# Galactic Scale Gas Flows in Colliding Galaxies: 3-Dimensional, N-body/Hydrodynamics Experiments

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**Abstract.**
We present some results from three dimensional computer simulations of collisions between models of equal mass galaxies, one of which is a rotating, disk galaxy containing both gas and stars and the other is an elliptical containing stars only. We use fully self consistent models in which the halo mass is 2.5 times that of the disk. In the experiments we have varied the impact parameter between zero (head on) and $0.9R$ (where $R$ is the radius of the disk), for impacts perpendicular to the disk plane. The calculations were performed on a Cray 2 computer using a combined N-body/SPH program. The results show the development of complicated flows and shock structures in the direction perpendicular to the plane of the disk and the propagation outwards of a density wave in both the stars and the gas. The collisional nature of the gas results in a sharper ring than obtained for the star particles, and the development of high volume densities and shocks.

## 1 Introduction

Collisions between galaxies can produce large scale flows of both the stellar components and any gas present in the system. All close encounters and interpenetrating collisions result in the overall contraction of each galaxy as it passes close to, or through the other, and a subsequent expansion as the two move apart. At early times in the collision the gas and stars react in similar fashions to the changing gravitational potential, but the collisional nature of the gas soon becomes important (see Gerber and Lamb, 1993), leading to the development of shocks and regions of very high gas density, even if only one of the two galaxies contains any appreciable quantity of gas, and even if the two galaxies do not actually overlap at any time. The resulting distribution of the high density, shocked gas regions, as distinct from the underlying, somewhat different distribution of stars, is more likely to predict the actual optical morphology of interacting disk galaxies, because

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it is in the regions of high gas density and shocks that we expect large amounts of star formation to take place. In contrast, near-infrared observations are more likely to display the underlying old stellar populations in the galaxies (see Bushouse and Stanford, 1992).

The relationship between the distribution of the star forming regions in these galaxies and the physical properties of the natal gas is of particular importance to an understanding of global star formation in disturbed systems. At present, there is no comprehensive, physical theory of star formation which can be applied to predict what mass range of stars will be produced in different physical circumstances of the interstellar gas, or, for example, what morphological distribution of star forming regions might be expected, given the flow patterns and clumping in the gas. We are attempting to get some information relevant to this overall situation by using an empirical approach in which detailed numerical models are constructed and then compared to real interacting galaxies. The more realistic our models and the more detailed the observations, the more likely we will be able to learn something of true significance to this area of study. As a first step in this endeavor, we have constructed 3-D models of a gas-free elliptical and a gas-rich spiral using a combined N- body/hydrodynamics code, and performed experiments on a Cray 2 computer in which they collide at a variety of angles and impact parameters (see Gerber, Lamb and Balsara, 1991 for a preliminary discussion of these experiments and Gerber, 1993; Gerber, Lamb and Balsara, 1993, 1994, for a current discussion). These experimental collisions can then be compared to observations of real interacting systems, such as the Arp 147 system. In this particular case, a detailed comparison of one of our evolved models and the real system, which contains a galaxy with an incomplete ring in its disk formed by the collision, (Gerber, Lamb and Balsara, 1992) indicates good agreement between model and observed properties, and allows an examination of some of the generic features of this type of collision.

It has been suggested that there is an evolutionary connection between the AGN phenomenon and collisions and mergers of galaxies (see Norman and Scoville, 1988), and between starbursts and AGN (see Perry and Dyson, 1985). Some observational evidence for this has been provided by Simkin, Su and Schwartz (1980). A simplified summary of the suggested scheme follows: the interaction causes gas in the disk of a galaxy to flow towards the central regions. There, a change in physical conditions could lead to star formation, which might then feed an already existing black hole or cause the formation of one. Or, alternatively, the gas flow might fuel an existing black hole directly in some way. The overriding problem to be understood in these scenarios is how angular momentum and energy can be shed by the infalling gas so that a central reservoir of gas or stars can be formed around the black hole and be available to feed it at an appropriate rate. In this connection, a detailed study of the effects of radiating shocks in the gas will be very important. Another part of the puzzle is the possible star formation episode. Under what circumstances would the inflowing gas form stars, where in the flow would they form, and what might the properties of those stars be? For example, would they be supermassive, all very low mass, or follow a Salpeter mass function? A study of the energy budget in starburst galaxies suggests that the IMF is tipped towards the high mass end. At what radius might they form? Observations of nearby Seyferts suggests that in these objects star formation is concentrated in a ring of radius 1 Kpc centered on the nucleus. N- body calculations that merely assume a star formation rate as a function of collision rate between calculational 'gas' particles merely beg the question of star formation in these systems The central region of interest here in these active galaxies is only of the order of a few parsecs in dimension and thus beyond the limits of resolution by current instruments, so any information that we can obtain about the types and clustering of stars formed in other strongly disturbed environments is of possible interest. (We note here that galactic bars can also channel gas towards the central regions and that the same considerations concerning star formation in dense, shocked gas would also apply (see Shloshman, Begelman and Frank, 1989).

The overall rate of star formation in disk galaxies experiencing moderately strong interactions is enhanced by only a factor of approximately two, as observed optically by Bushouse (1986, 1987) and Kennicutt *et al* (1987), and as observed in the far- infrared using IRAS data by Bushouse, Lamb and Werner, 1988. However, the observed new star formation takes place preferentially in the nuclear regions (here of dimension one kpc), rather than in the spiral arms of the disk as in isolated disk galaxies. A moderately intense burst of star formation in the very central regions might go undetected by such relatively low resolution observations. Some galaxies that are classified as merger remnants do appear to have large amounts of star formation in their central regions. These are the extreme IRAS objects which have far-infrared fluxes exceeding $10^{12}$ $L_{\odot}$ (see Sanders *et al*, 1988, and references therein). Any star formation currently taking place could have been triggered by the first crossing of the two parent galaxies, when a large impulse inward would have been given to the gas and stars, approximately $10^8$ or more years ago. Alternately, the final merger of the two galactic nuclei may be implicated in the triggering of a large central star burst. (See Majewski *et al*, 1993, for some observational constraints on high far-IR systems). Much remains to be discovered about possible star formation in active galaxies and its relationship to the AGN phenomenon, however one of several good starting places is an investigation of the gas flows in colliding and merging systems and the changes in physical parameters that result.

The remainder of this paper is divided into three sections. In Section 2 we describe the model galaxies and the numerical methods used and in Section 3 we describe the particular subset of our experiments which we have chosen to discuss here. (These experiments appear to emulate the formation of a class of ring galaxies which have a relatively simple geometry and are thus easier to investigate for our present purposes). Lastly, in Section 4 we present those results of these calculations which are relevant to large scale gas flows and possibly to star formation.

## 2 Model Galaxies and Numerical Methods

We have chosen to start our investigation of galaxy collisions by modelling systems in which only one of the galaxies contains gas initially. Consequently, we built 3-D models of equal mass galaxies, one of which is a rotating, disk galaxy containing both gas and stars and the other is an elliptical containing stars only. We produced fully self consistent models in which the halo mass is 2.5 times that of the disk. The calculations were performed using a combined N-body/SPH program in which 50,000 N-body (star) particles and 22,000 SPH particles (representing the gas) were utilized. All computations were conducted on the Cray-2 supercomputer at the National Center for Supercomputing Applications, which is situated at the University of Illinois at Urbana-Champaign.

### 2.1 Numerical Methods

The computer code used to perform the experiments described here is a combined N-body/smooth particle hydrodynamics (SPH) code. That is, it represents both stars and gas using particles, but the evolution of the stars (and any collisionless dark matter) is followed by using N-body techniques, whereas the gas is represented by 'particles' that act as moving interpolation points (see Monaghan, 1985). The gas is considered to form a continuum and the density at any point is obtained by smoothing out the mass of nearby particles, which is done by using an analytic smoothing function. Forces are calculated by taking gradients of a smoothed estimate of the pressure. Shocks can be modelled in this scheme and this code utilizes an artificial viscosity formulation due to Balsara (1990) to accomplish this.

For the experiments described here, we have chosen to use an ideal gas equation of state and to assume that the gas cooling time is less than a typical time step in our calculation. Under these assumptions, each gas particle retains its initial temperature throughout the experimental run.

In the code discussed above the gravitational force is calculated by standard particle-mesh (PM) techniques (see Hockney and Eastwood, 1988). In this method, the gravitational potential is calculated at a restricted number of points on a three-dimensional grid and the force on an individual particle is determined by interpolation between values for surrounding grid points. In determining the gravitational potential on the grid both the stellar and SPH particles contribute. We chose to use a cubic grid with 64 points along each side.

The particular combination of SPH and PM techniques that we use is very suitable for modelling collisions between galaxies in 3- D. The grid method in PM is much faster than the alternative 'tree methods' and can be combined with the SPH, as demonstrated by Balsara (1990). The particle nature of both methods lends itself to the straightforward determination of the gravitational potential. However, one drawback of having a grid tied to the computational space is that the volume of interaction in the experiment must be limited. Thus, the method we have used here is ideally suited to the calculation of the first passage of one galaxy through another but is not suited, in its present formulation, to the calculation of distant encounters and to following energetic collisions to merger. More details of the combined code and its uses can be found in Balsara (1990, 1993), where tests of it are presented. Further discussion of the methods of SPH can be found in, among other places, Lucy (1977), Gingold and Monaghan (1982), Hernquist and Katz (1989), and Balsara (1990, 1993).

## 2.2 MODELS

The disk galaxy has a radially exponential disk of gas and stars surrounded by a massive, almost spherical halo of gravitating (star) particles, which can be considered to be either stellar or 'dark matter'. The elliptical galaxy consists of a spherical distribution of star particles only. The halo and disk were given 25,000 particles each and the latter were placed in rotation around the center such that Toomre's stability parameter, $Q$, (Toomre 1964) was 1.5 everywhere in the disk. The three dimensional density distribution, $\rho$, of the disk is

$$\rho(r,z) = \frac{M_{\text{exp}}}{4\pi H R_d^2} \exp\left(-\frac{r}{R_d}\right) \operatorname{sech}^2\left(\frac{z}{H}\right), \tag{1}$$

where $(r, z)$ are cylindrical coordinates, and $M_{\text{exp}}$ is the total mass of a radially infinite exponential disk with disk radial scale length, $R_d$. Here, the scale height, $H$, is set constant with radius, and the disk density distribution is cut off at 4.4 $R_d$; interior to which radius the integrated mass is $0.93 M_{\text{exp}}$. In the $z$ direction the density distribution of the disk is cut off after two scale heights, where the density is 0.07 of its value at $z = 0$.

The gaseous disk was modelled using approximately 22,000 SPH particles. (This is a factor of seven larger than has been used previously to model such systems). We have set the gas density distribution such that it follows that of the disk stars but has a total mass equal to only one tenth that of the stellar disk. The SPH particles were placed in circular orbits around the center of the disk with no dispersion in their velocities.

We chose to represent the gas-free elliptical galaxy by a spherical King model with 10,000 N-body particles. Its total mass was set equal to that of the disk galaxy (including its halo), and its radius is approximately that of the disk galaxy halo.

No galaxy model is ever perfectly stable, but a model must be stable over a long enough time that changes resulting from the collision can be distinguished from those that would occur in any case. Consequently, we evolved the models in isolation for a period considerably longer than the duration of our collision experiments. We found that the disk exhibited a tendency to produce a low amplitude sheared spiral pattern and expanded slightly in the direction normal to the plane, but no large scale changes in the galaxies occurred on this time scale.

More details of both the numerical methods and the starting models for these numerical experiments can be found in Gerber (1993) and in Gerber, Lamb and

Balsara (1993).

## 3 The Numerical Experiments

Collisions between galaxies will always lead to large scale gas flows if there is any gas present in either galaxy. The simplest experimental geometry for studying these flows is the case of one galaxy colliding with another along the latter's spin axis and through its center. If two disk galaxies were involved in such a collision then the orientation of the two spin axes would also be a consideration, so it is simpler to start with an investigation of the collision of an elliptical with a disk galaxy. Below we will discuss the results of such an investigation (see Gerber, Lamb and Balsara, 1993) together with the results of an investigation of such collisions in which the impact parameter has been increased in increments up to a value of $0.9R$, where $R$ is the radius of the disk, (see Gerber, Lamb and Balsara, 1994).

The experiment is started with the two galaxies placed at opposite corners of the computational cube, and the two centers of mass are given an initial relative velocity so as to produce a mildly hyperbolic orbit. The initial center of mass separation of the two galaxies is about 7 radial disk scale lengths.

As mentioned above, the particular collision geometry that we have chosen to model leads to the formation of a ring structure in the disk galaxy. Such structures were first investigated computationally for the star particles by Lynds and Toomre (1976). However, this excess density region exists in both the stars and gas, although the detailed structure is somewhat different for the two. The collisional nature of the gas results in a sharper ring than obtained for the star particles, and the development of high volume densities and shocks. The central collision produces a true ring, but off-center collisions result in the formation of only partial rings which have a 'horse-shoe' shape. Real galaxies in interacting pairs are observed to show these types of structure, and it has been natural and informative to associate them with the formation mechanism discussed here (see Lynds and Toomre, 1976; Theys and Spiegel, 1976, 1977; Huang and Stewart, 1988).

As seen in previous studies, our computations show that each galaxy contracts and then expands due to the increase and then decrease in gravitational potential as the other passes through it. The disk participates in that dynamics, initially contracting radially and bowing toward the incoming intruder. Soon after close approach, the inflow has produced a density maximum in time in the central parts of the galaxy. This is the point in time during the first passage that the inward velocities and densities in the center are the highest. The inward motion is stopped when the particles reach their centrifugal barrier. They then move outwards as a result of the declining potential. (Consequently, if one wishes to harness this material as fuel for a hypothetical, central black hole, the problem becomes one of removing large amounts of angular momentum from the gas, or from the stars which may be forming in it, so that it does not re-expand with the overall flow).

The outwardly propagating density wave in the disk is formed because the particles in the inner part of the disk start to move outwards even as Bthe outer parts of the disk are still contracting, creating a density peak where orbits crowd. For a more detailed discussion of this see Gerber, Lamb and Balsara, 1993; Struck-Marcell and Lotan, 1990; and Lynds and Toomre, 1976. As the stellar ring moves outward it expands in the radial direction and a large fraction of the total disk mass is contained within it. The gaseous ring has much sharper, thinner features because the gas is collision-dominated and has no radial velocity dispersion in the initial galaxy model.

When these calculations are done in 3-D with all components contributing to the gravitational potential, as in these experiments, one finds the development of complicated flows and shock structures in the direction perpendicular to the plane of the disk. Thus the outwardly propagating density wave contains a lot of structure which might be expected to lead to star formation, and perhaps considerable radiation directly from heated gas. The dynamical time scale of the collision (typically a few hundred million years) is approximately equal to one revolution time in the outer disk, consequently, the coupling of the particle rotation and density wave disturbance is clearly shown in the numerical results. More discussion of the overall gas flows is contained in Section 4.

We follow the collision as one galaxy passes through the other, only terminating the calculation when particles begin to be lost from the computational box. This is equivalent to following the collision through one dynamical timescale. Both the intruder and the disk galaxy's halo are quite disrupted by the interaction. We do not know the ultimate fate of these components, but enough energy has been pumped into the internal dynamics of these components to make them very diffuse by the end of the experiment. We find that as the density wave reaches the outer disk it has diffused in radius and is becoming somewhat indistinguishable from the background. Thus the timescale for the dynamical disturbance is a few times $10^8$ years. However, we note that clumping on scales at or below our experimental resolution, which may occur in the gas in real galaxies, could persist for much longer times, and may lead to star formation over a considerable period.

## 4 Large Scale Material Flows

On the grossest scales, the material flows in these galaxies consists of a flow inwards towards the center followed by an outflow which takes individual particles to larger radii than they started at. Superimposed on this is the detailed dynamical behavior that we outline below. On even smaller scales (ones below our resolution limit) there is the possibility of flows that may be very important for the formation of stars etc. and other observationally relevant behavior. However, we do not anticipate that the existence of such flows or the formation of stars will have a first order effect upon the flows described here. Rather, they should be considered as possibly producing perturbations on the current results.

In our experiments the two galaxies are of comparable mass and, as a consequence, the disk is bowed considerably out the plane during the collision. Thus, when the material from the inner disk expands radially, it is displaced in the $z$-direction from the infalling outer material. This results in the stars and gas moving in something close to a toroidal flow in the vicinity of the ring (as Lynds and Toomre speculated would happen). Incoming and outgoing gas flows meet and form a shock on the side of the disk away from the direction of approach (underside). Shocked gas is thus swept up through the disk and carried out with the outflowing material. As a result of this three-dimensional structure the gas ring is broader and less dense than it would have been if the motion had been constrained to only two dimensions. The gas and stars generally flow together throughout the experiments with the gaseous ring lagging the stellar ring only slightly during the outward propagation.

### 4.1 The Central Collision

Because of the azimuthal symmetry of this collision geometry, it is useful to average the particle properties over their angular position in the disk for many purposes. For example, we note that a plot of radial position versus radial velocity is multivalued in the ring region because the inner parts of the disk expand 'above' the infalling outer regions. A 3-D plot including the above parameters and the $z$-direction (see Gerber, 1993) allows one to explore the structure in more detail and to determine the locations of shocks in the flow. For a disk galaxy with dimensions the same as those of the Milky Way we find that the peak inflow velocity (at 22 Myr after the collision) is about 160 km $s^{-1}$ and the velocity difference between the two streams at this time is about 320 km $s^{-1}$. This relative velocity is equivalent to Mach 10 or greater but, owing to the complex morphology in the region and the multidimensional nature of the shock, it is not really possible to assign a unique shock strength.

As noted above, the full particle motions (where this applies to both the gas and stars) are approximately toroidal in the ring region and this will have an important role when considering possible star formation in these regions.

An analysis of the motion in the azimuthal direction shows a maximum value in the ring region. The collision with the other galaxy gives the particles an inward pull and angular momentum conservation forces azimuthal velocities to increase. Soon after collision the peak in the velocity curve is approximately 250 km $s^{-1}$ but this has increased to about 450 km $s^{-1}$ by 22 Myr after the collision. The value then falls during the continued expansion and the velocity curve behind the ring is relatively flat. We note that rotation velocities in the ring are greater than velocities on either side of it.

Soon after passage of the intruder galaxy the central density peaks at about a factor of 5 above its initial value. After this a ring forms and moves outward. The relative density increases within it are more dramatic and peaks at about

42 Myr after the collision, with a space density of about 10–20 greater than the initial value for the particles involved. At this point the surface density in the ring is about 7–8 times that in the original disk. These high values are maintained beyond 62 Myr after collision, but have declined by 82 Myr.

Further details on this central collision model can be found in Gerber (1993) and in Gerber, Lamb and Balsara (1993).

### 4.2 The Off-Center Collisions

It is in the results of the off-center collision experiments that the coupling between the rotational motion in the disk and the motion due to the collision can be seen clearly. Features in the 'modified ring' or 'horse-shoe' structure can be seen to spiral outward during the evolution, making approximately one revolution as the ring expands to the edge of the disk.

This coupling of the motions contributes to the detailed morphology that develops in the outwardly propagating density structure in that the differential rotation spreads out the dense region in the disk that is formed when the other galaxy passes through, forming an arc of dense material. Soon after the collision the densest part of the arc is at its apex. However, as the structure evolves, the densest region bifurcates and proceeds to migrate down each arm of the arc. The flows in these arcs are relatively complicated and can only be appreciated by viewing a time sequence of the morphological development of the density enhancements with a depiction of the flow patterns superimposed. In real ring galaxies, such as Arp 147 (see Schultz *et al*, 1991), there is evidence that star formation proceeds down the arms of the arc. Thus one is drawn to the conclusion that the star formation is related to the increase in density, and to the strong shocks that develop in these regions as a result of the complicated flows. For more discussion of this see Gerber (1993) and Gerber, Lamb and Balsara (1992, 1994).

### 4.3 Unequal Mass Galaxies in Collision

How dependent are these results on the mass ratio of the two galaxies involved? We have experimented with intruders one- fourth as massive as the target galaxy and find that much of the displacement in the $z$-direction between outflowing and infalling material disappears. The ring expansion velocity decreases and two rings can appear (for the head on collisions), as predicted by other N-body investigations and epicycle/impulse approximation studies (see Appleton and Struck-Marcell, 1987; Struck-Marcell and Higdon, 1993; Hernquist and Wiel, 1993; Gerber and Lamb, 1993). Thus, many of the strong three-dimensional effects we see in our model are due to the fact that we are colliding together two equal-mass galaxies. However, the equal-mass collisions produce the densest rings and if this translates into the most prominent (or observable) rings, then these experiments have a good chance of being applicable to real systems, such as VII Zw 466, Arp 146, and Arp 147.

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## References

Appleton, P. N. and Struck-Marcell, C.: 1987, *Astrophys. J.* **318**, 103.
Balsara, D. S.: 1990. Ph.D. thesis. University of Illinois, at Urbana- Champaign.
Balsara, D. S.: 1993, submitted for publication.
Bushouse, H. A.: 1986, *Astron.J.* **91**, 255.
Bushouse, H. A.: 1987, *Astrophys. J.* **320**, 49.
Bushouse, H. A., Lamb, S. A. and Werner, M. W.: 1988, *Astrophys. J. 325*, 74.
Bushouse, H. A., and Stanford, A.: 1992, *Astrophys. J. Suppl.* **79**, 213.
Gerber, R. A.: 1993, Ph.D. thesis. University of Illinois, at Urbana- Champaign.
Gerber, R. A. and Lamb, S. A.: 1993, submitted to *Astrophys. J.*
Gerber, R. A., Lamb, S. A. and Balsara, D. S.: 1992, *Astrophys. J. Lett.* **399**, L51-54.
Gerber, R. A., Lamb, S. A. and Balsara, D. S.: 1993, in preparation.
Gerber, R. A., Lamb, S. A. and Balsara, D. S.: 1994, in preparation.
Gingold, R. A. and Monaghan, J. J.: 1982, *J. Comp. Phys.* **46**, 429.
Hernquist, L. and Katz, N.: 1989, *Astrophys. J. Suppl.* **70**, 419.
Hernquist, L. and Weil, M. L.: 1993, submitted for publication.
Hockney, R. W. and Eastwood, J. W.: 1988, *Computer Simulations Using Particles*, Adam Hilger, Bristol.
Huang, S. and Stewart, P.: 1988 *Astron. Astrophys.* **197**, 14.
Kennicutt, R. C., Keel, W. C., van der Hulst, J. M., Hummel, E. and Roettiger, K.: 1987, *Astron. J.* **93**, 1011.
Lucy, L. B.: 1977, *Astron. J.* **82**, 1013.
Lynds, R. and Toomre, A.: 1976, *Astrophys. J.* **209**, 382.
Majewski, S. R., Herald, M., Koo, D. C., Illingworth, G. D. and Heckman, T. M.: 1993, *Astrophys .J.* **402**, 125.
Monaghan, J. J.: 1985, *Comp. Phys. Rep.* **3**, 71.
Norman, C. and Scoville, N.: 1988, *Astrophys. J.* **332**,124.
Perry, J. J. and Dyson, J.: 1985, *Mon. Not. R. Astr. Soc.* **213**, 665.
Sanders, D. B., Soifer, B. T., Elias, J. H., Madore, B. F., Matthews, K., Neugebauer, G. and Scoville, N. Z.: 1988, *Astrophys. J.* **325**, 74.
Schultz, A. B., Spight, L. D., Rodrigue, M., Colegrove, P. T. and DiSanti, M. A.: 1991, *BAAS*, **23**, 953.
Shlosman, I., Frank, J. and Begelman, M. C.: 1989, *Nature* **338**, 45 .
Simkin, S. M., Su, H. J. and Schwartz, M. P.: 1980, *Astrophys .J.* **237**, 404.
Struck-Marcell, C. and Higdon, J. L.: 1993, *Astrophys. J.*, in press.
Struck-Marcell, C. and Lotan, P.: 1990, *Astrophys. J.* **358**, 99.
Theys, J. C. and Spiegel, E. A.: 1976, *Astrophys. J.* **208**, 650.
Theys, J. C. and Spiegel, E. A.: 1977, *Astrophys. J.* **212**, 616.
Thompson, L. A. and Theys, J. C.: 1978, *Astrophys. J.* **224**, 796.
Toomre, A.: 1964, *Astrophys. J.* **139**, 1217.