

# WORKING PAPERS



## GEOGRAPHIC MARKET DEFINITION UNDER THE DOJ GUIDELINES

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WORKING PAPER NO. 129

August 1985

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FEDERAL TRADE COMMISSION  
WASHINGTON, DC 20580

GEOGRAPHIC MARKET DEFINITION UNDER  
THE DOJ MERGER GUIDELINES\*

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I. Introduction

There is a considerable body of literature discussing how geographic markets should be delineated for antitrust purposes. Noteworthy contributions include Elzinga and Hogarty (1973), Shrieves (1978), Horowitz (1981), and Stigler and Sherwin (1983).<sup>1</sup> The 1982 Department of Justice Merger Guidelines (DOJ Guidelines) and their revision in 1984 provide a new methodology for defining markets relevant for antitrust purposes and elaborate on how this definition should be applied in a geographic market context.

This paper has four purposes:

(1) We analyze the underlying economic model of the DOJ Guidelines' treatment of geographic markets. The basis of this model is the residual demand facing a given group of producers.<sup>2</sup> The price elasticity of the residual demand provides a basis for a new empirically implementable test for the extent of geographic markets.

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\* The authors thank Scott Harvey for assistance in finding and interpreting the data on the oil industry, Ken Elzinga, John Peterman, Mark Frankena, Phillip Nelson and Jim Hurdle for helpful comments, and Mary Brown and Dan O'Brien for excellent research assistance. The opinions expressed are those of the authors, not necessarily those of the FTC.

<sup>1</sup> For a collection of many of the papers addressing the problem of delineating relevant markets in antitrust, see Elzinga and Rogowsky (1984).

<sup>2</sup> By residual demand we mean the demand function specifying the level of sales made by the group as a function of the price they charge. The analysis of residual demand in a geographic context is developed below.

(2) We develop three models in which geographic location is a critical attribute. These models provide reasonable theoretical approximations to most conceivable actual geographic market situations. We show how to derive residual demand in these models and identify its properties.

(3) Using these models we show that the criteria most commonly used in defining relevant geographic markets such as the Elzinga-Hogarty (E-H) test based on shipments data (Elzinga and Hogarty (1973)), and price tests such as those proposed by Shrieves (1978), Horowitz (1981), and Stigler and Sherwin (1983) are not generally consistent with the Guidelines' market definition.

(4) Finally, we discuss how to estimate econometrically the residual demand facing a group of producers in a geographic area and present estimates of the residual demand facing refiners of gasoline in the eastern U.S. These estimates provide evidence bearing on the extent of relevant geographic markets for the production of gasoline in the U.S., an important issue in the antitrust analysis of recent mergers of oil companies.<sup>3</sup>

## II. "Antitrust Markets" vs. "Economic Markets"

In this section we discuss the Department of Justice's Merger Guidelines' definition of antitrust markets and relate this to the classical concept of markets which we will term economic markets.

### A. The 1982 DOJ Merger Guidelines' Definition of Relevant Market

The 1982 DOJ Merger Guidelines, as revised in 1984, adopt the following basic definition of a relevant market:

Formally, a market is defined as a product or group of products and a geographic area in which it is sold such that a hypothetical, profit-maximizing firm, not subject to price regulation, that was the only present and future seller of those products in that area would impose a 'small but significant and nontransitory' increase in price above prevailing or likely future levels.

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<sup>3</sup> For example, Texaco's acquisition of Getty and Chevron's acquisition of Gulf.

(1984 DOJ Merger Guidelines, p. 4).

The 1984 Guidelines further describe how this definition will be applied in the context of geographic market analysis of a merger:

In defining the geographic market or markets affected by a merger, the Department will begin with the location of each merging firm (or each plant of a multiplant firm) and ask what would happen if a hypothetical monopolist of the relevant product at that point imposed a 'small but significant and nontransitory' increase in price. If this increase in price would cause so many buyers to shift to products produced in other areas that a hypothetical monopolist producing or selling the relevant product at the merging firm's location would not find it profitable to impose such an increase in price, then the Department will add the location from which production is the next-best substitute for production at the merging firm's location and ask the same question again. This process will be repeated until the Department identifies an area in which a hypothetical monopolist could profitably impose a small but significant and nontransitory increase in price. (1984 DOJ Merger Guidelines ,pp. 13-14).

The statement accompanying the announcement of the DOJ Merger Guidelines in 1982 and the revision in 1984 provides considerable discussion of the rationale for this approach to market definition. We will focus on the theoretical foundations of the DOJ's approach.

"The" issue in antitrust analysis is the possibility of anticompetitive effects arising from the current structure or conduct or a change in structure or conduct in a market. In the antitrust analysis of a merger the central question is whether the merger will lead to the exercise of market power by the merged entity or some larger group of producers. The Guidelines' definition of a relevant antitrust market requires the determination of the smallest relevant group of producers and geographic area (with the parties to the merger as the focus of the group) that possesses market power. To see the utility of such an approach, consider first the classical approach to defining markets. In what follows we will use the term antitrust market in referring to relevant markets as defined by the Guidelines.

## B. "Economic Markets"

The classical definition of markets, which we will call economic markets, arises from Marshall (1920, p.324), who defined a market as an area where "prices of the same goods tend to equality with due allowance for transportation costs".<sup>4</sup> Put differently, a classically defined market is that area within which prices are linked to one another but can be treated independently of prices of goods not in the market, i.e, an area within which partial equilibrium analysis is valid. Previous research on the delineation of geographic markets for antitrust analysis has generally adopted a view of markets consistent with the classical approach.<sup>5</sup>

## C. A Comparison of Economic and Antitrust Markets

Corresponding to any antitrust market there will be an economic market and the relevant antitrust market will generally be included in (and perhaps coincide with) the appropriate economic market.<sup>6</sup> Antitrust markets will sometimes be significantly smaller

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<sup>4</sup> See also Stigler (1966, p.85) Transportation costs for shipment between the two areas may create a wedge between prices in the two areas. Prices cannot differ by more than (total) transportation costs. If prices differ by less than the transportation costs, the two areas will in fact be separated. If prices differ by exactly the required transportation costs, the two areas will be integrated. See Spiller and Huang (1984) for a more detailed discussion of the role of transportation costs in delineating economic markets.

<sup>5</sup> See Elzinga and Rogowsky (1984) and references therein.

<sup>6</sup> In some circumstances it is possible for the antitrust market to be larger than the economic market. For example, suppose that producers of widgets in adjoining areas X and Y produce widgets at identical constant costs and that widgets are sold competitively in each area. Under these assumptions, as long as there are costs of shipping widgets between the two areas, there will be no shipments between them. Because, by assumption, there are no shipments between X and Y and both markets have been competitive, X and Y are not in the same economic market (because the prices of widgets in X and Y have not been jointly determined). Consider now a merger of all the producers in the relevant economic market, X. If the transactions costs of shipments from Y to X are small relative to the price of widgets (e.g., considerably less than 5% of the price of widgets), a "monopoly" producer in X would have very little latitude to raise price. Therefore, under the Guidelines' methodology, if transactions costs are small relative to price, the relevant geographic market includes X and Y, and in these circumstances the relevant antitrust market is larger than the economic market.

than their corresponding economic markets. For example, suppose sprockets are a sufficiently good substitute in demand for widgets that the economic market includes both widgets and sprockets. If the elasticity of supply of sprockets is sufficiently low it would be possible for widget producers, acting in concert but independently of sprocket producers, to profitably raise the price of widgets,<sup>7</sup> and under the Guidelines, the relevant market in which to analyze a merger between two widget producers would be no larger than the widgets, even though the economic market comprises widgets and sprockets. Indeed, some subgroup of widget producers could themselves constitute an antitrust market.<sup>8</sup>

Similarly, if there are shipments of widgets from area Y to area X, X and Y will be in the same economic market, but not necessarily in the same antitrust market relevant to the analysis of a merger of two widget producers located in X. This would be the case if the supply elasticity of shipments from Y to X is sufficiently small that a concerted price increase by the producers in X would not be rendered unprofitable by any resulting increase in shipments from Y.<sup>9</sup>

Of course a proficient antitrust economist would be able to come to grips with basic antitrust issues, such as "Will prices be likely to increase after this merger?," using either the concept of economic markets or antitrust markets. Market definition, after all, merely specifies a relevant universe within which a complete antitrust analysis should be

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<sup>7</sup> We will elaborate further on this hypothetical below. However, notice that since widgets and sprockets are good substitutes in demand, an increase in the price of widgets will, because of the presumed low elasticity of supply of sprockets, result in an increase in the price of sprockets.

<sup>8</sup> This could be the case if all widget producers outside this subgroup had low supply elasticities.

<sup>9</sup> For example, although there are imports of sugar into the U.S., there is a binding import quota. Since the effective supply elasticity of imports of sugar is zero, imports of sugar are not part of the relevant antitrust market, for the purposes of analyzing the effect of a merger between U.S. sugar producers.

focused.<sup>10</sup> Why then should we bother with a "new" definition of markets for antitrust purposes? One economic reason is that the universe specified by the antitrust market is more closely related to the question at issue: are prices likely to rise because of the merger? The dispositive reason, however, for the superiority of the Guidelines' approach to market definition arises from the fact that the Guidelines specify structural thresholds (levels and changes in Herfindahl indices) that will trigger concern with mergers. It is precisely because economic markets and antitrust markets sometimes differ significantly that the use of economic markets for specifying these structural thresholds is inappropriate.

Of course, if the Herfindahl and change in Herfindahl in the economic market always exceeded the Herfindahl and change in Herfindahl in the relevant antitrust market by a fixed absolute or relative amount, there would be no problem in specifying structural guidelines based on economic markets that were equivalent to a different set of structural guidelines based on antitrust markets. Such an equivalence is not possible.<sup>11</sup>

In summary, the greatest advantage of the Guidelines' approach to market definition over the classical approach is that it ensures a coherence across markets in the meaning and implications of structural indices such as the level and change in the Herfindahl index.

### III. An Economic Model for Applying the DOJ Guidelines' Definition of Relevant Geographic Market

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<sup>10</sup> This argument is made in Fisher (1979).

<sup>11</sup> Consider again an economic market comprising widgets and sprockets and an antitrust market limited to widgets. Let there be 10 equal-sized widget producers and  $n$  equal-sized sprocket producers and let the percentage share of widgets in the (widget/sprocket) economic market be  $x$ . The Herfindahl in the antitrust (economic) market is equal to  $1000(10[x/10]^2 + n[(100-x)/n]^2)$ . A merger of two widget producers leads to a change in Herfindahl in the antitrust (economic) market of  $200(2(x/10)^2)$ .

### A. Residual Demand in a Non-Spatial Context<sup>12</sup>

In the analysis of geographic markets that we will develop in this paper, the critical concept will be the residual demand for the output of the group of producers located in some geographic area. To illustrate this concept in its simplest form, let us begin by abstracting from geographic issues entirely. Assume first that all widget producers and purchasers are located at X and that all widget transactions occur in the spot market. Therefore, the market demand for widgets is simply the demand for widgets by purchasers at location X, and the demand function, relating purchases of widgets to the price of widgets at X is defined as a function of the FOB price of widgets. Suppose that there are  $n$  producers of widgets located at X, labelled by  $i = 1, \dots, n$ . Arbitrarily divide the producers into two groups, labelled  $G_1$  and  $G_2$ , where  $G_1$  is the group of producers labelled by  $i = 1, \dots, n_1$  and  $G_2$  is the group of producers labelled by  $i = n_1 + 1, \dots, n$ .

Since widgets are homogeneous and all producers and purchasers are located at X, the price charged by members of  $G_1$  will always be the same as the price charged by the members of  $G_2$ . If the producers in  $G_2$  act competitively, we can define the supply curve for  $G_2$ , relating the amount  $G_2$  desires to sell in aggregate as a function of the market price. Then, the demand facing  $G_1$  at any price is simply the difference between demand and the supply of  $G_2$  at that price. This demand is termed the residual demand facing  $G_1$ . In Figure I, we depict the market demand curve DD, the supply curve by producers in  $G_2$ ,  $S_2S_2$ , and the residual demand curve facing producers in  $G_1$ ,  $D_1^R D_1^R$ . The residual demand curve facing group  $G_1$  is simply the horizontal difference between DD and  $S_2S_2$  at each price.

$G_1$  has market power only if the residual demand facing  $G_1$  is sufficiently

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<sup>12</sup> The approach in this section closely follows Landes and Posner (1981). Baker and Bresnahan (1984) also use this construct in estimating the degree of market power by a single firm.



inelastic.<sup>13</sup> Alternatively, if the residual demand facing group  $G_1$  is sufficiently elastic, a small but significant nontransitory price increase would not be profitable. Therefore, the delineation of relevant antitrust markets must be based on estimates of the price elasticity of residual demand, or, if data limitations preclude such a measurement, evidence bearing on the size of this elasticity. Below, we will discuss how the price elasticity of residual demand can be measured.

#### B. Determinants of the Elasticity of the Residual Demand Curve

The elasticity of the residual demand facing  $G_1$  depends on the elasticities of the market demand curve and of the supply curve of  $G_2$ . In particular, the more elastic are the market demand curve or the supply curve of  $G_2$ , or the smaller is the share of  $G_2$  in total output, ceteris paribus, the more elastic is the residual demand curve facing the  $G_1$  producers. This can be seen in Figure I.

Formally, if  $E_1^R$  is the price elasticity of the residual demand facing  $G_1$ ,  $E^D$  is the elasticity of the market demand,  $E_2^S$  is the elasticity of  $G_2$ 's supply,  $Q$  is the total quantity demanded, and  $q_2$  is the quantity supplied by  $G_2$ , then

$$(1) E_1^R = E^D[Q/(Q-q_2)] + E_2^S[q_2/(Q-q_2)].^{14}$$

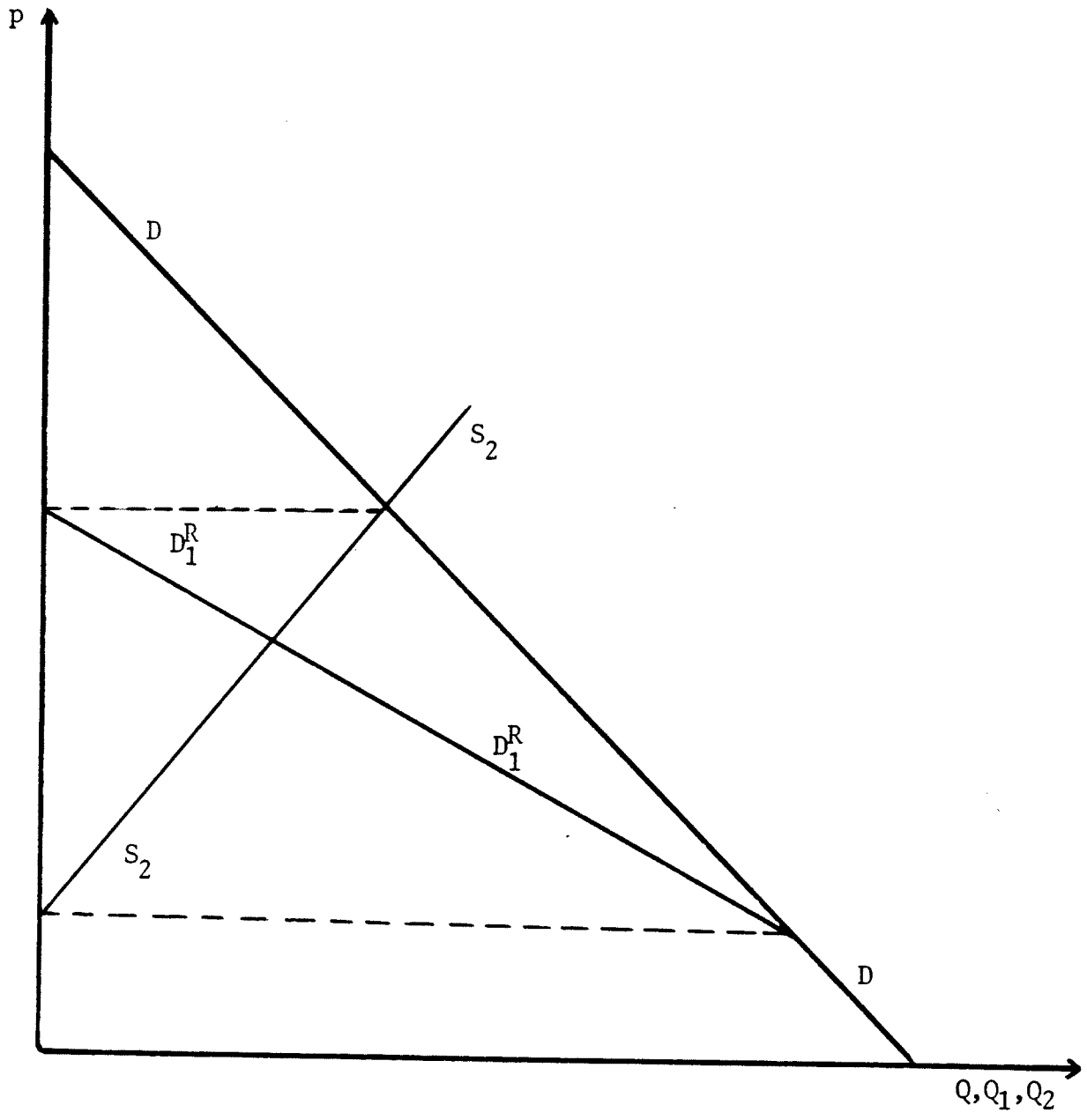
We have shown that the delineation of antitrust markets must be based on estimates of the price elasticity of residual demand. Before developing the analysis of residual demand in a geographic context, we briefly discuss the two most common empirical approaches to geographic market delineation.

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<sup>13</sup> What constitutes a "sufficiently inelastic" residual demand, however, remains an open question. The 1982 Guidelines proposed a "5% Rule" which implied, for constant costs, a benchmark elasticity of 10 (20) for a linear (log-linear) residual demand function. The 1984 revision of the Guidelines abandoned the "5% Rule" for the imprecise "small but significant" standard. Below, we will consider two criteria: the "5% Rule" and a 10% rule. The benchmark elasticities for a 10% rule (assuming constant costs) for a linear (log-linear) residual demand function is 5 (10).

<sup>14</sup> In this paper all elasticities are expressed as absolute values.

FIGURE 1



#### IV. Shipments and Price Tests

##### A. The Elzinga-Hogarty Test

Perhaps the most widely used empirical method for delineating relevant geographic markets is the Elzinga-Hogarty test (Elzinga-Hogarty, 1973). This test specifies two criteria ("little-in-from-outside" (LIFO) and "little-out-from-inside" (LOFI)) based on shipments data. Clearly, substantial shipments between two areas will place them in the same economic market. Also, intuition suggests that a situation in which few widgets are shipped into or out from X raises the possibility that producers in X have potential market power. However, it has been recognized by Elzinga and Hogarty (in their original article) and others, that if the LIFO and LOFI conditions hold for an area X, it does not necessarily follow that X is always a relevant geographic market.<sup>15</sup> Alternatively, it is possible that there are significant shipments from Y to X but that the supply elasticity of producers in Y is very low. Then, if the market elasticity of demand for widgets is sufficiently low, the elasticity of the residual demand facing producers in X would be low enough to justify defining X as a relevant market. (See equation (1)). Therefore, as Elzinga and Hogarty and others have recognized, if the LIFO and LOFI benchmarks are surpassed, X may nonetheless constitute a relevant geographic market.

##### B. Price Tests

Shrieves (1978), Horowitz (1981), and Stigler and Sherwin (1983) suggest empirical tests that focus on the pattern and trend of prices prevailing in different areas. For

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<sup>15</sup> For example, suppose that there are identical constant average costs of production in the two areas X and Y. Assume further that the costs of transporting widgets from X to Y are insignificant. Under competitive conditions, prices in the two areas would be identical even though there are no shipments between the two areas. However, even if producers in X were cartelized, they could not raise the price in X because producers in Y would be able to increase sales into X at the pre-cartel price. In this example the residual demand curve facing producers in X is perfectly elastic, even though there might not be any shipments into or out from X. Therefore, as Elzinga and Hogarty and others have recognized, if the LIFO and LOFI conditions hold for area X, it does not necessarily follow that X is a relevant antitrust market.

example, if areas X and Y are in the same economic market, on average the prices in the two areas should move together, with the difference in prices in the two areas approximating marginal transportation costs. The empirical implementation of the Stigler and Horowitz tests requires econometric estimates of the relationship between prices over time in X and Y.<sup>16</sup> These tests share the same problems as the shipment tests: they are designed to identify economic markets, not antitrust markets. In particular, the relationship between prices in X and Y provides little information about the elasticity of the residual demand facing producers in X.<sup>17</sup>

Thus, in general shipments and price tests will not provide a method for delineating antitrust markets. However, in order to specify the residual demand facing a group of producers, their competitors (in the economic market) must be identified. Therefore, shipments and price tests can be of use in the process of identifying antitrust markets.

#### V. Residual Demand in a Geographic Context

In the hypothetical depicted in Figure I, the residual demand facing  $G_1$  does not have as a separate determinant the price charged by  $G_2$ . This is a critical attribute of residual demand: it is constructed so that the only widget price determining the level of residual demand for widgets facing  $G_1$  is the price by charged  $G_1$ , i.e., the residual demand for the sales of  $G_1$  does not have as separate determinants the prices charged by producers

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<sup>16</sup> In most circumstances, if the prices in two areas are independent of each other, the two areas would not be in the same antitrust market. See, however, fn. 6 above.

<sup>17</sup> Consider, for example, two alternative circumstances, one where producers in Y face a binding capacity constraint, and another where producers in Y have constant average costs. Assume further that in both circumstances there are substantial shipments from Y to X, so that prices are always highly correlated between areas Y and X. Assuming the demand in X to be relatively inelastic, when producers in Y face a binding capacity constraint, producers in X could, in principle, raise prices if they coordinate their actions. That will not be the case, however, if producers in Y have constant average costs.

outside  $G_1$ .<sup>18</sup>

In an explicit geographic context,  $G_1$  and  $G_2$  may have different locations, so even under competitive conditions they may charge different FOB prices. In addition, if purchasers are at different locations, they may pay different delivered prices. As a consequence, defining residual demand for  $G_1$  as a function only of "the" price set by  $G_1$  becomes more complicated. The models developed below show how the residual demand of the group of producers in a given geographic area can be derived and identify its properties. These three models provide a framework for the analysis of most situations that arise in a geographic market context, and so they provide a theoretical foundation for the specification and measurement of residual demand in a geographic context.

The first model is concerned with markets in which the flow of shipments between areas is predominantly one-way. The generic situation arises when there are two (or more) geographic areas, X and Y, for which the transportation costs within each area is significantly lower than the transportation costs between areas. Typically, in such situations if there are shipments of a homogeneous good between the two areas, the shipments are predominantly one-way.<sup>19</sup>

The second model is concerned with a situation in which the market areas served by two geographically separated groups of producers are adjacent, but the extent of actual overlap between the market areas (in terms of sales by both groups in the same approximate

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<sup>18</sup> Formally, if  $F_n(p_1, p_2, \dots, p_n)$  is the demand facing  $G_1$  written as a function of the prices of all producers, if all producers are in the same economic market, equilibrium requires that there be a relationship between the prices, expressed by  $H_2(p_1, \dots, p_n) = 0, \dots, H_{n-1}(p_1, \dots, p_n) = 0$ , so that  $p_2, \dots, p_n$  can be solved in terms of  $p_1$ . Later we will discuss residual demand with heterogeneous goods in a geographic context.

<sup>19</sup> A specific example arises in refined petroleum products in the eastern U.S. (the focus of our empirical analysis discussed later in this paper), where, for example, there are imports of refined petroleum products into the Northeast from the Gulf and outside the U.S., but there are virtually no exports from the Northeast.

location) is minimal. Local wholesale gasoline markets may be an example.<sup>20</sup> In markets approximated by this model, competition between two groups of producers in adjoining locations occurs, if at all, mostly with respect to the extent of the market areas that they serve.

Our last model allows the possibility that there may be a significant overlap in the market areas served by the two producer groups, i.e. that there may be substantial direct competition between the two groups. In such a situation an increase in the price of one producer group may not result in a change in the boundaries of the market areas served by the two groups.<sup>21</sup> As we will see below, the existence of a location in which both groups of producers have significant sales, a not uncommon situation, can greatly simplify the analysis of geographic markets. Finally, we will discuss how the analysis can be modified to include heterogeneous goods.

#### A. Model I

Consider two geographic areas denoted X and Y, each containing both producers and purchasers of some homogeneous product, say widgets. We assume that the transportation costs incurred for shipping widgets within either of the areas are small enough so that a delineation of relevant geographic markets would not comprise areas smaller than X or Y. We also assume that the costs incurred for transporting widgets between the two areas are

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<sup>20</sup> For example, wholesale gasoline terminals located in Washington, D.C., and Baltimore, Maryland, serve most gasoline retailers at and between the two locations because there are no other terminals located between the two locations. The areas served by the two terminal clusters are adjoining, but there is generally not much overlap between them.

<sup>21</sup> Consider, for example, a situation in which producers in New York City sell in and to the west of the New York City area and producers in Chicago sell in and to the east of the Chicago area. Assume further that both groups of producers have significant portions of their sales in the Cleveland area, which serves as an approximate geographic dividing line between the market areas of the two groups. Then, a ceteris paribus increase in the price of producers in New York (caused, e.g., by an increase in their costs) may reduce their sales in the Cleveland area, but leave the geographic areas served by the New York and Chicago producers, respectively, unaltered.

significantly higher than the costs of intra-area shipments. Finally, we assume that the pattern of shipments is predominantly one-way, with widgets shipped from Y to X. One application of this model is to products involved in international trade, although shipment patterns between different intra-national regions often are also characterized by one-way flows. The issue to be addressed is whether the relevant geographic market that includes the X-producers also includes the Y-producers.

We begin by adopting some notation.

**Definitions:**

$Q_z$  production in market  $Z = Y, X$

$M_{yx}$  net exports from Y to X

$C_z$  consumption in market  $Z = Y, X$

$p_z$  price in market  $Z = Y, X$

$r$  constant transportation costs (per unit) from Y to X.<sup>22</sup>

We assume that the demand for widgets in each area is a linear function of the delivered price of widgets in that area, and that the cost of widget production is such that the (competitive) supply curve of widgets in each area is a linear function of the net (of transportation costs) price received by producers.<sup>23</sup> Therefore, the supply and demand in each area  $Z = Y, X$  will be written as

$$\text{SUPPLY: } p_z = A_z + B_z Y_z$$

$$\text{DEMAND: } p_z = J_z + K_z C_z$$

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<sup>22</sup> Typically, this would be the most reasonable assumption. However, if the widget industry is large enough or its transportation requirements are sufficiently specialized to make the supply of transportation services to the widget industry upward sloping, or the import of widgets into Y faces an ad valorem or sliding scale tax or duty, the constant marginal transportation costs assumption would not hold. Making alternative assumptions, such as increasing marginal transportation costs, makes the analysis more complex, but does not alter the qualitative conclusions.

<sup>23</sup> The qualitative results are unaffected by a more general specification of demand and supply.

where  $A_y$ ,  $B_y$ ,  $J_y$  and  $K_y$  are parameters.<sup>24</sup>

The supply of exports (demand for imports) from Y to X can be derived from the supply and demand curves in area Y, assuming that producers in area Y act competitively. At any price  $p_y$ , the supply of exports to X, i.e., the excess supply in Y, is simply the difference between the supply and demand in Y ( $Q_y - C_y$ ) at price  $p_y$ .

In what follows we consider two kinds of geographic equilibria. First, we assume that Y-producers and X-producers act competitively. Then, we assume that X-producers cartelize, and analyze the conditions under which such a cartel could profitably raise price significantly above the competitive level, i.e., whether X alone is a relevant antitrust market. Under the assumption that Y-producers act competitively and there are shipments from Y to X, equilibrium requires that the net (of transportation costs) price that Y-producers receive for sales in X is the same as the price they receive for sales in Y, i.e.,

$$(2) \quad p_x - r = p_y$$

Equation (2) shows that  $p_y$  and  $p_x$  are jointly determined, reflecting the fact that areas Y and X are in the same economic market.

A second condition for a geographic equilibrium is that the supply of exports from Y to X is equal to the demand for imports in X from Y, i.e.,

$$(3) \quad M_{yx} + M_{xy} = 0$$

Using the demand functions for X and Y and the supply function for Y and the relationship between prices in X and Y given by (2), the residual demand function facing X, denoted  $Q_x^R(p_x)$  can be shown to be:

$$(4) \quad Q_x^R(p_x) = (k_x + k_y - b_y)p_x + [-J_x k_x - (k_y - b_y)r + (A_y b_y - J_y k_y)].$$

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<sup>24</sup>  $A_y$  and  $J_y$  also can be conceptualized as containing other variables that affect demand and supply such as income and cost variables.



Notice that the residual demand facing X depends only on the price charged by X,<sup>25</sup> and on variables that would shift demand in Y and X (variables that are suppressed in  $J_y$  and  $J_x$ , such as income in Y and X) and variables that would shift supply in Y (variables that are suppressed in  $A_y$ , such as the prices of inputs in Y). To measure this residual demand, we would need data on unit sales by X, prices charged by X-producers, and on other variables influencing demand in Y and X (suppressed in  $J_y$  and  $J_x$ ), and influencing costs in Y (suppressed in  $A_y$ ).<sup>26</sup> Alternatively, if estimates are available for  $E_x^D$ ,  $E_y^D$  and  $E_y^S$ , and data is available for consumption and production in X and Y, the price elasticity of residual demand,  $E_x^R$  can be estimated from:

$$(5) E_x^R = E_x^D(p_x C_x / p_x Q_x) + E_y^D(p_y C_y / p_x Q_x) + E_y^S(p_y Y_y / p_x Q_x).^{27}$$

As expected, the residual demand facing X-producers is more elastic, the more elastic are the demands in Y and X and the supply in Y, and the larger is the value of shipments from Y to X relative to the value of production in X.<sup>28</sup>

## B. Model II

In the preceding model the costs of transporting widgets to customers within a given area were assumed to be small relative to the costs of inter-region transport. In this section we explicitly consider the possibility that purchasers and producers in a given region may have different locations and that the costs of intra-region transport may be

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<sup>25</sup> In particular, (4) is not a function of  $p_y$ , this variable having been substituted out, reflecting how producers in Y will respond to an increase in the price in X.

<sup>26</sup> We will discuss how to measure residual demand in Section VII.

<sup>27</sup> Note that in the derivation of (4) to (7) the direction of shipments was not used. Therefore (4) - (7) are equally valid for a situation in which there are shipments from X to Y instead of from Y to X. If there are shipments from X to Y it is likely that the relevant geographic market containing area X is X itself. However, as can be seen from (5), if the demand for exports from Y to X is sufficiently elastic, it may not be possible for a cartel in X to profitably raise price by a small but significant amount.

<sup>28</sup> Observe, also, that if marginal transportation costs were not assumed to be constant, the elasticity of marginal transportation costs would also affect the elasticity of the residual demand faced by X.

relevant. We assume that producers and purchasers are located along a line, and that one group of producers, denoted  $x$ , is located at  $(0)$ , and the other group of producers, denoted  $y$ , is located at  $Y (>0)$ . For ease of exposition we will assume that purchasers of widgets are identical, except in location, and are uniformly distributed on the line between  $(0)$  and  $Y$ . As in the preceding model, we assume that purchasers have linear demand curves and that the producers' supply curve is linear.

**Definitions:**

Transportation costs (per widget) from 0 to $w$	$r(w)$ , $r' > 0$
Transportation costs from $Y$ to $w$	$r(Y-w)$
F.O.B. price of producers $z=x,y$	$p_z$
Delivered price paid by a purchaser located at $w$ to producer $z$	$P_z(w)$
Supply (marginal cost) curve of producer group $z$	$MC_z(X_z) = A_z + B_z X_z$ , with $b_z = 1/B_z$
Demand curve of a purchaser located at $w$	$D(P(w)) = -Jk + kP(w)$ ( $k < 0$ )

Solving for the residual demand facing  $x$  is somewhat complicated, so the details are left to the Appendix. For the case in which producers have constant costs ( $B_x=0$ ), residual demand is easily derived. As shown in the Appendix, in this case the residual demand facing  $x$ ,  $Q_x^R(p_x)$ , can be written:

$$(6) \quad Q_x^R(p_x) = -k(A_y + rY - p_x)[J - p_x - (A_y + rY - p_x)/4]/2r.^{29}$$

As with (4), the residual demand facing  $x$  depends on the (FOB) price they charge and on other variables affecting demand, such as income (suppressed in  $J$ ), and variables affecting the costs of  $y$  (suppressed in  $A_y$ ). Even though the supply elasticity of the  $Y$  is infinite (because we have assumed constant costs here), the residual demand curve facing  $x$

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<sup>29</sup> Recall  $k < 0$ . Notice that residual demand goes to zero when the delivered price to the consumers located farthest from  $Y$  ( $A_y + rY$ ) is equal to the FOB price of producers at 0,  $p_x$ .

is not perfectly elastic. This is because the dispersion of purchasers and positive transportation costs give  $x$  locational market power. If  $y$  had increasing costs, the residual demand curve facing  $x$  would be less elastic than is (6). As shown in the Appendix, the elasticity of  $Q_x^R$  at the competitive price  $p_x=A_x$  is given by:

$$(7) E_x^R = A_x[J-A_x+(A_y+rY-A_x)/2]/[(A_y+rY-A_x)(J-A_x-(A_y+rY-A_x)/4)].$$

The values of  $A_x$  (average costs of  $x$ ),  $J$  (the intercept of the individual purchasers' demand curves),  $r$  (transportation costs) and  $Y$  (the distance between the two production locations) determine whether the area served by  $x$  should be treated as a separate antitrust market.<sup>30</sup> In particular, the larger is  $A_y$  (average costs of  $Y$ ), or  $rY$  (the cost per widget of shipping widgets from  $Y$  to  $X=0$ ), the lower is  $E_x^R$ , increasing the possibility for  $x$  to profitably raise prices by more than a trivial amount.

### C. Model III

In the model of the preceding section the extent of direct competition (i.e., competition for the same purchasers), was minimal. Some markets combine the dominance of a producer group in its directly surrounding area with direct competition for customers not located in the vicinity of any significant producer group. In this section we incorporate the possibility of significant direct competition between the two producer groups. Again, we assume that the two producer groups are located at (0) and  $Y$ . As in the preceding section we also assume that purchasers are uniformly distributed between these two points, except, that  $Z$  purchasers are massed at location  $w'$ .<sup>31</sup> Because our model is one-dimensional, the point  $w'$  will be the boundary of the marketing areas of the two producer groups.<sup>32</sup> Direct competition between  $x$  and  $y$  occurs with respect to the  $Z$

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<sup>30</sup> Notice that the whole area between  $X$  and  $Y$  constitutes an economic market.

<sup>31</sup> We can think of location  $w'$  as being a city where there is a substantial (relative to other locations) number of purchasers of widgets.

<sup>32</sup> A more elaborate model (e.g., two-dimensional) would allow more general marketing areas, but would not change the basic qualitative conclusions derived from our simple model.

purchasers located at  $w'$ .

The details of the model are worked out in the Appendix, where we show that the residual demand facing  $x$  is:

$$(8) Q_x^R(p) = Sp_x + R, \text{ where}$$

$$(8a) R = -kw'[J-(r/2)w'] - [b_y - k(Y-w')][rw' - r(Y-w')] \\ + [A_y b_y + k(r/2)(Y-w')^2] - kZ(J-w')$$

$$(8b) S = [k(Z+Y) - b_y].^{33}$$

The residual demand facing  $x$  is linear, with its slope and location depending on cost conditions at  $y$  ( $b_y$  and  $A_y$ ). In general form this residual demand is analogous to (4), with the residual demand determined by the price charged by  $x$  and variables affecting demand and the costs of the  $Y$ .<sup>34</sup> The possibility of direct competition between the two producer groups for purchasers located at  $w'$  greatly simplifies the analysis of the earlier model, because the competition between the two groups becomes essentially localized at  $w'$ .

Since (8) is linear, the elasticity of the residual demand facing  $x$  is easily derived:

$$(9) E_x^R = -p_x / (p_x + R/S)$$

#### D. Price and Shipments Tests

In each of the three models prices of two producer groups were correlated,<sup>35</sup> so that price tests would (correctly) conclude that the two groups were in the same economic

<sup>33</sup> Notice in (10) that we are not assuming here that  $y$  has constant marginal costs ( $B_y=0$ ), since (10) requires  $b_y$  ( $=1/B_y$ ) to be finite. If  $y$  had constant marginal costs, the residual demand facing  $x$  would be perfectly elastic at the price  $A_y+r(Y-w')$  as long as output levels by  $x$  required sales to some of the  $Z$  purchasers at  $w'$ . If  $x$  raised price sufficiently, they would lose all their sales at  $w'$  and the model would be similar to the model of the preceding section. Independent of the cost conditions of  $y$ , the presence of significant direct competition between the  $x$  and  $y$  makes the residual demand facing  $x$  more elastic, ceteris paribus.

<sup>34</sup> Recall, as above, that the specification of the residual demand facing  $x$  requires that their competitors be identified.

<sup>35</sup> In models I and III, prices were perfectly (linearly) correlated.

market.<sup>36</sup> The Elzinga-Hogarty test could be of some value in delineating antitrust markets for markets approximated by Models I and III, because the relative magnitude of shipments can be used as an input into the determination of the price elasticity of residual demand. However, in markets approximated by Model II, since market areas served by the two producer groups are always distinct,<sup>37</sup> shipment tests would imply the two areas are always separate markets.

### Summary

We developed three models of spatial competition which reasonably approximate most "real world" situations. We showed that it is possible to define the residual demand facing a given group of producers and that the determinants of residual demand are the prices charged by the producer group, variables affecting demand, and variables affecting the costs of competing producers not in the group. In market situations in which there is substantial direct<sup>38</sup> competition between the relevant group of producers and other producers (Models I and III), residual demand is particularly easy to define and derive. Finally, we indicated how the residual demand analysis must be modified for heterogeneous products.

### VI. Residual Demand with Heterogeneous Goods

Consider two heterogeneous goods,  $x$  and  $y$ , that are substitutes in consumption and are produced and consumed in the same location. Write the (interrelated) demands for the two goods as:

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<sup>36</sup> Of course, they will not be able to discriminate between economic and antitrust markets.

<sup>37</sup> Although such a stark delineation of market areas would be an unusual occurrence, the model may be a reasonable approximation of some markets, such as wholesale gasoline terminalling and cement distribution. In some circumstances Model III will provide a better approximation.

<sup>38</sup> In the sense that a substantial amount of sales are made by both the group and its competitors to purchasers in some locations.

$$(10) \text{ a) } Q_x^D = f^x(p_x, p_y, d_x)$$

$$\text{ b) } Q_y^D = f^y(p_x, p_y, d_y),$$

where  $d_x$  and  $d_y$  represent other variables determining the demands for  $x$  and  $y$ . Let the supply function for the producers of  $y$  be:

$$(11) Q_y^S = g^y(p_y, s_y),$$

where  $s_y$  represents other variables determining the supply of  $y$ .

Equilibrium in the  $y$  market requires that  $Q_y^D = Q_y^S$ , which, using (10b) and (11), yields a relationship between  $p_x$  and  $p_y$ :

$$(12) p_y = h(p_x, d_y, s_y).$$

This is analogous to our analysis of spatially differentiated production above, where we used the fact that two groups of producers were in the same economic market in order to derive the relationship between their prices.<sup>39</sup> Then, (12) can be substituted into (10a), giving us the residual demand facing the producers of  $X$ ,  $Q_x^R(p_x)$ :

$$(13) Q_x^R(p_x) = f^x(p_x, h(p_x, d_y, s_y)) = f_x^R(p_x, d_y, s_y).$$

Notice that (13) is identical in form to the residual demand functions derived above, i.e., the residual demand facing the producers of  $X$  depends on the price they charge and on variables affecting the demands for  $X$  and  $Y$  and affecting the supply of  $Y$ .

Therefore, for heterogeneous goods, we can define a residual demand for one of the goods in a manner analogous to defining the residual demand facing a given group of producers. However, if the goods are sufficiently heterogeneous so that they are not close substitutes in demand (and thus, their prices will not be near-perfectly correlated), a different approach can be used. Consider the demand for one of the goods written as (10a). We could attempt to estimate that demand function. Now the own-price elasticity of demand measured from (10a) does not measure the own-price elasticity of the residual demand facing  $x$  because (10a) does not measure the reaction of the  $Y$  to changes in the price of  $X$

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<sup>39</sup> Recall, for example, equation (3) above.

(this reaction is summarized in (13). However, we can relate the own-price elasticity of (10a) to the own-price elasticity of the residual demand facing  $x$  (13).

Let  $E_j^i$  be the elasticity of demand for good  $i$  ( $=X, Y$ ) with respect to the price of good  $j$  ( $=X, Y$ ), let  $E_y^S$  be the supply elasticity of good  $Y$ , and let  $E_x^R$  be the elasticity of the residual demand facing producers of good  $X$ . Then it is easily seen from (12) and (13) that

$$(14) \quad E_x^R = E_x^X - E_y^X E_x^Y / (E_y^S + E_y^Y).^{40}$$

Observe that  $E_x^R = E_x^X$  only when the supply elasticity for  $Y$  is infinite. Otherwise (assuming that  $E_y^X$  and  $E_y^Y$  have the same sign),  $E_x^R < E_x^X$ .

In a geographic context with heterogeneous goods, residual demand can be defined using the earlier analysis plus the analysis of this section. If the goods in question are sufficiently heterogeneous so that they are not close to being perfect substitutes in demand, their prices will not be nearly perfectly correlated, and geographic residual demand can be measured as a function of the prices of all of the heterogeneous goods. The own-price elasticity of this "semi"-residual demand function will allow us to place an upper bound on the own-price elasticity of the "full"-residual demand function using an analysis similar to (14).

## VII. The Econometrics of Residual Demand:

### Gasoline Refining in the Eastern United States

#### A. The Econometrics<sup>41</sup>

The general specification of the residual demand function is given by

$$(15) \quad p_x = F(Q_x^R, d_x, d_y, s_y, t_x, e_x^D),$$

where  $y$  represents the other supply area(s),  $p_x$  the price at location  $x$ ,  $d_i$  ( $i=x, y$ ) a

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<sup>40</sup> Recall that all elasticities are taken as absolute values.

<sup>41</sup> The econometric problems related to the estimation of the residual demand faced by a single firm (discussed by Baker and Bresnahan (1984)) are similar to those discussed here.

vector of other variables affecting demand in  $i$ ,  $s_y$  a vector of variables affecting supply in  $y$ ,  $t_x$  a vector of variables affecting transportation costs, and  $e_x^D$  stochastic shocks to the residual demand. Assuming competition in  $X$ , the supply side can be described as

$$(16) \quad p_x = G(Q_x^S, s_x, e_x^S),$$

with  $s_x$  representing a vector of variables affecting supply in  $X$ , and  $e_x^S$  stochastic shocks to the supply in  $X$ . Equilibrium requires that the residual demand equals the quantity sold:

$$(17) \quad Q_x^R = Q_x^S.$$

From the system (15)-(17) we are primarily interested in the estimation of the residual demand (15). It will generally not be difficult to obtain the data necessary to estimate (15). Only the prices and sales of producers in region  $x$  (not those in the competing production locations) are needed. Interregional shipments data are not needed. However, as will be discussed below, much care has to be devoted to the specification of the vectors  $s_x$  and  $s_y$ . These vectors represent regional supply (cost) conditions.

Since the system (15)-(17) has two endogenous variables,  $p_x$  and  $Q_x^R$ , two stage least squares estimation is required.<sup>42</sup> Identification of the residual demand function requires that some of the cost shifters in  $s_x$  should not form part of the vector  $s_y$  in (15). When the relevant areas are national regions, knowledge of regional input costs and capacity conditions should allow the estimation of (15).<sup>43</sup> If the two production groups are located in the same area, regional differences in input costs may not be observed, and the

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<sup>42</sup> Joint estimation of (15)-(17) performed using three stage least squares. The advantage of using three stage least squares is that it takes into account the correlation in the error terms ( $e_x^D, e_x^S$ ). This method, however, would require careful treatment of the supply equation (16), otherwise, its misspecification will translate into its error term and pollute the estimation of (15). Since we are not interested in (16) by itself we would recommend against its use.

<sup>43</sup> Caution, however, should be exercised when using regional factor prices and capacity levels since these may in turn be endogenous variables.



identification problems may make the estimation of residual demand infeasible.

#### B. Gasoline Refining in the Eastern United States.

Recently, some of the mergers of major oil companies have required the examination of the possibility that the merger would lead to anticompetitive effects in the production of light refined products (gasoline, kerosene, etc.) in the eastern U.S.<sup>44</sup> In what follows we describe estimates we have made of residual demands for unleaded gasoline in the eastern U.S. Our estimates provide new evidence relevant to the delineation of geographic markets for gasoline in the eastern U.S. Unleaded gasoline was chosen as the relevant product because of the convenience of product homogeneity and the associated simplification of the choice of the relevant price. In the short run there are no good substitutes in demand for unleaded gasoline.<sup>45</sup>

##### 1. The Industry.

The refining of gasoline east of the Rockies is divided into three main geographic areas by the Department of Energy: PAD's I-III.<sup>46</sup> Gasoline is also imported from overseas into the eastern U.S., mainly from the Caribbean and Europe.<sup>47</sup> Recently, imports have become a significant proportion of total consumption in the Northeast.

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<sup>44</sup> For example, if the the northeastern U.S. is a relevant antitrust market, some of the mergers involved levels and changes in the Herfindahl indices that raised concerns when tested against the DOJ Merger Guidelines. In some instances, (e.g., Texaco's acquisition of Getty and Chevron's acquisition of Gulf) FTC approval of the merger was contingent on the divestiture of some refining assets in the Northeast.

<sup>45</sup> Automobiles and trucks produced since 1974 were required by regulation not to use leaded gasoline. The main factor affecting the demand for unleaded gasoline relative to the demand for leaded gasoline is the stock of pre-1974 automobiles which we capture with a trend in our estimates.

<sup>46</sup> PAD III contains the Southeast (with most production occurring in the Gulf Coast), and refines more than half of all the product produced in the U.S. Pad II, contains the Midwest (with most of the product originating in Oklahoma, Kentucky and Illinois), and refines about one-third of the total output of the U.S. PAD I contains the Northeast (with most refiners located in the Philadelphia area), and produces the remaining 15% of output.

<sup>47</sup> There are also some imports from Canada to the Midwest, but their magnitude has been historically small.

Gasoline produced in the Gulf Coast is shipped to both the Midwest and the Northeast. Shipments to the Northeast are sent by pipelines or by ocean-going tankers. Product flows to the Midwest from the Gulf Coast via pipelines and inland water cargoes. There are no significant shipments between the Northeast and the Midwest.

The economic market for the refining of unleaded gasoline east of the Rockies contains PAD's I-III. Prices are highly correlated (the correlation between weekly average price quotations in the Gulf and in New York is .995 for the period 1/1981 to 2/1985).<sup>48</sup> In addition, there are substantial shipments from the Gulf Coast to the other two areas, indicating that PAD's I and III and PAD's II and III (and therefore, PAD's I-III), are in the same economic market.

The costs of producers in the three regions differ. Residual fuel oil and natural gas are important energy inputs for northeast and Gulf Coast refiners, respectively. Crude oil refined in the Gulf Coast is predominantly produced in that area, but crude oil inputs of northeastern refiners are predominantly from abroad. Gasoline is refined as a by-product of producing heating oil and the production of heating oil is driven by the weather-determined demand for heating oil. Consequently, different relative production levels of heating oil, ceteris paribus, imply different marginal costs of refining gasoline.

Refining capacity has been falling during the 1980s in the whole country, with refiners in the northeast reducing capacity at a slightly higher rate than the rest. The reduction in refining capacity is a result of the combination of both an overall reduction in demand and outdated capacity. Because of the downward trend in capacity, it would not be expected that capacity played an important role in constraining production during the 1980's.

### C. Estimates of Residual Demands for Gasoline Refining

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<sup>48</sup> Of course this is partially explained by a major common input, oil.

There are three potential candidates for relevant antitrust markets in gasoline refining that contain the Northeast: a) The whole area east of the Rocky Mountains (PAD's I-III); b) the Gulf Coast together with the East Coast; and c) the Northeast alone.<sup>49</sup>

Above we discussed the technology of production and we observed that there are factors resulting in somewhat different cost factors in the different parts of the country. There are both common and regionally distinct demand shifters. Measures of general economic activity represent common demand shifters. On the other hand different climate conditions may affect differently travel related gasoline consumption in the different areas. Sales of refined unleaded gasoline are obtained by subtracting changes in gasoline inventory held by refiners from gasoline production.

Table 1 presents the variables used in the estimation for the different areas. The Data Appendix summarizes the sources of the data. We estimate monthly residual demand functions for a number of possible relevant antitrust markets for the period April 1981 to February 1985 (47 observations). The general specification that we use is as follows. All variables except the weather variables are expressed in natural logarithms.<sup>50</sup> Let  $P$  be the price and  $Q$  the quantity sold by the relevant group of producers,  $Y$  a vector of demand shifters,  $W$  a vector of cost shifters for potential competitors, and  $Z$  the vector of cost shifters for the producers whose residual demand is being estimated. Then, we estimate

$$(18) P_t = a + bQ_t + cW_t + dY_t + eP_{t-1} + fW_{t-1},^{51}$$

with the vector  $Z_t$  serving as instruments for  $Q_t$ . We chose to use price as the

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<sup>49</sup> The producing areas in PAD I are the Northeast and Appalachia. The Midwest and the Northeast combined should not be considered a potential antitrust market since arbitrage between the two would include the Gulf producers.

<sup>50</sup> Heating degree days are zero for some months.

<sup>51</sup> We include lagged values for price and competitors' cost shifters since in a monthly model it is reasonable to assume that there are adjustment processes in both demand and supply.

left-hand-side variable for two main reasons. First, if the set of producers for which the equation is estimated face a highly elastic residual demand, the coefficient of  $Q$  in equation (18) should be approximately 0. If, instead, we chose  $Q$  as the left-hand-side variable, and the residual demand is highly elastic, the standard error of  $P$  (as the right hand side variable) would be very large. The second reason for choosing price as the left-hand-side variable is that the extent of measurement errors in the variables is of prime consideration. Generally, data on prices is more likely to be subject to errors than data on sales. Thus, we expect to reduce measurement error biases by using price as the dependent variable. We begin by estimating the residual demand facing the producers in the whole region east of the Rockies. This region faces competition mostly from imports from the Caribbean, Europe, and to a limited extent, from Canada. The amount of imports has been historically very small as a proportion of production in the area. However, it is possible that the supply of imports to this region is sufficiently elastic that the residual demand for production in the area could be too elastic for the area to constitute an antitrust market.<sup>52</sup> The estimation of the residual demand for the whole area is presented in Table 2, columns 1 and 2.<sup>53</sup> Two specifications are presented. One uses the price in New York and the other the price in the Gulf. The estimates are robust to the specification of price. Tests for endogeneity of oil prices,<sup>54</sup> and for serial correlation

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<sup>52</sup> Recall equation (5) above.

<sup>53</sup> The details of the specification of the residual demand for the whole region east of the Rockies are as follows. First, we use as the cost shifter for outside suppliers the price of crude oil in the North Sea. Second, the set of instruments used in the estimation includes all the cost shifters for the domestic producers identified in Table 1. As discussed above there are reasons to believe that total capacity cannot be treated as an exogenous (to demand shocks) cost shifter. Thus, we use only lagged capacity in PAD I as an instrument for cost conditions. We also estimated the equation using as the importers' crude costs the North Africa crude oil spot price. The elasticity estimates using this variable were marginally higher.

<sup>54</sup> For example, if a shock to domestic demand had a significant contemporaneous effect on the world price of crude oil, the crude oil price would not be exogenous in the residual demand equation.

indicated that these were not a problem.<sup>55</sup> The point estimates are all of the right sign.<sup>56</sup> The demand shifters are also of the right sign,<sup>57</sup> but the coefficient of industrial production is insignificant.<sup>58</sup>

Both specifications result in an estimate of price elasticity of approximately 1,<sup>59</sup> which, as will be discussed further below,<sup>60</sup> is consistent with the region east-of-the-Rockies constituting a relevant antitrust market. Observe that the cost shifters for imported gasoline are important determinants of the residual demand,

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<sup>55</sup> To test for the endogeneity of the crude oil price we perform the specification test suggested in Hausman (1978). The test statistic, which is distributed Chi-squared with 10 degrees of freedom, has the value of 1.841 for the New York price specification and of 2.109 for the Gulf Coast price one. Both test statistics are clearly insignificant, failing to reject the exogeneity of the crude oil price variable. We also performed two tests for serial correlation. The tests do not reject the hypothesis of the absence of serial correlation. Because of the lagged dependent variable in the equation the Durbin h test is reported. Since the Durbin h test cannot always be calculated we regress the residuals of the regression on all right hand side variables and the residuals lagged. If the coefficient of the residual lagged is significantly different from zero then it provides evidence of serial correlation in the equation (see Judge, *et al* (1982)). Both the Durbin h statistic and the t-statistic for the coefficient of the lagged residual show no evidence of serial correlation.

<sup>56</sup> For example, the coefficient of foreign crude oil price is positive, as expected, since its increase should imply an increase in outside producers' costs and thus increase the residual demand for domestic refiners

<sup>57</sup> To avoid multicollinearity we included only one temperature variable in the equation (Chicago).

<sup>58</sup> We also used personal income and the results were essentially the same. Personal income and industrial production seem to have a very small short run effect on the demand for gasoline in our monthly model. Current industrial production was insignificant but had a negative coefficient. We present the results with industrial production lagged twice since that specification provides the best fit. The results, however, are robust to with respect to which income variable is chosen as well as which lag is used.

<sup>59</sup> Recall that all elasticities are taken as absolute values. The elasticity is calculated as one minus the estimated coefficient of lagged price divided by the estimated coefficient of quantity. Observe that since the point estimate of the elasticity is the ratio of two normal variables, there is no simple confidence interval that can be provided for the elasticity.

<sup>60</sup> Where we will discuss statistical tests of the price elasticities.

indicating the effect of imports on the effective demand faced by domestic producers. The effect of misspecifying the residual demand by excluding these variable is indicated in the results presented in Table 2, columns 3 and 4. We observe that, as predicted, the elasticity estimated in this way falls short of the elasticity estimated from the residual demand.

Columns 1 and 2 of Table 3 present estimates of the residual demand faced by refiners in PAD III and PAD I, combined. For this conjectured antitrust market the outside producers include those in the Midwest and the foreign producers.<sup>61</sup> As for the whole east-of-the-Rockies region, we present specifications using both the the New York price and the Gulf Coast price. We find no significant difference between the two specifications. Again, all the estimated coefficients for cost and demand shifters are of the the right sign. Also, no serial correlation is present in these two equations. The price elasticity of residual demand for the two regions combined is slightly above one, ranging from 1.1 to 1.3, which is consistent with the combined area constituting a relevant antitrust market.

The estimate of residual demand facing producers in the Gulf Coast (PAD III) is presented in Table 3, columns 3 and 4.<sup>62</sup> The estimated price elasticity of the residual demand for PAD III is between 1.3 and 1.6.

Table 4 presents the estimates of the residual demand facing refiners in PAD I and in the Northeast.<sup>63</sup> We again observe that the estimates are relatively robust to the

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<sup>61</sup> We account for the cost conditions of the midwestern producers by including in the equation the heating degree days in Chicago. The cost conditions of the foreign producers is accounted for by including the price of crude oil in the North Sea.

<sup>62</sup> Again, cost shifters for outside producers are accounted for by North Sea crude oil prices and heating degree days in the Midwest.

<sup>63</sup> For these estimates a trend was included with its estimated coefficient being negative and significant. Since imports became an increasingly important factor in the Northeast during the period of our estimation, this trend may reflect this development. While not statistically significant, the estimated coefficient of industrial production is negative. The remaining coefficients have their predicted signs.

specification, with the estimated elasticities for PAD I slightly below those for the Northeast. The estimated residual demand elasticity for Northeast refiners is between 2.0 and 2.2.

### 3. Price Elasticity Tests

As discussed in previous sections, if the firms in the conjectured antitrust market have a perfectly elastic supply curve and the residual demand is locally log-linear as in our empirical specifications, a price elasticity less than 20 (10) would provide the firms with the ability to profitably raise price by 5% (10%). We have chosen to use two benchmarks for critical elasticity values, 20 and 10.

As mentioned above, the estimated residual demand own-price elasticities are obtained by dividing two point estimates. This results in a nonlinearity that makes the derivation of simple confidence intervals impossible. One-sided tests, however, can be easily performed. Moreover, for the purposes of testing for the existence of potential market power, one-sided tests provide the relevant information. We are interested in knowing whether the estimated elasticities are statistically significantly smaller (in absolute value) than some critical value. The test for whether a ratio of estimates is below a given number can be constructed as follows. Let  $a$  and  $b$  be two positive unknown parameters, and  $c$  a positive constant. Let the null hypothesis be  $a/b=c$ , and the alternative hypothesis  $a/b<c$ . Then the null hypothesis can be rewritten as  $a-bc=0$  and the alternative hypothesis as  $a-bc<0$ . The test for the reformulated null hypothesis is  $(A-Bc)/(\text{Var}(A)+c^2\text{Var}(B)-2c\text{Cov}(A,B))$  (where  $A$  and  $B$  are the estimated values of  $a$  and  $b$ , respectively), which is distributed  $t$  with degrees of freedom equal to the number of observations (47) minus the estimated coefficients (8 or 10).<sup>64</sup> The critical  $t$  value is 1.69 for a 5% confidence level. Thus, if the absolute value of the  $t$ -statistic exceeds 1.69 we can reject the null hypothesis ( $a/b=c$ ) in favor of the alternative hypothesis

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<sup>64</sup> See Kendall and Stuart (1973) for a discussion of this test.

(a/b < c).

For each region this test is presented in Table 5. We test whether the estimated elasticities are statistically different from 1, 5, 10 and 20. The alternative hypotheses are that the elasticities are less than 5, 10, or 20. For the critical value of 1 the alternative hypothesis is that the elasticity exceeds or falls short of 1 depending on whether the estimated value is above or below 1.

Table 5 presents evidence suggesting that the Northeast, alone, may be a relevant antitrust market. Although under both specifications the elasticity is significantly larger than 1, and we cannot reject the hypothesis that the elasticity for the Northeast is 5. Moreover, the significance level at which the hypothesis that the elasticity is equal to 10 is rejected is relatively low (for one specification the t-statistic is only 1.39). However, the test clearly indicates that the elasticity is less than 20. Similar results are obtained for PAD I, which adds to the Northeastern producers those in Appalachia. Thus, the Northeast and PAD I could constitute relevant antitrust markets for the 5% benchmark, but not for the 10% benchmark.

The elasticity for PAD I and Gulf Coast producers together is significantly less than 5, and is not significantly different from 1. Clearly, the elasticity for PAD I, II and III together is also less than 5 and not significantly different from 1. Thus, under the 10% benchmark, the antitrust market centered around the Northeast will include only producers in PADs I and III, and exclude those in the Midwest. However, if 20 is the crucial elasticity (i.e. under the 5% benchmark), the Northeast, alone, will constitute an antitrust market.<sup>65</sup> Table 5 also shows that the Gulf Coast, alone, constitutes a relevant antitrust market. The point estimate of the elasticity for their residual demand is

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<sup>65</sup> Because our specification assumes constant elasticity, any elasticity smaller than twenty makes a price increase of at least 5% profitable in a simple model with constant costs.



between 1.3 and 1.6, and that point estimate is significantly smaller than 10.<sup>65</sup>

Using Table 5, some consistency tests can be performed. First, as more producers are added to the relevant set, the residual demand elasticity faced by the group should fall.<sup>66</sup>

We observe that pattern in our results. First, adding to the Northeast refiners those in the Appalachian region slightly reduces the estimated elasticity of the residual demand. Moreover, adding producers in PAD III reduces the elasticity as does adding producers in PAD II. A less rigorous test consists in relating elasticities to market shares in the economic market. If all suppliers share the same supply elasticity, then those with larger market shares should face smaller residual demand elasticities. This result is confirmed by comparing PADs I and III. Finally, a third consistency test results from comparing the estimated elasticities using Ordinary and Two Stage Least Squares. Table 6 presents these results. We observe that the OLS elasticities are slightly above the 2SLS ones, as would be expected if the correlation between residual demand and cost shocks are zero.

#### 1. Comparison With Price and Shipments Tests

Price correlation tests would suggest that the whole area east-of-the-Rockies should be an antitrust market since prices are very highly correlated. As expected, this method is not able to discriminate the areas with potential market power from those without it. The Elzinga-Hogarty test would suggest that the Northeast is not a relevant antitrust market, since it imports a substantial share of its consumption. However, the Elzinga-Hogarty test would suggest that the Gulf and the Northeast combined form a relevant antitrust market, since the amount this region imports from the outside is relatively small and the amount it ships to PAD II is also not large. Finally, the Elzinga-Hogarty

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<sup>65</sup> The estimated elasticity, moreover, is significantly smaller than 7, with the t statistic for that test being, for the two specifications reported in Table 5, equal to -1.776 and -2.227.

<sup>66</sup> This result is independent of the cost structure of the different firms.

test would not identify the Gulf Coast as a potential antitrust market by itself, since it exports more than half its production.

#### 4. Summary

To summarize, we have presented estimates of residual demand elasticities which, under the 5% benchmark, provide support for defining the Northeast as a relevant antitrust market. In addition, the estimated elasticities suggest that the whole area east of the Rocky mountains is too large to constitute the smallest relevant antitrust market.<sup>67</sup> We identified two clear candidates for antitrust markets, one the Gulf Coast by itself and another the Gulf Coast and the Northeast combined, depending on where the overlaps arise in a particular merger.

#### VIII. Final Comments

In this paper we have developed the underlying economic model of the DOJ Merger Guidelines' treatment of geographic markets. This model is based on the residual demand facing a given group of producers. The Guidelines' criterion for specifying a geographic market can be expressed as an area for which the own-price elasticity of the residual demand facing producers in that area is sufficiently small. We discussed, in the context of three alternative spatial models, the differences between the approach presented here and the criteria most commonly used to define relevant antitrust geographic markets. We also discussed the problems involved in the estimation of the price elasticity of the residual demand faced by a set of producers. Finally, we presented estimates of the residual demand for unleaded gasoline for the eastern U.S. and subregions. These estimates provide new evidence relevant to the delineation of antitrust markets for some of the recent mergers of major oil companies.

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<sup>67</sup> A smaller relevant market may be appropriate, for example, in considering a merger in which the refining overlaps are only in the Northeast and the Gulf Coast.

TABLE 1

<u>Variable Type</u>	<u>Geographic Area</u>
<u>Cost Shifters</u>	
Crude Oil Price	Gulf Coast: Louisiana Crude Oil Price Northeast and Overseas: North Sea and North Africa Crude Oil Spot Prices
Energy Use	Gulf Coast: Wellhead Natural Gas Average Price  Northeast: Residual Fuel Oil Price
Transportation Cost (crude oil)	Northeast: Spot Tanker Rate, large
Capacity	Northeast, Gulf and Midwest: Total Refining Capacity
By-Product Production	Northeast: New York Heating Degree Days  Gulf Coast: Atlanta Heating Degree Days  Midwest: Chicago Heating Degree Days
<u>Demand Shifters</u>	
Industrial Production	Northeast, Gulf Coast and Midwest
Personal Income	Northeast, Gulf Coast and Midwest
Quarterly Seasonal Dummies	Northeast, Gulf Coast and Midwest
Travel-related gas use	Northeast: New York Average Temperature  Gulf Coast: Atlanta Average Temperature  Midwest: Chicago Average Temperature
<u>Gasoline Price</u>	Northeast: New York Spot Price Average  Gulf Coast: Houston Spot Price Average
<u>Sales</u>	Northeast, Midwest and Gulf Coast: Total gasoline production in the area minus inventory changes held by refiners

TABLE 2<sup>68</sup>

	Residual Demand, PADs I, II & III		Demand PADs I, II & III	
	N.Y. Price	Gulf Price	N.Y. Price	Gulf Price
Constant	5.129 (4.111)	4.362 (3.707)	5.038 (3.351)	4.397 (3.014)
Quantity	-.483 (-3.901)	-.417 (-3.493)	-.521 (-3.518)	-.467 (-3.167)
Price (-1)	.569 (4.806)	.567 (5.011)	.709 (8.652)	.727 (9.065)
Ind. Prod. (-2)	.047 (.565)	.065 (.851)	.097 (1.014)	.106 (1.131)
Crude Oil Price (North Sea)	.425 (4.244)	.442 (4.694)	--	--
Crude Oil Price (-1) (North Sea)	-.218 (-2.079)	-.200 (-1.947)	--	--
Temperature (Chicago)	.0010 (2.226)	.0009 (1.977)	.00052 (.956)	.0003 (.553)
I (1st Quarter)	-.00041 (-.002)	.014 (.825)	-.010 (-.466)	-.00015 (-.007)
II (2nd Quarter)	.0173 (1.189)	.028 (2.089)	.0339 (1.990)	.0433 (2.627)
III (3rd Quarter)	.0204 (1.172)	.0283 (1.711)	.0387 (1.894)	.0467 (2.326)
SSE	.0262	.0237	.0408	.0394
DW	2.07	1.76	1.80	1.51
Durbin h	-.511	1.43	--	--
t-statistic (Ser. Corr.)	-.934	.349	.183	1.28
Price Elasticity	.893	1.038	.559	.584

<sup>68</sup> In Tables 2-4: non-weather variables are in natural logs, two stage least squares estimation, t-statistics in parentheses, and 47 observations.

TABLE 3

	Residual Demand PAD III		Residual Demand PADs I and III	
	Gulf Price	N.Y. Price	Gulf Price	N.Y. Price
Constant	2.834 (2.339)	3.713 (2.692)	4.049 (2.937)	5.143 (3.359)
Quantity	-.262 (-2.116)	-.338 (-2.509)	-.382 (-2.762)	-.447 (-3.22)
Price (-1)	.577 (4.11)	.552 (3.567)	.516 (3.589)	.488 (3.119)
Ind. Prod. (-2)	.0174 (.217)	-.0063 (-.0682)	.0368 (.447)	.0106 (.112)
Crude Oil Price (North Sea)	.455 (4.417)	.431 (3.845)	.452 (4.376)	.430 (3.806)
Crude Oil Price (-1) (North Sea)	-.190 (-1.682)	-.193 (-1.637)	-.174 (-1.510)	-.181 (-1.512)
Temperature (Atlanta)	-.00034 (-.242)	-.00048 (-.3088)	.000057 (.04)	.000033 (.025)
Heating Degree Days (Chicago)	-.000044 (-1.130)	-.000045 (-1.035)	-.000036 (-.924)	-.000034 (-.783)
Heating Deg. Days (-1) (Chicago)	-.0000054 (-.212)	-.000022 (-.799)	-.000011 (-.416)	-.000027 (-.984)
I	.0192 (.875)	.0098 (.393)	.0130 (.585)	.0014 (.540)
II	.0264 (1.489)	.0208 (1.062)	.0231 (1.271)	.0157 (.779)
III	.0208 (1.151)	.0124 (.626)	.0205 (1.119)	.0117 (.584)
SSE	.0254	.0290	.0262	.0305
DW	1.72	1.96	1.75	1.95
Durbin h	1.8	.352	2.733	--
t-statistic (Ser. Corr)	.458	-.883	-.107	-1.127
Price Elasticity	1.618	1.327	1.267	1.073

TABLE 4

	Residual Demand PAD I		Residual Demand Northeast	
	N.Y. Price	N.Y. Price	N.Y. Price	N.Y. Price
Constant	3.147 (2.309)	3.141 (1.943)	3.152 (2.294)	2.957 (1.884)
Quantity	-.343 (-2.222)	-.352 (-1.841)	-.314 (-2.190)	-.300 (-1.765)
Price (-1)	.268 (1.390)	.274 (1.421)	.315 (1.744)	.327 (1.878)
Ind. Prod. (-2)	-.0366 (-.376)	-.0279 (-.280)	-.0900 (-1.010)	-.0841 (-.984)
Time	-.0036 (-2.670)	-.0036 (-2.709)	-.0031 (-2.322)	-.0031 (-2.413)
Crude Oil Price (North Sea)	.502 (4.308)	.501 (4.359)	.505 (4.504)	.504 (4.667)
Crude Oil Price (-1) (North Sea)	-.220 (-1.805)	-.227 (-1.891)	-.229 (-1.953)	-.236 (-2.104)
I	.0183 (.786)	.0106 (.525)	.0188 (.842)	.0120 (.637)
II	.0038 (.183)	-.00095 (-.049)	.02121 (.106)	-.0019 (-.101)
III	.0121 (.498)	.0073 (.323)	.0135 (.570)	.0085 (.398)
Heating Deg. Days (Atlanta)	-.000013 (-.259)	--	-.0000086 (-.178)	--
Heating Deg. Days (-1) (Atlanta)	-.000026 (-.604)	--	-.000025 (-.611)	--
SSE	.03223	.03333	.02978	.02919
DW	1.94	1.94	2.03	2.07
Durbin h	--	--	-.631	--
t-statistic (Ser. Corr.)	-.618	-.586	-.804	-.803
Price Elasticity	2.131	2.060	2.178	2.243

**Table 5**  
t-Statistics for One Sided Tests<sup>70</sup>

		Elasticity	Critical Values			
			1	5	10	20
PAD's I, II & III						
	Gulf Price	1.038	.124	-2.939	-3.239	-3.372
	N.Y. Price	.893	-.379	-3.383	-3.667	-3.789
PAD's I & III						
	Gulf Price	1.267	.794	-2.305	-2.557	-2.665
	N.Y. Price	1.073	.250	-2.830	-3.052	-3.143
PAD III						
	Gulf Price	1.618	1.323	-1.611	-1.889	-2.008
	N.Y. Price	1.327	.831	-2.084	-2.323	-2.422
PAD I						
	1	2.131	1.840	-1.326	-1.797	-2.016
	2	2.060	1.673	-1.134	-1.503	-1.676
Northeast						
	1	2.178	1.764	-1.252	-1.738	-1.970
	2	2.243	1.713	-.993	-1.390	-1.581

<sup>70</sup> See text for description of test.

TABLE 6

OLS vs. Two Stage Least Squares Residual Demand Elasticities

	OLS	2SLS
PAD <sup>a</sup> I, II & III		
Gulf Price	1.282	1.038
N.Y. Price	1.129	.893
PAD <sup>a</sup> I & III		
Gulf Price	1.797	1.267
N.Y. Price	1.532	1.073
PAD III		
Gulf Price	2.188	1.618
N.Y. Price	1.881	1.327
PAD I		
Equation 1	4.536	2.131
Equation 2	4.549	2.060
Northeast		
Equation 1	4.280	2.178
Equation 2	4.271	2.243



<u>Variable</u>	<u>Data Appendix Source</u>
Gulf Coast Spot Unleaded Gas Price	<u>Platt's Oil Price Handbook and Oilmanac, 1981-1984, and Platt's Oilgram Price Report, monthly, 1984-1985.</u>
N.Y. Spot Unleaded Gas Price	Same
Spot Tanker Rates	Same
Louisiana South Posted Crude Oil Price (Amoco Light)	Same
Northsea Spot Crude Oil Price	Same
African Bonny Spot Crude Oil Price	Same
N.Y. Spot Residual Fuel Oil Price	Same
Industrial Production	<u>Survey of Current Business, monthly, 1981-1985</u>
Refinery Production, Unleaded Gasoline	<u>Petroleum Supply Monthly, Energy Information Administration, 1981-1985</u>
Refinery Inventories, Finished Unleaded Gasoline	Same
Refinery Operable Capacity	Same
Wellhead Natural Gas Price	<u>Natural Gas Monthly, Energy Information Administration, 1981-1985</u>
Temperature (Atlanta, N.Y., Chicago)	<u>Local Climatological Data: Monthly Summary, National Oceanic and Atmospheric Administration, 1981-1985</u>
Heating Degree Days (Atlanta, N.Y., Chicago)	Same

### Theoretical Appendix

In this Appendix we provide the mathematical details for Models II and III of Section V.

#### AI. Model II

Let us first assume that all producers, denoted  $x$ , are located at 0, and show how the demand facing them is derived in this simple case. Because by assumption widgets are homogeneous, all producers' FOB prices will be identical. If all producers face the same transportation costs, all producers' delivered prices will be identical at each location also. Therefore,

$$(A.1) \quad P(w) = p+r(w)$$

The purchasers located at any given distance  $w$  have demands that depend on the delivered price they pay. Using (A.1) we can express those demands in terms of the FOB price. The total demand facing  $x$  is the sum of the demands of purchasers at all locations. We can write this total demand, denoted  $Q_x^D(p_x)$  as

$$(A.2) \quad Q_x^D(p_x) = \int_0^{\hat{w}(p)} [-Jk+k(p_x+r(w))]dw.$$

In expression (A.2) location  $\hat{w}(p_x)$  is the furthest location at which a purchaser will buy widgets if the FOB price is  $p_x$ . This point  $\hat{w}(p_x)$  is given as the solution to the equation

$$(A.3) \quad -Jk+k(p_x+r(\hat{w})) = 0,$$

i.e.,  $\hat{w}$  is determined as the distance where the delivered price of widgets is so high that the demand for widgets at that distance is zero. Using (A.3) in (A.2),

$$(A.4) \quad Q_x^D(p_x) = -2k((J-p_x)^2/2r).$$

Now, let us return to the original model in which producers are located at both (0) and  $Y$ . We will derive the residual demand facing  $x$ . Notice first that if  $y$  acts competitively and  $x$  cannot price-discriminate, then the purchasers will all be divided (by their location) into two groups according to some location  $w^*$ . Purchasers to the left of

$w^*$  purchase from  $x$  and purchasers to the right of  $w^*$  purchase from  $y$ . Let  $w^*$  be some arbitrary market division point. Then the demand facing  $x$  can be expressed as a function of  $w^*$ :

$$(A.5) \quad Q_x^D(p_x; w^*) = -kw^*[J - p_x - rw^*/2]$$

The market division point  $w^*$  is going to depend on the FOB price charged by  $Y$ ,  $p_y$ , and on the FOB price charged by  $x$ , which we denote by  $p_x$ . The market division point will occur at the location where the delivered price of the two producers is the same. Therefore, the market division point can be expressed as a function of  $p_x$  and  $p_y$ :

$$(A.6) \quad w^* = (p_y + rY - p_x) / 2r.$$

Now, substituting (A.6) into (A.5), we can write the total demand facing  $x$  as a function of the price they charge,  $p_x$ , and the price charged by  $Y$ ,  $p_y$ , as

$$(A.7) \quad Q_x^D(p_x, p_y) = -k[(p_y + rY - p_x) / 2r][(J - p_x) - [(p_y + rY - p_x) / 4]]$$

Notice that this demand function is not the residual demand function facing  $x$ , because it is written as function of  $p_y$ , the price charged by  $Y$ . To derive the residual demand facing  $x$  we must solve for the relationship between  $p_y$  and  $p_x$ --i.e., we must make use of the fact that  $x$  and  $y$  are in the same economic market, and that an increase in the price charged by  $x$  will cause a change in the price charged by  $y$ .

For any price  $p_x$ , a delivered price schedule for those producers is determined giving the delivered prices charged at each distance. The extent of those producers' market area is determined by the division point between the two groups of producers, i.e., the point where the delivered price charged by each group is the same. The higher is the (FOB) price charged by  $x$ , the higher is their delivered price schedule and the smaller is the extent of their market area. If  $y$  has increasing marginal production costs ( $B_y > 0$ ), the extent of the market served by  $y$  will cause an increase in their FOB price,  $p_y$ . From the supply curve for  $y$  and the fact that in equilibrium  $y$  must supply all customers between  $Y$  and  $w^*$ , the relationship between  $p_y$  and  $p_x$  is determined and is defined implicitly by the equation:

$$(A.8) \quad 0 = p_y - A_y + B_y k[(p_y + rY - p_x)/2r][J - p_x - (p_y + rY - p_x)/4].$$

Equations (A.7) and (A.8) jointly determine the residual demand facing  $x$ ,  $Q_x^R(p)$ , i.e., for each price charged by  $x$ , the equilibrium price charged by  $y$  is determined by (A.8) and can be substituted into (A.7). Since expressing the residual demand curve analytically is complex in this case, it is useful to discuss the case where marginal costs are constant at each location. That is, let

$$MC_i(Q_i) = A_i, \quad i = x, y.$$

Then, the competitive price at  $Y$  equals  $A_y$  and (A.6) becomes:

$$(A.6a) \quad w^* = (A_y + rY - p)/2r.$$

Substitute (A.6a) into (A.5) and the residual demand function (11) is derived.

## II. Model III

The total number of customers at  $w'$  served by  $y$  will be denoted  $z_y (0 < z_y < Z)$ . In this situation the demand facing  $y$  when  $w'$  is the market division point is:

$$(A.9) \quad Q_y^D(p_y, w') = \int_0^{Y-w'} [-Jk + k(p_y + rw)] dw + z_y [-Jk + k(p_y + r(Y-w'))] \\ = -k[(X_y - w^*)[(J - p_y) - (r/2)(Y - w')]] \\ + z_y [J - (p_y + r(Y - w'))], \quad \text{where } 0 < z_y < Z.$$

Using the supply function for  $y$  and setting supply from those producers equal to demand for their production, we have the following equation for  $z_y$ :

$$(A.10) \quad z_y = -K \{ [b_y - k(Y - w')] p_y - [A_y b_y - k(Y - w') J + k(r/2)(Y - w')^2] \} / \\ [J - (p_y + r(Y - w'))].$$

The condition  $z_y > 0$  requires

$$(A.11) \quad p_y > \{ A_y b_y - k(Y - w') J + k(r/2)(Y - w')^2 \} / [b_y - k(Y - w')],$$

while  $z_y < Z$  requires

$$(A.12) \quad p_y < \{ [A_y b_y - k(Y - w') J + k(r/2)(Y - w')^2] \\ - k[J - r(Y - w') Z] \} / [b_y - k(Y + Z - w')].$$

The condition  $w'=w^*$  requires

$$(A.13) \quad p_y + r(Y - w') = p + rw'$$

Now consider the demand for the sales of  $x$ :

$$(A.14) \quad Q_x^D(p_x, z_y) = \int_0^{w'} [-Jk + k(p_x + rw)] dw + (Z - z_y) [-Jk + k(p_x + rw')] \\ = -kw' [J - p_x - (r/2)w'] - k(Z - z_y)[J - (p_x + rw')], \text{ where } z_y < Z.$$

The first term in (A.14) is total demand to the left of  $w'$  and the second term in (A.14) is total demand (facing  $x$ ) at  $w'$ . If  $(Z - z_y) > 0$ , (i.e.,  $x$  also sell in  $w'$ ) then the elasticity of total demand facing  $x$  is the share weighted average of the elasticity of total demand to the left of  $w'$  and the elasticity of residual demand at  $w'$ . Thus, for example, if the residual demand at  $w'$  is elastic and sales at  $w'$  are a significant portion of the sales of  $x$ , then the total demand facing  $x$  will be elastic.

The endogenous variable in (A.14),  $(z_y)$ , can be substituted out from (A.10) and  $p_y$  can be substituted out from (A.13), giving us the residual demand facing  $X$ :

$$(A.15) \quad Q_x^R(p_x) = [k(Z + Y) - b_y]p_x - kw'[J - (r/2)w'] \\ - [b_y - k(Y - w')][rw' - r(Y - w')] \\ + [A_y b_y + k(r/2)(Y - w')^2] - kZ(J - rw').$$

Define:

$$(A.16) \quad R = -kw'[J - (r/2)w'] - [b_y - k(Y - w')][rw' - r(Y - w')] \\ + [A_y b_y + k(r/2)(Y - w')^2] - kZ(J - w')$$

$$(A.17) \quad S = [k(Z + Y) - b_y].$$

Then, rewrite (A.15) to obtain:

$$(A.18) \quad Q_x^R(p) = Sp_x + R.$$

That is, the residual demand facing  $x$  is linear, with its slope and location depending on cost conditions at  $Y$  ( $b_y$  and  $A_y$ ). In general form this residual demand is analogous to (4).

Observe that for  $w^*=w'$ ,  $p_x$  in (A.15) must satisfy (A.11)-(A.13), i.e.,

$$(A.19) \quad (a) \quad p_x > r(Y-w') - rw' \\ + \{A_y b_y - k(Y-w')J + k(r/2)(Y-w')^2\} / [b_y - k(Y-w')] \\ (b) \quad p_x < r(Y-w') - rw' \\ + \{[A_y b_y - k(Y-w')J + k(r/2)(Y-w')^2] \\ - k[J - r(Y-w')Z] / [b_y - k(Y+Z-w')]\}.$$

From (A.15) and the supply curve of X the competitive (FOB) price of x can easily be derived:<sup>70</sup>

$$(A.20) \quad p_x^c = [A_x + B_x R] / [1 - B_x S]$$

From (A.18) the price elasticity of the residual demand facing x can be written

$$(A.21) \quad E_x^R = -p_x / (p_x + R/S).$$

Thus, the elasticity of the residual demand at the competitive price is:

$$(A.22) \quad (E_x^R)^c = -(A_x + B_x R)S / (R + A_x S).$$

Using the marginal cost curve for x, the optimal cartel price for those producers,  $p_x^m$ , can also easily be derived:

$$(A.23) \quad p_x^m = [A_x + B_x R] / [2 - B_x S] - R / [2 - B_x S]S$$

Then, combining (A.20) and (A.23),

$$(A.24) \quad [p_x^m - p_x^c] / p_x^c = -(R + A_x S) / [S(A_x + B_x R)(2 - B_x S)] \\ = 1 / ((2 - B_x S)E_x^R)^c. \quad ^{71}$$

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<sup>70</sup> Notice that  $p_x^c$  must satisfy (A.20) in order for the condition  $w^*=w'$  to hold. For example, if  $A_x$  is too large (A.20) will not hold because for large  $A_x$  equilibrium will require  $w^* < w'$ . Similarly, if  $A_x$ ,  $B_x$  and  $r$  are sufficiently small relative to  $A_y$ ,  $B_y$  and  $r$ , and  $Z$  is sufficiently small, equilibrium will require  $w^* > w'$ . Notice that as  $Z \rightarrow \infty$ ,  $p_x^c \rightarrow (J - rw')$ , and from (A.13)  $p_y^c \rightarrow [J - r(Y - w')]$  (assuming that  $0 < z_y < Z$ ).

<sup>71</sup> Thus, if  $B_x = 0$  (i.e., constant marginal costs for the X-producers), the knowledge of the elasticity of demand at the competitive price provides an exact statistic for measuring the potential price increase if the X-producers succeed in forming a perfect cartel.

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