as for instance a nuclear power plant, or a hydroelectric project. In such cases, because SAGE assumes that all technologies may be implemented in any size, it may happen that the model's solution shows some technology's capacity at an unrealistically small size. However, in the SAGE context, such a situation is relatively infrequent and innocuous, since the scope of application is at the regional level, and thus large enough that small capacities are unlikely to occur. Of course, it is possible to implement a version of SAGE where certain capacities are allowed only in multiples of a given size, by introducing integer variables. Such a facility was just recently added to the basic MARKAL model and could be adapted for SAGE. This approach should, however, be used sparingly because it greatly increases solution time. *More simply, a user may add ad hoc bounds (Investment Bound* $_{UP}$ *(k,r,t) = 0) if the solution shows capacities that are clearly too small.*

It is the linearity property that allows the SAGE equilibrium to be computed using Linear Programming techniques. In the case where economies of scale or some other non-convex relationship is important to the problem being investigated, the optimization program would no longer be linear or even convex. The fact that SAGE's equations are linear, however, does not mean that all production functions behave in a linear fashion. Indeed, the SAGE production functions are capable of reproducing highly non-linear behavior by representing non-linear functions as a stepped sequence of linear functions. As an example, a supply of some resource may be represented in SAGE as a sequence of segments, each with rising, but constant marginal cost. The modeler defines the 'width' of each segment so that the resulting supply curve may simulate any non-linear convex function.

3.2.3.2 Maximization of total surplus: Marginal Cost Equals Marginal Price

The *total surplus* of an economy is the sum of the suppliers' and the consumers' surpluses. The term *supplier* designates any economic agent that produces (and sells) one or more commodities (i.e., in SAGE an energy form, an emission permit, and/or an energy service). A *consumer* is a buyer of one or more commodities. Some agents may be both suppliers and consumers, but not for the same commodity. Therefore, for a given commodity, the Reference Energy System defines a set of suppliers and a set of consumers.

It is customary to represent the set of suppliers of a commodity by their inverse supply function, which plots the marginal production cost of the commodity (vertical axis) as a function of the quantity supplied (horizontal axis). In SAGE, as in other linear optimization models, the supply curve of a commodity is not explicitly expressed as a function of factor inputs (such as aggregate capital, labor and energy concepts in the Cobb-Douglas or Constant Elasticity of Substitution production function). However, because Linear Programming is used, it is a standard result that the inverse supply function is step-wise constant and increasing (Figure 3.1 for the case of a single commodity[1]). Each horizontal step of the inverse supply function indicates that the commodity is produced by a certain technology or set of technologies in a strictly linear fashion. As the quantity produced increases, one or more resources in the mix (technological potential or resource availability) is exhausted, and therefore the system must start using a different (more expensive) technology or set of technologies in order to produce additional units of the commodity, albeit at higher unit cost. Thus, each change in production mix generates one step of the staircase production function. The width of any particular step depends upon the technological potential and/or resource availability associated with the set of technologies represented by that step.

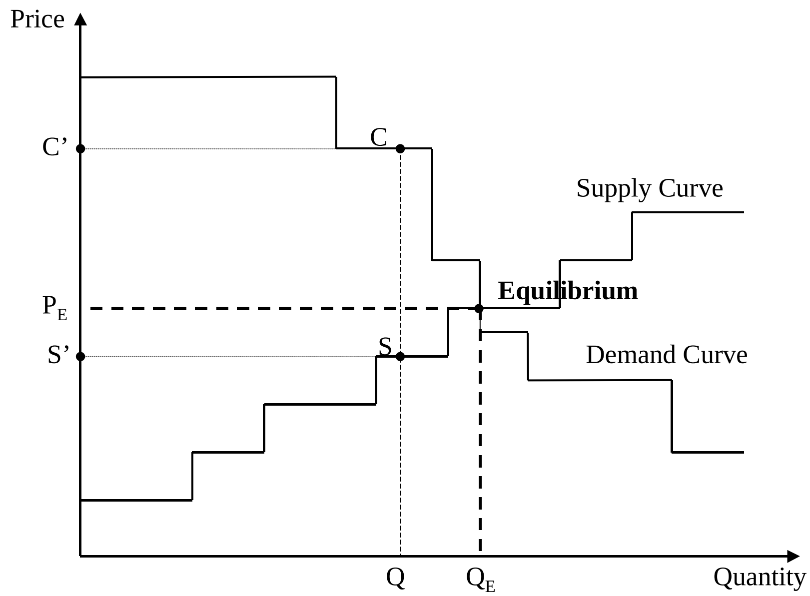

Figure. 3.1. *Equilibrium in the case of an energy form: the model implicitly constructs both the supply and the demand curves*

In an exactly symmetric manner, the set of consumers of a commodity have their implicit inverse demand function, which is a step-wise constant, decreasing function of the quantity demanded. As shown in Figure 3.1 the supply-demand equilibrium is at the

[1] This is so because in Linear Programming, the shadow price of a constraint remains constant over a certain interval, and then changes abruptly, giving rise to a stepwise constant functional shape.

intersection of the two functions, and corresponds to an equilibrium quantity Q_E and an equilibrium price P_E[1]. At price P_E, suppliers are willing to supply the quantity Q_E and consumers are willing to buy exactly that same quantity $Q_{E.}$ Of course, the SAGE equilibrium concerns many commodities, and the equilibrium is a multi-dimensional analog of the above definition, where Q_E and P_E are now vectors rather than scalars.

The above description of the SAGE equilibrium is valid for any energy form that is entirely endogenous to SAGE. In the case of a commodity that is an energy service, SAGE does not implicitly construct the demand function. Rather, the user *explicitly* defines the demand function. Each energy service demand is a constant elasticity function of the form:

$$D/D_0 = (P/P_0)^{-e} \qquad (3.3\text{-}1)$$

Where $\{D_0, P_0\}$ is a reference pair of demand and price values for that energy service over the forecast horizon, and e is the price elasticity of energy service demand chosen by the user (Note, though not shown, this price elasticity also has a time dimension). The pair $\{D_0, P_0\}$ is obtained by solving SAGE for a reference (base case) scenario. More precisely, D_0 is the demand projection estimated by the user in the reference case based upon explicitly defined relationships to economic and demographic drivers, and P_0 is the shadow price of that energy service demand obtained by running a reference case scenario of SAGE (Figure 3.2).

Using Figure 3.1 as a reference, the definition of the suppliers' surplus corresponding to a certain point S on the inverse supply curve is the difference between the total revenue and the total cost of supplying a commodity, i.e. the gross profit. In figure 3.1, it is thus the area between the horizontal segment SS' and the inverse supply curve. Similarly, the consumers' surplus for a point C on the inverse demand curve, is defined as the area between the segment CC' and the inverse demand curve. This area is a consumer's analog to a producer's profit, more precisely it is the cumulative opportunity gain of all consumers who purchase the commodity at a price lower than the price they would have been willing to pay. For a given quantity Q, the total surplus (suppliers' plus consumers') is simply the area between the two inverse curves situated at the left of Q. It should be clear from Figure 3.1 that the total surplus is maximized exactly when Q is equal to the equilibrium quantity Q_E. Therefore, we may state (in the single commodity case), "the equilibrium is reached when the total surplus is maximized". This result is referred to as the *Equivalence Principle*.

In the multi-dimensional case, the proof of the above statement is less obvious, and requires a certain qualifying property (called the integrability property) to hold (Samuelson, 1952, Takayama and Judge, 1972). A sufficient condition for the integrability property to be satisfied is realized when the cross price elasticities of any two energy forms are equal (i.e. $\partial \boldsymbol{P_j} / \partial \boldsymbol{Q_i} = \partial \boldsymbol{P_i} / \partial \boldsymbol{Q_j} \quad \boldsymbol{for\ all\ i, j}$).

[1] As may be seen in figure 3.1, the equilibrium is not unique. In the case shown any point on the vertical segment contain the equilibrium is also an equilibrium, with the same Q_E but a different P_E. In other cases, the multiple equilibria will have the same price and different quantities.

In the case of commodities that are energy services, these conditions are satisfied in SAGE, because we have assumed zero cross price elasticities. In the case of an energy form, where the demand curve is implicitly derived, it is also possible to show that the integrability property is always verified[1] and thus the equivalence principle is valid.
In summary, the equivalence principle guarantees that the SAGE supply-demand equilibrium maximizes total surplus. And the total surplus concept has long been a mainstay of social welfare economics because it takes into account both the surpluses of consumers and of producers.[2]

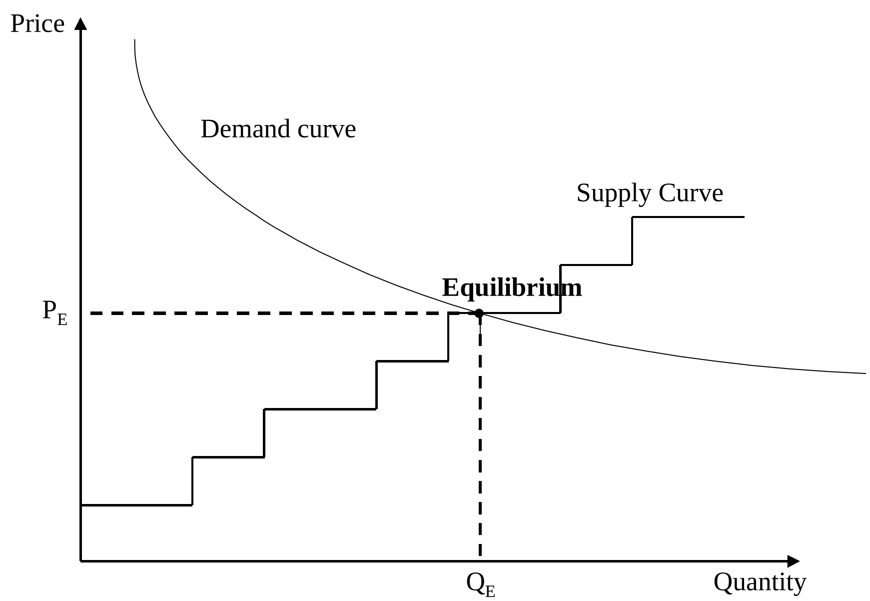

Figure 3.2. *Equilibrium in the case of an energy service: the user, usually with a simple functional form, explicitly provides the demand curve.*

In older versions of MARKAL, and in other so-called “least-cost” models, energy service demands are completely inelastic and only the cost of supplying these energy services is minimized. Such a case is illustrated in Figure 3.3 where the “inverse demand curve” is a vertical line. The objective of such models was simply the minimization of the total cost of meeting exogenously specified levels of energy service.

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[1] This results from the fact that in SAGE each price $\boldsymbol{P_i}$ is the shadow price of a balance constraint, and may thus be expressed as the derivative of the objective function $\boldsymbol{F}$ with respect to the right-hand-side of a This results from the fact that in SAGE each price $\boldsymbol{P_i}$ is the shadow price of a balance constraint, and may thus be expressed as the derivative of the objective function $\boldsymbol{F}$ with respect to the right-hand-side of a constraint, i.e. $\partial \boldsymbol{F} / \partial \boldsymbol{Q_i}$. When that price is further differentiated with respect to another quantity $\boldsymbol{Q_j}$, one gets $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_i} \bullet \partial \boldsymbol{Q_j}$, which, under mild conditions is always equal to $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_j} \bullet \partial \boldsymbol{Q_i}$, as desired.

[2] See e.g. Samuelson, P, and W. Nordhaus (1977)

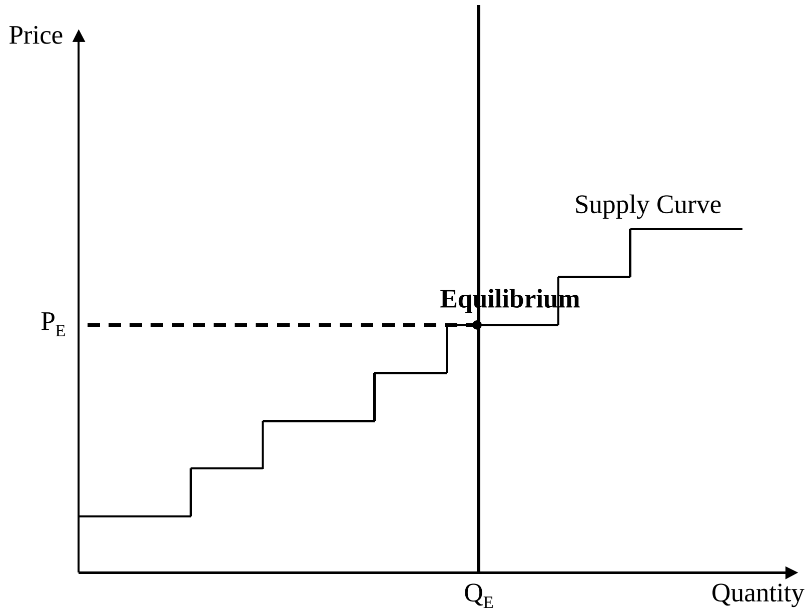

***Figure 3.3**: Equilibrium when an energy service demand is fixed*

3.2.3.3 Competitive energy markets with limited foresight

Competitive energy markets are characterized by perfect information and an atomization of the economic agents, which together preclude any of them from exercising market power. That is, neither the level any individual producer supplies, nor the level any individual consumer demands, affects the equilibrium market price. In the usual, myopic, or time-stepped, version of SAGE, the perfect information assumption is limited to the current five-year period, so that each agent has perfect knowledge of the market's parameters in the current decision period, and no information at all regarding future periods. Hence, the equilibrium is computed by maximizing total surplus sequentially, one period at a time. This type of behavior is called myopic in comparison to an assumption where all agents have perfect information over the entire time horizon. In the latter case, the equilibrium is called inter-temporal or dynamic.

The SAGE time-stepped equilibrium computation proceeds as follows:

- Maximize total surplus for period 1 only, and then freeze all investments for period 1;
- Maximize total surplus for period 2, knowing that capacity associated with period 1 investments remain in period 2, and
- Repeat for all periods till last the last period.

Remark: SAGE versions where all agents have foresight over more than one period may be constructed, in which case the equilibrium would be constructed by maximizing at once the net present value of the sequence of total surpluses at all periods. For example, the MARKAL model has traditionally been run with perfect foresight over all periods.

3.2.3.4 Marginal value pricing

The duality theory of Linear Programming indicates that for each constraint of the SAGE linear program, there is a dual variable. This dual variable (when an optimal solution is reached) is equal to the marginal change of the objective function per unit increase of the constraint's right-hand-side. As already noted in chapter 2, the dual variable of the balance constraint of a commodity (whether it be an energy form, a service demand, or an emission) represents the market price of the commodity. Equally in SAGE, the price of a commodity is equal to its *marginal value* in the system, also called *shadow price*[1]. The fact that the price of a commodity is equal to its' marginal value is an important feature of competitive markets. Duality theory does not necessarily indicate that the marginal value of a commodity is equal to the marginal cost of *producing* that commodity. For instance, in the equilibrium shown in Figure 3.4, the equilibrium price does not correspond to *any* marginal supply cost. In this case, the price is determined by demand and the term *marginal cost pricing* (so often used in the context of optimizing models) doesn't apply. Therefore the term *marginal value pricing* is a more generally appropriate term to use.

Marginal value pricing does not imply that suppliers have zero profit. Profit is exactly equal to the suppliers' surplus, and Figures 3.1 through 3.4 show that it is not generally equal to zero. Only the last few units produced may have zero profit, if, and when, their production cost equals the equilibrium price, and even in this case zero profit is not automatic.

In SAGE, the shadow prices of commodities play a very important diagnostic role. If some shadow price is clearly out of line (i.e. if it seems much too small or too large compared to commodity market prices), this indicates that the model's RES may contain some errors. The examination of shadow prices is just as important as the analysis of the quantities produced when analyzing the results of a SAGE run. Note that the SAGE endogenous market share algorithm employs the marginal cost (shadow prices) of grouped technologies competing to service a market (e.g. alternative fuel cars versus conventional cars). Thus the model itself employs these endogenous prices between periods to adjust the technology market shares.

[1] The term Shadow Price is often used in the mathematical economics literature, whenever the price is derived from the marginal value of a commodity. The qualifier 'shadow' is used to distinguish the competitive market price from the price observed in the real world, which may be different, as is the case in regulated industries or in sectors where either consumers or producers exercise market power. When the equilibrium is computed using LP optimization, as is the case for SAGE, the shadow price of each commodity is computed as the dual variable of that commodity's balance constraint, as will be further developed in section 3.4.

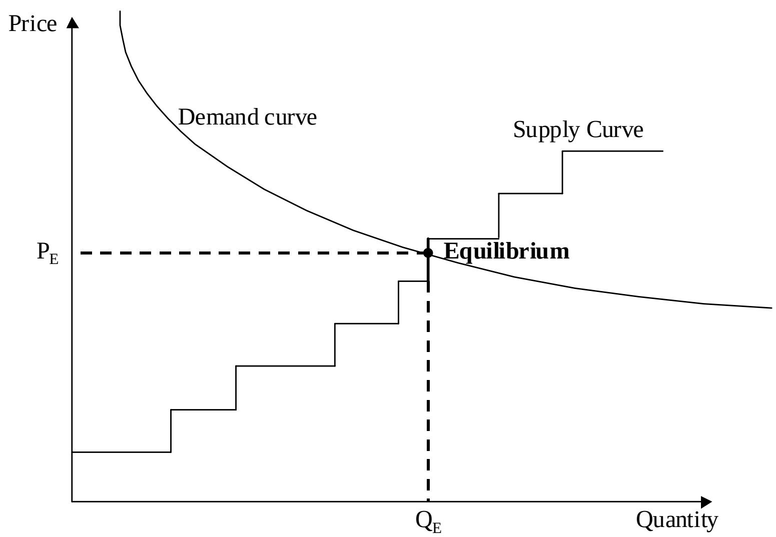

Figure 3.4. *Case where the equilibrium price is not equal to any marginal supply cost.*

3.2.3.5 Profit maximization

An interesting property may be derived from the assumptions of competitiveness. While the initial objective of the SAGE model is to maximize the overall surplus, it is also true that each economic agent in SAGE maximizes its own ‘profit’. This property is akin to the famous ‘invisible hand’ property of competitive markets, and may be established rigorously by the following theorem which we state in an informal manner:

Theorem: Let P* be the pair of equilibrium vectors. If we now replace the original SAGE linear program by a market where the commodities are priced at prices P*, and we let each agent maximize its own profit, there exists a vector of optimal quantities produced or purchased by the agents that is equal to Q*[1].

This property is very important because it provides an alternative justification for the class of equilibria based on the maximization of total surplus. It is now possible to shift the model’s rationale from a global, societal one (surplus maximization) to a local, decentralized one (individual utility maximization). Of course, the equivalence suggested by the theorem is valid only insofar as the marginal value pricing mechanism is strictly enforced—that is, neither individual producers’ or individual consumers’ behavior affect market prices—both groups are price takers. Clearly, many markets are not competitive in the sense the term has been used here. For example, the behavior of a few, state-owned oil producers have a dramatic affect on world oil prices. Market power may also be exhibited in cases where a few consumers dominate a market. The entire annual crop a

[1] Unfortunately, the resulting Linear Program has multiple optimal solutions. Therefore, although Q* is an optimal solution, it is not necessarily the one found when the modified program is solved.

given region's supply of coffee beans may, for example, be purchased by a handful of purchasers.

3.2.4 SAGE technology market share mechanism

As already mentioned, SAGE incorporates a special mechanism for estimating technology market shares for groups of competing technologies based upon each technology's relative cost of providing an energy service or energy carrier. For example, many conventional and renewable technologies are included in a SAGE database. Based on energy resource and technology cost assumptions, one technology alone, say a natural gas fired combined cycle unit, may represent the least cost means of producing electricity and gain 100-percent of the market for generating electricity. Such a result, however, would be at odds with observed market behavior for many reasons. For example, the level of geographic aggregation may mask significant intra-regional differences in resource availability and technology cost, and profit maximization may not be the sole motivating factor determining investment behavior. This section summarizes how market shares are computed in a typical optimizing model such as MARKAL versus how market shares are calculated in SAGE.

In the MARKAL model, the market shares of technologies are determined purely on a profit (or utility) maximization basis. If a number of end-use technologies compete for the satisfaction of some energy service, MARKAL (along with other optimizing models) will determine at each period the "winning" technology by implicitly computing the life cycle costs of all competitors and selecting the one with the lowest cost as the winner. Three factors may prevent the "winning" technology from capturing the entire market:

- First, some technologies may have residual capacities (from investments in previous periods), which give them a cost advantage over other technologies (essentially by eliminating the portion of the life cycle cost representing the annualized investment cost). These technologies may thus capture each a share of the market (up to their residual capacity).

- Second, some technologies may have user specified upper bounds on their capacity, that limit the pace at which they may be introduced in a given market.

- Finally, the use of a technology may be resource constrained, that is someplace "upstream" in the RES a needed commodity cannot be produced at a higher level (due to constraints imposed on it by the system), or due to some restriction imposed limiting emissions.

Apart from these factors, it may be said (speaking loosely) that the 'winner takes all'.

SAGE, however, has been designed with greater parametric control over technology choice specifically to prevent a single technology (the one with the smallest annualized life-cycle cost) from winning the whole market. It allows multiple technologies to capture some market share, by setting up suitably derived market share constraints. The mechanism works as follows.

- In a first step, the model is run without any upper bound on technology market shares (we call that a "free mode" run), but with a small (epsilon) *lower* bound on each technology's capacity to force its participation in the market. In the optimal solution to this run, the competing technologies fall into three groups:
    - Group 1: technologies whose capacities are exactly at their lower bounds;
    - Group 2: technologies whose capacities are exactly at their upper bounds;
    - Group 3: technologies whose capacities are at neither bound. These correspond to "basic" variables of the Linear Programming optimal basis.

Consider the *reduced costs* of the technologies' investment variables. Linear Programming theory defines the concept of reduced cost of a technology's investment variable as the amount by which the *initial* investment cost must be reduced in order for the investment variable to move away from its present lower or upper bound[1]. Linear Programming theory indicates that the reduced cost of a technology's investment variable in group 1 is positive, and that the reduced cost of each technology in group 3 is negative. To see this, consider the case of a technology whose capacity is at its lower bound in the optimal solution (group 1). This means that the model would 'prefer' to decrease this technology's capacity below the lower bound. For this situation to change (i.e. for the model to 'want' to further invest in the technology) the unit investment cost must be decreased, hence the positive reduced cost. A symmetric situation prevails when the technology is at its upper bound, in which case the model 'wants' to invest more in that technology. To make the model 'want' to reduce the technology's capacity below the upper bound the initial unit investment cost must be increased hence the negative reduced cost. Finally, if a technology is not at any bound (it is then said to be *marginal*, or *basic*, see section 3.4), the reduced cost is set to zero for mathematical reasons.

Note that technologies in groups 2 and 3 have a positive market share, but those in group 1 have no market share if the lower bound is indeed equal to 0.

The SAGE market sharing approach assigns some market share to a technology in group 1 *if it's reduced cost is suitably small* (which indicates that the technology was "close" to being adopted by the LP in the first place). There are several versions of the SAGE market share algorithm, each proposing various degrees of choice to the user for the weighing of the reduced costs. A more complete description of how to select an option is included in the manual accompanying the VEDA-SAGE commercial software mentioned in chapter 1.

Remark: the market share algorithm is used only for end-use and electricity generation technologies in SAGE. Primary energy supply technologies are left free to compete.

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[1] Even if the user specifies no lower bound, every SAGE variable has a lower bound of zero. Upper bounds on SAGE variables, if any, are user defined.

3.3 Elastic demands and the computation of the supply-demand equilibrium

SAGE does more than minimize the cost of supplying energy services. Instead, it computes a supply-demand equilibrium where both the supply options and the energy service demands are computed by the model. The equilibrium is driven by the user-defined specification of demand functions, which determine how each energy service demand varies as a function of the market price of that energy service. The SAGE code assumes that each demand has constant own price elasticity in a given time period, and that cross price elasticities are null. Economic theory establishes that the equilibrium thus computed corresponds to the maximization of the net total surplus, defined as the sum of the suppliers and of the consumers' surpluses (Samuelson, 1952, Takayama and Judge, 1972). The total net surplus has been often considered a valid metric of societal welfare in microeconomic literature, and this fact confers strong validity to the equilibrium computed by SAGE. In this section, we explain how Linear Programming computes the equilibrium. Additional technical details may be found in Tosato (1980) and in Loulou and Lavigne (1995).

The model, as implemented, provides both reference case projections and the capability of analyzing scenarios where some strain on the various economic sectors occurs resulting in increases in the marginal cost of some energy services (for example, severe emission reductions may have such an impact). In SAGE, demands self-adjust in reaction to changes (relative to the reference case) of their own marginal values, and therefore, the model goes beyond the optimization of the energy sector only. It still falls short of computing a general equilibrium: to do so would require a mechanism for adjusting the main macroeconomic variables as well, such as consumption, savings, wages, and interest rates, which SAGE does not. However, the model captures a major element of the feedback effects not previously accounted for in bottom-up energy models.

3.3.1 Theoretical considerations: the Equivalence Theorem

The computation approach is based on the equivalence theorem[1] that may be stated as: *"A supply/demand economic equilibrium is reached when the sum of the producers and the consumers surpluses is maximized"*. Figure 3.2 provides a graphical illustration of this theorem in a case where only one commodity is considered. The equilibrium point (i.e. the intersection of the inverse supply and the inverse demand curves) is seen to be the point at which the area between the two curves is maximized (this area is the sum of the producers' and the consumers' surpluses, called the *net total surplus*). As discussed in section 3.2.3.2, this property may be extended to the case of multiple commodities, which prevails in SAGE.

[1] The Equivalence Theorem is valid when both the inverse supply and the inverse demand functions are *integrable*. In the case of a supply function generated by an optimization model such as MARKAL or SAGE, it is easily shown that the inverse supply function is integrable (this is so precisely because the corresponding integral is equal to MARKAL's objective function). However, the integrability of the inverse demand function requires that the cross-elasticities be symmetric, a fact that is trivially verified when all cross-elasticities are assumed to be zero, as they are in SAGE and MARKAL.

3.3.2 Mathematics of the SAGE equilibrium

3.3.2.1 Defining demand functions

For each demand category, define a demand curve, i.e. a function determining demand as a function of price. In SAGE, a constant elasticity relationship is used, represented as:

$$DM_i(p) = K_i \bullet p_i^{E_i} \qquad \textbf{(3.3-1)}$$

Where DM_i is the i^{th} demand, p_i is its price, taken to be the marginal cost of procuring the i^{th} commodity, and E_i is the own price elasticity of that demand. Note that although the time index *t* has been omitted in this notation, all quantities in Equation 3.3-1 are time dependent in the SAGE context, including the elasticities. Constant K_i may be obtained if one point (p^0_i, DM^0_i) of the curve is known (the reference case). Thus Equation 3.3-1 may be rewritten as:

$$\boldsymbol{DM_i(p_i) / DM_i^0 = (p_i / p_i^0)^{E_i}} \qquad \textbf{(3.3-2)}$$

Or its inverse: $\boldsymbol{p_i(DM_i) = p_i^0 (DM_i / DM_i^0)^{1/E_i}}$

Where the superscript '0' indicates the reference case and the elasticity E_i is negative.

3.3.2.2 Formulating the SAGE equilibrium

Without elastic demands, the SAGE model may be written as the following Linear Program

$$Min \quad c \cdot X \qquad (3.3-3)$$

$$s.t. \quad \sum_k CAP_{k,i}(t) = DM_i(t) \qquad i = 1, I; \; t = 1, T \qquad (3.3-4)$$

$$and \quad B \cdot X \geq b \qquad (3.3-5)$$

where X is the vector of all variables. In words:

- (3.3-3) expresses the total discounted cost to be minimized.
- (3.3-4) is the set of demand satisfaction constraints (where the CAP variables are the capacities of end-use technologies, and the *DM* right-hand-sides are the exogenous demands to satisfy). The number of demand constraints equals the number of service demand categories times the number of time periods.
- (3.3-5) is the set of all other constraints.

The role of SAGE is to compute a supply/demand equilibrium among equations 3.3-3 through 3.3-5 where both the supply side and the demand side adjust to changes in prices, and the prices charged by the supply model are the marginal costs of the demand categories, (i.e. p_i is the marginal cost of producing demand DM_i.) A priori, this seems to be a difficult task, because the prices used on the demand side are computed as part of the solution to equations 3.3-3, 3.3-4, and 3.3-5. The Equivalence Theorem, however, states that such an equilibrium is reached as the solution of the following mathematical program, where the objective to maximize is the net total surplus:

$$Max \sum_i \sum_t \left(\left[p_i^0(t) / DM_i^0(t) \right]^{1/E_i} \bullet \int_a^{DM_i(t)} q^{1/E_i} \cdot dq \right) - c \cdot X \qquad (3.3-6)$$

$$s.t. \quad \sum_k CAP_{k,i}(t) - DM_i(t) = 0 \qquad i = 1, \ldots, I; \; t = 1, \ldots, T \qquad (3.3-7)$$

$$and \quad B \cdot X \geq b \qquad (3.3-8)$$

Where *X* is the vector of all SAGE variables, (3.3-6) expresses the total net surplus, and (3.3-7) is simply (3.3-4) rewritten to make clear that *DM* is now a vector of *variables* rather than fixed demands.

The integral is easily computed, yielding the following maximization program:

$$Max \sum_i \sum_t \left(\left[p_i^0(t) / DM_i^0(t) \right]^{1/E_i} \bullet DM_i(t)^{1+1/E_i} / (1+1/E_i) \right) - c \cdot X \qquad (3.3-6)'$$

$$s.t. \quad \sum_k CAP_{k,i}(t) = DM_i(t) \qquad i = 1, I; \; t = 1, T \qquad (3.3-7)'$$

$$and \quad B \cdot X \geq b \qquad (3.3-8)'$$

3.3.2.3 Linearization of the Mathematical Program

The Mathematical Program embodied in (3.3-6)’, (3.3-7)’ and (3.3-8) has a non-linear objective function. Because the latter is separable and concave in the DM_i variables, each of its terms is easily linearized by piece-wise linear functions which approximate the integrals in (3.3-6). This is the same as saying that the inverse demand curves are approximated by stair-case functions, as illustrated in fig. 3.5. By so doing, the resulting optimization problem becomes linear again. The linearization proceeds as follows.

a) For each demand category i, the user selects a range within which it is estimated that the demand value $DM_i(t)$ will remain, even after adjustment through price effects (for instance the range could be equal to the reference demand $DM^o_i(t)$ plus or minus 50%).

Select a grid that divides each range into a number n of equal width intervals. Let $ß_i(t)$ be the resulting common width of the grid, $ß_i(t)= R_i(t)/n$. See Fig. 3.5 for a sketch of the non-linear expression and of its step-wise constant approximation. The number of steps, n, should be chosen so that the step-wise constant approximation remains clsoe to the exact value of the function.

For each demand segment $DM_i(t)$ define n step-variables (one per grid interval), denoted $s_{1i}(t)$, $s_{2i}(t)$, …, $s_{ni}(t)$. Each s variable is bounded below by 0 and above by $ß_i(t)$. One may now replace in equations (3.3-6) and (3.3-7) each $DM_i(t)$ variable by the sum of the n-step variables, and each non-linear term in the objective function by a weighted sum of the n-step variables, as follows:

$$DM_i(t) = \sum_{j=1}^{n} s_{j,i}(t) \qquad (3.3\text{-}9)$$

and

$$DM_i(t)^{1+1/E_i} \cong \sum_{j=1}^{n} A_{j,s,i}(t) \bullet s_{j,i}(t) / \beta_i(t) \qquad (3.3\text{-}10)$$

The resulting Mathematical Program is now fully linearized.

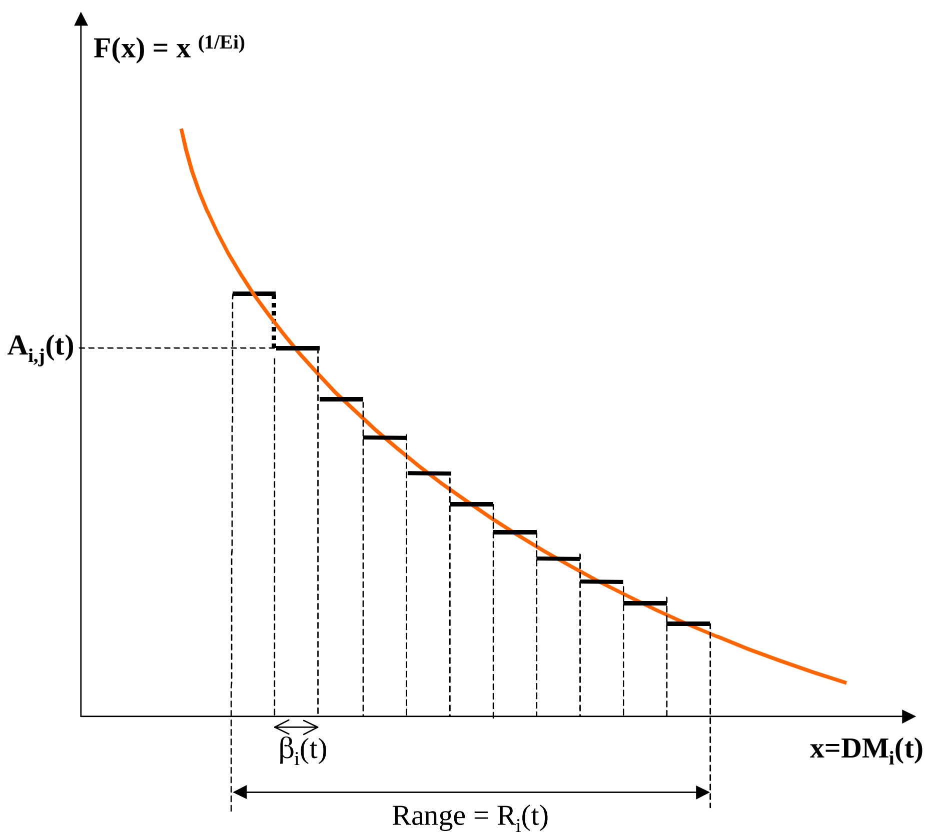

***Figure3.5**. Step-wise approximation of non-linear terms in the objective function*

3.3.2.4 Calibration of the demand functions

Besides selecting elasticities for the various demand categories, the user must evaluate each constant $K_i(t)$. To do so, we have seen that one needs to know one point of each demand function at each time period, $\{ p^0_i(t), DM^0_i(t) \}$. To determine such a point, we perform a single preliminary run of the inelastic SAGE model (with exogenous $DM^0_i(t)$), and we use the resulting shadow prices $p^0_i(t)$ for all demand constraints, at all time periods.

3.3.2.5 Computational considerations

Each demand category that is to be made elastic to its own price requires the definition of as many variables as there are steps in the discrete representation of the demand curve. Each such variable has an upper bound, but is otherwise involved in no other constraint except its associated demand constraint and the objective function. Therefore, the linear program is augmented by a number of variables, but does not have more constraints than the initial inelastic model's LP (with the exception of the upper bounds). It is well known that the number of variables has little impact on computational time in Linear Programming, even if the variables are upper bounded. Therefore, the inclusion in SAGE of elastic demands has a very minor impact on computational time or on the tractability of the resulting LP. This is an important observation in view of the very large scale LP representing the SAGE global model.

3.3.3 Interpreting the price results of SAGE

It is important to note that, instead of maximizing the net total surplus, SAGE minimizes its negative (plus a constant), obtained by changing the signs in expression (3.3-6). For this and other reasons, it is inappropriate to pay too much attention to the meaning of the absolute objective function values. Rather, examining the difference between the objective function values of two scenarios is a far more useful exercise. That difference is of course, the negative of the difference between the net total surpluses of the two scenario runs.

Also note again that the popular interpretation of shadow prices as the marginal cost of model constraints is inaccurate. Rather, the shadow price of a constraint is, by definition, the incremental value of the objective function per unit of that constraint's right hand side (RHS). The interpretation is that of an amount of *surplus loss* per unit of the constraint's RHS. The difference is subtle but nevertheless important. For instance, the shadow price of the electricity balance constraint is not necessarily the marginal cost of producing electricity. When the RHS of the constraint is increased by one unit, one of two things may occur: either the system indeed produces one more unit of electricity, or else the system *consumes one unit less* of electricity (perhaps by choosing more efficient end-use devices or by reducing an electricity-intensive energy service). It is therefore correct to speak of shadow prices as the marginal *value* of a resource, rather than the marginal *cost* of procuring that resource.

Finally, bear in mind that the elastic SAGE objective function maximizes net total surplus. For that reason, differences in objective function values for alternative energy policy cases indicate differences in the gain or loss of total welfare. In contrast, comparing only the supply costs of the alternative runs in an LP problem ignores the loss of surplus (welfare) due to reductions in energy service demand.

3.4 Additional economic aspects of the SAGE equilibrium

This section is not needed for a basic understanding of the SAGE model and the general reader may skip it in its entirety. This section does, however, provide additional insights into the microeconomics of the SAGE equilibrium. In particular, it reviews the theoretical foundation of Linear Programming and the accompanying Duality Theory. This knowledge may help the user more fully appreciate the role shadow prices play in the economics of the SAGE model.

3.4.1 A brief primer on Linear Programming and Duality Theory

A Linear Program may always be represented as the following *Primal Problem.* In canonic form:

$$\text{Max } c'x \qquad \textbf{(3.4-1)}$$

$$s.t. \quad Ax \leq b \qquad \textbf{(3.4-2)}$$

$$x \geq 0 \qquad \textbf{(3.4-3)}$$

Where x is a vector of *decision variables*, $c'x$ is a linear function representing the *objective* to maximize, and $Ax \leq b$ is a set of inequality *constraints*. Assume that the LP has a finite optimal solution, x^*, and that each decision variable, x^*_j falls into one of three categories. X^*_j is equal to its:

- Lower bound (as defined in a constraint), or
- Upper bound, or it is strictly
- Between the two bounds.

In the last case, the variable x^*_j is called *a basic variable*.

For each primal problem, there corresponds a *Dual problem* derived as follows:

$$Min\ b'y \qquad \textbf{(3.4-4)}$$

$$A'y \geq c \qquad \textbf{(3.4-5)}$$

$$y \geq 0 \qquad \textbf{(3.4-6)}$$

Where A' is the transposed A matrix. Note that the number of dual variables equals the number of constraints in the primal problem. In fact, each dual variable y_i may be assigned to its corresponding primal constraint, which we represent as: $A_i x \leq b_i$, where A_i is the ith row of matrix A.

Duality Theory can be represented by three theorems[1]: *weak duality, strong duality, and complimentary slackness.*

Weak Duality Theorem

If x is any feasible solution to the primal problem and y is any feasible solution to the dual, then the following inequality holds:

$$c'x \leq b'y \qquad \textbf{(3.4-7)}$$

[1] Their proofs may be found in most textbooks on Linear Programming (see e.g. Chvatal (1983) or Hillier and Lieberman (1990)).

The weak duality theorem states that the value of the dual objective is never smaller than the value of the primal objective.

Strong duality theorem
If the primal problem has a *finite, optimal* solution x^*, then so does the dual problem (y^*), and both problems have the same optimal objective value:

$$c'x^* = b'y^* \tag{3.4-8}$$

Note that the optimal values of the dual variables are also called the *shadow prices* of the primal constraints.

Complementary Slackness theorem
At an optimal solution to an LP problem:

- If y^*_i is > 0 then the corresponding primal constraint is satisfied at equality (i.e. $A_i x^*=b_i$ and the i^{th} primal constraint is *tight*. Conversely,
- If the i^{th} primal constraint is *slack* (not tight), then $y^*_i = 0$
- If x^*_j is basic, then the corresponding dual constraint is satisfied at equality, (i.e. $y^*A'_j=c_j$, where A'_j is the j^{th} row of A', i.e. the j^{th} column of A. Conversely, if the j^{th} dual constraint is slack, then x^*_j is equal to one of its bounds.

3.4.2 Sensitivity analysis and the economic interpretation of dual variables

It may be shown that if the j^{th} RHS b_j of the primal is changed by an infinitesimal scalar amount d, and if the primal LP is solved again, then its new optimal objective value is equal to the old optimal value plus $y_j^* \bullet d$, where y_j^* is the optimal dual variable value. Loosely speaking, one may say that *the partial derivative of the optimal primal objective function's value with respect to the RHS of the* i^{th} *primal constraint is equal to the optimal shadow price of that constraint.*

Economic Interpretation of the Dual variables and Constraints
If the primal problem consists of maximizing the surplus (objective function $c'x$), by choosing an activity vector x, subject to upper limits on several resources (the b vector) then:

- Each a_{ij} coefficient of the dual problem matrix, A, then represents the consumption of resource b_j by activity x_i;
- The optimal dual variable value y^*_j is the unit price of resource j, and
- The total optimal surplus derived from the optimal activity vector, x^*, is equal to the total value of all resources, b, priced at the optimal dual values y^* (strong duality theorem).

Furthermore, each dual constraint $y^*A'_j \geq c_j$ also has an important economic interpretation. Based on the Complementary Slackness theorem, if an LP solution is optimal then for each x^*_j that is not equal to its upper or lower bound (i.e. each basic variable x^*_j), there corresponds a *tight* dual constraint $y^*A'_j = c_j$, which means that the profit coefficient c_j must equal the cost incurred to purchase the resources needed to produce one unit of x_j[1]. If a variable is not basic, then by definition it is equal to its lower bound or to its upper bound. In both cases, the unit profit c_j need not be equal to the cost of required resources.

More precisely:

- if x^*_j is at its lower bound, then profit c_j is *less* than the resource cost, and
- if x^*_j is at its upper bound, profit c_j is *larger* than the cost of resources.

The Complementary Slackness theorem also indicates that for each optimal dual variable that is basic, the corresponding primal constraint is tight, and for every primal constraint that is slack, the corresponding optimal dual variable is equal to zero.

3.4.3 SAGE Dual constraints (price formation equations)

In SAGE, the objective function has the dimension of a surplus (akin to a profit) to maximize[2]; therefore, the dual variables have the dimension of a price, and the price formation equations are expressed in dollars per unit of the corresponding constraint. The reader may consult the SAGE LP matrix provided in Table A1: SAGE_MATRIX to verify that the equations below are indeed the correct dual constraints defined in section 3.4.1.

Dual constraint corresponding to an INVestment variable [INV(k,t,r)]

A glance at the SAGE LP matrix shows that, in the absence of emission constraints, for a given technology *k*, the only matrix coefficient in the *INV(k,t)* column has value *1*, which appears at the intersection with the capacity transfer constraint *CPT(k,t)*. Therefore, the dual constraint reads:

ANNUALIZEDINVCOST(k,t) ≥ shadow price (i.e. system value) of CPT(k,t) constraint

Recall that the shadow price of *CPT(k,t)* is the value of one additional unit of capacity for period *t*. The Complementary Slackness theorem indicates that at an LP optimal solution, the above constraint must hold with equality whenever the model invests in technology *k* at an intermediate level (basic).

Economic balance of running a technology [ACT(k,t)]

The ACT*(k,t)* column of the matrix has coefficients with the balance constraint and with the emission constraints (if any). Therefore, the dual constraint of variable ACT*(k,t)* is:

*Sum {over all c} of: output(c,k,t)*Shadow Price of Commodity(c,t) -*
*Sum {over all c'} of: input(c',k,t)*Shadow Price of Commodity(c',t) -*
*Sum {over all f} of: Delivcost(f,k,t) *Input(f,k,t) –*
*Sum {over all p} of: emact(p,k,t)*Shadow Price of Pollutants p ≤ Varom(k,t)*

[1] In economists terms, *marginal cost equals marginal revenue which, for perfectly competitive markets, is equal to the market price of* x^*_j.
[2] To be precise, SAGE minimizes the negative of the total surplus. This however does not alter the interpretation of the dual problem discussed in this section.

At optimal, the above constraint must hold with equality (is tight) whenever the technology is indeed active at a basic level, and therefore the net value of the outputs minus the value of the inputs of the technology is equal to the variable cost of running the technology.

Economic profitability of a mining, an import or an export node [SRC(c,r,l,t)]
The following are the dual constraints of a mining, an import, and an export variable, respectively.

Shadow Price of Commodity (c,t) ≤ Miningcost(c,l,t)
Shadow Price of Commodity (c,t) ≤ Importprice(c,l,t)
Shadow Price of Commodity (c,t) ≥ Exportprice(c,l,t)

At optimal, these dual constraints are tight whenever the mining, export, or import variable is basic. This ensures that the system mines, imports, or exports commodities whose system value is equal to the procurement price. In all cases, if the dual constraint is not tight (e.g. if the system value of an import is less than its price), the Complementary Slackness condition guarantees that the activity (mining, import, or export) is at its lower bound.

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4 Reference Guide

The purpose of this Reference Guide is to lay out the full details of the SAGE model, including data specification, internal data structures, and mathematical formulation of the SAGE Linear Program. As such it provides the SAGE modeler/programmer with sufficiently detailed information so that the syntax and purpose of the data components, and model equations and variables are fully understood; and link these to the relevant GAMS source code listed in Appendix B where appropriate. Thus, understanding this chapter is a necessary prerequisite for considering programming changes in the SAGE source code.

The Chapter is divided into eight sections, as follows:

- 4.1 – Notation, syntax, conventions and definitions: lays the groundwork for understanding the rest of the material in the Reference Guide;
- 4.2 – Sets: explains the meaning and role of various sets that identify how the various entities are grouped according to their nature (e.g. demand devices, power plants, energy carriers, etc.) in a SAGE model;
- 4.3 – Parameters: presents the user-provided numerical data as well as the internally constructed data structures used by the model to derive the coefficients of the L.P. matrix;
- 4.4 – Variables: defines each variable that may appear in the matrix, both explaining its nature and indicating how if fits into the matrix structure,
- 4.5 – Equations: states each equation in the model, both explaining its nature role, and providing its explicit mathematical formulation,
- 4.6 – Description of the market share and the technology learning mechanisms, two features that distinguish SAGE from its forebear MARKAL, inasmuch as both are strongly linked to the time-stepped nature of SAGE
- 4.7 – Description of the inter-regional trade of commodities as treated in SAGE, and
- 4.8 – Description of the execution sequences and controls of the SAGE system.

Collectively, the information assembled in this Reference Guide, combined with Appendix A: Table A1 SAGE_MATRIX (which provides a spreadsheet "snapshot" of the row/column intersections and associated coefficient parameters of the matrix generator), provides a comprehensive specification of the SAGE model and its implementation.

4.1 Notation, syntax, conventions and definitions

4.1.1 Basic notation and conventions

A detailed understanding of the SAGE variables and equations must start an appreciation of the set definitions used in the model (section 4.2). The use of carefully defined sets enables SAGE to succinctly define the elements of the SAGE Reference Energy System (RES). Set membership defines the nature of the named item. To illustrate, an energy carrier VVVVV would be a member of the set XXX (the set of all energy carriers). An energy carrier that is also a fossil fuel belongs to two sets, XXX and TTT (the set of all fossil energy carriers). Sets can also be defined as the intersection or union of other sets.

Set membership determines the parameters or input data permitted for each component[1] of the SAGE RES. Thus if a technology is described as a resource supply option different parameters are used to describe it than those used for demand devices. The set definitions also serve as indices for SAGE parameters, variables and equations.

The sections that follow describe the sets and parameters used by the model (section 4.3), the model variables (section 4.4) and the equations (section 4.5) that define the SAGE linear programming model. Special aspects of the model formulation such as the Market Share and Endogenous Technology Learning, as well as the trade options in SAGE, are also highlighted (sections 4.6 and 4.7), and the GAMS environment in which SAGE operates explained (section 4.8).

To assist the reader, the following conventions are employed consistently throughout this chapter:

- Sets, and their associated index names, are in lower case, usually within parentheses;
- Literals, explicitly defined in the code such as VVV, are in quotes;
- Parameters, and scalars (constants, i.e., un-indexed parameters) are in capital letters;
- Equations are in upper case with a prefix of MR_, and
- Variables are in upper case with a prefix of R_.

SAGE tracks most activities (capacity, new investments, technology operation, non-electric/heat commodity flows) at an annual level. Each parameter, variable and equation is related to a typical year within a period (SAGE is typically run using 5-year periods). For electricity and low-temperature heat, however, sub-annual flows and technology operation are needed to track the seasonal/time-of-day dependent load curves resulting from various energy service demands. Collectively, the season/time-of-day splits are referred to as time-slices. SAGE includes 6 time-slices, combining 3 seasonal divisions (Winter (W), Intermediate (I), and Summer (S)), with 2 time-of-day divisions (Day (D), and Night (N)).

4.1.2 GAMS Modeling Language & SAGE Implementation

SAGE consists of variables and equations constructed from the specification of sets and parameter values depicting the energy system of each distinct region in the model. To construct a SAGE model, a preprocessor first translates all data defined by the modeler into special internal data structures representing the coefficients of the SAGE variables of section 4.3 in the equations of section 4.5. This step is called the Matrix Generation. Once the model is solved a report writer assembles the results of the run for analysis by the modeler. The matrix generation, report writer and control files are written in the

[1] The SAGE Reference Energy System does not vary from one region to another, what does vary are data, parameter, and scenario specific assumptions.

General Algebraic Modeling System (GAMS)[1], a powerful high-level language specifically designed to facilitate the process of building large-scale optimization models. It accomplishes this by relying heavily on the principles of sets, and compound indexed parameters, variables and equations. Thus the fit with the overall concept of the RES specification embodied in SAGE is very strong, and GAMS is very well suited to the SAGE paradigm.

Furthermore, code specification in GAMS is remarkably similar to the mathematical description of the equations provided in Section 4.5. Thus the approach taken to implement the model is to “massage” the input data by means of a (rather complex) preprocessor that handles the necessary exceptions that need to be taken into consideration to construct the matrix coefficients in a form ready to be applied to the appropriate variables in the respective equations. GAMS also integrates seamlessly with a wide range of commercially available optimizers which are charged with the task of optimizing the SAGE Linear Program. This step is called the Optimization step. Currently CPLEX and XPRESS are the optimizers employed to solve SAGE.

As described in Chapter 2 and 3, SAGE is a specialized version of the MARKAL modeling system. In reality MARKAL is an amalgamation of a number of model variants, of which SAGE is a particular instance. To accommodate this situation most of the model is based upon the same basic GAMS code. However, conditional execution and dynamic substitution of components of the code are used to fully establish the appropriate executable code, in this case SAGE. This is possible because GAMS is a two-pass system, first fully resolving the conditional execution and dynamic substitution components, and then executing the resulting code. For example, a single region MARKAL model uses EQ_ as a prefix for all equations, while a SAGE multi-region model has MR_ as the prefix. Thus into every GAMS equation routine (those files beginning with MMEQ) either ‘EQ_’ or ‘MR_’ is a literal passed as the first parameter. The GAMS compiler then substitutes this so that the final SAGE code has the desired MR_ prefix for all the equations. This is necessary owing to the fact that an additional index (region) must be added to each of the model equations and variables when performing a multi-region run. This approach is further elaborated, and examples provided, in Section 4.4. However, the modeler does not need to be concerned about this aspect of SAGE. The GAMS code provided in Appendix B (SAGE GAMS Source Code) is the complete SAGE code established from the core MARKAL components when the %SAGE% environment variable is set to ‘YES,’ including all the various SAGE add-ons (e.g., Market Share Algorithm and inter-period Endogenous Technology Learning).

Regionalization in SAGE is handled by adding a prefix to equations (MR_<root>) and variables (R_<root>), and a suffix to set and parameter (<root>_R) names. During a multi-region model run the matrix generator and report writer buffer into/from regional data structures to isolate the specific information associated with each region. These parallel data structures have the identical name as their single region counterparts, but with the appropriate prefix/suffix added to their names and the 1st index for all such

[1] *GAMS A User’s Guide*, A. Brooke, D. Kendrick, A. Meeraus, R. Raman, GAMS Development Corporation, December 1998.

mapped sets, parameters, equations and variables designating the region (via the index (reg) or (r)). In all the descriptions of the model components in this Chapter the regional prefix and index are provided, but the suffix associated with sets and parameters are omitted; though it is understood to be part of the associated data structures, in almost all cases. Where there is an exception to this rule it is noted, for example if trade options identify both from/to regions (and commodities) then more than one regional index is specified.

4.2 Sets

In elaborating the details of the construction of the derived sets of the SAGE model standard mathematical terminology and expressions are employed, as summarized in table 4.1 below. As noted in the table, there are two extensions to standard notation, a dollar sign ($), is lifted from the GAMS language, and a '/' used as a shorthand for enumerating multiple similar entities (e.g., energy and material).

Table 4.1: Mathematic Symbols Used for Derived Set Specification[1]

Symbol	Description
$\subset$	A set is a subset of another set.
$\in$	An element is a member of a set.
$\notin$	An element is not a member of a set.
$\cap$	The "anding" or intersection of two sets, where an element must exist in both sets.
$\cup$	The "oring" or union of two sets, where an element must exist in at least one of the listed sets.
$\sum$	Summation over a set or series of sets.
$\leq$	Less than or equal to condition.
$=$	Equal to condition.
$\geq$	Greater than or equal to condition.
$\wedge$	The "anding" of two conditional criteria, where both must be true.
$\vee$	The "oring" of two conditional criteria, where at least one must be true.
$	Indicates that a condition controls whether or not a particular calculation is performed.[2]
/	A shortcut employed to reduce the need to repeat identical specifications of equations, for example for energy and material (e.g., ent/mat).

Components of a SAGE model are broken into five primary groups corresponding to energy carriers (e.g., coal, oil, gas, electricity), demands for energy services (e.g., residential heating, commercial cooling, passenger transport), emissions (e.g., CO_2, SOx, NOx), and technologies which are split into resource supply (e.g., imports, exports, coal mining, oil/gas production) and actual facilities (processes, devices, e.g., power plants

[1] A more elaborate version of this table also appears in Section XX as figure YY, expanded for those symbols needed when defining the equations.
[2] For example, 1$(dcn) + TE(ent)$(cen) would indicate that the transmission efficiency is 1 for decentralized (dcn) plants and the user provided overall commodity efficiency (TE(ent)) if centralized (cen).

(which produce electricity and/or heat), processes (which convert standard energy forms (or point in the RES) from one to another (e.g., a refinery, pipeline)), end-use devices meeting energy services (e.g., cars, air conditioners)). These splits correspond to the highest-level sets in SAGE as noted in table 4.2, below. Besides indicating how components fit into the underlying RES, membership in these groups also controls which other subset groupings are permitted (e.g., energy carriers are split between fossil, renewable, etc.; technologies into power plants, demand devices, etc.), as well as what actual data can be associated with a RES component (e.g., emission rates only for emissions, resource supply cost only for resources, investment cost for new technologies).

Table 4.2: Main RES Components Sets

RES Component	Main Sets	Key Subsets	Description
Demand Services	dm		All demand sectors.
Energy Carriers/ Materials	ent	enc elc lth mat	All commodities (other than environmental indicators) passing through the RES, broken out into basic energy carriers (enc) along with electricity (elc)and low-temperature heat (lth), and materials (mat).
Environmental Indicators	env	gwp	Emissions or other environmental indicators associated with the energy system, associated into groups by the modeler using a global warming or other multiplier.
Technologies	srcencp tch	 con dmd prc	All resource supply options. All technologies, broken out into Conversion Plants (con), Demand Devices (dmd), and Processes (prc).

Note that sets are not data in the traditional sense since no numeric value is associated with them. Rather set membership, determines how SAGE “looks at” each element of SAGE. The complexity, scope, detail, and size of the model is determined by the number and types of elements included in the sets. In order to include technologies, fuels, emissions and energy service demands in the model they must be properly declared as members of the relevant sets.

Two types of sets employed in the SAGE-modeling framework are user-defined sets and internal sets. The sets of the first group are the responsible of the modeler, who assigns the RES components to the appropriate sets, such as the primary groupings just mentioned in table 4.2 above. Membership in the internal sets is determined by the code for the matrix generator and the report writer. They serve to both ensure proper exception handling (e.g., from when is a technology available, in which time-slices is a technology permitted to operate), as well as sometimes just to improve the performance or smooth the complexity of the actual model code. As noted early and elaborated in table 4.3 of Section 4.2.2, for this documentation a one-letter index is used instead of the actual set name employed in the code when the sets, parameters, equations and variables are presented in the various tables and mathematical formulas.

As noted in the introduction to this Chapter, when running multi-region SAGE the sets and parameters have a suffix ("_R") appended to their names. However, this suffix is not indicated in the tables of this chapter.

The two distinct groups of sets in SAGE, User-defined and Internal Source Code defined, are discussed in the following subsections.

4.2.1 Overview of User-defined Sets

As noted in the previous section, sets are defined according to their role in the RES as

- Demands
- Emissions
- Energy Carriers
- Materials
- Technologies

In the subsequent section (4.2.1.7) each of the user input sets are defined in detail, including an explanation of the implications on the model when an item is assigned to each set.

The SAGE use of sets and naming conventions employed for commodities and technologies is fully explained in Volume 2: Data Implementation Guide.

4.2.1.1 Demand Sets

The elements of the master demand set (dm) define the different categories (or end-use sectors) consuming energy service included in the model. Each demand service must be assigned to one of six main end-use demand categories, depicted in figure 4.1, where:

- **AGR** is the set of energy services associated with the agricultural sector;
- **COM** is the set energy services associated with the commercial sector;
- **IND** is the set of energy services associated with the industrial sector;
- **NON** is the set of energy services associated with the non-energy sector;
- **RES** is the set of energy services associated with the residential sector, and
- **TRN** is the set of energy services associated with the transportation sector.

Within each of the sectors there may be any number of sub-sectors for specific demand for energy services. For example, in the commercial sector there may space conditioning (both heating and cooling), lighting, hot water, etc. By convention the sub-sector names begin with the 1st character of the demand sector they belong to, and the devices that service them are expected to begin with the 1st characters of the primary sub-sector that it services.

To elaborate with an example let us look at two sub-sectors in SAGE, commercial lighting ("CLA") and cooking ("CCK"). For each sector the subset of all technologies (tch) that provide commercial lighting/cooking are identified as a subset of all demand technologies (dmd) by the root 'CLA/CCK' respectively. Note that as mentioned in section 2.5, and fully developed in Volume 2, the rest of the device name either reflects the nature of the technology and/or the primary commodity it consumes, followed by the year in which the device is first available. The technologies listed below are those available in the year 2005 and beyond.

Commercial Lighting	CLA
CLACFL005	COM.LIGH: .05.ELC.FLUO.LAMP.COMPACT.
CLAFLA005	COM.LIGH: .05.ELC.FLUORESCENT.RAPIDSTART.
CLAFLB005	COM.LIGH: .05.ELC.FLUORESCENT.ELECTRODELESS
CLAFLE005	COM.LIGH: .05.ELC.FLUORESCENT.BASELINE.
CLAHAE005	COM.LIGH: .05.ELC.HALOGEN.
CLAHID005	COM.LIGH: .05.ELC.MERCURY.
CLAICE005	COM.LIGH: .05.ELC.INCANDESCENT.
CLAKER005	COM.LIGH: .05.KER.LAMP.
CLASUL005	COM.LIGH: .05.ELC.SULFER.

Commercial Cooking	CCK
CCKBIO005	COM: .05.BIO.COOKING.
CCKCOA005	COM: .05.COA.COOKING.
CCKELC005	COM: .05.ELC.COOKING.
CCKKER005	COM: .05.KER.COOKING.
CCKLPG005	COM: .05.LPG.COOKING.
CCKNGA005	COM: .05.NGA.COOKING.

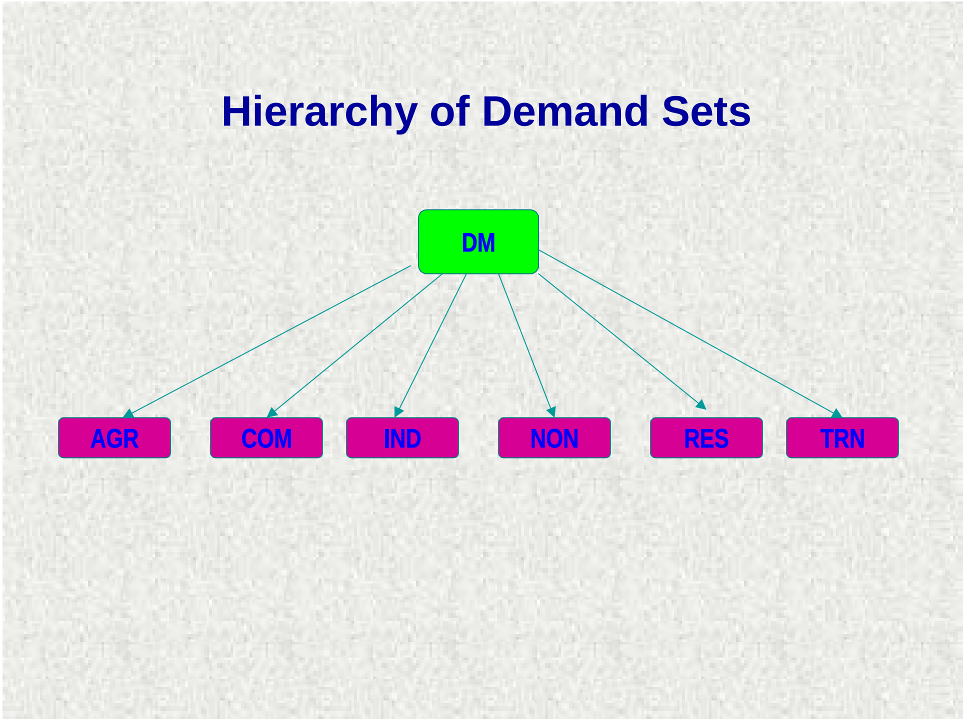

Figure 4.1: Hierarchy of Demand Sector Sets

There is one other aspect of characterizing demands that are serviced by devices consuming electricity or low-temperature heat, which is indicating whether or not the shape of the load curve for the demand is uniformly spread over the year or demand specific. In the former case (unifdist) the default regional load shape (QHR(Z)(Y)) is used, in the latter the user provides the demand specific shape by means of the demand load fractions (DM_FR).

4.2.1.2 Emissions Sets

The elements of Emission Indicator sets (env) identify the environment indicators that are to be tracked throughout the RES by the model according to the activity, capacity, or new investments of technologies. The modeler may also use the Indicator to monitor any other entity that can be directly associated with components of the RES (e.g., land-use).

Besides the complete list, a subset may be defined that represents greenhouse gas emissions that are subject to global warming potential estimates. The individual emissions can then be associated with an aggregate indicator by means of an emission-to-emission mapping table (ENV_GWP). The same technique may be used to compile sector or total emissions by providing an ENV_GWP entry of units between two indicators.

4.2.1.3 Energy Carrier Sets

As discussed in Section 2.4, energy carriers flow between the various technologies and other entities in the Reference Energy System establishing the inter-connectivity between the technologies.

Energy carriers are organized into sets according to the hierarchy tree shown in figure 4.2 below.

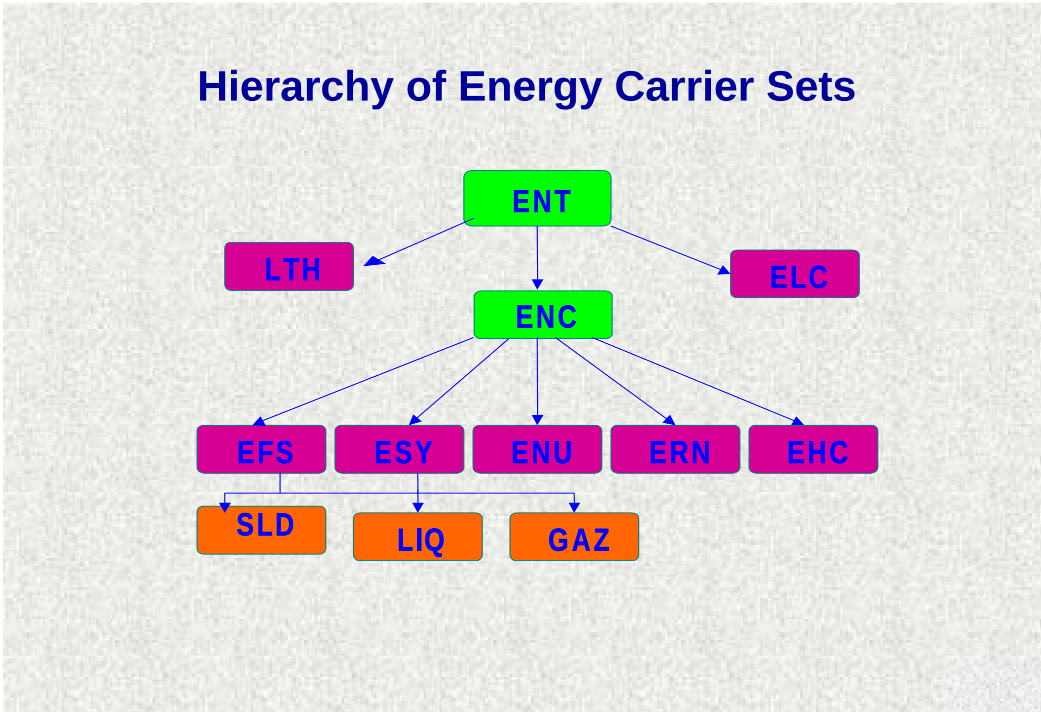

Figure 4.2: Hierarchy of Energy Carrier Sets[1]

As part of describing an energy carrier to the model the modeler must assign it to one of several types (e.g., fossil, nuclear, renewable) according to the nature of the energy carrier, and in the case of fossil or synthetic energy carriers the form (solid, liquid or gas). The modeler needs to be aware of the consequence of including an energy carrier in certain sets, most notably renewables (ern). For those energy carriers characterized as renewable for which no physical resource option is provided (via srcencp) it is assumed that such commodities (solar, wind) are unlimited and they are tracked in a non-binding balance equation (MR_BAL_N), as opposed to a greater than or equal to constraint (MR_BAL_G) employed for the other energy carriers (including those renewables for which a physical resource limit is provided). Thus under normal circumstances the production of all energy carriers is expected to equal or exceed the demand for said commodity.

This generic description holds for any MARKAL model variant. For SAGE a long list of energy carriers are already established and assigned their appropriate set memberships. In addition, as presented in Section 2.4.2, strict naming conventions have been established so as to better impart the nature of the energy carrier by indicating the kind of energy (e.g., coal, oil, gas, electricity) and the sector in which this particular instance of the energy carrier appears (e.g., upstream, "feeding" residential or commercial, etc). A new

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[1] Though not depicted in the schematic, in the GAMS code the set ENT actually encompasses all primary flow commodities (other than emissions and demand services) and thus includes both energy and materials (the latter not shown).

energy carrier in SAGE has to follow the above requirements, all aspects of which are handled by VEDA-SAGE, as discussed in Volume 2: Data Implementation Guide.

4.2.1.4 Material Sets

Materials are organized into sets according to the hierarchy tree shown in figure 4.3 below, divided by the way their material is measured (by weight verses volume).

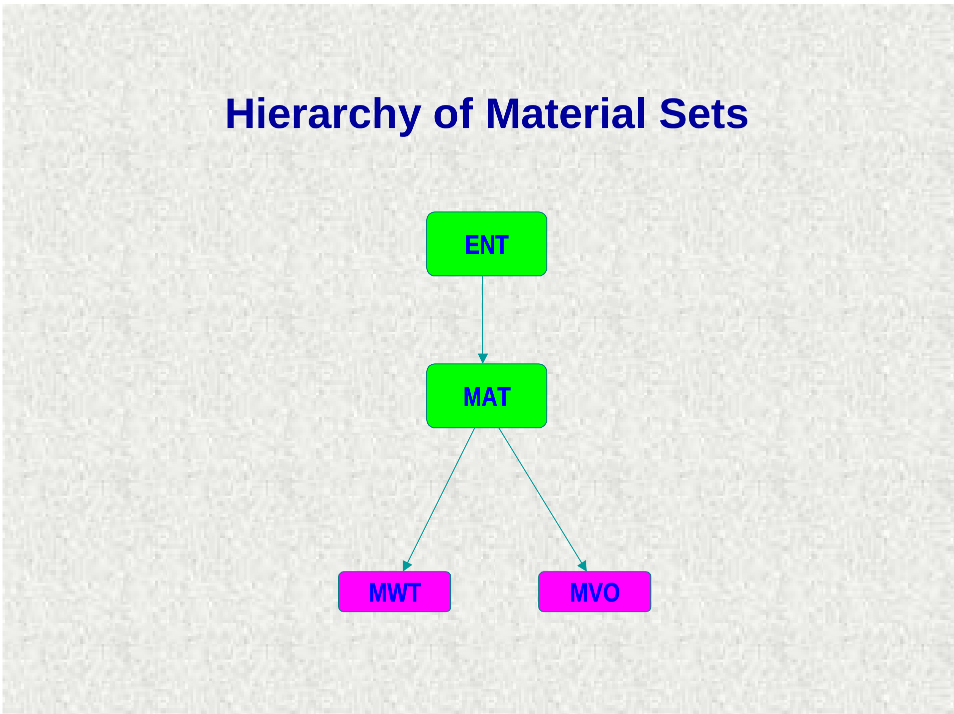

Figure 4.3: Hierarchy of Material Sets

The modeler needs to be aware of the consequence of describing a commodity as a material. All materials are subject of a material balance *equality* constraint (MR_BAL_E). Thus under normal circumstances the production of a material is forced to equal the demand for that material. As noted previously, in the model the set (ent) actually encompasses all primary commodities (other than emissions and demand services) and thus includes both energy and materials.

4.2.1.5 Technology Sets

Technologies are central to the definition of a RES and most of the user-provided input data is associated with them. In addition, there is quite a large number of distinct technology types (e.g., resource activities, power plants, refineries, demand devices) that can be depicted in SAGE. The primary nature of a technology is determined by the sets to which it is assigned, though in some cases data instances further refine some aspect of a technology (e.g., peaking only devices) to the model. As a result there is a more complex hierarchy and inter-relationship between the technology sets.

An underlying principle of the SAGE RES philosophy is that commodities flow between technologies. For the most part, technologies consume one or more commodity and produce others, different commodities. More precisely, no technology, other than storage plants (stg), can have the same commodity as an input and an output, at least not using the same commodity name. This is what enables the RES connectivity to be sorted out by the model code without the need to explicitly define actual inter-connections, links and nodes, embodied in the RES. For certain classes of technologies noted below commodities may only be produced or consumed, but not both.

As previously noted technologies are divided into four main groups as listed and depicted (in figure 4.4) below, then elaborated upon in the rest of this section.

- Resources (srcencp) – are the extraction, import or export of a commodity that enable resources to enter/leave the RES. Unlike most technologies resources do not consume any commodities (except for export). The resource technologies are treated separately and distinctly for the rest of the technologies, and have fewer parameters.
- Processes (prc) – are technologies that convert commodities into others, excluding electricity, heat, and demand services.
- Conversion plants (con) – are technologies that produce electricity and/or low-temperature heat. These technologies require special handling, as their activity is at a sub-annual or time-slice level.
- Demand devices (dmd) – are the only technologies that can meet the demands for energy services. As such they typically consume commodities but output a service rather than another commodity.

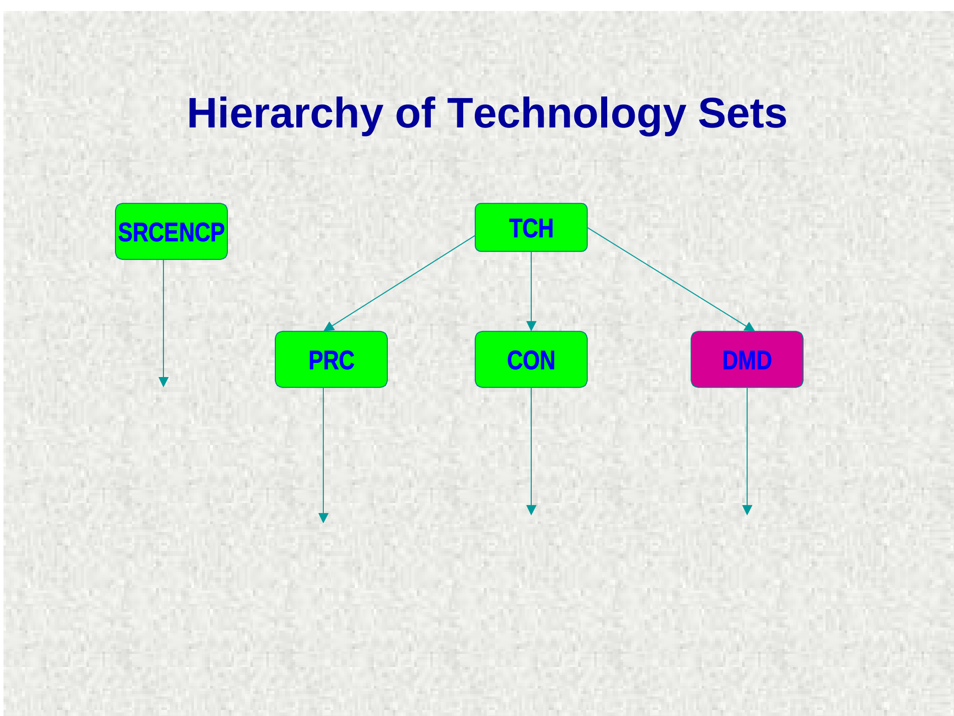

Figure 4.4: Main Hierarchy of Technology Sets

The main technology sets are further sub-divided to qualify the specific nature of each technology. Many of these sub-sets affect the matrix generator while some are strictly used for grouping technologies for reports. In Section 4.2.3 each of the user sets is discussed, with table 4.4 providing detailed descriptions of the purpose of the set and how membership influences the model. The basic groups of the technology sets are shown in figure 4.5 below. Subsequently each of the four main core technology groups are presented in the subsequent sub-sections.

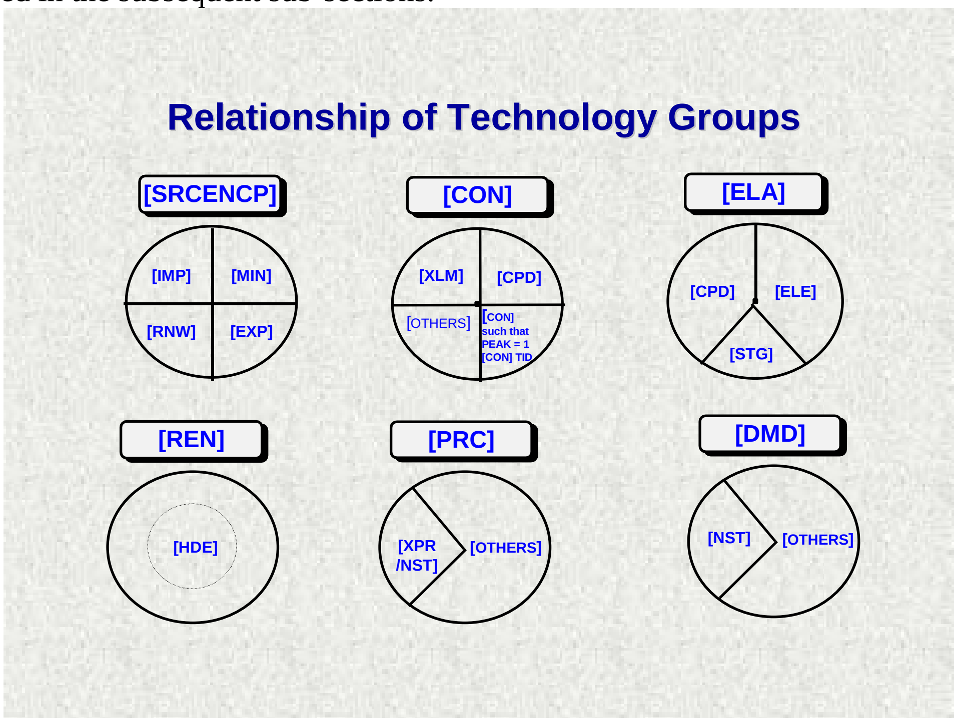

Figure 4.5: Relationship of Technology Groups

Resource Supply Sets

Resource supply options (srcencp) define the way commodities enter or leave the RES for each region. They are divided into five primary groups according to the nature of the resource supply activity as described here.

- **EXP**ort – which enables a commodity to be exchanged between two regions by allowing the commodity to leave the current region. As part of the regional energy balance exports are consumption activities, with any associated emissions leaving the region with the commodity (if desired, by means of a negative emission coefficient).
- **IMP**ort - which enables a commodity to be exchanged between two regions by allowing said commodity to enter the current region. In the regional energy

balance imports are production activities, with any associated emissions charged to the region (if desired).

- **MIN**ing **–** or extraction is the domestic production of a resource such as mining of coal or the production of oil or gas.
- **R**e**N**e**W**able **–** identifies those sources producing physically renewable resources such as municipal solid waste, biomass, etc..
- **ST**oc**K**piling **–** is the movement into/out of a stockpile of a commodity. The commodity must first be supplied to the system by means of a resource activity or process. The stockpiled commodities are accumulated, passed from period to period and drawn down as necessary. Stockpiling is most often used for representing nuclear fuel cycles. [This feature is not used in SAGE at the current time.]

The modeler is strongly encouraged to follow the approach outlined here when naming the individual resource options.

- The 1st three characters of the resource name should correspond to the primary group names just discussed.
- The next 3-6 characters should correspond to the primary output (or input in the case of exports) commodity associated with the resource activity.
- The last character is a unique alphanumeric that serves as the supply step cost indicator, so that a number of similar resource supply options can be defined for the same commodity.

The reason for encouraging the modeler to follow this approach is because the GAMS code maps the 7-10 character name into the three pieces just discussed. In the case of the resource type the 3-character short names are sometimes referenced in the code (e.g., ‘IMP’ enters the balance equation with a +coefficient, while ‘EXP’ enters with a –coefficient). Since the resources are assigned into the trees according to the 5 groups the preprocessor code sorts this out, but confusion (as noted later) can arise. Similarly, only the last character is stripped off for the price step, leaving the other 3-6 characters for the energy carrier name. Again, although while the preprocessor code sorts, not having the middle characters corresponding directly to the name of main commodity output again may result in confusion. Where this is particularly a concern is if there is a need to look into the actual GAMS run listing file, usually only necessary if the model is infeasible. If the rules just outlined are not followed there may be a difference between what one sees in the variable and equation listings and what was originally entered. While the report generator sorts this out (for the most part), again following these naming conventions is advised.

There is one other aspect of naming resource supply options that the modeler needs to be aware of. For bi-lateral trade the price step (last character) of the corresponding import/export options in the two regions **MUST** be the same, though the commodities traded are permitted to have different names in each region.

The hierarchy for the resource supply options is shown in figure 4.6.

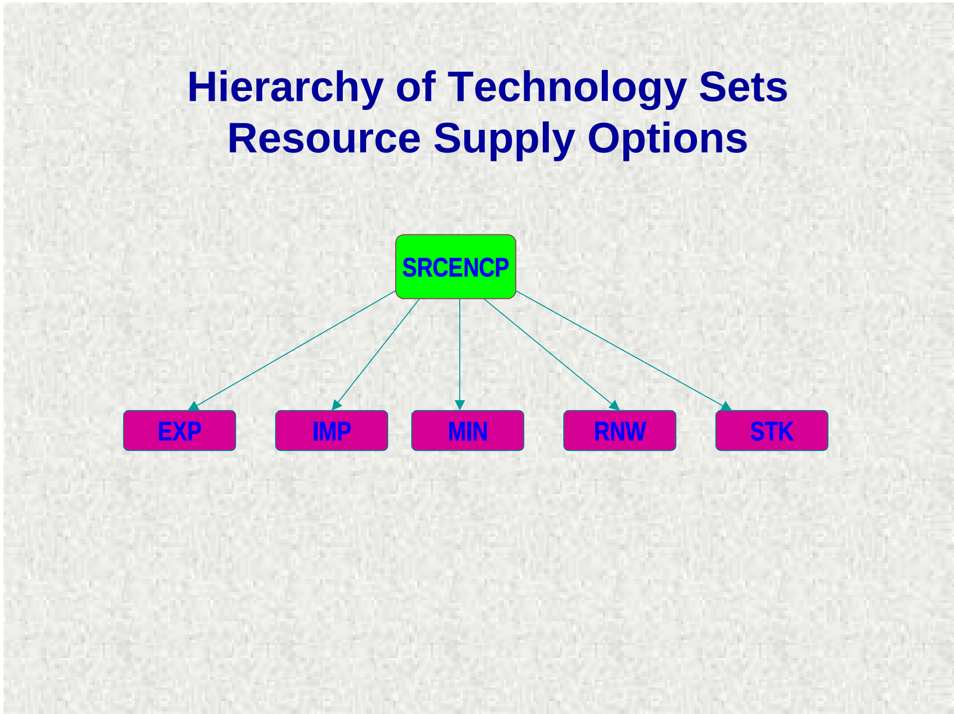

Figure 4.6: Hierarchy of Resource Supply Sets

Process Sets

Processes are the most general type of technology that can be depicted in SAGE. They can take in (take in) any commodity (except low-temperature heat) and output any other (except electricity and low-temperature heat). They range from pipelines to gasification plants or refineries. They are also commonly used to model emissions reduction options, including sequestration.

Under most circumstances the inputs and outputs are in fixed proportions relative to the activity of the process, with the exception of flexible limit processes (identified by means of the LIMIT parameter) that permit variable levels of the individual outputs from a process. Processes are divided into two primary groups according to whether they primarily handle energy or materials. That is not to say that a process cannot use or produce both energy and materials, but it is the modeler’s responsibility to ensure that all units and relationships are consistently defined. The material processes are further split into those that are modeled in terms of volume versus weight.

All process can be further qualified as to their operational nature (e.g., night storage (nst) or externally load managed (xpr), and reporting type (renewable (rnt) and dummy (dum)). Night storage processes (nst) strictly consume off-peak electricity during the night to produce their annual output; an example might be hydrogen production. This hierarchy is shown in figure 4.7 below. See Definition of User Input Sets, table 4.4, for further elaboration as to the role of each of the sets and the implication on the model structure.

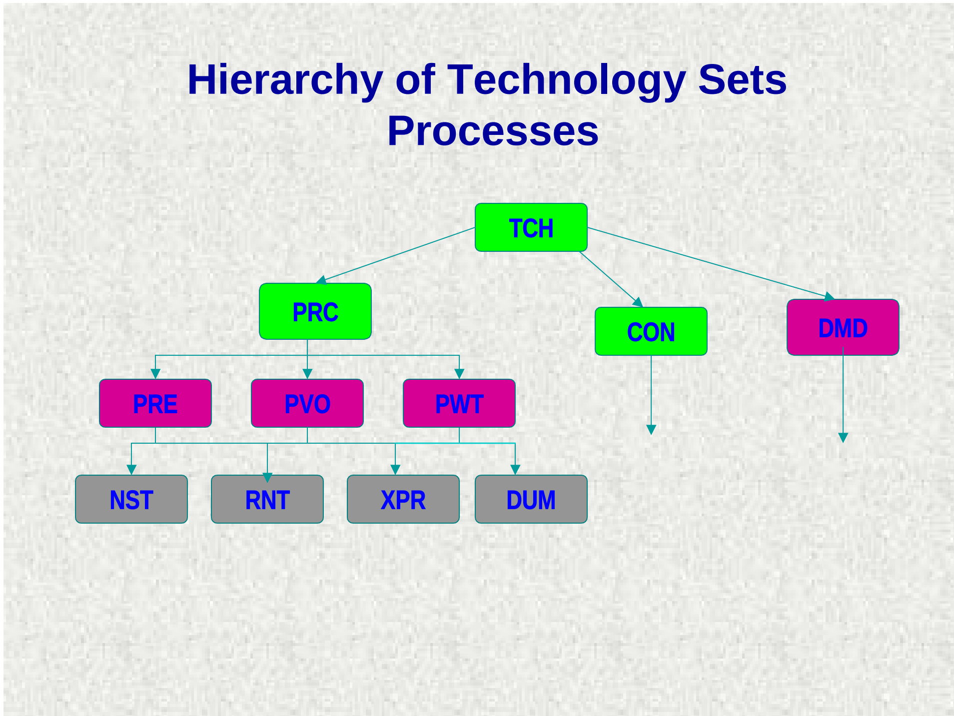

Figure 4.7: Hierarchy of Process Sets

Conversion Plant Sets

Conversion plants are the most elaborate and complicated technologies depicted in a SAGE model. This is to be expected, as they are the technologies that handle the production of the time-sliced energy carriers electricity and low-temperature heat. But besides that, there are several types of conversion plants that can be modeled including ones that only produce electricity (ele and stg), those that can produce both electricity and heat (cpd), and heating only plants (hpl). Among these three major groups the power plants, except for storage plants, can be characterized as centralized (cen) or decentralized (dcn), In addition these facilities can be either base load (bas) or non-load management (nlm) plants. Such plants can contribute to meeting the base load constraint, but the former are forced to operate at the same level day and night. This is often the case for most nuclear and large coal-fired power plants. All the plant types, other than coupled heat and power, may also be characterized as externally load managed (xlm). Such plants operate according to a fixed operational profile, rather than the flexible nature under which most plants operate. This feature is most often used for renewable technologies where the operation of the plant is dependent upon external factors (such as weather that affects solar and wind technologies). The conventional electricity only plants may also be characterized as hydroelectric (hde), which permits them to be subject to annual or seasonal reservoir availability, as will be seen later. Finally, electricity and heat grid interchange links (lnk and hlk respectively) can be identified. Such technologies accept electricity or heat form one grid and release it to another. This hierarchy is shown in figure 4.8 below.

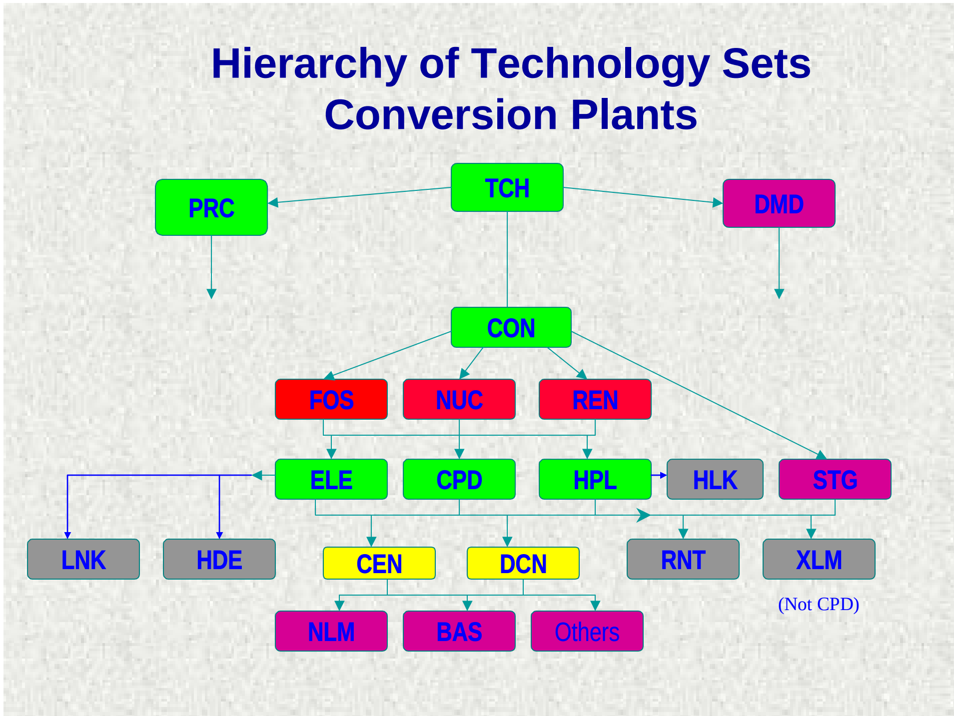

Figure 4.8: Hierarchy of Conversion Plant Sets

Besides the sets shown in figure 4.8 there are other characteristics that can be assigned to conversion plants: some plants may be designated as a peaking only device. Such plants (which cannot be coupled heat and power or externally load managed facilities) are identified by means of a parameter (PEAK_TID(CON)),. Also, as noted by the red boxes, the grouping of power plants according to what type of energy carrier they consume (that is fossil, nuclear and renewable) is derived by the preprocessor according to the commodity that provides the highest share of input to the plant.

See Definition of User Input Sets, table 4.4, for further elaboration as to the role of each of the sets and the implication on the model structure.

Demand Device Sets

Demand devices are modeled in a simpler manner than other technologies in that their level of activity is derived directly from the installed capacity in place based upon a fixed capacity utilization factor. In addition they can only accept commodities in fixed proportions and typically provide demand services to a single sector, although there is a provision for devices servicing multiple demands (e.g. a furnace may feed hot water and heat). The only set characterization that can be associated with demand devices is indicating that they are renewable technologies (rnt) or night storage (nst). The latter indicates that the technology consumes off-peak (night-time) electricity, for example an electric car that is charged in the evening. This hierarchy is shown in figure 4.9. See

Definition of User Input Sets, table 4.4, for further elaboration as to the role of each of the sets and the implication on the model structure.

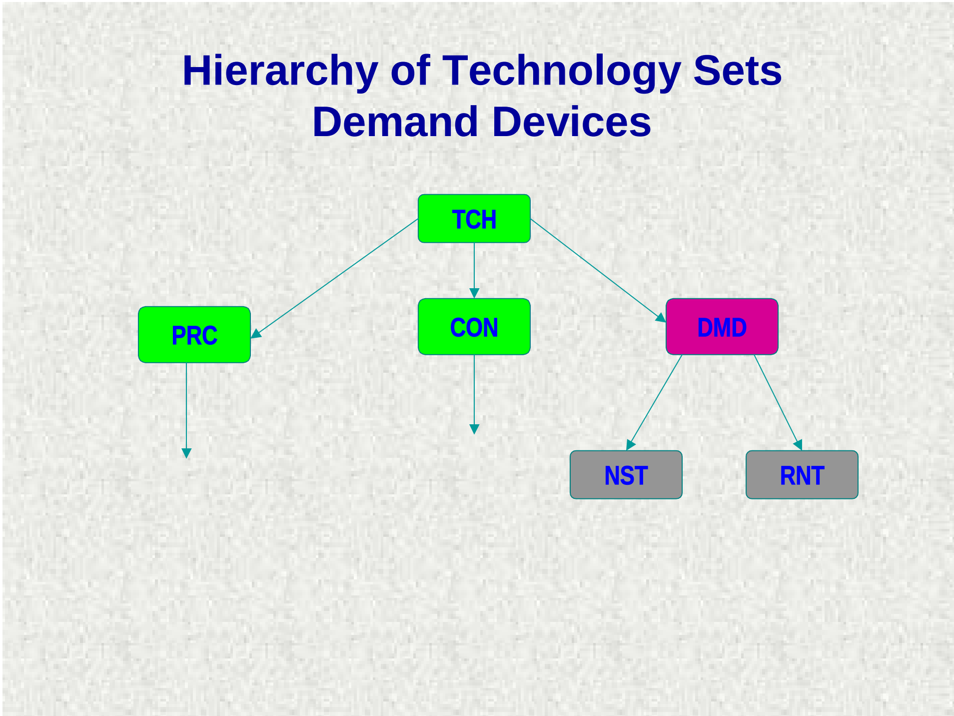

Figure 4.9: Hierarchy of Demand Device Sets

4.2.1.6 Other User-Defined Sets

There are two other sets that may be supplied by the modeler.

The first is the list of user-defined ad hoc constraints (referred to in the GAMS code as the set (adratio)). These constraints are used to define special circumstance to the model that cannot be readily defined using the conventional constraints handled by the model. Some examples might include:

- Grouping competing technologies into market share groups to ensure that a particular set of technologies does not acquire more or less than a given market share;
- Establishing a renewable energy portfolio standard that requires a percent of total generation to be met by renewable technologies, and
- Tying the capacity of two technologies to one another (e.g., applying an emission reduction technology to existing power plants, using two separate demand devices to represent on physical device (e.g., a heat pump serving both heating and cooling demands)).

The set of user-defined constraints (adratio) is simply the list of all such constraints. These specialized constraints are discussed in Section .4.5

Another closely related set is the list of Market Share groups (mkt_id) employed as part of the market share mechanism (MKTSHR). As described in some detail in Section 4.6.1, the market share mechanism is used to reallocate shares to closely competing technologies serving a particular market (e.g., all alternative fueled vehicles).

In order to employ the Endogenous Technology Learning mechanism (SETL), the modeler needs to provide the list of learning technologies as the set (teg). The technology learning capability is described in Section 4.6.2.

The other set that may be defined by the user is the list of commodity-based tax/subsidies (taxsub) that may be imposed on the energy system. The affect of a tax/subsidy is to increase or decrease the relative cost of a technology/commodity at various points in the system.

4.2.2 Sets and Indexes

As a reminder a short table of indexes is provided below. The process of constructing the parameters, variables and equations that control the actual intersections in the matrix is controlled by subsets of the referenced indexes, particular with regard to the technology references. For the most part these indexes correspond directly to user input sets discussed in section 4.2.3. The reader is referred to said section for a fuller description of each index, and for an indication of how entries in the sets influence the resulting model.

Table 4.3: Definition Indexes and their associated Sets

Index (Input Set)[1]	Description
a (adratio)	The name of the user-defined **A**d hoc constraints (referred to in the code by the set (adratio)) that are used to guide market splits, group limits for technologies, etc.
b (bd)	**B**ound or right-hand-side constraint type (‘LO’,’FX’,’UP’, and variations thereof (e.g., ‘MIN’,’FIX’,’MAX’)).
c (p)	The **C**ost, or step indicator, for the resource supply options.
d (dm)	All useful energy **D**emand sub-sectors.
e (ent/mat)	All **E**nergy carriers and all materials.
g (trd)	The name of the globally traded (as opposed to than bi-lateral) commodities (ENT or ENV).
m (mkt_id)	The **M**arket share group indicators.
p (tch)	All the technologies or **P**rocesses in the energy system.
r (reg)	The **R**egions to be tracked.

[1] (Input Set) identifies the user input set corresponding to the 1-character indexes used in the definition of the matrix components.

Index (Input Set)[1]	Description
s (srcencp/sep)	The resource **S**upply options, including those serving as the trade variables into/out of the regions.
t (year/tp)	The **T**ime periods requested for the current model run.
txs	The tax/subsidy on a technology/commodity combination.
u (step)	The steps that each elastic demand curve (consumer **U**tility) is approximated with.
v (env)	All emission or en**V**ironmental indicators.
x (ie)	Export/import e**X**change indicator. Note: the definition of the set IE is internal to the model and fixed at "EXP" for exports and "IMP" for imports. But for any trade variables defined over ie there are only variables for the instances for which the modeler has explicitly provided data.
w (td/z+y)	The season/time-of-day compound index (season (z) and time-of-day (z)) identifying the time-slice **W**hen an activity occurs.
z	The (pre-defined) seasons for heat generation.

4.2.3 Synopsis of User-Defined Sets

In Table 4.4 the main sets seen by the modeler are listed. For each set a short description is provided, along with an indication of the Master sets and any Alias/Linked sets associated with the main sets. The Master set indicates the primary higher-level set under which this set resides, and determines the domain over which the members of the set must belong. If no Master set is specified then, either the set has a predefined fixed list of possible values (e.g., BD, SRC), or is at the highest level of the set hierarchy and thereby its domain is determined by the values explicitly provided by the modeler. The Alias/Linked sets are either directly mapped to the main set, or directly derived subsets of the main set. For the most part such subsets have compound names built from the various main sets from which they are derived, and are created as the intersection or union of the named sets, possibly subject to certain other conditions. These sets are used subsequently in the code to conditionally control execution of the matrix generator and/or report writer, or to improve the performance of the code for frequently used indexes and conditions. The Linked sets are further defined in Table 4.5: Definition of Internal Sets.

The Description column has two purposes. It first provides a brief statement of what the set is all about, and then a series of bullet points indicating how the set impacts the model structure.

Table 4.4: Definition of User Input Sets

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]

[1] The SetID column uses upper case for the input sets, breaking with the convention employed in the rest of this document, to distinguish them from the single-character indexes.

[2] ***The SetIDs are used in the GAMS code as indexes into other sets, the parameters and the matrix rows/columns. For those SetIDs that correspond to indexes used in this document the index character is given within parentheses after the SetID.***

[3] The user input sets are often combined into these Alias/Linked related sets for use in the GAMS code.

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
ADRATIO (a/m)	mkt_id	The list of modeler use-defined ad hoc constraints, or ADRATIOs, as well as the SAGE market share subset group (mkt_id) that may replace said user-defined constraint according to the AMSWTCH control switch. • For each user-defined constraint (not involved in an active MKTSHR group), an MR_ARATx equation is created (as long as a RAT_RHS entry is provided).
ANSWTCH (ADRATIO, AMS) (a/m)	adratio/ mkt_id	Indication of how to treat individual user-defined ad hoc constraints with respect to the market share groups, according to which of the three values for the ADRATIO/MKTSHR switch is activated as described here. • AMS = 'A', always generate the user-defined constraint (MR_ARAT). • AMS = ' B', generate as a user-defined constraint (MR_ARAT) or handle via the market share algorithm depending upon whether MKTSHR is active. [Default, if MKTSHR is active surpress generation of the MKT_ID=ADRATIO MR_ARAT equations.] • AMS = 'X', ignore the user-defined constraint (MR_ARAT) if MKTSHR is active, regardless of whether or not a corresponding MKT_ID exists.
BAS	basnuc cenbas con cpdbas elabas elaxbas	Base load power plants, that is plants that are forced to operate at the same levels day and night, when operated in a season. • Contributor to meeting the base load constraint • No BAS entries eliminates the constraint
BD (b)		Bound type = 'LO', 'FX', 'UP', 'NON'. • Indicates the nature of a bound (or RHS of a user constraint) that is applied to a variable (or equation), as well as the direction of the demand elasticity specification. For example, the modeler may wish to disallow investment in new nuclear capacity. This can be accomplished by putting an upper bound of zero on the appropriate investment variable.
BI_TRDENT		The 2-tuple of region/commodity involved in bi-lateral trade. For each commodity an associated import/export supply option (srcencp) must also be specified for the same price set (p). Note that the trade is unidirectional, that is only from export to import region. • Controls the generation of the bi-lateral commodity trade constraint (MR_BITRD) between two regions.

[1] In the Description column the bulleted entries address the implications for the model of including an item in the Set.

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
BI_TRDELC (Z)(Y)		The 2-tuple of region/electricity involved in bi-lateral trade of electricity by season/time-of-day. For each form of the electricity energy carrier an associated import/export supply option (srcencp) must also be specified for the same price set (p). Note that the trade is unidirectional, that is only from export to import region, and that it only occurs for the time-slices specified. • Controls the generation of the bi-lateral electricity trade constraint (MR_BITRDE) between two regions for the time-slices specified. • The time-sliced electricity trade variable (R_TSEPE) is set equal to the traditional trade variable (R_TSEP), which is then used in the objective function, emission constraints, etc.; the seasonal variable appears directly in all the electricity specific constraints (MR_BALE, MR_BAS, MR_EPK).
CEN	con cenbas cencpd elacen	Centralized electric (ela), heat (hpl) and coupled-production (cpd) power plants. These plants have associated investments in transmission and distribution infrastructure whenever new capacity is added, and the energy generated is subject to transmission losses. • The cost of expanding the centralized grid (ETRANINV for electricity, DTRANINV for heat) is added to the investment cost (INVCOST) associated with each unit of new capacity, along with the distribution cost for electricity (EDISTINV).
CON	pon tch	All conversion plants, that is technologies producing electricity and/or possibly low-temperature heat. • The technology is modeled at a finer operational level, season/time-of-day (td=z,y) for electricity plants (ela, R_TEZY/T_RCZYH) and season (z) for heating plants (hpl, R_HPL).
CPD	cencpd con cpdbas dcncpd ela tpcpd	The subset of conversion plants that produce both electricity and low-temperature heat. • The technology output is modeled at the season/time-of-day level. • If the plant has a flexible pass-out turbine (CEH(Z)(Y)), then a separate variable is generated for the dynamic heat output (R_TCZYH) , otherwise the heat produced is calculated directly from the electricity production variable (R_TEZY).
DCN	con dcncpd	Those electric (ela), heat (hpl) and coupled production (cpd) power plants that are decentralized with only distribution infrastructure investments required whenever new capacity is added. • For each unit of new investment in capacity the distribution system cost is added (EDISTINV).
DM (d)		All useful energy demand sub-sectors. • An equation is created (MR_DEM) to ensure that the output of all demand devices (dmd) servicing the demand meet or exceed the demand. • The reporting subsystem breaks the demands and demand devices (dm, dmd) out into sectors passed upon the 1st character of their name (Agriculture, Commercial, Industrial, Non-energy, Residential, Transportation).

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
DMD	tch dmd_dm dmd_dms dmdnst tpdmd	The demand devices. Demand devices are different from other technologies in that they usually do not produce an energy carrier (or material), but rather provided end-use demand services. • Only demand devices can deliver demand services, and thus satisfy the demand constraint. • Demand devices are assumed to operate if built. Therefore in the model their activity/output is determined directly from the level of installed capacity (R_CAP) and there is no separate variable for the activity (as is the case with process (prc) and conversion (con) technologies. • The reporting subsystem breaks the demands and demand devices (dm, dmd) out into sectors passed upon the 1st character of their name (Agriculture, Commercial, Industrial, Non-energy, Residential, Transportation).
DUM		Suppresses the technology from the reporting for production and use of commodities by said technologies.
ELA	con elabas elacen elanuc elastg elaxbas elaxlm elaxstg tpela	This set is actually established by the pre-processor directly from the input data as the combination of the various types of power plants that can generate electricity (conventional (ele), coupled production (cpd) and storage (stg)). • All such power plants are modeled at the season/time-of-day level (R_TEZY). • For each power plant not externally load managed (xlm) the activity is a function of the installed capacity (R_CAP) and availability (AF/AF(Z)(Y)) by means of a utilization equation (MR_TEZY). • Identifies which technologies contribute to the time-slice based electricity equations for the production portion of the balance (MR_BALE) equation, the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
ELC	e ent	Electricity energy carriers, where for regions with multi-grids there may be more than one. • There are a series of time-slice based electricity equations generated for the production/consumption balance (MR_BALE), the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
ELE	ela	The subset of conventional conversion plants that produce only electricity, excluding storage (stg) facilities (e.g., pumped hydro). • For all such power plants not subject to external load management (xlm), activity is modeled by season/time-of-day (R_TEZY) as a function of the installed capacity (R_CAP) and the associated availability factor (AF/AF(Z)(Y)). This is controlled by means of a utilization equation (MR_TEZY). • Identifies which technologies contribute to the time-slice based electricity equations for the production portion of the balance (MR_BALE) equation, the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
ECV	enc enc_n	Those energy carriers that are considered conservation. • Conservation energy carriers are tracked by a non-binding balance equation (MR_BAL_N) that simply monitors the total fossil equivalent (FEQ) consumption.
EFS	enc enc_g	Those energy carriers that are considered fossil energy carriers. • Controls the generation of the non-binding fossil (MR_FOSSIL/TFOSSIL) and non-renewable (MR_NONRNW/TNONRNW) accounting equations • Used when building production/consumption tables in VEDA by fuel types.
EHC	enc enc_g	Those energy carriers that are considered high-temperature process heat. • Used for standard MARKAL reports T03 and T05, and as when building production/consumption tables in VEDA.
ENC	ent mat	All energy and material commodities, except electricity and heat.
ENT (e)	elc enc lth mat	All energy and material commodities, including electricity and heat.
ENV (v)	gwp	All emission indicators.
ERN	enc Ernxmac mac	Those energy carriers that are considered renewable. • Renewable energy carriers for which no resource supply option is provided are considered “free’ (e.g., solar sunlight, the wind) and are tracked using a non-binding fossil equivalent (FEQ) accounting equation (MR_BAL_N). • Renewable energy carriers for which a resource option is provided (via RNWencp) are handled as all other energy carriers by means of a greater than or equal to balance constraint (MR_BAL_G). They are often tracked by a parallel fossil energy equivalent (FEQ) accounting renewable energy carrier.
ESY	enc enc_g	Those energy carriers that are considered synthetic. The set has no explicitly impact on the matrix or reports, except to complete the list of all standard energy carriers (enc).
FEQ		The set is a short name of Fossil Equivalent, and contains a single reserved element “FEQ.” • The fossil equivalent value corresponds to the reciprocal of the average efficiency (actually inverse thereof) of the fossil fuel power plants in a region, and used for reporting the primary energy equivalent of renewable energy use.
GAZ	efs ent	Gaseous fossil energy carriers. • Used for standard MARKAL tables T02-T05, and VEDA reporting.
G_TRADE (g)		Any commodity, including emissions, that is to be traded globally. {Not currently used in SAGE, as all trade options are specified bi-laterally.}

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
HDE	ela	Hydroelectric power plants. • Identifies those power plants that are subject to annual or seasonal reservoir management (MR_ARM/SRM). • Used for construction of power plant tables by type in VEDA.
HEATCOOL		Indicator whether the peaking constraint for the heating/cooling will be modeled for the winter (heating) or summer (cooling). Default is "W"inter.
HLK	hpl	Heat grid inter-connection or link conversion technologies. These technologies consume and produce differently named heat energy carriers. • All heat grids are modeled at the seasonal level (R_THZ). • These technologies contribute to the season based heat equations for the production/consumption portions of the heat balance (MR_BALDH) and the peaking requirement (MR_HPK) equations.
HPL	con tphpl	The subset of conversion plants that produce only low-temperature heat. • For each heating plant not externally load managed (xlm) the activity is tracked by season (Z) as a function of the installed capacity (R_CAP) and availability (AF/AF(Z)(Y)) by means of a utilization equation (MR_THZ). • Identifies which technologies contribute to the season based heat equations for the production portion of the heat balance (MR_BALDH) and the peaking requirement (MR_HPK) equations.
LNK	ela	Electric grid inter-connection or link conversion technologies. These technologies consume and produce differently named electric energy carriers, as opposed to storage (stg) technologies which consume/produce the same electricity. • All such grids are modeled at the season/time-of-day level (R_TEZY). • Identifies which technologies contribute to the time-slice based electricity equations for the production/consumption portions of the balance (MR_BALE) equation, the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
LIQ	efs ent	Liquid fossil energy carriers. • Used when building production/consumption tables in VEDA by fuel types.
LTH	ent h	Low-temperature heat energy carriers, where for regions with multi-grids there may be more that one. • There are a series of seasonally based heat equations generated for the production/consumption balance (MR_BALDH) and the peaking requirements (MR_HPK).
MAT	ent mvo mwt	Those commodities designated as materials. • The production and use of such commodities are modeled using an equality constraint to ensure that the amount produced does not exceed the amount consumed (MR_BAL_E). • Used when building production/consumption tables in VEDA by fuel types.
MKT_GRP		The mapping of the individual technologies subject to the market share algorithm (when MKTSHR activated, see section 4.4) indicating the group to which the technology is assigned.

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
MKT_ID		The set of user provided names for the groups of technologies to be involved in the market share algorithm (when MKTSHR activated, see section 4.4). MKT_IDs often correspond directly to the user-defined Ad hoc constraint (adratio) that they substitute for when the market share algorithm is activated.
MVO	mat	Those materials tracked by volume.
MWT	mat	Those materials tracked by weight.
NLM		Subset of conversion plants that can meet system base load requirements, but does not have to have the same day/night production.
NST	Dmdnst prcnst	Processes (prc) and demand devices (dmd) that consume electricity only at night while supplying energy and demand services annually.
NUR		List of reactor types. • Used when building tables in VEDA by reactor types.
P (c)		The cost step associated with each resource supply option (srcencp). Taken directly from the name of the resource option by grabbing the last character. Note that for the specification of bi-lateral trade the same cost step (p) must be designated.
PRC	pon prcnst tch tpprc zpr	The process technologies. Those technologies not explicitly characterized as belonging to the power sector (con) or as demand devices (dmd). • For each process not subject to external load managed (xpr, or only 1 period in length and no investment cost (INVCOST)), activity is modeled annually (R_ACT) as a function of the installed capacity (R_CAP) and the associated availability factor (AF). This is controlled by means of a utilization equation (MR_UTLPRC).
REG (r)		Each of the regions in the model. Note that for each region self-contained input datasets are generated containing the complete problem specification.
RNT		Those technologies that are to be excluded from the MARKAL PRICER non-binding total system cost excluding renewable technologies. • Used when building production/consumption tables in VEDA by fuel types.
SLD	efs ent	Solid fossil energy carriers. • Used when building production/consumption tables in VEDA by fuel types.
SRCENCP (s)	Sep (src,ent,p) tpsep	List of the resource supply options, including imports and exports. The single SRCENCP member names are actually composed of three sub-components (src,ent,p) that are derived by the preprocessor (sep). • For each resource supply option there is an associated variable (R_TSEP). • The resource supply variable appears in all appropriate balance, peaking, base load, and emissions constraints, as well as the objective function. • It is recommended that the modeler use the same characters for the commodity part of the name (srcENTp) as the primary output (OUT(ENT)r)) to avoid a possible mix-up with respect to the actual names employed in the model and those provided by the modeler.]

Set ID/ Index[1,2]	Alias/ Linked[3]	Description[1]
STG	con ela	Conversion plants that produce electricity only during the day, consuming off-peak nighttime electricity to charge the storage (e.g., pumped storage). • In the model structure no nighttime activity variable (R_TEZY) is generated, but rather it the corresponding daytime variable is used to determine the amount of night electricity needed to refill the reservoir.
TAXSUB (txs)		The name of the tax/subsidy constraints for commodity/technology combinations.
TEG		Those technologies subject to the endogenous technology learning algorithm when SETL active (see section 4.4).
TCH (p)	con dmd prc tptch	The master list of all technologies (processes (prc) + conversion plants (con) + demand devices (dmd); collectively sometimes referenced as general processes), except for resource supply options (srcencp).
TP (t)	ts tc/rtp year	The periods for the current model run, where TS is the current time-step when running SAGE sequentially, solving period-by-period. • The series of technology related sets (e.g., tptch, tpcon, tpdmd, tpprc, tpsep) establish the periods for which a technology is available according to the year in which it can 1st be deployed (START). • An alias (tc) is used for dual period index parameters such as those needed for the capacity transfer constraint.
XLM	Elaxlm elaxstg nucxlm hplxlm	Conversion plants that can not operate in a flexible manner, that if built are then used according the to associated capacity utilization factor (CF/CF(Z)(Y)) rather than up to a maximum availability factor (AF/AF(Z)(Y)). Technologies that are subject explicitly to external factors such as weather (e.g., solar, wind) are usually characterized as these externally load managed plants. • For such conversion plants no activity variable (R_TEZY/TCZYH/HZ) is generated, but rather the activity is derived directly from the level of installed capacity (R_CAP) as a function of a capacity utilization factor (CF/CF(Z)(Y)); therefore no utilization equation (MR_TEZY) is needed.
XPR	zpr	Process technologies which are external load managed and thus that cannot operate in a flexible manner, that is if built then used according to an associated capacity utilization factor (CF), rather than up to a maximum availability factor (AF). Dummy technologies, which simply change the name of a commodity in the RES, are often characterized as these externally load managed processes. • For such processes no activity variable (R_ACT) is generated, but rather the activity is derived directly from the level of installed capacity (R_CAP) as a function of a capacity utilization factor (CF); therefore no utilization equation (MR_UTLPRC) is needed. • The preprocessor adds to the list of externally load managed processes all processes (prc) that have a lifetime (LIFE) of less or equal to one period in length or for which no investment cost is provided. This is done to eliminate above mentioned unneeded variables and equations.
YEAR (t)	tp	The periods associated with the input data and thereby periods available for the model runs, though a run may be done for a shorter time horizon if desired.

4.2.4 Internal Sets

This section presents the key internal sets, built from primary sets discussed above and used to control the execution of the model code and the coefficient instances of the matrix.

As part of defining the internal control sets various rules are applied to combine and limit the resulting set. In table 4.5 the shorthand expression (that is in some cases not fully elaborated for every required part of the condition, but reflecting the important aspects of said requirement(s)) of the Set Rules use the following logical operator conventions:

1. * - “and” meaning that the element is a member of both sets (intersection);
2. + - “or” meaning that the element is a member of either set (union);
3. # - “not” meaning except for the condition stated, and
4. <>= - means that a logical condition of less/greater than and/or equal to is evaluated.

Table 4.5: Definition of Internal Sets[1]

Internal Control Set[2]	Main Input Sets	Set Rule[3]	Description
BASNUC	con	Bas*nuc	Nuclear plants that are characterized as base load.
CENBAS CENCPD	con cpd	Cen*bas cen*cpd	Centralized electricity and coupled-production power plants.
CPDBAS	cpd	Cpd*bas	Base loaded coupled heat and power plants.
DCNCPD	cpd	Dcn*cpd	Decentralized coupled heat and power plants.
DMD_DM	dmd,dm	dmd* OUT(DM), where OUT(DM) is main	Identifies the primary demand sector (dm) serviced by each demand device (dmd) according to which output fraction (DMD_OUT) is the largest, or the 1st one encountered if more that one demand receives the same fraction. [Set in DMD_DM.ANS]
DMD_DMS	dmd,dm	dmd* OUT(DM)	All the demand sectors (dm) serviced by a demand device (dmd) according to the device output specification (DMD_OUT). [Set in MMFILL.INC]
DMDNST	dmd	Dmd*nst	Demand devices that are night storage technologies.

[1] Sets without a rule are static and declared in MMINIT, most others are established in MMSETS; except where noted in the description. While this list, in tandem with the Input Sets in table xxx, is comprehensive, it is not exhaustive.

[2] ***The Internal sets are used in the GAMS code as indexes into other sets, the parameters and the matrix row/columns.***

[3] In some cases the Set Rule is only given for the 1st Internal Set, as indicated by …, with the others listed following an identical approach but with different condition criteria.

Internal Control Set[2]	Main Input Sets	Set Rule[3]	Description
ELA ELABAS ELACEN ELANUC ELASTG ELAXBAS ELAXLM ELAXLSTG	con	ela+stg+cpd ela*bas ela*cen ela*nuc ela*stg Ela-bas ela*xlm elaxlm*stg	The subset of conversion plants that produce electricity, including conventional power plants, storage plants, and coupled heat and power facilities. These are then further qualified by additional sets criteria for coding purposes as part of the matrix generator and report writer.
ERNXMAC	ern	Not mac	Renewable energy carriers for which no physical resource supply (RNWencp) activity are defined. These energy carriers (e.g., solar, wind, geothermal) have no balance equations constructed, but instead are characterized at the technology level. Their fossil equivalent (FEQ) is tracked by means of non-binding balance constraints (MR_BAL_N). This set also includes the fossil equivalent energy carriers associated with physical renewables that are input in material rather than energy units (e.g., tons of biomass).
FOS	con	Not stg	All conversion plants that consume a fossil energy carrier. [Set in ANS2GAMS.ANS by examining the inputs (INP(ENT)c) to the technology.]
HPLXLM	con	hpl*xlm	Heating plants that are externally load managed.
IE	src srcencp		The exchange resource options for EXPort and IMPort of commodities.
IE_ELC	Sep	(‘IMP’encp+ ’EXP’encp)* elc	An indication of which resource supply options are EXPort and IMPort of electricity.
MAC	ern ernxmac	‘RNW’encp* ern	Renewable energy carriers for which a physical resource supply activity is defined and therefore the material must be accounted for. These energy carriers (e.g., MSW, biomass) have energy balance equations (MR_BAL_G) constructed, as opposed to the rest of the renewable energy carriers (ern), which are considered unlimited from the resource availability point of view (which are instead characterized at the technology level). These renewables are often modeled in physical units (e.g., tons of biomass), and shadowed by a fossil equivalent (feq) accounting energy carrier.
NUC	con	Not stg	All conversion plants that consume a nuclear energy carrier. [Set in ANS2GAMS.ANS]
NUCXLM	con	nuc*xlm	Nuclear plants that are externally load managed.
P (c)			List of all 1-digit numbers and upper case letters available to identify the individual supply curves or the cost steps; corresponds directly to the last position of the members of set SRCENCP.
PON	tch	prc+con	All process (prc) and conversion (con) technologies.
PRCNST	prc	Prc*nst	Processes that are night storage technologies.

Internal Control Set[2]	Main Input Sets	Set Rule[3]	Description
QHRZ	z	y=‘D’+’N’	Fraction of the year for each season, that is the sum of the day and night portions (QHR(Z)(Y)).
REN	con	Not stg	All conversion plants that consume a renewable energy carrier. [Set in ANS2GAMS.ANS]
RTY			Equation type = FIX, MAX (GE), MIN (LE), OBJ.
SEP	src,ent,p	srcencp	Parsed pieces of resource supply names, where 1st three characters correspond to the source (src), the last character to the supply step (p), and the remaining characters to the commodity (ent). It is strongly recommended that the modeler uses the same characters for the SEPent root as the primary output (SEP_OUT) to avoid missing internal and input/output SRCENCP names, as well as sticking with the 1st three character naming convention (below). [Established from SRCENCP in ANS2GAMS.ANS.]
SRC		srcencp	Type of resource supply option[1]: • EXP = Export • IMP = Import • MIN = Mining/Extraction • RNW = Renewable (physical) • STK = Stockpiling (for nuclear material, not used in SAGE)
TB	tp	tp=1st	First period of the model run.
TD	z,y		The combination of all season/time-of-day timeslice combinations = Intermediate Day (‘ID’), Intermediate Night (‘IN’), Summer Day (‘SD’), Summer Night (‘SN’), Winter Day (‘WD’), Winter Night (‘WN’). The modeler assigns the percentage of the year that corresponds to each of the six possible divisions via the global parameter QHR(TD). To eliminate a season/time-of-day do not provide said QHR(TD).
TLAST	t	t=last	Last period of the model run.
TPTCH TPCON/CPD/ DMD/ELA/ ELAXLM/ HPL/PRC/ TEG/ZPR	tp,tch	tp*tch* (VAL(tp) > START) …	List of periods for which each technology is available, beginning with the period in which the technology is 1st available and running through the modeling horizon.
XPRT	src	srcencp	The EXPort resource option = ‘EXP’.
Y (td)			The time-of-day values = Day/Night (‘D’/’N’), corresponding to an index in the set of the time-slices (td).

[1] These specific short acronyms **MUST** be used as the root first 3 characters of the SRCENCP resource name.

Internal Control Set[2]	Main Input Sets	Set Rule[3]	Description
Z (td)			The season values = Intermediate/Summer/Winter (‘I’/’S’/’W’), corresponding to an index in the set timeslices (td).
ZPR	prc	xpr + (LIFE<=1 period or no INVCOST)	All externally load managed processes (xpr) and regular processes (prc) with a lifetime (LIFE) <= 1 period (5 years) or no investment are included in this extended set, which is used for all subsequent processing. Note that such processes do not have an activity variables (R_ACT) generated in the model, rather the capacity variable (R_CAP) is used directly applying the fixed capacity factor (CF).

4.3 Parameters

The parameters comprise the data input to SAGE defining the RES by indicating the inputs and outputs to/from each technology, describing the operations and limits of the individual technologies, and representing the demands for energy services. SAGE parameters are described below in two sub-sections, 1) User Input Parameters (also called primary parameters) and 2) Matrix Coefficients and Internal Model Parameters. The User Input Parameters are those parameters that the modeler sees and define in the SAGE templates and in the front-end scenario development system, VEDA-SAGE. The Matrix Coefficient and Internal Model Parameters are those parameters either entered by the user or derived by the GAMS pre-processor which appear in the actual matrix generation portion of the code.

In section 4.3.1 the User Input Parameters are briefly described and presented in table 4.7. The Matrix Coefficient and Internal Model Parameters are then described in section 4.3.2 and presented in table 4.11. This table has an entry for every entity appearing on the Appendix A: Table A1 SAGE_MATRIX that provides a schematic snapshot of the matrix structure. Finally a few other important Key Internal Parameters are presented in table 4.12. Collectively these tables highlight almost all the parameters embodied in the model and the associated code, except for some temporary parameters local to a single GAMS routine.

As noted in the introduction to this Chapter, when running multi-region SAGE the parameters discussed here have “_R” appended to their names. During multi-region runs the matrix generator and report writer buffer into/from a regional data structure to isolate the specific information associated with each region. These parallel data structures have names identical to their single region counterparts, but with the “_R” suffix added to their names, and the 1st index for all such mapped parameters designating the region (via the index REG or R). In all the descriptions of parameters in this section the regional suffix and index are omitted, though it is understood to be part of the associated data structures, in almost all cases. Where there is an exception to this rule it is noted, for example if trade options identify both from/to regions (and commodities) or for parameters that are strictly related to trade.

4.3.1 User Input Parameters (Primary Parameters)

Each User Input Parameter is explained in terms of the:

- Indexes defining the structure;
- Purpose (as noted in table 4.6);
- Alias or Internal code Name;
- Related Internal Parameters;
- Units, Range & Default;
- Instance indicating when the parameter is expected to be provided, and
- Description.

The Alias/Internal Name is provided only if a name different from the primary parameter (seen by the modeler) appears in the GAMS code. The Related Parameters column serves to map the primary parameters to the key associated internal parameters used in the GAMS code, and vice versa. Only the more significant Related Parameters are listed in the table. The Units, Range and Default column indicates the units in which the data is to be entered, consistency of which is totally the responsibility of the modeler. The Range and Default indicate the expected range for the value provided, and the Default value set in the code if the parameter value is not provided by the user. The Instance column indicates when a parameter is expected to be provided or omitted, as well has highlights any special conditions that may be imposed on the parameter. The Description column has two purposes. It first provides a brief statement of what the parameter is all about, and then a series of bullet points indicating how the parameter impacts the model structure.

Each parameter imparts a particular kind of information about the RES and its components. To advise the user as to the nature or purpose of each parameter, each has been assigned to one of the following groups.

Table 4.6: Purpose of User Input Parameters

Nature or Purpose	ID[1]	Description
Costs	C	All the monetary parameters.
Demand	D	Characterize the demand for energy services.
Environmental Indicators	E	Used to track emissions associated with resource activities and technologies.
Endogenous Technology Learning	L	Those parameters associated with the inter-period Technology Learning algorithm.
Miscellaneous	M	Global and other parameters not covered by one of the other groups.

[1] The 1-character indicator that appears within [] in table xxx to indicate the purpose of the attribute.

Nature or Purpose	**ID[1]**	**Description**
Technical Operational Characteristics	O	Those parameters describing the operating characteristics of resource options and technologies and the infrastructure (e.g., transmission and distribution system), including limits imposed on them.
Market Share	S	Those parameters associated with the inter-period Market Share algorithm.
Topology & Trade	T	Depict the flow of commodities into and out of technologies and regions, thereby the commodities consumed/produced.
User Constraints	U	Information related to those specialized constraints explicitly built by the modeler to associate variables of the model in ways not otherwise possible.
Model Variant and Switches	V	Indicators of which model features are to be activated during the current run (e.g., multi-region, time-stepped, market share, endogenous technology learning, and flexible demands), [In a separate table 4.14 in section 4.7.]

As mentioned in the introduction to Chapter 4, to assist the reader with recognizing the nature of the various model components, the following conventions are employed in the tables:

- Sets, and their associated index names, are in lower case, usually within parenthesis;
- Literals, values explicitly defined in the code, are in quotes;
- Parameters, and scalars (un-indexed parameters) are in all capital letters;
- Equations are in upper case with a prefix of MR_, and
- Variables are in upper case with a prefix of R_.

As already mentioned, the region suffix (_R) and index (reg, r) are not shown in the tables, except for trade related parameters, but implied for the multi-region SAGE model runs.

Table 4.7: Definition of User Input Parameters[1,2]

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
AF (p,t) [O]	TCH_AF1	*TEZY_CAP* *THZ_CAP* *TUTL_CON* *UPRC_ACT* *UPRC_CAP*	• Hours of production / # of hours in the year. • [0,1]; default = 1, if no AF(Z)(Y) provided.	• Required for all process (prc) and conversion (con) technologies. • Omit if seasonal availability (AF(ZY)(Y)) provided, or	The annual availability of a process (prc) or conversion plant (con). That is the percent of the year that the capacity is available to operate. • The activity of a conventional process (R_ACT) or conversion plant (R_TCZY, R_TEZY, R_HZ) is related to the total installed capacity (R_CAP) as a function of the availability factor by the utilization equation (MR_UTLPRC, MR_TEZY, MR_HZ). • If a technology is externally load managed (xpr, xlm) and a capacity factor (CF/CF(Z)(Y)) is not provided then

[1] *Italics* identify parameters that are directly referenced when generating matrix coefficients. In some cases it is the Input Parameter, in which case a superscript of "row/column" indicating the cell in the SAGE_MATRIX spreadsheet is specified, or referenced Alias/CodeName that appears in the equation generation code, but most often it is a Related Parameter that is conditionally constructed and/or combines several input components that serves as the matrix coefficient. In this table those parameters directly referenced in the equations are highlighted in this manner, with all such parameters (including those listed in the Parameter and Alias columns) presented in table xxx as well. Note that there is at least 1 italic entry for each parameter, as all input data is somehow represented in the matrix.
[2] Note that some parameters that are specific only to MARKAL and are not employed in SAGE at this time have been omitted from this table.
[3] A Parameter without indexes is considered a Scalar in GAMS, that is it just assumes a single numeric value. The documentation index letters are used here, except when a parameter only applies to a subset of the master index sets.
[4] The purpose indicates the main role played by the parameter, as noted in table 4.6.
[5] Most Parameters are named differently in the GAMS code than as presented to the modeler. An Alias is a parameter where only the name is different, that is it is not subject to condition or a combination of parameters.
[6] Only the most prominent related parameters are listed here, which encompasses most but not necessarily all those parameters associated with a user parameter entry in this table.
[7] An indication of the circumstances for which the parameter is to be provided or omitted, and any special circumstances.
[8] References to other model components are enclosed in (), after naming them; (set) in lower case, (PARAMETER) in upper case, and (MR_/R_) are references to model equations/variables. This holds for the most part, with the exception that some file names, literals (e.g., 'BPRICE') and MARKAL variants that are also in upper case. The bulleted items indicate how the parameter impacts the matrix.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				technology is external load managed (xpr, xlm) or if peaking device (PEAK(CON)_TID) • The capacity factor (CF) is set to the availability factor (AF) if a plant is externally load managed and CF is not provided. • Peaking devices (PEAK(CON)_TID) apply a peak duration factor (PD(Z)D) to the time-slice (QHR) instead of an availability factor.	the availability factor is assigned as the capacity factor. For such technologies no activity variable is created, rather the capacity factor is applied to the capacity variable to determine the technology's fixed operation.
AF(Z)(Y) (p,w,t) [O]	TCH_AF3	*TEZY_CAP* *THZ_CAP*	• Hours of production / # of hours in the time-slice (td). • [0,1]; no default,	• When seasonal variability of conversions plants (con)	The annual seasonal/time-of-day of a conversion plant (con). That is the percent that the capacity is available to operate in each time-slice. • The seasonal/time-of-day availability of a conversion

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
			but set to AF if not provided.	exists. • Omit if annual availability (AF) provided, or technology is externally load managed (xpr, xlm) or if peaking device (PEAK(CON)_TID). • Once one is specified the plant will operate only for those time-slices for which AF(Z)(Y) is provided. • Heating (hpl), base load (bas) and storage (stg) plants should only have a day ('D') slice provided = seasonal availability / # of hours in the season (z).	plant (R_TCZY, R_TEZY, R_HZ) is related to the total installed capacity (R_CAP) as a function of the time-sliced availability factor by the utilization constraint (MR_TEZY, MR_THZ). • The time-sliced availability factors take presence over the annual AF. When a seasonal AF(Z)(Y) is specified for a power plant, then its availability in each time-slice is conditioned upon whether or not that time-slice is explicitly provided. • If a technology is externally load managed (xlm) and a time-sliced capacity factor (CF(Z)(Y)) is not provided then the availability factor is assigned as the capacity factor. For such technologies no activity variable is created, rather the capacity factor is applied to the capacity variable to determine the technology's fixed operation. • Note that if a conversion plant is externally load managed (xlm) and a capacity factor (CF/CF(Z)(Y)) is not provided then the availability factor is assigned as the capacity factor.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				• Note that since AF(Z)(Y) is the percent for each time-slice they may very well sum to more than 1. • Peaking devices (PEAK(CON)_TID) apply a peak duration factor (PD(Z)D) to the time-slice (QHR) instead of an availability factor.	
AF_TID (p) [O]		*TEZY_CAP* *THZ_CAP* *TUTL_CON*	• Hours of production / # of hours in the time-slice (td). • [0,1]; default = 1	• Provided when not all the unavailability of a conversion technology (con) can be scheduled by the model, that is the portion of the unavailability that corresponds to forced outage. • AF_TID can	So, AF/AF_TID imply that o Unavailability = 1 – AF o Forced Outage = AF_TID * (1-AF) o Remaining (scheduled) outage = (1-AF_TID) * (1-AF) • Results in the introduction of a maintenance equation (MR_UTLCON) and variable (R_M) that ensures that the unavailability of the technology in any time-slice that the model can schedule is reduced by the forced outage.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				only be specified with AF (not AF(Z)(Y)).	
BASELOAD (e,t) [O]	T_BASELOAD	*BAS_CAP* *BAS_SEP* *BAS_TCZY* *BAS_TEZY*	• Hours of production / # of hours in the time-slice (td). • [0,1]; no default.	• To be provided if the base load set (bas) is provided. • Omit otherwise.	The electric base load requirement for each electricity energy carrier. The value indicates the maximum amount (percent) of the highest nighttime demand that can be met by base load (bas) or non-load management (nlm) power plants. • For each electricity energy carrier for which the parameters is provided a seasonal base load constraint (MR_BAS) is generated.
$BI\_TRDCST_{25s/28s}$ (r,x,e,c) [C]			• Monetary units (M4$2000). • [$.001-1000]; no default.	• Omit if not desired.	The additional "transaction' cost for bilateral trading a particular commodity. • Cost multiplier in the regional objective function (MR_MTSOBJ) for a traded commodity (R_TSEP).
BI_ TRDCSTELC (r,x,e,c,w) [C]	$BI\_TRDCSTE_{25t,28t}$		• Monetary units (Million US$2000). • [$.001-1000]; no default.	• Omit if not desired.	The additional "transaction' cost for bilateral trading of electricity for each time-slice. • Cost multiplier in the regional objective function (MR_MTSOBJ) for traded electricity (R_TSEPE).
BOUND(BD) (p,,b,t) [O]	TCH_BND_{35f}		• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Omit if capacity of a technology is not to be explicitly limited.	The limit on total installed capacity (R_CAP = residuals (RESID) + new investments still available (R_INV)) in a period. • Sets a hard bound on the capacity variable (R_CAP) according to the bound type (bd).
BOUND(BD)O (p,b,t)	$TCH\_BNDO_{35E,13Y}$		• Units of activity	• Omit if activity	The limit on total annual activity in a period.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[O]			(e.g., GWh, PJ). • [open]; no default.	of a technology is not to be explicitly limited.	• Sets a hard bound on the activity of a process (R_ACT) according to the bound type (bd). • Sets an annual limit on the activity of a conversion plant by means of a utilization equation (MR_BNDCON) that sums the time-sliced activity variables (ela=R_TEZY+R_TCZYH, hpl=R_THZ).
BOUND(BD)Or (s,b,t) [O]	*SEP_BND*$_{35S}$		• Units of activity (e.g., PJ). • [open]; no default.	• Omit if activity of a resource supply is not to be explicitly limited.	The limit on total activity of a resource supply option in a period. • Sets a hard bound on the activity of a resource option (R_TSEP) according to the bound type (bd).
CAPUNIT (p) [O]	TCH_CAPU	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *BAS_CAP* *DEM_CAP* *EPK_CAP* *HPKW_CAP* *TENV_ACT* *TENV_CAP* *TEZY_CAP* *THZ_CAP* *TUTL_CON* *UPRC_CAP*	• Scalar, units of annual production activity per unit of capacity. • [open]; default = 1.	• Can omit if the default of 1 is wanted. • Affects the units of: o technologies FIXOM, INVCOST, BOUND(BD), IBOND(BD), RESID, INP(ENC)_TID o energy carriers DTRANINV,	Defines the units of annual production activity. When set to unity the units of capacity and production are identical. Traditionally used to allow conversion technology capacity to be in terms of kilo/gigawatts while annual production is defined in terms of petajoules or BBTU's. • The CAPUNIT, in tandem with the capacity factor (CF/CF(Z)(Y)), is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • CAPUNIT is applied directly to the activity bound (TCH_BNDO) when bounding output for externally load managed processes or conversion plants (xpr/xlm).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				ETRANINV, EDISTINV • For SAGE, the gigawatts-to-petajoule CAPUNIT = 31.536.	
CCAP0 (p) [L]			• Capacity units (either PJ/a, Gw, or demand units. • [any], none.	• Required if SETL activated (see section 4.4) for each technology to be subjected to endogenous learning that already has some installed capacity base.	Initial cumulative capacity when the learning algorithm starts up. • The existing cumulative installed capacity for a learning technology.
CEH(Z)(Y) (cpd,w,t) [O]	*T_CPDCEH*	*ANC_TCZY* *BAL_TCZY* *BALE_CZY* *BAS_TCZY* *BND_TCZY* *TENV_CZY* *TEZY_CZY* *HPKW_CPD* *INV_INV*	• Units of electricity lost per unit of heat gained. • [0,1]; no default.	• Required for pass-out turbines.	The pass-out turbine operates by diverting heat from the turbine to the low-temperature heat grid. As heat is produced, electricity production decreases at this rate. The electricity reduction is limited by the ELM parameter. • The heat production variable (R_TCZYH) is generated for each time-slice for which CEH(Z)(Y) is provided. • Total electric production in a time-slice is thus a function of two parameters and variables:

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
					$[R\_TEZY_{r,t,p,w} + (CEH_{r,p,w,t} * (1/ELM_{r,p} - 1) * R\_TCZYH_{r,t,p,w})]$
CF (p,t) [O]	TCH_CF2	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *DEM_CAP* *EPK_CAP* *HPKW_CAP* *TENV_CAP*	• Decimal fraction. • [0,1]; default = 1.	• To be provided for externally load managed processes (xpr) or conversion plants (xlm) when a seasonal capacity factor is not provided (CF(Z)(Y)), or demand devices (dmd). • The time-sliced capacity factors (CF(Z)(Y)) take presence over the annual CF if both are specified. • Note that since CF(Z)(Y) is the percent for each time-slice they may very well sum to more than 1. • Note that if a	The capacity factor is the fixed percent of utilization of the installed capacity, as opposed to an availability factor (AF) that allows for flexible operation up to said limit. • The capacity factor, in tandem with the CAPUNIT, is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • The capacity factor is applied directly to the activity bound (TCH_BNDO) when bounding output for externally load managed processes or conversion plants (xpr/xlm).

Input Parameter (Indexes[3]) [Purpose][4]	**alias/internal Name[5]**	**Related Parameters[6]**	**Units/ Range & Defaults**	**Instance[7] (Required/Omit/ Special Conditions)**	**Description[8]**
				conversion plant is externally load managed a capacity factor is not provided then the availability factor (AF) is assigned as the capacity factor.	
CF(Z)(Y) (p,w,t) [O]	TCH_CF1	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *BAS_CAP* *EPK_CAP* *HPKW_CAP* *TENV_CAP*	• Decimal fraction. • [0,1]; no default.	• To be provided for externally load managed conversion plants (xlm) when a seasonal capacity factor is not provided (CF(Z)(Y)). • The time-sliced capacity factors take presence over the annual CF if both are specified. • Once one is specified the plant will operate only for those time-slices for	The time-sliced capacity factor is the fixed percent of utilization of the installed capacity in a season/time-of-day, as opposed to an availability factor (AF(Z)(Y)) that allows for flexible operation up to said limit. Note that when the season CF(Z)(Y) is specified for a power plant, then its availability in each time-slice is conditioned upon whether or not that time-slice is explicitly provided for the technology. • The capacity factor, in tandem with the CAPUNIT, is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • The capacity factor is applied directly to the activity bound (TCH_BNDO) when bounding output for externally load managed processes or conversion plants (xpr/xlm).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				which CF(Z)(Y) is provided. • As heating plants are operated on a seasonal basis in the model, rather than day/night as for electricity, CF(Z)(Y) should only be provided for y = 'D' as seasonal availability / # of hours in the season. • Note that if a conversion plant is externally load managed and a capacity factor is not provided then if seasonal availability factors are provided (AF(Z)(Y)) they are assigned to the capacity	

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				factor.	
COST (s,c) [C]	SEP_COST1	*ANC_TSEP*	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• Provided whenever a cost is incurred to supply a resource. • EXPorts are assigned a negative value by the matrix generator.	Annual cost of supplying a resource. • The resource supply variable (R_TSEP) is entered into the regional objective function (MR_MTSOBJ). • For electricity (elc) imports ('IMP') transmission and distribution O&M charges are added to the COST.
COST_TID (s) [C]	SEP_STKCOS	*ANC_ZSTK*	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• Provided whenever there is a value associated with a stockpiled resource.	Salvage value of stockpiled energy carrier at the end of the modeling horizon. • The stockpile terminal level variable (R_ZSTK) is entered into the regional objective function (MR_MTSOBJ).
DELIV(ENT/MAT) (s,e,t) DELIV(ENT/MAT) (p,e,t) [C]	SEP_DELIV TCH_DELIV	*ANC_ACT* *ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• If an auxiliary charge is to be levied for the consumption of a commodity by a supply option or technology. • Note that a corresponding commodity input parameter (one of the INP(ENT)s)	Annual cost for the delivery of a commodity to a resource activity or a technology. • Said cost is multiplied by the activity level (R_TSEP, R_ACT, R_TCZYH, R_TEZY, R_TZ) and enters the objective function as an additional variable cost.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				must be provided.	
DHDE(Z) (e,,z,t) [O]	T_DHDE	*BALH_CAP* *BALH_THZ* *HPWK_CAP* *HPKW_HLK*	• Decimal fraction. [0,1]; no default.	• To be provided if some demand device (dmd) consumes low-temperature heat (lth).	Distribution efficiency for low-temperature heat in each season (z). • Applied to the consumption level for demand devices (dmd) and grids (hlk) using low-temperature in the balance (MR_BALDH) and peaking equations (MR_HPKW) to raise the production requirements appropriately to account for the loss.
DISCOUNT [C]		*COST_INV* CRF CRF_RAT PRI_DF PRI_DISC	• Decimal fraction. • [0,..99]; no default.	• Always required.	Overall long-term annual discount rate for the whole economy. • Used in the calculation of the capital recovery factor (CRF). • Also used to report the discounted costs (e.g., total system cost) to a base year, including taking into consideration any technology-based discount rate (DISCRATE) via a CRF ratio (CRF_RAT).
DISCRATE (p) [C]	TCH_DISC	*COST_INV* CRF CRF_RAT	• Decimal fraction. • [0,..99]; no default.	• Only provided when a technology-based discount or "hurdle" rate is to be applied.	The discount rate associated with an individual technology, or all technologies in a sector. • Used in the calculation of the capital recovery factor (CRF) to determine the annual payments for investments. • Also used to report the discounted costs (e.g., total system cost) to a base year, including taking into consideration the relationship to the standard discount rate (DISCOUNT) via a CRF ratio (CRF_RAT).
DTRANINV (lth,p) [C]	T_DTRANINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of heat plant capacity (e.g.,	• Provided if additional charges are to be	Investment in low-temperature heat transmission equipment for centralized (cen) heat producing plants (hpl/cpd). It is assumed that grid expansion and/or upgrading is required whenever new capacity is added.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
			2000 M$/PJa). • [open]; no default.	levied for the heat transmission system for central (cen) plants.	• The total cost of investing in the expansion of the heating grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the regional objective function (MR_MTSOBJ).
DTRANOM (lth,t) [C]	T_DTRANOM	*ANC_CAP* *ANC_TCZY* *ANC_THZ*	• Base year monetary units per unit of heat (e.g., 2000M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the heat transmission system for central (cen) plants.	Operating and maintenance costs associated with low-temperature heat transmission equipment for centralized plants. • The total O&M cost associated with a heating grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, THZ, or R_CAP if xlm).
EDISTINV (elc,t) [C]	T_EDISTINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of electricity capacity (e.g., 2000 M$/GW). • [open]; no default.	• Provided if additional charges are to be levied for the electric distribution system.	Investment in electricity distribution equipment. It is assumed that grid expansion and/or upgrading is required whenever new capacity is added. • The total cost of investing in the expansion of the electricity distribution grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the regional objective function (MR_MTSOBJ).
EDISTOM (elc,t) [C]	T_EDISTOM	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	• Base year monetary units per unit of electricity (e.g., 2000 M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the electricity distribution system.	Operating and maintenance costs associated with electricity distribution equipment. • The total O&M cost associated with an electricity distribution grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, THZ, or R_CAP if xlm).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
EFF (dmd,t) [O]	DMD_EFF	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *EPK_CAP* *HPKW_CAP*	• Decimal fraction equal to the useful energy service met per unit of commodity consumed. • [0,1] usually; default = 1.	• Provided when a value other than the default of 1 is necessary. • For some demands in energy units, which is the norm, the efficiency is usually a decimal fraction. • For transportation the devices (cars, trucks, training) may be providing billion passenger kms or tons of freight and thus the efficiency must also convert from energy units to demand service units. • For devices such as heat pumps, refrigerators,	The technical efficiency of a demand device. • Applied to the consumption level for demand devices when entering the balance (MR_BAL, MR_BALE, MR_BALDH) and peaking equations (MR_EPK, MR_HPKW) to raise the consumption requirements appropriately, and to the variable costs (DELIV, VAROM) according to activity (R_CAP taking into consideration CAPUNIT and CF) in the regional objective function (MR_MTSOBJ).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				lights, the efficiency may be greater than 1.	
ELCFEQ (c) [D]		T_ELCFEQ	• Factor. • [1 - any]; no default.	• The fossil equivalent value is set to that the average of the fossil fuel power plants in the region, and used for reporting the primary energy equivalent of renewable energy use.	The fossil equivalent corresponds to the average efficiency (inverse thereof actually) of the fossil fuel power plants in a region. • Used for reporting the primary energy equivalent of renewable energy use.
ELF (d,t) [D]	DM_ELF	*EPK_CAP*	• Decimal percentage. • [0,1]; no default.	• Provided for any demand (dm) serviced by a device that consumes electricity, when a value other than the default of 1is necessary.	The electricity load factor is a fraction that indicates how much of the electric demand that occurs during the peak time-slice actually coincides with the peak moment. • Controls the amount of the electricity consumption by the associated demand devices (dmd) that shall enter the peak equation (MR_EPK).
ELM (cpd,w,t) [O]	T_CPDELM	*ANC_TCZY* *BAL_TCZY* *BALE_CZY* *BALH_CZY* *BAS_TCZY*	• Decimal percentage. • [0,1]; default = 1.	• The indicator that a coupled production plant is a pass-out turbine.	The electric-loss maximum defines the percentage loss beyond which the total electric production may not be reduced. • Results in the creation of a heat production variable (R_TCZYH).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
		BND_TCZY *TENV_CZY* *TEZY_CZY* *HPKW_CPD* *INV_INV*		• Usually set to about .1.	• Total electric production in a time-slice is thus a function of two parameters and variables: $[R\_TEZY_{r,t,p,w} + (CEH_{r,p,w,t} * (1/ELM_{r,p} - 1) * R\_TCZYH_{r,t,p,w})]$
ENV_ACT (p,v,t) [E]	ENV_TACT	*TENV_ACT* *TENV_CAP* *TENV_CZY* *TENV_EZY* *TENV_HZ* env_a env_c	• Quantity of annual emissions (tons or ttons) per unit of technology annual activity. • [open]; no default.	• Provided if a technology emits (or removes) an environmental indicator according to its activity. • ENVSCAL may be used to scale all environmental indicators if desired.	Amount of emissions discharged, or reduced (if negative), per unit of output from a process or conversion technology. Note that this coefficient is output based, so if the source data ties the emissions to a commodity it is the modeler's responsibility to convert the factor by reflecting the units of energy consumed per unit output (e.g., the inverse of the commodity efficiency). • Providing ENV_ACT results in the associated activity variable (R_ACT, R_TCZY/EZY/HZ) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied. • If the technology is externally load managed (xpr/xlm) then ENV_ACT is assigned to ENV_CAP by applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)), and time-slice fraction (QHR) if power plant.
ENV_BOUND(BD) (v,b,t) [E]	*EM_BOUND* *EM_FIX*		• Tons or thousand tons. • [open]; no default.	• When there is to be an annual limited imposed.	Limit on the total production of an emissions indicator in a period. • Sets a bound on the total annual emissions variable (R_TENV).
ENV_CAP (p,v,t)		*TENV_CAP*	• Quantity of annual emissions (tons or ttons) per unit of	• Provided if a technology emits (or removes) an	Amount of emissions discharged, or reduced (if negative), per unit of installed capacity. • Providing ENV_CAP results in the associated

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[E]			technology capacity. • [open]; no default.	environmental indicator according to its capacity. • ENVSCAL may be used to scale all environmental indicators if desired.	capacity variable (R_CAP) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied. • If the technology is externally load managed (xpr/xlm) then ENV_ACT is assigned to ENV_CAP by applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)) , and time-slice fraction (QHR) if power plant.
ENV_COST$_{25/28H}$ (v,t) [E]			• Monetary unit per unit of emission indicator. • [open]; no default.	• When there is to be a charge applied to an emissions indicator.	An emissions "tax" applied to an indicator, and "backed out" of the total system cost (in MARKAL reports). • When provided the annual emissions variable (R_TENV) enters the regional objective function (MR_MTSOBJ).
ENV_CUMMAX (v) [E]	*ENV_CUM*		• Tons or thousand tons. • [open]; no default.	• When there is to be a cumulative limited imposed.	Total limit on an emissions indicator over the entire modeling horizon. • Sets the right-hand-side of the total emissions constraint (MR_ENV). • Note that the annual emissions variable (R_ENV) is multiplied by the period length (NYRSPER) when entered into MR_ENV.
ENV_GWP$_{29H}$ (gwp,v,t) [E]			• Scalar. • [open]; no default.	• GWPs are included in the environmental indicator set (env), and identified by inclusion (in the	The global warming potential that relates the relative contribution that various greenhouse gases make to the warming of the atmosphere. • For each emissions indicator in the 1st position an emission accounting equation (MR_TENV) is created where the entries are the total emissions (R_EM) associated with the various contributing greenhouse

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				1st position) in this table. • Provided for each GHG emissions indicator (env) that is to be accumulated according to its global warming potential. • The ENV_GWP table can also be used to accumulate several emission indicators to create a sector or region total if desired.	gases.
ENV_INV (p,v,t) [E]	*TENV_INV*		• Quantity of annual emissions (tons or ttons) per unit of technology capacity. • [open]; no default.	• Provided if a technology emits (or removes) an environmental indicator according to its investment. • ENVSCAL may be used to	Amount of emissions discharged, or reduced (if negative), per unit of investment in new capacity. • Providing ENV_INV results in the associated investment variable (R_INV) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				scale all environmental indicators if desired	
ENV_SCAL$_{29H}$ [E]			• Scalar. • [.000001 - 1]; default = 1.	• Provided if marginals are not reported for constrained emissions.	A global emissions scaling factor that is applied unilaterally to all emission indicators (env). Sometimes used to force reporting of marginal values on emission constraints. • Applied to the period emission total variable (R_EM).
ENV_SEP (s,v,t) [E]	*ENV_TSEP* *TENV_SEP*		• Quantity of annual emissions (tons or ttons) per unit of resource production. • [open]; no default.	• Provided if a resource option emits (or removes) an environmental indicator. • ENVSCAL may be used to scale all environmental indicators if desired	Amount of emissions discharged, or reduced (if negative), per unit of resource activity. • Providing ENV_SEP results in the associated resource supply variable (R_TSEP) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied.
ERESERV (elc,t) [O]	T_ERESERV	*EPK_ACT* *EPK_CAP* *EPK_HPL* *EPK_SEP* *EPK_TCZY* *EPK_TEZY*	• Decimal fraction. • [0,1]; no default.	• ERESERV should be provided for each electricity energy carrier (elc) for which a peaking requirement is to be imposed on	The electricity peaking factor looks to ensure that enough capacity is in place to meet the highest average electricity demand during the day of any season. It must encompass both an estimate of the level above that highest average demand that corresponds to the actual peak (moment of highest electric demand) + a reserve margin of excess capacity. This is to ensure that if some plants are unavailable said demand for electricity can still be met.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				the system.	Note that because ERESERV must encompass both the estimate from average to peak and the actual additional peak reserve it is usual substantially above that of a tradition utility reserve margin. • Providing ERESERV results in the creation of an electricity peaking constraint (MR_EPK) that enforces the above requirement. The equation is generated for each season according to the daytime demand for electricity.
ETRANINV (elc,t) [C]	T_ETRANINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of electric capacity (e.g., 2000 M$/GW). • [open]; no default.	• Provided if additional charges are to be levied for the electric transmission system for centralized power plants (cen).	Investment in electricity transmission equipment. It is assumed that grid expansion and/or upgrading is required whenever new capacity is added. • The total cost of investing in the expansion of the electricity transmission grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the regional objective function (MR_MTSOBJ).
ETRANOM (elc,t) [C]	T_ETRANOM	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	• Base year monetary units per unit of electricity (e.g., 2000 M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the electricity transmission system.	Operating and maintenance costs associated with electricity transmission equipment. • The total O&M cost associated with an electricity transmission grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, THZ, or R_CAP if xlm).
FIXOM (p,t)	TCH_FIXOM	*ANC_CAP*	• Base year monetary units per unit of installed capacity	• Provided if fixed operating an maintenance	Annualized fixed operating and maintenance cost for ALL the capacity in place in a period, charged regardless of whether or not the technology operates.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[C]			(e.g., 2000 M$/GW or PJa). • [open]; no default.	charges are to be levied for the total installed capacity.	• The cost is applied to the capacity variable (R_CAP) in the regional objective function (MR_MTSOBJ).
FR(Z)(Y) (d,w) [D]	DM_FR	*BALH_CAP* *BALE_CAP* *EPK_CAP* *HPKW_CAP*	• Decimal fraction. • [0,1]; default QHR(Z)(Y).	• Potentially provided for demand sectors (dm) for which some device consumes electricity or heat. • When not provided the demand timing matches that of the supply side, so all time-slices are set to the standard season/time-of-day splits (QHR(Z)(Y)) indicating uniform distribution (UNIFDIST). • If a FR(Z)(Y)	The percent of the annual demand for an energy service in each time-slice (w) when some device servicing the demand consumes electricity or heat. This thus describes the timing or shape of the load curve. • The capacity of the electricity and heat demand device is thus chosen such that it is sufficient to meet the demand according to FR(Z)(Y)/QHR(Z)(Y). • The coefficient thus contributes to determining the amount of electricity and heat that must be provided (via the balance equations (MR_BALE, MR_BALDH)) and the peaking needs (MR_EPK, MR_HPK) in each time-slice according to the device activity (R_CAP adjusted for capacity factors, efficiency and market allocation).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				is provided, and others not, for those omitted it is assumed that there is no associated electric/heat demand in said time-slice.	
FR(Z)(Y)(ELC) (s,w) [O]	SEP_FR	*BALE_SEP* *BAS_SEP* *EPK_SEP*	• Decimal fraction. • [0,1]; default QHR(Z)(Y).	• Only provided for electricity imports/exports. • When not provided all time-slices are set to the standard season/time-of-day splits (QHR(Z)(Y)). • If a FR(Z)(Y)(ELC) is provided, and others not, for those omitted it is assumed that there is no associated electricity import/export	The fraction of annual electricity imports or exports that occur in each time-slice. • The amount of electricity credit to (for imports) or against (for exports) the electricity balance (MR_BALE) and peaking (MR_EPK) in a time-slice is determined by the FR(Z)(Y)(ELC) parameter.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				during said time-slice.	
GROWTH (p,t) [O]	*TCH_TGR*		• Percent annual growth. • [-x,+y]; no default.	• Provided if the rate of increase in total capacity of a technology is to be limited. • The expansion limit constraint may need to be seeded by a maximum initial build (see GROWTH_TID).	The maximum annual growth rate of total installed capacity in a period. • A technology growth constraint (MR_GRTCH) is created for each period for which the parameter is provided. • The constraint limits the rate at which capacity can expand such that the total capacity (R_CAP) in the current period does not exceed (1+GROWTH)**NYRSPER. For example, a 10% annual growth rate in a 5-year period model would an increase of (1+.1)**5 = 1.61051 times the current level of installed capacity.
GROWTH_TID (p) [O]	*TCH_GRTI*		• .Units of capacity (e.g., GW, PJa). • [open]; no default.	• A growth constraint cannot be applied if technology has not been deployed (that is the growth from 0 is 0 no matter what the rate).	The growth constraint seed indicates the initial permitted maximum capacity level for the purposes of getting the growth constraint going. • GROWTH_TID is the right-hand-side of the technology growth constraint (MR_GRTCH). • Thus for new technologies not yet introduced to the energy system the constraint takes the form of 0 – permitted growth < seed.
GROWTH_TIDr (s) [O]	*SEP_GRTI*		• .Units of a commodity (e.g., PJ). • [open]; no default.	• A growth constraint cannot be applied if a resource has not been tapped (that is the growth	The growth constraint seed indicates the initial permitted maximum production level for the purposes of getting the growth constraint going. • GROWTH_TIDr is the right-hand-side of the resource growth constraint (MR_GRSEP). • Thus for new resources not yet introduced to the

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				from 0 is 0 no matter what the rate).	energy system the constraint takes the form of 0 – permitted growth < seed.
GROWTHr (s,t) [O]	*SEP_TGR*		• Percent annual growth. • [-x,+y]; no default	• Provided if the rate of increase in total resource production is to be limited. • The expansion limit constraint may need to be seeded by a maximum initial build (see GROWTH_TIDr).	The maximum annual growth rate of total production from a resource option in a period. • A resource growth constraint (MR_GRSEP) is created for each period for which the parameter is provided. • The constraint limits the rate at which a resource can expand such that the total annual production (R_TSEP) in the current period does not exceed (1+GROWTHr)**NYRSPER. For example, a 10% annual growth rate in a 5-year period model would permit an increase of (1+.1)**5 = 1.61051 times the current level of production.
GobhZscal [M]			• Scalar, • [.000001 – 1], default 1.	• Provided to scale the objective function value. • SAGE uses the default.	The internal scaling of the model to improve stability and speed the solution of large problems. • Appears only in the individual regional objective functions (MR_MTSOBJ).
HRESERV (lth) [O]	T_HRESERV	*HPKW_ACT* *HPKW_CAP* *HPKW_CPD*	• Decimal fraction. • [0,1]; no default.	• HRESERV should be provided for each low-temperature heat energy carrier (lth) for which a peaking	The low-temperature heat peaking factor looks to ensure that enough capacity is in place to meet the highest average heating demand in the winter (heatcool) season. It must encompass both an estimate of the level above that highest average demand that corresponds to the actual peak (moment of highest heat demand) + a reserve margin of excess capacity. This is to ensure that if some plants are

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				requirement is to be imposed on the system.	unavailable said demand for heat can still be met. Note that because HRESERV must encompass both the estimate from average to peak and the actual additional peak reserve it is usual substantially above that of a tradition utility reserve margin. • Providing HRESERV results in the creation of a low-temperature heat peaking constraint (MR_HPKW) that enforces the above requirement.
IBOND(BD) [O]	*TCH_IBND*		• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Omit if investment in a technology is not to be explicitly limited.	The limit on new investments (R_INV) in a period. • Sets a hard bound on the investment variable (R_INV) according to the bound type (bd).
INP(ENC)_TID (con,enc) INP(MAT)_TIDp (prc,mat) [T]	CON_INP1 PRC_INP1 prc_enc snk_enc	*BAL_INV* T08	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is consumed as part of putting a new investment in place.	Energy/material required at the time an investment in new capacity is made. Typically used to represent the initial load of fuel into the core of a nuclear reactor. • Said amount enters the balance equation (MR_BAL) in the period in which the investment takes place, except for nuclear energy carriers (enu) which is charged to the previous period. • If more than one commodity is consumed at investment time then each must have a corresponding INP(ENT/MAT)_TID parameter. Contributes to defining the flow of commodities through the RES.
INP(ENT)c (con,ent,t)	CON_INP2 con_enc	*ANC_CAP* *ANC_TCZY* *ANC_TEZY*	• Units of commodity consumed per unit of production.	• If a commodity is consumed by a power plant.	Energy/material consumed to generate one unit of electricity and/or low-temperature heat leaving the facility, that is, after any on-sight consumption. Expressed as the

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
INP(MAT)c (con,mat,t) [T]	con_elc con_lth	*ANC_THZ* *BAL_CAP* *BALE_CAP* *BALE_EZY* *BALE_HPL* *BALH_CAP* *BALH_THZ* *EPK_CAP* *EPK_HPL* *EPK_TEZY* *HPKW_CAP* *HPKW_HLK* TCH_DELIV	• [open]; no default.	• At least on input must be specified.	inverse of the efficiency of the power plant. • Said amount then enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK, MR_HPKW) equations, and contribute to the regional objective function (MR_MTSOBJ) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the facility (R_TCZY, R_TEZY, R_HZ). • If more than one commodity is consumed by a power plant then each must have a corresponding INP(ENT/MAT)c parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)p (prc,ent,t) {But not ent=LTH} INP(MAT)p (prc,mat,t) [T]	PRC_INP2 prc_enc prc_elc	*ANC_ACT* *ANC_CAP* *BAL_ACT* *BAL_CAP* *BALE_ACT* *BALE_CAP* *EPK_TEZY* TCH_DELIV	• Units of commodity consumed per unit of production. • [open]; no default.	• If a commodity is consumed by a process. • At least on input must be specified.	Energy/material consumed to generate one unit output from a process. • Said amount then enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contribute to the regional objective function (MR_MTSOBJ) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the process (R_ACT). • If more than one commodity is consumed by a process then each must have a corresponding INP(ENT/MAT)p parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)r (s,ent,t)	SEP_INP sep_ent	*ANC_TSEP* *BAL_SENT* *EPK_SEP*	• Units of commodity consumed per unit of	• If an auxiliary commodity is	Energy/material consumed to produce one unit output from a resource activity.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
{But not ent=LTH} INP(MAT)r (s,mat,t) [T]		TCH_DELIV	production. • [open]; no default.	consumed as part of a resource supply option.	• Said amount then enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contribute to the regional objective function (MR_MTSOBJ) if there is a delivery cost (DELIV) for the commodity, according to the amount of resource produced (R_TSEP). • If more than one commodity is consumed by a resource activity then each must have a corresponding INP(ENT/MAT)r parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)x (s,ent,t) {But not ent=LTH} INP(MAT)x (s,mat,t) [T]	srcencp sep	*ANC_TSEP* *BAL_SENT* *EPK_TEZY*	• Indicator. • [1]; no default.	• If a commodity is exported. • Identifies the name of the commodity, and should always be specified with a value of 1. • Only one INP(ENT/MAT)x should be specified per export option.	Energy/material "consumed" as part of an exporting activity, that is the actual exported commodity. • Said amount then enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contribute to the regional objective function (MR_MTSOBJ) if there is a delivery cost (DELIV) for the commodity, according to the amount of resource produced (R_TSEP). • Contributes to defining the flow of commodities through the RES.
INVCOST (p,t) [C]	TCH_INVCOS	*COST_INV* CRF FL/FRLIFE FRACLIFE INV_INV	• Base year monetary units per unit of capacity (e.g., 2000 M$/GW or PJa).	• Provided for all technologies for which an investment cost	The total cost of investments associated with the installation of a unit of new capacity in a given period. • The technology investment cost is spread (COST_INV) over the lifetime (LIFE) of the technology

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
		PRI_INV	• [open]; no default.	applies. • If no investment cost or lifetime (LIFE) is provided for processes (prc) then they are considered externally load managed (xpr).	by applying a capital recover factor (CRF), and adjusting for any fraction of the last period an investment is available (FRACLIFE/FRLIFE), if necessary. CRF in turn depends upon the discount rate (DISCOUNT or DISCRATE) as well as the technical lifetime (and period length NYRSPER). • If the technology is a conversion technology (con), then additional costs for the transmission (if cen, ETRANINV) and distribution (EDISTINV) infrastructure is added to the facility investment cost to determine to total investment cost (INV_INV). • The spread annualized cost (COST_INV) is applied to the investment variable (R_INV), as well as the existing residual capacity still available (R_RESID), in the regional objective function (R_MTSOBJ). Note that the investment cost is taken from the input data in the period in which the investment takes place, and from the 1st period of all the remaining residual capacity.
LIFE (p) [O]	*TCH_LIFE*	*BAL_INV* *COST_INV* *CPT_CAP* *CPT_INV* CRF CRF_RAT FL/FRLIFE FRACLIFE INV_INV *PRI_INV*	• Number of years. • [open]; for SAGE the following defaults are set if LIFE is not provided: o prc = 30 (except dum) o con = 40 o dmd =20.	• Should be provided for all technologies, except dummy (dum) processes. • If no investment cost (INVCOST) or lifetime is provided for processes (prc)	The number of years that a technology is available from the year of initial installation. • The technology investment cost (INVCOST) is spread (COST_INV) over the lifetime of the technology by applying a capital recover factor (CRF), and adjusting for any fraction of the last period an investment is available (FRACLIFE/FRLIFE), if necessary. • The spread annualized cost (COST_INV) is applied to the investment variable (R_INV), as well as the existing residual capacity still available (R_RESID), in the regional objective function (R_MTSOBJ).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				then they are considered externally load managed (xpr).	• The availability of new investments (R_INV) is tracked by means of the capacity transfer equation (MR_CPT), where said investment remains available until the lifetime is reached, including the fraction thereof if LIFE is not a multiple of the number of year per period (NYRSPER).
LIMIT (prc,t) [O]	PRC_LIM	*BAL_ACT* *BAL_CAP*	• Sum of all outputs per unit of production (e.g., PJ/PJ). • [0,1]; no default.	• Provided for processes that permit flexibility among the possible outputs.	Flag indicating that a process is to be modeled permitting flexible output of the various commodities produced by the process. When LIMIT is provided the commodity output fractions (OUT(ENC)p) are interpreted as maximum shares, rather than a rigid fixed ratio. The value of LIMIT defines the overall efficiency of such processes. • The LIMIT parameter results in a new process/commodity variable (R_LOUT) being created for each output. • The individual output flows are controlled by a limit equation (MR_LIM) for each commodity. • The overall efficiency of the process is ensured by a process balance equation (MR_PBL) that sums the individual output flows (R_LOUT) and makes sure that they do not exceed the LIMIT * process activity (R_ACT).
LSPILL (p,p,t) [L]			• Fraction. [.01 - 1], none.	• Provided if SETL activated (see section 4.4) for learning technologies that have learning components.	The amount of learning that “spills” from one technology to another to contribute to the latter’s learning rate. There is also a regional form of LSPILL_R that handles “spill” between regions and technologies. • Results in a portion of the cumulative investment of a learning component to be taken into consideration when determining the current costs for a “dependent” learning

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
					technology.
MA(ENC)_TID (dmd,enc) MA(MAT)_TID (dmd,mat) [T]	DMD_MATI snk_enc	*BAL_INV* T08	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is consumed as part of putting a new investment in place.	Energy/material required at the time an investment in new capacity is made. • Said amount enters the balance equation (MR_BAL) in the period in which the investment takes place. • If more than one commodity is consumed at investment time then each must have a corresponding MA(ENT/MAT)_TID parameter. • Contributes to defining the flow of commodities through the RES.
MA(ENT) (dmd,ent,p) MA(MAT) (dmd,mat,p) [T]	DMD_MA dd_ma	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *EPK_CAP* *HPKW_CAP* TCH_DELIV	• Units of commodity per unit of device output (e.g., PJ/PJ). • [open]; no default.	• If a commodity is consumed by a demand device.	The share of energy/material consumed to service one unit demand from a demand device. • Said amount then enters the balance (MR_BAL, MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPKW) equations, and contribute to the regional objective function (MR_MTSOBJ) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the facility (R_TCZY, R_TEZY, R_HZ). • If more than one commodity is consumed by a power plant then each must have a corresponding MA(ENT/MAT) parameter. • Contributes to defining the flow of commodities through the RES.
MED-BASEANNC [C]	MEDBANNC		• Base year monetary units (e.g., 2000 M$). • [open]; no default.	• Obtained from $SET MARKALED 'BPRICE' run as	Annualized costs from the MED reference run used to calculate the change in annual producer/consumer surplus in the reports.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				written to <reg>.EDD file.]	
MED-BASEOBJ [C]	BASEOBJ		• Base year monetary units (e.g., 2000 M$). • [open]; no default.	• Obtained from $SET MARKALED 'BPRICE' run as written to <reg>.EDD file.	Total system cost for the MED reference run used to report change in total producer/consumer surplus.
MED-DMBPRICE (d,t) [C]	*DMBPRICE*	*DM_ELAST* *DM_STEP* *DM_VAR*	• Base year monetary units (e.g., 2000 M$). • [open]; no default.	• Obtained from $SET MARKALED 'BPRICE' run as written to <reg>.EDD file.	Marginal cost of the individual flexible demads for the MED reference run. • Used in the calculation of the cost to the objective function (MR_MTSOBJ) as a result of up/down movement of the elastic demands (R_ELAST) according to MED-ELAST/STEP/VAR.
MED-ELAST(BD) (d,b,t) [D]	*DM_ELAST*	*DMBPRICE* *DM_STEP* *DM_VAR*	• Scalar exponent. • [open]; no default.	• An elasticity is required for each step/direction the demand is permitted to	Elasticity of demand indicating how much the demand rises/falls in response to a unit change in the marginal cost of meeting the elastic demands. Contributes to determining (according to MED-ELAST/STEP/VAR) the amount the demand that can

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				move. A different value may be provided for each direction, thus curves may be asymmetric.	move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (MR_MTSOBJ). The core calculation is shown in the footnote[1], and is multiplied by MED-BPRICE when entering the objective function.
MED-STEP(BD) (d,b,t) [D]	*DM_STEP*	*DMBPRICE* *DM_ELAST* *DM_VAR*	• Count. • [open]; no default.	• The number of steps is required for each step/direction the demand is permitted to move. • A different value may be provided for each direction, thus curves may be	The number of steps to use for the approximation of the change in producer/consumer surplus when running MED. • Contributes to determining (according to MED-ELAST/STEP/VAR) the amount the demand can move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (MR_MTSOBJ). [See MED-ELAST(BD) for calculation.]

[1]

$$\sum_{b,u=1}^{MED\_STEP(BD)_{r,d,b,t}} \left\{ \left\langle -\left\{ \begin{matrix} [1+(MED\_VAR_{r,d,b,t}/MED\_STEP(BD)_{r,d,b,t}*(u-.5))]** \\ (1/MED\_ELAST_{r,d,b,t}) \end{matrix} \right\} \$b='UP'+ \left\{ \begin{matrix} [1-(MED\_VAR_{r,d,b,t}/MED\_STEP(BD)_{r,d,b,t}*(u-.5))]** \\ (1/MED\_ELAST_{r,d,b,t}) \end{matrix} \right\} \$b='LO' \right\rangle * R\_ELAST_{r,t,d,b} \right\}$$

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				asymmetric.	
MED-VAR(BD) (d,b,t) [D]	*DM_VAR*	*DM_BPRICE* *DM_ELAST* *DM_STEP*	• Percent • [0,1]; no default.	• A variance is expected for each step/direction the demand is permitted to move. • A different value may be provided for each direction, thus curves may be asymmetric.	Variance of demand indicating how much the demand rises/falls in response to a unit change in the marginal cost of meeting the elastic demands. • Contributes to determining (according to MED-ELAST/STEP/VAR) the amount the demand can move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (MR_MTSOBJ). [See MED-ELAST(BD) for calculation.]
MKT_CE (m)			• Either percent (if MKTSHR = INVPCT) or monetary units (if INV) or activity units (if ACT). • [depends on MKTSHR type], default =.2.	• Required if MKTSHR is activated (see section 4.4) for each MKT_ID group, unless default will do. • Must be provided for each group if MKTSHR is not INVPCT.	The "close enough" criteria qualifying a technology to get a share of a market based on how close to being competitive the technology is when it does not make it into the preliminary solve for a period. • Involved in each market share group evaluation to determine the candidates to be included in the re-assignment of the shares for the competing technologies.
MKT_GAMA (m)			• Scalar. • [1-5]; default = 2.	• Provided if MKTSHR is activated (see	The degree of optimization to be applied when determining the share for qualifying candidates in a market share group.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				section 4.4) for each MKT_ID group, unless default will do.	• Involved in each market share group evaluation to determine how much of a share each of the candidates is to receive.
MKT_LO (m)			• Either percent (if MKTSHR = INVPCT) or monetary units (if INV) or activity units (if ACT). • [depends on MKTSHR type], default =.00001.	• Provided if MKTSHR is activated (see section 4.4) for each MKT_ID group, unless default will do.	Tiny lower bound applied to the investment variable (R_INV) if MKTSHR is INV or INVPCT, or capacity (R_CAP) if MKTSHR is ACT. • Applied to each MKT_GRP technology to force a marginal value on each candidate.
MKT_PREF (m,p)			• Scalar. • [.001 - 5]; default = 1.	• Provided if MKTSHR is activated (see section 4.4) for each MKT_ID group, if a weighted preference is desired unless default will do.	A preference or weighting factor applied to an individual technology when determining its share as part of the market share algorithm. • Involved in each market share group when determining how to split the reallocation group and how much of a share the candidate is to receive.
MKT_REAL (m)			• Scalar. • [.001 - 1]; default = .2.	• Provide if MKTSHR is activated (see section 4.4) for each MKT_ID group, unless	The size of the market to subject to the reallocation algorithm. • Involved in each market share group when determining how big the market to be reallocated is to be.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				default will do.	
MKT_RARC (m)			• Scalar. • [.000001 - 1]; default = .1.	• Provided if MKTSHR is activated (see section 4.4) for each MKT_ID group, unless default will do.	The size of the market to subject to the 2nd reallocation pass if one of the reallocated shares exceeds the smallest basic technology that had penetrated during the initial solve. • Involved in each market share group 2nd reallocation algorithm.
MM-SCALE [M]			• Scalar. • [.000001 to 1]; default = 1.	• Default applies in SAGE.	The internal scaling of the model when running MACRO to scale the energy component (EC) relative to the overall economy. • Appears in every equation of the model.
MO(ENC)_TID (dmd,enc) MO(MAT)_TID (dmd,mat) [T]	DMD_MOTI REL_ENC dd_mo	*BAL_INV*	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is released as part of decommissioning an investment made during the modeling horizon.	Energy/material released per unit of investment at the end of the useful lifetime (LIFE) of a demand device. • Said amount enters the balance (MR_BAL) for the commodity according to the level of investment (R_INV). • If more than one commodity is produced at the end of the lifetime (LIFE) of a demand device then each commodity must have a corresponding MO(ENC/MAT)_TID parameter. • Contributes to defining the flow of commodities through the RES.
MO(ENC) (dmd,enc,t) MO(MAT) (dmd,mat,t)	DMD_MO dd_mo	*BAL_CAP*	• Units of commodity released per unit of energy service provided (e.g., PJ/PJ).	• If a commodity is produced as part of servicing a demand.	Energy/material released per unit of activity of a demand device. • Said amount enters the balance (MR_BAL) for the commodity according to the level of operation of the

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[T]			• [0,1]; no default.		device(R_CAP with consideration for capacity factor and units). • If more than one commodity is produced by a device then each commodity must have a corresponding MO(ENC/MAT) parameter. • Contributes to defining the flow of commodities through the RES.
OUT(DM) (dmd,d,t) [T/D/O]	DMD_OUT DMD_OUTX dmd_dm dmd_dms	*BALE_CAP* *BALH_CAP* *DEM_CAP* *EPK_CAP* *HPKW_CAP*	• End-use demand service met per unit of device activity. • [0,1]; no default.	• Provided for each demand sector serviced by a demand device. • Each device is expected to service at least one sector. • The total of the OUT(DM) should sum to 1.	The fraction of the output from a demand device that services a particular demand. • The amount that output (R_CAP taking into consideration the capacity factor and units) contributing to the meeting to the demand (MR_DEM). • When electricity/heat is involved the load is adjusted according to the weighted DM_FR for each demand serviced, affecting the balance (MR_BALE, MR_BALDH) and peak (MR_EPK, MR_HPKW) equations. • If more than one demand is serviced by a device then each demand must have a corresponding OUT(DM) parameter. Contributes to defining the flow of commodities through the RES.
OUT(ELC)_TID (con,elc) OUT(LTH)_TID (con.,lth) [T]	*CON_GRID* CON_HGRD tch_enc	*ANC_TCZY* *ANC_TEZY* *ANC_THZ* *BALE_CAP* *BALE_EZY* *BALE_HPL* *BALH_CAP*	• Indicator. • [1]; no default.	• Identifies the grid to which a conversion technology is connected. • A conversion	An indicator (always 1) as to which electric/low-temperature heat grid a power plant is connected. • The grid controls to which electric/heat balance MR_BALE/MR_BALDH) and peaking (MR_EPK/MR_HPK) equations a conversion plant contributes, and which transmission and distribution costs (ETRANOM, EDISTOM, DTRANOM) are to be

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
		BALH_CYZ *BALH_THZ* *EPK_CAP* *EPK_HPL* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP* *HPKW_CPD* *HPKW_HLK* INV_INV TCH_DELIV T04		plant may only feed a single electricity/heat grid.	charged to the regional objective function (MR_MTSOBJ). • Contributes to defining the flow of commodities through the RES.
OUT(ENC)c (con,enc,t) OUT(MAT)c (con,mat,t) OUT(ENC)p (prc,enc,t) OUT)MAT)p (prc,mat,t) [T]	CON_OUT con_enc PRC_OUT prc_enc	*BAL_ACT* *BAL_CAP* *BAL_TCZY* *BAL_TEZY* *BAL_THZ*	• Units of commodity released per unit of activity (e.g., PJ/PJ). • [0,1 usually]; no default.	• If a commodity is produced by a process or conversion technology.	The amount of a commodity released per unit of activity. In the case of conversion plants this is a by-product of the electric/heat generation activity. For processes this is the fixed or maximum (in the case of flexible LIMIT processes) primary output from the process. • Said amount then enters the balance (MR_BAL) equation according to the level of operation of the facility (R_TCZY, R_TEZY, R_HZ). • If more than one commodity is produced by a process or as a bi-product from a conversion plant then each commodity must have a corresponding OUT(ENC/MAT)_TIDc/p parameter. • Contributes to defining the flow of commodities through the RES.
OUT(ENT)r (s,ent,t)	sep tpsep	*ANC_TSEP* *BAL_TSEP* *BALE_SEP*	• Indicator. • [1]; no default.	• Identifies the commodity produced, as well	For resource activities, other than exports, an indicator (should be 1) as to what is the commodity is produced from a resource supply option.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
{not ent = lth} OUT(MAT)r (s,mat,t) [T]		*BAS_SEP* *CUM_TSEP* *EPK_SEP*		as the grid for electricity.	• The commodity enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contributes to the regional objective function (MR_MTSOBJ) if there is a resource cost (COST) for the commodity, according to the amount of resource produced (R_TSEP). • Contributes to defining the flow of commodities through the RES.
OUT(MAT)_TIDc (con,mat) OUT(MAT)_TIDp (prc,mat) [T]	CON_OUT1 PRC_OUT1 prc_enc rel_enc	*BAL_INV*	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is produced as part of retiring a new investment at the end of its lifetime (LIFE).	Material released per unit of investment at the end of the useful lifetime (LIFE) of a process or conversion technology. • If more than one commodity is produced at the end of the lifetime then each commodity must have a corresponding OUT(MAT)_TIDc/p parameter. • Contributes to defining the flow of commodities through the RES.
PD(Z)D (con,z,t) {except con=cpd} [O]	TCH_PD	EPK_TEZY HPKW_ACT PEAK_TID TEZY_CAP THZ_CAP	• Fraction. • [0,1]; default = 1, when PEAK_TID (CON) provided.	• Most often left to the default of 1. • Not available for coupled heat and power plants (cpd).	For power generating plants consider peaking devices (PEAK_TID(CON)) whose contribution to the peaking constraint is only to be credited according to its operational level, the fraction of the daytime in a season for electricity or season for heat that they are available. • The activity of such plants is controlled by the utilization equation (MR_TEZY, MR_THZ) by limiting the activity (R_TEZY, R_THZ) to the peak duration * fraction of the year (QHR(Z)(Y), QHRZ). • In the peaking constraint (MR_EPK, MR_HPKW) the activity variable enters the equation taking into consideration both the percent capacity that can be

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
					credited to the peak (PEAK(CON)) and the duration of operation within the time-slice.
PEAK(CON) (con,z,t) [O]	PEAK1	*EPK_CAP* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP* *HPKW_CPD*	• Fraction. • [0,1]; default = 1.	• For most plants it is assumed that all the capacity is available to meet the peak, however for intermittent technologies (e.g., solar, wind) a value less 1than is normally provided.	For power generating plants, the fraction of the plant's capacity that should be credited towards the peaking requirement. • The multiplier applied to the capacity variable (R_CAP), or activity variable (R_TEZY/R_THZ) if peaking device (PEAK_TID(CON)) taking into consideration the duration factor (PD(Z)D), in the peaking constraint (MR_EPK, MR_HPKW).
PEAK(CON)_TID (con) [O]	PEAK_TID	*BAS_TEZY* *EPK_CAP* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP* *HPKW_CPD* *PD(Z)D* *TEZY_CAP* *THZ_CAP*	• Indicator. • [1]; no default.	• Used for peaking-only devices that have low capital costs but high operating costs, which sometimes results in the model building but not using such technologies. If such decisions were made by the model only to	An indicator that a power generating plant is only to be credited towards the peaking requirement based upon its operation, not its capacity. • The indicator serves as a switch which inhibits the generation of an activity variable (R_TEZY, R_THZ) a off-peak times, thus allowing such plants to only operate at potential peak times. • The activity of such plants is controlled by the utilization equation (MR_TEZY, MR_THZ) by limiting the activity to the peak duration (PD(Z)D) * fraction of the year (QHR(Z)(Y), QHRZ). • In the peaking constraint (MR_EPK, MR_HPKW) the activity variable enters the equation taking into consideration both the percent capacity that can be credited to the peak (PEAK(CON)) and the duration of

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				meet the peaking requirements, then activating this tends to result in the running of such plants, or the selection of some other technology. • Not permitted for coupled production (cpd) or base load (bas) plants.	operation within the time-slice.
PEAKDA(PRC) (prc,t) [O]	PEAK_PRC	*EPK_ACT* *EPK_CAP*	• Fraction. • [0,1]; default = 1.	• Only provided if some part of the demand for electricity is know not to occur during the peak time.	For processes that use electricity the fraction of the consumption that is coincident with the peak. • A multiplier adjusting the amount of electricity consumed (R_ACT/R_CAP (xpr) consumption) that contributes to peaking requirements (MR_EPK).
PEAKDA(SEP) (s,t) [O]	*PKDA_SEP*	*EPK_SEP*	• Fraction. • [0,1]; default = 1.	• Only provided if some part of the import/export/ consumption of electricity does not occur during the peak time.	For the import and export of electricity, and for ancillary use of electricity, the fraction of the supply/export/consumption that is coincident with the peak. • A multiplier adjusting the amount of electricity delivered/exported/consumed (R_TSEP, latter adjusted for consumption) that contributes to meeting (for imports) or add to the requirements for peak capacity

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				• For resources consuming electricity during the extraction process, it indicates the fraction of said consumption that is coincident with the peak.	(MR_EPK).
PRAT (p) [L]			• Ratio. • [.01 - 1], none.	• Required if SETL activated (see section 4.4) for each technology to be subjected to endogenous learning.	Progress ratio, or the driver for determining the amount by which the investment cost changes for every doubling of capacity. • Used to determine the current investment costs based upon the anticipated progress as a function of the cumulative capacity installed.
QHR(Z)(Y) (w) [M]	QHR_{18D}	$QHRZ_{8D}$	• Fraction. • [0,1]; no default.	• Must be provided for each time-slice to be represented. • Omit for any time-slice NOT to be tracked.	The fraction of the year represented by each time-slice. The year may be divided up into six time-slices according to season (Z=Winter/Intermediate/Summer) and time-of-day (Y=Day/Night). The time-slices are used to shape electricity and heat load to discriminate between times of high and low demand. • The electricity and heat balance (MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPK) equations are only generated for those time-slices specified. • All power plants (con) operate based upon these

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
					fractions of the year, with variables (R_TCZYH, R_TEZY, R_HZ) only created for those time-slices specified. • The power sector generation in each time-slice is matched upon against the load accumulated according to the various demands shapes (DM_FR(Z)(Y)), which default to QHR(Z)(Y) when not provided.
RAT_RHS (a,b,t) [U]	*RAT_RTY1* *RAT_RTY*		• Fraction. • [open]; no default.	• MR_ARAT user constraints are only generated when the user explicitly provides a value (including 0) for the RHS in a period.	RAT_RHS has three purposes. It is a designator as to the sense ('LO','FX','UP','NON', corresponding to MR_ARAT1,2,3,4 respectively) of a user-defined Ad hoc constraint, indicates in which periods the constraint applies (whenever a value is explicitly provided for period), and the value of the RHS. • A user-defined constraint (MR_ARAT1-4, according to the sense of the equation) generated by applying the associated multipliers (the RAT_* parameters below) directly to the appropriate associated variables.
RAT_ACT$_{6E,6P,6Q,6R}$ (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided. • Note that the same coefficient is used for all time-slices when RAT_ACT is used for	The coefficient in a user-defined constraint (MR_ARAT1/4) to be applied to a technology according to its annual activity. • The multiplier to be applied to the activity variable (R_ACT (for prc), R_CAP (for dmd), R_TCZYH (pass-out cpd), R_TEZY (ela) , R_THZ (hpl)) for each time-slice. • For externally load managed and demand technologies (xlm, xpr, dmd) the coefficient is applied to the capacity variable (R_CAP) according to the operation of the technology.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				conversion plants. If the coefficient is to vary according to time-slice, then the individual RAT_TCZY/EZY/HZ parameters need to be used.	
RAT_CAP$_{6F}$ (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint according to capacity. • An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to its total installed capacity (R_CAP).
RAT_HPL$_{6R}$ (a,hpl,z,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period and season for which a coefficient is provided. • The multiplier is applied to the seasonal activity variable (R_HZ) only for those seasons for which the parameter is provided.	The coefficient in a user-defined constraint according to the seasonal (z) activity of a heating plant (hpl). • An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to the seasonal activity of a heating plant (R_THZ).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
RAT_INV$_{6J}$ (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint according to new investment in a technology. • An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to the level of new investment (R_INV).
RAT_SEP (a,s,t) [U]	*RAT_TSEP*$_{6R}$		• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint according to the level of production from a resource supply option (srcencp). • An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to the resource activity level (R_TSEP).
RAT_TCZY$_{6P}$ (a,cpd,w,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period and time-slice for which a coefficient is provided. • The multiplier is applied to the activity variable (R_TCYZH) only for those time-slices for which the parameter is provided.	The coefficient in a user-defined constraint according to the time-slice (w) activity of a coupled heat and power (cpd) pass-out turbine (ELM/CEH). • An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to the time-slice activity of the heat output from a pass-out turbine (R_TCZYH).
RAT_TEZY$_{6Q}$ (a,ela,w,t)			• Constant.	• An intersection is created for	The coefficient in a user-defined constraint according to the time-slice (w) activity of an electric plant (ela).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[U]			• [open]; no default.	each period and time-slice for which a coefficient is provided. • The multiplier is applied to the activity variable (R_TCYZH) only for those time-slices for which the parameter is provided.	• An intersection is created in the user-defined equation (MR_ARAT1/4) for a technology according to the time-slice activity of the electricity output (R_TEZY).
REG_XCVT(ent)$_{11S, 11T, 21I}$ (r,ent) *REG_XCVT(env)* $_{21I}$ (r,v) *REG_XCVT(mat)* $_{11S, 11T, 21I}$ (r,mat) [T]			• Constant. • [open]; default = 1.	• Provided if the units of any commodity are deferent between regions.	Commodity conversion factor by region for bilateral and global trade. • Applied to the import/export (R_TSEP, R_TSEPE) and global (R_GTRD) trade variables in the bi-lateral (MR_BITRD, MR_BITRDE) and global (MR_GTRD) constraints.
REG_XMONY$_{24M,24N}$ (r) [T/C]			• Constant. • [open]; • Default = 1.	• Provided if the monetary units in any region are deferent.	Monetary conversion factor by region for the total costs of the energy system in each region. • Applied to the regional total system cost variable (R_MTSOBJ, R_objZ) for each region as it enters the final objective function (MR_OBJ).

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
REH (cpd,t) [O]	*T_CPDREH*	*ANC_TEZY* *BALH_CYZ* *HPKW_CPD* *HPKW_HLK*	• Unit of electricity produced per unit of heat produced. • [0,1]; no default.	• Provided if a coupled-heat and power plant is a back-pressure turbine.	For back-pressure turbine coupled production plants (cpd), that is those having a fixed electricity-to-heat ratio, the ratio of electricity per unit of heat produced. • Since there is a fixed relationship between the amount of electricity and heat, the electric production variable (R_TEZY) is used to represent both. REH is thereby applied to R_TEZY to reflect the heat contribution from such plants to the heat balance (MR_BALH) and peaking (MR_HPKW) constraint. • If there are O&M charges for the heat distribution system (DTRANOM) then the variable also appears in the regional objective function (MR_MTSOBJ).
RESID (p,t) [O]	*TCH_RES*		• Unit of capacity (e.g., GW, PJa). • [0,1]; no default.	• Provided for each period for which capacity installed prior to the beginning of the model horizon is still in place.	For technologies available at the beginning of the modeling horizon, that is those that were in place before the 1st year of the model, the "retirement" trajectory as those technologies reach their useful lifetime. Thus for each period RESID is the exogenous specification of the residual capacity left around for those technologies inherited by the model when it started. • The RESID appears as the right-hand-side of the capacity transfer constraint (MR_CPT1/2/3). • As the annualized investment cost associated with the RESID technologies is charged to the system as long as it stays available, a variable (RESID) is included in the objective function against which to apply said costs (rather than having a large constant in the objective function).
SC0			• Monetary units.	• Required if	Cost of the initial capacity when the learning algorithm

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
(p) [L]			(M$2000) • [any], none.	SETL activated (see section 4.4) for each technology to be subjected to endogenous learning.	starts up. • Used to determine the current investment costs based upon this initial cost and the subsequent amount of cumulative capacity installed.
SRCENCP[1] [T]	sep tpsep	*ANC_TSEP* *ANC_ZSTK* *BAL_SENT* *BALE_SEP* *BAS_SEP* *CUM_TSEP* *EPK_SEP*	• List. • [alpha-numeric according naming convention]; no default.	• Provided for each resource supply options, including bi-lateral imports/exports. • A mapping set (sep) splits the single SRCENCP name into its 3 components (src,e,c). This short form is often used in the code. • [Note, it is recommended that the modeler use the same	List of resource supply options. • For each element of the set SRCENCP (sep) a resource variable is created (R_TSEP), beginning from the year the variable is 1st available. • The resource variable enters the balance (MR_BAL, MR_BALE) and if electricity (elc) the peaking constraint (MR_EPK) and base load (MR_BAS) constraints, as well as the objective function (MR_MTSOBJ). • If the total amount available from a source over the entire time horizon is limited by a cumulative (CUM) proven reserves, then the resource variable also enters a special cumulative constraint (MR_CUM). • For the stockpile activities there is a special variable created for the recovered material in the stockpile at the end of the model horizon (R_ZSTK).

[1] SRCENCP is actually a list created from the user input, and managed as a Set in the GAMS code. However, the entries themselves, constructed with the named outputs from the various resource options, control the entry in the various cost, balance and peaking equations. Therefore it is listed here as well.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				characters for the SEPent root as the primary output (OUT(ENT)r/x)) to avoid mixing internal and input/output SRCENCP names.]	
START (p/s) [O]	TCH_STRT SEP_STRT tptch tpsep et al		• Year. • [1890-2200]; default = 1st year of the model horizon.	• Used when establishing the master control set for technologies (tptch), and technology subsets (e.g., tpela, tpcpd, tpsep) that is used throughout the code to control from which period a technology is available.	The year in which a technology or resource option is initially available. • Controls from which period technology-based equations (e.g., MR_CPT, MR_TEZY, MR_THZ) and technology and resource based variables (all with p, and R_TSEP respectively) are to be generated.
STARTYRS [M]		*PRI_DF* PRI_DISC	• Number of years. • [open]; default = 2.5.	• For SAGE used just for reporting the total discounted	Shift from the 1st year of the model to the year to which all discounted costs are to be calculated. • A period runs from Jan 1 to Dec 31 over some number of years. Thus a 5year period is represented by the

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				system cost, for MARKAL part of calculating the cost coefficients for the objective function.	middle of the middle year, 2.5 years from the beginning or June 30th of the 2 year. As discounting begins from the 1st day of the 1st period, to move to said mid-point the discounting calculation needs to be shifted accordingly.
TE(ENT) (ent,t) TE(MAT) (mat,t) [O]	T_TE T_TEENC TE_CON	*BAL_TSEP* *ANC_ZSTK* *BAL_ACT* *BAL_CAP* *BAL_TCZY* *BAL_TEZY* *BAL_THZ* *BALE_CAP* *BALE_SEP* *BALE_CZY* *BALE_EZY* *BAS_CAP* *BAS_SEP* *BAS_TCZY* *BAS_TEZY* *EPK_CAP* *EPK_SEP* *EPK_TEZY*	• Fraction. • [0,1]; default = 1.	• Usually a non-unity value is only provided for electricity and low-temperature heat transmission systems (from central station plants).	The overall efficiency of the infrastructure associated with each energy carrier/material. The purpose of the commodity based efficiency factor is to reflect losses encountered due to transmission or other movement of a commodity. When less than the default o f 1, the affect being to increase the amount of the commodity that may be produced. • The transmission efficiency appears in most every equation tracking energy/material (MR_BAL, MR_BALDH, MR_BALE, MR_EPK, MR_HPK, MR_BAS), where it is applied to the technology and resource activity variables (all with p, and R_TSEP respectively).
TRD_BND$_{36I}$ (e/v) [T]			• Units of the commodity. • No default.	• Provided if a bound is to be applied.	Limit on the import or export of a globally traded commodity into/from a region. • Applied to the global trade (R_GTRD) variable for the region, period and import/export operation.
TRD_COST$_{25S,28S,25I}$			• Base year monetary	• Provided if a	Additional (transaction) cost associated with global trade

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
(e/v) [T/C]			units (e.g., 2000 M$). • [open]; default = 1.	transaction cost is to be added to the cost of bi-lateral for globally traded commodity.	in a commodity. • Applied to the import/export (R_TSEP) and global (R_GTRD) trade variables in the regional objective function (MR_MTSOBJ).
TRD_FROM$_{21D}$ (e/v) [T]			• Year. • [open]; default = 1st.	• Provided if trade in a global commodity is not to start in the 1st period.	Year from which global trade in a commodity may commence. • The global trade constraint (MR_GTRD) is only generated beginning from the TRD_FROM period. • Entries for the global trade variable (R_GTRD) are only made in the associated balance (MR_BAL_G/E) and emissions (MR_TENV) equations beginning from the TRD_FROM period.
TRNEFF(Z)(Y) (cpd,w,t) [O]	T_CPDEFF	*BALH_CZY* *HPKW_CPD*	• Decimal percentage. • [0,1]; default = 1.	• Provided if default of 1 is not desired, the time-slices have different efficiencies, or some time-slice is available. • Note that this efficiency is used in place of the commodity-based transmission efficiency (TE) for coupled heat	Transmission efficiency of low-temperature heat from coupled heat and power plants (cpd) according to season and time-of-day. • Affects the low-temperature heat balance (MR_BALDH) and peaking (MR_HPK) constraints by adjusting the amount of heat delivered (R_TCZYH, R_TEZY) to reflect the loss.

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
				and power plants.	
TSUB_BND(BD) (txs,b,t) [C]					Bound applied to a tax/subsidy as the total tax/subsidy permitted in a period.
TSUB_COST$_{25/28T,32D}$ (txs,t) [C]			• Monetary units. • No default.	• Provided if a tax (+) or subsidy (-) is to be associated with a commodity /technology.	Cost of a tax/subsidy applied to selected energy carriers and the technologies producing and/or consuming it. • The cost is applied to the accumulated total in the objective function (MR_MTSOBJ).
TSUB_ENT (txs,e.t) *TSUB_MAT* (tx,mat,t) [C]			• Indicator. • (0,1), no default.	• Provided or the commodity into/from the technology to which the tax (+) or subsidy (-) is to be associated.	Commodity to which a tax/subsidy applies. • All commodities listed for a particular tax/subsidy constraint are accumulated in each tax/subsidy equations (MR_TXSUB) based upon the amount flowing in/out of the identified resources and technologies.
TSUB_SEP$_{32S}$ (txs,p,t) [C]			• Indicator. • (0,1), no default.	• Provided or the technology to which the tax (+) or subsidy (-) is to be associated.	Contribution of a resource activity to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed. • All resources listed for a particular tax/subsidy constraint are accumulated in each tax/subsidy equations (MR_TXSUB).
TSUB_TECH$_{32E/F/J/O/P/Q/R}$ (txs,p,t)			• Indicator. • (0,1), no default.	• Provided or the technology to which the tax (+) or subsidy (-) is	Contribution of a technology to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed. • All technologies listed for a particular tax/subsidy

Input Parameter (Indexes[3]) [Purpose][4]	alias/internal Name[5]	Related Parameters[6]	Units/ Range & Defaults	Instance[7] (Required/Omit/ Special Conditions)	Description[8]
[C]				to be associated.	constraint are accumulated in each tax/subsidy equations (MR_TXSUB).
UNIFDIST (d) [D]	DM_UDIS	*BALE_CAP* *BALH_CAP* DM_FR *EPK_CAP* *HPKW_CAP*	• Indicator. • [0,1]; default = yes, if no demand fraction (FR(Z)(Y)) is to be provided.	• Results in the demand fractions (FR(Z)(Y)) being set = QHR(Z)(Y).	An indicator that the electric and heat load shapes are a unified distribution matching that of the standard time-slices (QHR(Z)(Y)). When non-standard the demand shape if specified by time-slice by means to the demand fractions (FR(Z)(Y)). • FR(Z)(Y) controls the percentage of the annual service demand met by electricity/heat that occurs in each time-slice. As such it controls the amount required for the balance (MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPKW) constraints.
VAROM (p,t) [C]	TCH_VAROM	*ANC_ACT* *ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_THZ*	• Base year monetary units per unit of activity (e.g., 2000 M$/PJ). • [open]; default = 1.	• Provided if there are variable costs associated with the operation of a technology.	The variable operating and maintenance costs associated with the activity of a technology. • The cost to be applied to the activity variable (R_ACT (for prc), R_CAP (for dmd), R_TCZYH (pass-out cpd), R_TEZY (ela) , R_THZ (hpl)) to charge the variable costs to the regional objective function (R_MTSOBJ). • For externally load managed and demand technologies (xlm, xpr, dmd) the cost applied to the capacity variable (R_CAP) according to the operation of the technology.

4.3.2 Matrix Coefficients and Internal Model Parameters

The parameters described in this section relate to the coefficients and internal parameters used in the SAGE GAMS code to control the translation of the model equations and data input to a matrix ready for solution. These constitute those parameters actually referenced in the equation models themselves, and the key internal parameters used in the SAGE GAMS code when building the matrix coefficient parameters. As such they constitute the interim and final LP matrix coefficients calculated as part of the model generation process.

Section 4.3.2.1 first lists and describes all of the sets and parameters appearing on the SAGE_MATRIX.XLS matrix overview (table 4.8). This includes input data parameters that appear directly in the equation generation code as well. In Section 4.3.2.2 the internal parameters that are crucial to building the matrix coefficients (e.g., the capital recovery factor (CRF) for annualizing the investment costs) are described.

4.3.2.1 Matrix Control Sets and Parameters

There are quite a few input parameters and attributes associated with technologies that affect which variables and matrix intersections are populated in the matrix. Under normal circumstances both activity and capacity variables are generated, annual or at the time-slice level according to the nature of the technology. Some of the key aspects controlling how a technology is modeled are presented in table 4.8 below.

Table 4.8: Matrix Component Control

Input	Type[1]	Implications
BAS	S	Conversion technologies characterized as base load are modeled using only a daytime production vectors (R_TCZY, R_TEZY). The level of said vectors reflect both the day and nighttime production, which by definition are equal.
CPD	S	Coupled heat and power technologies are characterized as either back-pressure (REH) or pass-out (ELM/CEH). In the case of the former, the heat output is directly derived from the electric production (R_TEZY), in the case of the latter the heat output is modeled (R_TCZYH) and the electricity then derived. These variables are modeled at the time-slice level, and the associated parameters must both take into consideration the type of the plant as well as for which time-slices the technology is available.
DMD	S	Demand devices are strictly modeled by means of a capacity variable (R_CAP), with the activity derived by applying the appropriate capacity factors (CF) and unit conversion (CAPUNIT).

[1] P = parameter describing the technical nature of a technology, S = set characterizing the nature of a technology.

Input	**Type[1]**	**Implications**
ELE	S	Electric generating plants are modeled at the time-slice level, and the associated parameters must take into consideration the type of the plant (e.g., conventional, external load managed, base load, storage, coupled heat and power) as well as for which time-slices the technology is available.
HPL	S	Heat generating plants are modeled at the season level, and the associated parameters must take into consideration the type of the plant (e.g., conventional, externally load managed) as well as for which seasons the technology is available.
NST	S	Night storage technologies are special processes (prc) and demand devices (dmd) that consume off-peak electricity at night and provide commodities or demand services during the day.
PEAK_TID (CON)	P	Peaking only technologies have their activity controlled to ensure they operate at peak times only (e.g., daytime).
START	P	The period from which a technology or resource option is first available is used to inhibit generation of all related equations and variable before that time.
XPR/XLM	S	Externally load managed process and conversion plants are forced to operate according to the explicit capacity factors (CF/CF(Z)(Y)) provided by the user. As such no activity variable needs to be generated, but rather the activity level is determined from the installed capacity (R_CAP).

To manage this complexity a series of internal parameters are built by the preprocessor of the matrix generator to ensure that each technology is properly modeled. These parameters indicate which technologies belong in the various constraints and under what circumstances, thus enabling them to control the permitted intersections in the matrix and make the equation code rather straightforward.

To the extent possible parameter prefixes "match" the root of the equation to which the coefficient applies. These are summarized in table 4.10 below.

Table 4.9: Description of Matrix Parameters Prefixes[1]

Parameter Prefix	Description
ANC_	Annualized costs appearing in the regional objective function (MR_MTSOBJ).
BAL__	Contribution (+ for production, for consumption) to the energy and material balance constraints (MR_BAL).
BALE_	Contribution (+ for production, for consumption) to the electricity balance constraints (MR_BALE), including an indication of in which time slices.
BALH_	Contribution (+ for production, for consumption) to the low temperature heat balance constraints (MR_BALDH), including an indication of in which season.
BAS_	Contribution (+ for base load, for non base load) to the electricity base load constraint (MR_BAS).
BND_	Annual bound to be applied to time sliced variables (R_TCZYH, R_TEZY, R_THZ) by means of associated constraints (MR_BNDCON).
CPT_	Capacity transfer constraint (MR_CPT).
DM_	Demand for energy services (MR_DEM).
ENV_	Total emission over the entire modeling horizon (MR_ENV).
EPK_	Electricity peaking constraint (MR_EPK).
HPKW_	Heat peaking constraint (MR_HPK).
PRC_	Limit processes (MR_LIM, MR_PBL).
PRI_	Discounted costs (MR_PRIC).
RAT_	User-defined constraints (MR_ARAT).
REG_	Multi region linkage (MR_MTSOBJ, MR_OBJ).
SEP_	Limits applied to resources (MR_CUM and R_TSEP annual bound).
TCH_	Technology characterization, thus apply to various constraints.
TENV_	Annual emissions constraint (MR_TENV).
TEZY_	Electricity plant utilization (MR_TEZY).
THZ_	Heating plant utilization (MR_THZ).

[1] Only those prefixes that apply to more that 1 parameter are listed here.

Parameter Prefix	**Description**
TRD_	Bi-lateral trade (MR_BITRD, MR_BITRDE).
UPRC_	Process utilization constraint (MR_UTLPRC).

Similarly, the suffix on many of the parameter names also follows a convention. Whenever possible it reflects the variables to which the parameter is applied, as noted in table 4.10 below.

Table 4.10: Description of Matrix Parameters Variable Name Suffix

Parameter Suffix	**Description & Variables**
ACT	Related to the annual activity of a technology. Most often relates to the annual activity of a process (R_ACT). But also an annual value to used in all time-slices for conversion technologies (R_TCZY, R_TEZY, R_HZ), or the activity rather that the capacity of demand devices (R_CAP).
CAP	Related to the total installed capacity of a technology (R_CAP).
INV	Related to the newly installed capacity of a technology (R_INV).
TCZY	Related to the production of heat (and electricity) from coupled heat and power pass-out turbines in each time-slice (R_TCZYH).
TEZY	Related to the production of electricity from power plants in each time-slice (R_TEZY).
THZ	Related to the production of low-temperature heat from heating plants in a season (R_THZ).
TSEP	Related to the annual production of a resource supply option (R_TSEP).
ZTSK	Related to the final released of a stockpiled commodity (R_ZSTK). [Note in SAGE at this time.]

The table 4.11 below describes each of the parameters and sets involved in the matrix. A heavy emphasis is put on identifying the key input parameters from which these important control parameters are derived. The matrix intersections to which the parameter applies are referenced as the row and column intersection in the SAGE_MATRIX, and can be often be derived from the prefix/suffix comprising many of the internal parameter names. The input parameters are themselves defined in table 4.7.

Table 4.11: Definition of Matrix Coefficient Parameters and Controls[1,2]

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
ANC_ACT$_{25E}$ (t,prc)	I	Annualized cost of operating a processes (R_ACT) comprising the variable operating costs (VAROM), and any delivery (DELIV) according to how much of the commodity is consumed in the current period (taking into consideration both the input amount (INP(ent/mat)p) and any lead or pre-capacity based commodity requirements (LED(enc/mat)). Note that technologies for which SAGE does not construct an activity variable the capacity variable is used instead; see ANC_CAP.
ANC_CAP$_{25F}$ (t,p)	I	Annualized cost associated with the total capacity in place (R_CAP) of a technology. Note that in the case of technologies for which SAGE does not build an activity variable. That is for demand devices (dmd) and externally load managed processes or conversion plants (xpr, xlm) the coefficient associated with the activity of the technology must to associated with the capacity variable instead of the instead of the capacity variable as would normally be the case.

[1] Those parameters and algorithms related to the inter-period SAGE Market Share (MKTSHR) and Endogenous Technology Learning (SETL) facilities are described in Sector 4.6. They are not included here as they do not explicitly appear in the actual matrix of the model passed to the solver(s).
[2] Some parameter not employed in SAGE, but used by MARKAL, have been omitted from this table at this time.
[3] For certain parameters there may be substantively different conditions/code to warrant splitting out the details of an internal parameter according to the main user sets. The documentation index letters are used here, except when a parameter only applies to a subset of the master index sets.
[4] Type indicates the nature of the parameter/control as follows:

- A – Alias of a user input parameter;
- I – Internal parameters, most often derived from a combination of user input parameters and other conditions;
- S – Set (either internal or by user), and
- U – User input parameter.

[5] Text references in upper case to (INPUT PARAMETER) correspond to those parameters listed in table xxx. Sets (set) may also be noted in lower case. Model equations and variables (MR_<equa>/R_<var>) mentioned here are described in Section xxx.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
(t,con)		For conversion plant (con), the annual fixed operation and maintenance cost (FIXOM) as input by the modeler. For externally load-managed plants (xlm), any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ent/mat)c) and capacity-to-activity and unit conversion factors (CAPUNIT/CForCF(Z)(Y)). Note that for electricity and district heat the transmission (ETRANOM, DTRANOM) and distribution (EDISTOM, only when electricity) are also accounted for. Furthermore, note that the season/day night nature of the operation of these plants is also taken into consideration (via CF(Z)(Y) and QHR).
(t,dmd)		For demand devices (dmd), the annual fixed operation and maintenance cost (FIXOM) plus any associated variable operating (VAROM) and delivery (DELIV) costs, taking into consideration the appropriate consumption/performance (MA(ent/mat), EFF) information and capacity-to-activity and unit conversion factors(CAPUNIT, CF).
(t,prc)		For processes (prc), the annual fixed operation and maintenance cost (FIXOM) as input by the modeler. For externally load-managed processes (xpr), and those processes with no investment cost (INVCOST) and life (LIFE) specified, any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ent/mat)p/LED(ent/mat)) and capacity-to-activity and unit conversion factors (CAPUNIT/CF).
ANC_TCZY_{25O} (t,cpd)	I	For coupled heat and power plant which are pass-out turbines (ELM/CEH(Z)(Y)) according to the amount of heat produced (R_TCYZH), the annual variable operating (VAROM) and delivery (DELIV) are accounted for, taking into consideration consumption/performance (INP(ent/mat)c, ELM/CEH(Z)(Y)) and transmission (ETRANOM, DTRANOM for centralized plants (cen)) and distribution (EDISTOM, only when electricity). The code must then do some exception handling for base load (bas) technologies (these modeled using only a 'D'ay variable).
ANC_TEZY_{25P} (t,ela)	I	For standard electric power plant (ela), according to the amount of electricity produced (R_TEYZ), the annual variable operation and maintenance cost (FIXOM) along with any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ent/mat)c) and capacity-to-activity and for electricity and district heat the transmission (ETRANOM, DTRANOM for centralized plants (cen)) and distribution (EDISTOM). The code must then do some exception handling for peaking-by-activity (PEAK_TID) (with variables on for peaking times) and base load (bas) and storage (STG) technologies (these modeled using only a 'D'ay variable).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
ANC_THZ$_{25Q}$ (t,hpl)	I	For standard heating plants (hpl) according to the amount of heat produced (R_THZ), the annual variable operating (VAROM) and delivery (DELIV) are accounted for, taking into consideration consumption/performance (INP(ent/mat)c) and transmission (DTRANOM for centralized plants (cen)). The code must then do some exception handling for base load (bas) technologies (these modeled using only a 'D'ay variable).
ANC_TSEP$_{25R}$ (t,e,c)	I	The cost (COST) of obtaining a resource according to its activity (R_TSEP). This includes the operation and maintenance cost of power lines (ETRANOM, EDISTOM) if importing electricity. Assumes a negative value for exports. If there are delivery costs (DELIV) associated with the consumption of auxiliary commodities (INP(ENT)r) they are reflected as well.
ANC_ZSTK$_{25U}$ (t,enc,c)	I	Value of stockpiled material at the end of the modeling horizon (R_ZSTK) according to the unit cost (COST_TID).
BAL_ACT$_{7E,32E}$ (t,t,prc,enc)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)p) or produced (OUT(ENT/MAT)p) in a time period by processes as a function of the overall activity (R_ACT) of the process. Any transmission losses (TE(ENT/MAT)) are taken into consideration. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes needed.
BAL_CAP$_{7F,32F}$ (t,t,p,enc)	I	The amount of each commodity (other than electricity and heat) consumed (MA(ENT/MAT), INP(ENT/MAT)p) or auxiliary commodity produced (MO(ENT/MAT), OUT(ENT/MAT/p) in a time period by demand devices (dmd) or externally load managed technologies (xlm/xpr) based upon total installed capacity (R_CAP). The capacity factor (CF/CF(Z)(Y)) and unit (CAPUNIT), along with efficiency (EFF) for demand devices is applied as well. Any transmission losses (TE(ENT/MAT)) are taken into consideration. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes needed.
BAL_INV$_{7J,32J}$ (t,t,p,e)	I	The amount of each commodity (other than electricity and heat) required (MA(ENT/MAT)_TID, INP(ENT/MAT)_TID) at the time an investment is made in new capacity (R_INV). Any transmission losses (TE(ENT/MAT)) are taken into consideration. For nuclear fuel (enu) the commodity flow occurs in the previous time period (INP(ENC)_TID), for all others in the period the investment takes place. Thus double period indexes needed.
BAL_SENT$_{7S,9S,18S}$ (t,s,e)	I	The amount of an auxiliary commodity (other than electricity and heat) consumed (INP(ENT/MAT)r) in a time period by a resource supply activity (R_TSEP). Any transmission losses (TE(ENT/MAT)) are taken into consideration.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
BAL_TCZY$_{7P,32P}$ (t,cpd,enc,w)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by a coupled production pass-out turbine according to the heat generation in each time-slice (R_TCZYH). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_TEZY$_{7Q,32Q}$ (t,p,enc,w)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by an electric generation plant according to the level of activity in each time-slice (R_TEZY). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_THZ$_{7R,32R}$ (t,hpl,enc,z)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by a heating plant according to the heat generated in a season (R_THZ). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_TSEP$_{7S,32S}$ (t,t,s)	I	The amount of a commodity (other than electricity and heat) delivered as part of a resource activity (OUT(ENT/MAT)r) in a time period by a resource supply activity (R_TSEP). For stockpiled resources (‘STK’) there is a carry-over of the current amount to the next period, thus double period indexes are needed. Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_ZSTK$_{7V}$ (t,enc,c)	I	The amount of a commodity (other than electricity and heat) salvaged from a stockpile (R_ZSTK) in the final period, taking into consideration any transmission losses (TE(ENT/MAT)).
BALE_ACT$_{9E,32E}$ (t,t,prc,elc,w)	I	The amount of electricity consumed (INP(ENT)p) in a time period by processes as a function of the overall activity (R_ACT) of the process. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes needed.
BALE_CAP$_{9F,32F}$ (t,t,p,elc,w)	I	The amount of electricity consumed (MA(ENT), INP(ENT)p) in a time period by demand devices (dmd) or externally load managed processes (xlm) based upon total installed capacity (R_CAP). The capacity factor (CF) and unit (CAPUNIT), along with efficiency (EFF) for demand devices is applied as well. In addition, for generation of electricity (OUT(ELC)_TID) from externally load managed power plants (xlm) is accounted for according to the appropriate capacity factor (CF(Z)(Y)) and unit, and time-slice duration (QHR(Z)(Y)). The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes needed.
BALE_CZY$_{9P}$ (t,cpd,elc,w)	I	The amount of electricity (OUT(ELC)_TID) produced (R_TCZYH) in a time period by a coupled production pass-out turbine according to the heat generation in each time-slice (adjusted for base load (bas) plants appropriately) by applying the electricity/heat relationship parameters (ELM/CEH(Z)(Y). Any transmission losses (TE(ENT)) are taken into consideration.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
BALE_EZY$_{9Q,32Q}$ (t,ela,elc,w)	I	The amount of electricity (OUT(ELC)_TID) produced (R_TEZY) in a time period by an electric plant in each time-slice (QHR(Z)(Y), adjusted for base load (bas) and peaking (PEAK(CON)_TID) plants appropriately). Any transmission losses (TE(ENT)) are taken into consideration for centralized (cen) plants. The parameters also has consumption of electricity (INP(ENT)c) by storage (stg) and grid link (lnk) technologies.
BALE_HPL$_{9R,32R}$ (t,hpl,elc,w)	I	The amount of electricity consumed (INP(ENT)p) in a time period and season (QHR(Z)) by heating plants as a function of the heat produced (R_HPL).
BALE_SEP$_{9S,32S}$ (t,s,w)	I	The import/export of electricity, taking into consideration the time-slice fractions (SEP_FR), as well as losses (TE(ENT)) during importing.
BALH_CAP$_{8F,32F}$ (t,t,p,lth,w)	I	The amount of low-temperature heat consumed (MA(ENT)) in a time period by demand devices (dmd) based upon the activity from total installed capacity (R_CAP). The capacity factor (CF), efficiency (EFF) and unit (CAPUNIT), as well as the split to the demand (OUT(DM)) and associated season load shape (DM_FR), are applied as well. In addition, the season distribution efficiency (DHDE(Z)) is applied. In addition, for generation of heat (OUT(LTH)_TID) from externally load managed power plants (xlm) is accounted for according to the appropriate capacity factor (CF(Z)(Y)) and unit (CAPUNIT) and time-slice duration (QHR(Z)(Y)).
BALH_TCZY$_{8P,32P}$ (t,cpd,lth,w)	I	The amount of heat (OUT(ELC)_TID) produced (R_TCZYH for pass-out turbine, R_TEZY for back-pressure turbines) in a time period by a coupled production plants in each time-slice (adjusted for base load (bas) plants appropriately) by applying the electricity/heat relationship parameters (ELM/CEH(Z)(Y). Any seasonal transmission losses (TRNEFF(Z)(Y)) are taken into consideration for the centralized (cen) plants.
BALH_THZ$_{8R,32R}$ (t,hpl,lth,z)	I	The amount of heat (OUT(LTH)_TID) produced (R_THZ) in a time period by a heating plant in a season (QHR(Z), adjusted for base load (bas) and peaking (PEAK(CON)_TID) plants appropriately). Any transmission losses (TE(ENT)) are taken into consideration for centralized (cen) plants.
BAS_CAP$_{10F}$ (t,tch,elc,w)	I	Reflects the base load (bas) and non-base load power plant contribution (BASELOAD) to the base load constraint (MR_BAS) for externally load managed (xlm) plants based upon the total installed capacity (R_CAP). The capacity factor (CF(Z)(Y)) and unit (CAPUNIT), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).
BAS_TCZY$_{10P}$ (t,cpd,elc,w)	I	Reflects the base load (bas) and non-base load contribution (BASELOAD) to the base load constraint (MR_BAS) from couple heat and power (cpd) plants based upon the generation (R_TEZY according to REH for back-pressure and R_TCYZH taking into consideration ELM/CEH(Z)(Y)), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
BAS_TEZY$_{10Q}$ (t,ela,elc,w)	I	Reflects the base load (bas) and non-base load contribution (BASELOAD) to the base load constraint (MR_BAS) from electricity (ela) plants based upon the generation (R_TEZY), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).
BAS_SEP$_{10S}$ (t,s,z)	I	Reflects the base load ('IMP') and non-base load ('EXP') contribution (BASELOAD) to the base load constraint (MR_BAS) from electricity exchanges based upon the level of resource activity (R_TSEP), as well as the time-slice duration (SEP_FR(Z)(Y)) and any transmission losses (TE(ENT)).
BI_TRDCST$_{25S,28S}$ (r,s)	U	The addition cost, or transaction cost, to be applied to bi-laterally traded commodities (BI_TRDENT) according to the level of resource activity (R_TSEP).
BI_TRDCSTE$_{25T,28T}$ (r,s,w)	A	The addition cost, or transaction cost, to be applied to bi-laterally traded electricity (BI_TRDELC) according to the level of exchange in a time-slice (R_TSEPE).
BND_TCZY$_{13P}$ (cpd,w)	I	The electric portion of the annual bound (BOUND(DB)O) to be applied to the output from coupled heat and power pass-out turbine (R_TCZYH), taking into consideration ELM/CEH(Z)(Y) and base load (bas) characteristics if appropriate. Both it and the heat portion are subject to the total annual production bound (MR_BCON).
BND_TEZY$_{13Q}$ (ela,w)	I	Is an indicator of when a time-slice electricity production variable (R_TEZY) is to be generated taking into consideration base load (bas), peaking (PEAK(CON)_TID), storage (stg), and externally load managed (xlm) characteristics of a conversion technology. It thereby controls the actual time-sliced variables summed in the annual production constraint (MR_BCON) according to the limit (BOUND(BD)O).
BND_THZ$_{13R}$ (hpl,z)	I	An indicator of when a seasonal heat production variable (R_THZ) is to be generated taking into consideration peaking (PEAK(CON)_TID), and externally load managed (xlm) characteristics of a conversion technology. It thereby controls the actual seasonal variables summed in the annual production constraint (MR_BCON) according to the limit (BOUND(BD)O).
COST_INV$_{25J}$ (t,t,p)	I	Technology annualized investment costs, where all the conditions associated with calculating the full lumpsum investment cost have already been taken into consideration. Thus COST_INV distributes said costs annually according to the capital recovery factor (CRF) and the lifetime (LIFE) of the technology, including any fraction of a period (FRLIFE). Note that COST_INV has dual period indexes where the 1st represents the initial period in which the investment took place, and the 2nd is the current period for which the payment is being calculated.
CPT_CAP$_{14F}$ (t,p)	I	An indicator of from when a technology is available (START) and needs a capacity transfer constraint (MR_CPT) to accumulate the total installed capacity (R_CAP). The latter is the case whenever a lifetime (LIFE) is provided.

Matrix Controls & Coefficients (indexes)[3]	Type[4]	Description & Calculations[5]
CPT_INV$_{14J}$ (t,t,p)	I	An indicator of from when a technology is available (START) for new investment and a capacity transfer constraint (MR_CPT) is needed (LIFE provided), as well as the amount of the investment (R_INV) available in the period. The latter is 1 in all periods from the installation period until the last period, where it may reflect that only a fraction (FRLIFE) is available owing to the lifetime not being a multiple of the period length (NYRSPER).
CUM_TSEP$_{15S}$ (t,s)	I	Indicator that a resource option is subject to a cumulative resource limit (CUM). Set to the number of years per period (NYRSPER), except stockpiles (‘STK’), to drive the cumulative resource constraint (MR_CUM).
DD_MA$_{22D}$ (dmd,e)	S	A mapping set that indicates which commodities are consumed (MA(ENT/MAT), MA(ENT/MAT)_TID) by a demand device.
DEM_CAP$_{16F}$ (t,dmd,d)	I	The amount of useful energy demand services (dm) delivered from a demand device, taking into consideration the capacity factor (CF) and unit (CAPUNIT), as well as the split for devices serving more than one demand sector (OUT(DM)). [NOTE: in SAGE the individual OUT(DM) are normalized to ensure that they sum to 1.]
DM_DEM$_{16D,16Y,35G}$ (d,t)	A	The projected demand for useful energy services that must be met (MR_DEM right-hand-side) in the reference run, and from which the elastic demands (R_ELAST) move in response to price.
DM_ELAST$_{25G}$ (d,b,t)	A	The elasticity (MED-ELAST) associated with a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and regional objective function (MR_MTSOBJ).
DM_STEP$_{25G,35G}$ (d,b)	A	The number of divisions of the demand curve (MED-STEP) to be used for a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and regional objective function (MR_MTSOBJ).
DM_VAR$_{25G,35G}$ (d,b,t)	A	The variance (MED-VAR) associated with a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and regional objective function (MR_MTSOBJ).
DMBPRICE$_{25G}$ (d,t)	A	The marginal price of meeting the demand in the reference run. Used for determining the change in cost (DM_ELAST,DM_STEP,DM_VAR) associated with moving along the demand curve (R_ELAST) in the regional objective function (MR_MTSOBJ).
EM_BOUND$_{35H}$ (v,t)	U	The bound on annual emissions (R_EM).
ENV_COST$_{25H,28H}$ (v,t)	U	The tax/subsidy to be applied to an emission (R_EM) in the regional objective function (MR_MTSOBJ).
ENV_CUM$_{17Y}$ (v)	A	The cumulative limit (ENV_CUMMAX) on the production of an emission indicator over the entire modeling horizon (MR_CUM).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
ENV_GWP$_{29H}$ (v,v,t)	U	The global warming potential, or other constant factor, to be applied to one emissions variable (R_EM) in order to compile another.
ENV_SCAL$_{17H,17Y}$	U	A multiplier that is applied to scale all emissions variables (R_EM), turned to if the marginals are only being reported as 'EPS'.
EPK_ACT$_{18E}$ (t,t,prc,elc,z)	I	The amount of electricity required (INP(ENT)p) by a process (prc) activity (R_ACT) that occurs during the peak time (PEAKDA(PRC)), taking into consideration any lead requirements (LED) and night storage (nst) operation in the peaking constraint (MR_EPK).
EPK_CAP$_{18F}$ (t,t,p,elc,z)	I	The amount of electricity (MA(ENT)) required (CF, EFF, CAPUNIT) by a demand device (dmd) activity (R_CAP) taking into consideration the shape of the load being served (DM_FR, OUT(DM)), as well as the peaking coincidence of the demand (ELF) and the length of the season (QHRZ), ignoring night storage (nst) devices. The electricity consumed by process (INP(ENT)p) that are externally load managed (xpr) taking into consider the capacity factor (CF) and unit (CAPUNIT), as well as LED requirements and peak coincidence (PEAKDA(PRC)). In addition, the parameter handles the contribution (OUT(ELC)_TID) to the peaking requirement (ERESERV) made by the individual power plants capacity (R_CAP) taking into consideration the peak contribution (PEAK(CON)) capacity unit (CAPUNIT), and transmission losses (TE(ENT)), as well as inhibiting for peaking only plants (PEAK(CON)_TID, forcing activity) in the peaking constraint (MR_EPK).
EPK_HPL$_{18R}$ (t,hpl,elc,z)	I	The amount of electricity required (INP(ENT)c) by a heating plant (hpl) activity (R_HZ) that occurs during the peak time (PEAKDA(PRC)), taking into consideration the length of the season (QHRZ) in the peaking constraint (MR_EPK).
EPK_SEP$_{18S}$ (t,s,elc,z)	I	The amount of electricity (INP(ENT)x) exported (R_TSEP) by a resource option (srcencp) that occurs during the peak time (PEAKDA(SEP)), taking into consideration any restrictions on the amount of the annual total available in a season (SEP_FR). The parameter also covers the amount of electricity imported (OUT(ENT)r) by a resource option (srcencp) that occurs during the peak time (PEAKDA(SEP)), taking into consideration any restrictions on the amount of the annual total available in a season (SEP_FR) in the peaking constraint (MR_EPK).
EPK_TEZY$_{18Q}$ (t,t,ela,elc,w)	I	The contribution (OUT(ELC)_TID) to the peaking requirement (ERESERV) from peaking only plants (PEAK(CON)_TID) according to activity (R_TEZY) that occurs at peaking time (PD(Z)D) and the amount of the output to credit (PEAK(CON)), taking into consideration any efficiency loss (TE(ENT)) in the peaking constraint (MR_EPK).
GobjZscal$_{25M,28N}$	U	A scalar applied to the regional total system cost to scale the entire model for the regional objective function (MR_MTSOBJ).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
HPKW_ACT$_{22R}$ (t,con,lth)	I	The contribution (OUT(LTH)_TID) to the peaking requirement (HRESERV) from peaking only plants (PEAK(CON)_TID) according to activity (R_THZ) that occurs at peaking time (PD(Z)D) and the amount of the output to credit (PEAK(CON)) according to the season length (QHRZ) in the peaking constraint (MR_HPKW).
HPKW_CAP$_{22F}$ (t,con,lth)	I	The amount of heat (MA(ENT)) required (CF, EFF, CAPUNIT) by a demand device (dmd) activity (R_CAP) taking into consideration the shape of the load being served (DM_FR, OUT(DM)), as well as the distribution losses (DHDE(Z)) and the length of the season (QHRZ), ignoring night storage (nst) devices. In addition, the parameter handles the contribution (OUT(LTH)_TID) to the peaking requirement (HRESERV) made by the individual power plants capacity (R_CAP) taking into consideration the peak contribution (PEAK(CON)) capacity unit (CAPUNIT), and transmission losses (TE(ENT)), as well as inhibiting for peaking only plants (PEAK(CON)_TID, forcing activity). These demands and capacity contributions are "balanced" in the peaking constraint (MR_HPKW).
HPKW_CPD$_{22F}$ (t,cpd,lth)	I	The amount of heat (OUT(LTH)_TID) related capacity available from coupled heat and power plants (R_TEZY for back-pressure, R_TCZYH for pass-out), taking into consideration operation characteristics (CEH(ZY)(Y)/ELM/REH), reserve margin (HRESERV) and transmission efficiency (TRNEFF(Z)(Y)) that contributes to the peaking requirement (MR_HPKW).
HPKW_HLK$_{22R}$ (t,hlk,lth)	I	The consumption (R_THZ) of heat (INP(ENT)c) by grid links (hlk) taking into consideration distribution efficiency (DHDE(Z)) and seasonal fraction (QHRZ) in the peaking constraint (MR_HPKW).
LSPILL (p,p,t)	U	Amount of "learning" contribution from one technology (and region) involved in the inter-period ETL calculation. The LSPILL_R version of the parameter allows for learning experience to be shared across periods as well.
NYRSPER$_{17H,29J}$	I	The number of year per period, as determined by the difference between the 1st and 2nd year of the model.
PRC_LIM$_{23D,23E,23F}$ (prc,t)	U	The overall efficiency of a flexible output process, applied to the activity variable (R_ACT) to ensure that the sum of all the individual commodity outputs (R_LOUT) is properly limited in the limit equation (MR_LIM).
PRC_OUT$_{23D,23E}$ (prc,enc,t)	U	The maximum share (OUT(ENC)p) that is applied to the overall process activity (R_ACT) to limit the individual commodity flows leaving such processes (R_LOUT) in the process operation equation (MR_PBL).
QHR$_{9S,18D}$ (w)	U	The duration of each time-slice as a fraction of the year, where omitted prevents the generation of equations and variables in said slices.
QHRZ$_{8D,9D,10D,30D,\ ,31D}$ (z)	I	The duration of each season as a fraction of the year, based upon the individual time-slice splits (QHR), such that both the day and night are omitted no equations or variables are generated in said season.
RAT_ACT$_{6E}$ (a,p,t)	U	The multiplier to be applied to the activity variable (R_ACT, R_TCZY, R_TEZY, R_THZ) in a user-defined constraint (MR_ARAT).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
RAT_CAP$_{6F}$ (a,p,t)	U	The multiplier to be applied to the capacity variable (R_CAP) in a user-defined constraint (MR_ARAT).
RAT_HPL$_{6R}$ (a,hpl,z,t)	U	The multiplier to be applied to the activity of a heating plant (R_THZ) in a season for a user-defined constraint (MR_ARAT).
RAT_INV$_{6J}$ (a,p,t)	U	The multiplier to be applied to the investment variable (R_CAP) in a user-defined constraint (MR_ARAT).
RAT_RTY$_{6Y}$ (a,b)	A	The indicator specifying the sense (bd) of a user-defined constraint (MR_ARAT).
RAT_RTY1$_{6D}$ (a,b,t)	A	The right-hand-side value associated with a user-defined constraint (MR_ARAT), where the equation is only generated for those periods in which a value is provided.
RAT_TCZY$_{6P}$ (a,cpd,w,t)	U	The multiplier to be applied to the activity of a coupled heat and power plant (R_TCZY, R_TEZY) in a time-slice and for a user-defined constraint (MR_ARAT).
RAT_TEZY$_{6Q}$ (a,ela,w,t)	U	The multiplier to be applied to the activity of an electric generating plant (R_TEZY) in a time-slice and for a user-defined constraint (MR_ARAT).
RAT_TSEP$_{6S}$ (a,s,t)	A	The multiplier to be applied to the resource supply variable (R_TSEP) in a user-defined constraint (MR_ARAT).
REG_XCVT$_{11S,12T,21I}$ (r,e)	U	A multiplier applied to a commodity in a region in the trade constraints (MR_BITRD, MR_BITRDE, MR_GTRD) to convert said commodity to a standard common unit, if needed. For SAGE the default (1) used.
REG_XMONY$_{24M, 24N}$ (r)	U	A multiplier applied to the costs in a region to convert the regional object function (MR_MTSOBJ) terms to a standard common unit, if needed. For SAGE the default (1) used.
SEP_BND$_{35S}$ (s,b,t)	A	The resource bound (BOUND(BD)Or) applied directly to limit the annual production from a supply option (R_TSEP) in each period for which a value is provided.
SEP_CUM$_{15D,15Y}$ (s)	A	The cumulative resource limit (CUM) applied as the right-hand-side of the cumulative resource constraint (MR_CUM) to limit the total supply of an option (R_TSEP) over the entire modeling horizon.
SEP_GRTI$_{19Y}$ (s)	A	The “seed” used as the right-hand-side of the resource growth constraint (MR_GRSEP) to set the maximum penetration of a resource (R_TSEP) if it was not used in the previous period.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
SEP_STRT$_{19D}$ (s)	A	The initial period from which a resource supply option (R_TSEP) is available, here ensuring that the growth constraint does not start prematurely.
SEP_TGR$_{19S}$ (s,t)	A	The annual growth of a resource option (R_TSEP) permitted, as controlled by the resource growth constraint (MR_GRSEP) in a period when a value is provided.
TCH_BND$_{35F}$ (p,b,t)	A	The limit (BOUND(BD)) directly imposed on the total installed capacity (R_CAP) in each period for which a value is provided.
TCH_BNDO$_{33D,13Y,15E,35F}$ (p,b,t)	A	The limit (BOUND(BD)O) imposed on annual activity in each period for which a value is provided. For regular processes (prc) bound is applied directly to the activity variable (R_ACT). Conventional conversion plants are limited annually by means of a constraint (MR_BNDCON) that sums the individual time-slice activity variables (R_CTZY, R_TEZY, R_HZ). For externally load-managed technologies (xlm/xpr) as applied to the capacity (R_CAP) taking into consider the capacity factor (CF) and unit CAPUNIT).
TCH_CAPU$_{27F,35S}$ (p,t)	A	The capacity to activity conversion factor to ensure that the units of annual production activity match that of the overall model. When set to unity the units of capacity and production are identical. Traditionally used to allow conversion technology (con) capacity to be in terms of kilo/gigawatts while annual production is defined in terms of petajoules.
TCH_CF2$_{23F,27F,35F}$ (p,w,t)	A	The fixed capacity utilization factor (CF/CF(Z)(Y)) controlling the activity of demand devices (dmd) and externally load managed technologies (xlm, xpr) according to the total installed capacity (R_CAP). The seasonal CF(Z)(Y) takes precedence over the annual CF, with the latter assigned to the former when the time-sliced values are not provide for conversion plants. Also, the maximum availability factor (AF/AF(Z)(Y)) is assigned to the capacity factor for externally load managed technology when it is provided and the CFs not.
TCH_GRTI$_{6E}$ (p)	A	The "seed" used as the right-hand-side of the technology growth constraint (MR_GRSEP) to set the maximum penetration of capacity (R_CAP) if it was not used in the previous period.
TCH_IBND$_{35J}$ (p,b,t)	A	The limit (IBOND(BD)) directly imposed on investments in new capacity (R_INV) in each period for which a value is provided.
TCH_LIFE$_{14D}$ (p)	A	The number of periods (and fractions thereof) that a technology is available (LIFE) from the year of initial installation (R_INV), where the number of years per period (NYRSPER) is determined in the code from the 1st two periods of the model run. Used for control the need for and longevity of new investment in the capacity transfer equation (MR_CPT). The LIFE also impacts the annualize cost of new investments as embodied by the capital recovery factor (CRF).

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
TCH_RES$_{14D,14Y}$ (p)	A	The amount of pre-existing or residual capacity (RESID) originally available prior to the modeling horizon and still around in the current period. The RESID serves as the right-hand-side of the capacity transfer equation (MR_CPT) as well appearing in the regional objective function (MR_MTSOBJ) where the annualized investment payments are charged against the level (R_RESID, fix bounded).
TCH_STRT$_{20D}$ (p)	A	The period from which a technology is available (START), controlling when equations and variables are permitted. It is imbedded into most all the other technology related internal parameters so that it does not need to be tested repeatedly throughout the code.
TCH_TGR$_{19Y}$ (p,b,t)	A	The annual growth of installed capacity (R_CAP) permitted, as controlled by the resource growth constraint (MR_GRTCH) in a period when a value is provided.
TENV_ACT$_{29E}$ (t,p,v)	I	Amount of emissions (ENV_ACT) discharged, or reduced (if negative), per unit of output from a process (prc) and credited into the annual emissions constraint (MR_TENV). If the coefficient ENV_ACT was originally provided a conversion plant (con) or demand device (dmd) it is assigned to an associated internal parameter (TENV_CAP/TCZY/TEZY/THZ).
TENV_CAP$_{29F}$ (t,p,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of installed capacity (R_CAP) as tracked in the annual emissions constraint (MR_TENV). If the technology is a demand device or is externally load managed (xpr/xlm) and ENV_ACT then it is assigned to ENV_CAP applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)) , and time-slice fraction (QHR) if power plant.
TENV_INV$_{29J}$ (t,p,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of new investment in capacity (R_INV) as tracked in the annual emissions constraint (MR_TENV).
TENV_TCZY$_{29P}$ (t,cpd,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of pass-out turbine production of heat (R_TCZYH) as tracked in the annual emissions constraint (MR_TENV) by applying the heat ratio factor CEH(Z)(Y)/ELM. It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.
TENV_TEZY$_{29Q}$ (t,ela,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of electricity production (R_TEZY) as tracked in the annual emissions constraint (MR_TENV). It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.
TENV_HZ$_{29R}$ (t,hpl,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of heat production (R_THZ) as tracked in the annual emissions constraint (MR_TENV). It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
TENV_SEP$_{29S}$ (t,s,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of resource activity. Providing ENV_SEP results in the associated resource supply variable (R_TSEP) entering the equation tracking the emissions (MR_TENV) with the coefficient applied.
TEZY_CAP$_{30F}$ (t,ela,w)	I	The amount of the total installed capacity (R_CAP) available in a time-slice (AF/AF(Z)(Y)), taking into consideration any forced outage (AF_TID); as well as base load (bas), storage (stg) and peaking only (PEAK(CON)_TID) attributes. With regard to the later the peak duration (PD(Z)(D) is used in place of the availability.
TEZY_TCZY$_{30P}$ (t,cpd,w)	I	An indicator for pass-out turbine production of heat (R_TCZYH) electric output (CEH(Z)(Y)/ELM) in a particular period and time-slice, taking into consideration base load (bas), as it contributes to the utilization constraint (MR_TEZY).
TEZY_TEM$_{30Q}$ (t,cpd,w)	I	Maintenance indicator (AF_TID) forcing the inclusion of a forced maintenance variable (R_M) in the utilization constraint (MR_TEZY).
TEZY_TEZY$_{30Q}$ (t,cpd,w)	I	An indicator that a conventional conversion plant (non-xlm) is permitted to run (R_TEZY) in a particular period and time-slice in the conversion plant utilization equation (MR_TEZY), taking into consideration base load (bas), storage (stg), and peaking only (PEAK(CON)_TID) attributes.
THZ_CAP$_{31F}$ (t,hpl,z)	I	The amount of the total installed capacity (R_CAP) available in a season (AF/AF(Z)(Y)) and period in the conversion plant utilization equation (MR_TEZY), taking into consideration any forced outage (AF_TID); as well as peaking only (PEAK(CON)_TID) attributes. With regard to the later the peak duration (PD(Z)(D) is used in place of the availability.
THZ_THZ$_{31R}$ (t,hpl,z)	I	An indicator that a conventional heating plant (non-xlm) is permitted to run (R_THZ) in a particular period and season in the conversion plant utilization equation (MR_THZ), taking into consideration base load (bas), storage (stg), and peaking only (PEAK(CON)_TID) attributes.
TRD_BND$_{35F}$ (r,g,ie,b,t)	A	The bound on the import/export of a globally traded commodity (R_GTRD) in a region/period.
TRD_COST$_{25I,25S,28I,28S}$ (r,e/v)	A	An additional (transaction) cost applied to traded commodities (R_GTRD, R_TSEP, R_TSEPE) in the regional objective function (MR_MTSOBJ).
TRD_FROM$_{21D}$ (r,e/v)	A	The year from which trade in a global commodity is permitted to take place.

Matrix Controls & Coefficients (indexes) [3]	Type [4]	Description & Calculations[5]
TSUB_BND$_{36U\ 28U,32D}$ (txs,t)	U	Bound applied to a tax/subsidy as the total tax/subsidy permitted in a period.
TSUB_COST$_{25u\ 28U,32D}$ (txs,t)	U	Cost of a tax/subsidy applied to selected energy carriers and the technologies producing and/or consuming it.
TSUB_SEP$_{32S}$ (txs,s,t)	A	Contributions of a resource supply option to a tax/subsidy, where the value is applied to the level of resource activity.
TSUB_TCH$_{32E,32F,\ 32J,32P,32Q,32R}$ (txs,p,t)	A	Contribution of a technology to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed.
UPRC_ACT$_{34E}$ (t,prc)	I	An indicator that a conventional process (non-xpr) is permitted to run (R_ACT) in a particular period, and thus appear in the process utilization equation (MR_UTLPRC).
UPRC_CAP$_{34F}$ (t,prc)	I	The amount of the total installed capacity (R_CAP) available in a period (AF) in the process equation (MR_UTLPRC).
ZPR$_{34D}$ (prc)	S	The expanded list of externally load managed processes (xpr) plus those process with a short lifetime (LIFE <= 1 period).

4.3.2.2 *Other Key Internal Parameters*

Besides the parameters presented in the previous two tables, there are several other internal parameters that are central to the calculation of key other parameters. These are listed in table 4.12 below.

Table 4.12: Definition of Other Key Internal Parameters

Internal Parameter (index(es))	Related Parameters	Description
CRF (p)	DISCOUNT DISCRATE LIFE	The capital recovery factor for each technology; based upon the applicable discount rate, the technology lifetime, and the number of years per period. The CRF is calculated as *x=1/(1+DISCOUNTorDISCRATE), and then* $CRF=\{1-x\}/\{1-x^{LIFE}\}$.
CRF_RAT (p)	CRF DISCOUNT LIFE	The ratio of the capital recovery factor for technologies with "hurdle rates," calculated as the technology CRF / the CRF using the standard system-wide discount rate.
FRACLIFE (p)	COST_INV PRI_DISC	The fraction of a period that an investment is available; such that 1 for all periods up until the period from installation until the lifetime (LIFE), and the (number of years in the last period of existence / number of year per period (NYRSPER)) if LIFE is not a multiple of NYRSPER.
FLLIFE (p)	COST_INV	Last period during which a technology is available based upon its LIFE.
FRLIFE (p)	COST_INV LIFE	Adjustment to the capital recovery factor (CRF) if the lifetime (LIFE) is not a multiple of the number of years per period (NYRSPER).
NYRSPER	TP	The number of years per period. As SAGE permits only periods of equal length the value of NYSPER is determined by the number of years between the first two periods provided for the set YEAR.
SALV_INV (t,p)	PRI_INV	The amount of remaining investment salvaged, that is credited against the lump-sum cost investment charge, when calculating totally discounted system cost for reporting.
SET_CC (t,p)		Endogenous technology learning (SETL) cumulative cost.
SET_SC (t,p)		Endogenous technology learning (SETL) specific cost.
SIGN (src)		For EXPorts = -1, 1 for the other four resources options (IMPort, MINing, ReNeWable, StocKpile).
T04/E/H/O (t,p,*)	BAL/BALE/ BALH_*	Electricity and heat production by plant, along with production of auxiliary commodities, for the VEDA reports.
T08/ T08ENT (e,p,t)/(e,p)	BAL_*	Energy/material consumption by technology for the VEDA reports.
T27V (t,p,v)	TENV_*	Emissions activity by technology and resource for the VEDA reports.

4.4 Variables

This section details the role of each primal decision variable of the SAGE Linear Program. These variables, along with the sets and parameters described in previous sections, enter the mathematical expressions that constitute the SAGE constraints, which will be stated and explained in section 4.5. The primal decision variables of a Linear Program are often referred to as *Columns*, whereas constraints are referred to as *Rows*. This terminology stems from the position of these two types of entity in the matrix of the Linear Program, as explained in section 3.4. For the same reason, the phrase *Row/Column Intersection* is often used to designate the coefficient of a single SAGE variable in a particular constraint. A fairly complete rendition of the SAGE LP matrix is provided as table A1 SAGE_MATRIX of Appendix A.

The same syntax conventions used in the earlier parts of Chapter 4 are used here as well. Namely:

- Sets, and their associated index names, are in lower case, usually within parenthesis;
- Literals, values explicitly defined in the code, are in quotes;[1]
- Parameters, and scalars (un-indexed parameters) are in all capital letters;
- Equations are in upper case with a prefix of MR_, and
- Variables are in upper case with a prefix of R_.

Table 4.15 lists the indexes referenced in this section. As noted previously in section 4.2the process of constructing the parameters, variables and equations that control the intersections in the SAGE_MATRIX is controlled by subsets of the referenced indexes, particularly with regard to the technology references. For the most part these indexes correspond directly to user input sets discussed in Section 4.2. The reader is referred to that section for a ful description of each index, and for an indication of how entries in the sets influence the resulting model.

Table 4.15 – Documentation Indexes and associated user sets

Index	Input Set[2]	Description
a	adratio	The name of the user-defined ***A**d hoc* constraints (referred to in the code by the set (adratio)) that are used to guide market splits, group limits for technologies, etc.
b	bd	**B**ound or right-hand-side constraint type ('LO','FX','UP', and variations thereof (e.g., 'MIN','FIX','MAX')).
c	p	The **C**ost, or step indicator, for the resource supply options. Corresponds to the last character of each member of the resource supply set (srcencp).
d	dm	All useful energy **D**emand sub-sectors.
e	ent/mat	All **E**nergy carriers and all materials.

[1] Examples of literals in the code are:

- 'EXP' and 'IMP' are reserved for exports and import;
- 'LO', 'FX', and 'UP' identify lower, fixed and upper bound types, and
- 'S', 'I', and 'W' for Summer, Intermediate, and Winter seasons, and 'D' and 'N' for day and night components of the sub-annual timeslices.

[2] Input Set identifies the user input set corresponding to the 1-character indexes used in the definition of the matrix components in this chapter..

Index	Input Set[2]	Description
g	trd	The name of the **G**lobally traded (as opposed to bi-lateral) commodities (ENT or ENV).
m	mkt_id	The **M**arket share group indicators.
p	tch	All the technologies or **P**rocesses in the energy system.
r	reg	The **R**egions to be tracked.
s	srcencp/sep	The resource **S**upply options, including those serving as the trade variables into/out of the regions.
t	year/tp	The years for which data is provided, and the (possible) subset thereof of **T**ime periods requested for the current model run.
txs	taxsub	Name of each **T**a**X**/**S**ubsidy imposed on the system.
u	med-step(bd)	The steps employed for the discretization of the price elastic demand curve, and for expressing the loss of **U**tility. Provided for each demand sector for which flexible demands are employed.
v	env	All emission or en**V**ironmental indicators.
x	ie	Export/import e**X**change indicator. Note: the definition of the set IE is internal to the model and fixed at "EXP" for exports and "IMP" for imports. But for any trade variables defined over ie there are only variables for the instances for which the user has explicitly provided data.
w	td/z+y	The season/time-of-day compound index (season (Z) and time-of-day (Y)) identifying the time-slice **W**hen an activity occurs.
z	z	The (pre-defined) seasons for heat generation.

The variables of the SAGE model correspond to the columns of the LP matrix. Each variable is indexed by region, and by the appropriate set of other indices necessary to fully and uniquely identify the model component. The variables in the model are generally tied to a region, a technology or commodity, and a time period[1]. Most variables track the timing of new investments in technologies, accumulate the total installed capacity in a period, and determine the activity (output) of each technology in a time period.

In SAGE, as in all models built upon the MARKAL framework, technologies are portrayed based upon their level of activity, total installed capacity and amount of new investment. Dividing the activity level by the technology's efficiency derives consumption requirements associated with each technology. In general, there are no "consumption" (input flow) or "production" (output flow) variables in the model (except for exports and imports). Depending upon the nature of a technology either there are individual flow variables for the primary output commodities (e.g., imports, electricity/heat from conversion plants, commodities from flexible (LIMIT) processes), or the various outputs are derived from technological activity or capacity levels (e.g., commodities from regular process, ancillary commodities, useful energy services from demand devices). Thus the energy and material balance equations, as well as peaking and base load requirements and the variable operation costs, are constructed by either employing the flow variables where they exist, or applying the appropriate coefficients to determine the

[1] The only exceptions are the regional total system cost variable with only a region index, the objective function variable without any index; and the tax/subsidy variable with the named tax/subsidy as an index.

amount of each commodity into or out of the technologies based upon the activity or on total installed capacity of the technologies.

The variables of the model are listed alphabetically in table 4.16, with a sequence number in the first column. Column 2 of the table indicates the variable name and the associated indexes needed to uniquely define the instances of the variable. The last column describes the variable, with particular emphasis on the conditions that trigger the creation of each variable. The modeler is referred to Appendix Table A1 SAGE_MATRIX[1] for a global view of the variables involved in the model, which constraints they are involved in, and what parameters (and sets) determine the actual coefficient of each row/column intersection. By looking down a variable column of the SAGE_MATRIX one can quickly see all the non-zero intersections, thereby identifying the constraints and associated parameters determining the coefficients of that variable in the various constraints[2]. Also, recall that every set and parameter appearing on the SAGE_MATRIX is explained in the Set & Parameters section 4.2, table 4.11.

Note that in the source code, regionalization is handled by adding a prefix to equation (MR_<root>)[3] and variable (R_<root>) names, and a suffix to set and parameter (<root>_R) names. During multi-region runs the matrix generator and report writer generate the parameters and process the model results region-by-region. For matrix generation this is accomplished by moving the single region data into parallel multi-region data structures. The multi-region model variables and constraints, and parameters, if needed, are moved into their corresponding single region instances for the reports. These parallel data structures have the identical names as their single region counterparts, but with the appropriate prefix/suffix added to their names and the 1st index designating the region (via the index reg/r). In all the descriptions of sets and parameters in this Chapter the regional suffix is omitted, though it is understood to be part of the associated data structures. Where there is an exception to this rule it is noted, for example if trade options identify both from/to regions.

Table 4.16: Model Variables[4,5]

VAR Ref	Variable (Indexes)	Variable Description

[1] Appendix Table A1 SAGE_MATRIX is provided both as a text table for those who have a magnifying glass handy and as an inserted Excel file.
[2] For each intersection the user can check the rows in table 4.17 of this chapter to see in which equation source code routines the variable occurs. In addition the variables may have bounds applied in the MMBNDS.INC/MTS/MED routines.
[3] In a single region model the energy balance equation is named EQ_BAL_G, while for a multi-region SAGE run the regionalized equation has the EQ_ prefix replaced by MR_; for variable the R_ prefix is added to the root variable name, such as single region INV variable is named R_INV for multi-region SAGE.
[4] The VAR Ref is the sequence number according to the variable's alphabetic ordering in this chapter. Each variable is also listed in the corresponding order in the SAGE model matrix spreadsheet presented in Appendix A: Table A
[5] In the Description column references within parentheses refer to (sets), (PARAMETERS), (MR_/R_ model rows/columns), and ("literal").

VAR Ref	Variable (Indexes)	Variable Description
1	R_ACT (r,t,p)	Activity of a process technology (prc). Note that only processes have R_ACT variables. For power sector technologies (con), variables are defined for each time-slice are employed instead, and for demand devices (dmd) an activity variable is not generated as the activity of demand devices is derived directly from the associated capacity variable.
2	R_CAP (r,t,p)	Total installed capacity of each technology. This variable thus represents the total residual capacity plus investments in new capacity made up to and including the current period and that are still available to the model (i.e. whose life has not expired yet).
3	R_ELAST (r,t,d,b,u)	The movement (up or down) of an energy service demand in response to energy service price when running the model with elastic demands (also referred to as MED).
4	R_EM (r,t,v)	The level of each emission indicator.
5	R_GTRD (r,t,w,x)	The total amount of a globally traded commodity imported/exported by a region, where x = 'IMP'/'EXP'.
6	R_INV (r,t,p)	The addition of new capacity (new investment) in a period. The investment is assumed to take place at the beginning of the period, and is thereby fully available to the model in the period in which the investment takes place.
7	R_LOUT (r,t,p,e)	The amount of each energy carrier and/or material produced by flexible (LIMIT) "refineries" and other mixing processes (prc) producing multiple commodities that are not necessarily produced in fixed proportions.
8	R_M (r,t,p)	The scheduled maintenance for power plants for which the user has specified that some fraction of annual unavailability is scheduled [AF_TID] and thus not allowed to be optimally scheduled by the model.
9	R_MTSOBJ (r)	The total cost of each region's energy system, the sum of which (over all regions) constitutes the SAGE objective function to be minimized.
10	R_OBJTOT	The SAGE objective function to be minimized, consisting of the sum of the regional total costs.
11	R_RESID (r,t,p)	The exogenously specified (RESID) level of investment in technologies made prior to the modeling horizon that are still available in a period. The variable is used to charge the annualized payments for these investments to the energy system objective function. [Note that this is SAGE variable is not subject to optimization, as it is completely determined by the RESID input parameter.[Note that in MARKAL there is no such variable, and the cost of the "sunk" investment is not included in the model's objective function.]
12	R_TCZYH (r,t,p,w)	The amount of low-temperature heat (lth) produced by a coupled heat and power plant (cpd) in each time-slice (td) that is modeled as a pass-out turbine with a flexible electricity/heat relationship (CEH).
13	R_TEZY (r,t,p,w)	The amount of electricity (elc) produced by each power plant (ela) in each time-slice (td).
14	R_THZ (r,t,p,z)	The amount of low-temperature heat (lth) produced by a heating plant (hpl) in each season (z).
15	R_TSEP (r,t,s)	The level of resource production, including imports and exports, arising from each step (price level) of the supply curve (srcencp).

VAR Ref	Variable (Indexes)	Variable Description
16	R_TSEPE (r,t,s,w)	For bi-lateral electricity trade (bi_trdelc, srcencp), the amount of electricity (elc) exchanged at each time-slice (td).
17	R_TXSUB (r,txs)	Tax/subsidy level accumulated from all technology/commodity pairs involved in a tax and subsidy (cost per unit of activity) imposed on the energy system.
18	R_ZSTK (r,e,c)	Carry-over from the final time period of stockpiled material (src = "STK") that needs to be credited to the objective function. *{Not currently used in SAGE.}*

In the rest of this section each of the variables is described in more detail. On the variable sub-header record the variable name and index is given, along with a reference to the associated column in the Appendix Table A1 SAGE MATRIX. It is helpful to either print the SAGE MATRIX or have it open as you use the description of the structure of SAGE. For each variable the following information is provided.

Description: A short description of the nature of the variable.

Purpose: A brief explanation of why the variable is needed what it is used for .

Occurrence: A general description of the data circumstances and model rules that control the instances of the variable in the various constraints.

Units: The units associated with the variable.

Bounds: An indication of how limits (lower, upper, fixed) can be directly and indirectly applied to the variable.

4.4.1 R_ACT(r,t,p)

Table A1, Column e

Description: The level of **ACT**ivity of each traditional process (prc)[1]. The variable represents the total annual process output, thereby determining the production of each of the energy carriers and materials produced by a process, as well as the energy and material required by the process and any emissions based upon activity.

Purpose: To track the level of activity associated with each process (prc). The R_ACT variable enables the variable costs to be separated from the fixed, and to permit a process to run at less than its full installed capacity. The total variable operating and maintenance cost (VAROM) is tied to the level of activity of each process.

Occurrence: This variable is generated for each process in all time periods beginning from the period that the technology is first available (START), whenever LIFE is provided and is at least one period in length, and the process is not described as externally load managed (xpr). [In either of these excluded cases the activity is determined directly from the

[1] Reminder: demand device (dmd) activity is derived directly from the capacity variable (R_CAP), and conversion plants (con) have time-sliced activity variables (R_TCZY, R_TEZY, R_THZ).

current installed capacity (R_CAP) by applying a capacity factor (CF), which fixes the activity; as opposed to an availability factor (AF) that permits the process to operate at or below its capacity, in the process utilization constraint (MR_UTLPRC)]. The activity variable also serves to ensure that the level of each commodity produced by flexible process (see MR_LIM/MR_PBL), does not exceed its maximum permitted share of the total output. Since the amount of each commodity consumed or produced by a process is a function of the process activity, the R_ACT variable appears in the commodity balance equation (MR_BAL_E/G, MR_BALE1/2), as well as contributing to the peaking requirements (MR_EPK). Since R_ACT represents the overall activity of the process, it appears in the objective function (MR_MTSOBJ) to account for the variable operating and maintenance costs (VAROM) associated with the process. R_ACT may also appear in user-defined constraints (MR_ADRAT if RAT_ACT), emission (MR_TENV if ENV_ACT) and tax/subsidy (MR_TXSUB, TSUB_TCH) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the model user for activity of processes. If activity units differ from capacity units, the conversion is specified by the capacity unit parameters (CAPUNIT = capacity unit/activity unit). In SAGE, process activity units are either PJ per annum. The same units must be used for bound (BOUND(BD)O) parameters related to the technology activity.

Bounds: The process activity variable may be directly bounded by specifying an annual activity bound (BOUND(BD)O), where BD = LO/UP/FX. It may be indirectly limited by bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)), owing to the fact that the activity variable can not exceed the capacity variable times an availability factor.

4.4.2 R_CAP(r,t,p)

Table A1, Column f

Description: The level of total installed **CAP**acity available in each period, defined for each process (prc), conversion (con) and demand (dmd) technology.

Purpose: To accumulate the total level of capacity in each period as a function of the remaining residual capacity (RESID) from technologies deployed prior to the modeling horizon, plus those investments in new technologies (R_INV) made earlier than or at current period t that have not yet exhausted their useful lifetime (LIFE).

Occurrence: This capacity variable is generated for each and every technology in all time periods, beginning from the period that the technology is first available (START). The variable appears in the capacity transfer constraint (MR_CPT) that oversees the tracking of vintages and lifetimes of technologies, and all constraints (MR_TEZY, MR_THZ, MR_UTLPRC) that relate the activity of technologies (R_ACT, R_TCZY, R_TEZY, R_HZ) to the installed capacity. R_CAP directly contributes to the electricity and heat peaking constraint (MR_EPK/MR_HPKW). Note also that R_CAP may be used as a surrogate for activity for externally load managed, or rigid, processes (xpr) and power plants (xlm), substituting for activity variables with the appropriate capacity factors

(CF/CF(Z)(Y)) applied. Furthermore, as it is assumed that all demand devices (dmd) operate in accordance with the level of the installed capacity according to a fixed utilization factor (derived from CAPUNIT, CF, EFF and MA(ENT)). For demand devices (dmd), R_CAP thus also serves to define the level of demand services provided by a technology to help satisfy the demand constraint (MR_DEM according to OUT(DM)) It appears in a growth constraint (MR_GRTCH) if requested (GROWTH). Since R_CAP represents the total installed capacity of each technology, it also appears in the object function (MR_MTSOBJ) to account for the fixed operating and maintenance costs (FIXOM) associated with the technology. R_CAP may also appear in a user-defined constraint (MR_ADRAT if RAT_CAP) and emissions (MR_TENV if ENV_CAP) constraints if the associated data is provided.

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. The same units are used for the investment (R_INV) variable, and for the residual capacity (RESID), and capacity bound (BOUND(BD)) parameters related to the technology.

Bounds: The capacity variable may be directly bounded by specifying a capacity bound (BOUND(BD)), with BD = LO/UP/FX. It may be indirectly limited by capacity growth rates (GROWTH), or by bounds on investment variables IBOND(BD)..

4.4.3 R_ELAST(r,t,d,b,u)

Table A1, Column g

Description: The uth component of the movement (up or down) of an **ELAST**ic demand for a useful energy service in response to the associated own price elasticity. The demand in a particular run is equal to the demand in the base case plus the sum of the components of the demand variation, as explained in sub-section 3.3.2.3.

Purpose: To allow for the step-wise constant representation of the consumer demand curve for each energy service, if that option is selected by the user.

Occurrence: The elastic demand variable appears in the demand equation (MR_DEM) whenever the number of steps (MED-STEP(BD)) is provided for a direction (‘LO’/’UP’), and in each period for which an elasticity (MED-ELAST(BD)) is provided for that direction. The variable also appears in the objective function for each step/elasticity specified by the analyst.

Units: PJ, bkmt, or any other unit in which a demand for energy services is specified.

Bounds: For each direction (b) and step (u) an upper bound is set such that R_ELAST does not exceed an amplitude that is a function of the reference case demand level (DEMAND), a percent variation parameter (MED-VAR), and the desired number of steps (MED-STEP) for the discrete approximation for that direction, according to the formula Upper Bound = (DEMAND)*(MED-VAR)/(MED-STEP).

4.4.4 R_EM(r,t,v)

Table A1, Column h

Description: The total level of the various **EM**ission indicators in each period.

Purpose: To accumulate the total (net) level of emissions emanating from all sources, adjusted for those leaving the energy system (due to exports or "removal" (e.g., scrubbing, sequestration)). If the user associates a charge (e.g., emissions "tax," ENV_COST) with an emission indicator, the tax is applied to this variable in the objective function.

Occurrence: Emissions indicators are tracked for each indicator in each period for any period in which emission is produced or absorbed by any technology or resource. Emissions are tied to technologies according to their activity (ENV_ACT), capacity (ENV_CAP), or investment (ENV_INV), and resources according to their activities (ENV_SEP), where corresponding coefficients are applied to the associated variables for each technology/resource emitting, or consuming, the indicator. Since R_EM represents the total level of emissions, it may also appear in the objective function (MR_MTSOBJ) if an emission tax (ENV_COST) is specified.

Units: Tons, Thousand tons, or any other unit defined by the analyst as a measure of the emission per unit of activity, installed capacity, or investment. The same units are used if a bound (ENV_BOUND(BD), ENV_CUM, ENV_MAXEM) is imposed. Coherent the units must be used if an emission "tax" is to be applied (ENV_COST).

Bounds: The total emissions may be directly bounded by specifying a cap (ENV_MAXEM) or bound (ENV_BOUND(BD)) on annual emissions, or a cumulative limit on an indicator over the entire modeling horizon.

4.4.5 R_GTRD

Table A1, Column I

Description: The total amount of a **G**lobally **TR**a**D**ed commodity imported/exported to/from a region.

Purpose: To permit trade of a commodity (energy, material or emissions) between exporting and importing regions.

Occurrence: The global trade variable enters the commodity balance equation (MR_BAL_G, MR_BAL_E, MR_TENV), the objective function (MR_MTSOBJ) where any transaction cost (TRD_COST) is applied, and the global trade balance equation (MR_GTRD) where the total exported must equal the total imported. Note that trade starts beginning at the TRD_FROM period.

Units: Units of the commodity traded, e.g., PJ for energy, Tons for material, Tons for emissions or emission permits.

Bounds: The amount of a global traded commodity exported/imported from a region, in a period, may be limited by the TRD_BND parameter.

4.4.6 R_INV(r,t,p)

Table A1, Column j

Description: The level of **INV**estment in new technologies, which is added to the existing capacity in place, available during the current time period and subsequently for the technical lifetime of the technology (LIFE). The level of the variable represents the total investment occurring during the entire period, but is credited and available from the beginning of the period.

Purpose: To enable the vintage of a technology to be tied to the period in which the investment is made so that the technical lifetime and associated capital stock turnover can be monitored and modeled. Having a separate primal variable R_INV (new construction) also serves the purpose of separating the contribution to the total discounted system cost of investment costs from the contribution of other costs.

Occurrence: This variable is generated for each technology in (prc)/(con)/(dmd) in all time periods beginning from the period that the technology is first available (START), whenever the lifetime (LIFE) parameter have been specified or set by the preprocessor. If any cost parameter is provided, but LIFE is not, then LIFE is set equal to 1 period. If the LIFE parameter is not specified, and no cost parameter is provided then the investment variable is not generated.. Whenever LIFE is greater than 1 period the investment variable appears in the capacity transfer constraint (MR_CPT) by which the investments are accumulated over time to define the current installed capacity (R_CAP) in each period. Since R_INV represents the investment in new capacity of each technology, it also appears in the object function (MR_MTSOBJ) to account for the investment costs (INVCOST) associated with the technology. R_INV may also appear in a user-defined constraint (MR_ADRAT if RAT_INV), energy and material balance (MR_BAL_G/E) and emissions (MR_TENV if ENV_INV) constraints if the associated data is provided; as long as LIFE has been provided or set by the preprocessor.

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. The same units are used for the investment (R_INV) variable, and for the residual capacity (RESID), cost (INVCOST, FIXOM), and investment bound (IBOND(BD)) parameters related to the technology.

Bounds: The investment variable may be directly bounded by specifying an investment bound (IBOND(BD)). It may be indirectly limited by bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or activity (BOUND(BD)O).

4.4.7 R_LOUT(r,t,p,e)

Table A1, Column k

SAGE_MATRIX_V52.XLS

Description: The output of an energy carrier (enc) or material (mat) from a flexible (**L**IMIT) multiple **OUT**put process (prc).

Purpose: Variable output "limit" processes permit the model to determine the level of each output energy carrier (OUT(ENC)p) or material (OUT(MAT)p) independently of one another, as opposed to fixed proportions which is the case for regular processes.

Occurrence: There is thus a collection of R_LOUT variables for each flexible process. The output parameters (OUT(ENC)p/OUT(MAT)p) serve to both identify which commodities are produced by the process, as well as indicate the maximum share each commodity can have of the overall output. Note that while the sum of these maximum shares may be more than one, the total of the actual shares represented by R_LOUT must not exceed the overall efficiency (LIMIT) of the flexible process. Each R_LOUT variable is thus tied to the activity variable (R_ACT) of such processes according to its maximum share (MR_LIM), with the overall efficiency controlled by the flexible process output balance constraint (MR_PBL), and contributes to the balance equation for each output commodity (MR_BAL_E/G).

Units: Most often PJ, but may be any other unit defined by the analyst for activity of processes. Coherent units must be used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The individual flow of each commodity is bounded by the OUT(ENC/MAT)p parameter. The flow may also be indirectly limited by an annual activity bound (BOUND(BD)O) on the process, bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

4.4.8 R_M

Table A1, Column l

Description: The portion of annual unavailability of conversion plants representing the scheduled maintenance.

Purpose: To optionally split the portion of annual unavailability between scheduled (by the model) verses forced outage. Under normally circumstances MARKAL is left free to schedule the unavailability, but operations such as reloading reactor core or maintenance may not be scheduled by the model.

Occurrence: The variable enters the conversion plant activity (MR_TEZY) and utilization (MR_UTLCON) constraints if a scheduled outage (AF_TID) is not 1.

Units: Percent.

Bounds: None, other than that imposed by the utilization constraint (MR_UTLCON).

4.4.9 R_MTSOBJ(r)

Table A1, Column m

Description: This variable embodies the total system cost for a region. Since SAGE is a time-stepped model, the total cost is expressed as an annual cost for the current period only (as opposed to a perfect foresight utilization of MARKAL which uses a present value of all periods' costs).

Purpose: To pass along the expression of the total system cost of each region to the final global objective function (MR_OBJ). The regional cost variables may be adjusted (REG_XMONY) if different monetary units are employed in some regions.

Occurrence: The variable appears in the equation defining each regional total costs (MR_MTSOBJ) corresponding to the equation defining the SAGE objective function.

Units: Million 2000 US$, or any other unit in which costs are tracked. Note that a regional monetary conversion factor (REG_XMONY) may be applied if different monetary units are used for the various regions.

Bounds: None

4.4.10 R_OBJTOT

Table A1, Column n

Description: The SAGE **g**lobal **TOT**al **OBJ**ective function variable corresponding to the sum of the total costs in each region for the current time period.

Purpose: To represent via a variable the global SAGE objective function (MR_OBJ).

Occurrence: The variable appears as the right-hand-side of the global objective function (MR_OBJ).

Units: Million 2000 US$, or any other unit in which costs are tracked.

Bounds: None

Remarks: R_objZ is the variable name of the RHS of the MR_OBJ objective function when SAGE is run in a clairvoyant manner using the standard MARKAL code.

R_RESID(r,t,p)

Table A1, Column o

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Description: The **RESID**ual capacity still available during the modeling horizon.

Purpose: To represent via a variable the level of residual capacity still in place in a period, so as to allow the annual charges for the past investments to be accounted for in the objective function. Note that this variable has a purely accounting role, as its value is fully determined by the input data parameter RESID.

Occurrence: The variable appears in the objective function for each period for which a echnology's residual capacity is still available. The variable is fixed explicitly at the level provided in the input data (RESID), and corresponds directly to the constant value set as the right-hand-side of the capacity transfer constraint (MR_CPT).

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. Coherent units are used for the investment (R_INV) variable, and for the residual capacity (RESID), cost (INVCOST, FIXOM), and capacity bound (BOUND(BD)) parameters related to the technology.

Bounds: The variable is fixed explicitly at the level provided in the input data (RESID).

4.4.11 R_TCZY(r,t,p,z,y)

Table A1, Column p

SAGE_MATRIX.V52.XLS

Description: The amount of low-temperature heat (lth) produced in a **T**ime period by a **C**oupled heat and power (cpd) pass-out turbine (CEH(Z)(Y)/ELM) in each time-slice (td = **z,y**). Such power plants have the flexibility to trade-off the amount of electricity versus heat produced (CEH(Z)(Y)), by controlling how much steam is sent to the turbines as opposed to the heat grid.

Purpose: The variable tracks the amount of low-temperature heat produced from a flexible pass-out turbine in each time-slice, and thus depicts the activity of such technologies. There are variables for each time-slice as the level at any time-slice is independent of that in another time-slice. Once these levels are established the amount of electricity that can be produced is then determined, so the R_TCZY variable is also used to represent the electric production from the plant where appropriate.

Occurrence: For coupled heat and power pass-out turbines **t**here is a R_TCZY heat variable for each time-slice. The variable represents the amount of heat produced, and thus appears in each low-temperature heat related constraint (MR_BALDH, for the grid connected (OUT(LTH)_TID)), and to determine the amount of energy consumed per unit production (MR_BAL_G/E). The activity is derived directly from the current installed capacity (R_CAP) by applying an availability factor (AF) that limits the plants' maximum output (MR_TEZY) in each time-slice. Once the amount of heat that is to be produced from the plant is determined, the R_TCZY variable is also used as a surrogate for the electricity production from the power plant in all the electricity-related constraints (MR_BALE1/2, MR_BAS) since it is a direct function of the amount of heat production. In the case that the power plant is defined as base load (bas) then only a single daytime variable (y='D') is created, and this same variable is used for both the day and night (y='N') to ensure that these plants operate identically day and night. As it defines the overall activity of the technology it also appears in the objective function (MR_MTSOBJ) to represent the variable operating and maintenance costs (VAROM). R_TCZY may also appear in a user-defined constraint (MR_ADRAT if RAT_TCZY or RAT_ACT), emissions (MR_TENV if ENV_ACT) and tax/subsidy (MR_TXSUB, TSUB_TCH) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of conversion technologies. Coherent units are used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in constraint (MR_BNDCON1/2/3) to sum the time-sliced activity variables. Indirect limits may be imposed on the activity of pass-out turbines if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

4.4.12 R_TEZY(r,t,p,z,y)

Table A1, Column q

<u>SAGE_MATRIX_V52.XLS</u>

Description: The amount of electricity (elc) produced in a **T**ime period by an **E**lectric generating plant (ela) in each time-slice (td = **z,y**).

Purpose: The variable tracks the amount of electricity produced in each time-slice for electricity generating facilities (electric only (ele), coupled heat and power (cpd) and storage plants (stg)), and thus depicts the activity of such technologies. There are variables for each time-slice as the level at any time-slice is independent of that in another time-slice.

Occurrence: For all power plants, **t**here is a R_TEZY electricity variable for each time-slice. The variable represents the amount of electricity produced, and thus appears in each electricity related constraint (MR_BALE1/2 and MR_BAS for the grid connected (OUT(ELC)_TID) and in MR_EPK if peak contribution is a function of the plant activity rather than capacity via PEAK(CON)_TID). This variable also determines the amount of energy consumed per unit production (MR_BAL_G/E). For standard power plants the activity is upper bounded by the current installed capacity (R_CAP), by applying an availability factor (AF/AF(Z)(Y)) that limits the plants' maximum output in each time-slice. In the case that the power plant is defined as base load (bas) then only a single daytime variable (y='D') is created, and this same variable used for both the day and night (y='N') to ensure that those plants operate at the same level day and night.

For externally load managed (or rigid) plants the R_TEZY is not generated, but instead the user-chosen capacity factor (CF/CF(Z)(Y)) is applied to the capacity variable (R_CAP) to determine the activity in each time-slice. As the level of R_TEZY defines the overall activity of the technology in each time-slice it also appears in the objective function (MR_MTSOBJ) to represent the variable operating and maintenance costs (VAROM). R_TEZY may also appear in a user-defined constraint (MR_ADRAT if RAT_TEZY or RAT_ACT), emissions (MR_TENV if ENV_ACT), and tax/subsidy (MR_TXSUB, TSUB_TCH) constraints if the associated data is provided

Units: Most often PJ, but may be any other unit defined by the analyst for activity of conversion technologies. Coherent same units are used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in a constraint (MR_BNDCON1/2/3) that sums the time-sliced activity variables. Indirect limits may be imposed on the activity of power plants if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

4.4.13 R_THZ(r,t,p,z)

Table A1, Column r

<u>SAGE_MATRIX_V52.XLS</u>

Description: The amount of low-temperature heat (lth) produced in a **T**ime period by a **H**eating plant (hpl) in each season (**z**).

Purpose: The variable tracks the amount of low-temperature heat produced in each season by each heat generating facility, and thus depicts the activity of such technologies. There are variables for each season as the level in any season is independent of that in another.

Occurrence: For power plants generating low-temperature heat **t**here is a R_THZ heat variable for each season. The variable represents the amount of low-temperature heat produced, and thus appears in each heat related constraint (MR_BALDH, for the grid connected (OUT(LTH)_TID)); and in MR_HPKW if peak contribution is a function of the plant activity rather than capacity via PEAK(CON)_TID)). This variable also determines the amount of energy consumed per unit production (MR_BAL_G/E, MR_BALE1/2 if the plant consumes electricity). The activity is upper-bounded from the current installed capacity (R_CAP) by applying an availability factor (AF/AF(Z)'D') that limits the plants' maximum output (MR_THZ) in each season. For externally load managed, or rigid, plants the R_THZ is not generated, but instead a capacity factor (CF/CF(Z)'D') is applied to the capacity variable (R_CAP) to determine the activity in each season. As the level of R_THZ defines the overall activity of the technology in each season it also appears in the objective function (MR_MTSOBJ) to report the variable operating and maintenance costs (VAROM). R_THZ may also appear in a user-defined constraint (MR_ADRAT if RAT_THZ or RAT_ACT), emissions (MR_TENV if ENV_ACT), and tax/subsidy (MR_TXSUB, TSUB_TCH) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of conversion technologies. The same units are used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in constraint (MR_BNDCON1/2/3) to sum the seasonal activity variables. Indirect limits may be imposed on the activity of power plants if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

4.4.14 R_TSEP(r,t,s)

Table A1, Column s

SAGE_MATRIX_V52.XLS

Description: The level of activity in a **T**ime period of each resource supply or export option (srcencp = src,ent,p = **sep**). The variable represents the total annual amount of the main energy carrier or material produced, imported, or consumed for exports, by the resource option.

Purpose: To track the level of activity associated with each resource supply option. The total annual cost of procuring (and the revenue from exporting) a resource (COST) is tied to the level of activity of each supply/export option.

Occurrence: This variable is generated for each resource supply/export option in all time periods beginning from the period that the option is first available (START). The variable appears in the commodity balance equation (MR_BAL_E/G, MR_BALE1/2), as well as contributes to the base load and peaking requirements (MR_BAS/MR_EPK) if involving electricity (elc). If the source is imports or exports (src = ‘IMP/EXP’) and the commodity is involved in bi-lateral trade (BI_TRD(ENT/MAT)) then it also appears in the exchange balance constraints (MR_BITRD). If electricity is traded according to season/time-of-day, then the variable balances against the total over all such time-slices (R_TSEPE) by means of an annual total constraint (MR_REGELC). Since R_TSEP represents the overall activity of the supply option, it appears in the objective function (MR_MTSOBJ) to account for the cost of the resource (COST). R_TSEP may also appear in a user-defined constraint (MR_ADRAT if RAT_SEP), emissions (MR_TENV if ENV_SEP) and tax/subsidy (MR_TXSUB, TSUB_SEP) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options. The same units are used for the cost (COST) parameter related to the resource option.

Bounds: The resource supply option may be directly bounded by specifying an annual activity bound (BOUND(BD)r). It may also be limited by bounds specified on growth rates (GROWTHr) between periods, as well as cumulative limits (CUM) imposed on the resource over the entire time horizon.

4.4.15 R_TSEPE(r,t,s,z,y)

Table A1, Column t

SAGE_MATRIX_V52.XLS

Description: The amount of electricity exchanged (BI_TRDELC, src = 'IMP/EXP', elc, p = **sep**) in a **T**ime period between two regions in each time-slice (td = z,y).

Purpose: To track the level of activity associated with each bi-lateral exchange of electricity in a time-slice.

Occurrence: This variable is generated for each bi-laterally traded electricity exchange in all time periods beginning from the period that the option is first available (START) for each time-slice for which such exchanges are permitted (BI_TRDELC). Since the traded electricity is tracked according to season/time-of-day, it must be balanced against the total annual production (R_TSEP), which is then used to link up with the rest of the model, by means of an annual constraint (MR_REGELC). But as the variable represents the amount of electricity in each time-slice, it appears directly in the appropriate electric balance (MR_BALE1/2), peak (MR_EPK) and base load (MR_BAS) constraints rather than the annual resource variable.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options.

Bounds: Only the annual exchange variables (T_SEP) may be limited by means of a direct bound on the level (BOUND(BD)r), as well as by bounds specified on growth rates (GROWTHr) between periods, or cumulative limits (CUM) imposed on the exchange over the entire time horizon.

4.4.16 R_TXSUB(r,txs)

Table A1, Column u

SAGE_MATRIX_V52.XLS

Description: The total level of any tax/subsidy introduced to the energy system.

Purpose: To accumulate the revenue (+taxes) or expenditures (-subsidy) associated with a user-named tax/subsidy.

Occurrence: This variable is generated for each tax/subsidy (txsub) for which a cost (TSUB_COST) is provided by the user. The commodities (TSUB_ENT) and technologies (TSUB_SEP and/or TSUB_TCH) that are involved with this tax/subsidy must be elaborated. Then according to the TSUB_SEP/TCH value and the activity of the individual technologies the total tax/subsidy is determined.

Units: Million 2000 US$ or whatever monetary unit is used in the model.

Bounds: The total tax/subsidy level may be limited by the TSUB_BND parameter if desired.

4.4.17 R_ZSTK

Table A1, Column v

SAGE_MATRIX_V52.XLS

Description: The total amount of stockpiled material, if any, at the end of the modeling horizon.

Purpose: To accumulate the net remaining energy/material in stockpiles.

Occurrence: This variable appears in the commodity balance equations (MR_BAL_E/G) and the objective function (MR_MTSOBJ) when a price is associated with the stockpiled material.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options.

Bounds: A resource limit (BOUND(BD)Or) may be applied to the stockpiled material in any period.

Remarks: Not used in SAGE at this time.

4.5 Equations (constraints and objective function)

The equations (constraints and objective function) of the SAGE model correspond to the rows of the LP matrix. Each equation is indexed by region, and by the appropriate set of other indexes necessary to fully and uniquely identify the model component. Each equation represents a logical condition that the SAGE Linear Program must satisfy. The condition is expressed as a mathematical relation of the form:

$$LHS \ \Re \ RHS$$

Where:
LHS is a linear expression involving the SAGE variables and attributes
$\Re$ is one of the three relational signs: =, ≥, or ≤
RHS is an expression involving only constants and attributes (may be 0 in several cases).

As discussed in section 3.3, the objective of SAGE is to maximize at each time period the total surplus (consumers plus producers surplus) for all regions together. This is equivalent to minimizing the period's total cost, comprising: annual costs, annualized investment costs, and a cost representing the utility loss incurred when demands for energy services are reduced (due to their price elasticity). In the implementation of SAGE, the cost minimization formulation is adopted. As already mentioned, when SAGE is run in its normal, time-stepped manner, the optimization is performed one period at a time (as opposed to the standard MARKAL model which optimizes the sum of discounted total costs over all periods).

The equations of the model are listed alphabetically in table 4.17, with a sequence number in the first column. Column 2 of the table indicates the equation name and the associated indexes, elaborated in table 4.15, needed to define the unique instances of the equation. The last column describes the equation, with particular emphasis on conditions that control the creation or not of that equation. The modeler is referred to Appendix Table A1 SAGE_MATRIX[1], that presents a global view of the all the equations involved in the model, which variables are involved in each equation, and what parameters (and sets) determine the actual coefficient of each potential equations/variable intersection. The rows (equations) are ordered alphabetically in this spreadsheet as well as the table presented here and the detail specification sheets for each equation that follow. Thus by simply looking across the associated row of the Table A1 SAGE_MATRIX one can quickly see all the variables and parameters that might influence an intersection in the matrix for each equation. However, the parameters listed in the Table A1 SAGE_MATRIX are those that are actually used in the code when creating the coefficient, and thus may not correspond directly to user input data. Every set and parameter appearing on the Table A1 SAGE_MATRIX is explained in the Set and Parameters section 4.2, table 4.11 of this chapter. From that table the user can dig deeper into the details to gain a full understanding of the particulars of the matrix coefficients, including the input parameter(s) they are dependent upon, if desired.

[1] Appendix Table A1 SAGE_MATRIX is provided both as a text table for those who have a magnifying glass handy and as an inserted Excel file for the rest of us.

Table 4.17: Model Constraints[1,2]

EQ Ref	Constraints (Indexes)	Constraint Description	GAMS Ref
1	MR_ADRAT (r,t,a)	The specialized, user-defined constraints that link variables of the model together for some special purpose (e.g., set market shares for a group of technologies).	MMEQARAT
2	MR_BAL_G/E (r,t,e)	Thie balance equation that ensures that the production of each commodity equals or exceeds the total consumption of that commodity. When the commodity is an energy carrier (enc), then the MR_BAL_G constraint permitting production to exceed consumption is applied. When the commodity is a material (mat), then the MR_BAL_E constraint applies which forces production to match consumption. [e = enc/mat]	MMEQBAL
3	MR_BALDH (r,t,e,z)	The balance equation that ensures that the total amount of low-temperature heat produced in each season meets or exceeds that demanded in that season. [e = lth]	MMEQBALH
4	MR_BALE1/2 (r,t,e,w)	The balance equation that ensures that the total amount of electricity produced in each time-slice (season/time-of-day) meets or exceeds that demanded in that time-slice. Equation 1 is for daytime constraints (w = z,'D') and equation 2 is for the nighttime constraints (w = z,'N'). [e = elc]	MMEQBALE
5	MR_BAS (r,t,e,z)	Ensures that those power plants designated as base load (bas) operate at the same level in the day and night, while not exceeding a percentage of the highest nighttime electricity demand (according to BASELOAD). [e = elc]	MMEQBAS
6	MR_BITRD (r,e,r,t,e,c)	Matches up the bi-lateral trade of a commodity between two regions. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (c) must be used.	MMEQUA.REG
7	MR_BITRDE (r,e,r,t,e,c,w)	Matches up the bi-lateral trade of electricity between two regions during a particular time-slice. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (c) must be used. [e = elc]	MMEQUA.REG
8	MR_BNDCON1/2/3 (r,t,p)	Bound on the (total) annual output from a conversion technology [BOUND(BD)O], that is the sum over all time-slices. The 1/2/3 indicator correspond to the sense of the equation = LO/FX/UP. [p = con]	MMEQBCON

[1] The EQ Ref is the sequence number according to the equation's alphabetic ordering in this chapter. Each equation is also listed in the corresponding order in the SAGE model matrix spreadsheet presented in Appendix A: Table A1.SAGE_MATRIX_V52.XLS

[2] The GAMS Ref column identifies the main routines in which the equations are constructed. A series of core pre-processor routines (MMSETS (prepare control sets), MMFILL (perform interpolation), FRACLIFE (calculate fractional period lifetimes), MMCOEF (derive the matrix coefficients)), along with some minor associated subroutines, prepare the input data for direct use in the actual equation specification routines listed here. [Note that all core code has an extension of .INC unless noted otherwise (e.g., multi-region specific code has .REG extension, SAGE time-stepped specific code has .MTS extension).

EQ Ref	Constraints (Indexes)	Constraint Description	GAMS Ref
9	MR_CPT1/2/3 (r,t,p)	The capacity transfer constraint that ensures that the total capacity (R_CAP) in a period correctly reflects the remaining residual capacity from before the modeling horizon (RESID), plus all investments (R_INV) made in the current and earlier periods whose LIFE has not yet expired. The 1/2/3 corresponds to p = dmd/con/prc respectively, where for dmd the constraint is an inequality (≥, allowing for excess capacity for end-use demand devices), while for all other technologies the constraint is an equality.	MMEQCPT
10	MR_CUM (r,s)	Any limit on the total production of a commodity from a resource activity over the entire modeling horizon (CUM).	MMEQCUM
11	MR_DEM (r,t,d)	The demand constraint that ensures that demands for useful energy services (DEMAND) are satisfied.	MMEQDEM
12	MR_ENV (r,v)	The total cumulative emissions over the entire modeling horizon. Built by summing the period emissions variables (R_EM * NYRSPER).	MMEQENV
13	MR_EPK (r,t,e,z)	The electricity peaking constraint ensures that there is enough capacity in place to meet the highest average electricity demand during the day of any season + estimated level above that for the actual peak (moment of highest electric demand) + a reserve margin of excess capacity (ERESERV includes both), to ensure that if some plants are unavailable, the demand for electricity can still be met. [e = elc]	MMEQEPK
14	MR_GRSEP (r,t,s)	The limit imposed on the rate at which a resource supply option can expand between periods (GROWTHr).	MMEQGRS
15	MR_GRTCH (r,t,p)	The limit imposed on the rate at which total installed capacity can expand between periods (GROWTH).	MMEQGRT
16	MR_GTRD (t,g)	The balance of the globally traded commodities between all the producers and consumers of these commodities.	MMEQUA.REG
17	MR_HPKW (r,t,e,z)	The low-temperature heat peaking constraint that ensures that there is enough capacity in place to meet the highest average seasonal heating demand + estimated level above that for the actual peak (moment of highest heat demand) + a reserve margin of excess capacity (HRESERV includes both) to ensure that if some plants are unavailable that demand for heat can still be met. [e = lth]	MMEQHPK
18	MR_LIM (r,t,p,e)	For flexible output "mixing" processes (LIMIT) the constraint that controls the levels of the individual outputs (OUT(ENC)p) from such processes. [p = prc]	MMEQLIM
19	MR_MTSOBJ	The SAGE time-stepped minimum annualized system cost objection function, solved successively for each period of the model run.	MMEQOBJ.MTS
20	MR_OBJ	The MARKAL total discounted system cost objective function (perfect foresight mode).	MMEQUA.REG

EQ Ref	Constraints (Indexes)	Constraint Description	GAMS Ref
21	MR_PBL (r,t,p)	For flexible output "mixing" processes (LIMIT) the constraint that ensures that the sum of all the outputs (OUT(ENC)p) are in line with the overall efficiency of such processes. [p = prc]	MMEQPBL
22	MR_PRICE	An accounting equation tracking the total discounted system cost for SAGE reporting purposes. [Corresponds to the MARKAL objective function.]	MMEQPRIC
23	MR_REGELC (r,t,s)	Balance equation tying the time-sliced electricity trade variables (R_TSEPE) to the regular annual electricity variable (R_TSEP). [s = "IMP/EXP"elcp]	MMEQUA.REG
24	MR_TENV (r,t,v)	The total amount of emissions generated in each period from all sources (resources and technologies) of a given emission indicator.	MMEQTENV
25	MR_TEZY (r,t,p,w)	The constraint that ensures that the season/time-of-day activity of a power plant (R_TEZY), that is the amount of electricity generated, is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF/AF(Z)(Y)/CF/CF(Z)(Y)), taking into consideration any scheduled maintenance (R_M). [p = ela]	MMEQTEZY
26	MR_THZ (r,t,p,z)	The constraint that ensures that the seasonal activity of a heating plant (R_THZ), that is the amount of heat generated, is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF/AF(Z)(Y)/CF/CF(Z)(Y)), taking into consideration any scheduled maintenance (R_M). [p = hpl]	MMEQTHZ
27	MR_TXSUB (r,t,txsub)	The charging or crediting of a tax/subsidy (TSUB_COST) for commodities (TSUB_ENT) consumed/produced by technologies (TSUB_TCH). The total tax/subsidy is subtracted from the total system costs in the reports, and reported separately.	MMEQTAXS
28	MR_UTLCON (r,t,p)	The adjustment to the availability of a power plant to account for the fact that some portion of outage is due to scheduled maintenance. Normally the model will schedule all unavailability, but if the user specifies that some fraction of annual unavailability is scheduled [AF_TID] then this constraint is generated to adjust the amount of unavailability that the model is free to schedule. [p = con]	MMEQUCON
29	MR_UTLPRC (r,t,p)	The constraint that ensures that the annual activity of conventional process (R_ACT) is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF). [p = prc]	MMEQUPRC

In elaborating the details of the construction of the individual equations of the model standard mathematical terminology and expressions are employed, as summarized in table 4.18 below. As noted in the table, there are two extensions to standard notation, a dollar sign ($), is lifted from the GAMS language, and a '/'.

Table 4.18: Mathematical Symbols Used for Equation Specification

Symbol	Description
$\subset$	A set is a subset of another set.
$\in$	An element is a member of a set.
$\notin$	An element is not a member of a set.
$\cap$	The "anding" or intersection of two sets, where an element must exist in both sets.
$\cup$	The "oring" or union of two sets, where an element must exist in one of the listed sets.
$\sum$	Summation over a set or series of sets.
$\leq$	Less than or equal to condition.
$=$	Equal to condition.
$\geq$	Greater than or equal to condition.
$\wedge$	The "anding" or intersection of two or more logical expressions, where all expression must be true.
$\vee$	The "oring" or union of two or more logical expressions, where at least one expression must be true.
$\forall$	A "such that" (or "for all")condition that explains under what circumstance an equation is generated.
$\$$	Indicates that a condition controls whether or not a particular calculation is performed.[1]
/	A shortcut employed to reduce the need to repeat identical specification of equations, for example for energy and material (e.g., ent/mat).

Before turning to the individual equations a general example is presented here to explain the syntax employed when specifying the details of the actual equations.

Equation specification header indicating the equation sequence number, the equation name (with the multi-region prefix (MR_), the indexes enumerating the specific instances of the equation (e.g., r = region, t = time period, e = energy or material). Then any overall conditions controlling the generation of the equation are specified.

$$\text{EQ\#}: MR\_ < eqname >_{r,t,e} \; \forall e \in < conditional\,criteria >$$

Multiplier applied to a part of the matrix intersections. The same XX parameter is named XXENT if it relates to energy and XXMAT if it relates to material. Note that all indexes must either appear on the equation declaration or in a $\sum$ expression (as shown in the next part of the example) indicating that said index is to be looped over for each qualifying element either summing or creating unique instances of the associated variable(s).

[1] For example, 1\$(dcn) + TE(ent)\$(cen) would indicate that the transmission efficiency is 1 for decentralized (dcn) plants and the user provided overall commodity efficiency (TE(ent)) if centralized (cen).

$$XX\ (ENT\ \ /\ MAT\ )_{r,e,t}\ *\ \bigg\langle$$

Matrix intersection constructed by summing over some index(es) p (e.g., processes), according to a certain criterion that <parname> exists for the current indexes, applying coefficient parameters to a variable (whose name always has a R_ prefix) for each index of the looping set x, depending upon additional criteria.

$$\left[\sum_{\substack{p \in <set> \forall \\ <pname>_{r,p,e,t} \\ p \neq '<literal>'}} \begin{array}{l} \left(<pname1>_{r,p,e,t} * R\_<vname1>_{r,t,p}\right) \$ <cond1> + \\ \left(<pname2>_{r,p,e,t} * R\_<vname2>_{r,t,p}\right) \$ <cond2> \end{array}\right] +$$

Rest of the intersections in the left-hand-side of the equation.

$$\ldots$$

Relational operator that indicates the sense of the equation, which may vary depending upon the nature of the equation (e.g., energy balance is ≥ while material balance is =).

$$\{\leq ; = ; \geq\}$$

Right-hand-side of the equation; usually 0 otherwise an expression involving constants and parameters (e.g., RESID, SEP_CUM).

$$RHS;$$

The rest of this section provides a detailed description of each of the equations of SAGE[1]. On the equation sub-header record the equation name and index is given, along with a reference to the associated row in the Table A1 SAGE_MATRIX. For each equation the following information is provided on "fact" sheets:

Description: A short description of the nature of the equation.

Purpose: A brief explanation of what the equation is for and why it is needed.
Primal: What the level implies.
Dual: What the cost formulation implies.

Occurrence: A general description of the data circumstances and model rules that control the instances of the constraint, or not, and the variables in the constraint.

Units: The units associated with the constraint.

Type: The sense of the equation (≤,=,≥)..

Interpretation of the results:
Primal: Guidelines of how to interpret the primal results, or the slack.
Dual: Guidelines of how to interpret the dual results, or the shadow price.

For each equation the "fact" sheet is followed by the mathematical specification of the equation. To the fullest extent possible these detailed equation descriptions indicate the user input parameters that the matrix intersection depends upon. However, at certain times intermediate parameters are employed. The nature of the parameters can be discerned by the position of the period index (t). User input is provided by means of spreadsheet tables where the periods run along the columns, and thus time is the last index for these parameters. However internal parameters take advantage of special preprocessor Sets that map the allowed period and technology combinations (e.g., tptch(t,p)), dependent upon when the technology first becomes available (START), in the left-most position of the parameter. Thus internal parameters have ttime as the 2nd index, right after the region. Note that the Table A1 SAGE_MATRIX depicting the model structure presents the parameters used in the code when the coefficients are created, and thus does not correspond directly to what appears in the mathematical descriptions presented here.

4.5.1 MR_ARAT1/2/3/4(r,t,a)

Table A1, Row 6
SAGE_MATRIX_V52.XLS

Description: These specialized constraints impose user-defined relationships among the variables of the model. (e.g., set market shares for a collective group of technologies). The '1/2/3/4' in the equation listing does not indicate that only 4 such constraints are

[1] Note that the generation of all constraints is conditioned upon which period is currently being solved, and the inclusion of any technology is conditioned upon the initial availability of said in a period. These conditions are true across the board, but are **NOT** reflected in the detail equations presented in this section.

allowed, but rather the type of the equation (see below). Indeed, in a large model, there may be hundreds of such user-defined constraints.

Purpose:

Primal: These user-defined constraints are unique in that their structure is completely defined by the modeler. They can be used to associate any components of the model by relating the variables associated with activity (RAT_SEP, RAT_ACT, RAT_TEZY, RAT_TCZY, RAT_HPL), or capacity (RAT_CAP), or investment (RAT_INV) to one another. One important restriction of a user-defined constraint is that it may only involve variables with the same period index. In other words, constraints that tie variables from different time periods are not allowed. The sense and right-hand-side of a user-defined equation is specified by means of the RAT_RHS parameter. For a user-defined equation each parameter provided represents the coefficient to be applied to the associated variables when constructing the equation. For example, to equate the capacity of two technologies (e.g., scrubbers to be installed on an existing power plant) one would provide a RAT_RHS'scrubit',t,'fx' = 0, and RAT_CAP'scrubit',t,'p1' = 1 and RAT_CAP'scrubit',t,'p2' = -1. The resulting equation MR_ARAT2(r,t,'scrubit') would be:

R_ CAP'scrubit',t,'p1' - R_ CAP'scrubit',t,'p2' = 0.

Dual: The dual variable (DVR_ARAT) represents the decrease in system cost resulting from a one-unit relaxation of the specialized constraint imposed by the user. Otherwise known as the shadow price or marginal cost of the constraint.

Occurrence: This equation is generated in all time periods for which a right-hand-side is provided (RAT_RHS), along with at least one variable coefficient (RAT_ACT, RAT_CAP, etc.).

Units: The units of the user-defined constraints should be compatible with those of the variables composing the constraint. It is fully the responsibility of the user to ensure that consistent units are employed for each user-defined constraint.

Type: *Binding*

The type of the equation is under user control according to the sense (b = 'LO', 'FIX', 'UP', 'NON') entered when specifying the RAT_RHS parameter

Interpretation of the results:

Primal: The level of a user-defined constraint represents the slack (difference between the RHS and LHS) if a binding constraint, or the level if a non-binding equation is specified..

Dual: As explained in section 3.4, the dual variable (DVR_ARAT) of the user-defined constraint (shadow price) indicates the amount that the objective function would change if the RHS of the constraint is increased by one unit. A positive value indicates an increase of the optimal cost.

Remarks: Note that in the description below the OR operator ($\vee$) is used to indicate that if RAT_ACT is provided it is assigned to the associated RAT_TEZY, RAT_TCZY, RAT_HPL parameter by the preprocessor.

GAMS Routine: MMEQARAT.INC

MATHEMATICAL DESCRIPTION

User-defined equations are always generated when Market Share algorithm is not activated, or when the user-defined constraint/MKTSHR switch = ‘A’lways. However, if the MKTSHR switch is activated and the user-defined constraint /MKTSHR switch = ‘B’oth and there is both an ADRATIO = MKT_ID, then the user-defined constraint is suppressed and the Market Share algorithm is applied instead.

$$\text{EQ\#1}: MR\_ARAT1/2/3/4_{r,t,a}\ \forall RAT\_RHS_{r,a,b,t}$$

$$\wedge \left(\begin{array}{l} AMSWTCH_{r,a} = 'A' \vee \\ \left\langle \begin{array}{l} MKTSHR = 'YES' \wedge \\ (MKT\_ID = a \wedge AMSWTCH \neq 'B') \end{array} \right\rangle \end{array} \right)$$

where 1/2/3/4 corresponds to equation type (b=’LO’ ≥, ‘FX’ =, ‘UP’ ≤, ‘NON’ non-binding).

All resource supply options for which the RAT_SEP parameter is provided are included in the associated user-defined constraint based upon their activity level.

$$\left[\sum_{s} \left(RAT\_SEP_{r,a,s,t} * R\_TSEP_{r,t,s} \right) \right] +$$

All technologies for which the RAT_INV parameter is provided are included in the associated user-defined constraint based upon the level of new investment in the technology.

$$\left[\sum_{p} \left(RAT\_INV_{r,a,p,t} * R\_INV_{r,t,p} \right) \right] +$$

All technologies for which the RAT_CAP parameter is provided are included in the associated user-defined constraint based upon the total installed capacity of the technology.

$$\left[\sum_{p} \left(RAT\_CAP_{r,a,p,t} * R\_CAP_{r,t,p} \right) \right] +$$

All technologies for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon activity level of the technology. However, as noted below there are quite a few different ways the parameter needs to be applied depending on the nature of the technology.

All demand devices for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the activity derived according to the amount of installed capacity.

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} RAT\_ACT_{r,a,p,t} * CF_{r,p,t} * \\ CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{array} \right) \right] +$$

All conventional (non externally load managed) processes for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the activity variable representing total output from the process.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(RAT\_ACT_{r,a,p,t} * R\_ACT_{r,t,p} \right) \right] +$$

All externally load managed processes for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the total output of all commodities as derived from the level of installed capacity of the process.

$$\left[\sum_{p \in xpr}\begin{pmatrix} RAT\_ACT_{r,a,p,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \sum_{e}\left(OUT(ENC/MAT)p_{r,p,e,t} * R\_CAP_{r,t,p}\right) \end{pmatrix}\right] +$$

All conventional electric generation plants for which the RAT_TEZY parameter is provided for some time-slice are included in the associated user-defined constraint based upon the time-slice activity of the power plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provide), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_TEZY if the same coefficient applies to all time-slices (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_TEZY=RAT_ACT in the preprocessor). [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{w,p \in ela \\ p \notin xlm \\ except\, y='N' \wedge (bas \cup stg) \\ except((y='N' \wedge z='I') \wedge \\ PEAK\_(CON)\_TID_{r,p})}} \begin{pmatrix} \begin{pmatrix} RAT\_ACT_{r,a,p,t} \vee \\ RAT\_TEZY_{r,a,p,w,t} \end{pmatrix} * \\ R\_TEZY_{r,p,t,w} \end{pmatrix}\right] +$$

All externally load managed electric generating plants for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the total output of electricity over all permitted time-slices (according to the CF(Z)(Y) provided) as derived from the level of installed capacity of the plant.

$$\left[\sum_{w,p \in xlm} \begin{pmatrix} RAT\_ACT_{r,a,p,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ QHR(Z)(Y)_{r,w} * R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

The heat component from all coupled heat and power pass-out turbines for which the RAT_TCZY parameter is provided for some time-slice are included in the associated user-defined constraint based upon the time-slice activity of the power plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provided), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_TEZY if the same coefficient applies to all time-slices (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_TEZY=RAT_ACT in the preprocessor).

$$\left[\sum_{\substack{p \in cpd,w \\ when\, ELM_{r,p}}} \begin{pmatrix} \begin{pmatrix} RAT\_ACT_{r,a,p,t} \vee \\ RAT\_TCZY_{r,a,p,w,t} \end{pmatrix} * \\ R\_TCZYH_{r,p,t,w} \end{pmatrix} \right] +$$

All conventional heating generation plants for which the RAT_THZ parameter is provided for some season are included in the associated user-defined constraint based upon the seasonal activity of the plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provide), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_THZ if the same coefficient applies to all seasons (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_THZ=RAT_ACT in the preprocessor). [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{z,p \in hpl \\ except((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p})}} \left(\left(\begin{array}{l} RAT\_ACT_{r,a,p,t} \vee \\ RAT\_THZ_{r,a,p,z,t} \end{array} \right) * \\ R\_THZ_{r,p,t,z} \right) \right]$$

The sense of the user-defined equation is determined by the RAT_RHS parameter setting such that the equation type is: 'LO' ≥, 'FX' =, 'UP' ≤, 'NON' non-binding.

$$\{\leq; =; \geq; non-binding\}$$

$$0$$

4.5.2 MR_BAL_G/E(r,t,e)

Table A1, Row 7
SAGE_MATRIX_V52.XLS

Description: **BAL**ance equation for standard commodities (energy different from electricity or low temperature heat, and materials), ensures that production plus imports is in balance accordance with the demand for the commodity.

Purpose:

Primal: The balance equation ensures the conservation of energy and materials throughout the energy system. The equation is generated for each energy carrier, other than renewable energy carriers (ern) for which no resource supply option (src = 'RNW') is provided, for example wind and solar, and each material of the underlying Reference Energy System. It sums all sources of the commodity including production from resource options, processes and other technologies, along with imports, and ensures that this total is greater than or equal to the total amount consumed by all technologies and exports. In addition there may be movements into (consumption) or out of (production) stockpiles (src = 'STK'). For energy carriers slack is permitted, as it may be the case that as a result of some rigid processes one particular commodity forces another commodity to be over produced at levels above the total amount actually needed. However, the analyst should carefully investigate any slack on the balance equations as any energy carrier for which there is slack on the balance equation will have a 0 shadow price. For materials a strict equality constraint is imposed, therefore no excess production is allowed.

Dual: The dual variable (DVR_BAL) represents the marginal value of one unit of the commodity to the energy system. It thus represents to the price of that commodity that the model must pay for the each unit of the commodity consumed, and receives for each unit produced.

Occurrence: This equation is generated in all time periods during which some resource option or technology produces or consumes the commodity. Note that standard energy carriers do not include electricity and low-temperature heat, which are handled in separate equations that reflect the time-sliced nature of these commodities.

Units: PJ, or Bvkm, or any other unit in which a commodities are tracked.

Type: *Binding*

The equation is a greater than or equal (≥) if the commodity is an energy carrier (enc), or an equality (=) if the commodity is a material (mat).

Interpretation of the results:

Primal: The level of the balance equation represents the slack or overproduction of an energy carrier. In most cases there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of some processes which ties the fraction of one energy carrier to those of the other, if one energy carrier in such a group is needed at a higher level that may result in an excess of the associated energy carriers. Under normal circumstance the user should look to redress this situation by employing more flexible output processes (e.g., LIMIT), or parallel input processes that would allow the model to optimize the commodity mix (within limits the user imposes). For materials since the constraint is an equality the level of the equation is by definition 0.

Dual: For energy carriers, the dual variable (DVR_BAL) of the balance constraint (shadow price) indicates the amount that the energy system pays for the each unit of the commodity consumed, and accrues for each unit produced. A zero dual value implies that the supply of the commodity exceeds its consumption.. For materials, the dual value may be positive or negative.

Remarks: For renewable energy carriers for which no resource supply option is provided (src = 'RNW') a non-binding accounting equation is generated which tracks the total consumption of such energy carriers. In this case the level of this row is assumed to be the fossil equivalent (feq) of the consumed renewable, but no active constraint is imposed.

GAMS Routine: MMEQBAL.INC

MATHEMATICAL DESCRIPTION

$$\mathrm{EQ\#2}: MR\_BAL\_l_{r,t,e} \; \forall e \in (enc \vee mat),$$

where l = G (≥) for energy carriers, and E (=) for materials

PRODUCTION - commodities entering the system from Resource Supply options and produced by Technologies as main outputs or by-products.

System-wide commodity transmission efficiency applied to the total production, reflecting overall loss and forcing higher production levels.

$$TE(ENT/MAT)_{r,e,t} * \quad \langle$$

Production of the current commodity from resource supply options, other than EXPorts, identified by means of the OUT(ENT)r indicator parameter.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x \neq 'EXP'}} R\_TSEP_{r,t,s} \right] +$$

Import of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'IMP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right] +$$

Production of the current commodity from conventional (non-externally load managed and non-LIMIT) processes that deliver the current commodity according to fixed out shares.

$$\left[\sum_{\substack{p\in prc \\ p\notin\left(xpr\vee LIMIT_{r,p,t}\right)}}\begin{pmatrix} OUT(ENC)p_{r,p,e,t} * \\ R\_ACT_{r,t,p} \end{pmatrix}\right]+$$

Production of the current commodity from flexible output processes (identified by means of the LIMIT parameter) that delivers the current commodity according to the level determined by the model. [The level of the R_LOUT variable for each commodity produced from a flexible process is controlled by the MR_PBL and MR_LIM equations.]

$$\left[\sum_{p\in prc\wedge LIMIT_{r,p,t}} R\_LOUT_{r,t,p,e}\right]+$$

Production of the current commodity from externally load managed processes, where the activity of the process (and thus the amount of each commodity produced) is directly determined from the level of installed capacity by means of a fixed capacity factor. [Note that flexible (LIMIT) processes may not be designated as externally load managed.]

$$\left[\sum_{p\in xpr}\begin{pmatrix} R\_CAP_{r,t,p} * CF_{r,p,t} * \\ OUT(ENC)p_{r,p,e,t} * CAPUNIT_{r,p} \end{pmatrix}\right]+$$

Annual production of the current commodity as a fixed ratio by-product of conventional (non-externally load managed) electric generation, summed over the season/time-of-day activity of power plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in (ela, \notin xlm), w \\ except\ y='N' \wedge p \in (bas \cup stg) \\ except\ ((y='N'+z='I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \begin{pmatrix} OUT(ENC)c_{r,p,e,t} * \\ R\_TEZY_{r,t,p,w} \end{pmatrix} \right] +$$

Annual production of the current commodity as a by-product from externally load managed power plants, where the activity of the plant (and thus the amount of each commodity produced per unit of electricity or heat generated) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice.

$$\left[\sum_{p \in xlm} \begin{pmatrix} OUT(ENC)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ \sum_{z,y} \left(QHR(Z)(Y)_{r,p,z,y} * CF(Z)(Y)_{r,p,z,y,t} \right) * \\ R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Annual production of the current commodity as a by-product of low-temperature heat generation for conventional (non-externally load managed) facilities, summed over the seasonal activity of conventional power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p\in hpl\notin xpr,z \\ except((z='S'\vee'I')\wedge \\ PEAK(CON)\_TID_{r,p}))}} \begin{pmatrix} OUT(ENC)c_{r,p,e,t}* \\ R\_THZ_{r,t,p,z} \end{pmatrix}\right]+$$

Annual production of the current commodity as a by-product of low-temperature heat generation from coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of power plants. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p\in cpd,w \\ when CEH(Z)(Y)_{r,p,w,t} \\ except(y='N'\wedge p\in bas)}} \begin{pmatrix} OUT(ENC)c_{r,p,e,t}* \\ R\_TCZYH_{r,t,p,w} \end{pmatrix}\right]+$$

Production of the current commodity as a fixed proportion of the activity of demand devices. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{p\in dmd} \begin{pmatrix} MO(ENC)_{r,p,e,t}*CAPUNIT_{r,p}* \\ CF_{r,p,t}*R\_CAP_{r,t,p} \end{pmatrix}\right]$$

$\rangle$

$\{\geq ;=\}$, ≥ for energy carriers, and = for materials

CONSUMPTION - commodities leaving the system as Exports or consumed by Technologies according to their activity level.

EXPorts of a commodity, identified as the consumption of the commodity by means of INP(ENT)x.

$$\left[\sum_{\substack{s \forall INP(ENT)x_{r,s,e,t} \\ x='EXP'}} R\_TSEP_{r,t,s} \right] +$$

Export of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'EXP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right] +$$

Consumption of the current commodity as an auxiliary input required by resource supply options, according to the amount needed to produce another commodity (e.g., electricity for mining coal).

$$\left[\sum_{s \neq e} \begin{pmatrix} INP(ENT)r_{r,s,e,t} * \\ R\_TSEP_{r,t,s} \end{pmatrix} \right] +$$

Consumption of a commodity by conventional (non-externally load managed) processes according to the amount of said commodity needed per unit of overall activity of the process.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \begin{pmatrix} INP(ENT)p_{r,p,e,t} * \\ R\_ACT_{r,t,p} \end{pmatrix} \right] +$$

Consumption of the current commodity by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p \in xpr} \begin{pmatrix} INP(ENT)p_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} * R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Annual consumption of the current commodity as required for a unit of electric generation from conventional (non-externally load managed) facilities, summed over the season/time-of-day activity of power plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p\in ela,w\\ except\ y='N'\wedge p\in(bas\cup stg)\\ except\ ((y='N'\wedge z='I')\wedge\\ PEAK(CON)\_TID_{r,p}))}}\begin{pmatrix}INP(ENT)c_{r,p,e,t}*\\ R\_TEZY_{r,t,p,w}\end{pmatrix}\right]+$$

Annual consumption of the current commodity by externally load managed power plants, where the activity of the plant (and thus the amount of each commodity required per unit of electricity or heat produced) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice.

$$\left[\sum_{p\in xlm}\begin{pmatrix}INP(ENT)c_{r,p,e,t}*CAPUNIT_{r,p}*\\ \sum_{z,y}\left(QHR(Z)(Y)_{r,p,z,y}*CF(Z)(Y)_{r,p,z,y,t}\right)*\\ R\_CAP_{r,t,p}\end{pmatrix}\right]+$$

Annual consumption of the current commodity as required for a unit of low-temperature heat generation, summed over the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p\in hpl,z \\ p\notin xlm \\ except((z='S'\vee'I')\wedge \\ PEAK(CON)\_TID_{r,p}))}} \begin{pmatrix} INP(ENT)c_{r,p,e,t}* \\ R\_THZ_{r,t,p,z} \end{pmatrix}\right]+$$

Annual consumption of the current commodity as required for a unit of low-temperature heat generation from coupled production pass-out turbines (identified by means of the ELM parameter), summed over the seasonal/time-of-day activity of power plants (as determined by CEH(Z)(Y)). [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p\in cpd,w \\ when ELM_{r,p,t} \\ except(y='N'\wedge p\in bas)}} \begin{pmatrix} INP(ENT)c_{r,p,e,t}* \\ \left(CEH(Z)(Y)_{r,p,w,t}/ELM_{r,p,t}\right)* \\ R\_TCZYH_{r,t,p,w} \end{pmatrix}\right]+$$

Consumption of the current commodity as required for a unit of output from demand devices. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} / EFF_{r,p,t} * R\_CAP_{r,t,p} \end{array} \right) \right]$$

Where x = the source (src) part of the resource name (srcencp) for Imports/exports ('IMP/EXP').

4.5.3 MR_BALDH(r,t,e,z)
Table A1, Row 8

SAGE_MATRIX_V52.XLS

Name: **BAL**ance equation for low-temperature **D**istrict **H**eat energy carriers (lth) in each season.

Purpose:

Primal: Like all balance equations, this one for low-temperature heat guarantees the conservation of energy throughout the energy system. The equation ensures that the production of low-temperature heat in each season is equal to or exceeds the demand for heat. It sums over all sources of low-temperature heat and ensures that this total is greater than or equal to the amount consumed by all demand devices (dmd) [Note: low-temperature heat may ONLY be consumed by demand devices]. Slack is permitted, as it may be the case owing to some rigid coupled heat and power plant (REH) that the amount of low-temperature heat generated may exceed the amount demanded in a season. However, the analyst should carefully investigate any slack on the low-temperature heat balance.

Dual: The dual variable (DVR_BALDM) represents the marginal value of one unit of the low-temperature heat to the energy system in each season. It thus represents the price that the model must pay for the each unit of low-temperature heat produced/consumed.

Occurrence: This equation is generated in all time periods and for all seasons during which some demand device consumes the low-temperature heat.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding*

The equation is a greater than or equal (>=) constraint for each season.

Interpretation of the results:

Primal: The level of the low-temperature heat balance equation represents the slack or overproduction of low-temperature heat in a season. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of back-pressure turbines if electric demand forces the operation of such a plant it is possible that production may exceed the demand for low-temperature heat. Under normal circumstance the user should look to examine and possibly redress this situation by employing a flexible output coupled heat and power (CEH(Z)(Y)), or determine any other reason for the excess (e.g., lower limits forcing the operation of a power plant).

Dual: The dual variable (DVR_BALDH) of the low-temperature heat balance constraint (shadow price) indicates the amount that the energy system must pay per unit of low-temperature heat consumed and receive per unit produced, in each season. A zero dual value implies that the supply of heat exceeds the demand by the rest of the energy system. The value is positive when the primal equation is tight, which is normally the case.

Remarks: Note that high temperature process heat is modeled as a annual energy carrier as it is for industrial use and not subject to seasonal variations.

GAMS Routine: MMEQBALH.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#3}: MR\_BALDH_{r,t,e,z} \ \forall e \in lth \wedge QHRZ_{r,z}$$

PRODUCTION – low temperature heat generated from either heating or coupled production power plants feeding the current heating grid.

Production of low temperature heat to the heat grid from conventional heating plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ except((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_THZ_{r,t,p,z} \right] +$$

Production of low temperature heat from externally load managed plants, where the activity of the plant (and thus the amount of heat produced) is directly determined from the level of installed capacity by means of a fixed capacity factor for each season. [Note that as heating plants are modeled by season (not time of day) CF(Z)(Y) should only be specified for y = 'D'.]

$$\left[\sum_{p \in (hpl \cap xlm)} \left(\begin{array}{l} CAPUNIT_{r,p} * \\ \left(QHRZ_{r,p,z} * CF(Z)(Y)_{r,p,z,'D',t} \right) * \\ R\_CAP_{r,t,p} \end{array} \right) \right] +$$

Production of low temperature heat to the heat grid from coupled heat and power pass-out turbines, for each time-slice (w=z,y) for which the electricity loss to heat gain ratio (CEH(Z)(Y)) is provided, as well as for which a time-sliced transmission efficiency (TRNEFF(Z)(Y)) is provided if centralized, and 1 otherwise. [Special conditions apply for base load plants that force them to operate the same day and night, thus only a day

$$\left[\sum_{\substack{p \in cpd, z \\ CEH(Z)(Y)_{r,p,z,y,t} \\ except(p \in bas \wedge y = 'N')}} \left(\begin{array}{l} TRNEFF(Z)(Y)_{r,p,z,y,t} \$ p \in cen * \\ R\_TCZYH_{r,t,p,z,y} \end{array} \right) \right] +$$

Production of low temperature heat to the heat grid from coupled heat and power back-pressure turbines (as designated by REH), for each time-slice (w=z,y); for which a transmission efficiency (TRNEFF(Z)(Y)) is provided if centralized, and 1 otherwise. [Special conditions apply for base load plants that force them to operate the same day and night, thus only a day variable exists.]

$$\left[\sum_{\substack{p\in cpd,z \\ REH_{r,p} \\ except(p\in bas \wedge y='N')}} \left(\begin{array}{l} (TRNEFF(Z)(Y)_{r,p,z,y,t}\$ p\in cen / \\ REH_{r,p})*R\_TCZYH_{r,t,p,z,y} \end{array} \right)\right] +$$

$$\{\geq\}$$

CONSUMPTION – of low temperature heat by demand devices, and heating grid transfers.

Consumption of low temperature heat by demand devices. The shape of the sector heating demand if controlled by FR(Z)(Y), and if the demand device services more than one demand sector then the output share (OUT(DM)) is taken into consideration.

$$\left[\sum_{p\in dmd} \left(\begin{array}{l} R\_CAP_{r,t,p} * \\ CAPUNIT_{r,p} * MA(ENT)_{r,p,e,t} \\ CF_{r,p,t} / (DHDE(Z)_{r,p,z,t} * EFF_{r,p,t}) * \\ \sum_{d,y} \left(OUT(DM)_{r,p,d,t} * FR(Z)(Y)_{r,d,z,y} \right) \end{array} \right)\right] +$$

Transfer of low temperature heat to another conventional (non load managed) heat grid.

$$\left[\sum_{\substack{p \in hpl \\ p \notin xpr}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} / DHDE(Z)_{r,e,z,t} * \\ R\_THZ_{r,t,e,z} \end{pmatrix}\right] +$$

Transfer of low temperature heat to another non load managed heat grid. [Note that as heating plants are modeled by season (not time of day) CF(Z)(Y) should only be specified for y = 'D'.]

$$\left[\sum_{\substack{p \in hpl \\ p \in xlm}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ \left(QHRZ_{r,p,z} * CF(Z)(Y)_{r,p,z,'D',t}\right) * \\ R\_CAP_{r,t,p} \end{pmatrix}\right]$$

4.5.4 MR_BALE1/2(r,t,e,w)

Table A1, Row 9
SAGE_MATRIX_V52.XLS

Name: **BAL**ance equation for **E**lectricity energy carriers (elc) in each time-slice (w=z,y), where equation 1 corresponds to day and equation 2 to night in each season.

Purpose:

Primal: Like all balance equations, this one for electricity guarantees the conservation of energy throughout the energy system. The equation ensures that the production (plus imports) of electricity in each time-slice is equal to or exceeds the demand (including exports) in the same time-slice. Equation 1 is for daytime constraints (w=z,'D') and equation 2 is for the nighttime constraints (w=z,'N'). Slack is permitted (but very unlikely), and may occur due to some rigid coupled heat and power plant (REH) or to the existence of lower limits forcing the operation of a power plant. However, the analyst should carefully investigate any slack on the electricity balance.

Dual: The dual variable (DVR_BALE) represents the marginal value of one unit of electricity to the energy system in each time-slice. It thus represents the price that the model must "pay" per of electricity consumed, and receives per unit produced.

Occurrence: This equation is generated in all time periods and for all time-slices during which some resource option or technology produces or consumes the electricity.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding*
The equation is a greater than or equal (>=) constraint for each time-slice.

Interpretation of the results:

Primal: The level of the electricity balance equation represents the slack or overproduction of electricity in a time-slice. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of back-pressure turbines if heat demand forces the operation of such a plant it is possible that production may exceed the demand for electricity. Under normal circumstance the user should look to redress this situation by employing a flexible output coupled heat and power (CEH(Z)(Y)), or determine any other reason for the excess (e.g., base load constraint forces over-production at night, lower limits forcing the operation of a power plant).

Dual: The dual variable (DVR_BALE) of the electricity balance constraint (shadow price) indicates the amount that the energy system must pay per unit of electricity consumed, and receives per unit produced in each time-slice. A zero dual value implies that the supply of the commodity exceeds the demand for that commodity by the rest of the energy system. The value is positive when the primal equation is tight, which is normally the case.

Remarks: An important purpose of the electricity balance constraint is to bring into alignment the load shapes associated with the various demand sectors that consume electricity (DM_FR(Z)(Y)) with the fractions of the year that the power plants operate (G_YRFR(Z)(Y)), taking into account annual and/or seasonal availability of each power plant (AF/AF(Z)(Y)/CF/CF(Z)(Y)).

GAMS Routine: MMEQBALE.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#4}: MR\_BALEl_{r,t,e,w} \; \forall (e \in elc) \wedge QHR_{r,w} exists$$

where l = 1 for days (w(z,'D')) and l = 2 for nights (w(z,'N'))

PRODUCTION – of electricity from imports and power plants feeding the current electricity grid.

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='IMP'}} \left[TE(ENT)_{r,e,t} * \left\{ \begin{array}{l} \left(\begin{array}{l} \left\langle \begin{array}{l} QHR(Z)(Y)_{r,w} \$ (no\,SEP\_FR_{r,s,w,t}) \vee \\ (SEP\_FR_{r,s,w,t} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \end{array} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,w} \right) + \\ \left(R\_TSEPE_{r,t,s,w} \$ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,w} \right) \end{array} \right\} \right] +$$

Production to the current electricity grid by conventional (non-externally load managed) electric generating plants for each season/time-of-day the plant is available (AF(Z)(Y), which is set from AF if only annual availability is specified). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela \\ p \notin xlm \\ AF(Z)(Y)_{r,p,w,t}}} TE\_CON_{r,t,p,e} * \left\langle \begin{array}{l} \left(\begin{array}{l} *(QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) \$ p \subset bas * \\ *R\_TEZY_{r,t,p,z,y} \end{array} \right) \$ \left(\begin{array}{l} y='D' \wedge \\ (not\, z='I' \wedge PEAK\_TID_{r,p}) \end{array} \right) + \\ \left(\begin{array}{l} R\_TEZY_{r,t,p,z,y} \$ (p \notin (bas \vee stg \vee PEAK\_TID_{r,p})) + \\ (R\_TEZY_{r,t,p,z,'D'} * (QHR(Z)(Y)_{r,w} / QHRZ_{r,z})) \$ p \in (bas \vee stg) \end{array} \right) \$ y='N' \end{array} \right\rangle \right] +$$

Production to the current electricity grid by coupled heat and power pass-out turbines for each season/time-of-day the plant is available (AF(Z)(Y), which is set from AF if only annual availability is specified), and a electricity loss per unit of heat ratio (CEH(Z)(Y)) is provided. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Special conditions apply to base load facilities where only daytime variables (to force day=night generation) are permitted.]

$$\left[\sum_{\substack{p \in cpd \\ ELM_{r,p} \\ AF(Z)(Y)_{r,p,w,t}}} \left\langle \begin{array}{l} TE\_CON_{r,t,p,e} * \begin{pmatrix} CEH(Z)(Y)_{r,p,w,t} * \\ (1 - ELM_{r,p}) / ELM_{r,p} \end{pmatrix} * \\ \left[\begin{array}{l} \begin{pmatrix} ((QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) \$ p \in bas) * \\ R\_TCZYH_{r,t,p,z,y} \end{pmatrix} \$ (y = 'D' \vee (y = 'N' \wedge p \notin bas)) + \\ (R\_TCZYH_{r,t,p,z,'D'} * (QHR(Z)(Y)_{r,w} / QHRZ_{r,z})) \$ (y = 'N' \wedge p \in bas) \end{array} \right] \end{array} \right\rangle \right] +$$

Production of electricity from externally load managed plants and grid interchanges, where the activity of the plant is directly determined from the level of installed capacity by means of a fixed capacity factor (CF(Z)(Y)) for each time-slice.

$$\left[\sum_{p \in (ela \wedge xlm)} \begin{pmatrix} (QHRZ_{r,z} * CF(Z)(Y)_{r,p,w,t}) * \\ CAPUNIT_{r,p} * TE\_CON_{r,t,p,e} * \\ R\_CAP_{r,t,p} \end{pmatrix} \right]$$

$$\{\geq\}$$

CONSUMPTION – of electricity by resource options and technologies, and as part of grid interchanges.

Export of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)x_{r,s,e,t} \\ x='EXP'}} \left\{ \begin{array}{l} \left(\left\langle \begin{array}{l} QHR(Z)(Y)_{r,w} \$ (no SEP\_FR_{r,s,w,t}) \vee \\ SEP\_FR_{r,s,w,t} \end{array} \right\rangle * \$ \\ R\_TSEP_{r,t,s} \right) \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,w} \right) + \\ \left(R\_TSEPE_{r,t,s,w} \$ \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,w} \right) \end{array} \right\} \right] +$$

Consumption of electricity as part of supplying another resource. The annual resource production electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e',t} \\ e' \neq e}} \left(\begin{array}{l} QHR(Z)(Y)_{r,w} * INP(ENT)r_{r,s,e,t} * \\ R\_TSEP_{r,t,s} \end{array} \right) \right] +$$

Consumption of electricity by a conventional (non externally load managed) process. The annual process electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{p\in prc\\ p\notin xpr}}\left(\begin{array}{l}QHR(Z)(Y)_{r,w} * INP(ENT)p_{r,s,e,t} * \\ R\_ACT_{r,t,p}\end{array}\right)\right]+$$

Consumption of electricity from the current grid by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p\in xpr}\left(\begin{array}{l}INP(ENT)p_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} * R\_CAP_{r,t,p}\end{array}\right)\right]+$$

Transfer of electricity from the current grid to a conventional (non externally load managed) grid interchange technology, taking into consideration that base load plants only have daytime variables (to force equal day/night operation).

$$\left[\sum_{\substack{p\in \ln k\\ p\notin xlm}}\left(\begin{array}{l}\left(INP(ENT)c_{r,p,e,t} * R\_TEZY_{r,t,p,w}\right)\$ p\notin bas * \\ \left(\begin{array}{l}(QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) * \\ R\_TEZY_{r,t,p,z,'D'}\end{array}\right)\$ p\in bas\end{array}\right)\right]+$$

Transfer of electricity from the current grid to an externally load managed grid interchange technology for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p\in \ln k\\ p\in xlm}}\left(\begin{array}{l}INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF(Z)(Y)_{r,p,w,t} * R\_CAP_{r,t,p}\end{array}\right)\right]+$$

Consumption of electricity as required for a unit of low-temperature heat generation, split into season/time-of-day based upon the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ except\,((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(\left(\begin{array}{l} INP(ENT)c_{r,p,e,t} * \\ \left(QHR(Z)(Y)_{r,w} / QHRZ_{r,z} \right) * \end{array} \right) * \\ R\_THZ_{r,t,p,z} \right) \right] +$$

Consumption of electricity by pumped storage facilities at night, based upon the daytime production.

$$\left[\sum_{\substack{p \in stg \\ y='N'}} \left(\begin{array}{l} INP(ENT)c_{r,p,e,t} * TE(ENT)_{r,e,t} * \\ R\_TEZY_{r,t,p,z,'D'} \end{array} \right) \right] +$$

Consumption of electricity by demand devices according to the amount of electricity needed per unit output (a function of the market share of electricity to the device (MA(ENT)) and the device efficiency (EFF)) and the seasonal/time-of-day shape (DM_FR) of the demand load being serviced by the share of the output from the device to the sector (DM_OUTX).

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} / EFF_{r,p,t} * \\ \sum_{d} \left(\begin{array}{l} DM\_FR(Z)(Y)_{r,d,w} * \\ DM\_OUTX_{r,p,d,t} \end{array} \right) * \\ CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{array} \right) \right]$$

Where l = where l = 1 for days (w(z,'D')) and l = 2 for nights (w(z,'N')), and

x = the source (src) part of the resource name (srcencp) for Imports/exports ('IMP/EXP').

4.5.5 MR_BAS(r,t,e,z)

Table A1, Row 10

SAGE_MATRIX_V52.XLS

Name: The **BAS**e load constraint for electricity energy carriers (elc) in each season (z).

Purpose:

Primal: The base load constraint ensures that those power plants designated as base load (bas) operate at the same level in the day/night and do not exceed a specified percentage of the highest nighttime electricity demand (according to BASE LOAD). The main purpose of the constraint is to ensure that enough power plants that are characterized as base load are available in each period, but do not by themselves meet all the off-peak demand for electricity. Such class of power plants (e.g., hydro, large coal-fired, nuclear) cannot be easily started/shutdown/restarted, but instead tend to be run at full capacity almost all of the time. In SAGE this results in a single activity variable (R_TEZY) being generated (for the day (y='D')) and the appropriate day and night share multipliers applied to ensure continuous operation (other than for maintenance).

Dual: The dual variable (DVR_BAS) represents the pressure on the cost of the system to use other than base load power plants to meet the highest nighttime electricity demand.

Occurrence: This equation is generated in all time periods and for each season during which power plants or imports, produce electricity.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding*

The equation is a less than or equal (<=) constraint for each season.

Interpretation of the results:

Primal: The level of the base load equation represents the level below the maximum percent of highest nighttime demand that is met by base load power plants. If zero, than the maximum base load constraint is tight and demand is fully met only by base load plants.

Dual: The dual variable (DVR_BAS) of the base load constraint (shadow price) indicates the "cost" that the energy system must incur because it cannot install more base load power plants. A zero dual value implies that the amount of electricity from the base load plants is below the permitted percent of maximum nighttime demand. The value is negative when the primal equation is tight. It thus represents the amount by which the objective function would be reduced if one more unit of base load power generation were permitted in the season with the highest nighttime demand.

Remarks: The technique used of employing a daytime electricity generation variable (R_TEZY), and then apply the appropriate splits means that the analyst should avoid referring directly to the levels reported in the (GAMS) solution listing, but rather strictly rely on the level reported in the results tables, which take into account the need to apply the appropriate factors to the variables. Note that non-load management power plants (nlm) are included among the plants contributing to the base load constraint, but that these plants may operate normally, not being forced to operate at equal levels day and night.

GAMS Routine: MMEQBAS.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#5}: MR\_BAS_{r,t,e,z}\ \forall (e \in elc) \wedge QHRZ_{r,z} exists$$

Contribution to meeting the base load requirement, that is that the nighttime electric generation from base load plants does not exceed a specified percent of total nighttime demand for electricity in each season.

Import of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='IMP'}} \left\{ \begin{array}{l} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left(\left\langle \begin{array}{l} QHR(Z)(Y)_{r,z,'N'} \$ (noSEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'} \right) \end{array} \right\} \right] +$$

Nighttime production sent to the current electricity grid by conventional (non-externally load managed) base load electric generating plants. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Remember, base load plants only have daytime variables from which night production is calculated based upon the length of the time-slice (QHR(Z)N).]

$$\left[\sum_{p \in bas} \left(\begin{array}{l} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ \left(QHR(Z)(Y)_{r,z'N'} / QHRZ_{r,z} \right) * \\ R\_TEZY_{r,t,p,z,'D'} \end{array} \right) \right] +$$

Nighttime production sent to the current electricity grid by externally load managed base load electric generating plants, where activity is determined by the level of installed capacity. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p\in bas\\ p\in xlm}}\left(\begin{array}{l} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ CAPUNIT_{r,p} * CF(Z)(Y)_{r,z'N'} * \\ QHR(Z)(Y)_{r,z,'N'} * R\_CAP_{r,t,p} \end{array}\right)\right] +$$

Nighttime production to the current electricity grid by base load coupled heat and power pass-out turbines, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Remember, base load plants only have daytime variables from which night production is calculated based upon the length of the time-slice (QHR(Z)N). However only daytime CEH(Z)(Y) should be provided.]

$$\left[\sum_{\substack{p\in bas\cup cpd\\ ELM_{r,p}}}\left(\begin{array}{l} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ QHR(Z)(Y)_{r,z'N'} / QHRZ_{r,z} * \\ \left(CEH(Z)(Y)_{r,p,z'D',t} * (1 - ELM_{r,p}) / ELM_{r,p}\right) * \\ R\_TCZYH_{r,t,p,z,'D'} \end{array}\right)\right] +$$

Nighttime production sent to the current electricity grid by non-load managed electric generating plants that can contribute to the base load requirements. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{p\in nlm}\left(\begin{array}{l} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ R\_TEZY_{r,t,p,z,'N'} \end{array}\right)\right] +$$

Nighttime production to the current electricity grid by non-load managed coupled heat and power pass-out turbines that can contribute to the base load requirements, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p \in nlm \cup cpd \\ ELM_{r,p}}} \begin{pmatrix} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ \left(CEH(Z)(Y)_{r,p,z'N',t} * (1 - ELM_{r,p}) / ELM_{r,p}\right) * \\ R\_TCZYH_{r,t,p,z,'N'} \end{pmatrix}\right]$$

$$\{\leq\}$$

Electric generation by non-base load power plants at night.

Export of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction)

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='EXP'}} \begin{Bmatrix} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left(\left\langle \begin{matrix} QHR(Z)(Y)_{r,z,'N'} \$ (no\ SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) \end{Bmatrix}\right] +$$

Nighttime production sent to the current electricity grid by conventional (non-externally load managed) non-base load electric generating plants. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p\in ela\\ p\notin bas}}\begin{pmatrix}TE\_CON_{r,t,p,e}*BASELOAD_{r,e,t}*\\ \left(QHR(Z)(Y)_{r,z'N'}/QHRZ_{r,z}\right)*\\ R\_TEZY_{r,t,p,z,'N'}\end{pmatrix}\right]+$$

Nighttime production sent to the current electricity grid by externally load managed non-base load electric generating plants, where activity is determined by the level of installed capacity. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p\in ela\\ p\notin bas\\ p\in xlm}}\begin{pmatrix}TE\_CON_{r,t,p,e}*BASELOAD_{r,e,t}*\\ CAPUNIT_{r,p}*CF(Z)(Y)_{r,z'N'}*\\ QHR(Z)(Y)_{r,z,'N'}*R\_CAP_{r,t,p}\end{pmatrix}\right]+$$

Nighttime production to the current electricity grid by non-base load coupled heat and power pass-out turbines at night in each season, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p\in cpd\\ p\notin bas\\ ELM_{r,p}}}\begin{pmatrix}TE\_CON_{r,t,p,e}*BASELOAD_{r,e,t}*\\ QHR(Z)(Y)_{r,z'N'}/QHRZ_{r,z}*\\ \left(CEH(Z)(Y)_{r,p,z'N',t}*(1-ELM_{r,p})/ELM_{r,p}\right)*\\ R\_TCZYH_{r,t,p,z,'N'}\end{pmatrix}\right]$$

Where x = the source (src) part of the resource name (srcencp) for Imports/exports ('IMP/EXP').

4.5.6 MR_BITRD(r,e,r,t,e,c)

Table A1, Row 11

SAGE_MATRIX_V52.XLS

Description: The **BI**-lateral **TR**a**D**e constraint matches up the trade in a commodity between two regions. Note that the name of the commodity may be different between the two regions, if desired, but the same cost step (trade route) is to be used.

Purpose:

Primal: The bi-lateral trade constraint ensures that the amount of a commodity delivered from a source region equals the amount received by the destination region. It includes provisions to convert the units (REG_XCVT) if necessary. Note that the individual export and import supply options must also be specified in each region as part of the list of resource options (srcencp).

Dual: The dual variable (DVR_BITRD) represents the marginal cost associated with moving the commodity from one region to another.

Occurrence: This equation is generated in each time period from which both resource options first become available in their respective regions, and the mapping set indicating the pairing of the importing/exporting of the commodity between the regions is provided (bi_trdent).

Units: PJ, or any other unit in which a commodity is tracked.

Type: *Binding*
The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the bi-lateral trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual: The dual variable (DVR_BITRD) of the bi-lateral trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of trade. A zero dual value implies that there is no cost associated with producing and delivering the commodity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: This bi-lateral trade constraint is an annual constraint. Electricity may be traded by season/time-of-day if desired by means of the bi-lateral electricity trade constraint (MR_BITRDE). Note that to force trade between regions one of the resource variables in the region (R_TSEP) can be bounded.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 6}: MR\_BITRD_{r,e,t,r',e',c} \ \forall \left(BI\_TRDENT_{r,e,t,r',e',c} \wedge \left(TPSEP_{r,t,'EXP',e,c} \wedge TPSEP_{r',t,'IMP',e',c} \right) \right)$$

Exports from region r, possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEP_{r,t,"EXP",e,c} \right\rfloor -$$

Imports to region r’, possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r',e'} * R\_TSEP_{r',t,"IMP",e',c} \right\rfloor$$

$$\{=\}$$

$$0$$

Where bi_trdent = mapping of permitted bi-lateral trade options
tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

4.5.7 MR_BITRDE(r,e,r,t,e,c,w)

Table A1, Row 12

<u>SAGE_MATRIX_V52.XLS</u>

Description: The **BI**-lateral **TR**a**D**e of **E**lectricity constraint matches up the trade in a commodity between two regions in each time-slice. Note that the name of the commodity may be different between the two regions, if desired, but the same cost step (trade route) is to be used.

Purpose:

Primal: The bi-lateral trade of electricity constraint ensures that the amount of a electricity delivered from a source region in each time-slice equals the amount received into the destination region. It includes provisions to convert the units (REG_XCVT) if necessary. Note that the individual export and import supply options must also be specified in each region as part of the list of resource options (srcencp).

Dual: The dual variable (DVR_BITRDE) represents the marginal cost associated with moving the electricity from one region to another.

Occurrence: This equation is generated in each time period from which both exchange options first become available in their respective regions, and the mapping set (bi_trdelc) indicates that the pairing of the importing/exporting of electricity between the regions is permitted in the time-slice.

Units: PJ, or any other unit in which electricity is tracked.

Type: *Binding*

The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the bi-lateral electricity trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual: The dual variable (DVR_BITRDE) of the bi-lateral electricity trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of that trade. A zero dual value implies that there is no cost associated with producing and delivering the electricity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: This bi-lateral electricity trade constraint is a time-sliced constraint. Electricity may also be traded annually if desired by means of the conventional commodity trade constraint (MR_BITRD). Note that there is currently no way to bound the amount of electricity exchanged between regions for a particular time-slice, unless distinct time-slice based resource options (srcencp with SEP_FR for only 1 season/time-of-day for each possible exchange) are specified in each region. The commodity conversion factor (REG_XCVT) needs to be used to make any necessary adjustments in the time-slice fractions if different between two regions. Note that this constraint works in tandem with the MR_REGELC constraint, which hooks the time-slice based bi-lateral electric trade variables (R_TSEPE) with the traditional annual resource variable (R_TSEP).

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#7}: MR\_BITRDE_{r,e,t,r',e',c,w} \; \forall \left(\begin{array}{l} e \in elc \wedge \\ BI\_TRDELC_{r,e,t,r',e',c,w} \wedge \\ \left(\begin{array}{l} TPSEP_{r,t,"EXP",e,c} \wedge \\ TPSEP_{r',t,"IMP",e',c} \end{array} \right) \end{array} \right)$$

Exports from region r, possibly with unit conversion applied for the electricity. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEPE_{r,t,'EXP',e,c,w} \right\rfloor -$$

Imports to region r', possibly with unit conversion applied for the commodity, and subject to any transmission losses. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r',e'} * R\_TSEPE_{r',t,'IMP',e',c,w} \right\rfloor$$

$$\{=\}$$

$$0$$

Where bi_trdelc = mapping of permitted bi-lateral electric trade options, including permitted time-slices

tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

4.5.8 MR_BNDCON1/2/3(r,t,p)

Table A1, Row 13

SAGE_MATRIX_V52.XLS

Name: Annual **B**ou**ND** on the output from a **CON**version technology.

Purpose:

Primal: The annual bound constraint imposes a limit on the total output from conversion technologies for all relevant time-slices. In the case of electric generation and coupled heat and power plants the (up to 6) season/time-of-day time-slices are summed, for heating plants the winter season.

Dual: The dual variable (DVR_BNDCON) represents the cost pressure exerted on the system to move the bound by one unit.

Occurrence: The equation is generated for each period for which the user specifies an output limit (BOUND(BD)O) on a conversion technology.

Units: PJ, or any other unit in which activity of power plants is tracked.

Type: *Binding*

The type of the equation is a function of the sense (b = 'LO','FIX','UP') specified by the user when providing the bound parameter.

Interpretation of the results:

Primal: The level indicates any slack on a constraint imposing a production limit above/below a minimum/maximum permitted level.

Dual: The dual variable (DVR_BNDCON) of the annual conversion bound constraint represents the amount by which the objective function would be changed if the bound were increased by one unit. A zero dual value implies that the annual output from a power plant is below the permitted maximum or above the required minimum level. The value is negative when the limit is a maximum, positive when a lower limit is imposed, and the constraint..

Remarks: The only way that the user can limit the level of production in an individual time-slice (or collection less than the entire year, e.g., a season) is to apply and user-defined constraint using the appropriate time-sliced based parameters (RAT_TCZY, RAT_TEZY, RAT_HPL). Note also that if the technology is externally load managed (XLM), which results in only a capacity variable (R_CAP) being generated, then the annual activity bound is applied directly to R_CAP properly taking into consideration the capacity factor(s) (CF/CF(Z)(Y)), unit (CAPUNIT) and time-slices (QHR(Z)(Y)).

GAMS Routine: MMEQBCON.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#8}: MR\_BCONl_{r,t,p} \ \forall \left(\begin{array}{l} p \in con \wedge p \notin xlm \wedge \\ \sum_{b} BOUND(BD)O_{r,p,b,t} \end{array} \right)$$

where l the nature of the constraint such that l = 1 for b=‘LO’ (≥), 2 for b=‘FX’ (=), and 3 for b=‘UP’ (≤).

Total annual electricity generation from standard power plants, summed over the season/time-of-day activity of power plants when the plant is available (AF(Z)(Y)). [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela, w \\ except\ y='N' \wedge p \in (bas \cup stg) \\ AF(Z)(Y)_{r,p,w,t} \\ except\ ((y='N' \vee z='I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_TEZY_{r,t,p,w} \right] +$$

Total annual production of low-temperature heat generation from coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of power plants. As total production is bounded the total electricity output and this hybrid heat term are summed. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd, w \\ ELM_{r,p} \\ except(y='N' \wedge p \in bas)}} \begin{pmatrix} CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} * \\ R\_TCZYH_{r,t,p,w} \end{pmatrix} \right] +$$

Total annual production of low temperature heat from heating plants. [Special conditions apply to peaking plants, where only daytime winter variables are generated.]

$$\left[\sum_{\substack{p \in hpl, z \\ except((z='I' \vee 'S') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_THZ_{r,t,p,z} \right]$$

$$\{\leq; =; \geq\}$$

Limit on total annual production from the conversion plant.

$$BOUND(BD)O_{r,p,b,t}$$

4.5.9 MR_CPT1/2/3(r,t,p)

Table A1, Row 14

SAGE_MATRIX_V52.XLS

Description: **C**a**P**acity **T**ransfer of a technology across time periods, accounting for all existing residual capacity and new investments made up to the present time period that remain available.

Purpose:

Primal: The capacity transfer relation ensures that the existing capacity of each technology in each time period is the result of investments in the current and previous time periods within its lifetime, plus the residual capacity from investments done before the initial model year and still active. Its main purpose is to properly monitor the turnover of the capital stock. *This is an inter-temporal equation and does not contribute to the Reference Energy System of the model.*

Dual: The dual variable (DVA_CPT) represents the marginal value to the energy system in each period of an additional unit of capacity of technology ready for production.

Occurrence: This equation is generated in all time periods beginning from the START period for each technology for which the LIFE or residual capacity (RESID) has been specified. If LIFE is not specified, or is less-than-or-equal to 1, and no RESID exists, then the MR_CPT equation is not generated, as there is no carryover of capital stock between periods.

Units: Gw, or PJ/a, or Bvkm/a, or any other unit in which the capacity of a technology is specified (e.g., parameters INVCOST, RESID, FIXOM, etc).

Type: *Binding*

The equation is a less than or equal (≤) if the technology is a demand device (dmd), see Interpretation of Results below.

The relation is an equality (=) if the technology is a conversion plant (con) or a process (prc).

Interpretation of the results:

Primal: The level of capacity transfer constraint in the solution output (primal or slack value) is the value of the left hand side (LHS) of the CPT equation, which is the difference between the capacity in place in time period t minus the new capacities built / purchased in previous time periods still in operation. In processes and power plants the level always has to be equal to the residual capacity (RESID), which is the right hand side of the constraint (RHS); demand devices may have a LHS that may be lower than the RHS. This may happen when the RESID is larger than required by demands, or when the demand fluctuates up and down between successive time periods, etc.

Dual: The dual variable (DVR_CPT) of the capacity transfer constraint in the solution (shadow price) indicates the annuity to be paid in the time period t to pay back the investment cost incurred to build a new unit of capacity of the technology p in time period t. In the case of end-use technologies, the dual value is zero when there is excess capacity at some period, and positive if some new investment is made by the model in the current time period. For process and conversion technologies, the value may be positive or negative.

Remarks: The user should omit the lifetime parameter of a technology (and thereby avoid generating extra relations) in cases where (a) there are no competing technologies, e.g., a demand category such as air transport which might have only one technology (jet) to satisfy the demand; or where (b) the technology is not a real device but acts only to transfer energy carriers, e.g., a demand technology which supplies oil to the petrochemicals industry or a process technology which mixes fuels (such technologies are usually referred to as *dummy technologies*)

GAMS Routine: MMEQCPT.INC, FRACLIFE.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#}9: MR\_CPTl_{r,t,p}\ \forall\ LIFE_{r,p}\ exists$$

where l the nature of the constraint such that l = 1 for demand devices (dmd) where the constraint is (≥), 2 for conversion plants and 3 for processes where the constraint is (=).

Total installed capacity in place in the current period.

$$\lfloor R\_CAP_{r,t,p} \rfloor -$$

Sum of all investments made in earlier periods that have not yet reached their technical lifetimes (LIFE), and thus are still available. [Note that an earlier investment may only be available for part of the period if the lifetime is not a multiple of the number of years per period (NYRSPER). This is handled by CPT_INV that is discussed further below.]

$$\left[\sum_{u=t-LIFE}^{t}\left(CPT\_INV_{r,u,p} * R\_INV_{r,u,p}\right)\right]$$

$$\{\geq;=\}$$

The residual capacity installed prior to the start of the modeling horizon that is still available in the current period.

$$RESID_{r,p,t}$$

Where	l	1 for p = dmd, 2 for p = con, 3 for prc; where the sense of the equations is ≥, = , = respectively;
	t,u	are time periods, where t is the current period and u is the period in which the investment was originally made;
	CPT_INV	= 1 for u = t-LIFE+1, FRACLIFE for the last period (that means investments made t-LIFE periods earlier, where it may be a fraction of the period if LIFE is not a multiple of the period length (NYRSPER)), and
	FRACLIFE	= 1 except for the last period where it is (# of years the technology exists in the period/NYRSPER).

4.5.10 MR_CUM(r,s)

Table A1, Row 15

SAGE_MATRIX_V52.XLS

Name: The **CUM**ulative or total limit on the production of a commodity from a resource activity over the entire modeling horizon (CUM).

Purpose:
Primal: The cumulative resource constraint limits the total production from a coalmine or oil/gas well, or a contracted maximum for imports/exports. If is most often used to indicate total proven reserves.
Dual: The dual variable (DVR_CUM) represents the pressure exerted on the system to allow more of the resource option.

Occurrence: The equation is generated for each resource option (srcencp) for which the user specifies a cumulative limit (CUM).

Units: PJ, or any other unit in which a commodities are tracked.

Type: *Binding*
The type of the equation is a less than or equal to (≤).

Interpretation of the results:
Primal: The level indicates the cumulative production from a resource supply option.
Dual: The dual variable (DVR_CUM) of the cumulative resource constraint (shadow price) indicates how desirable is an additional unit of the resource option. A zero dual value implies that the limit has not been reached at period t (yet). The value is negative when

the imposed limit is reached. This thus represents the amount by which the cost objective function would be reduced if one more unit of production were permitted.

Remarks: Owing to the fact that SAGE in a myopic, time-stepped model the benefits arising from the introduction of such constraints are limited, as the model has no look-ahead. In a clairvoyant model like MARKAL the model can choose to “hold” scarce resources for later use if it sees a benefit (lower overall cost) in doing so. But in SAGE the model may simply run up against the limit in an out year, owing to its inability to take into consideration the potential loss of the resource option. Note that since the variables in the model represent annual production, the resource variable must be multiplied by the number of years in each period when monitoring the cumulative amount.

GAMS Routine: MMEQCUM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#10}: \boldsymbol{MR\_CUM_{r,s}} \ \forall \boldsymbol{CUM_{r,s}} \ \boldsymbol{exists,}$$

$$\boldsymbol{except\, src = 'STK'}$$

Each annual resource supply option is multiplied by the number of years per period to determine the total cumulative amount supplied over the modeling horizon.

$$\sum_{t} NYRSPER * R\_TSEP_{r,s,t}$$

$$\{\leq\}$$

Cumulative limit imposed by the modeler.

$$CUM_{r,s}$$

Where NYRSPER is the number of years in each period.

4.5.11 MR_DEM(r,t,d)

Table A1, Row 16

<u>SAGE_MATRIX_V52.XLS</u>

Name: The **DEM**and constraint that ensures that all sector demands for useful energy services (DEMAND) are satisfied.

Purpose:

Primal: The demand constraint ensures that for each energy service, the total output of energy service from demand devices to the sector they serve exceeds the level of the demand for that energy service. In addition, SAGE employs price elastic demands, which may rise/fall (relative to the base case level) in response to changes in the implicit price of the service. See section 3.3 for a detailed discussion on elastic demands.

Dual: The dual variable (DVR_DEM) represents the marginal cost of meeting the final level of the demand for energy services in each sector, i.e. the price of each energy service.

Occurrence: The equation is generated for each demand sector (DM) in each period for which a demand for energy services (DEMAND) has been provided by the user. Note that this is true even for elastic runs where demands are determined endogenously by the model. Also, the elastic demand variables (R_ELAST) are generated only for the demand sectors for which the appropriate input data is provided, in particular the number of step-wise approximation blocks (MED_STEP) to be used for estimating the demand curve, and the elasticities (MED_ELAST).

Units: PJ, bvmt, or any other unit in which a demand for energy services is specified.

Type: *Binding*
The type of the equation is equal to or greater than (≥).

Interpretation of the results:

Primal: The level indicates the final level of useful energy services delivered to a demand sector. The final level is both a function of the output of the individual demand devices servicing the sector, as well as the movement up or down of the demand level from the reference case level in response to the own price elasticities (MED_ELAST) provided by the analyst.

Dual: The dual variable (DVR_DEM) of the demand constraint represents to cost of meeting the last unit of demand services for the sector, i.e. the market price of the energy service. A zero dual value implies that the output of the various demand devices exceeds the final level, and such a situation should be examined and resolved by the analyst. Normally the value is negative and thus represents the amount by which the objective function would be reduced if one less unit of demand were required.

Remarks: The analyst controls the activation of the elastic demand formulation by means of the model variant selected at run time (Elastic Demand YES’), and the inclusion of the required elastic data.

GAMS Routine: MMEQDEM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#11:} MR\_DEM_{r,t,d} \; \forall DEMAND_{r,d,t}$$

Useful energy demand provided to the current demand sector according to any fraction output shares (OUT(DM)). Remember, there are no activity variables for demand devices, rather activity is directly derived from the total installed capacity based upon the capacity factor.

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} OUT(DM)_{r,p,d,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} * R\_CAP_{r,t,p} \end{array} \right) \right] -$$

The growth in demand owing to price relaxation, where each step of the approximation of the producer/consumer curve is "climbed" in sequence (j). The own price elasticity (change in cost per unit change in demand) and variance associated with each step of the curve is reflected in the objective function.

$$\left[\sum_{j\$ \left(\begin{array}{l} (MED\_STEP_{r,d,'UP',t} \\ \wedge MED\_ELAST_{r,d,'UP',t}) \end{array} \right)} R\_ELAST_{r,t,'UP',d} \right] +$$

The reduction in demand owing to price pressure, where each step of the approximation of the producer/consumer curve is "descended" in sequence (j). The own price elasticity (change in cost per unit change in demand) and variance associated with each step of the curve is reflected in the objective function.

$$\left[\sum_{j\$\left((MED\_STEP_{r,d,'LO',t} \wedge MED\_ELAST_{r,d,'LO',t})\right)} R\_ELAST_{r,t,'LO',d} \right]$$

$$\{\geq\}$$

Reference demand for energy services.

$$DEMAND_{r,d,t}$$

4.5.12 MR_ENV(r,v)

Table A1, Row 17

SAGE_MATRIX_V52.XLS

Name: The total cumulative emissions or other **ENV**ironmental indicators (env) over the entire modeling horizon. If desired the level may be capped by means of the ENV_CUM input parameter.

Purpose:

Primal: The cumulative emissions equation monitors the total production of an environmental indicator from all resource options and technologies.

Dual: The dual variable (DVR_ENV) represents the cost pressure exerted on the system to allow more of the environmental indicator option.

Occurrence: The equation is generated for each emission or other environmental indicator option. The equation is generated regardless of whether or not the user (srcencp) specifies a cumulative limit (CUM).

Units: Tons, kton or any other unit in which emission indicators are tracked.

Type: *Binding*
The type of the equation is a less than or equal to (≤), but with an unlimited (infinity) right-hand-side value if no limit is provided by the user.

Interpretation of the results:

Primal: The level indicates cumulative production from an emissions indicator over the entire modeling horizon.

Dual: The dual variable (DVR_ENV) of the cumulative emissions equation (shadow price) indicates how much the energy system would want to pay in order to relax the emission limit by one unit. A zero dual value implies that the limit has not been reached (yet). In particular, this is the case if no limit has been specified by the user. The value is negative when the imposed limit is reached. This thus represents the amount by which the objective function would be reduced if one more unit of emission were permitted.

Remarks: Since SAGE is generally run in time-stepped mode the benefits arising from the introduction of such constraints are limited, as the model has no look-ahead. In a clairvoyant model like MARKAL the model can choose to take some early action to limit to total emissions if it sees a benefit (lower overall cost) in doing so. But in SAGE the model may simply run up against the limit in an out year, owing to its inability to take into consideration the potential restriction on total emissions. Note that since the variables in the model represent annual production, the resource variable must be multiplied by the number of years in each period when monitoring the cumulative amount.

GAMS Routine: MMEQENV.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#12}: MR\_ENV_{r,v}$$

Total period emissions determined by multiplying the annual emissions by the number of years per period (NYRSPER) are summed over the entire modeling horizon.

$$\left[\sum_{t} NYRSPER * R\_EM_{r,t,v} \right]$$

$$\{\leq\}$$

Total period emissions determined by multiplying the annual emissions by the number of years per period (NYRSPER) are summed over the entire modeling horizon.

$$CUM_{r,v}$$

Where	NYRSPER	is the number of years in each period,
	CUM	is either +INF if not provided, or the cumulative limit provided by the user.

4.5.13 MR_EPK(r,t,e,z)

Table A1, Row 18

<u>SAGE_MATRIX_V52.XLS</u>

Name: The **E**lectricity **P**ea**K**ing constraint ensures that there is enough capacity in place to meet the electricity demand during the day of the season with the highest demand.

Purpose:

Primal: The electricity peaking constraint ensures that there is enough capacity in place to meet the electricity demand during the day of the season with the highest demand taking into consideration an estimate of the level above that demand that corresponds to the actual peak moment plus a reserve margin of excess capacity (ERESERV includes both), to ensure that if some capacity is unavailable the highest demand for electricity (including unforeseen capacity reductions) can still be met.

Dual: The dual variable (DVR_EPK) represents the marginal value associated with the capacity needed to ensure that enough power plants are in place to meet the peak demand for electricity in each season (z). It thus represents the additional price that the model must "pay" for each unit of electricity guaranteed to be delivered in the season with peak demand.

Occurrence: This equation is generated for all time periods and for the "W'inter and 'S'ummer seasons (z), during which some resource option or technology produces or consumes the electricity.

Units: PJ, or any other unit in which electricity is tracked. Note that although the constraint is thought of in terms of capacity, it is actually modeled in energy units, based upon consumption and production arising from the associated capacity.

Type: *Binding*
The equation is a greater than or equal (>=) constraint for each season.

Interpretation of the results:

Primal: The level of the electricity peaking equation represents the slack or excess capacity over and above the amount required based upon the season in which the highest demand occurs, plus the electricity reserve (ERESERV) margin provided by the analyst. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of capacity above the minimum level required. If there is slack the user should check the residual capacity levels (RESID) to ensure that they properly reflect the level in place prior to the modeling period, and review the value of the ERESERV parameter (remembering that is has a dual role of both estimating how much above the level of highest <u>average</u> electric demand the peak moment is, plus the traditional requirement for extra 'just in case' capacity.

Dual: A zero dual value implies that the available capacity exceeds the requirements imposed by the constraint. The value is negative when the primal equation is tight, which is normally the case, and indicates how much the objection function would be lowered if the reserve constraint was one unit lower.

Remarks: As already noted, the electricity peaking constraint is highly dependent upon the reserve margin (ERESERV), which is usual substantially above that of a tradition utility reserve margin.

GAMS Routine: MMEQEPK.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#13}: MR\_EPK_{r,t,e,z}\ \forall \begin{pmatrix} e \in elc \wedge \\ z \neq 'I' \wedge \\ QHRZ_{r,z} exists \end{pmatrix}$$

PEAK Contribution –imports and peak technology capacity credit towards the peaking requirement, as well as grid interchanges.

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice. The contribution is subject to transmission losses as well as the reserve margin requirements.

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e,t}\\ x='IMP'}} \begin{Bmatrix} PEAKDA(SEP)_{r,s,e,t} * TE(ENT)_{r,e,t} / (1+ERERSEV_{r,e,t}) * \\ \left(\begin{matrix} \left\langle \begin{matrix} 1\$(noSEP\_FR_{r,s,z,'D',t}) \vee \\ (SEP\_FR_{r,s,z,'D',t} / QHR(Z)(Y)_{r,z,'D'} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \end{matrix} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'D'} \right) + \\ \left(\begin{matrix} \left\langle R\_TSEPE_{r,t,s,z,'D'} / QHR(Z)(Y)_{r,z,'D'} \right\rangle \$ \\ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'D'} \end{matrix} \right) \end{Bmatrix}\right] +$$

Deliverable capacity from non-peaking (PEAK_TID) power plants, and grid exchange link technologies, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements. The contribution is subject to transmission losses as well.

$$\left[\sum_{\substack{p\in ela\\ p\notin PEAK\_TID_{r,p}}} \left(\begin{matrix} TE(ENT)_{r,e,t} * PEAK(CON)_{r,e,t} / \\ (1+ERERSEV_{r,e,t}) * \\ CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{matrix} \right)\right] +$$

Contribution from peaking only power plants, where the amount to credit to the peak in a season is a function of the activity of the plant with the explicitly specified peak duration factor (PD(Z)D), as well as the capacity credit factor (PEAK). The contribution is subject to transmission losses as well as the reserve margin requirements.

$$\left[\sum_{\substack{p \in ela \\ p \in PEAK\_TID_{r,p}}} \left(\begin{array}{l} TE(ENT)_{r,e,t} * PEAK(CON)_{r,p,t} / \\ \left(\begin{array}{l} QHR(Z)(Y)_{r,z,'D'} * \\ (1 + ERERSEV_{r,e,t}) * PD(Z)D_{r,p,t} \end{array} \right) * \\ R\_TEZY_{r,t,p,z,'D'} \end{array} \right) \right]$$

$$\{\geq\}$$

CONSUMPTION – of electricity by resource options and technologies, and as part of grid interchanges.

Export of electricity occurring during peak times (PEAKDA(SEP)), either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no

$$\left[\sum_{\substack{s\forall OUT(ENT)x_{r,s,e,t}\\ x='EXP'}} \begin{array}{c} PEAKDA(SEP)_{r,s,t}* \\ \left\{\begin{array}{l} \left(\left\langle \begin{array}{l} 1\$(no\, SEP\_FR_{r,s,z,'D',t})\vee \\ SEP\_FR_{r,s,z,'D',t}/QHR(Z)(Y)_{r,z,'D'} \end{array}\right\rangle * \right. \\ \left. R\_TSEP_{r,t,s} \right)\$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'D'}\right)+ \\ \left(\left\langle R\_TSEPE_{r,t,s,z,'D'}/QHR(Z)(Y)_{r,z,'D'}\right\rangle\$ \right. \\ \left. \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'D'} \right) \end{array}\right\} \end{array}\right]+$$

Consumption of electricity during peak time as part of supplying another resource. The annual resource production electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e',t}\\ e'\neq e}} \begin{pmatrix} PEAKDA(SEP)_{r,s,t}*INP(ENT)r_{r,s,e,t}* \\ R\_TSEP_{r,t,s} \end{pmatrix}\right]+$$

Consumption of electricity by a conventional (non externally load managed) process. The annual process electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{p\in prc\\ p\notin xpr}} \begin{pmatrix} QHR(Z)(Y)_{r,w}*INP(ENT)p_{r,s,e,t}* \\ R\_ACT_{r,t,p} \end{pmatrix}\right]+$$

Consumption of electricity from the current grid by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p \in xpr} \begin{pmatrix} INP(ENT)p_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} * R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Transfer of electricity from the current grid to a conventional (non externally load managed) grid interchange technology, taking into consideration that base load plants only have daytime variables (to force equal day/night operation).

$$\left[\sum_{\substack{p \in \ln k \\ p \notin xlm}} \left(\begin{pmatrix} INP(ENT)c_{r,p,e,t} * \\ R\_TEZY_{r,t,p,e,w} \end{pmatrix} \$ p \notin bas * \\ \begin{pmatrix} (QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) * \\ R\_TEZY_{r,t,p,z,'D'} \end{pmatrix} \$ p \in bas \right)\right] +$$

Transfer of electricity from the current grid to an externally load managed grid interchange technologies for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p \in \ln k \\ p \in xlm}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p,e,t} * \\ CF(Z)(Y)_{r,p,w,t} * R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Consumption of electricity as required for a unit of low-temperature heat generation, split into season/time-of-day based upon the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ except\ ((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * \\ \left(QHR(Z)(Y)_{r,w} / QHRZ_{r,z}\right) * \\ R\_THZ_{r,t,p,z} \end{pmatrix} \right] +$$

Consumption of electricity by pumped storage facilities at night, based upon the daytime production.

$$\left[\sum_{\substack{p \in stg \\ y='N'}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * TE(ENT)_{r,e,t} * \\ R\_TEZY_{r,t,p,z,'D'} \end{pmatrix} \right] +$$

Consumption of electricity by demand devices, shaped according to the season/time-of-day profile of the demand via DM_FR(Z)(Y).

$$\left[\sum_{p \in dmd} \begin{pmatrix} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \sum_{d} (DM\_FR(Z)(Y)_{r,d,w} * DM\_OUTX_{r,p,d,t}) / \\ \left(QHR(Z)(Y)_{r,p,z,'D'} * EFF_{r,p,t}\right) * R\_CAP_{r,t,p} \end{pmatrix} \right]$$

Where x = the source (src) part of the resource name (srcencp) for

Imports/exports (‘IMP/EXP’).

4.5.14 MR_GRSEP(r,t,e)

Table A1, Row 19

SAGE_MATRIX_V52.XLS

Name: The inter-temporal **GR**owth constraint on resource supply.(**S**rc**E**nc**P**) options.

Purpose:

Primal: To impose a limit on the rate at which a resource supply option can expand (GROWTHr) between periods. The growth constraint needs a non-zero starting point (GROWTH_TIDr) from which to expand if the option has not yet been used.

Dual: The dual variable (DVR_GRSEP) represents the marginal value associated with the limit imposed by the growth rate. It thus represents to the desirability of permitting the resource to expand at a higher rate than that imposed by the growth constraint.

Occurrence: This equation is generated for all time periods for which the growth rate parameter (GROWTHr) is provided.

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding*
The equation is a less than or equal (<=) constraint.

Interpretation of the results:

Primal: The level of the resource growth equation indicates how much growth in a resource supply option has occurred between two periods.

Dual: The dual variable (DVR_GRSEP) of the resource growth constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of growth in the resource supply option. A zero dual value implies that the supply of the resource has not reached the limit imposed on the rate of growth between two periods. The value is negative when the primal equation is tight, and the system desires more of the resource in the current period.

Remarks: The initial potential level of production from the resource options (GROWTH_TIDr) needs to be provided as a maximum starting level or “seed” in case the resource option has not yet been tapped.

GAMS Routine: MMEQGRS.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 14}: MR\_GRSEP_{r,t,s} \ \forall GROWTHr_{r,s,t}$$

Supply of a resource in the previous period, with annual growth rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\begin{bmatrix} -(GROWTHr_{r,s,t}**NYRSPER)* \\ R\_TSEP_{r,t-1,s} \end{bmatrix} +$$

Supply of the resource in the current period based.

$$\begin{bmatrix} R\_TSEP_{r,t,s} \end{bmatrix}$$

$$\{\leq\}$$

Initial permitted level for a resource to "seed" the growth constraint. Needed if the resource option was not used in the previous period.

$$GROWTH\_TIDr_{r,s}$$

4.5.15 MR_GRTCH(r,t,p)

Table A1, Row 20

<u>SAGE_MATRIX_V52.XLS</u>

Name: The inter-temporal **GR**owth constraint on the expansion of total installed capacity of a **T**e**CH**nology between periods.

Purpose:

Primal: To impose a limit on the rate at which total installed capacity can expand (GROWTH) between periods. The growth constraint needs a non-zero starting point (GROWTH_TID) from which to expand if the technology has not yet been deployed.

Dual: The dual variable (DVR_GRTCH) represents the marginal value associated with the limit imposed by the growth rate. It thus represents to the desirability of permitting that technology to expand at a higher rate than that imposed by the growth constraint.

Occurrence: This equation is generated for all time periods for which the growth rate parameter (GROWTH) is provided.

Units: GW, PJa, or any other unit in which capacity is tracked.

Type: *Binding*
The equation is a less than or equal (<=) constraint.

Interpretation of the results:

Primal: The level of the technology growth equation indicates how much growth in a technology has occurred between two periods.

Dual: The dual variable (DVR_GRTCH) of the technology growth constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of growth in the total installed capacity of a technology. A zero dual value implies that the increase in capacity between two periods has not reached the level imposed by the growth limit. The value is negative when the primal equation is tight, and the system desires more of the technology in the current period.

Remarks: The initial capacity level of the technology (GROWTH_TID) needs to be provided as a maximum starting level or "seed" in case the technology has not yet been deployed.

GAMS Routine: MMEQGRT.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#15}: MR\_GRTCH_{r,t,p}\ \forall GROWTH_{r,p,t}$$

,

Total installed capacity in the previous period, with annual growth rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\left[\begin{array}{l} -(GROWTH_{r,p,t} ** NYRSPER)* \\ R\_CAP_{r,t-1,p} \end{array}\right] +$$

Total installed capacity in the current period based.

$$\left[R\_CAP_{r,t,p} \right]$$

$$\{\leq\}$$

Initial permitted level for a technology to "seed" the growth constraint. Needed if the technology was not installed in the previous period.

$$GROWTH\_TID_{r,p}$$

4.5.16 MR_GTRD(r,g)

Table A1, Row 21

<u>SAGE_MATRIX_V52.XLS</u>

Name: The **G**lobal **TR**a**D**e of a commodity (g = enc, mat or env) among all regions in the model.

Purpose:

Primal: To allow for trading in a globally available commodity between all regions in the model. The equation ensures the balance between all producers ('EXP') and consumers ('IMP') of each globally traded commodity.

Dual: The dual variable (DVR_GTRD) represents the cost associated with the traded commodity.

Occurrence: This equation is generated in each time period beginning from the initial trading period (TRD_FROM) and for which some region produces or consumes the commodity.

Units: PJ (energy), Mton (materials or emissions), M$ (permits) or any other unit in which a commodity is tracked.

Type: *Binding*
The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the bi-lateral electricity trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual: The dual variable (DVR_BITRDE) of the bi-lateral electricity trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of that trade. A zero dual value implies that there is no cost associated with producing and delivering the electricity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: Flows into individual regions may be limited by the TRD_BND parameter.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#16:}\, MR\_GTRD_{t,g}\ \forall t \geq TRD\_FROM_g$$

Imports (SIGN = +1) and Exports (SIGN = -1) of a globally traded commodity into/from each region. A region based conversion factor may be applied, defaulted to 1.

$$\left[\sum_{r,ie} SIGN_{ie} * REG\_XCVT_{r,e} * G\_TRD_{r,t,g,ie} \right]$$

$$\{=\}$$

$$0$$

4.5.17 MR_HPKW(r,t,e,z)

Table A1, Row 22

<u>SAGE_MATRIX_V52.XLS</u>

Name: The low-temperature **H**eat (lth) **P**ea**K**ing constraint ensures that there is enough capacity in place to meet the heating demand during the **W**inter, or season set by the analyst for the heating/cooling grid (heatcool).

Purpose:

Primal: The heat peaking constraint ensures that there is enough capacity in place to meet the heating demand during the season designed by the analyst. It takes into consideration an estimate of the level above the demand that corresponds to the actual peak moment plus a reserve margin of excess capacity (HRESERV includes both), to ensure that if some plants are unavailable the highest demand for heat can still be met.

Dual: The dual variable (DVR_HPKW) represents the marginal value associated with the capacity needed to ensure that enough heating plants are in place to meet the peak demand for heat in the heating/cooling season (z=heatcool). It thus represents the additional price that the model must "pay" for each unit of heat provided in the peaking season.

Occurrence: This equation is generated for all time periods and for the heat/cool seasons (z=heatcool), during which some demand device consumes heat.

Units: PJ, or any other unit in which heat is tracked. Note that although the constraint is thought of in terms of capacity, it is actually modeled in energy units.

Type: *Binding*
The equation is a greater than or equal (>=) constraint for the (heatcool) season.

Interpretation of the results:

Primal: The level of the low-temperature heat peaking equation represents the slack or excess capacity over and above the amount required based upon the highest demand, plus the heating reserve (HRESERV) margin provided by the analyst. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of capacity above the minimum level required. If there is slack the user should check the residual capacity levels (RESID) to ensure that they properly reflect the level in place prior to the modeling period, and review the value of the HRESERV parameter (remembering that is has a dual role of both estimating how much above the level of highest <u>average</u> heat demand the peak moment is, plus the traditional requirement for extra 'just in case' capacity.

Dual: A zero dual value implies that the available capacity exceeds the requirements imposed by the constraint. The value is negative when the primal equation is tight, which is normally the case, and indicates how much the objection function would be lowered if the reserve constraint was one unit lower.

Remarks: As already noted, the electricity peaking constraint is highly dependent upon the reserve margin (HRESERV), which is usual substantially above that of a tradition utility reserve margin. Only demand devices (dmd) can consume low-temperature heat. Note also that the analyst can set the season in which the peaking constraint is to be modeled so that the "heating" grid may be used as a "cooling" grid in hot climates (e.g., middle east).

GAMS Routine: MMEQHPK.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#17}: MR\_HPKW_{r,t,e} \ \forall \left(\begin{array}{l} e \in lth \wedge \\ z \in HEATCOOL_{r,e} \wedge \\ \sum\limits_{p \in dmd} MA(ENT)_{r,p,e,t} \end{array} \right)$$

Contribution from conventional (non externally load managed and non peaking only) heating plants and grid interchange technologies, where the amount to credit to the peak in a season is a function of the installed capacity of the plant taking into consideration the capacity credit factor (PEAK) as well.

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ p \notin PEAK\_TID_{r,p}}} \left(\begin{array}{l} PEAK(CON)_{r,p,t} / (1 + HRERSEV_{r,e,t})* \\ CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{array} \right) \right] +$$

Contribution from peaking only heating plants, where the amount to credit to the peak in a season is a function of the activity of the plant with the explicitly specified peak duration factor (PD(Z)D), as well as the capacity credit factor (PEAK).

$$\left[\sum_{\substack{p\in hpl\\ p\in PEAK\_TID_{r,p}}} \left(\begin{array}{l} PEAK(CON)_{r,p,t} / \\ \left(\begin{array}{l} QHRZ_{r,z} * \\ (1+HRERSEV_{r,e,t}) * PD(Z)D_{r,p,t} \end{array} \right) * \\ R\_THZ_{r,t,p,z} \end{array} \right)\right] +$$

Contribution from coupled heat and power pass-out turbine plants, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements, as well as the distribution efficiency.

$$\left[\sum_{\substack{p\in cpd\\ ELM_{r,p}}} \left(\begin{array}{l} PEAK(CON)_{r,p,t} * CAPUNIT_{r,p} * \\ ELM_{r,p} * TRNEFF(Z)(Y)_{r,p,z,'D',t} / \\ \left(\begin{array}{l} CEH(Z)(Y)_{r,p,z,'D',t} * \\ (1+HRERSEV_{r,e,t}) * PD(Z)D_{r,p,t} \end{array} \right) * \\ R\_CAP_{r,t,p} \end{array} \right)\right] +$$

Contribution from coupled heat and power backpressure turbine plants, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements, as well as the distribution efficiency.

$$\left[\sum_{\substack{p \in cpd \\ REH_{r,p}}} \left(\begin{array}{l} PEAK(CON)_{r,p,t} * CAPUNIT_{r,p} * \\ TRNEFF(Z)(Y)_{r,p,z,'D',t} / \\ \left(REH(Z)(Y)_{r,p} * \left(1 + HRERSEV_{r,e,t} \right) \right) * \\ R\_CAP_{r,t,p} \end{array} \right) \right]$$

$$\{\geq\}$$

Transfer of heat from the current grid to a conventional (non externally load managed) grid interchange technology.

$$\left[\sum_{\substack{p \in hlk \\ p \notin xlm}} \left(INP(ENT)c_{r,p,e,t} * R\_THZ_{r,t,p,e,z} \right) \right] +$$

Transfer of heat from the current grid to an externally load managed grid interchange technologies for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p\in \ln k \\ p\in xlm}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF(Z)(Y)_{r,t,p,w} * R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Consumption of electricity by demand devices according to the amount of electricity needed per unit output (a function of the market share of electricity to the device (MA(ENT)) and the device efficiency (EFF)) and the seasonal/time-of-day shape (DM_FR) of the demand load being serviced by the share of the output from the device to the sector (DM_OUTX), as well as the loss on the heat grid.

$$\left[\sum_{p\in dmd} \begin{pmatrix} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \sum_{d,y} \begin{pmatrix} DM\_FR(Z)(Y)_{r,d,z,y} * \\ DM\_OUTX_{r,p,d,t} \end{pmatrix} / \\ \left(QHRZ_{r,p,z} * EFF_{r,p,t} * DHDE(Z)_{r,z,t} \right) * \\ R\_CAP_{r,t,p} \end{pmatrix}\right]$$

4.5.18 MR_LIM(r,t,p,e)

Table A1, Row 23

SAGE_MATRIX_V52.XLS

Name: For flexible output “mixing” process (**LIM**IT) the constraint that controls the levels of the individual outputs according to the maximum level provided by the analyst for each commodity produced (OUT(ENC)p) from such processes

Purpose:

Primal: For flexible output “mixing” processes this constraint ensures that the output of the individual commodities (R_LOUT) does not exceed their maximum share (OUT(ENC)p) permitted.

Dual: The dual variable (DVR_LOUT) represents the marginal value associated with the limit imposed restricting the maximum amount of a particular commodity that can be produced per unit of overall activity from a flexible process. It thus represents the desirability of permitting a higher percentage of the commodity to be produced by the process.

Occurrence: This equation is generated for all time periods for which the flexible process (LIMIT) produces the commodity (OUT(ENC)p).

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding*
The equation is a less than or equal (<=) constraint.

Interpretation of the results:
Primal: The level of the limit equation indicates how much below the maximum permitted production of a commodity a process is actually outputting.
Dual: The dual variable (DVR_LIM) of the limit constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of the commodity from the flexible process. A zero dual value implies that this commodity is not driving the actual operation of the process. The value is negative when the primal equation is tight, and the system desires more of the commodity from the process. It may be worth examining the maximum output share for such limited commodities to determine whether that a bit more of the most desired commodity(ies) can be squeezed out of the flexible process.

Remarks: This equation works in tandem with the overall limit constraint (MR_PBL) that ensure that the total output from a flexible process does not exceed its overall efficiency (LIMIT). Note that the output share (OUT(ENC)p) for all non-LIMIT processes fixes the share, and thus the sum of all OUT(ENC)p is the overall efficiency of the process. But for these flexible processes the individual shares (OUT(ENC)p) represent the maximum and thus they may sum to more than 1.

GAMS Routine: MMEQLIM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#18:}\ MR\_LIM_{r,t,p,e}\ \forall \begin{pmatrix} LIMIT_{r,p,t} \wedge \\ OUT(ENC)_{r,p,e,t} \end{pmatrix}$$

Level of the current commodity from a flexible limit process.

$$\left[R\_LOUT_{r,t,p,e}\right] -$$

Maximum share of the total output from conventional (non externally load managed) limit process of the current commodity.

$$\left[OUT(ENC)p_{r,p,e,t} * R\_ACT_{r,t,p}\right] \$(not\ p \in xpr) -$$

Maximum share of the total output from non externally load managed limit process of the current commodity.

$$\begin{bmatrix} OUT(ENC)p_{r,p,e,t} * CAPUNIT_{r,p} * \\ CF_{r,p,t} * R\_CAP_{r,t,p} \end{bmatrix} \$(p \in xpr)$$

$$\{\leq\}$$

$$0$$

4.5.19 R_MTSOBJ

Table A1, Row 24

<u>SAGE_MATRIX_V52.XLS</u>

Description: As discussed below (MR_OBJ) the SAGE **M**inimum **T**ime-**S**tepped **OBJ**ective function looks to maximize the total surplus (consumers plus producers surplus) for each region, in each period. This is equivalent to minimizing the region's total cost, comprising: annual costs, annualized investment costs, and a cost representing the loss incurred when demands for energy services are elastic. In the implementation of SAGE, the cost minimization formulation is adopted. Furthermore, when SAGE is run in its normal, time-stepped manner, the optimization is performed one period at a time (as opposed to the MARKAL model which optimizes the sum of total discounted costs over all periods).

Purpose:

Primal: The regional SAGE objective function expresses each region's total cost at the current period. The regional costs are then aggregated into a single total cost for all regions together (see MR_OBJ). When SAGE optimizes cost at period t, all past decisions are frozen at the values found when optimizing for previous periods. The individual regional levels are accumulated in the R_MTSOBJ variables that are then summed over the regions in the overall SAGE objective function (MR_OBJ).

Occurrence: This equation is generated for each region in each time period and includes all terms to which a cost is applied.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding*
The equation is constructed as a balance equation that equates (=) the total cost to a single variable for each region (R_MTSOBJ).

Remarks: As the SAGE objective function works with annualized costs for a single period at a time it is not discounted.

GAMS Routine: MMEQOBJ.MTS

MATHEMATICAL DESCRIPTION

$$\text{EQ\#20}: MR\_MTSOBJ_{r}$$

Establish the current sequential time period to be solved.

$$\sum_{t \subset ts} \Big\langle$$

Annual cost of supplying domestic resources, plus any delivery costs associated with ancillary commodities, according to the level of the resource activity.

$$\sum_{\substack{s \in tpsep \\ x \neq 'IMP \vee 'EXP \\ e \notin (enu \cup \\ ecv \cup lth)}} \left[R\_TSEP_{r,t,s} * \left\langle \begin{array}{l} COST_{r,s,t} + \\ \sum_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle \right] +$$

Annual cost of imports and export, plus any delivery costs associated with ancillary commodities involved in the import/export process, according to the level of the resource activity. Note that if the trade is expressed as bi-lateral trade then any resource supply cost (COST) is ignored and only the transport/transaction cost applied. Also, if importing electricity then transmission and distribution O&M costs are incurred.

$$\sum_{\substack{s \in tpsep \\ x = 'IMP \vee 'EXP' \\ e \notin (enu \vee \\ ecv \vee lth)}} \left[R\_TSEP_{r,t,s} * \left\langle \begin{array}{l} SIGN_x * \begin{pmatrix} COST_{r,s,t} \$ not\ BI\_TRDENT + \\ BI\_TRDCST_{r,s,t} \$ BI\_TRDENT \end{pmatrix} + \\ (ETRANOM_{r,e,t} + EDISTOM_{r,e,t}) \$ \begin{pmatrix} e \in elc \wedge \\ x = 'IMP' \end{pmatrix} + \\ \sum_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle \right] +$$

Annual cost of imports and export electricity by time slice, according to the level of activity. Any bi-lateral, time-sliced transport/transaction cost is applied here, and any other delivery or transmission and distribution costs are captured in the

$$\left[\sum_{\substack{s \subset tpsep,w \\ e \in elc \\ BI\_TRDELC}} \begin{pmatrix} R\_TSEPE_{r,t,s,w} * \\ SIGN_x * BI\_TRDCSTE_{r,s,w} \end{pmatrix} \right]^{+}$$

Transaction cost associated with a globally traded commodity charge in the

$$\left[\sum_{\substack{e \in ent \vee env \vee mat \\ x = 'IMP' \vee 'EXP' \\ G\_TRADE_e \\ t \geq TRD\_FROM}} \begin{pmatrix} R\_GTRD_{r,t,e,x} * \\ TRD\_COST_{r,e,t} \end{pmatrix} \right]^{+}$$

Stockpile credit in the final period.

Fixed operating and maintenance costs for conventional (non externally load managed) process and conversion technologies as a function of the total installed capacity. [For externally load managed technologies the FIXOM is included when determining the total operating costs (below).]

$$\left[\sum_{p \left(\begin{array}{l} (\in tpprc \wedge \notin xpr) \vee \\ (\in tpcon \wedge \notin xlm) \end{array} \right)} \left(FIXOM_{r,p,t} * R\_CAP_{r,t,p} \right) \right] +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) processes as a function of the total process activity.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(\begin{array}{l} R\_ACT_{r,t,p} * \\ \left\langle \begin{array}{l} VAROM_{r,p,t} + \\ \sum_{e \subset INP(ENT)p_{r,p,e,t}} \left(INP(ENT)p_{r,p,e,t} * DELIV(ENT)r_{r,p,e,t} \right) \end{array} \right\rangle \end{array} \right) \right] +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) electric power plants as a function of the time-sliced generating activity. The transmission and distribution O&M costs are added on top of the plant operating costs. [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\sum_{\substack{p \in tpela, w \\ p \notin xlm \\ except\ p \in (bas \cup stg) \wedge \\ (z='N') \\ except\ (z='I' \vee y='N') \wedge \\ PEAK\_(CON)\_TID_{r,p}}} \left(R\_TEZY_{r,t,p,w} * \left\langle \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + VAROM_{r,p,t} + ELCOM \right\rangle \right) +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) heating plants as a function of the seasonal generating activity. The transmission O&M costs are added on top of the plant operating costs. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\sum_{\substack{p \in tphpl, z \\ p \notin xlm \\ except\ (z='I' \vee 'S') \wedge \\ PEAK\_(CON)\_TID_{r,p}}} \left[R\_THZ_{r,t,p,z} * \left\langle \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + VAROM_{r,p,t} + \sum_{e \in lth} DTRANOM_{r,e,t} \right\rangle \right] +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for coupled heat and power pass-out turbines as a function of the time-sliced generating activity. The transmission and distribution O&M costs for both electricity and heat are added on top of the plant operating costs. [Special conditions apply to base load which only have daytime variables (to force day=night generation for base load plants.]

$$\sum_{\substack{p \in tpcpd,w \\ p \in ELM_{r,p} \\ except(y='N' \wedge \\ e \in bas)}} \left[\begin{array}{l} R\_TCZYH_{r,t,p,w} * \left(CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p} \right) * \\ \left\langle \begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ +(1 - ELM_{r,p}) * ELCOM + \sum_{e \in lth} DTRANOM_{r,e,t} + VAROM_{r,p,t} \end{array} \right\rangle \end{array} \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for demand devices as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{p \in tpdmd} \left[\begin{array}{l} R\_CAP_{r,t,p} * \\ \left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{c} \sum_{e \subset MA(ENT)_{r,p,e,t}} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} / \\ EFF_{r,p,t} * DELIV(ENT)_{r,p,e,t} \end{array} \right) + \\ VAROM_{r,p,t} \end{array} \right] \end{array} \right\rangle \end{array} \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed processes as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{\substack{p \in tpprc \\ p \in xpr}} \left[\begin{array}{l} R\_CAP_{r,t,p} * \\ \left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{l} \sum_{e \subset INP(ENT)p_{r,p,e,t}} \left(INP(ENT)p_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} \end{array} \right] \end{array} \right\rangle \end{array} \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed power plants as a function of the total installed capacity, and the operation (dictated by CF(Z)(Y)) thereof.

$$\sum_{\substack{p \in tpcon \\ p \in xlm}} \left[\left\langle \begin{array}{l} R\_CAP_{r,t,p} * \\ \sum_{w} \left(CF(Z)(Y)_{r,p,w,t} * QHR_{r,w} \right) * CAPUNIT_{r,p} * \\ \left[\begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} + ELCOM\$p \in ela + LTHOM\$p \in hpl \end{array} \right] * \\ \quad + FIXOM_{r,p,t} \end{array} \right\rangle \right] +$$

Annualized investment costs associated with new investments still available in the current period, when t' is the vintage period in which the investment took place. The calculated investment cost (COST_INV) consists of the actual investment cost (INVCOST), as well as electricity transmission and distribution system investment costs if appropriate (see below), taking into consideration the capital recovery factor (CRF), as determined by the lifetime (LIFE) and discount rate (DISCOUNT and DISCRATE if technology based), as discussed below.

$$\left[\sum_{p} \sum_{t'=t-LIFE_{r,p}}^{t} \left(COST\_INV_{r,t,t',p} * R\_INV_{r,t',p} \right) \right] +$$

Annualized investment costs associated with residual capacity (that is capacity installed prior to the 1st modeling period) still available in the current period. The calculated investment cost (COST_INV) consists of the actual investment cost (INVCOST), as well as electricity transmission and distribution system investment costs if appropriate (see below), taking into consideration the capital recovery factor (CRF), as determined by the lifetime (LIFE) and discount rate (DISCOUNT and DISCRATE if technology based), as discussed below.

$$\left[\sum_{\substack{p \in RESID_{r,t,p} \\ t' = first\ period}} \left(COST\_INV_{r,t',t',p} * R\_RESID_{r,t,p} \right) \right] +$$

Emission taxes or other costs associated with the level of an emission indicator.

$$\left[\sum_{v} \left(ENV\_COST_{r,v,t} * R\_EM_{r,t,v} \right) \right] +$$

Taxes and subsidies associated with particular commodities, and the consumption and/or production thereof.

$$\left[\sum_{txs} \left(TSUB\_COST_{r,v,t} * R\_TXSUB_{r,t,txs} \right) \right] +$$

The costs associated with growth or reduction in demand as a function of the elastic demand own price elasticity and variance as represented by a step curve.

$$MED\text{-}STEP(BD)_{r,d,b,u,t} \sum_{b,d,u=1} \left\{ \begin{array}{l} R\_ELAST_{r,t,d,b,u} * MED\text{-}BPRICE_{r,d,t} * \\ \left\langle \begin{array}{l} -\left\{ \begin{array}{l} \left[1+ \left(MED\text{-}VAR_{r,d,b,t} / MED\text{-}STEP_{r,d,b,t} * (u-.5) \right) \right] ** \\ \left(1/ MED\text{-}ELAST_{r,d,b,t} \right) \end{array} \right\} \$ b='UP' + \\ \left\{ \begin{array}{l} \left[1- \left(MED\text{-}VAR_{r,d,b,t} / MED\text{-}STEP_{r,d,b,t} * (u-.5) \right) \right] ** \\ \left(1/ MED\text{-}ELAST_{r,d,b,t} \right) \end{array} \right\} \$ b='LO' \end{array} \right\rangle \end{array} \right\}$$

$$\Big\rangle$$

$$\{=\}$$

$$R\_MTSOBJ_{r}$$

Where SIGN = -1 for x='EXP'; otherwise 1

ELCOM = if $e \in elc \wedge p \in ela$ then
$EDISTOM_{r,e,t} + ETRANOM_{r,e,t} \$(p \subset cen)$;
$0\$(p \in stg)$

LTHOM = if $e \in lth \wedge p \in (p \in cpd \wedge cen)$ and p is a back-pressure turbine (REH) then
$DTRANOM_{r,e,t} / REH_{r,p}$; 0 otherwise

ELCINV = if $e \in elc \wedge p \in ela$ then
$EDISTINV_{r,e,t} \$(p \notin stg) +$
$ETRANINV_{r,e,t} \$(p \subset cen)$

LTHINV = if $e \in lth$
if $p \in hpl$ then $DTRANINV_{r,e,t}$;
if $p \in (p \in cpd \wedge cen)$
if $p \in REH_{r,p}$ then
$DTRANINV_{r,e,t} / REH_{r,p}$
else

$$DTRANINV_{r,e,t} * \frac{ELM_{r,p}}{CEH(Z)(Y)_{r,p,w,t}} \quad 0$$

otherwise

CRF = where $x=1/(1+DISCOUNTorDISCRATE)$, *and then* $CRF=\{1-x\}/\{1-x^{LIFE}\}$

FRACLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
(years in last period of LIFE/NYRSPER)

FRLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
1 if current period is not the last (fractional period)
otherwise
CRF adjustment for the fraction of the period

COST_INV = $$\frac{\left\langle INVCOST_{r,p,t} + (ELCINV + LTHINV)\right\rangle *}{FRACLIFE_{r,p} * CRF_{r,t,p} * FRLIFE_{r,p}}$$

4.5.20 MR_OBJ

Table A1, Row 25

<u>SAGE_MATRIX_V52.XLS</u>

Description: As discussed in section 3.3, the **OBJ**ective of SAGE is to maximize the total surplus (consumers plus producers surplus) for all regions together. This is equivalent to minimizing the period’s total cost, comprising: annual costs, annualized investment costs, and a cost representing the loss incurred when demands for energy services are elastic. In the implementation of SAGE, the cost minimization formulation is adopted. Furthermore, when SAGE is run in its normal, time-stepped manner, the optimization is performed one period at a time (as opposed to the MARKAL model which optimizes the sum of total discounted costs over all periods).

Purpose:
Primal: The SAGE objective function looks to minimize total system cost. It does so in a time-stepped manner, optimizing for each period successively. When SAGE optimizes cost at period t, all past decisions are frozen at the values found when optimizing for previous

periods. [In contrast, MARKAL minimizes the total discounted cost over all periods in one step.]

Occurrence: This equation is generated at each time period.

Units: Million 2000 US$, or any other unit in which costs are tracked. Note that a regional monetary conversion factor (REG_XMONY) may be applied if different monetary units are used for the various regions.

Type: *Binding*
The equation is actually constructed as a balance equation that equates (=) the total producer plus consumer surplus to a single variable across all regions (RMTSOBJ) that is minimized by the solver.

Remarks: The same equation serves as the clairvoyant object function over the entire modeling horizon for a multi-region MARKAL run, but with a differently named regional and total system cost variables (R_objZ/R_TOTobjZ) involved. However, as the SAGE objective function works with annualized costs for a single period at a time it is not discounted. [In the case of MARKAL it aggregates the total global system cost using discounting]

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

EQ#20: *MR_OBJ*

Regional producer/consumer surplus, with optional monetary conversion factor applied. [For MARKAL R_objZ is the regional minimized total discounted system cost.]

$$\left[\sum_{r}\left(REG\_XMONY_{r} * R\_MTSOBJ_{r}\right)\right]$$

$$\{=\}$$

Objective function variable minimized by the solver. [For MARKAL R_TOTOBJ is the global minimized total discounted system cost.]

4.5.21 MR_PBL(r,t,p)

Table A1, Row 26

SAGE_MATRIX_V52.XLS

Name: For flexible out "mixing" **P**rocesses this **B**alance constraint **L**imits the sum of all the outputs (OUT(ENC)p) from such a process to ensure that they are compatible with the overall efficiency of such processes (LIMIT).

Purpose:
Primal: To ensure that the sum of all outputs from a flexible "mixing" process respects the overall efficiency of that process. For such processes a variable is created for each individual output commodity (R_LOUT), limited to a maximum share (OUT(ENC)p) of the process activity (R_ACT), which are then summed here.
Dual: The dual variable (DVR_PBL) represents the marginal cost associated with the limit on the overall activity from a flexible process.

Occurrence: This equation is generated for all time periods for which the flexible process (LIMIT) produces the commodity (OUT(ENC)p).

Units: PJ, or any other unit in which commodities are tracked.

Interpretation of the results:
Primal: The level of the process balance equation indicates how much below the maximum combined output from a flexible process the total output is.
Dual: The dual variable (DVR_PBL) of the process balance constraint (shadow price) indicates the cost pressure on the operation of a flexible plant. A zero dual value rarely occurs unless there is no cost associated with the commodities consumed by the process. The value may be positive or negative.

Type: *Binding*
The equation an equality (=) constraint.

Remarks: This equation works in tandem with the individual flexible output commodity limit constraint (MR_LIM) that ensures that no commodity output rises above the maximum output of the commodity (OUT(ENC)p).

GAMS Routine: MMEQPBL.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#21: } MR\_PBL_{r,t,p} \ \forall \begin{pmatrix} p \in prc \wedge \\ LIMIT_{r,p,t} \end{pmatrix}$$

Overall efficiency of a flexible limit process applied to the total activity of conventional processes or the activity derived from the capacity for externally load managed processes.

$$\left[\begin{array}{l} LIMIT_{r,p,t} * \\ \left(\begin{array}{l} R\_ACT_{r,t,p} \$ (p \notin xpr) \vee \\ \left[\begin{array}{l} \left(CAPUNIT_{r,p} * CF_{r,t,p} \right) * \\ R\_CAP_{r,t,p} \end{array} \right] \$ \, p \in xpr \end{array} \right) \end{array} \right] -$$

Sum over all the commodities produced from a flexible process.

$$\left[\sum_{e} \left(R\_LOUT_{r,t,p,e} \right) \right]$$

$$\{=\}$$

$$0$$

4.5.22 MR_PRICE **Table A1, Row 27**

SAGE_MATRIX_V52.XLS

Description: The standard MARKAL objective function. Used in SAGE strictly as an accounting equation tracking the total discounted system cost for SAGE reporting purposes.

4.5.22 MR_REGELC(r,t,s)

Table A1, Row 28

<u>SAGE_MATRIX_V52.XLS</u>

Description: The **REG**ional **EL**e**C**tricity trade constraint maps the time-sliced import or export of electricity by a region, to the conventional annual electricity import/export variable (R_TSEP).

Purpose:

Primal: In order to link bi-lateral time-sliced electricity trade into the parts of SAGE that are not time-slice dependent (e.g., the objective function, emissions accounting) the individual season/time-of-day trade variables (R_TSEPE) are summed to the annual electricity import/export variable (R_TSEP).

Dual: The dual variable (DVR_REGELC) represents the cost of allowing one more/less unit of the time-sliced electricity trade.

Occurrence: This equation is generated in each time period from the first period the resource option is available.

Units: PJ, or any other unit in which electricity is tracked.

Type: *Binding*
The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the regional trade constraint indicates the amount by which the time-sliced electricity trade does not match up with the associated annual import/export variable. If non-zero the constraint, and thus model, is infeasible. This may be due to some conflicting limits in the regions involved imposed on the annual import/export variables.

Dual: The dual variable (DVR_REGELC) of the regional time-sliced trade constraint (shadow price) indicates the pressure on the system to supply one more/less unit of electricity trade. A zero dual value implies that there is no cost associated with producing and delivering the commodity, which is highly unlikely. The value may be positive or negative.

Remarks: Note that this constraint works in tandem with the MR_BITRDE constraint, which links the time-sliced electric trade variables between the two regions involved. While this constraint ties the bi-lateral time-sliced electricity trade variables (R_TSEPE) to the annual variable (R_TSEP), the former is used directly in the time-sliced electricity constraints (MR_BALE1/2, MR_BAS, MR_EPK).

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#23}: MR\_REGELC_{r,t,s} \; \forall s \in tpsep \wedge$$

$$\left[\begin{array}{l} s \subset ('IMP' \vee 'EXP + e = elc + c) \wedge \\ \sum_{r',e',w} \left(\begin{array}{l} BI\_TRDELC_{r,e,r',e',c,w} \vee \\ BI\_TRDELC_{r',e',r,e,c,w} \end{array} \right) \end{array} \right]$$

Standard annual resource supply option corresponding to electricity import/export.

$$\lfloor R\_TSEP_{r,t,s} \rfloor -$$

Bi-lateral, time-slice electricity trade summed of each time-slice.

$$\left[\sum_{w} \left(R\_TSEPE_{r,t,s,w} \right) \right]$$

$$\{=\}$$

$$0$$

Where bi_trdelc = bi-lateral time-sliced electric trade options
tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

4.5.23 MR_TENV(r,t,v)

Table A1, Row 29

SAGE_MATRIX_V52.XLS

Description: The **T**otal amount of an **ENV**ironmental indicator (emissions) generated in each period from all sources (resources and technologies).

Purpose:

Primal: The environmental equation tracks all sources of emissions from resources and technologies and accumulates the total level of emissions in each period. For each source of emissions the analyst associates the emissions rate with some aspect of the technology, that is overall activity of resource options or technologies (ENV_SEP, ENV_ACT) or time-sliced activity for conversion technologies (ENV_TEZY, ENV_TCZY, ENV_HPL) if desired, installed capacity (ENV_CAP), or new investment (ENV_INV). The model then applies the emission rate directly to the associated variable (R_TSEP, R_ACT, R_TEZY, R_TCZYH, R_HPL, R_CAP, R_INV) respectively. The net sum of all the emissions is accumulated in variable (R_EM) for each period.

Dual: The dual variable (DVR_TENV) represents the marginal value of one unit of the emissions indicator arising from the energy system whenever a limit is imposed and reached upon the total level of the emissions. It thus represents the endogenous price of this emission.

Occurrence: This equation is generated in all time periods during which some resource option or technology produces the environmental indicator.

Units: Tons, or thousand tons or any other unit in which emissions are tracked.

Type: *Binding*

The equation is an equality (=) constraint, with all emissions accumulated in a total emissions variable (R_EM) for each indicator in each period. Note that to limit total emissions in a period a bound is applied to the R_EM variable.

Interpretation of the results:

Primal: The interpretation of the level of the emission equation depends upon whether a limit is imposed upon the total emissions variable or not. If so, it represents the slack (level above/below) the limit (lower/upper) imposed of the total emissions variable. If there is no limit imposed then the constraint will always balance to 0.

Dual: The dual variable (DVR_TENV) of the emission constraint (shadow price) is normally 0, unless a limit on emissions is imposed (on variable R_TENV) and reach. It thus represents the endogenous price of this emission.

Remarks: For activity related emissions the analyst must remember that the parameter is applied to the output variable of the technologies. This being the case any efficiency loss associated with the technology needs to be embedded in the parameter to properly account for emissions based upon the amount of the source commodity consumed, while taking into consideration the nature of the way the commodity is used where appropriate. Also, for exports or processes that serve as emission sinks (e.g., sequestration) or are emission reduction options (e.g., a scrubber for which the emissions have already be included in the emission accounting constraint), the analyst needs to enter a negative emissions indicator.

GAMS Routine: MMEQTENV.INC

$$\text{EQ\# 24}: MR\_TENV_{r,t,v}$$

Annual production of the current emission indicator as a function of the level of activity resource supply options, where exports are subtracted from the balance.

$$\left[\sum_{s \forall ENV\_SEP_{r,s,v,t}} \begin{pmatrix} SIGN_x * ENV\_SEP_{r,s,v,t} * \\ R\_TSEP_{r,t,s} \end{pmatrix}\right] +$$

Import or export of a globally traded emissions (or permit).

$$\left[\left(\sum_{ie} SIGN_{ie} * R\_GTRD_{r,t,v,ie}\right) \$ \begin{pmatrix} v \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix}\right] +$$

Production of the current emissions indicator based upon activity from conventional (non-externally load managed and non-LIMIT) processes.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(ENV\_ACT_{r,p,v,t} * R\_ACT_{r,t,p}\right)\right] +$$

Production of the current emissions indicator based upon the level of activity from externally load managed processes, where the activity of the process (and thus the amount of each emission produced) is directly determined from the level of installed capacity by means of a fixed capacity factor.

$$\left[\sum_{p \in xpr} \left(\left(ENV\_ACT_{r,p,v,t} {}^{*} CAPUNIT_{r,p} {}^{*} CF_{r,p,t} \right) {}^{*} R\_CAP_{r,t,p} \right) \right] +$$

Annual production of the current emissions indicator as a function of electric generation from conventional (non-externally load managed) power plants, summed over the season/time-of-day activity of the plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela, w \\ p \notin xlm \\ except\ y='N' \wedge p \in (bas \cup stg) \\ except\ ((y='N' + z='I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(ENV\_ACT_{r,p,v,t} {}^{*} R\_TEZY_{r,t,p,w} \right) \right] +$$

Annual production of the current emissions indicator as a function of generation from externally load managed power plants, where the activity of the plant (and thus the amount of each emissions indicator produced per unit of electricity or heat generated) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice (CF(Z)(Y)).

$$\left[\sum_{p \in xlm, w} \left(\begin{array}{l} ENV\_ACT_{r,p,v,t} * CAPUNIT_{r,p} * \\ QHR(Z)(Y)_{r,p,w} * CF(Z)(Y)_{r,p,z,w} * \\ R\_CAP_{r,t,p} \end{array} \right)\right] +$$

Annual production of the current emissions indicator as a function of the amount of heat generation for conventional (non-externally load managed) heating plants, summed over the seasonal activity of conventional power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl, z \\ p \notin xpr \\ except\ ((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(ENV\_ACT_{r,p,v,t} * R\_THZ_{r,t,p,z} \right)\right] +$$

Annual production of the current emissions indicator according to the activity of coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of power plants. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd, w \\ when\, CEH(Z)(Y)_{r,p,w,t} \\ except\,(y='N' \wedge p \in bas)}} \left(ENV\_ACT_{r,p,v,t} * R\_TCZYH_{r,t,p,w}\right)\right] +$$

Production of the current emissions indicator as a function of the total level of installed capacity for a technology.

$$\left[\sum_{p}\left(ENV\_CAP_{r,p,v,t} * R\_CAP_{r,t,p}\right)\right] +$$

Production of the current emissions indicator as a function of the total level of new investment in a technology.

$$\left[\sum_{p}\left(ENV\_INV_{r,p,v,t} * R\_INV_{r,t,p}\right)\right] +$$

Production of the current emissions indicator as a function of the activity of demand devices. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{p \in dmd}\left(\begin{array}{l}\left(ENV\_ACT_{r,p,v,t} * CAPUNIT_{r,p} * CF_{r,p,t}\right) * \\ R\_CAP_{r,t,p}\end{array}\right)\right] +$$

Application of any GWP factor for GHG accounting that relates one emission indicator to another.

$$\left[\sum_{v' \in gwp} \left(ENV\_GWP_{r,v',v,t} * EM_{r,v}\right) / ENV\_SCAL_r\right] +$$

The annual total of the current emission indicator, optionally scaled. Note that the variable may be bounded (EM_BOUND) to cap emissions in a region.

$$\left\lfloor EM_{r,v} / ENV\_SCAL_r \right\rfloor$$

$$\{=\}$$

$$\mathbf{0}$$

Where ENV_SCAL = scale parameter applied to all emissions to scale the model or see marginals reported.

4.5.25 MR_TEZY(r,t,p,w)

Table A1, Row 30

<u>SAGE_MATRIX_V52.XLS</u>

Description: The **T**ime-slice **E**lectric generation utilization constraint that ensures that the season(**Z**)/time-of-day(**Y**) activity of a power plant (R_TEZY), that is the amount of electricity generated, is in line with the total installed capacity (R_CAP), according to the availability of the technology (AF/AF(Z)(Y)), taking into consideration any scheduled maintenance (R_M).

Purpose:

Primal: The electric generation utilization equation oversees the basic operation of a power plant. As such it ensures that the production of electricity in each time-slice from all the installed capacity is in accordance with the availability of the technology at each point of time. The determination of the availability is dependent upon the nature of technology (e.g., conventional or externally load managed (xlm)), and whether the analyst defines the availability annually or by time-slice (AF/AF(Z)(Y) or CF/CF(Z)(Y)), and if unavailability can be scheduled by the model or some portion of the availability is pre-scheduled (AF_TID≠1) and thus removes some of the flexibility for the model to schedule downtime.

Dual: The dual variable (DVR_TEZY) represents the cost pressure to permit the power plant to operate more in a particular time-slice. It thus represents the cost to the energy system of limiting the operation of a power plant.

Occurrence: This equation is generated in all time periods and time-slices for which a technology is available.

Units: PJ, or any other unit in which heat is tracked.

Type: *Binding*
The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under utilization of a power plant in a time-slice and period. As conventional power plants normally operate at different levels at various times of the day it would not be surprising to see slack with respect to the utilization of the capacity, particularly at night.

Dual: The dual variable (DVR_TEZY) of the utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more unit of production from a power plant in a time-slice. A zero dual value implies that the plant is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of power plant modeled, conventional plants where a maximum availability factor is proved (by means of AFs), and externally load managed plants where the operation is fixed with respect to the total capacity installed (by means of CFs). These AF/CFs are then applied along with the time-slices (QHR) to determine the actual amount of electricity produced by each power plant in each time-slice. Since CF-based plants have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_TCZYH,R_TEZY) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQTEZY.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#25}: MR\_TEZY_{r,t,p,w} \ \forall \left\{ \begin{array}{l} AF(Z)(Y)_{r,p,w,t} \ \wedge \\ p \in ela \wedge p \notin xlm \wedge except \\ \left[\begin{array}{l} [p \in (bas \cup stg) \wedge y = 'N'] \vee \\ [PEAK\_TID(CON)_{r,p} \wedge (y = 'N')] \end{array} \right] \end{array} \right\}$$

Electric production from conventional electric generating plants.

$$\left[R\_TEZY_{r,t,p,w} \right]^{-}$$

Electric production from coupled heat and power pass-out turbines, where the amount of electricity is dependent upon the level of heat output.

$$\left[\begin{pmatrix} CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p} * \\ R\_TCZYH_{r,t,p,w} \end{pmatrix} \$ \begin{pmatrix} p \in cpd \wedge \\ ELM_{r,p} \end{pmatrix} \right] -$$

Available capacity from plants for which availability is specified by season/time-of-day, as opposed to annually, apportioned to the current time-slice.

$$\left[\left\langle \begin{array}{l} CAPUNIT_{r,p} * \begin{pmatrix} QHR(Z)(Y)_{r,p,w} \$ p \notin bas \vee \\ QHRZ_{r,p,z} \$ p \in bas \end{pmatrix} * \\ AF(Z)(Y)_{r,p,w,t} * R\_CAP_{r,t,p} \end{array} \right\rangle \$ \begin{pmatrix} not \\ AF_{r,p,t} \end{pmatrix} \right] -$$

Available capacity from plants for which annual availability is specified, and perhaps scheduled outage.

$$\left[\left\langle \begin{array}{l} CAPUNIT_{r,p} * \begin{pmatrix} QHR(Z)(Y)_{r,p,w} \$ p \notin bas \vee \\ QHRZ_{r,p,z} \$ p \in bas \end{pmatrix} * \\ \left[\left(1 - AF_{r,p,t}\right) * \left(1 \$ (not\, AF\_TID_{r,p}) + AF\_TID_{r,p}\right) \right] * \\ R\_CAP_{r,t,p} \end{array} \right\rangle \$ AF_{r,p,t} \right] -$$

Available capacity from plants for which annual availability is specified, and where a fraction of outage is scheduled (AF_TID). Here adding in the scheduled portion of the outage.

$$\left[\left\langle \begin{array}{l} \begin{pmatrix} \left(QHR(Z)(Y)_{r,p,w} / QHRZ_{r,p,z}\right) \\ \$ p \notin bas \vee (1) \$ p \in bas \end{pmatrix} * \\ R\_M_{r,t,p,z} \end{array} \right\rangle \$ \begin{pmatrix} AF\_TID_{r,p} \neq 1 \wedge \\ AF_{r,p,t} \end{pmatrix} \right] -$$

Available capacity from plants peaking only plants (PEAK_TID) according to the peak operation percentage for the current time-slice.

$$\left[\begin{pmatrix} CAPUNIT_{r,p} * QHR(Z)(Y)_{r,w} * \\ PD(Z)D_{r,p,z,t} * R\_CAP_{r,t,p} \end{pmatrix} \$ PEAK\_TID_{r,p} \right]$$

$$\{\leq\}$$

$$0$$

4.5.26 MR_THZ(r,t,p,z)

Table A1, Row 31

<u>SAGE_MATRIX_V52.XLS</u>

Description: The **T**ime-slice **H**eat generation utilization constraint that ensures that the seasonal(**Z**) activity of a heating plant (R_THZ), that is the amount of heat generated, is in line with the total installed capacity (R_CAP), according to the availability of the technology (AF/AF(Z)(Y)), taking into consideration any scheduled maintenance (R_M).

Purpose:

Primal: The heat generation utilization equation oversees the basic operation of a power plant. As such it ensures that the production of heat in each season all the installed capacity is in accordance with the availability of the technology. The determination of the availability is dependent upon the nature of technology (e.g., conventional or externally load managed (xlm)), whether the analyst defines the availability annually or by time-slice (AF/AF(Z)(Y) or CF/CF(Z)(Y)), and whether or not unavailability can be scheduled by the model or some portion of the availability is pre-scheduled (AF_TID≠1) and thus removes some of the flexibility for the model to schedule downtime.

Dual: The dual variable (DVR_THZ) indicates the cost pressure to permit the heating plant to operate more in a particular season. It thus represents the cost to the energy system of limiting the operation of a heating plant in a given time slice.

Occurrence: This equation is generated in each season and time period from which a technology first becomes available. Note that for peaking only heating plants (PEAK(CON)_TID) the constraint is only generated in the heating/cooling (heatcool, default 'W'inter) primary season.

Units: PJ, or any other unit in which heat is tracked.

Type: *Binding*

The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under utilization of a heating plant in a season and period.

Dual: The dual variable (DVR_THZ) of the utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more unit of production from a heating plant in a season. A zero dual value implies that the plant is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of power plant modeled, conventional plants where a maximum availability factor is proved (by means of AFs), and externally load managed plants where the operation is fixed with respect to the total capacity installed (by means of CFs). These AF/CFs are then applied along with the seasonal time-slices (QHRZ) to determine the actual amount of heat produced by each power plant in each season. Since CF-based plants have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_TCZYH, R_THZ) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQTEZY.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#26: } MR\_THZ_{r,t,p,z} \ \forall \left(\begin{array}{l} AF(Z)(Y)_{r,p,'z'D',t} \wedge \\ p \in hpl \wedge p \notin xlm \wedge except \\ \left[\begin{array}{l} [PEAK\_TID(CON)_{r,p} \wedge \\ (z = 'I' \vee 'S')] \end{array} \right] \end{array} \right)$$

Heat production from conventional heating plants.

$$\left[R\_THZ_{r,t,p,z} \right] -$$

Available capacity from plants for which availability is specified by season/time-of-day, as opposed to annually, apportioned to the current time-slice.

$$\left[\left(\begin{array}{l} CAPUNIT_{r,p} * QHRZ_{r,z} * \\ AF(Z)(Y)_{r,p,z,'D',t} * R\_CAP_{r,t,p} \end{array} \right) \$ \left(\begin{array}{l} not \\ AF_{r,p,t} \end{array} \right) \right] -$$

Available capacity from plants for which annual availability, and perhaps scheduled outage is specified.

$$\left[\left\langle \begin{matrix} CAPUNIT_{r,p} * QHRZ_{r,z} * \\ \left[1-\left(1-AF_{r,p,t}\right)*AF\_TID_{r,p}\right]\$AF\_TID_{r,p} * \\ R\_CAP_{r,t,p} \end{matrix} \right\rangle \$ AF_{r,p,t} \right] -$$

Available capacity from plants for which annual availability is specified, and where a fraction of outage is scheduled (AF_TID). Here adding in the scheduled portion of the outage.

$$\left[R\_M_{r,t,p,z} \$ \begin{pmatrix} AF\_TID_{r,p} \neq 1 \wedge \\ AF_{r,p} \end{pmatrix} \right] -$$

Available capacity from plants peaking only plants (PEAK_TID) according to the peak operation percentage for the current time-slice.

$$\left[\begin{pmatrix} CAPUNIT_{r,p} * QHR(Z)(Y)_{r,w} * \\ PD(Z)D_{r,p,'W',t} * R\_CAP_{r,t,p} \end{pmatrix} \$ PEAK\_TID_{r,p} \right]$$

$$\{\leq\}$$

$$0$$

4.5.27 MR_TXSUB(r,txsub)

Table A1, Row 32

SAGE_MATRIX_V52.XLS

Description: The charging or crediting of a tax/subsidy (TSUB_COST) for commodities (TSUB_ENT) consumed/produced by technologies (TSUB_TCH). The total tax/subsidy is subtracted from the total system costs in the reports, and reported separately.

Purpose:

Primal: To allow a tax (+TSUB_COST) or subsidy (-TSUB_COST) to be associated with a commodity entering or leaving a technology. For example, to give a credit to green power the "green" electricity leaving each such generating facility would be listed.

Dual: The dual variable (DVR_TXSUB) represents the added cost of one more unit of the tax or the added benefit of one more unit of the subsidy to the overall energy system.

Occurrence: This equation is generated in each time period when at least one tax or subsidy cost is provided.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding*

The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the tax or subsidy accumulated from all the commodities and technologies associated with the tax/subsidy. Note that the tax/subsidy is applied to the level of incoming and/or outgoing commodities listed for a particular tax/subsidy.

Dual: The dual variable (DVR_TXSUB) of the tax/subsidy constraint (shadow price) indicates the amount that the energy system would be benefit if one less unit of the tax were generated, or loose if one less unit of the subsidy was available. A zero dual value is not possible unless no tax/subsidy is produced. The value is negative when the primal equation is tight and a tax is imposed, indicating how much the objective function would decrease, and positive if a subsidy is available indicating how much more direct cost would be incurred without the subsidy.

Remarks: The level of the tax subsidy is included in the overall objective function, but split out from the total energy system costs as they are considered direct energy system expenditures.

GAMS Routine: MMEQTAXS.INC

MATHEMATICAL DESCRIPTION

As the MR_TXSUB equation is built up directly from the three primary balance equations:

- MR_BAL_G/E for energy and materials respectively;
- MR_BALE1/2 for electricity, and
- MR_BALDH for heat.

Rather than reproducing those entire equations here these equations are simply referred in this equation specification, with the TSUB_TCH multiplier applied as appropriate.

$$\text{EQ\#27:}\ MR\_TXSUB_{r,t,txs}\ \forall TSUB\_COST_{r,txs,t}$$

Tax/subsidy component that involves energy, other than electricity or heat, or material from or to resource supply options. Refer to EQ#2 for MR_BALcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \notin elc \\ s \in TSUB\_SEP_{r,txs,s,t}}} \begin{pmatrix} TSUB\_SEP_{r,txs,s,t} * \\ MR\_BALcoef \end{pmatrix}\right] +$$

Tax/subsidy component that involves electricity from or to resource supply options. Refer to EQ#4 for MR_BALEcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \in elc \\ s \in TSUB\_SEP_{r,txs,s,t}}} \begin{pmatrix} TSUB\_SEP_{r,txs,s,t} * \\ MR\_BALEcoef \end{pmatrix}\right] +$$

Tax/subsidy component that involves regular energy, other than electricity or low-temperature heat, or material from technologies. Refer to EQ#2 for MR_BALcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \notin (elc \vee lth) \\ p \in TSUB\_TCH_{r,txs,p,t}}} \begin{pmatrix} TSUB\_TCH_{r,txs,p,t} * \\ MR\_BALcoef \end{pmatrix} \right] +$$

Tax/subsidy component that involves electricity from power plants. Refer to EQ#4 for MR_BALEcoef coefficient details.

$$\left[\sum_{\substack{e \in elc \\ e \in TSUB\_ENT_{r,txs,e} \\ p \in TSUB\_TCH_{r,txs,p,t} \\ p \in ela}} \begin{pmatrix} TSUB\_TCH_{r,txs,p,t} * \\ MR\_BALEcoef \end{pmatrix} \right] +$$

Tax/subsidy component that involves low-temperature heat from power plants. Refer to EQ#3 for MR_BALDHcoef coefficient details.

$$\left[\sum_{\substack{e \in lth \\ e \in TSUB\_ENT_{r,txs,e} \\ p \in TSUB\_TCH_{r,txs,p,t} \\ p \in (hpl \cup cpd)}} \begin{pmatrix} TSUB\_TCH_{r,txs,p,t} * \\ MR\_BALDHcoef \end{pmatrix}\right] -$$

The total accumulated amount of the tax/subsidy from all sources (commodity/technology combinations. The variable enters the objective function with the (+/-) TSUB_COST applied.

$$\left\lfloor R\_TAXSUB_{r,txs,t} \right\rfloor$$

$$\{=\}$$

$$0$$

4.5.28 MR_UTLCON(r,t,p)

Table A1, Row 33

SAGE_MATRIX_V52.XLS

Description: The adjustment to the availability of a power plant to account for the fact that some portion of outage is due to scheduled maintenance. Normally the model will schedule all unavailability, but if the user specifies that some fraction of annual unavailability is scheduled [AF_TID ≠ 1] then this constraint is generated to adjust the amount of unavailability that the model is free to schedule.

Purpose:

Primal: The conversion plant utilization equation ensures that the sum of all scheduled maintenance in all seasons is in line with the portion of the capacity which is unavailable due to scheduled outages, such as fuel reloading or maintenance.

Dual: The dual variable (DVR_UTLPRC) represents the pressure to permit a process to operate more than is currently possible. It thus represents the cost to the energy system of limiting the operation of a process.

Occurrence: This equation is generated in each time period from which a technology first becomes available if scheduled maintenance is specified for the plant (AF_TID ≠ 1) and time-sliced availability factors (AF(Z)(Y)) are not provided.

Units: PJ, or any other unit in which process activity is tracked.

Type: *Binding*
The equation is a greater than or equal (≥) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or excess scheduled maintenance owing to idling of a conversion plant in a period.

Dual: The dual variable (DVR_UTLCON) of the conversion plant utilization maintenance constraint (shadow price) indicates the amount that the energy system would have to pay if the scheduled maintenance was increased by one unit. A zero dual value implies that the plant operation is not limited by its scheduled outage. The value is positive when the primal equation is tight.

Remarks: The MR_UTLCON equation is only generated when the AF_TID ≠ 1, thus the equation is eliminated when the model is left to schedule all unavailability.

GAMS Routine: MMEQUCON.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#28}: MR\_UTLCON_{r,t,p} \; \forall \begin{pmatrix} p \in con \wedge \\ (AF\_TID_{r,p} \neq 1) \wedge \\ AF\_TID_{r,p,t} \end{pmatrix}$$

Total scheduled maintenance in all seasons.

$$\left[\sum_{z} \left(R\_M_{r,t,p,z} \right) \right] -$$

The portion of the capacity that is unavailable due to scheduled outage.

$$\left[\left\langle \left(1 - AF_{r,p,t}\right) * \left(1 - AF\_TID_{r,p}\right) \right\rangle * CAPUNIT_{r,p} * R\_CAP_{r,t,p} \right]$$

$$\{\geq\}$$

$$0$$

4.5.29 MR_UTLPRC(r,t,p)

Table A1, Row 34

SAGE_MATRIX_V52.XLS

Description: The constraint that ensures that the annual activity of conventional process (R_ACT) is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF).

Purpose:

Primal: The process utilization equation oversees the basic operation of a process (prc). As such it ensures that the annual activity for all the installed capacity is in accordance with the availability of the process. The determination of the actual availability of a process is dependent upon the nature of technology (e.g., conventional or externally load managed (xpr)).

Dual: The dual variable (DVR_UTLPRC) represents the cost pressure to permit a process to operate more than is currently possible. It thus represents the cost to the energy system of limiting the operation of a process.

Occurrence: This equation is generated in each time period from which a technology first becomes available.

Units: PJ, or any other unit in which process activity is tracked.

Type: *Binding*
The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under utilization of a process in a period.

Dual: The dual variable (DVR_UTLPRC) of the process utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more unit of production from a process. A zero dual value implies that the process is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of processes modeled, conventional processes where a maximum availability factor is proved (by means of AFs), and externally load managed processes where the operation is fixed with respect to the total capacity installed (by means of CF). Since CF-based processes have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_ACT) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQUPRC.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#28}: MR\_UTLPRC_{r,t,p} \ \forall \begin{pmatrix} p \in prc \wedge \\ p \notin xpr \end{pmatrix}$$

Annual activity of a process.

$$\left\lfloor R\_ACT_{r,t,p} \right\rfloor -$$

Total amount of available capacity of a process.

$$\left\lfloor \left(CAPUNIT_{r,p} * AF_{r,p,t} \right) * R\_CAP_{r,t,p} \right\rfloor$$

$$_{1}\{\leq\} \quad 0$$

4.6 Inter-period Dynamic Market Share and Endogenous Technology Learning Facilities

SAGE has embedded algorithms, under modeler control, that can be employed to examine the results of the last period solved and augment the solution by introducing new bounds on competing technologies or adjusting the investment cost of a technology based upon the cumulative capacity installed to date. The former capability is referred to as SAGE dynamic Market Share (MKTSHR) and the latter as Endogenous Technology Learning (SETL). The data requirements for both facilities were presented in Tables 4.7 and 4.11 earlier in this chapter, and are repeated here, where the algorithm employed is also described.

4.6.1 Market Share

The nature of the market share facility is for the modeler to identify market segments or "groups of competing technologies" (e.g., all alternative fueled passenger vehicles), then employ an algorithm after each period is solved to redistribute a portion of each such market segment to technologies that are nearly competitive (as measured by a function of their reduced costs). Below first each of the input parameters are explained in a bit more detail than was provided earlier in the tables, then the algorithm is elaborated.

4.6.1.1 Input parameters needed[1]

The input parameters are presented here in "logical" order based upon the order in which the modeler would tend to initiate, control and specify the Market Share algorithm and components.

"SAGE Activation" To request time-stepped running of the model the SAGE switch must be activated in the <case>.GEN/SLV templates[2]. The market share algorithm is applied between periods during a time-stepped model run only. A fatal error is given if the algorithm is requested and SAGE is not activated.

[$SET SAGE 'YES/NO', default 'NO']

"Market Share Algo." To request a SAGE that uses the market share algorithm, this switch must be activated in the <case>.GEN/SLV templates[3]. There are currently 3 versions of the algorithm implemented. Only one may be selected currently for a SAGE-TS run although the option to choose a different one for each market group will be implemented later.

– INV – uses the reduced cost associated with the investment variable;

[1] The input switch, set or parameter name, and default are mentioned in [] at the end of each component.
[2] This is accomplished at runtime by selecting MARKAL-SAGE from the ANSWER run form.
[3] Adjusted "semi-permanently" in VEDA-SAGE via the Run/Edit GEN/RegGEN/RegSLV option, or at Run time by requesting Edit of the GEN file before submitting. Make sure to include 'value'!!!

– **INVPCT –** uses (investment variable reduced cost/current investment cost), and

– **ACT –** uses the marginal cost associated with the activity of a technology (for DMDs only at this time!) The user should be aware that the 'ACT' option is currently limited in its use because of the implementation. Since an 'ACT' variable is the sum of all activities from a specific technology which may be available in multiple periods, a lower bound on such variables may not give the expected results since the technology may be attractive in one period but be seriously uneconomic in future periods.

Only one of these options may be selected per SAGE run.

[$SET MKTSHR 'INV/INVPCT/ACT', no default]

"Market Share Log" Indication of whether the full or only a partial trace of the market share algorithm actions are to be reported in the MKTSHR.LOG file[1].

It is suggested that the 'ALL' option be used first until the modeler has confidence that the case has been defined as intended, then switch to the 'MIN' option.

[$SET MKTSHRLOG 'MIN/ALL', default ALL].

"Market Group ID" A list of the market group names corresponding to each of the clustered candidates to be evaluated for a share of each such group.

Note that special treatment is given to the user-defined constraints when the market share is activated according to AMSWTCH, below.

[MKT_ID(*)]

"Use-defined/ Market Share Control" When the Market Share algorithm is activated certain groups may be in conflict with some user-defined constraints, and vise versa. How to handle such situations is controlled according to the provided by the modeler for AMSWTCH such that:

- 'A' - always generate the user-defined constraint, suppressing an associated MRK_ID group with the same name;
- 'B' – generate as an user-defined constraint when MKTSHR is not active or if no matching MKT_ID, but suppress if there is the same MKT_ID and the algorithm is activated, and
- 'X' - ignore the user-defined constraint if MKTSHR is active, regardless of MKT_ID match.

[1] Adjusted "semi-permanently" in VEDA-SAGE via the Run/Edit GEN/RegGEN/RegSLV option, or at Run time by requesting Edit of the GEN file before submitting. Make sure to include 'value'!!!

[AMSWTCH(adratio,'A/B/X'), default 'B'.]

"Market Groups" The 2-tuples of market group ID and technology that are to make up each market share group.

A critical test is performed to ensure that no technology appears in more than one market group. If this requirement is violated the model run is halted and a message presented on the screen and in the MKTSHR.LOG file. The reason the run is halted is that a different market share or lower bound would be developed from each group for the variable otherwise and it is unclear which market share should be honored.

[MKT_GRP(mkt_id,tch)]

"Close Enough" Definition of "close enough" to get a share of market is based on how close to being competitive a technology is. The "close enough" parameter is intended to partially correct for regional and market aggregation errors. For example, if we knew how much relevant prices varied in a region we could set "close enough" to be close to that the variation in a region in $/MMBtu or $/PT. Alternatively, we could use the rule that anything within 20% of the competitive price gets a share. Let π be the duals (reduced cost) on the technologies of interest; more accurately, let π be the measure of closeness used which could be the dual or reduced cost or a function of the reduced cost (reduced cost/investment cost) used to measure closeness.

A technology in a market group is allocated a share only if $abs(\pi_i) \leq CE_i$ (CE = close enough value). We make sure that no π equals zero! If any is equal to zero, a warning message is written out for the modeler that the variable is ignored in the market share reallocation, and goes on.

[MKT_CE(mkt_id), default = .2.]

Tech. Preference" It may be desirable to allow a weighted preference for a technology, α. This value will be applied to each technology as part of determining the market share to be credited to each technology in a group.

Why is a market preference weight needed? For certain markets, consumers make their decisions based on more than just least cost – some of which are cultural/regional in nature. For example, in the U.S., vehicle efficiency and cost are often subordinate to performance. Such regional biases can be reflected by choosing appropriate weights.

[MKT_PREF(mkt_id,tch), default = 1.]

"Gamma"	An indication of the degree of optimization to be applied when determining the shares for qualifying candidates in a market share group, δ. The larger the gamma, the more the shares are based on only cost. For end use markets, the gamma should be less than 0.3. For industrial and electricity generation choices, gamma should reflect consumers that are more optimizing. Gamma of about 2 is more appropriate for refineries and electric utility choices. [MKT_GAMA(mkt_id), default = 2.]
"Reallocation %"	Once the size of the entire market for market share group has been determined, the reallocation algorithm apportions this percent of the total market for re-distribution to the competing "non-winners" who were "close enough" in the initial solve in each period. [MKT_REAL(mkt_id), default = .2.]

4.6.1.2 Market Share Algorithm[1]

In this section the sequential actions taken by the code, and the implications, are elaborated to provide the modeler with an understanding of the way the algorithm is applied.

"Market Candidates"	Out of the list of market candidates originally provided by the modeler (MKT_GRP), a check is done to ensure that a technology has not been explicitly bounded to 0; if it is bounded to 0, it is unavailable to the model in the current period. Those not bounded to zero constitute the viable market candidates.
"Initial Solve"	With a small lower bound set for each of the market candidates, the model is solved so as to obtain a reduced cost for every non-basic candidate.
"Market Qualifiers"	A check is done to ensure that the "close enough" criteria are met, that is that the reduced cost of the candidates is below the criteria (and are not basic or upper bounded). These candidates then qualify for the reallocation algorithm. Those whose reduced costs are higher than the "close enough" criteria are deemed uncompetitive and thus not included in the group of qualified candidates.
"Size of Reallocation Group"	Based upon all the candidates in a market group, the market size is determined according to the level of the variables associated with each technology in the group for the period. A proportion of this market, according to the reallocation % (MKT_REAL), is then calculated for

[1] Each of the steps outlined in the algorithm are reflected in the MKTSHR.LOG file, in accordance with the level of details requested by the $MKTSHRLOG switch.

redistribution to the qualifying market candidates. This pool then corresponds to the market size, MKT_SZ.

"Market Share" Let π be the duals (reduced cost) on the technologies of interest. Let K be all candidates in the group, J be the set of technologies that are "close enough" to get a share of the market set aside. Let η be the modeler provided technology preference weight. Then

$$\text{Market Share}_j = \eta * \text{abs}(\pi_j)^{-\delta} / \{ \Sigma(\alpha * \text{abs}(\pi_k)^{-\delta}) \}$$
$$\text{for } k \in K,\ j \in J,\ \delta \subset (.1, 5)$$

where the larger the η, the more optimizing the choices. This is tracked as MKT_SHR(mkt_id,tch).

"Reallocation Value" Each qualifying market candidate, former "non-winner" meeting the "close enough" criteria, is then assigned a potential market share value:

Market penetration level for technology = MKT_SZ(mkt_id) * MKT_SHR(mkt_id,tch). This target level for all qualifying candidates is named MKT_VAL(tch).

"Integrity Test" If within a market group any proposed market value (MKT_VAL) exceeds the level obtained by any basic or upper bounded variable then a 2nd pass at assigning market shares is done. In this case all such basic or upper bounded variables having lower shares then the non-winner share are added to the market reallocation pool, and the market size (MKT_SZ) is increased by their level. Each of these former "winners" is then assigned a "dummy" reduced cost slightly smaller than the smallest reduced cost obtained by the "non-winners." This retains the relative attractiveness of the former "winners" but re-distributes the pool between all these closely competing technologies. The former "winners" are assigned upper bounds according to the share of the market suggested by the algorithm.

"Assignment of Lower Bound" Each of the original market qualifying candidates are assigned lower bounds at their target market level to ensure that they at least obtain the desired level in the solution.

"Resolving with Market Shares" With the market share criteria in place, the model is resolved for the current period. The final values obtained are then written to the LOG.

Thus as a result of the Market Share algorithm competing technologies are assured a share of the market based upon their relative attractiveness, and any preference imposed by the modeler.

4.6.2 Endogenous Technology Learning

The inter-period endenous technology learning feature of SAGE enables the modeler to identify new technologies for which a determination is to be made as to their future investment cost based upon the total cumulative build up of said technology from the beginning of the modeling horizon up until the current period being solved. Below first each of the input parameters are explained in a bit more detail than was provided earlier in the tables, then the algorithm is elaborated.

4.6.2.1 Input parameters needed[1]

The input parameters are presented here in "logical" order based upon the order in which the modeler would tend to initiate, control and specify the Market Share algorithm and components.

"SETL Activation"	To request inter-period learning the model must be run time-stepped, activitate by means of the $SET SAGE 'YES' switch, and then the SETL switch activated and the appropriate data provided. [$SET SETL 'YES/NO', default 'NO']
"Identification of Learning Technologies"	The subset of all technologies that are to be subject to the endogenous learning algorithm must be explicitly identified by the modeler. [TEG(tch)]
"Initial Cost"	The initial commerical cost associated with each learning technology. This initial investment cost will be subjected to the learning algorithm, and reduced according to the progress ratio specified by the modeler and the cumulative deployment level of the technology. [SC0(tch)]
"Progress Ratio"	The cost associated with learning technologies is a function of the cumulative deployment of the technology subject to the progress ratio. The progress ratio indicates how much this investment cost is to be lowered based upon each doubling of installed capacity. Note that another expression of this ration is called the "learning rate," and is defined as 1 – PRAT. [PRAT(tch)]

[1] The input switch, set or parameter name, and defaults are mentioned in [] at the end of each component.

“Initial Capacity Threshold” Learning with respect to actually reducing the cost of a new technology is assumed to begin only once the total cumulative capacity exceeds some minimal level as expressed by this parameter. Below this level the “initial cost” (SC0) continues to apply.

[CCAP0(tch)]

“Learning Spill” Between technologies, and regions, learning based upon progress with components associated with one technology may have beneficial “spill” learning effects on another technology. To allow for such shared learning (of components or across technologies/regions) the modeler can interrelate such technologies by means of this parameter. Note that the technology “spilling” does not need to be a learning technology itself, and that spill rate may change over time.

[LSPILL(tch,teg,tp).LSPILL_R(reg1,tch,reg2,teg,tp)]

4.6.2.2 Endogenous Learning Algorithm

The SAGE endogenous technology learning algorithm employs a standard representation of learning progress whereby the cost of a technology is assumed to be reduced by a certain amount for every doubling of the capacity. The progress ratio (PRAT) is the key parameter that controls this relationship.

Between each period, after all market share adjustments have been addressed, the solution for the current period is examined and the investment cost associated with the learning technologies calculated and prepared for the next period solve as described next.

Let $j \in J$ be the learning technologies, SC0j be the initial investment cost associated with each learning technology until such time as the cumulative capacity reaches the learning threshold (CCAP0[1]), and CCj be the total cumulative capacity installed to date (including any deployed prior to the modeling horizon (as expressed via RESID). Then the basic estimate of the cost of investing in technologies benefiting from learning is determined by:

$$SC_i = SC0_i * CC_j^{-(\ln(PRATj)/\ln(2))}$$

In this form, we accommodate international and cross-technology learning by means of a “spill” parameter (LSPILL_R(r’,t,r,j,year)), which is the fraction of the capacity of technology (t) in one region (r’) contributes to learning in the current region (r) of the learning technology (j). Note that the code sets LSPILL_R(r,j,r,j) = 1 (that is the learning technology learns from itself).

$$SC_{r,i} = SC0_{r,i} * [CC_{r,j} + SUM((r',t), lspill\_r_{r',t,r,j} * CC_{r',t})]^{-(\ln(PRATj)/\ln(2))}$$

Note that in both of the examples above the period index has been left off, but LSPILL_R may be expressed by period and thus could change over time.

So SC_i is the specific cost owing to the cumulative capacity buildup and learning rate which then corresponds to the new investment cost (INVCOST) to be used when calculating the actual annualized investment cost (COST_INV) for each learning technology in each region. Note that for conversion technologies (con) the investment transmission + distribution (ETRANINV/EDISTINV) is added to the total cost.

Thus although there is a “pot” for all capacity to go into, the learning that takes place in a particular region is a function of the capacity built in that region plus some weighted sum of the learning in other regions.

At the current time the initial values for the INVCOST and the SETL parameters are dumped before the solve loop begins in the list (LST) file. During each iteration SETL_CC/SC are output to the LST file. [So searching for SETL_CC is the way to see what is going on SETL wise.]

4.7 Bi-lateral and global trade in SAGE

In the early days of MARKAL the model was exclusively used to model a single autonomous region such as a country or a municipality. All movements of commodities (energy, materials, permits, etc.) into and out of this area of study were handled by the import/export supply step curves the modeler would specify. With this approach these so call SRCENCP options (where SRC = 'IMP'/'EXP', ENC = an energy carrier or material, and P = the price step) needed to be established for each such commodity, price and availability consideration that was to be reflected in the model.

But as MARKAL has evolved into a multi-region and now global modeling framework there is a fundamental shift resulting from the endogenizing of the trade activity. Thus in SAGE, instead of the modeler explicitly establishing a price/quantity supply step curve for the importing (and perhaps exporting) of a commodity, when actual trade between 2 or more players (regions) is represented then the price of the associated commodity is determined by the cost of extracting and/or processing the commodity in preparation for exporting, along with any additional transportation costs between specific regions.

To support endogenous trade a series of new Sets and Parameters have been introduced that permit the modeler to identify the commodities that are to be traded, and for bi-lateral trade the partners engaged in the trade. In addition, electricity trade can be further refined by time-slice if desired.VEDA-SAGE has elaborate, but simple to use, data handling facilities for managing the trade options and specification of associated data. See the SAGE User's Guide for details.

The trade data and implications on the model have been presented previously in sections 4.2 and 4.3 and 4.4 and 4.5 but are again discussed here as a specialized topic. The index abbreviations and associated SAGE sets are presented the same as those in table 4.15 of section 4.4. The variable letters and equation numbers correspond to the sequencing of the variables and equations in sections 4.4 and 4.5 respectively, and the table A1 SAGE_MATRIX.

4.7.1 Global Trade

4.7.1.1 Specifying Global Trade

Global trade allows the modeler to simply identify commodities potentially available for trade between any regions in a model. As such the commodity in question MUST have the same name in all the regions that are to be involved in the trade of this commodity. Then those regions that produce said commodity compete based upon their commodity supply costs against one another to provide the commodity to those regions needing it. This form of trade is on a level playing field, with all takers seeing the market at the same cost, with only the possibility to add a region specific transaction cost to the traded commodity price. Trade commences from a certain period as specified by the modeler for each commodity. The only way to limit the export or import levels from/to a region is by way of introducing an explicit bound on such activities. Table 4.19 below provides an overview of each of the Global trade data component. As already noted this information is also presented in tables in section 4.2 and 4.3 covering all user input data.

The price of globally traded commodities is determined from the costs of producing and processing such commodities, along with any region-based transportation and/or transaction costs.

Table 4.19: Global Trade Sets and Parameters

Sets & Parameter (Indexes)	Units/ Range & Defaults[1]	Instance[2] (Required/Omit/ Special Conditions)	Description[3]
G_TRADE (enc/mat/env)			Any commodity, including emissions but excluding electricity, that is to be traded globally.
REG_XCVT (r,enc/mat/env)	• Scalar. • Default = 1.	• Provided if different commodity units exist in a region.	Multiplier applied when balancing a globally traded commodity. • Applied to the global trade (R_GTRD) variable for the region in the global trade equation (MR_GTRD).
REG_XMONY (r)	• Scalar. • Default = 1.	• Provided if different monetary units exist in a region.	Multiplier applied in the objective function to each globally traded commodity. • Applied to the global trade (R_GTRD) variable for the region in the objective function (MR_PRICE for MARKAL, MR_MTSOBJ for SAGE).
TRD_BND (r,enc/mat/env,bd,t)	• Units of the commodity. • No default.	• Provided if a bound is to be applied.	Limit on the import or export of a globally traded commodity into/from a region. • Applied to the global trade (R_GTRD) variable for the region, period and import/export operation.
TRD_COST (r,enc/mat/env,bd,t)	• Base year monetary units. • [open]; default = 1.	• Provided if a transaction or transportation cost is to be added to the cost of a bi-lateral and/or globally traded commodity.	Additional (transaction) cost associated with global trade in a commodity. • Applied to the import/export (R_TSEP) and global (R_GTRD) trade variables in the regional objective function (MR_PRICE/MTSOBJ).
TRD_FROM (enc/mat/env)	• Year. • [open]; default = 1st.	• Provided if trade in a global commodity is not to start in the 1st period.	Year from which global trade in a commodity may commence. • The global trade constraint (MR_GTRD) is only generated beginning from the TRD_FROM period. • Entries for the global trade variable (R_GTRD) are only made in the associated balance (MR_BAL_G/E) and emissions (MR_TENV) equations beginning from the TRD_FROM period.

[1] For parameters.
[2] For parameters.
[3] The description provides an indication of how the parameter affects the model structure.

4.7.1.2 Implications on the Model Structure

The global trade facility requires the introduction of a new variable and equation, and entries in several other model equations. Table 4.20 below lists the variables and equations involved in the global trade facility. This information is also presented in section 4.4 and 4.5 where all variables and equations of SAGE are presented. After the table the math of the individual equations, or equation intersections, is presented. This is obviously a subset of the complete equation specification elaborated in section 4.5.

Table 4.20: Global Trade Variable and Equations

Variables & Equations[1] (Indexes) [#][2]	Nature	Description
MR_BAL_G/E (r,t,enc/mat) [2]	Add Variable	The balance equation ensures that the production of each commodity equals or exceeds the total consumption of said commodity. When the commodity is an energy carrier (enc), then the MR_BAL_G constraint permitting production to exceed consumption is applied. When the commodity is a material (mat), then the MR_BAL_E constraint applies which forces production to match consumption. The trade variable (R_GTRD) enters the equation is +/- for imports/exports. [Note that for imports any commodity overall transmission loss is applied.]
MR_GTRD (t,enc/mat/env) [16]	New	The balance of the globally traded commodities between all the producers and consumers of said commodities.
MR_PRICE (r) [22]	Add Variable	The MARKAL clairvoyant total discounted system cost for each region, summed to the objective function. The trade cost (TRD_COST) is applied to trade variable (R_GTRD).
MR_MTSOBJ (r) [19]	Add Variable	The SAGE time-stepped minimum annualized system cost for a region, summed to the objection function, and solved successively for each period of the model run. The trade cost (TRD_COST) is applied to trade variable (R_GTRD).
MR_TENV (r,t,env) [24]	Add Variable	The total amount of emissions generated in each period from all sources (resources and technologies) of said emissions indicator. The trade variable (R_GTRD) enters the equation is +/- for imports/exports.
R_GTRD (r,t,enc/mat/env,ie) [I]	New	The total amount of a globally traded commodity imported/exported to/from a region.

4.7.1.3 Mathematical Description of Global Trade

[1] In multi-region MARKAL all variables begin with a prefix of R_, and all equations have a prefix of MR_.
[2] [#] corresponds to the equation/variable number in the Table A1 SAGE_MATRIX.

The mathematics related to the part of each of the equations affected by the new global trade capability, listed in table 4.20, is presented here as a reduced subset of the complete mathematics elaborated in section 4.5.

$$4.5.2: MR\_BAL\_l_{r,t,e} \ \forall e \in (enc \vee mat),$$

where l = G (≥) for energy carriers, and E (=) for materials

PRODUCTION - commodities entering the system from Resource Supply options and produced by Technologies as main outputs or bi-products.

$$TE(ENT/MAT)_{r,e,t} * \langle$$

{various production activities …}

Import of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'IMP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right]$$

$$\rangle$$

$\{\geq;=\}$, ≥ for energy carriers (enc), and = for materials (mat)

CONSUMPTION - commodities leaving the system as Exports or consumed by Technologies according to their activity level.

{various consumption activities …}

Export of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'EXP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right]$$ [1]

$$4.5.16: MR\_GTRD_{t,g} \ \forall \begin{bmatrix} (g \subset enc \cup mat \cup env) \wedge \\ (t \geq TRD\_FROM_g) \end{bmatrix}$$

Imports (ie='IMP', SIGN = +1) and Exports (ie='EXP', SIGN = -1) of a globally traded commodity into/from each region. A region-based conversion factor may be applied, defaulted to 1.

$$\left[\sum_{r,ie} SIGN_{ie} * REG\_XCVT_{r,e} * G\_TRD_{r,t,g,ie} \right]$$

$$\{=\}$$

$$0$$

[1] In the code imports/exports are multiplied by SIGN(IE), where IE('MP'/'EXP') = 1/-1, but here the exports are depicted as the RHS of the equation.

$4.5.19 and 22: MR\_MTSOBJ_r$ and MR_PRICE_r [1]

{various cost components of the regional objective function …}

Transaction cost associated with a globally traded commodity charge in the current region.

$$\left[\sum_{\substack{e \in \left(\begin{array}{l} (ent \vee env \vee mat) \wedge \\ G\_TRADE \end{array} \right) \\ ie = 'IMP' \vee 'EXP' \\ t \geq TRD\_FROM}} \left(\begin{array}{l} R\_GTRD_{r,t,e,ie} * \\ TRD\_COST_{r,e,t} \end{array} \right) \right]$$ [2]

$4.5.24: MR\_TENV_{r,t,env}$

{various cost components of the regional emissions equations …}

[1] MR_PRICE is of course discounted, which is NOT reflected in the equation snippet.

[2] Note that SIGN does not play a role here, so if exports are to reduce the objective TRD_COST for an exporting region should be entered as a negative value. Furthermore, with only a single TRD_COST for both imports/exports the user needs to take care to avoid double-counting and should (probably) enter ½ the regional transaction/transportation estimates so that the sum approximates the total added cost.

Import or export of a globally traded emissions (or permit).

$$\left[\left(\sum_{ie} SIGN_{ie} * R\_GTRD_{r,t,env,ie} \right) \$ \left(\begin{matrix} env \in g\_trade \wedge \\ t \geq TRD\_FROM \end{matrix} \right) \right]$$

4.7.2 Bi-lateral Trade

4.7.2.1. Specifying Bi-lateral Trade

The bi-lateral trade capability allows the modeler to explicitly identify commodities available for trade between specific regions in a model. It builds on the conventional resource supply mechanism traditionally available in MARKAL models. That is it employs the same import/export options (SRCENCP = IMP/EXPentp, where ent may be energy or materials, and p is the price step or here the trade link) and variables (R_TSEP). The big difference is that bi-lateral trade is endogenous, and thus the price of the traded commodity is determined by the model, rather than specified exogenously (see next paragraph). So to use the bi-lateral trade the modeler must first provide entries in the resource supply set (SRCENCP) and associated data (e.g., OUT(ENT)r, BOUND(BD)r; but see next paragraph with regard to COST) for both the importing and exporting regions.

The named commodities, and their units, may be different between the pair of regions involved in the trade, but the price index (p) must be the same for each region. Thus the price index (p) identifies a particular trade route for each traded commodity. Furthermore, the bi-lateral trade specification is directional, so if a commodity can move in either direction then two trade set entries must be specified accordingly.

To accommodate the requirement that the price of bi-laterally traded commodities be determined by the costs associated with producing and processing such commodities in the exporting regions, plus any transaction/transportation costs, any conventional resource supply cost specified for the import/export options (via COST) are ignored (this is “nulled out” as reported in the QC_CHECK.LOG file). Note that any delivery costs (DELIV) associated with the consumption of commodities as part of the import/export activities are still applied. In addition, when electricity is involved in bi-lateral imports are still subject to the transmission investment (ETRANINV) and operating and maintenance (ETRANOM) costs associated with the grid. A transport and/or transaction cost can be specified for bi-lateral trade by means of the BI_TRDCST/BI_TRDCSTELC parameters (the latter for time-slice electricity trade, as discussed in the next paragraph).

Speaking of electricity. Besides permitting annual trade in electricity by means of the conventional resource supply variable (R_TSEP), time-sliced bi-lateral electricity trade may also be modeled (by means of MR_REGELC and R_TSEPE). For this a separate specification set (BI_TRDELC) is provided where the individual time-slices during which electricity trade is permitted can be enumerated. This set is directional, and time-slice sensitive. Due to the former, entries must be provided in both directions if the electricity can flow either way, and owing to the latter if a time-slice is omitted then no trade is permitted in that time-slice. Note that the conventional SRCENCP set entries **MUST** also be made for the time-sliced electric trade options. The data options specifically related to bi-lateral trade are presented in table 4.21 below. These are a subset of the full specification of the sets and parameters of the model presented previously in sections 4.2 and 4.3 respectively.

Table 4.21: Bi-lateral Trade Sets and Parameters

Sets & Parameter (Indexes)	Units/ Range & Defaults[1]	Instance[2] (Required/Omit/ Special Conditions)	Description[3]
BI_TRDENT (r,e,r,e,p)			The 2-tuple of region/commodity involved in bi-lateral trade. For each commodity an associated import/export supply option (srcencp) must also be specified for the same price set (p). Note that the trade is unidirectional, that is only from export to import region. • Controls the generation of the bi-lateral commodity trade constraint (MR_BITRD) between two regions.

[1] For parameters.
[2] For parameters.
[3] The description provides an indication of how the parameter affects the model structure.

Sets & Parameter (Indexes)	Units/ Range & Defaults[1]	Instance[2] (Required/Omit/ Special Conditions)	Description[3]
BI_TRDELC (r,elc,r,elc,p)			The 2-tuple of region/electricity involved in bi-lateral trade of electricity by season/time-of-day. For each form of the electricity energy carrier an associated import/export supply option (srcencp) must also be specified for the same price set (p). Note that the trade is unidirectional, that is only from export to import region, and that it only occurs for the time-slices specified. • Controls the generation of the bi-lateral electricity trade constraint (MR_BITRDE) between two regions for the time-slices specified. • The time-sliced electricity trade variable (R_TSEPE) is set equal to the traditional trade variable (R_TSEP), which is then used in the objective function, emission constraints, etc.; the seasonal variable appears directly in all the electricity specific constraints (MR_BALE, MR_BAS, MR_EPK), as well as the objective function (MR_PRICE/MTSOBJ).
BI_TRDCST (r,ie,ent,p)	• Monetary units. • [open]; no default.	• Omit if not desired.	The additional "transaction' cost for bilateral trading a particular commodity. • Cost multiplier in the regional objective function (MR_PRICE/MTSOBJ) for a traded commodity (R_TSEP).
BI_ TRDCSTELC (r,ie,e,p,td)	• Monetary units. • [open]; no default.	• Omit if not desired.	The additional "transaction' cost for bilateral trading of electricity for each time-slice. • Cost multiplier in the regional objective function (MR_PRICE/MTSOBJ) for traded electricity (R_TSEPE).

4.7.2.2. Implications on the Model Structure

The bi-lateral trade facility requires the introduction of a new variable for time-sliced electricity trade, new equations to match up the trading partners, and entries in several other model equations. Table 4.22 below lists the variables and equations involved in the global trade facility. After the table the math of the individual equations, or equation intersections, is presented. These are a subset of the full specification of the variables and equations of the model presented previously in sections 4.4 and 4.5 respectively. Note that since the traditional MARKAL trade variables are employed there is minimal impact on the existing model equations, other than for time-sliced electricity trade.

Table 4.22: Bi-lateral Trade Variable and Equations

Variables & Equations[1] (Indexes)	Nature	Description
MR_BALE1/2 (r,t,elc,td) [4]	Add bi-elc Variable	The balance equation that ensures that the total amount of electricity produced in each time-slice (season/time-of-day) meets or exceeds that demanded in said time-slice. Equation 1 is for daytime constraints (w = z,'D') and equation 2 is for the nighttime constraints (w = z,'N'). The time-sliced bi-lateral electricity trade variable (R_TSEPE) needs to be included, where permitted, with the appropriate sign.
MR_BAS (r,t,elc,z) [5]	Add bi-elc Variable	Ensures that those power plants designated as baseload (bas) operate at the same level in the day and night, while not exceeding a percentage of the highest nighttime electricity demand (according to BASELOAD). The time-sliced bi-lateral electricity trade variable (R_TSEPE) needs to be included, where permitted, with the appropriate sign.
MR_BITRD (r,e,r,t,e,p) [6]	New	Matches up the bi-lateral trade of a commodity between two regions. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (p) must be used.
MR_BITRDE (r,elc,r,t,elc,td) [7]	New	Matches up the bi-lateral trade of electricity between two regions during a particular time-slice. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (p) must be used.
MR_EPK (t,elc,z) [18]	Add bi-elc Variable	The electricity peaking constraint ensures that there is enough capacity in place to meet the highest average electricity demand during the day of any season + estimated level above that for the actual peak (moment of highest electric demand) + a reserve margin of excess capacity (ERESERV includes both), to ensure that if some plants are unavailable said demand for electricity can still be met. The time-sliced bi-lateral electricity trade variable (R_TSEPE) needs to be included, where permitted, with the appropriate sign.
MR_MTSOBJ (r) [19]	Add bi-elc Variable	The SAGE time-stepped minimum annualized system cost for a region, summed to the objection function, and solved successively for each period of the model run. The trade cost (BI_TRDCST/ELC) is applied to the regular/time-sliced electricity trade variable (R_TSEP/TSEPE), where desired.
MR_PRICE (r) [22]	Add bi-elc Variable	The MARKAL clairvoyant total discounted system cost for each region, summed to the objective function. The trade cost (BI_TRDCST/ELC) is applied to the regular/time-sliced electricity trade variable (R_TSEP/TSEPE), where desired.
R_TSEPE (r,t,ie,elc,p,w) [T]	New	The time-sliced electricity trade variable, which is mapped to the regular annual electricity variable (R_TSEP) by means of the MR_REGELC equation. The R_TSEPE variable does however replace the R_TSEP electric variable in the balance, peaking, et al equations.
MR_REGELC (r,t,ie,elc,p) [23]	New	Balance equation tying of the time-sliced electricity trade variables (R_TSEPE) to the regular annual electricity variable (R_TSEP).

[1] In multi-region MARKAL all variables begin with a prefix of R_, and all equations have a prefix of MR_.

4.7.2.3. Mathematical Description of Bi-lateral trade

The mathematics related to the part of each of the equations affected by the bi-lateral trade capability, listed in table 4.22, is presented here. These are a subset of the full specification of the variables and equations of the model presented previously in sections 4.4 and 4.5 respectively.

$$4.5.4: MR\_BALEl_{r,t,e,td} \ \forall \begin{pmatrix} (e \in elc) \wedge \\ QHR_{r,w} exists \end{pmatrix}$$

where for td(z,y) l = 1 y = 'D' (day) and 2 for y = 'N' (night)

PRODUCTION – of electricity from imports and power plants feeding the current electricity grid.

{various electricity production activities …}

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='IMP'}} \left\{ TE(ENT)_{r,e,t} * \left(\left(\left\langle \begin{array}{l} QHR(Z)(Y)_{r,td} \$ (no\, SEP\_FR_{r,s,w,t}) \vee \\ (SEP\_FR_{r,s,td,t} \end{array} \right\rangle * R\_TSEP_{r,t,s} \right) \$ \left(not \sum_{r',e',p} BI\_TRDELC_{r',e',r,e,p,td} \right) + \left(R\_TSEPE_{r,t,s,td} \$ \sum_{r',e',p} BI\_TRDELC_{r',e',r,e,p,td} \right) \right) \right\} \right]$$

$$\{\geq\}$$

CONSUMPTION - commodities leaving the system as Exports or consumed by Technologies according to their activity level.

{various electricity consumption activities …}

Export of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s\forall OUT(ENT)x_{r,s,e,t}\\ x='EXP}} \left\{\begin{array}{l}\left(\left\langle\begin{array}{l}QHR(Z)(Y)_{r,td}\$(noSEP\_FR_{r,s,td,t})\vee\\ SEP\_FR_{r,s,td,t}\end{array}\right\rangle * \$\\ R\_TSEP_{r,t,s}\end{array}\right)\\ \left(not\sum_{r',e',p} BI\_TRDELC_{r,e,r',e',p,td}\right)+\\ \left(R\_TSEPE_{r,t,s,td}\$\sum_{r',e',p} BI\_TRDELC_{r,e,r',e',p,td}\right)\end{array}\right\}\right]$$

4.5.5: $MR\_BAS_{r,t,e,z}\ \forall(e \in elc) \wedge QHRZ_{r,z} exists$

Contribution to meeting the base load requirement, that is that the nighttime electric generation from base load plants does not exceed a specified percent of total nighttime demand for electricity in each season.

{various base load plant nighttime electricity generation …}

Import of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='IMP'}} \left\{ \begin{array}{l} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left(\left\langle \begin{array}{l} QHR(Z)(Y)_{r,z,'N'} \$ (no\,SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',p} BI\_TRDELC_{r',e',r,e,p,z,'N'} \right) \end{array} \right\} \right]$$

$$\{\leq\}$$

Electric generation by non-base load power plants at night.

{various non-base load plant nighttime electricity generation …}

Export of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='EXP'}} \left\{ \begin{array}{l} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left(\left\langle \begin{array}{l} QHR(Z)(Y)_{r,z,'N'} \$ (no\ SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',p} BI\_TRDELC_{r,e,r',e',p,z,'N'} \right) \end{array} \right\} \right] +$$

$$\boldsymbol{MR\_BITRD_{r,e,t,r',e',p} \ \forall}$$

$$\boldsymbol{\left(\begin{array}{l} BI\_TRDENT_{r,e,t,r',e',p} \wedge \\ \left(TPSEP_{r,t,'EXP',e,p} \wedge TPSEP_{r',t,'IMP',e',p} \right) \end{array} \right)}$$

Exports from region r, possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the price index (p) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEP_{r,t,"EXP",e,c} \right\rfloor -$$

Imports to region r', possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the price index (p) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r',e'} * R\_TSEP_{r',t,''IMP',e',c} \right\rfloor$$

$$\{=\}$$

$$0$$

Where bi_trdent = mapping of permitted bi-lateral trade options
tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

4.5.4: $MR\_BITRDE_{r,e,t,r',e',p,td}\ \forall$

$$\left(\begin{array}{l} e \in elc \wedge BI\_TRDELC_{r,e,t,r',e',p,td} \wedge \\ \left(TPSEP_{r,t,''EXP'',e,p} \wedge TPSEP_{r',t,''IMP'',e',p} \right) \end{array} \right)$$

Exports from region r, possibly with unit conversion applied for the electricity. [Note that while the commodities may have different names in each region, the cost index (p) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEPE_{r,t,'EXP',e,p,td} \right\rfloor -$$

Imports to region r', possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the price index (p) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r',e'} * R\_TSEPE_{r',t,'IMP',e',p,td} \right\rfloor$$

$$\{=\}$$

$$0$$

Where bi_trdelc = mapping of permitted bi-lateral electric trade options, including permitted time-slices

tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

$$4.5.5: MR\_BAS_{r,t,e,z} \; \forall (e \in elc) \wedge QHRZ_{r,z} exists$$

Contribution to meeting the base load requirement, that is that the nighttime electric generation from base load plants does not exceed a specified percent of total nighttime demand for electricity in each season.

{various base contribution power plants …}

Import of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e,t}\\ x='IMP'}} \left\{\begin{array}{l} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left(\left\langle\begin{array}{l} QHR(Z)(Y)_{r,z,'N'} \$ (no\,SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{array}\right\rangle * \$ \\ R\_TSEP_{r,t,s} \right) \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'}\right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'}\right) \end{array}\right\}\right] +$$

$$\{\leq\}$$

Electric generation by non-base load power plants at night.

{various non-base load generation contributing to the overall base load requirements …}

Export of nighttime electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction)

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e,t}\\ x='EXP'}} \begin{Bmatrix} TE(ENT)_{r,e,t} * (1-BASELOAD_{r,e,t}) * \\ \left(\left\langle \begin{matrix} QHR(Z)(Y)_{r,z,'N'} \$ (no\ SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) \end{Bmatrix} \right] +$$

$$4.5.13: MR\_EPK_{r,t,e,z} \ \forall \begin{pmatrix} e \in elc \wedge \\ z \neq 'I' \wedge \\ QHRZ_{r,z} exists \end{pmatrix}$$

PEAK Contribution –imports and peak technology capacity credit towards the peaking requirement, as well as grid interchanges.

{various peak contribution power plants …}

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice. The contribution is subject to transmission losses as well as the reserve margin requirements.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ x='IMP'}} \begin{Bmatrix} PEAKDA(SEP)_{r,s,e,t} * TE(ENT)_{r,e,t} / (1+ERERSEV_{r,e,t}) * \\ \begin{pmatrix} \left\langle \begin{matrix} 1\$(noSEP\_FR_{r,s,z,'D',t}) \vee \\ (SEP\_FR_{r,s,z,'D',t} / QHR(Z)(Y)_{r,z,'D'} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \end{pmatrix} \$ \\ \left(not \sum_{r',e',p} BI\_TRDELC_{r',e',r,e,p,z,'D'} \right) + \\ \begin{pmatrix} \left\langle R\_TSEPE_{r,t,s,z,'D'} / QHR(Z)(Y)_{r,z,'D'} \right\rangle \$ \\ \sum_{r',e',p} BI\_TRDELC_{r',e',r,e,p,z,'D'} \end{pmatrix} \end{Bmatrix} \right]$$

$$\{\geq\}$$

CONSUMPTION – of electricity by resource options and technologies, and as part of grid interchanges.

{various demand for electricity during the day to contributing to the peaking requirements …}

Export of electricity occurring during peak times (PEAKDA(SEP)), either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$
\left[\sum_{\substack{s \forall OUT(ENT)x_{r,s,e,t} \\ x='EXP'}} \begin{Bmatrix} PEAKDA(SEP)_{r,s,t} * \\ \left(\left\langle \begin{matrix} 1\$(no\,SEP\_FR_{r,s,z,'D',t}) \vee \\ SEP\_FR_{r,s,z,'D',t} / QHR(Z)(Y)_{r,z,'D'} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',p} BI\_TRDELC_{r,e,r',e',p,z,'D'} \right) + \\ \left(\left\langle R\_TSEPE_{r,t,s,z,'D'} / QHR(Z)(Y)_{r,z,'D'} \right\rangle \$ \\ \sum_{r',e',p} BI\_TRDELC_{r,e,r',e',p,z,'D'} \right) \end{Bmatrix} \right]
$$

$4.5.19$ *and* $22: MR\_MTSOBJ_r$ *and* MR_PRICE_r [1]

{various cost components of the regional objective function …}

Annual cost of imports and export, plus any delivery costs associated with ancillary commodities involved in the import/export process, according to the level of the resource activity. Note that if the trade is expressed as bi-lateral trade then any resource supply cost (COST) is ignored and only the transport/transaction cost applied. Also, if importing electricity then transmission and distribution O&M costs are incurred.

[1] MR_PRICE is of course discounted, which is NOT reflected in the equation snippet.

$$\sum_{\substack{s \in tpsep \\ x='IMP' \vee 'EXP' \\ e \notin (enu \vee \\ ecv \vee lth)}} \left[R\_TSEP_{r,t,s} * \left\langle \begin{matrix} SIGN_x * \begin{pmatrix} COST_{r,s,t} \$ not\ BI\_TRDENT + \\ BI\_TRDCST_{r,s,t} \$ BI\_TRDENT \end{pmatrix} + \\ (ETRANOM_{r,e,t} + EDISTOM_{r,e,t}) \$ \begin{pmatrix} e \in elc \wedge \\ x='IMP' \end{pmatrix} + \\ \sum_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{matrix} \right\rangle \right] +$$

Annual cost of imports and export electricity by timeslice, according to the level of activity. Any bi-lateral, time-sliced transport/transaction cost is applied here, and any other delivery or transmission and distribution costs are captured in the

$$\left[\sum_{\substack{s \subset tpsep, w \\ e \in elc \\ BI\_TRDELC}} \begin{pmatrix} R\_TSEPE_{r,t,s,w} * \\ SIGN_x * BI\_TRDCSTE_{r,s,w} \end{pmatrix} \right]$$

$$4.5.23: MR\_REGELC_{r,t,s}\ \forall s \in tpsep \wedge$$

$$\left[s \subset ('IMP' \vee 'EXP' + e = elc + p) \wedge \sum_{r',e',td} \begin{pmatrix} BI\_TRDELC_{r,e,r',e',p,td} \vee \\ BI\_TRDELC_{r',e',r,e,p,td} \end{pmatrix} \right]$$

Standard annual resource supply option corresponding to electricity import/export.

$$\left[R\_TSEP_{r,t,s} \right] -$$

Bi-lateral, time-slice electricity trade summed of each time-slice.

$$\left[\sum_{td} \left(R\_TSEPE_{r,t,s,td} \right) \right]$$

$$\{=\}$$

$$0$$

Where bi_trdelc = bi-lateral time-sliced electric trade options
tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

4.7.3 Trade Output and Reporting

The bi-lateral and global trade activity and costs from each run are echoed in the <case>.LST (associated with the actual solve of the multi-region model) and prepared for post-processing with VEDA. Since the import/export trade flows are basically just another energy flow, their levels are associated with the VAR_FIN/VAR_FOUT results parameters in VEDA. Examples of trade related VEDA results tables can be found in the SAGE User's Guide.

Besides the specific trade parameters and results reflected in VEDA, following the results section (solution listing) of a multi-region run the information below is "dumped" and can be found in the <case>.LST file. Note that since the values are simply dumped the marginals reported (those entries qualified by .M) have not been undiscounted!

```
---- 58083 EQUATION  MR_BITRD.M    Bi-Trade Balance             =E=

               ( ALL       0.000 )

---- 58083 EQUATION  MR_BITRDE.M   Bi-Trade Balance for electricity =E=

               ( ALL       0.000 )

---- 58083 EQUATION  MR_GTRD.M     Global Trade Balance         =E=
```

```
          CO2
2000      0.287
2010      1.182
2020      9.800

----  58083 VARIABLE  R_NTXTRD.L    Region's Global Trade

                  CO2
TESTX1.1990 -1.33191E+6
TESTX1.2000   -4093.878
TESTX1.2010   24345.999
TESTX1.2020   27163.617
TESTX2.1990 -1.33191E+6
TESTX2.2000    4093.878
TESTX2.2010  -24345.999
TESTX2.2020  -27163.617

----  58083 VARIABLE  R_NTXTRD.M    Region's Global Trade

                  ( ALL       0.000 )
```

As part of handling trade the QA_CHECK.LOG routine reflects some checks and changes that are necessary. Specifically, checks are done to ensure that bi-lateral traded commodities are not also involved in global trade, and that SRCENCP entries exist in each region. Also the “P”rice index used must match up with the corresponding SRCENCP entry in each region for the “IMP”/”EXP” and ent/mat involved in any specified bi-lateral trade. In addition checks are made to ensure that not both annual and bi-lateral electricity trade are specified for an energy carrier/material.

As noted earlier, when involved in bi-lateral trade the associated SRCENCP entries must exist in each region, but any resource supply cost (COST) associated with the import/export is ignored (“nulled out), as the price of such traded commodities is determined by the model based upon the resource supply and processing options that are employed are part of making such commodities available in the exporting region. The “nulling” of the COST is also reported in the LOG. An example of this as found in the QA_CHECK.LOG is shown below.

```
*============================================================*
     Multi-region Run: Checking Trade Tables
*============================================================*

                ***** QUALITY ASSURANCE LOG *****

 *** Bi-lateral Electric Trade SRCENCP COSTs Ignored ***
   * NOTE        - REG/ELC:REG2/TELC:P DEMO/ELC:UTOPIA/ELC:1
```

4.7.4 VEDA-SAGE[1] Handling of Bi-lateral Trade[2]

VEDA-SAGE focuses only on bi-lateral trade, employing a very visual approach to supporting the specification of the permitted trade patterns and associated data. When the modeler indicates that he/she wants to work on trade related input information, by means of the Module-2 pull-down menu Trade option, the form in Figure A1 is presented. This form provides access to the two basic functions of establishing the permitted trade patterns, by means of the Trade links matrix shown in Figure A2, and entering data. With respect to the latter, either a single period matrix oriented view (see Figure A3) or a time series spreadsheet view (see Figure A4) may be employed.

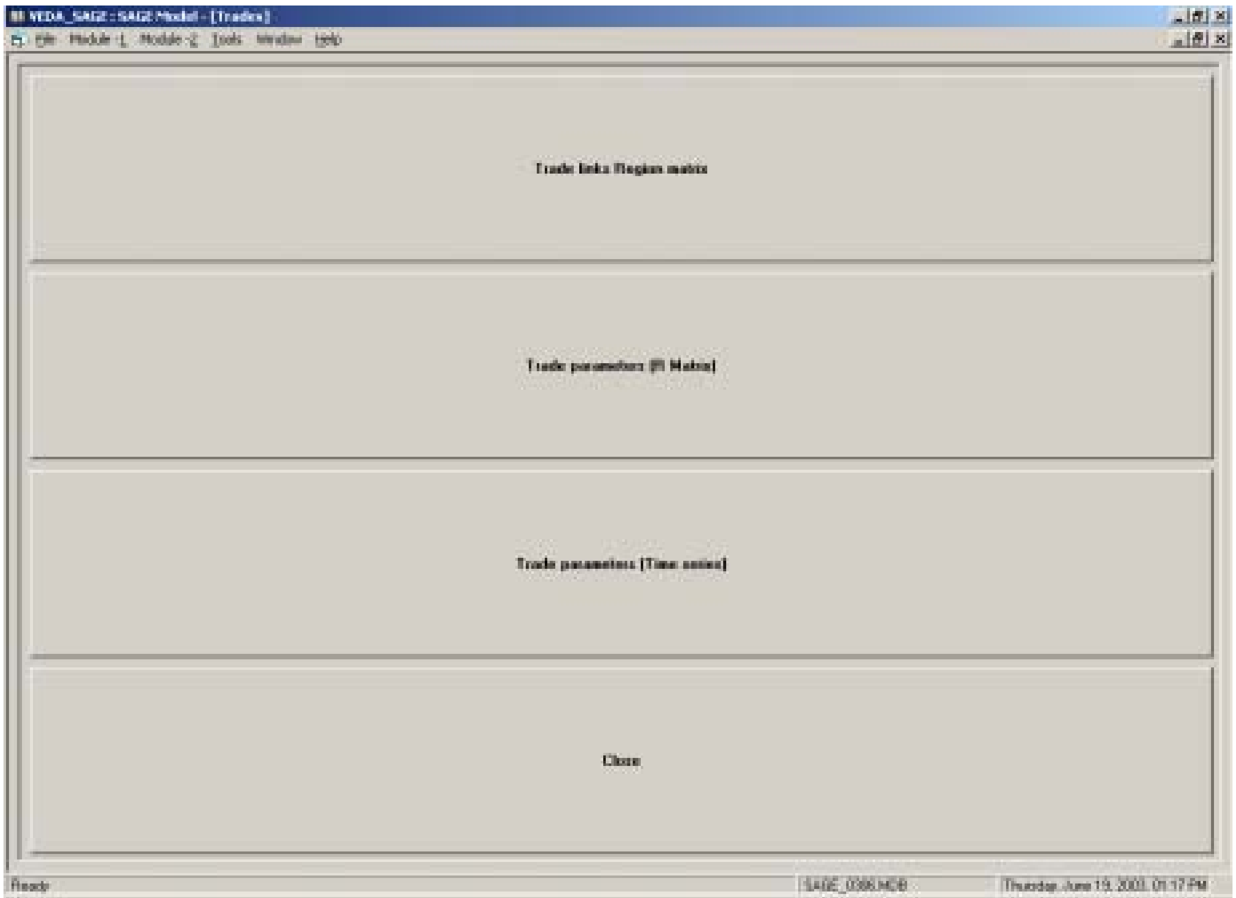


Figure A1: VEDA-SAGE Trade Options Menu

[1] VEDA-SAGE currently fully supports all aspects of working with the multi-region SAGE model when coupled with the spreadsheet template subsystem. It is expected that VEDA-SAGE will fully support all variants of MARKAL, as well as TIMES, in the near future. It does generate and run both of the latter, but additional features are needed for general-purpose use. However, at the moment it remains a SAGE-dedicated platform.

[2] This appendix is just meant to summarize the trade related data, running and results aspects of VEDA-SAGE. The reader is referred to the VEDA-SAGE and VEDA-BE User's Guides for complete information on using SAGE under VEDA.

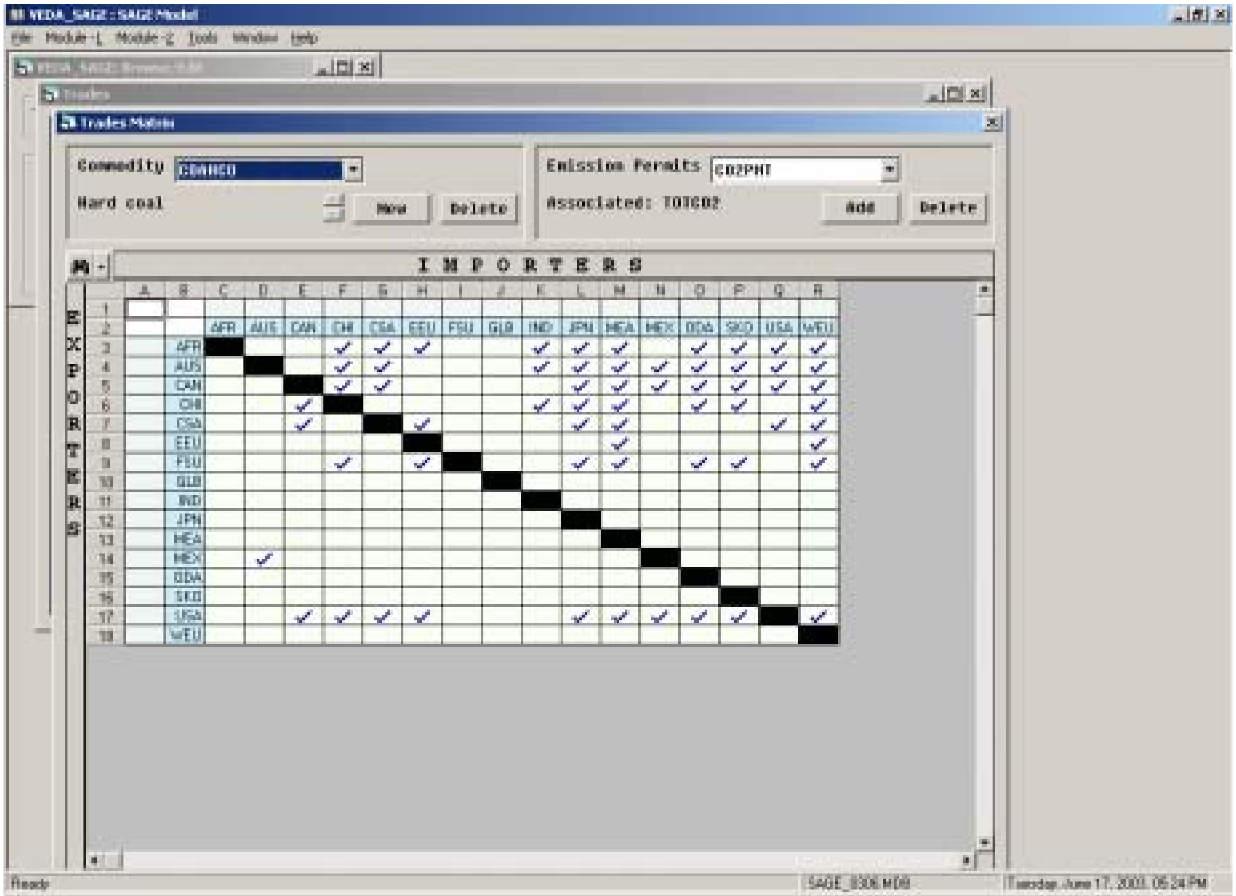


Figure A2: VEDA-SAGE Trade Specification Matrix

The Trade Specification matrix allows the user to very conveniently identify the individual import/export regions involved in bi-lateral trade commodity by commodity. The modeler simply identifies the commodity and then checks off the valid trading partners. It is assumed that the commodity name is the same in both regions, though by double-clicking on the cell above/next to the region name a region-specific commodity may be entered. Note that since VEDA-SAGE is SAGE-centric at the current time it does not provide for unit conversion (as all commodities in all regions are the same unit in SAGE). VEDA-SAGE then automatically generates a “P”rice index for each trade route and the associated SRCENCP entries for each region.

With the permitted trade routes established data can be entered. As just mentioned there are to ways to approach providing the trade data. As shown in Figure A3 a matrix approach presents the same matrix layout as the specification form shown in Figure A2, but with the cells into which data can be entered ready for input, and the unlinked pairs darkened out. Up top the user must identify the commodity, parameter (and any qualifier (e.g., bound type)), and period for which data is to be entered. Then where appropriate the individual data values may be entered. If data is entered into a shaded cell, the trade routes, as defined in the Trade Specification Matrix, are updated to include said new link.

Note that since data (including bounds) is interpolated if flat or linearly changing values are to be entered then values only need to be provided for the 1st and last periods.

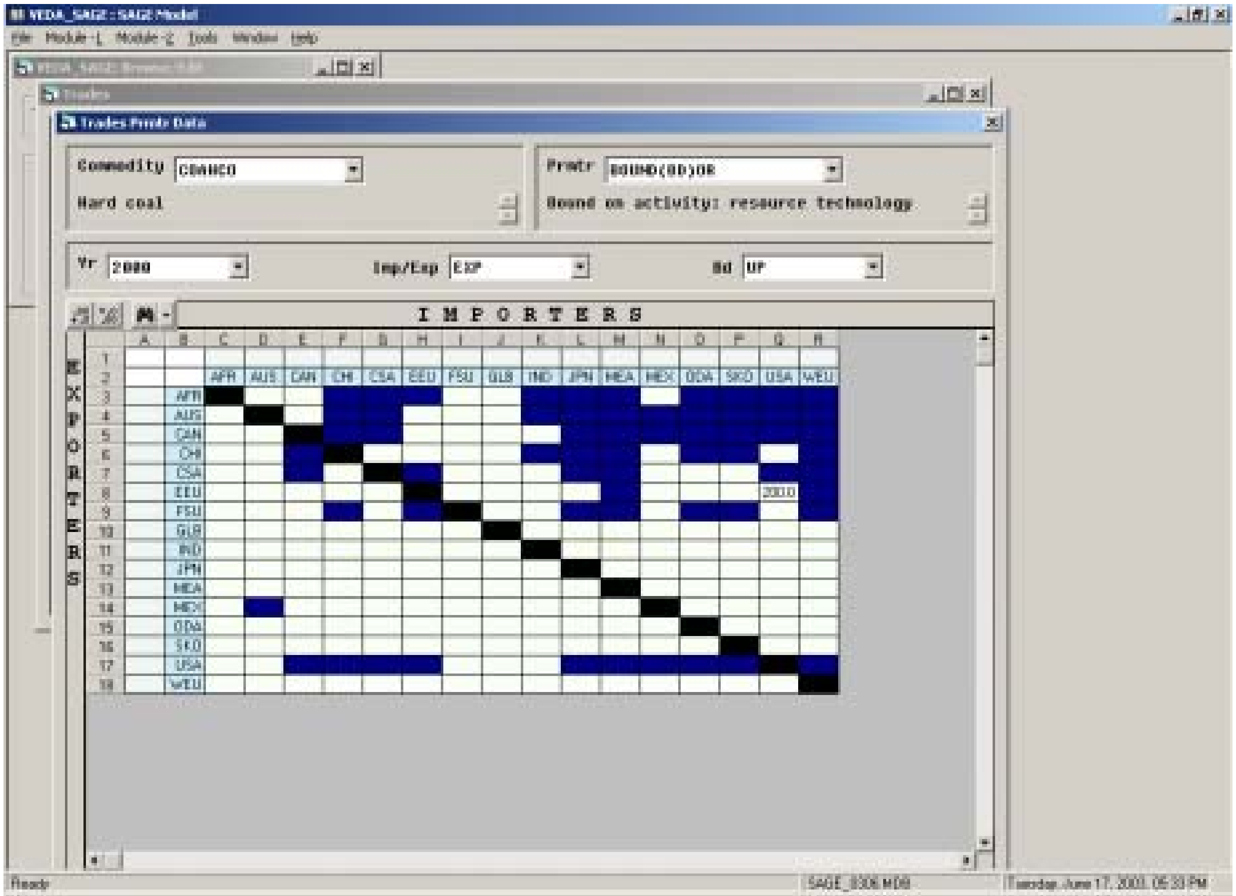


Figure A3: VEDA-SAGE Matrix-view Data Specification

The other means to enter trade data is in a spreadsheet or time series oriented manner, as shown in Figure A4. Here again the individual parameter and commodity that is to be worked on must be entered first, with importer and/or exporter if desired, to establish the data view. Then pressing the magnifying glass button brings up the basic spreadsheet. If data already exists for the specified parameter said information is displayed, as shown in Figure A4.

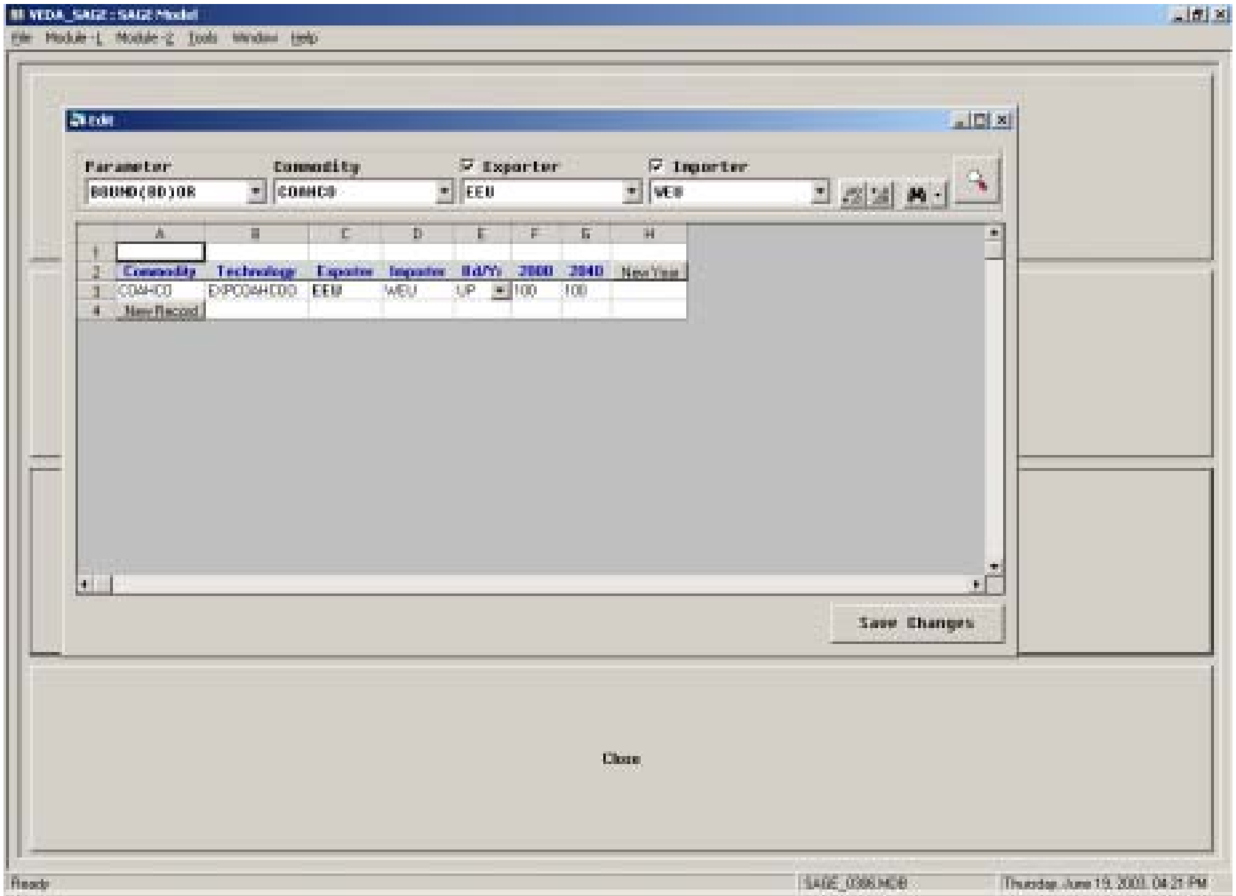


Figure A5: VEDA-SAGE Spreadsheet of Existing Data

Otherwise, to complete the preparation of the spreadsheet the New Year button is hit and the desired years for which data is to be entered selected, as shown in Figure A5. If the importer/exporter was specified up top then your all set. Now each trade parameter can be specified by hitting new record and completing series of dialog boxes that prompt the user for the export/import region, as shown in Figure A6 if not specified up top, and any qualifier needed (e.g., bound type, emissions indicator). Then the data can be entered into the appropriate period cells of the spreadsheet. Using this approach changing data can be more conveniently entered than in the matrix view.

Note that all VEDA-SAGE spreadsheet forms allow for convenient cut-and-paste to/from Excel to facilitate moving data from outside sources.

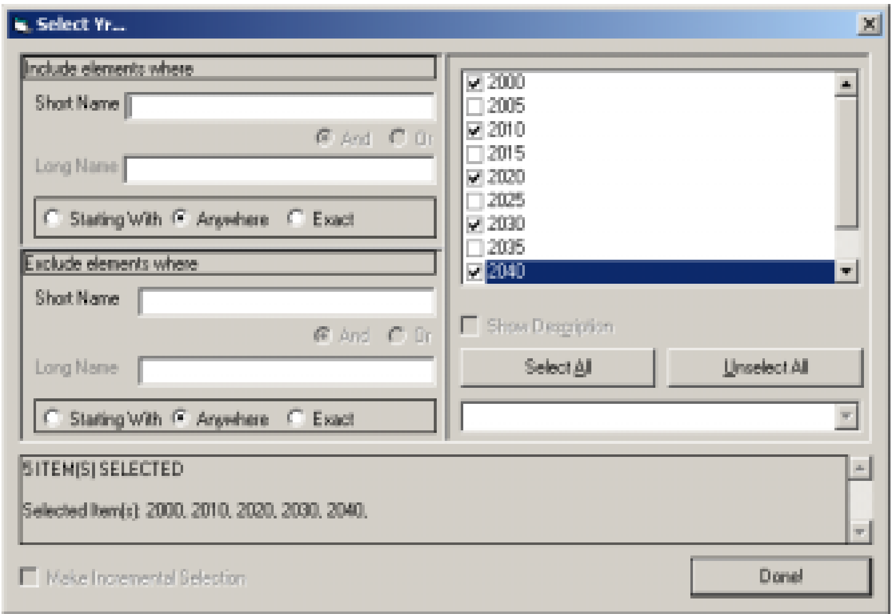


Figure A5: VEDA-SAGE Spreadsheet Year Selection

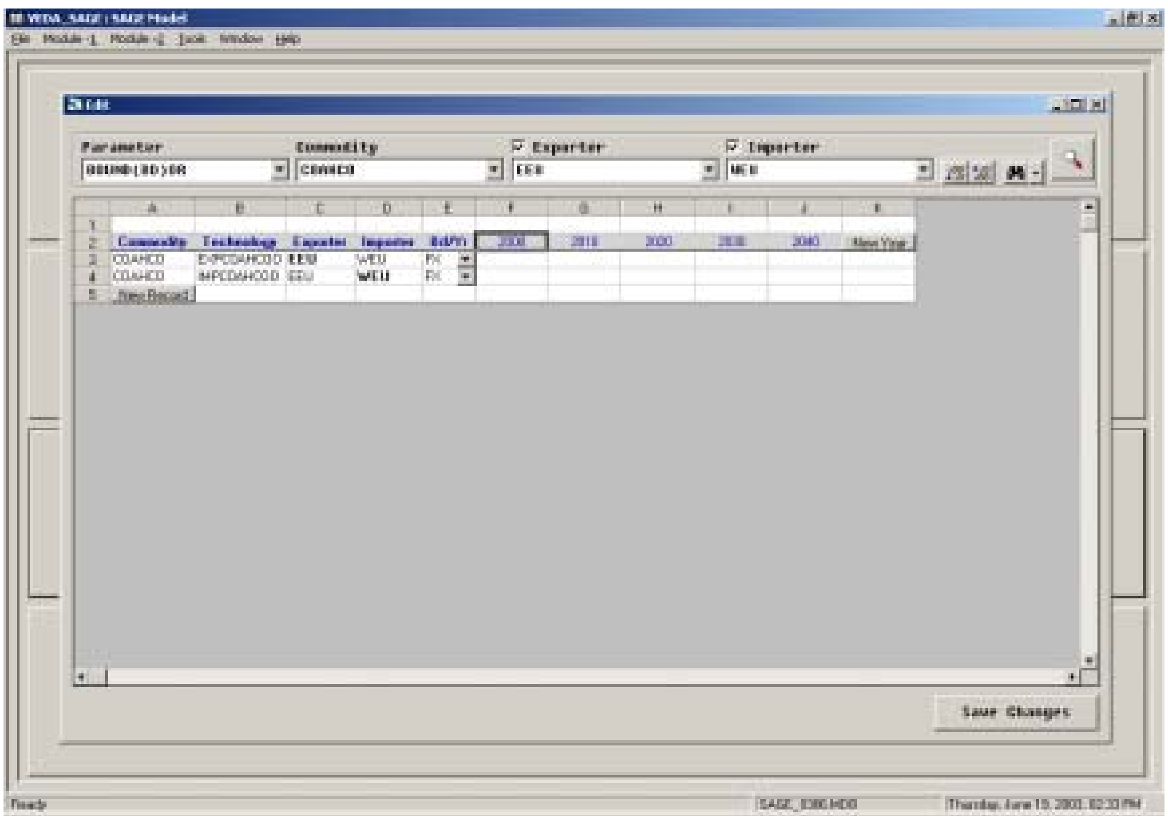


Figure A6: VEDA-SAGE Spreadsheet Times Series Specification

Once all data specification has been completed a model run can be submitted. The VEDA-SAGE run form is shown in Figure A7. SAGE is the only model variant supported at the moment, but the run options may be specified by means of the Set Options button, with the resulting form shown in Figure A8.

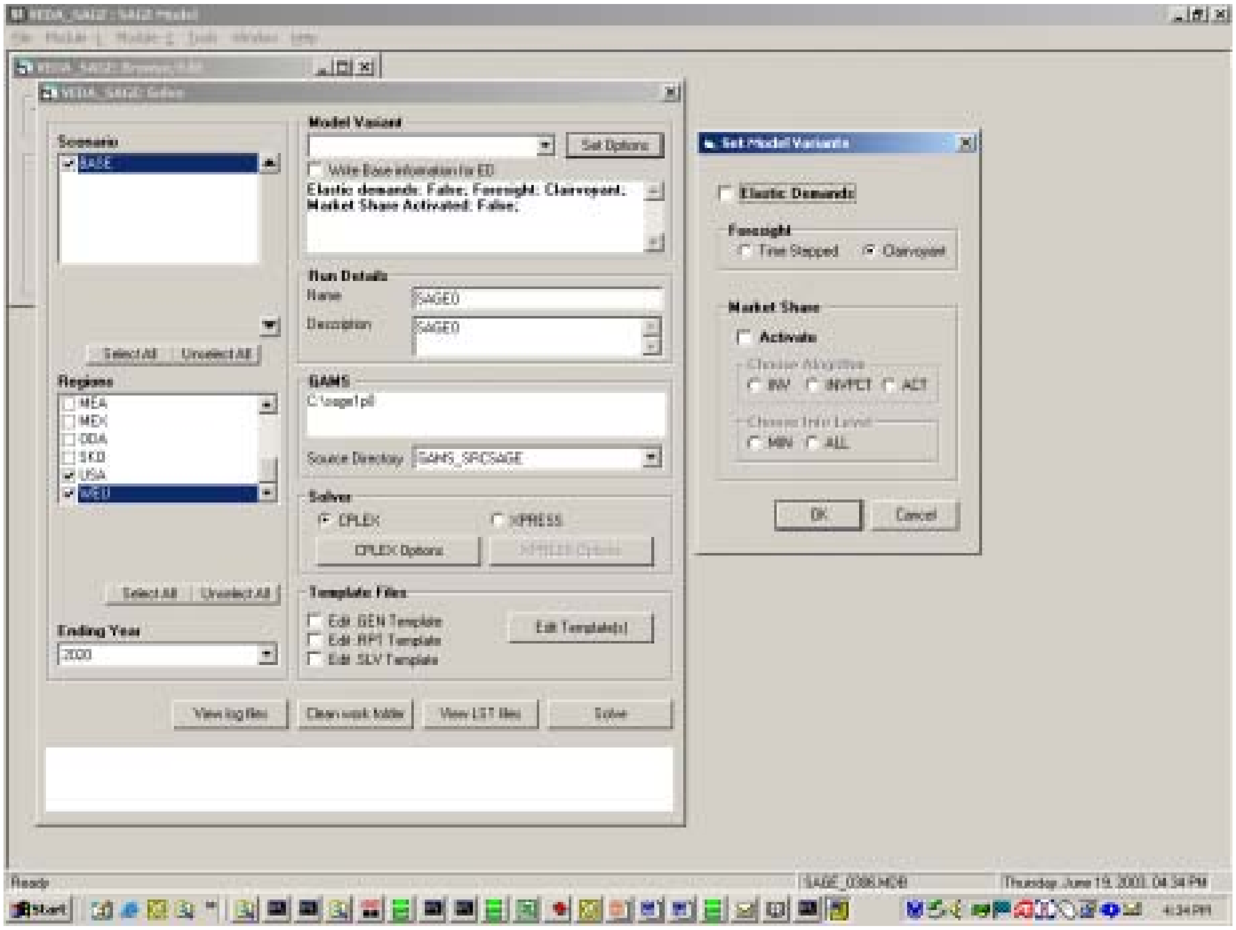


Figure A7: VEDA-SAGE Run & Variant Setting Forms

On the Run form the scenarios to be combined, regions to be included, path to the working and source code directories can all be specified. If desired the Solver (CPLEX or XPRESS) options may be adjusted using the form presented in Figure A8. Upon saving the desired options VEDA-SAGE writes a CPLEX/XPRESS.OPT file to the run directory.

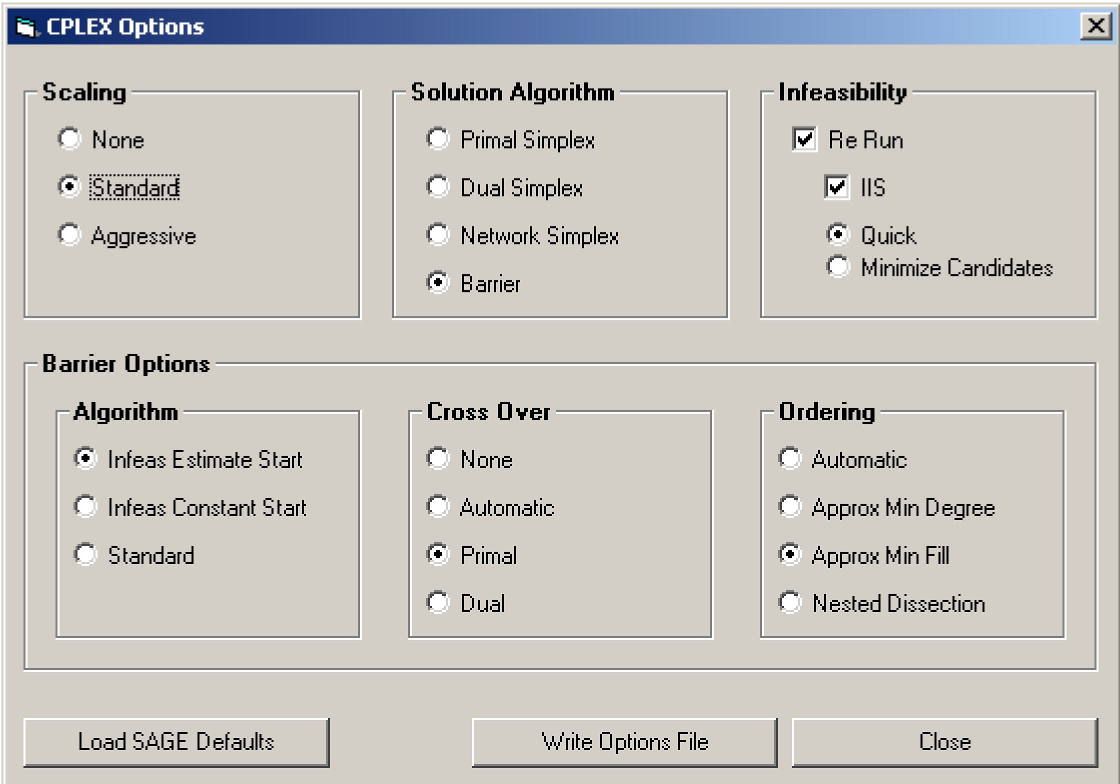


Figure A8: VEDA-SAGE CPLEX Options Form

Finally, if the GEN/SLV/RPT templates are to be adjusted manually (e.g., to activate the equation listing) the corresponding check box(es) can be activated. The requested files are then presented inside of Workpad for editing as desired, as shown in Figure A9.

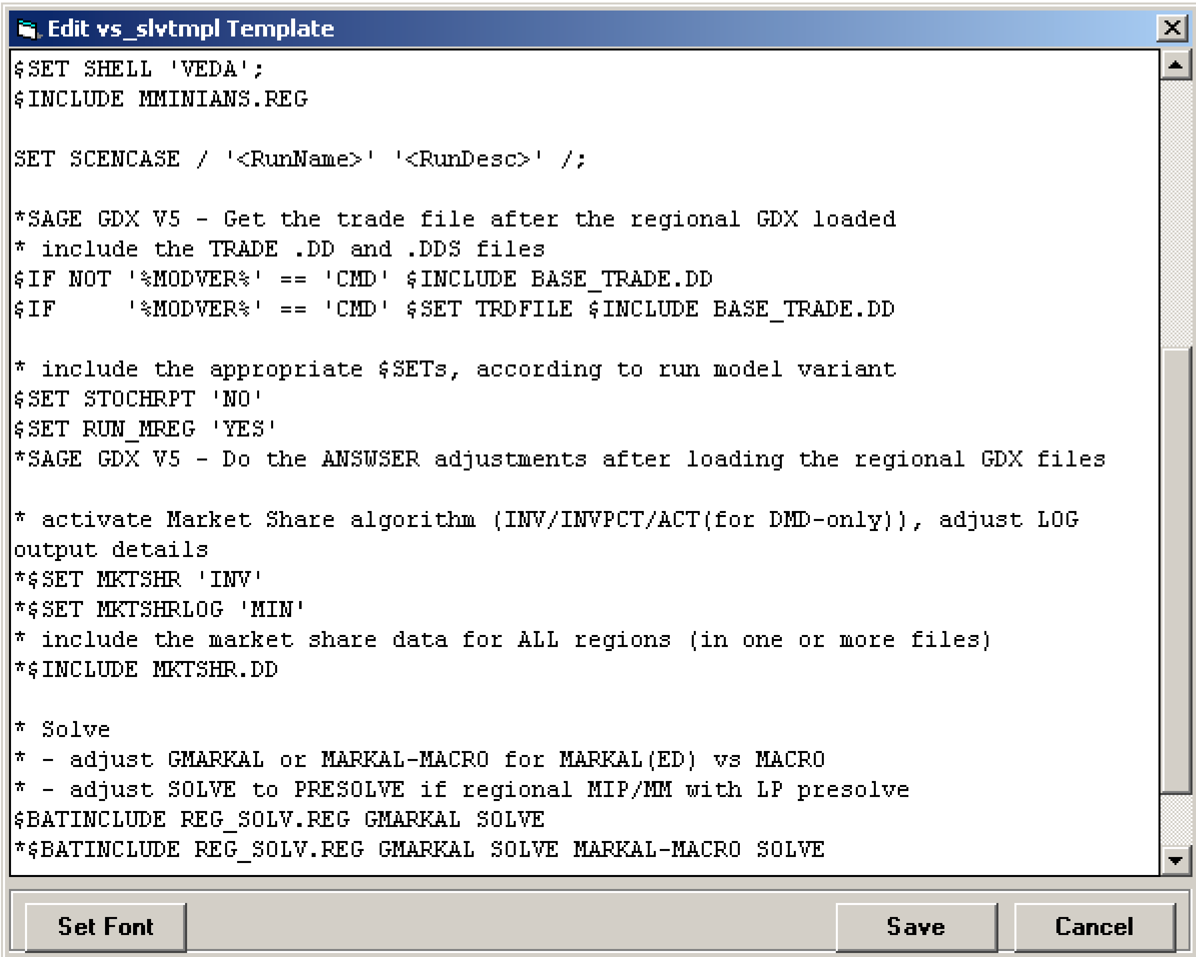


Figure A8: SLV Command Directive File Template Edit

VEDA-BE is used for examine the results of model runs. When activated, which VEDA-SAGE requests automatically, the GAMS report writing producers a 3-tuple of <case>.VD* files with the set information (members with descriptions, and subsets) and model result data. This information can then be loaded into VEDA by means of the Import option shown in Figure A9.All the runs found in the current list of folders to be examined whose file layout matches the SAGE (MARKAL) structure are presented for selection.

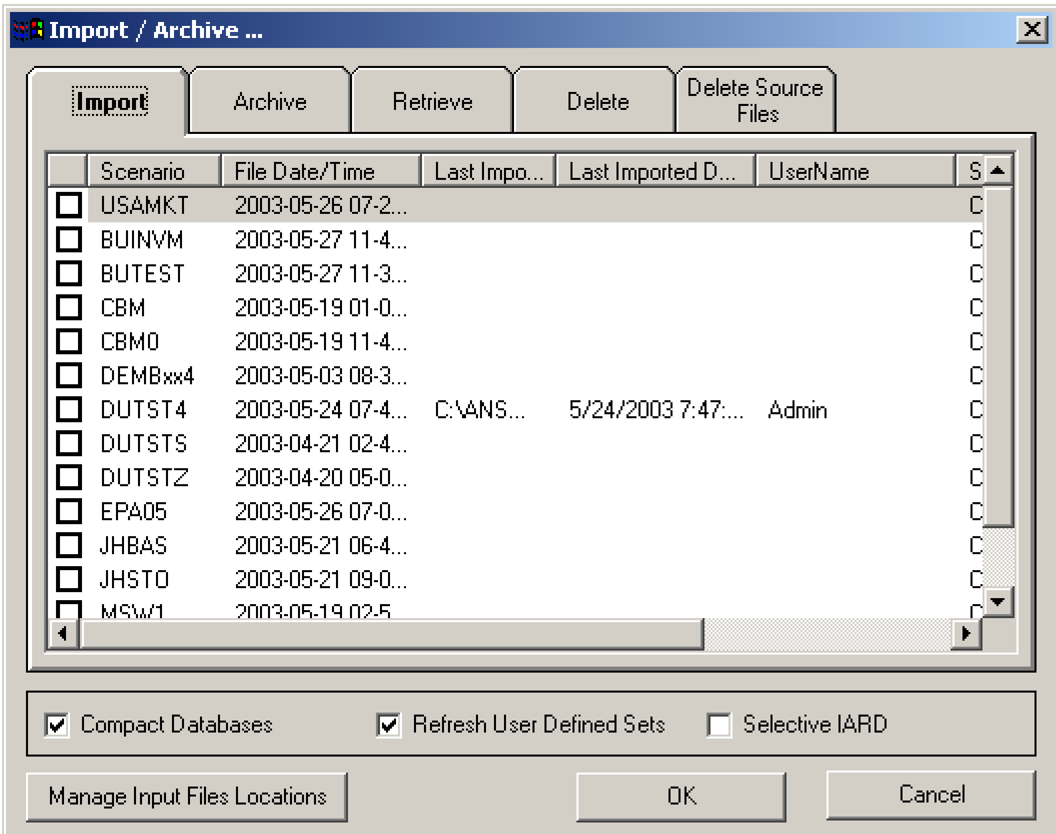


Figure A9: VEDA-BE Results Importing

Once imported tables may be constructed from the various attributes, shown in Figure A10. The attributes related to trade include Cost_TRD, which indicates the marginal price of each bi-lateral trade commodity according to the exporter region, and the VAR_FIN (exports), and VAR_FOUT (imports) for the SRCENCP options. Such a table, which for this Demo problem includes both internal bi-lateral trade as well as conventional SRCENCP supply curves, is shown in Figure A11. VEDA-BE provides power filter facilities that make selecting such subsets (e.g., all technologies beginning with IMP or EXP) trivial.

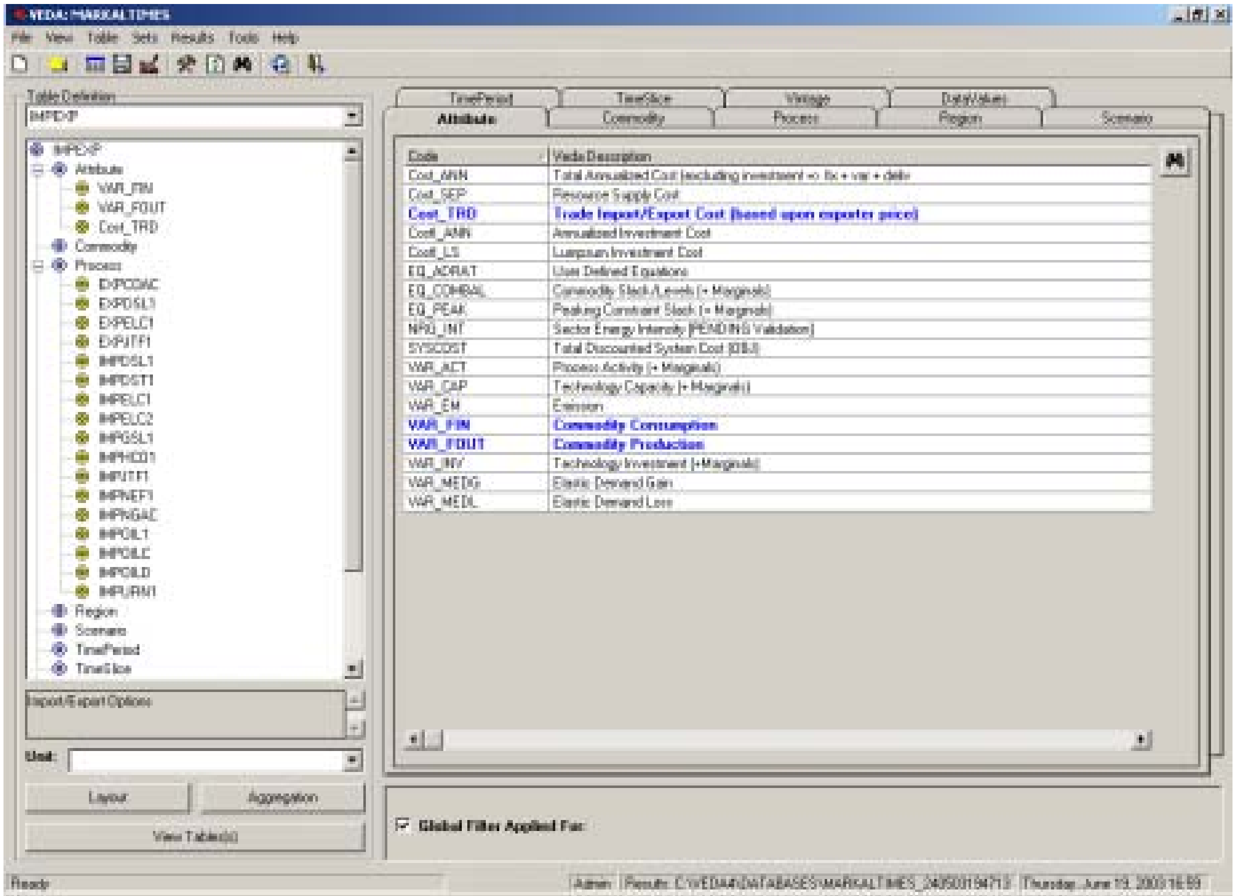


Figure A10: VEDA-BE Import/Export Table Specification

The resulting table is presented within a DataCube as shown in Figure A12. Note that there are primal (PV) and dual (DV) values presented for many entries under each period. Once in the cube the user can dynamically reconfigure the table say by commodity instead of region, as shown in Figure A13.

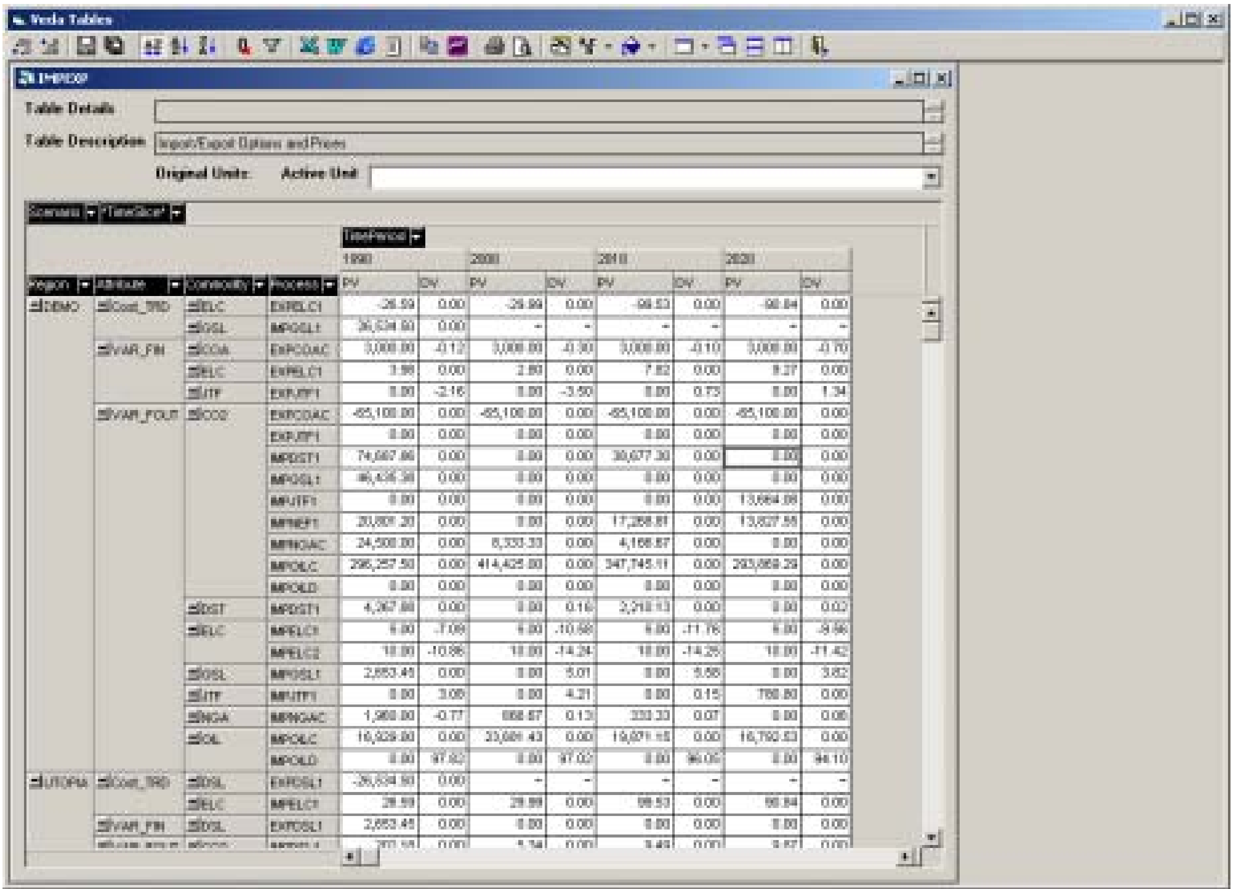
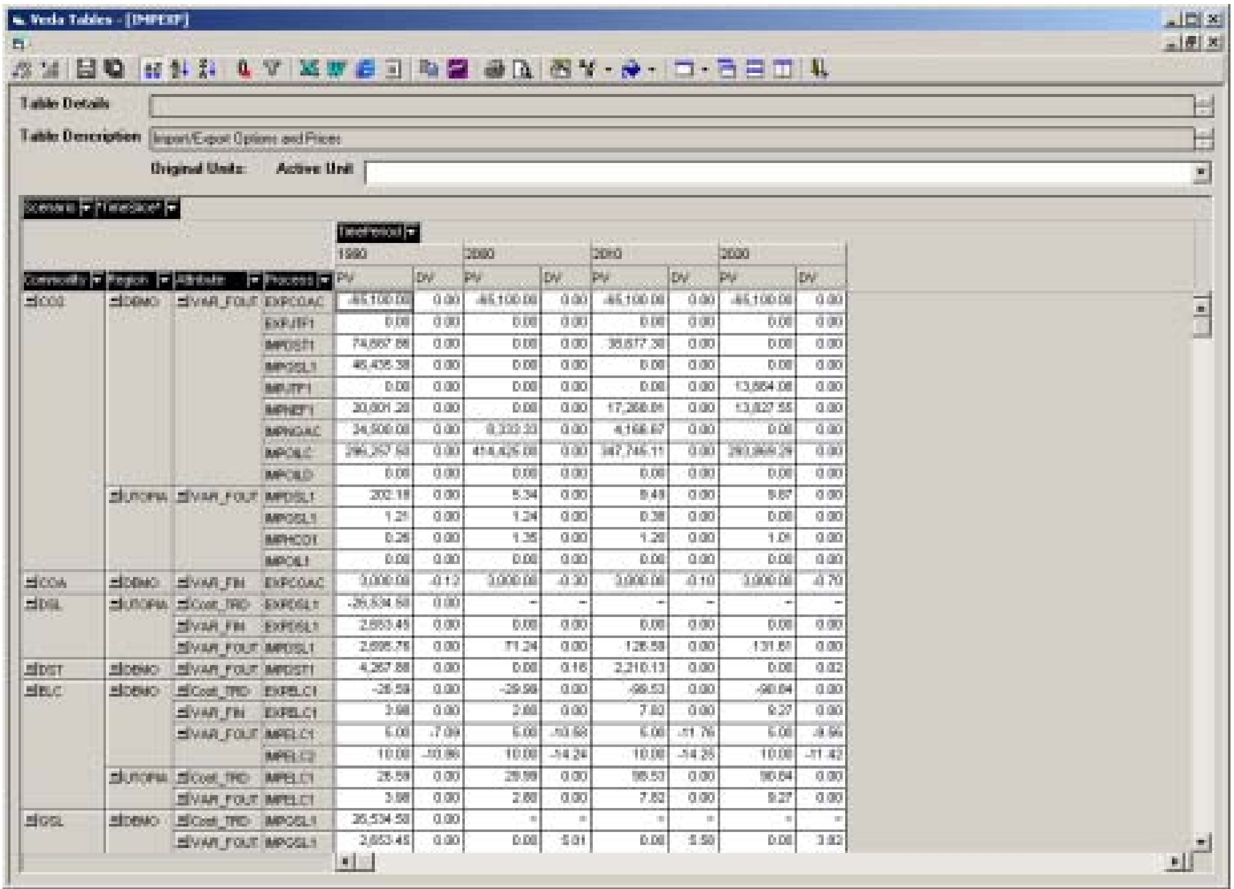


Figure A11: VEDA-BE Import/Export Table – by Region



Commodity	Region	Attribute	Process	1990 PV	1990 DV	2000 PV	2000 DV	2010 PV	2010 DV	2020 PV	2020 DV
CO2	DEMO	VAR_FOUT	EXPCOAC	-65,100.00	0.00	-65,100.00	0.00	-65,100.00	0.00	-65,100.00	0.00
			EXPJTF1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			IMPDST1	74,667.66	0.00	0.00	0.00	38,817.38	0.00	0.00	0.00
			IMPGSL1	46,435.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			IMPJTF1	0.00	0.00	0.00	0.00	0.00	0.00	13,864.08	0.00
			IMPNEF1	20,801.20	0.00	0.00	0.00	17,268.01	0.00	13,827.55	0.00
			IMPNGAC	34,500.00	0.00	8,333.33	0.00	4,166.67	0.00	0.00	0.00
			IMPOILC	396,357.50	0.00	414,625.00	0.00	387,745.11	0.00	293,869.29	0.00
			IMPOILD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	UTOPIA	VAR_FOUT	IMPDSL1	202.18	0.00	5.34	0.00	8.48	0.00	9.87	0.00
			IMPGSL1	1.21	0.00	1.24	0.00	0.38	0.00	0.00	0.00
			IMPHCO1	0.26	0.00	1.35	0.00	1.20	0.00	1.01	0.00
			IMPOIL1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COA	DEMO	VAR_FIN	EXPCOAC	3,000.00	-0.12	3,000.00	-0.30	3,000.00	-0.10	3,000.00	-0.70
DSL	UTOPIA	Cost_TRD	EXPDSL1	-26,534.50	0.00	-	-	-	-	-	-
		VAR_FIN	EXPDSL1	2,653.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		VAR_FOUT	IMPDSL1	2,695.76	0.00	71.24	0.00	126.58	0.00	131.81	0.00
DST	DEMO	VAR_FOUT	IMPDST1	4,267.88	0.00	0.00	0.16	2,210.13	0.00	0.00	0.02
ELC	DEMO	Cost_TRD	EXPELC1	-28.59	0.00	-29.99	0.00	-99.53	0.00	-90.64	0.00
		VAR_FIN	EXPELC1	3.98	0.00	2.80	0.00	7.82	0.00	9.27	0.00
		VAR_FOUT	IMPELC1	5.00	-7.09	5.00	-10.58	5.00	-11.76	5.00	-9.96
			IMPELC2	10.00	-10.86	10.00	-14.24	10.00	-14.25	10.00	-11.42
	UTOPIA	Cost_TRD	IMPELC1	28.59	0.00	29.99	0.00	99.53	0.00	90.64	0.00
		VAR_FOUT	IMPELC1	3.98	0.00	2.80	0.00	7.82	0.00	9.27	0.00
GSL	DEMO	Cost_TRD	IMPGSL1	26,534.50	0.00	-	-	-	-	-	-
		VAR_FOUT	IMPGSL1	2,653.45	0.00	0.00	5.01	0.00	5.50	0.00	3.82

Figure A12: VEDA-BE Import/Export Table – by Commodity

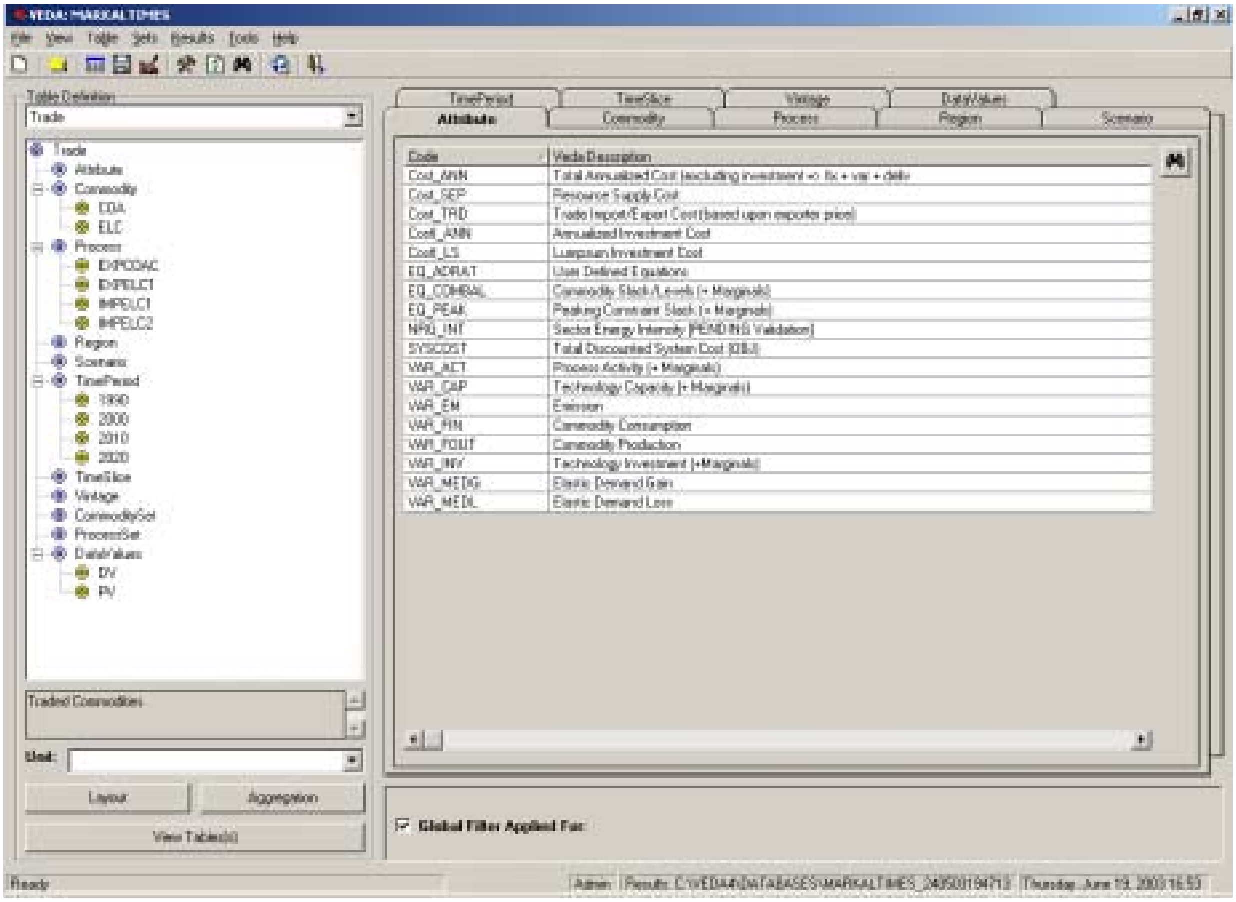
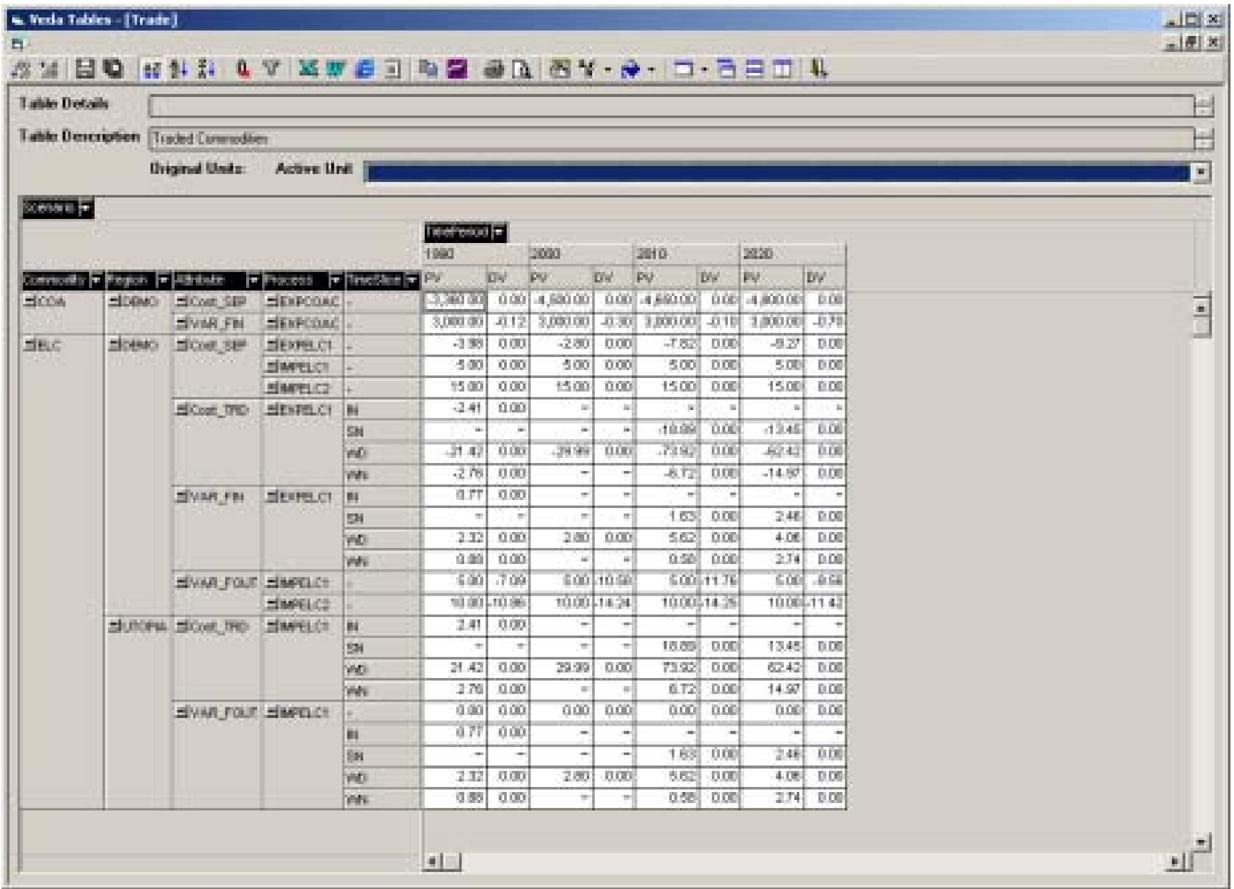


Figure A10: VEDA-BE Attribute List



Commodity	Region	Attribute	Process	TimeSlice	1990 PV	1990 DV	2000 PV	2000 DV	2010 PV	2010 DV	2020 PV	2020 DV
COA	DEMO	Cost_SEP	EXPCOAC	-	-3,380.00	0.00	-4,580.00	0.00	-4,680.00	0.00	-4,800.00	0.00
		VAR_FIN	EXPCOAC	-	3,000.00	-0.12	3,000.00	-0.30	3,000.00	-0.10	3,000.00	-0.70
ELC	DEMO	Cost_SEP	EXPELC1	-	-3.98	0.00	-2.80	0.00	-7.82	0.00	-9.27	0.00
			IMPELC1	-	5.00	0.00	5.00	0.00	5.00	0.00	5.00	0.00
			IMPELC2	-	15.00	0.00	15.00	0.00	15.00	0.00	15.00	0.00
		Cost_TRD	EXPELC1	IN	-2.41	0.00	-	-	-	-	-	-
				SN	-	-	-	-	-10.09	0.00	-13.45	0.00
				WD	-21.42	0.00	-29.99	0.00	-73.92	0.00	-62.42	0.00
				WN	-2.76	0.00	-	-	-6.72	0.00	-14.97	0.00
		VAR_FIN	EXPELC1	IN	0.77	0.00	-	-	-	-	-	-
				SN	-	-	-	-	1.63	0.00	2.46	0.00
				WD	2.32	0.00	2.80	0.00	5.62	0.00	4.06	0.00
				WN	0.88	0.00	-	-	0.58	0.00	2.74	0.00
		VAR_FOUT	IMPELC1	-	5.00	-7.09	5.00	-10.50	5.00	-11.76	5.00	-9.58
			IMPELC2	-	10.00	-10.96	10.00	-14.24	10.00	-14.25	10.00	-11.42
	UTOPIA	Cost_TRD	IMPELC1	IN	2.41	0.00	-	-	-	-	-	-
				SN	-	-	-	-	10.09	0.00	13.45	0.00
				WD	21.42	0.00	29.99	0.00	73.92	0.00	62.42	0.00
				WN	2.76	0.00	-	-	6.72	0.00	14.97	0.00
		VAR_FOUT	IMPELC1	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
				IN	0.77	0.00	-	-	-	-	-	-
				SN	-	-	-	-	1.63	0.00	2.46	0.00
				WD	2.32	0.00	2.80	0.00	5.62	0.00	4.06	0.00
				WN	0.88	0.00	-	-	0.58	0.00	2.74	0.00

Figure A9: VEDA-BE Traded Commodity Table

4.8 GAMS Execution Environment, Model Variants and other Control Switches

SAGE is composed of a series of the various components that include:

- VEDA-SAGE for assembling and managing input data and scenarios;
- "DOS" command scripts that manage system files and invoke the appropriate GAMS routines;
- GAMS command directive files that set model variants and identify input files;
- data extracted from VEDA-SAGE (the <scenario>_<region>.DD/DDS files);
- the GAMS actual source code;
- the solver(s), called from the SOLVE.* GAMS routines using either the default or an override solver;
- output files, including the run listing file, the quality control and other log files, and the results files passed to VEDA, and
- VEDA for processing the model results.

Diagram 4.10 below depicts the interaction of the various components of the model execution environment (VEDA-SAGE and VEDA are not explicitly shown or discussed here, see Volume 2), and the table 4.13 give a brief explanation of each component.

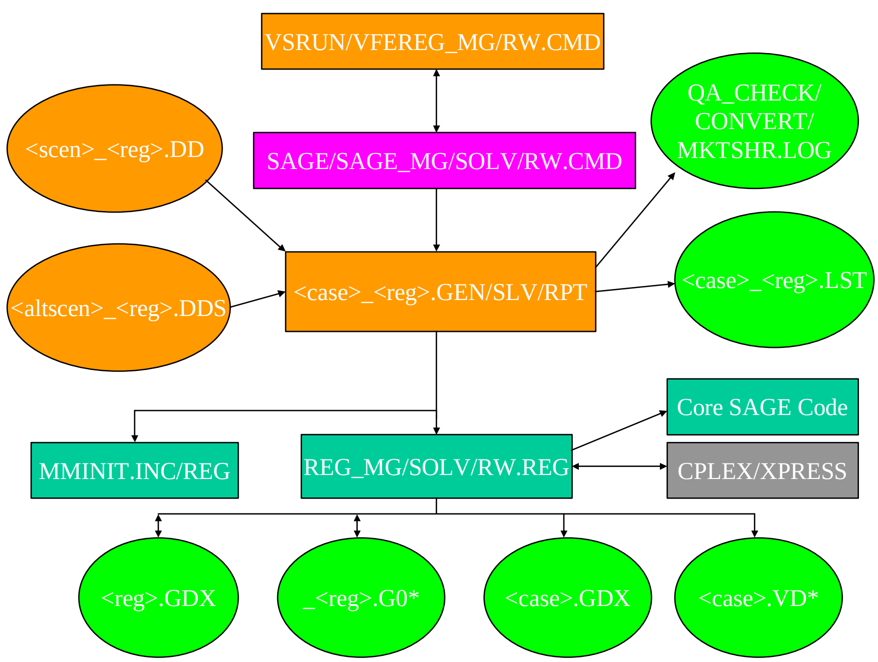


Figure 4.10: SAGE Execution Environment[1]

[1] The orange boxes are dynamically produced by VEDA-SAGE at job submission time, the violet boxes are the main "DOS" command script files that oversee the job run, blue boxes are SAGE GAMS source code, the gray boxes is the solvers, and the green boxes are output files generated from the system.

Table 4.23: Components of the SAGE Execution Environment

Component	Nature of Component	Purpose
VSRUN/ VFEREG_MG VFEREG_RW .CMD	"DOS" Command Script	Command scripts generated by VEDA-SAGE with the job name and pathing information (to the source code) for the current run. Associated script files with the list of regions for which the matrix generator and report writer are to be run are also produced to guide those job steps.
SAGE SAGE_MG SAGE_SOLV SAGE_RW .CMD	"DOS" Command Script	Main driver overseeing overall execution and the execution of the matrix generator, solve step and report writer. Command scripts generated by VEDA-SAGE, above, call SAGE.CMD with pathing information and are called by SAGE.CMD to process the regions' matrix generation and report writing
<scen>_<reg>. DD	Reference Data for each Region	The full RES data specification generated by VEDA-SAGE for each region, plus <scen>_TRADE.
<altscen>_<reg>. DDS	Alternate Data for each Region	Augmenting information to be applied to the reference data generated by VEDA-SAGE for each region, plus <scen>_TRADE if appropriate.
<case>_<reg>. GEN/SLV/RPT	GAMS Command Directives	The GAMS commands identifying the DD/DDS files associated with each region, setting the desired model variant switches (discussed below), and calling the main matrix generation/solve/report writer drivers (REG_MG/SOLV/RW.REG). [Note that only <case>.SLV exists, that is there is not one for each region.]
MMINIT.INC/ REG	GAMS Initialization Routines	The modules that declare all "empty" GAMS data structures appearing in the model code. This ensures that no execution error occurs for models that do not use certain features in a particular run.
REG_MG/ REG_SOLV/ REG_RPT.REG	GAMS Main Component Drives	The main matrix generation/solve/report writer drivers called from the <case>_<reg>.GEN/SLV/RPT files for each region.
<reg>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data declarations for each region. Generated during matrix generation and accessed during the solve and report writing.
_<reg>.G0*	GAMS Save	The GAMS save files contain the model declaration for each region. Generated during the matrix generation and accessed by the report writer.
<case>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data and model results from the optimization. Generated during the solve and accessed by the report writer.
<case>.VD/VDE/ VDS	VEDA Results	The VEDA data, SAGE RES elements and set membership information passed by the report writer for subsequent processing.
Core GAMS Code	GAMS Code	The individual routine making up the core MARKAL code (those routines ending in INC) and the SAGE specific code (those routines ending in MTS and MKT).
CPLEX/XPRESS	Solver	The solver used to actually solve the model. The model is successively solved for each period.
<case>_<reg>. LST	GAMS Output Listing	GAMS writes a execution log (including error message, if any) to the LST file for each region (and TRADE). The <case>_TRADE file also contains the complete solution listing.

Component	Nature of Component	Purpose
QA_CHECK/ CONVERT/ MKTSHR.LOG	SAGE LOG	During execution 3 LOG files are written. QA_CHECK reports on the correctness and consistency of the underlying data describing each regional energy system. CONVERT reports on the actions taken to adjust the lifetimes of technologies for which the technical lifetime (LIFE) is not a multiple of the number of years per period (NYRSPER). MKTSHR.LOG give a trace of the actions taken between periods to apply the Market Share algorithm.

As noted in the introduction, SAGE has been developed based on the MARKAL modeling framework. As a result it shares most of the same source code modules with other MARKAL-based models. GAMS dynamic substitution for environment variables is employed to fully establish the appropriate executable code. This is possible as GAMS is a two-pass system, first fully resolving the source code (substituting the string associated with the %env_variables% environment variables), and then executing the resulting code. The GAMS code provided in Appendix B is the code established from its MARKAL components when the %SAGE% environment variable is set to 'YES,' and all the various possible SAGE add-ons (e.g., time-stepped simulation mode, Market Share Algorithm and inter-period Endogenous Technology Learning) are activated. The SAGE relevant switches are presented in table 4.14 below.

Table 4.24: Descriptions of SAGE Model Variant Switches

Environment Variable	Values	Description and Implications
MARKALED	• BPRICE • YES	• Conduct the elastic demand formulation reference run to establish the base price for meeting each of the demand services, and the annual cost of the energy system. A <region>.EDD file with this information is written for each region. • Activate the elastic demand formulation, accessing the <region>.EDD file for the reference prices and associated information, and to include the flexible demand variables in the demand equations (MR_DEM) and objective function (MR_MTSOBJ).
MKTSHR[1]	• INV • INVPCT • ACT	• Activate the market share algorithm based upon the marginal cost of the investment variable. • Activate the market share algorithm based upon the marginal cost of the investment variable as a share of current investment cost in the period. • Activate the market share algorithm based upon the activity of the technology (for demand devices only at this time).
MKTSHRLOG[2]	• MIN • ALL	• Write the minimal trace of the decisions made as part of the market share algorithm to the MKTSHR.LOG file. • Write the full trace of all the decisions made as part of the market share algorithm to the MKTSHR.LOG file.

[1] Some kind of example of each choice would be helpful. Especially INVPCT Requires that SAGE be activated.
[2] Requires that SAGE and MKTSHR be activated.

SAGE	• YES • any other value	• Activate the time-stepped technique for successively optimizing of the energy system period by period, rather than clairvoyantly over the entire modeling horizon as is done with MARKAL. • Solve in a single step with perfect foresight over the entire modeling horizon.
SETL[1]	• YES • any other value	• Activate the endogenous technology-learning algorithm that derives the current investment cost of a technology as a function of cumulative capacity in place up to and including the previous period. • Do not activate the technology learning algorithm.

Below is a small example showing a sample GAMS control directives file in figure 4.11 (generated dynamically by VEDA-SAGE according to the variant switches needed), and both the original GAMS source code and the associated fully resolved executable code in figures 4.12, 4.13, and 4.14 respectively.

[1] Requires that SAGE be activated.

Figure 4.11: DUTSTX.SLV solve command directive file for SAGE run with elastic demands and market share algorithm activated

```
* ----- Identify the “shell” and run name
$TITLE VEDA-SAGE SAGE: CASE DUTSTX
* ----- GAMS options. Set LIMROW=x, LIMCOL=y to activate an equation listing
OPTION LIMROW=0, LIMCOL=0, SOLPRINT=ON, SYSOUT=OFF,
ITERLIM=2000000;
OPTION RESLIM=50000, PROFILE=0, SOLVEOPT=REPLACE;
* ----- Uncomment if switching from the default CPLEX solver to XPRESS
*OPTION LP=XPRESS;
* ----- Turn off the listing of the data and source code
$OFFLISTING
* ----- Permit multiple instances of declarations
$ONMULTI
* ----- %MODVER% a GAMS environment variable indicating if BAT/CMD execution,
passed on call line.
*       It is used to ensure backward compatibility between older and current versions of the
model.
* ----- Identify the source code version (note: Title header printed in LST file comes from
MMINIT.INC)
$IF NOT '%MODVER%' == 'CMD' $LOG   *** Running MARKAL/SAGE Std ***
$IF     '%MODVER%' == 'CMD' $LOG   *** Running MARKAL/SAGE V5.1b ***
* ----- Include the files containing “empty” declaration of all sets and parameters used in the
code
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.INC
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.REG
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.VFE
$IF NOT '%MODVER%' == 'CMD' $CLEAR G_TRADE TRD_FROM TRD_COST
BI_TRDENT
                                        BI_TRDCST BI_TRDELC BI_TRDCSTE
REG_XMONY
                                        REG_XCVT REG_XMACRO
* ----- Get the complete list of all “master” sets for all regions generated during the
individual region matrix
*       generation pre-processing
$INCLUDE REG_FULL.SET
* ----- Indicate that VEDA-SAGE is the shell
$SET SHELL 'VEDA';

* ----- Declare the case/run name
SET SCENCASE  / 'DUTSTX'  'BASE scenario run for Demo only to 2020' /;

*----- Get the trade file, either now if BAT execution or after the regional GDX loaded for
CMD (SAGE)
$IF NOT %MODVER% == 'CMD' $INCLUDE
C:\AnswerV5\Gams_WrkVD\DUTRADE.DD
$IF  %MODVER% == 'CMD' $SET TRDFILE $INCLUDE C:\VEDA
```

Figure 4.12: MMEQUA.REG equation declaration and call to MMEQDEM.INC for the actual inclusion of the demand equation

```
*****************************************************************************
*************
*                                                                           *
*   MMEQUA.REG has the actual equation declarations and specifications for a multi-region
*
*      run. Most of the details regarding the actual value of the coefficients are handled in
MMCOEF.INC.   *
*
*
*****************************************************************************
*************

*=============================================================================
==========*
*  Declare the individual equations and provided informative text to be
*
*   printed in the *.LST files for the user
*
*=============================================================================
==========*
 EQUATIONS

 ...

 MR_DEM(*, *, *)          demand relation (=G=)
 ...

 .
```

Figure 4.13: MMEQDEM.INC and MMEQDEM.MED prior to substitution of the passed and global GAMS environment variables

```
*=========================================================================
==========*
*  MMEQDEM.INC Demand Equation
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
*    %4 - variable/coef prefix => 'DM' or 'S_' or 'R_'
*    %5 - REGional indicator => '' or '' or 'REG,'
*    %6 - regional scaling => '' or '' or '(REG)'
*    %7 - loop control set => 'TP' or 'TP_SOW(TP,SOW)' or 'TP'
*    %8 - DM or S indicator for table names => 'DM' or 'S' or 'DM'
*=========================================================================
==========*
* only generate if demand provided
%1_DEM(%5%7, DM)$(TS(TP) AND (%8_DEM%3(%5DM, %2TP) GT 0)) ..
MMSCALE%3%6 *
(  SUM(DMD$DMD_DMS%3(%5DMD, DM),
           DEM_CAP%3(%5TP, DMD, DM) * %4CAP(%5TP, %2DMD) )

* if requested include elastic demands
$ IF '%MARKALED%' == 'YES' $BATINCLUDE MMEQDEM.MED %1 '%2' '%3' '%4'
'%5'

)

  =G=

* Set the RHS to the demand level
  MMSCALE%3%6 * %8_DEM%3(%5DM, %2TP);

*************************************************************************
***
*   MMEQUADM.MED has the EQ_DEM part of the Elastic Demands
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
```

Figure 4.14: MMEQDEM.INC, including MMEQDEM.MED, after substitution of GAMS environment variables ready for execution

```
*----- GAMS call for the source code module
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.INC

*----- All environment variable parameters substituted in MMEQDEM.INC
MR_DEM(REG,TP, DM)$(TS(TP) AND (DM_DEM_R(REG,DM, TP) GT 0)) ..
MMSCALE_R(REG) *
* demand device production of useful demand services
(   SUM(DMD$DMD_DMS_R(REG,DMD
          DEM_CAP_R(REG,TP, DMD, DM) * R_CAP(REG,TP, DMD))

*----- Since %MARKALED% == 'YES' call is made to add the elastic demand parameters
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.MED

* growth in demand
  - SUM(DMSTEPS_R(REG,DM,'UP',JSTEP),
          R_ELAST(REG,TP,DM,'UP',JSTEP)$(DM_ELAST_R(REG,DM,'UP',TP) NE 0) )

*  reduction in demand
  + SUM(DMSTEPS_R(REG,DM,'LO',JSTEP),
           R_ELAST(REG,TP,DM,'LO',JSTEP)$(DM_ELAST_R(REG,DM,'LO',TP) NE 0) )

)
```

Appendix A: SAGE_MATRIX

Appendix B. GAMS Source code

SAGE is written in a modular fashion employing the General Algebraic Modeling System (GAMS). GAMS is a 2-pass compile and execute system that 1st builds the current code from the individual routines according to “control switches” ($SET environment variables) and parameters (%1-n) passed at invocation time. Once fully resolved this code is then executed. This mechanism was introduced in section 4.8, and is elaborated here. The overall organization of the SAGE model execution environment is also presented in this appendix.

The production version of the SAGE code is generated out of the full MARKAL modeling system to both isolate it from MARKAL and freeze the current production version of the code. The procedures for generating the fully resolved SAGE specific code are presented in detail in an EIA technical note (SAGE_Code-GenerateProcedures.DOC).

B.1 SAGE Source Code

The SAGE model is composed of a series of components that consist of:

- VEDA-SAGE for assembling and managing input data and scenarios;
- “DOS” command scripts that manage system files and invoke the appropriate GAMS routines;
- GAMS command directive files that set model variants and identify input files;
- data extracted from VEDA-SAGE (the <scenario>_<region>.DD/DDS files);
- the GAMS actual source code;
- the solver(s), called from the SOLVE.* GAMS routines using either the default or an override solver;
- output files, including the run listing file, the quality control and other log files, and the results files passed to VEDA, and
- VEDA for processing the model results.

Diagram B.1, below, depicts the interaction of the various components of the model execution environment (VEDA-SAGE and VEDA are not explicitly shown or discussed here, see Volume 2), and table B.1 gives a brief explanation of each component.

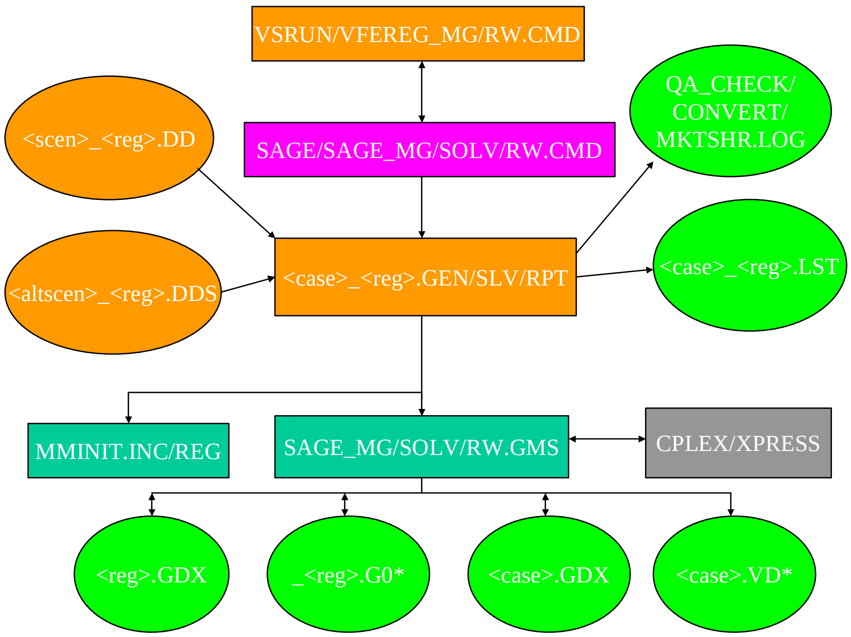


Figure B.1: SAGE Execution Environment[1]

Table B.1: Components of the SAGE Execution Environment

Component	Nature of Component	Purpose
VSRUN/ VFEREG_MG VFEREG_RW .CMD	"DOS" Command Script	Command scripts generated by VEDA-SAGE with the job name and pathing information (to the source code) for the current run. Associated script files with the list of regions for which the matrix generator and report writer are to be run are also produced to guide those job steps.
SAGE SAGE_MG SAGE_SOLV SAGE_RW .CMD	"DOS" Command Script	Main driver overseeing overall execution and the execution of the matrix generator, solve step and report writer. Command scripts generated by VEDA-SAGE, above, call SAGE.CMD with pathing information and are called by SAGE.CMD to process the regions' matrix generation and report writing. Other CMD routines are called from these for file management et al as needed. [Also, CURVER.CMD echo the current version of the source code to the screen during the run.]
<scen>_<reg>. DD	Reference Data for each Region	The full RES data specification generated by VEDA-SAGE for each region, plus <scen>_TRADE.

[1] The orange boxes are dynamically produced by VEDA-SAGE at job submission time, the violet boxes are the main "DOS" command script files that oversee the job run, blue boxes are SAGE GAMS source code, the gray boxes is the solvers, and the green boxes are output files generated from the system.

Component	Nature of Component	Purpose
<altscen>_<reg>. DDS	Alternate Data for each Region	Augmenting information to be applied to the reference data generated by VEDA-SAGE for each region, plus <scen>_TRADE if appropriate.
<case>_<reg>. GEN/SLV/RPT	GAMS Command Directives	The GAMS commands identifying the DD/DDS files associated with each region, setting the desired model variant switches (discussed below), and calling the main matrix generation/solve/report writer drivers (REG_MG/SOLV/RW.REG). [Note that only <case>.SLV exists, that is there is not one for each region.]
MMINIT.*/ MMINIANS.REG	GAMS Initialization Routines	The modules that declare all "empty" GAMS data structures appearing in the model code. This ensures that no execution error occurs for models that do not use certain features in a particular run.
SAGE_MG/ SAGE_SOL/ SAGE_RPT. GMS	GAMS Main SAGE Component Drives	The main matrix generation/solve/report writer drivers called from the <case>_<reg>.GEN/RPT files for each region, and the <case>.SLV solve directive file.
*.MKT	SAGE Market Share GAMS Code	The modules that control the market share algorithm. These routine have NOT been fully resolved but are compiled each time, resolving the code according to the MKTSHR related switches that are activated.
R_STATUS.ANS/ VEDA2GAMS.*/ REG_FULL.GMS	SAGE Market Share GAMS Code	Miscellaneous low level core MARKAL/SAGE source code routines that need to be resolved at execution time owing to the structure of the code.
<reg>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data declarations for each region. Generated during matrix generation and accessed during the solve and report writing.
_<reg>.G0*	GAMS Save	The GAMS save files contain the model declaration for each region. Generated during the matrix generation and accessed by the report writer.
<case>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data and model results from the optimization. Generated during the solve and accessed by the report writer.
<case>.VD/VDE/ VDS	VEDA Results	The VEDA data, SAGE RES elements and set membership information passed by the report writer for subsequent processing.
CPLEX/XPRESS	Solver	The solver used to actually solve the model. The model is successively solved for each period.
<case>_<reg>. LST	GAMS Output Listing	GAMS writes a execution log (including error message, if any) to the LST file for each region (and TRADE). The <case>_TRADE file also contains the complete solution listing.
QA_CHECK/ CONVERT/ MKTSHR.LOG	SAGE LOG	During execution 3 LOG files are written. QA_CHECK reports on the correctness and consistency of the underlying data describing each regional energy system. CONVERT reports on the actions taken to adjust the lifetimes of technologies for which the technical lifetime (LIFE) is not a multiple of the number of years per period (NYRSPER). MKTSHR.LOG give a trace of the actions taken between periods to apply the Market Share algorithm.

As noted previously, SAGE has been developed based on the MARKAL modeling framework. As a result it shares most of the same source code modules with other MARKAL-based models. GAMS dynamic substitution for environment variables is employed to fully establish the appropriate executable code. This is possible as GAMS is a two-pass system, first fully resolving the source code (substituting the string associated with the %env_variables% environment variables), and then executing the resulting code. The GAMS SAGE code is thus established from its MARKAL components when the %SAGE% environment variable is set to 'YES,' and all the various possible SAGE add-ons (e.g., time-stepped simulation mode, Market Share Algorithm and inter-period Endogenous Technology Learning) are activated. The SAGE relevant switches are presented in table 4.21 in section 4.8. An example of the originally MARKAL-based source code, and the resulting fully resolved SAGE code is given in figures B.2-B5.

Below is a small example showing a sample GAMS control directives file in figure B.2 (generated dynamically by VEDA-SAGE according to the variant switches needed), and both the original GAMS source code and the associated fully resolved executable code in figures B.3, B.4, and B.5 respectively.

Figure B.2: DUTSTX.SLV solve command directive file for SAGE run with elastic demands and market share algorithm activated

```
* ----- Identify the "shell" and run name
$TITLE VEDA-SAGE SAGE: CASE DUTSTX
* ----- GAMS options. Set LIMROW=x, LIMCOL=y to activate an equation listing
OPTION LIMROW=0, LIMCOL=0, SOLPRINT=ON, SYSOUT=OFF,
            ITERLIM=2000000;
OPTION RESLIM=50000, PROFILE=0, SOLVEOPT=REPLACE;
* ----- Uncomment if switching from the default CPLEX solver to XPRESS
*OPTION LP=XPRESS;
* ----- Turn off the listing of the data and source code
$OFFLISTING
* ----- Permit multiple instances of declarations
$ONMULTI
* ----- %MODVER% a GAMS environment variable indicating if BAT/CMD execution,
        passed on call line.
*       It is used to ensure backward compatibility between older and current versions of the
        model.
* ----- Identify the source code version (note: Title header printed in LST file comes from
        MMINIT.INC)
$IF NOT '%MODVER%' == 'CMD' $LOG   *** Running MARKAL/SAGE Std ***
$IF     '%MODVER%' == 'CMD' $LOG   *** Running MARKAL/SAGE V5.1b ***
* ----- Include the files containing "empty" declaration of all sets and parameters used in the code
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.INC
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.REG
$IF     '%MODVER%' == 'CMD' $INCLUDE MMINIT.VFE
$IF NOT '%MODVER%' == 'CMD' $CLEAR G_TRADE TRD_FROM TRD_COST
      BI_TRDENT BI_TRDCST BI_TRDELC BI_TRDCSTE REG_XMONY
      REG_XCVT REG_XMACRO
```

Figure B.2: DUTSTX.SLV solve command directive file for SAGE run with elastic demands and market share algorithm activated (cont.)

```
* ----- Get the complete list of all “master” sets for all regions generated during the
*        individual region matrix generation pre-processing
$INCLUDE REG_FULL.SET

* ----- Indicate that VEDA-SAGE is the shell
$SET SHELL 'VEDA';

* ----- Declare the case/run name
SET SCENCASE  / 'DUTSTX'  'BASE scenario run for Demo only to 2020' /;

*----- Get the trade file, either now if BAT execution or after the regional GDX loaded for
        CMD (SAGE)

$IF NOT %MODVER% == 'CMD' $INCLUDE  DUTRADE.DD
$IF  %MODVER% == 'CMD' $SET TRDFILE $INCLUDE DUTRADE.DD

*----- Include the appropriate $SETs, according to run model variant
*      Multi-region, elastic sensitivity run, using investment market share with limited log
reporting
$SET RUN_MREG 'YES'
$SET MARKALED ‘YES’
$SET MKTSHR 'INV'
$SET MKTSHRLOG 'MIN'
*----- Include the market share data for ALL regions (in one or more files)
$INCLUDE MKTSHR.DD

*----- Solve
$BATINCLUDE SAGE_SOL.GMS
```

Figure B.3: MMEQUA.REG equation declaration and call to MMEQDEM.INC for the actual inclusion of the demand equation

```
**************************************************************************
*
*   MMEQUA.REG has the actual equation declarations and specifications for a multi-region
*      run. Most of the details regarding the actual value of the coefficients are handled in
*        MMCOEF.INC.
*
**************************************************************************

*=======================================================================*
*  Declare the individual equations and provided informative text to be
*   printed in the *.LST files for the user
*=======================================================================*
  EQUATIONS

  ...

  MR_DEM(*, *, *)            demand relation (=G=)
  ...

  ;

*=======================================================================*
*  Generation of individual equations, passing the multi-region specs
*
*=======================================================================*

  ...

*=======================================================================*
*  Useful Energy Demands                                          *
*=======================================================================*
$BATINCLUDE MMEQDEM.INC MR '' '_R' 'R_' 'REG,' '(REG)' 'TP' DM

  ...
```

Figure B.4: MMEQDEM.INC and MMEQDEM.MED prior to substitution of the passed and global GAMS environment variables

```
*=============================================================================*
*  MMEQDEM.INC Demand Equation
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
*    %4 - variable/coef prefix => 'DM' or 'S_' or 'R_'
*    %5 - REGional indicator => '' or '' or 'REG,'
*    %6 - regional scaling => '' or '' or '(REG)'
*    %7 - loop control set => 'TP' or 'TP_SOW(TP,SOW)' or 'TP'
*    %8 - DM or S indicator for table names => 'DM' or 'S' or 'DM'
*=============================================================================*
* only generate if demand provided
%1_DEM(%5%7, DM)$(TS(TP) AND (%8_DEM%3(%5DM, %2TP) GT 0)) ..
MMSCALE%3%6 *
(  SUM(DMD$DMD_DMS%3(%5DMD, DM),
          DEM_CAP%3(%5TP, DMD, DM) * %4CAP(%5TP, %2DMD) )

* if requested include elastic demands
$ IF '%MARKALED%' == 'YES' $BATINCLUDE MMEQDEM.MED %1 '%2' '%3' '%4'
'%5'

)

  =G=

* Set the RHS to the demand level
  MMSCALE%3%6 * %8_DEM%3(%5DM, %2TP);

*****************************************************************************
*   MMEQUADM.MED has the EQ_DEM part of the Elastic Demands
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
*    %4 - variable/coef prefix => 'DM' or 'S_' or 'R_'
*    %5 - REGional indicator => '' or '' or 'REG,'
*****************************************************************************

* Growth in Demand
  - SUM(DMSTEPS%3(%5DM,'UP',JSTEP),
%4ELAST(%5TP,%2DM,'UP',JSTEP)$(DM_ELAST%3(%5DM,'UP',TP) NE 0) )

* Reduction in Demand
  + SUM(DMSTEPS%3(%5DM,'LO',JSTEP),
%4ELAST(%5TP,%2DM,'LO',JSTEP)$(DM_ELAST%3(%5DM,'LO',TP) NE 0) )
```

Figure B.5: MMEQDEM.INC, including MMEQDEM.MED, after substitution of GAMS environment variables ready for execution

```
*----- GAMS call for the source code module
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.INC

*----- All environment variable parameters substituted in MMEQDEM.INC
MR_DEM(REG,TP, DM)$(TS(TP) AND (DM_DEM_R(REG,DM, TP) GT 0)) ..
MMSCALE_R(REG) *
* demand device production of useful demand services
(   SUM(DMD$DMD_DMS_R(REG,DMD
          DEM_CAP_R(REG,TP, DMD, DM) * R_CAP(REG,TP, DMD))

*----- Since %MARKALED% == 'YES' call is made to add the elastic demand parameters
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.MED

* growth in demand
  - SUM(DMSTEPS_R(REG,DM,'UP',JSTEP),
          R_ELAST(REG,TP,DM,'UP',JSTEP)$(DM_ELAST_R(REG,DM,'UP',TP) NE 0) )

*  reduction in demand
  + SUM(DMSTEPS_R(REG,DM,'LO',JSTEP),
          R_ELAST(REG,TP,DM,'LO',JSTEP)$(DM_ELAST_R(REG,DM,'LO',TP) NE 0) )

)

   =G=

* Set the RHS to the demand level
MMSCALE_R(REG) * DM_DEM_R(REG,DM, TP);
```

The entire SAGE source code, along with a sample test problem, is available for downloading as a SAGE.ZIP file. The Zip file contains the SAGE source code and the necessary associated command (DOS) script files needed to perform an actual run of SAGE.

The code is organized according to matrix generation, solve, and report writer steps. In addition there are some initialization routines common to the various modules, which are discussed first. The nature and purpose of basic routines encompassing each subsystem are then briefly described.

B.1 SAGE Initialization Routines

In order to allow SAGE the flexibility to include only the desired model options, thereby omitting others, all possible GAMS sets and parameters are declared in "MMINIT" routines. Each and every set and parameter referenced anyplace in the SAGE code are initialized (to null) in these routines by naming them and defining their dimensionality (number of indexes) and scope (set controlling the domain)[2].

In table B.2 below the individual MMINIT routines are identified and the purpose elaborated.

Table B.2: SAGE Initialization Routines

Routine	Steps Used[3]	Purpose
MMINIT.INC	MG/SOL	Declare all the single region sets and parameters.
MMINIT.REG/ MMINIANS.REG	MG/SOL	Declare all the multi-region sets and parameters.
MMINT.ANS/VFE	MG/SOL/RW	Declare all the rest of the single region sets and parameters.
VEDA2GAMS.VFE/ REG	MG/SOL	Preprocess VEDA generated sets and parameters that need to be mapped to their GAMS equivalents, and initialize selected parameters.

B.2 SAGE Matrix Generator Source Code Overview

For each region the SAGE matrix generation is invoked to generate all the internal parameters governing the intersections to be created in the final matrix. The resulting GAMS Data Exchange file (<region>.GDX) contains all such information, along with the original input data, for each region. This information, along with the trade specification, is then grabbed by the solve step and assembled into the full multi-region matrix. The _<region>.G0* restart files are also passed to the report writer to re-establish the individual regional data needed for each report.

Each regional matrix generation step is controlled by the <case>_<region>.GEN GAMS command directive file generated by VEDA-SAGE. An overview of the calls and actions is given below.

- <case>_<region>.GEN
    - MMINIT.INC
    - MMINIT.GP (to be removed)

[2] For the multi-region sets and parameters employed in the solve step the domains have been set to the universe ('*'), rather than the specific known sets, owing to the approach taken of buffering the individual regional information into the multi-region structures.

[3] MG = Matrix Generation, SOL = Solve, RW = Report Writer

    - MMINIT.REG
    - MMINIT.ANS/VFE
    - <scenario>_<region>.DD
    - <altscenario>_<region>.DDS
    - $SET SAGE run switches
    - VEDA2GAMS.ANS
    - SAGE_MG.GMS
        - Perform QC checks
        - Derived the single region model coefficients
        - Move the region specific set and parameter information into their multi-region counterparts
- Files produced for each region
    - <case>_<region>.LST[4] with execution trace along with any error messages
    - QA_CHECK/CONVERT.LOG with quality control and any fractional LIFE adjustments
    - REG_FULL.SET, the incrementally built list of master set elements (e.g., ENT, TCH, DM) with values for all regions
    - <region>.GDX for the solve step

_<region>.G0* restart files for the report writer***B.3 SAGE Solve Source Code Overview***

The SAGE solve source code is invoked to generate the final matrix and solve the model. The GAMS Data Exchange file (<region>.GDX) produced by the regional matrix generation step are loaded and mapped to their regional counterpart. A loop of each time period first sets lower bounds on market share technologies, solves, adjusts market shares if necessary and re-solves (writing to the MKTSHR.LOG), then augments the investment costs for any learning technologies before going on to the next period.

Before the actual solve step is invoked a pre-step is done to assemble REG_FULL.GDX from REG_FULL.SET.

Each solve step is controlled by the <case>.SLV GAMS command directive file generated by VEDA-SAGE. An overview of the calls and actions is given below.

- <case>.SLV
    - MMINIT.INC
    - MMINIT.REG
    - MMINIT.ANS/VFE
    - $INCLUDE REG_FULL.SET to get the full list of all master sets assembled
    - MMINIANS.REG

[4] Note that the <case>_<region>.LST is generated first for each region in the matrix generation step, but later overwritten during the report writer step. If an error occurs during an regional matrix generation step the user must abort the run (via Ctrl-C in the DOS box) and examine the associate LST file using any text editor.

        - $SET SAGE run switches
        - SAGE_SOL.GMS
            - Perform QC checks on trade
            - Loop getting each <region>.GDX file, assembling the full matrix
            - Loop solving each period
                - Solve (via SOLVSHR.MKT)
                    - Make market share adjustments
                        - Resolve
                - Determine any SETL related adjustments to the investment cost of learning technologies for the next period
    - Files produced
        - <case>.LST with execution trace along with any error messages, as well as the solution dump and trade status
        - QA_CHECK/MKTSHR.LOG with trade and the actions taken by the market share algorithm
        - <case>.GDX for the report writer

B.4 SAGE Report Writer Source Code Overview

For each region the SAGE report writer is invoked to produce the VEDA-BE "dump" of model results. The individual <region>._G0* restart files and full GAMS Data Exchange file (<case>.GDX) containing the multi-region solution is processed to produce the region specific information. The resulting files can then be imported into VEDA-BE for subsequent analysis.

Each regional report writer step is controlled by the <case>_<region>.RPT GAMS command directive file generated by VEDA-SAGE. An overview of the calls and actions is given below.

- <case>_<region>.RPT
    - Load the <region>._G0* restart and <case>.GDX solution files
    - MMINIT.VFE
    - $SET SAGE run switches
    - SAGE_RPT.GMS
        - Buffer information from the multi-region solution for this region
        - Write the VEDA solution information
- Files produced for each region
    - <case>_<region>.LST with execution trace along with any error messages[5]
    - <case>.VDE/VDS/VD VEDA exchange files.

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[5] Note that the <case>_<region>.LST is generated first for each region in the matrix generation step, but later overwritten during the report writer step. If an error occurs during an regional matrix generation step the user must abort the run (via Ctrl-C in the DOS box) and examine the associate LST file using any text editor.