An Improved Representative Equation for the Dynamic Viscosity of Water Substance

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Experimental evidence for steam and other fluids has demonstrated the existence of an anomalous enhancement of the dynamic viscosity in the close vicinity of the critical point. A re-analysis of the experimental evidence for the viscosity of steam indicates that the observed behavior of the critical viscosity enhancement is consistent with current theoretical predictions. An interpolating equation for the dynamic viscosity of water substance is presented which is in good agreement with the experimental viscosity data in a large range of temperatures and pressures. The equation contains a smaller number of coefficients than the current international equation for the viscosity of water substance and incorporates the enhancement of the viscosity in the close vicinity of the critical point.

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Nomenclature

a	=	critical region equation of state parameter
a_k	=	coefficients in equation for η_0
a_{ij}	=	coefficients in equation for $\bar{\eta}$
b	=	critical region equation of state parameter
b_i	=	coefficients in equation for η_1
c_i	=	coefficients in equation for η_ℓ
d_i	=	coefficients in equation for η_v
E	=	parameter in equation for ξ
F	=	excess function defined in eq (2.2)
k	=	critical region equation of state parameter
n	=	number of data points
N	=	total number of data points
P	=	pressure
P_r	=	22.115 MPa
P^*	=	P/P_r
P_c	=	critical pressure
$\bar{P}_i$	=	coefficients in critical region equation of state
$\Delta \bar{P}$	=	$(P-P_c)/P_c$
q	=	parameter in equation for critical viscosity enhancement
r	=	parametric variable in critical region equation of state
t	=	temperature in °C
t_c	=	critical temperature in °C
T	=	temperature in K
T_r	=	647.27 K
T^*	=	T/T_r
T_c	=	critical temperature
$T_c^{(48)}$	=	critical temperature in terms of IPTS–48
$T_c^{(68)}$	=	critical temperature in terms of IPTS–68
$\Delta \bar{T}$	=	$(T-T_c)/T_c$
x	=	$\Delta \bar{T}/|\Delta \bar{\rho}|^{1/\beta}$
x_0	=	$-x$ at saturation boundary
X	=	$T^{*-1}-1$
y	=	objective function defined in eq (4.3)
Y	=	ρ^*-1
α	=	critical exponent of specific heat

β	=	critical exponent of coexistence curve
γ	=	critical exponent of compressibility
δ	=	critical exponent of chemical potential at $T=T_c$
η	=	dynamic viscosity
η_0	=	$\lim_{\rho \to 0} \eta$
η_1	=	$\lim_{\rho \to 0} (\partial \eta/\partial \rho)_T$
η_e	=	excess viscosity
$\bar{\eta}$	=	normal viscosity
$\bar{\eta}_c$	=	value of $\bar{\eta}$ at critical point
η_ℓ	=	viscosity of saturated liquid
η_v	=	viscosity of saturated vapor
$\Delta \eta$	=	critical viscosity enhancement
θ	=	parametric variable in critical region equation of state
Θ	=	Heaviside function
ξ	=	correlation length
ξ_0	=	correlation length amplitude above T_c at $\rho = \rho_c$
ξ_0'	=	correlation length amplitude along saturation boundary
ν	=	critical exponent of correlation length
ρ	=	density
ρ_r	=	317.763 kg/m³
ρ^*	=	ρ/ρ_r
ρ_c	=	critical density
$\Delta \bar{\rho}$	=	$(\rho-\rho_c)/\rho_c$
σ	=	standard deviation
τ	=	$T_c/T-1$
τ_0	=	$(q\xi_0')^{1/\nu}$
ϕ	=	exponent of critical viscosity enhancement factor
χ_T	=	$\rho(\partial\rho/\partial P)_T$ = "symmetrized" compressibility
χ_T^*	=	$\chi_T P_r/\rho_r^2$
$\bar{\chi}_T$	=	$\chi_T P_c/\rho_c^2$

1. Introduction

Authorized by the Eighth International Conference on the Properties of Steam, the International Association for the Properties of Steam (IAPS) issued in 1975 a new representation of the dynamic viscosity of water substance [I1][1]. This representation consists of a skeleton table and a recommended interpolating equation. The new formulation is a substantial improvement over the skeleton table and correlating formulae adopted earlier in 1964 [K1,M1,S1]. For a detailed discussion the reader is referred to articles by Hendricks et al. [H1], by Nagashima [N1] and by Scheffler et al. [S2]. The new representation has been incorporated in a revised edition of the ASME Steam Tables [M2].

The recommended interpolating equation was adopted by IAPS as the best equation available at that time. Nevertheless, the interpolating equation has a few unsatisfactory features. Firstly, it does not incorporate the enhancement of the viscosity observed in the critical region [N1,O1]. As a consequence, the interpolating equation does not yield an accurate representation of the viscosity in a small region around the critical point as recognized in the release issued by IAPS [I1]. Secondly, no attempts were made to assess the significance of the individual terms in the equation. As a result, the form of the equation has not been optimized and it contains a much larger number of coefficients than necessary.

[1] Symbols in brackets designate literature references.

In this paper we attempt to remedy the deficiencies of the interpolating equation mentioned above using current theoretical concepts for the representation of viscosity data of fluids near the critical point, combined with improved methods for the statistical analysis of data. The new equation, to be presented in this paper, was developed on the basis of the experimental viscosity data contained in the primary references recommended by IAPS's working group on transport properties [N1], but supplemented with detailed information concerning the behavior of the viscosity in the critical region [O1,R1,R2] as well as data from some other sources that have recently become available [K2,K3,N2].

2. Nature of the Viscosity Equation

2.1. The Normal Viscosity

In order to elucidate the nature of the new equation, it is convenient to introduce the notion of a normal viscosity. The normal viscosity $\bar{\eta}$ is defined as the viscosity without the extra enhancement observed in the immediate vicinity of the critical point. Hence outside a small region around the critical point, to be specified below, the actual viscosity η equals the normal viscosity $\bar{\eta}$.

The normal viscosity is most conveniently considered as a function of the temperature T and the density ρ. It is customarily treated as a sum of two distinct contributions

$$\bar{\eta}(\rho,T) = \eta_0(T) + \eta_e(\rho,T), \qquad (2.1)$$

where $\eta_0(T)$ represents the viscosity of the fluid in the limit of low densities and where $\eta_e(\rho,T)$ represents the increase in the viscosity at density ρ over the dilute gas value at the same temperature. The contribution $\eta_e(\rho,T)$ is often referred to as the excess viscosity. Many investigators have noted that the excess viscosity of fluids is a weak function of temperature over a substantial range of densities [D1,G1,H2,S3]. Hence, the excess viscosity, including that of water substance, has sometimes been approximated by a function of the density alone [S4,T1]. However, more detailed investigations have shown that the stratification of the isotherms, when $\eta_e(\rho,T)$ is plotted as a function of ρ at different temperatures, exceeds the experimental error of the data [G1,S5]. For an accurate representation over a wide range of conditions, the excess viscosity must be treated as a function of both density and temperature. This is especially important in the case of water substance, since the excess function displays some abnormal behavior at subcritical temperatures in both the vapor and the liquid phase.

Although the normal viscosity may be decomposed into two additive contributions as in eq (2.1), previous studies of the viscosity data of carbon dioxide [A1,E1] have indicated that the variation of the viscosity with density and temperature can be represented more concisely, if one uses an expression of the form

$$\ln\left[\frac{\bar{\eta}(\rho,T)}{\eta_0(T)}\right] = F(\rho,T). \qquad (2.2)$$

Expressing the excess factor $F(\rho,T)$ as a double power series in density and inverse temperature, Altunin and Sakhabetdinov [A1] found that the viscosity of carbon dioxide could be well represented by eq (2.2) over a wide range of conditions with only half the number of terms as compared to eq (2.1) with a similar power series for $\eta_e(\rho,T)$. These considerations led us to select the representation (2.2). Following Aleksandrov et al. [A2], we adopt the following functional form for $F(\rho,T)$

$$F(\rho,T) = \rho^* \sum_{i=0} \sum_{j=0} a_{ij} (T^{*-1} - 1)^i(\rho^* - 1)^j, \qquad (2.3)$$

where T^* is a reduced temperature and ρ^* a reduced density defined relative to some reference values T_r and ρ_r

$$T^* = T/T_r, \ \rho^* = \rho/\rho_r. \qquad (2.4)$$

Equation (2.3) has the same mathematical form as the representative equations currently recommended by IAPS for the viscosity [I1] and the normal thermal conductivity [I2,K4] for water substance. For consistency, we have adopted for T_r and ρ_r in eq (2.4) the same reference values

$$T_r = 647.27 \text{ K}, \ \rho_r = 317.763 \text{ kg/m}^3 \qquad (2.5)$$

as used in the current international equations [I1,I2].

With a few notable exceptions, such as the viscosity equations proposed by Hanley et al. [H3,H4,H5], the forms of the equations commonly used for either $\eta_e(\rho,T)$ or $F(\rho,T)$ are similar to the functional forms used for the representation of the compressibility factor Z [K5,K6]. In particular, the double power series adopted here is of the same mathematical form as the representation earlier used by Altunin and Gadetskii for the representation of the thermodynamic properties of carbon dioxide [A3]. As an alternative possibility we also investigated the viscosity of water substance using an expression for $F(\rho,T)$ similar to the one used by Jacobsen and Stewart [J1] for the representation of the thermodynamic properties of nitrogen. Among the many equations tested, we obtained the best overall representation of the selected viscosity data using the Altunin-Gadetskii form (2.3).

2.2. Viscosity in the Zero-Density Limit

In order to represent the normal viscosity by eq (2.2), the equation (2.3) adopted for the excess factor $F(\rho,T)$ must be complemented with an equation for the viscosity $\eta_0(T)$ in the limit of zero density.

It turns out that the viscosity of steam at subcritical temperatures has a negative pressure coefficient. The phenomenon was observed by Kestin and coworkers [K7,M3] using an oscillating disk viscometer and it has been confirmed using capillary viscometers as well [N1]. The differences between the values of the viscosity at atmospheric pressure and those in the limit of zero pressure increase with decreasing temperature. The equation for $\eta_0(T)$ should represent the viscosity data when extrapolated to zero density or pressure, taking into account the negative pressure effect. A careful study of the viscosity data for steam at low pressures was made by Aleksandrov et

al. [A2]. Starting from the expression for the viscosity as given by the kinetic theory of gases, but replacing the collision integral by an empirical power series expansion in inverse temperature, they proposed

$$\eta_0(T) = T^{*1/2} \left[\sum_{k=0}^{3} \left(\frac{a_k}{T^{*k}} \right) \right]^{-1}, \qquad (2.6)$$

where T^* is the same dimensionless temperature as defined in eq (2.4). With the values for the coefficients a_k listed in table A1 of Appendix A, this equation represents the extrapolated zero-density viscosity to well within the experimental accuracy in a range of temperatures from 0 °C to 1100 °C. It was therefore incorporated in the interpolating equation for the viscosity of water substance adopted by IAPS in 1975 [I1]. Since no new experimental information for the behavior of the viscosity of steam at low pressures has been reported subsequently, we have retained the same equation (2.6) for $\eta_0(T)$ in the present work.

FIGURE 1.   Schematic representation of the viscosity as a function of density at a temperature slightly above the critical temperature.

2.3. Viscosity in the Critical Region

Upon approaching the critical point an anomalous increase is observed in the thermal conductivity and viscosity of fluids. In the case of the thermal conductivity, the critical enhancement is encountered in a large range of temperatures and densities. The presence of a critical enhancement in the thermal conductivity of steam was confirmed experimentally by Le Neindre et al. [L1] and by Sirota et al. [S6]. At the critical density the anomalous behavior of the thermal conductivity extends up to temperatures twenty percent above the critical temperature. Equations representing the critical enhancement of the thermal conductivity of steam have been formulated [B1] and the effect is accounted for in a new interpolating equation for the thermal conductivity of steam adopted by IAPS in 1977 [I2,K4].

Theoretical and experimental evidence indicates that the viscosity of fluids also diverges at the critical point [S7]. However, unlike the thermal conductivity, the critical enhancement of the viscosity is restricted to a narrow range of temperatures and densities around the critical point. For steam, a critical enhancement in the viscosity has been observed by Rivkin et al. [R1,R2] and by Oltermann [O1]. As we shall see, the region of the critical behavior covers approximately one percent in absolute temperature and twenty percent in density relative to the temperature and density of the critical point.

In figure 1 we show schematically the variation of the viscosity with density at a temperature T slightly above the critical temperature. In the critical region we consider the viscosity $\eta(\rho,T)$ as the sum of a normal viscosity $\bar\eta(\rho,T)$ and a critical or anomalous viscosity $\Delta\eta(\rho,T)$

$$\eta = \bar\eta + \Delta\eta. \qquad (2.7)$$

Outside the critical region, the viscosity η is to be identified with the normal viscosity $\bar\eta$. Inside the critical region, $\bar\eta$ is
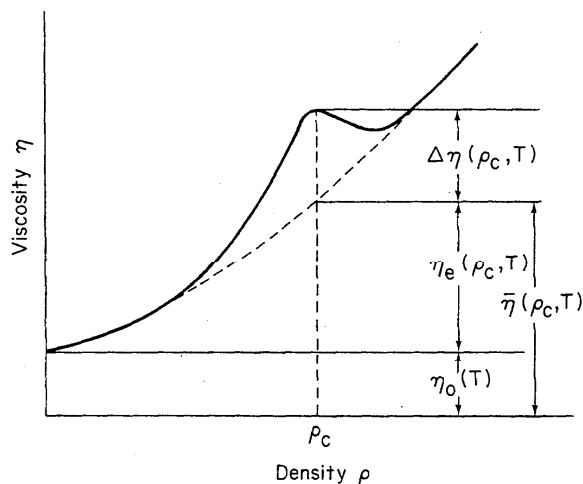
defined empirically by extrapolating the behavior of the normal viscosity outside the critical region smoothly into the critical region. In practice, this is accomplished by retaining the eqs (2.2) and (2.3) for $\bar\eta$ throughout the critical region. Experiment and theory indicate that the critical viscosity enhancement is a multiplicative enhancement, i.e., the anomalous viscosity $\Delta\eta$ is proportional to the normal viscosity $\bar\eta$ [C1,K8,O2,S7]. Hence, rather than $\Delta\eta$, we prefer to consider the relative critical enhancement $\Delta\eta/\bar\eta$ or the viscosity ratio $\eta/\bar\eta$.

In the vicinity of the critical point large-scale density fluctuations are present in the fluid. The spatial extent of these fluctuations is characterized by a correlation length ξ which diverges at the critical point [F1,S8]. The theory of dynamical critical phenomena predicts that the viscosity ratio $\eta/\bar\eta$ will diverge as ξ^ϕ where ϕ is a critical viscosity exponent [D2,H6,O3,S9]. We thus represent the asymptotic behavior of the viscosity by

$$\frac{\eta}{\bar\eta} = (q\xi)^\phi, \qquad (2.8)$$

where the parameter q is in principle a slowly varying function of density and temperature [B2,O4]. In practice, however, we treat q as an adjustable constant to be determined from the experimental viscosity data [B2]. Based on an approximate evaluation of the mode-coupling formulae for critical dynamics, several investigators have predicted an equation for the critical viscosity enhancement of the form [K8,O3,O4,P1]

$$\frac{\eta}{\bar\eta} = 1 + \frac{\Delta\eta}{\bar\eta} \simeq 1 + \phi\ln(q\xi) \simeq (q\xi)^\phi, \qquad (2.9)$$

with

$$\phi = \frac{8}{15\pi^2} \simeq 0.054. \qquad (2.10)$$

Theoretical values $\phi = 0.065$ and $\phi = 0.070$ have also been reported [G2,S9]. However, the latter calculations assume implicitly that the critical point is approached sufficiently closely so that $\Delta\eta >> \bar{\eta}$, a condition not satisfied in the temperature range where actual viscosity data are available. Experimental viscosity data near the gas-liquid critical point of a number of fluids are consistent with an exponent value ϕ close to 0.05 [B2,L2,S7,S10]. As we shall demonstrate, the experimental data for the critical viscosity enhancement for steam are also consistent with $\phi = 0.05$.

The asymptotic behavior (2.8) for the viscosity applies to temperatures and densities sufficiently close to the critical point so that $q\xi >> 1$. In order to represent the viscosity over a large range of temperatures and densities, we want to formulate an equation which connects smoothly the critical behavior as given by eq (2.8) with the normal behavior of the viscosity outside the critical region. For this purpose we propose

$$\eta = \bar{\eta}(q\xi)^\phi, \quad \text{for } q\xi > 1 \qquad (2.11a)$$

$$\eta = \bar{\eta}, \qquad \text{for } q\xi \leq 1. \qquad (2.11b)$$

An equation of this form has been used earlier to represent experimental viscosity data reported for nitrogen near the critical point [B2]. The relationship between this equation and those proposed by other investigators is discussed in a separate publication [B3].

A method for calculating the correlation length ξ of steam in the critical region as a function of temperature and density is presented in Appendix B.

2.4. Equation of State

The proposed equation for the viscosity of water substance represents the viscosity as a function of density and temperature. In practice, many experimental viscosity data are obtained as a function of pressure and temperature. In the formulation of the interpolating equation currently recommended by IAPS for the viscosity of water substance [I1], densities were calculated using either the 1967 IFC Formulation for Industrial Use [M2,S1] or the 1968 IFC Formulation for Scientific and General Use [I3].

Unfortunately, neither the 1967 IFC formulation nor the 1968 IFC formulation is suitable to represent the thermodynamic and transport properties of steam in the immediate vicinity of the critical point. First of all, the 1967 IFC and 1968 IFC formulations themselves lead to appreciably different predictions for the density in the vicinity of the critical point [N1]. Moreover, as discussed in Appendix B, the calculation of the correlation length ξ requires knowledge of the compressibility $(\partial\rho/\partial P)_T$ as well. Most importantly, recent developments have clearly demonstrated that the thermodynamic behavior of fluids exhibits a nonanalytic singularity at the critical point which can only be accommodated by the introduction of a so-called scaled equation of state [L3,S8]. In previous publications, it has been shown that scaled equations of state do represent the observed thermodynamic behavior of steam in the critical region as well [B4,L4,M4]. Finally, the reference values T_r and

ρ_r introduced in eq (2.5) are the values of the critical temperature and density implicitly assumed in the 1968 IFC Formulation for Scientific and General Use [I3]. However, we now know that these reference values are not equal to the actual temperature and density of the critical point of steam [L3,L4].

In order to represent the behavior of the viscosity in the critical region we have adopted a scaled parametric equation of state obtained by Murphy et al. [M4] from a fit to the experimental equation of state data of Rivkin and coworkers [R3,R4]. This equation has been subsequently extended into a complete fundamental equation by Levelt Sengers [L5]. The scaled equation of state used in this paper is specified in Appendix B; it was used in a region around the critical point bounded by

$$\left|\frac{T-T_c}{T_c}\right| \leq 0.05, \quad \left|\frac{\rho-\rho_c}{\rho_c}\right| \leq 0.25. \qquad (2.12)$$

Outside the range bounded by eq (2.12), we continued to use the 1968 IFC Formulation for Scientific and General Use as is done in the interpolating equation recommended by IAPS [I1]. A table of specific volume values for water substance calculated with the aid of the 1968 IFC formulation at integral pressures and temperatures is included in the paper of Nagashima [N1]. Nagashima has also discussed the effect of the difference of the 1967 and 1968 IFC formulations on the values calculated for the viscosity [N1]. Since the region of the critical behavior of the viscosity is wholly within the range bounded by eq (2.12), the same considerations apply to the normal viscosity as predicted by our equations.

Both the 1967 and the 1968 IFC formulations assume that the temperature is expressed in terms of the International Practical Temperature Scale of 1948 (IPTS–48). Most of the experimental viscosity data are also given in reference to IPTS–48. Since no internationally agreed equation of state in terms of the International Practical Temperature Scale of 1968 (IPTS–68) is currently available, the interpolating viscosity equation adopted by IAPS was actually formulated in terms of IPTS–48. For the same reason we have conducted our analysis also in terms of IPTS–48. However, the resulting equation can be readily used to calculate the viscosity in terms of IPTS–68 as indicated in Appendix D.

3. Experimental Viscosity Data

3.1. Primary Sources of Data prior to 1974

A survey of the experimental measurements for the viscosity of water substance, that were available to IAPS in 1974, was presented by Nagashima [N1]. A critical evaluation of these available data sets was made by IAPS's Working Group II on transport properties. The deliberations resulted in a recommended set of primary references, designated as "International Input", together with an assessment of the levels of uncertainty associated with the data. The criteria on which the selections and assessments were made, together with brief comments on the limits imposed on the range of applicability of certain primary references, can be found in reference [N1]. The list

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Table 1. Primary experimental data set (International Input)

Author	Year	Region	Method	Evaluated precision	Reference
White	1914	1	1	0.5%	[W1]
Coe	1944	1	1	0.3%	[C2]
Hardy (recalc.)	1949	1	1	0.5%	[H7]
Swindells	1952	1	1	0.1%	[S11]
Weber	1955	1	1	0.3%	[W2]
Roscoe	1958	1	2	0.1%	[R5]
Malyarov	1959	1	1	0.1%	[M5]
Hallet	1963	1	1	0.3%	[H8]
Korosi	1968	1	1	0.3%	[K9]
Korson (recalc.)	1969	1	1	0.3%	[K10]
Eicher (recalc.)	1971	1	1	0.3%	[E2]
Shifrin	1959	2	1	3 %	[S12]
Latto	1965	2	1	3 %	[L6]
Sato (recalc.)	1968/70	2	1	1.5%	[S13,S14]
Timrot (recalc.)	1973	2	1	0.4%	[T2]
Moszynski	1961	3	2	2 %	[M3]
Mayinger	1962	3	1	2 %	[M6]
Tanaka	1965	3	1	2 %	[T3]
Agayev	1968	3	1	1 %	[A4]
Nagashima	1969	3	1	2 %	[N3]
Kerimov	1969	3	1	1 %	[K11]
Rivkin	1970/72	3	1	1 %	[R6-R8]
Whitelaw	1960	4	1	4 %	[W3]
Kestin	1960	4	2	2 %	[K7]
Kestin	1963	4	2	0.8%	[K12]
Nagashima	1969	4	1	3 %	[N3]
Sato	1968/70	4	1	1.5%	[S13,S14]
Rivkin	1968/70/73	4	1	1 %	[R9-R12]
Timrot	1973	4	1	0.4%	[T2]
Whitelaw	1960	5	1	4 %	[W3]
Mayinger	1962	5	1	3 %	[M6]
Nagashima	1969	5	1	3 %	[N3]
Tanaka	1965	5	1	3 %	[T3]

Regions:

1 Water at atmospheric pressure.
2 Steam at atmospheric pressure.
3 Pressurized water.
4 Steam at t < 550° C .
5 Steam at t > 550° C .

Methods of measurements:

1 Capillary flow
2 Oscillating body or vessel

of primary references together with the estimated precision of the data is reproduced in table 1.

The experimental data values extracted from the primary references are contained in a technical report prepared for IAPS by Scheffler et al. [S15]. The data values listed in this report were used in our analysis. Preliminary fits of the proposed equation to the data revealed a number of minor inconsistencies. Some corrections are listed in the article of Nagashima [N1], but some additional changes to the primary data set were also found desirable.

The temperature uncertainties of ±2 °C in the work of Whitelaw [W3] lead to large uncertainties in the corresponding densities and, hence, to large uncertainties in the values for the dynamic viscosity calculated from the measured kinematic viscosities. The effect is most pronounced for the data at 380 °C, where an uncertainty of 2 °C in temperature may imply an uncertainty in the dynamic viscosity as large as 20%. To prevent degradation of the fit in the critical region, we deleted the first five points at 380 °C in reference [W3] which are subject to the largest uncertainties.

For the viscosity of water at elevated pressures and temperatures between 300 °C and the critical temperature, only three sources of experimental information are available; namely, the data reported by Tanaka [T3], by Nagashima et al. [N3] and by Rivkin et al. [R6,R7,R8] with estimated precisions of 1%–2%. However, the measurements of Nagashima et al., on the one hand and the measurements of Rivkin et al. on the other hand show mutual devations up to 5% in this range as illustrated in figure 12 in reference [N1]. It turns out that the data of Tanaka agree in general more closely with those of Rivkin et al. The data of Rivkin et al. also appear to be consistent with the data of Mayinger [M6] and with the more extensive and precise data of Kerimov, Agayev et al. [A4,K11] below 300 °C. Hence, to ensure consistency in the near critical region, we rejected the data of Nagashima et al. [N3] between 300 °C and the critical temperature in favor of the more precise measurements of Rivkin et al. [R6,R7,R8]. Actually, the 18 points of Nagashima et al. were retained in the selected data set for purposes of comparison, but their weights were reduced by a factor of one hundred.

Furthermore, 7 data points, not counting duplicates, were found to show deviations from preliminary fits to the surface in excess of three times their evaluated precision as given in table 1. They are the data at $t = 172.3$ °C, and $P = 49.749$ MPa in reference [T3], at $t = 375.0$ °C and $P = 22.658$ MPa, $t = 400.0$ °C and $P = 35.436$ MPa, $t = 425.0$ °C and $P = 30.498$ MPa, $t = 425.0$ °C and $P = 33.417$ MPa, $t = 450.0$ °C and $P = 35.406$ MPa in references [R9,R10] and at $t = 402.83$ °C and $P = 40.060$ MPa in reference [N3]. That the lack of adequate representation of these points is not indicative of an undetected trend in the data or failure of the proposed equation is indicated by the agreement between the fitted equation and adjacent data points from the same experimental sources. Six of the points lie in the most difficult region of measurement, where the kinematic viscosity has a low value and where the flow in the capillary viscometers used can easily become turbulent at low Reynolds numbers. Some of the discrepancies, in particular in the case of the first and last data points mentioned, may be caused by printing errors in the values reported for temperature, pressure or viscosity. The 7 suspected data points were retained in the analysis, but their weights were reduced by a factor of one hundred.

Using a new theoretical equation for the operation of capillary viscometers, Kestin, Sokolov, and Wakeham have recently made a re-analysis of some of the earlier, very precise measurements of the viscosity of water between 0° and 150 °C at near atmospheric pressures [K13]. Systematic errors introduced by the use of semi-empirical and possibly incorrect working equations were appreciably reduced. In particular, they re-evaluated completely the experimental measurements of Korson et al. [K10] and of Eicher and Zwolinski [E2]. Thus instead of the original values reported by Korson et al. and by Eicher and Zwolinski, we adopted the values deduced by Kestin et al. [K13] from these sources, retaining the error estimates quoted in table 1. From the work of Hardy and Cottington [H7] we took the values deduced for the ratios of the viscosity at temperature t relative to the viscosity at 20 °C and converted these ratios into absolute viscosities using the reference value

of 1002.0×10^{-6} Pa·s at 20 °C and atmospheric pressure [K13].

3.2. Additional Sources of Data

Since the selection of the primary references in 1974, the results of five new experiments have become available. The new references, together with the estimated precision of the data, are listed in table 2.

The viscosity measurements obtained by Nagashima et al. [N2] using a capillary flow assembly cover a temperature range from 250 °C to 600 °C and a pressure range up to 20.8 MPa. The kinematic viscosity is low in this region and the fluid flow can easily become turbulent, as mentioned earlier. However, we believe that extreme care was taken by the investigators and we have assigned an estimated error of 3% to the data as was done for the earlier work in this region by the same investigators [N3].

Rivkin et al. [R1,R2] and Oltermann [O1] have reported experimental studies of the enhancement of the viscosity in the critical region. The measurements of Rivkin et al. span a range in temperature from 374 °C to 390 °C and in pressure from 22.1 MPa to 27.5 MPa [R1,R2]. The measurements obtained by Oltermann cover a wider range of conditions; namely, a temperature range from 341 °C to 383 °C and a pressure range from 0.1 MPa to 23.8 MPa, but they provide less detailed coverage of the near critical region [O1]. A major objective of the present paper is to include a proper description of the critical viscosity enhancement. For this reason we shall discuss the viscosity data in the critical region in more detail below.

Kestin and collaborators have recently reported the results of two experimental studies of the viscosity of water at elevated pressures using a modified version of the oscillating-disk viscometer developed at Brown University [K2,K3]. The data were obtained relative to the viscosity of water at atmospheric pressure as a function of temperature. For the data obtained along five isotherms at temperatures between 10 °C and 40 °C and at pressures up to 31 MPa, we have adopted the error estimate

Table 2. Additions to the primary data set

Author	Year	Region[a]	Method[a]	Evaluated precision	Reference
Nagashima	1974	4/5	1	3.0%	[N2]
Rivkin (recalc.)	1975	3/4	1	1.0%[b]	[R1,R2]
Oltermann	1977	3/4	2	1.6%[b]	[O1]
Kestin	1977	3	2	0.3%	[K2]
Kestin	1978	3	2	0.7%	[K3]

[a]For designation of region and method see footnote in table 1.

[b]For evaluated precision in critical region see discussion in section 4.1.
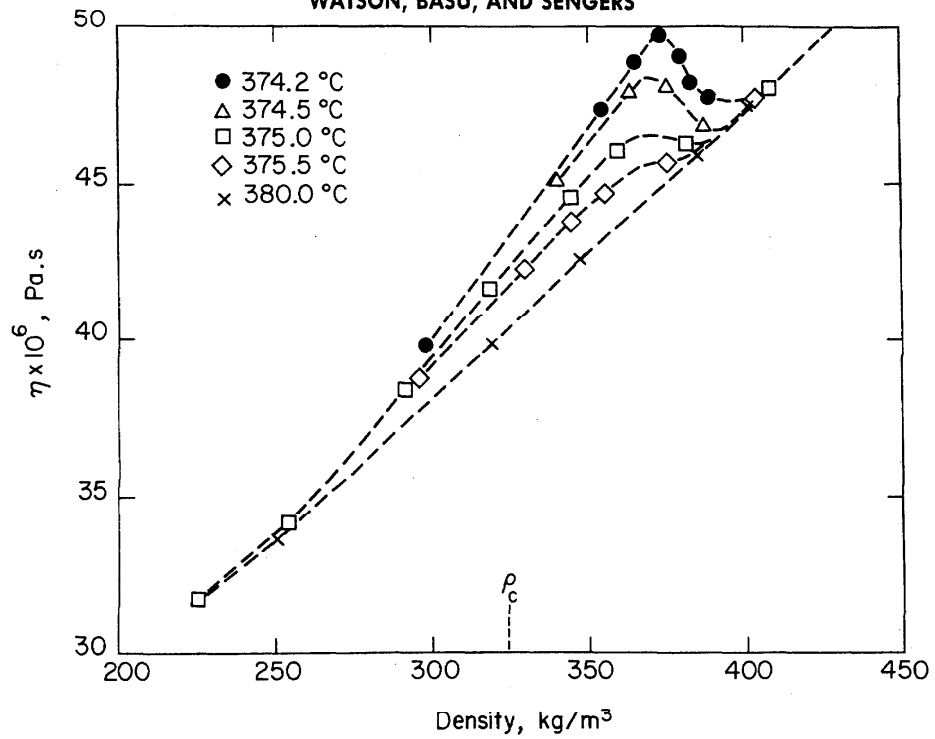
FIGURE 2.   Viscosity data along near critical isotherms as reported by Rivkin et al. [R1, R2].
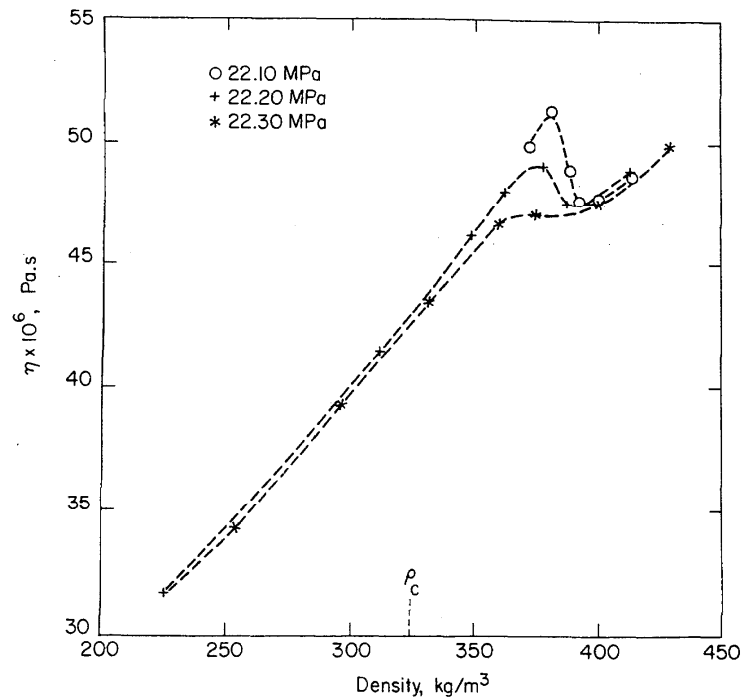

FIGURE 3.   Viscosity data along near critical isobars as reported by Rivkin et al. [R1,R2].

of 0.3% proposed by the authors [K2]. For the data along ten isotherms between 40 °C and 150 °C, Kestin et al. report an estimated error of 0.5% [K3]. For our analysis we have increased this estimate to 0.7%, since we found evidence for the existence of small, but significant, systematic deviations from the primary reference data in this region.

3.3. Data in the Critical Region

A detailed experimental study of the viscosity of steam in the critical region was made by Rivkin and coworkers at the All-Union Heat Engineering Institute in Moscow [R1,R2]. In this experiment the kinematic viscosity η/ρ was determined by measuring the flow rate through a capillary with an internal diameter of 0.3 mm and a length of 50 cm as a function of the pressure drop over the capillary [R2]. Actually, the use of a capillary flow viscometer in the critical region leads to many complications due to the large compressibilities in this region [S16]. However, the investigators took care in making measurements at various pressure differences ranging from 100 mm mercury to 8 mm mercury and verified that the measured kinematic viscosity became independent of the applied pressure difference below 12 mm mercury [R13]. We have, therefore, adopted the values determined by Rivkin et al. for the kinematic viscosity together with the error estimate of 1% suggested by the authors.

The data were obtained along eight isotherms at temperatures between 374.2 °C and 390.0 °C and along three isobars corresponding to 22.10 MPa, 22.20 MPa, and 22.30 MPa. In figures 2 and 3 we show the dynamic viscosity η as a function of the density ρ reported by Rivkin et al. for the near critical isotherms and isobars. The experiment yields the kinematic viscosity η/ρ as a function of pressure and temperature. In deducing values for the dynamic viscosity η an added uncertainty is introduced due to the fact that the densities have to be calculated from the given pressures and temperatures. Rivkin et al. used the 1968 IFC Formulation for Scientific and General Use for this purpose. However, as mentioned in section 2.4, this equation becomes inaccurate in the vicinity of the critical point. Hence, for a proper analysis of the dynamic viscosity as a function of the density, the densities have to be re-evaluated. In the region bounded by eq (2.12) we recalulated the densities using the scaled equation of state presented in Appendix B. Close to the critical point we expect that at a given temperature the anomalous viscosity $\Delta\eta/\overline{\eta}$ will reach a maximum at the critical density. This expectation is confirmed by measurements of the viscosity of other fluids near the critical point, where the viscosity was measured as a function of the density directly [B2,I4,Z1]. From figure 2 it is evident that this condition is not satisfied for the viscosity data reported by Rivkin et al. In our experience, this behavior is an indication that the wrong values of density were used.

The situation is further complicated by the fact that in the region where the anomaly is observed the values calculated for the density ρ and the correlation length ξ become very sensitive to the value adopted for the critical temperature T_c. An internationally agreed value for T_c is currently not available [L3]. In appendices B and C we have adopted the value $T_c = T_c^{(48)} = 646.996$ K deduced by Murphy et al. [M4] from

the equation of state data of Rivkin and coworkers [R3,R4]. However, because of the possibility of temperature gradients and other possible systematic errors, a careful analysis of experimental data near the critical point should preferably be conducted with a value for T_c determined in the same experiment. In the absence of such a determination, the critical behavior of thermophysical properties is usually analyzed using T_c as an adjustable parameter [L3]. For the viscosity experiment of Rivkin et al. we determined T_c by requiring that the maximum in $\Delta\eta/\overline{\eta}$ occurs at the critical density when the critical temperature is approached. This condition was optimally satisfied by using $T_c = T_c^{(48)} = 646.956$ K which differs by 0.04 K from the value deduced by Murphy et al. from the equation of state data. Rivkin et al. quote an accuracy of 0.02 °C in the measured temperatures and an accuracy of 0.01 MPa in the measured pressures. Since the slope dP/dT of the vapor pressure line equals 0.27 MPa/K at the critical point, a possible systematic error of 0.01 MPa in pressure implies an additional possible systematic error of 0.04 °C in temperature. Hence the apparent difference of 0.04 °C in the two critical temperature values mentioned above is well within the claimed accuracy of the experiment. The effect of this correction is further discussed elsewhere [B3].

In table 3 we present the viscosities deduced from the original data of Rivkin et al. [R1,R2], using in the region bounded by eq (2.12) the scaled equation of state presented in Appendix B with $T_c^{(48)} = 646.956$ K. These data were used for the present analysis. The behavior of the dynamic viscosity thus obtained as a function of the density at near critical isotherms and isobars is shown in figures 4 and 5, respectively.

A cursory examination of the experimental data indicated that the anomalous behavior of the viscosity is wholly contained inside a region bounded by

$$\left|\frac{T-T_c}{T_c}\right| \leq 0.023, \quad \left|\frac{\rho-\rho_c}{\rho_c}\right| \leq 0.25. \qquad (3.1)$$

The viscosity measurements reported by Oltermann [O1] were obtained using an oscillating-disk viscometer. Although as many as 96 data points were reported, most of the data relate to the behavior of the normal viscosity and only two data points appear to be located inside the region bounded by eq (3.1). Therefore, a re-evaluation of Oltermann's data using a scaled equation of state was not considered important.

3.4. Range of Experimental Data

The IFC formulations for the thermodynamic properties of water substance as well as the current international formulation for the viscosity of water substance cover a range of temperatures and pressures bounded by

$$0 < t < 800 \text{ °C}, \ 0 < P < 100 \text{ MPa}. \qquad (3.2)$$

Four of the primary sources listed in table 1 include viscosity data at higher temperatures and pressures. The measurements of Agayev and coworkers [A4,K11] extend to pressures of about 118 MPa at temperatures between 0 °C and 275 °C; the measurements of Nagashima et al. [N3] to 901 °C at pressures up

Table 3.   Viscosity data in the critical region deduced from the measurements of
            Rivkin et al. [R1,R2]

Nominal temperature °C	$T - T_c$ °C or K	P MPa	ρ kg/m^3	$\eta \times 10^6$ Pa·s
374.20	0.394	22.136	270.8	36.2
374.20	0.394	22.140	275.9	36.9
374.20	0.394	22.144	282.7	37.9
374.20	0.394	22.150	300.5	40.1
374.20	0.394	22.156	341.3	44.2
374.20	0.394	22.160	356.4	45.1
374.20	0.394	22.170	372.7	46.0
374.20	0.394	22.220	398.8	47.1
374.50	0.694	22.220	284.2	37.8
374.50	0.694	22.230	307.4	40.5
374.50	0.694	22.240	344.2	44.2
374.50	0.694	22.260	373.5	45.3
374.50	0.694	22.300	393.7	46.7
375.00	1.194	22.200	225.2	31.7
375.00	1.194	22.300	254.3	34.2
375.00	1.194	22.340	279.0	36.7
375.00	1.194	22.350	290.0	38.0
375.00	1.194	22.360	305.5	39.5
375.00	1.194	22.360	305.5	39.6
375.00	1.194	22.370	325.9	41.8
375.00	1.194	22.400	366.9	44.6
375.00	1.194	22.500	408.3	48.1
375.00	1.194	23.500	472.1	54.4
375.50	1.694	22.470	284.3	37.2
375.50	1.694	22.490	304.8	39.1
375.50	1.694	22.500	318.1	40.5
375.50	1.694	22.510	331.8	41.7
375.50	1.694	22.540	361.7	44.0
375.50	1.694	22.640	398.4	47.1
376.00	2.194	22.600	288.0	37.4
376.00	2.194	22.620	304.6	38.9
376.00	2.194	22.630	314.3	40.0
376.00	2.194	22.650	334.3	41.4
376.00	2.194	22.700	368.5	44.4
376.00	2.194	22.800	397.7	47.0
377.00	3.194	22.300	197.0	29.6
377.00	3.194	22.700	240.9	32.9
377.00	3.194	22.840	282.8	36.6
377.00	3.194	22.890	310.5	39.0
377.00	3.194	22.940	341.6	41.9
377.00	3.194	23.000	368.2	44.3
377.00	3.194	23.200	411.1	48.3

Table 3. (Continued)

Nominal temperature $^{\circ}C$	$T - T_c$ $^{\circ}C$ or K	P MPa	ρ kg/m^3	$\eta \times 10^6$ Pa·s
380.00	6.194	23.400	247.5	33.5
380.00	6.194	23.700	315.3	39.3
380.00	6.194	23.800	343.2	42.1
380.00	6.194	24.000	381.9	45.6
380.00	6.194	24.200	406.7	48.0
380.00	6.194	25.000	451.0	52.5
380.00	6.194	30.000	534.4	61.3
390.00	16.194	25.000	215.1	31.5
390.00	16.194	26.100	287.9	37.1
390.00	16.194	26.600	333.2	41.1
390.00	16.194	27.000	364.5	44.0
390.00	16.194	27.500	393.6	46.8
373.70	- 0.106	22.100	414.3	48.6
373.88	0.074	22.100	391.8	46.8
373.94	0.134	22.100	380.1	46.1
373.97	0.164	22.100	369.6	46.6
373.97	0.164	22.100	369.6	46.6
374.00	0.194	22.100	328.2	44.2
374.00	0.194	22.100	328.2	44.2
374.03	0.224	22.100	279.5	37.5
374.03	0.224	22.100	279.5	37.5
374.00	0.194	22.200	414.3	48.9
374.30	0.494	22.200	373.1	45.7
374.35	0.544	22.200	349.8	45.4
374.40	0.594	22.200	295.0	39.1
374.42	0.614	22.200	284.3	37.7
374.44	0.634	22.200	277.0	36.8
375.00	1.194	22.200	225.2	31.7
374.00	0.194	22.300	428.6	49.9
374.50	0.694	22.300	393.7	46.7
374.70	0.894	22.300	352.2	44.2
374.75	0.944	22.300	319.3	41.5
374.80	0.994	22.300	290.9	38.3
374.85	1.044	22.300	276.2	36.6
375.00	1.194	22.300	254.3	34.2
377.00	3.194	22.300	197.0	29.6

FIGURE 4.   Viscosity data along near critical isotherms obtained after re-evaluating the original
measurements of Rivkin et al. The curves indicate the values calculated from the
representative equation proposed in this paper.

to 22.9 MPa and those of Latto [L6] to 1071 °C at atmospheric
pressures. We retained the data points in the extended range
in our analysis, but increased their error estimates by 0.5%
to account for the increased uncertainties of the density values.

In addition, other sources are available, not included in
tables 1 and 2, that contain viscosity data outside the range

bounded by eq (3.2). Cappi [C3] has reported viscosity data
of water between 2 °C and 100 °C at pressures up to 1000
MPa, while Isdale and Spence [I5] have also measured the
viscosity of water at 25 °C and 50 °C at pressures up to 1000
MPa. Dudziak and Franck [D3] have measured the viscosity
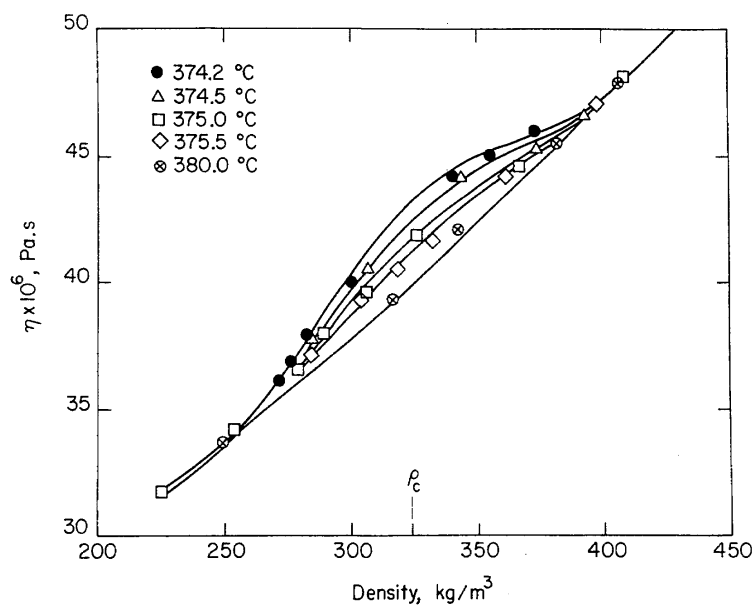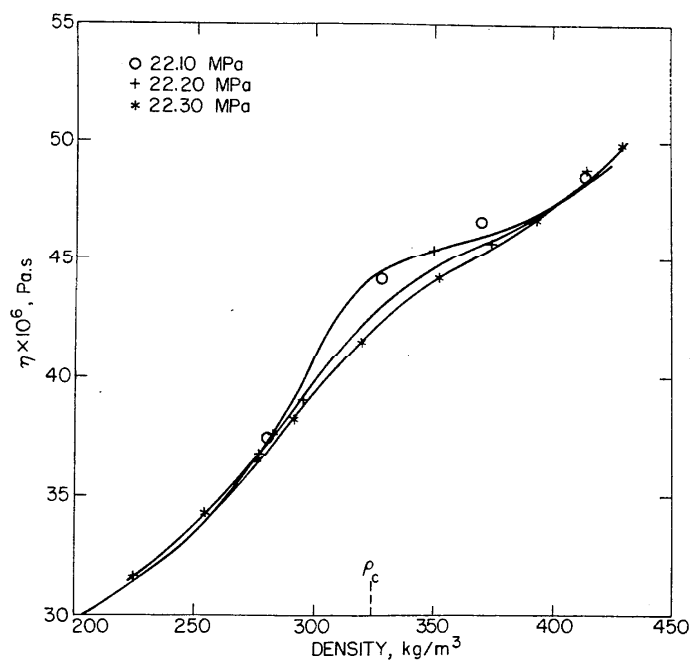of liquid water and supercritical water at pressures between

FIGURE 5.   Viscosity data along near critical isobars obtained after re-evaluating the original
measurements of Rivkin et al. The curves indicate the values calculated from
the representative equation proposed in this paper.

100 MPa and 350 MPa at temperatures up to 560 °C. Because of the lack of international agreement concerning the equation of state in this expanded pressure range, we have not included these data in determining the values of the coefficients in our viscosity equation. However, a comparison with these data values will enable us to investigate to what extent the proposed viscosity equation remains reliable ouside the range bounded by eq. (3.2).

4. Analysis and Results

4.1. Method of Analysis

In accordance with eqs (2.2) and (2.11), our proposed equation for the dynamic viscosity η assumes the form

$$\eta(\rho,T) = \eta_0(T)e^{F(\rho,T)}(q\xi)^\phi, \quad \text{for } \xi > q^{-1}$$

$$\eta(\rho,T) = \eta_0(T)e^{F(\rho,T)}, \quad \text{for } \xi \leq q^{-1}. \tag{4.1}$$

Taking the natural logarithm, we have

$$\ln\left(\frac{\eta}{\eta_0}\right) = F(\rho,T) + [\phi\ln(q\xi)]\Theta(q\xi-1), \tag{4.2}$$

where $\Theta(z)$ is the Heaviside step function such that $\Theta(z) = 1$ for $z \geq 0$ and $\Theta(z) = 0$ for $z < 0$. Thus with $\eta_0(T)$ given by eq. (2.6), the objective function y to be fitted to the data is

$$y = \ln\left(\frac{\eta}{\eta_0}\right) - F(\rho,T) - \phi[\ln q + \ln\xi]\Theta(q\xi-1). \tag{4.3}$$

On substituting eq (2.3) into eq (4.3) we note that the objective function y, except for the presence of the Heaviside step function, is linear in the coefficients a_{ij}, $\phi\ln q$ and ϕ. Hence, the optimum values of these coeffecients can be determined using the method of linear least squares. That is, we minimize the normalized sum of squares

$$S = \sum_{i=1}^{N}\left(y_i/\sigma_{y_i}\right)^2, \tag{4.4}$$

where N is the total number of data points, y_i the value of y for the ith datum point and $\sigma_{y_i}^2$ the estimated variance of y at the same data point. For data points outside a critical region bounded by eq (3.1), we approximate the weight factor $(1/\sigma_{y_i})^2$ as $(\eta_i/\sigma_{\eta_i})^2$, where η_i is the viscosity and σ_{η_i} the estimated uncertainty of the viscosity at the ith datum point. Since the estimated errors in the viscosity have been assessed as being proportional to the viscosity itself, we can actually express the weight factor at the ith point as $(1/\alpha_i)^2$, where α_i is the evaluated precision as listed in tables 1 and 2 with amendments discussed in sections 3.1, 3.2 and 3.4.

In the critical region it is not sufficient to consider only the uncertainty σ_η in the direct measurement of the viscosity, but in addition we need to assess the uncertainty in η due to the

uncertainties σ_p and σ_T in the measured values of pressure and temperature. Hence, in the region specified by eq (3.1) the weight factor was taken as $(\eta/\sigma)^2$ with

$$\sigma^2 = \sigma_\eta^2 + \left(\frac{\partial\eta}{\partial P}\right)_T^2 \sigma_P^2 + \left(\frac{\partial\eta}{\partial T}\right)_P^2 \sigma_T^2. \tag{4.5}$$

In practice, we first propagate the estimated errors in pressure and temperature into errors in η, ξ and $\bar{\eta}$ and then evaluate σ using eq (2.11). In calculating the weight factors for the experimental data of Rivkin et al., we adopted the reported error estimates $\sigma_\eta = 0.01\eta$, $\sigma_P = 0.01$ MPa and $\sigma_T = 0.02$ K [R1,R2]. Considering the experimental measurements of Oltermann [O1], it turns out that only two data points actually correspond to densities and temperatures inside the region defined by eq (3.1). Hence a simpler approach was adopted in this case and we attributed to Oltermann's data an estimated uncertainty of 1.6% outside the region specified by eq (3.1) and of 4% inside the region specified by eq (3.1).

It has been recommended that the value $\eta = 1002.0 \times 10^{-6}$ Pa·s be adopted as the standard value for the viscosity of water at 20 °C and atmospheric pressure [K13,M7,N1,S11]. To ensure that our equation remained consistent with this recommendation, the fit was constrained to reproduce this standard value at the reference point.

Two versions of the method of least-squares were employed in the analysis. The initial investigations were made using a modified Gram-Schmidt method with an iterative refinement of the solution following precedures proposed by Björck [B5]. The calculations were performed using double-precision arithmetic (18 decimal digits), but with the inner products accumulated in quadruple precision. Final optimization of the representative equation was achieved using a quadruple precision version of the multiple regression technique proposed by Wagner [W4]. In the latter method, the probability densities for the significance of the coefficients and for the significance of the regression as a whole were set at 0.95. Checks confirmed that the solutions obtained, when fitting the same form of the equation using the two procedures, were in agreement to within some 16 significant figures.

Using eqs (2.3) and (4.3), the objective functions tested in the analysis were selected from subsets of the following form

$$y = \ln\left(\frac{\eta}{\eta_0}\right) - \rho^* \sum_{i=0}^{5}\sum_{j=0}^{6}a_{ij}(T^{*-1}-1)^i(\rho^*-1)^j$$
$$- [\phi\ln q + \phi\ln\xi]\Theta(q\xi-1). \tag{4.6}$$

Variants of this equation with 18 and 19 non-zero coefficients a_{ij} were found that satisfied the statistical criteria and that represented the experimental viscosity data almost equally well. As mentioned in section 2.1, we also tested an objective function (4.3) using for $F(\rho,T)$ equations similar to those used by Jacobsen and Stewart [J1] for the compressibility factor of nitrogen. Again a fair representation of the experimental data was obtained with the exception of the region of anomalous behavior in the liquid phase at low temperatures and in the vapor phase at subcritical temperatures. No variant of either form was found that contained more than 19 terms significant at a 95% or greater confidence level.

Having determined the approximate form for $F(\eta,T)$ and, hence, for the normal viscosity $\bar{\eta}$, there remained the problem of optimizing the parameters q and ϕ for the critical enhancement of the viscosity. In the procedure described above, it was assumed that the boundary of the region where $q\xi \geq 1$ is known before determining q. Our approach was to use an iterative method, whereby an initial estimate of q yielded after fitting an improved estimate which, if necessary, could be refined further.

Among the many equations tested, the best overall representation of the selected data was obtained with an 18 term variant of eq (2.3). This equation was presented in an earlier report [W5]. While this equation fully represented the experimental viscosity data in a pressure range from 0 to 120 MPa, a further analysis revealed that it did not extrapolate satisfactorily to higher pressures. An examination of the other solutions was made and a variant with one additional term was found that yielded an almost equally good fit to the data, while being applicable over a much wider range of pressures.

4.2. Resulting Viscosity Equation

Using the procedure described in the preceding section, we arrived at an equation for the viscosity of the form

$$\ln\eta = \ln\bar{\eta} + \phi\ln(q\xi)\Theta(q\xi-1), \qquad (4.7a)$$

where

$$\ln\bar{\eta} = \ln\eta_0(T) + \rho^*\sum_{i=0}\sum_{j=0}a_{ij}(T^{*-1}-1)^i(\rho^*-1)^j, \qquad (4.7b)$$

with 19 non-zero coefficients a_{ij}. The dilute gas viscosity $\eta_0(T)$ is given by eq (2.6) with the coefficients a_k listed in table A1 of Appendix A. The non-zero coefficients a_{ij} are listed in table A2 of Appendix A. For the constants ϕ and q characterizing the critical viscosity enhancement we adopted the values [B3]

$$\phi = 0.05, \quad q^{-1} = 26.6 \times 10^{-10}\text{m}. \qquad (4.8)$$

The extent of the agreement obtained between this representative equation and the primary data from the sources listed in tables 1 and 2 can be assessed from the information provided in table 4, showing the average, mean, and rms percentage deviations as well as the number n of data points exceeding multiples of the evaluated precision. The deviations are almost all within the evaluated precision listed in tables 1 and 2. Exceptions are: the suspected data points in references [N3,R9,R10,T3] discussed in section 3.1, the viscosity value reported by Malyarov [M5] for water at 20 °C and the data of Mayinger [M6] at temperatures above 500 °C and at pressures in excess of 30 MPa. Similar deviations were found using any of the other equations tested.

Table 4. Summary of results

| Author	Ref.	No. of Pts.	Av. % Dev.	Mean % Dev.	rms % Dev.	n > 1σ	n > 2σ	n > 3σ
Swindells (1952)	[S11]	6	-0.03	0.03	0.05	1	0	0
White (1914)	[W1]	1	-0.02	0.02	0.02	0	0	0
Coe (1944)	[C2]	4	-0.01	0.01	0.02	0	0	0
Hardy (1949)	[H7]	11	0.06	0.07	0.09	0	0	0
Weber (1955)	[W2]	9	-0.00	0.03	0.04	0	0	0
Roscoe (1958)	[R5]	1	-0.01	0.01	0.01	0	0	0
Malyarov (1959)	[M5]	1	0.15	0.15	0.15	1	0	0
Shifrin (1959)	[S12]	114	-0.74	1.21	1.55	11	0	0
Kestin (1960)	[K7]	19	0.43	0.47	0.63	0	0	0
Whitelaw (1960)	[W3]	38	0.49	1.55	1.90	2	0	0
Moszynski (1961)	[M3]	41	-1.09	1.32	1.37	0	0	0
Mayinger (1962)	[M6]	48	1.20	1.74	2.58	13	2	0
Hallet (1963)	[H8]	1	-0.16	0.16	0.16	0	0	0
Kestin (1963)	[K12]	39	0.36	0.50	0.75	8	5	0
Latto (1965)	[L6]	65	0.14	0.80	1.09	2	0	0
Tanaka (1965)	[T3]	48	-0.13	0.96	1.55	2	0	0
Agayev (1968)	[A4]	277	0.07	0.31	0.39	4	0	0
Korosi (1968)	[K9]	11	-0.12	0.16	0.25	2	0	0
Rivkin (1968)	[R9]	108	0.19	0.62	0.91	15	0	0
Rivkin (1968)	[R9]	25	0.02	0.36	0.43	0	0	0
Rivkin (1968)	[R10]	127	0.21	0.65	1.00	16	0	0
Sato (1968)	[S13]	27	-0.22	0.40	0.52	0	0	0
Nagashima (1969)	[N3]	241	0.86	1.89	2.46	16	0	0
Kerimov (1969)	[K11]	190	0.19	0.30	0.38	1	0	0
Korson (1969)	[K10]	15	0.00	0.11	0.14	1	0	0
Eicher (1971)	[E2]	9	-0.02	0.04	0.04	0	0	0
Rivkin (1970)	[R6]	93	-0.08	0.29	0.39	1	0	0
Rivkin (1970)	[R11]	66	-0.29	0.45	0.54	3	0	0
Rivkin (1972)	[R7,R8]	76	-0.09	0.25	0.32	0	0	0
Rivkin (1973)	[R12]	52	-0.31	0.50	0.58	3	0	0
Timrot (1973)	[T2]	46	0.03	0.15	0.21	3	0	0
Sato (1970)	[S14]	88	0.16	0.38	0.50	1	0	0
Kestin (1977)	[K2]	70	0.04	0.08	0.10	1	0	0
Kestin (1978)	[K3]	111	-0.05	0.19	0.25	2	0	0
Nagashima (1974)	[N2]	53	-0.55	1.34	1.58	0	0	0
Rivkin (1975)	[R1,R2]	74	0.18	0.43	0.55	5	0	0
Oltermann (1977)	[O1]	95	0.73	0.83	1.25	8	3	0
Overall results:		2300	0.13	0.69	1.18	121	10	0
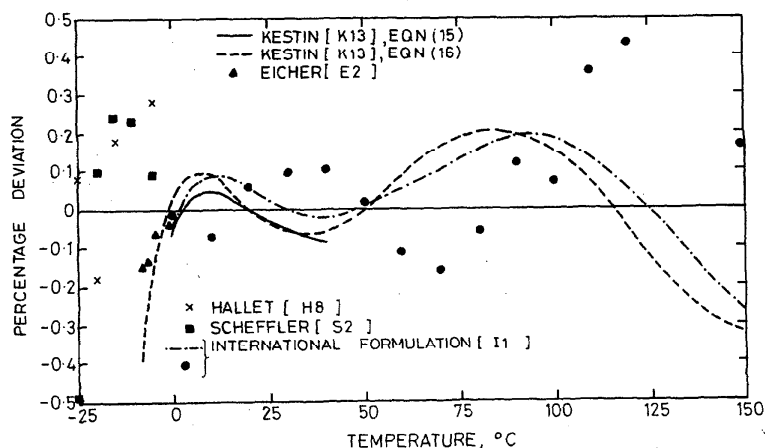
FIGURE 6. Comparison of selected data and interpolating equations for the viscosity of water at atmospheric pressure with the values calculated from the interpolating equation proposed in this paper.

It is of special interest to consider the behavior of the viscosity of water at atmospheric pressure up to 100 °C and at saturation between 100 °C and 150 °C which was recently reassessed by Kestin et al. [K13]. In figure 6 we compare the current international viscosity formulation [I1] and the correlating equation proposed by Kestin et al. [K13] with the values deduced from eq (4.7) for the viscosity of liquid water. We also show a comparison with the grid point values recommended by Scheffler et al. [S2] and with the experimental data obtained by Eicher and Zwolinski [E2] and by Hallet [H8] for the viscosity of water at atmospheric pressure in the metastable region below 0 °C. The values calculated from our equation (4.7) agree with the above sources to within the uncertainty levels of ±2% between −25 °C and −15 °C and ±1% between −15 °C and −5 °C as recommended by Scheffler et al. [S2] and to well within the uncertainty levels of ±0.15% between 0 °C and 40 °C and ±0.4% between 40 °C and 150 °C as recommended by Kestin et al. [K13].

The international formulation for the viscosity of water substance as adopted by IAPS consists of an interpolating equation and a skeleton table of critically evaluated experimental data [I1]. In table 5 we show the difference between the viscosity values calculated from eq (4.7), indicated by η_{WBS}, and the skeleton table values η_{IAPS}, relative to the tolerance of the skeleton table values. It is seen that our equation represents the skeleton table values to within their tolerances at all pressures and temperatures. The number 19 of significant coefficients a_{ij} in eq (4.7b) is substantially smaller than the 30 nonzero coefficients a_{ij} in the international formulation.

The viscosity values calculated from our representative equation (4.7) at integral pressures and temperatures is presented in table C1 of Appendix C. In order to reproduce these tabulated values the density is to be calculated with the 1968 IFC Formulation for Scientific and General Use [I3] and in the critical region with the linear model equation of state defined in Appendix B. Recommendations for calculating the viscosity in conjunction with the more widely used 1967 IFC Formulation for Industrial Use are given in Appendix D.

The representative equation (4.7) may be used at larger pressures than the pressures covered by the data sources in tables 1 and 2. As an illustration we compare in figures 7 and 8 the experimental data obtained by Cappi [C3], Isdale and Spence [I5] and Dudziak and Franck [D3] with the values calculated from our equation (4.7) in the pressure range of 100 MPa to 500 MPa at temperatures between 0 °C and 75 °C and in the pressure range of 100 MPa to 350 MPa at temperatures between 75 °C and 560 °C. In this extended pressure range the densities were calculated from an equation of state proposed by Jüza [J2]. The viscosity measurements of Dudziak and Franck at temperatures between 100 °C and 560 °C were obtained relative to the viscosity at a pressure of 80 MPa. We converted these data into absolute viscosities using for the viscosity at 80 MPa values calculated from an earlier version of our interpolating equation [E3,W5]. At the pressures considered the extrapolated values calculated from (4.7) represent the data from all three sources to within their evaluated precisions of 3%, 2%, and 5%, respectively. However, at pressures beyond 500 MPa, the extrapolated values differ systematically from the data of Cappi [C3] and Isdale and Spence [I5], the deviations becoming more pronounced with increasing pressures. We thus conclude that the domain of validity of the representative equation is given by

$$0\ °C \leq t \leq 150\ °C,\ 0 \leq P \leq 500\ \text{MPa}$$

$$150\ °C \leq t \leq 600\ °C,\ 0 \leq P \leq 350\ \text{MPa} \qquad (4.9)$$

$$600\ °C \leq t \leq 900\ °C,\ 0 \leq P \leq 300\ \text{MPa}.$$

The behavior of the viscosity as a function of density, calculated from the equation at temperatures between 200 °C and 800 °C is shown in figure 9. The behavior of the viscosity of liquid water as a function of density at temperatures between 0 °C and 150 °C is shown in figure 10. The density ranges in both figures correspond to the pressure ranges proposed above. The viscosity of water at low temperatures first decreases and

TABLE 5.   Comparison of the Calculated Viscosity Values η_{WBS} with the International Skeleton Table Values η_{IAPS}.   Quantity Listed: $(\eta_{WBS} - \eta_{IAPS})$/Tolerance.

P, MPa	0 °C	25 °C	50 °C	75 °C	100 °C	150 °C	200 °C	250 °C	300 °C	350 °C	375 °C
0.1	.10	-.05	-.01	.13	-.60	-.37	-.26	-.22	-.17	-.14	-.09
0.5	.10	-.10	.08	-.03	.03	.05	.00	-.05	-.02	.05	-.08
1.0	.09	-.10	.08	-.05	.01	.07	-.09	-.13	-.07	.05	-.07
2.5	.05	-.10	.07	-.02	.01	.06	-.01	-.25	-.12	.01	-.12
5.0	.05	-.11	.04	-.03	-.03	.09	.01	-.02	-.21	-.06	-.16
7.5	.06	-.11	.02	-.02	-.03	.06	-.04	-.02	-.23	-.07	-.18
10.0	.08	-.11	.03	-.00	-.11	.08	.05	.06	-.36	-.02	-.13
12.5	.10	-.13	.04	.02	-.08	.10	.06	.04	-.31	.04	-.13
15.0	.13	-.13	.05	.03	-.08	.06	.07	.10	-.35	.20	-.05
17.5	.17	-.15	.05	.02	-.09	.08	.07	.05	-.36	-.17	.03
20.0	.24	-.16	.07	.04	-.10	.04	.07	.09	-.39	-.11	.09
22.5	.30	-.16	.07	.06	-.07	.05	.07	.03	-.38	-.14	-.17
25.0	.31	-.16	.09	.05	-.08	.07	.06	.05	-.36	-.14	-.41
27.5	.32	-.16	.10	.07	-.08	.08	.05	.06	-.33	-.14	-.33
30.0	.35	-.17	.10	.06	-.06	.03	.04	.06	-.35	-.14	-.26
35.0	.43	-.17	.12	.10	-.07	.05	.07	.05	-.34	-.15	-.20
40.0	.42	-.17	.14	.12	-.06	.06	.02	.00	-.26	-.10	-.07
45.0	.44	-.18	.16	.13	-.04	.06	.03	.02	-.25	-.08	-.06
50.0	.43	-.17	.19	.15	-.06	.06	-.04	-.06	-.22	-.09	-.08
55.0	.40	-.17	.21	.15	-.04	.05	-.04	-.08	-.22	-.06	-.08
60.0	.34	-.18	.24	.17	-.02	.04	-.06	-.03	-.19	-.04	-.07
65.0	.30	-.17	.25	.19	-.05	.03	-.02	-.09	-.22	-.04	-.06
70.0	.23	-.17	.27	.22	-.03	.01	-.05	-.16	-.23	-.04	-.02
75.0	.20	-.16	.31	.22	-.02	-.01	-.09	-.16	-.19	-.05	-.01
80.0	.18	-.16	.32	.25	-.04	-.03	-.14	-.17	-.18	-.01	.01
85.0	.14	-.14	.34	.26	.01	-.06	-.13	-.18	-.18	-.03	.04
90.0	.12	-.13	.36	.26	.02	-.09	-.12	-.22	-.18	-.02	.09
95.0	.13	-.13	.38	.27	.00	-.12	-.17	-.26	-.20	.01	.07
100.0	.10	-.11	.41	.29	.01	-.10	-.18	-.32	-.22	-.01	.08

TABLE 5 (Continued)

| P, MPa \ t	400 °C	425 °C	450 °C	475 °C	500 °C	550 °C	600 °C	650 °C	700 °C	750 °C	800 °C
0.1	-.04	-.01	.04	.07	.06	.08	.07	-.00	-.04	-.10	-.11
0.5	-.01	-.02	-.03	-.04	-.07	-.06	-.15	-.09	-.12	-.00	.08
1.0	-.02	-.04	-.03	-.05	-.08	-.06	-.15	-.17	-.19	-.01	.01
2.5	-.04	-.06	-.04	-.04	-.06	-.05	-.14	-.11	-.13	-.01	.07
5.0	-.11	-.11	-.10	-.07	-.06	-.06	.04	-.08	-.11	-.01	.00
7.5	-.13	-.13	-.12	-.10	-.04	-.06	.07	.04	-.08	.04	.03
10.0	-.07	-.06	-.03	.02	-.05	-.06	.07	.00	-.12	-.00	-.02
12.5	-.07	-.05	-.02	.03	-.01	.01	.06	.07	-.15	-.03	-.05
15.0	-.10	-.06	-.01	.06	-.01	-.04	.06	-.03	-.16	-.05	-.01
17.5	-.02	-.02	.01	.12	.06	-.07	-.02	-.02	-.16	-.06	-.01
20.0	-.00	.04	.02	.11	.04	-.08	.02	.00	-.15	-.06	-.01
22.5	.04	.03	.01	.12	.05	-.18	.02	-.04	-.11	.13	-.08
25.0	.11	.04	-.06	.08	.09	-.19	-.01	-.07	-.15	-.01	.08
27.5	-.02	.02	-.09	.05	.07	-.18	-.11	-.08	-.10	.02	.03
30.0	-.09	.05	-.18	.07	.05	-.24	-.08	-.17	-.12	-.02	-.02
35.0	-.59	-.17	-.22	-.12	.09	-.23	-.12	-.18	-.28	-.06	-.02
40.0	-.69	-.73	-.21	-.23	-.07	-.10	-.12	-.28	-.22	-.13	-.14
45.0	-.61	-.63	.17	-.40	-.03	-.34	-.55	-.57	-.29	-.49	-.23
50.0	-.13	-.42	-.02	-.12	-.01	-.33	-.55	-.30	-.43	-.42	-.42
55.0	-.25	-.45	-.26	-.21	-.13	-.34	-.73	-.42	-.02	-.55	-.38
60.0	-.26	-.28	-.44	-.25	-.39	-.35	-.81	-.43	.13	-.12	-.40
65.0	-.33	-.09	-.49	-.17	-.38	-.20	.11	-.32	.07	.02	-.05
70.0	-.30	-.22	-.49	-.38	-.34	-.32	.12	.29	.67	.04	.04
75.0	-.28	-.12	-.37	-.35	-.23	-.28	.10	.31	.48	.22	.07
80.0	-.23	-.14	-.05	-.35	-.26	-.41	.62	.15	.50	.48	.23
85.0	-.26	-.22	-.08	-.25	-.21	-.30	.59	.65	.54	.54	.48
90.0	-.22	-.17	-.26	-.21	-.19	-.19	-.01	.61	.22	.57	.53
95.0	-.07	-.15	-.23	-.23	-.15	-.20	.09	.16	.67	.57	.65
100.0	-.13	-.09	.08	.21	-.03	-.09	-.22	.22	.72	.32	.71
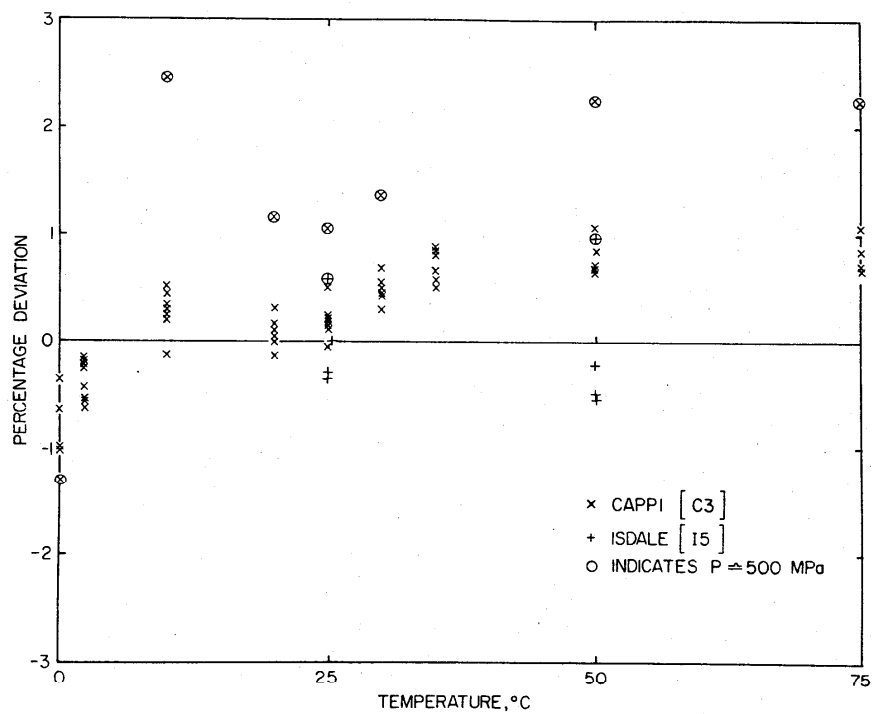
FIGURE 7. Comparison of the experimental data reported by Cappi [C3] and Isdale and Spence [I5] with the values extrapolated from our equation for water in the range 125 MPa ≤ P ≤ 500 MPa at 0 °C ≤ t ≤ 75 °C.
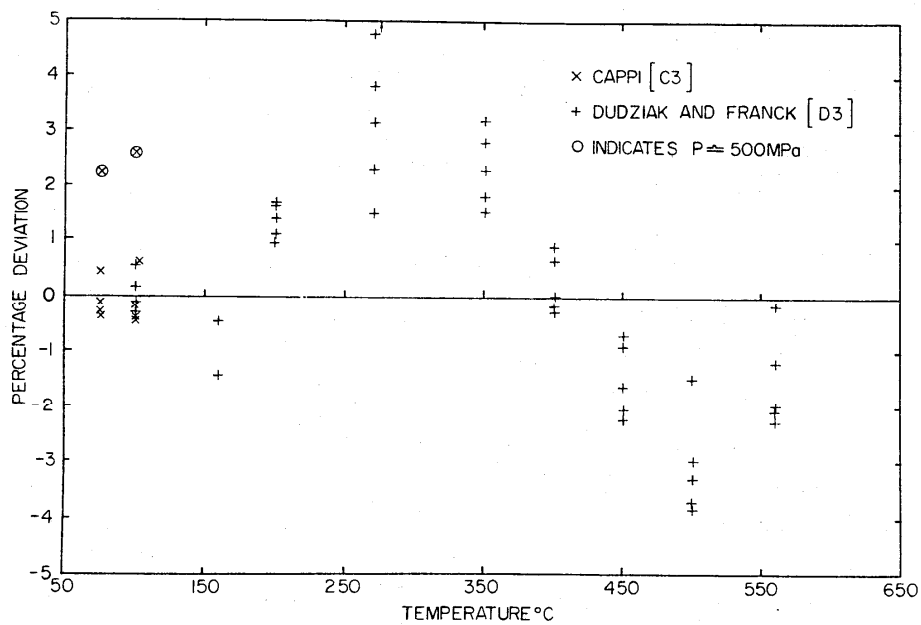
FIGURE 8. Comparison of the experimental data reported by Cappi [C3] and by Dudziak and Franck [D3] with the values extrapolated from our equation for water and steam in the range 125 MPa ≤ P ≤ 500 MPa at 50 °C ≤ t ≤ 650 °C.
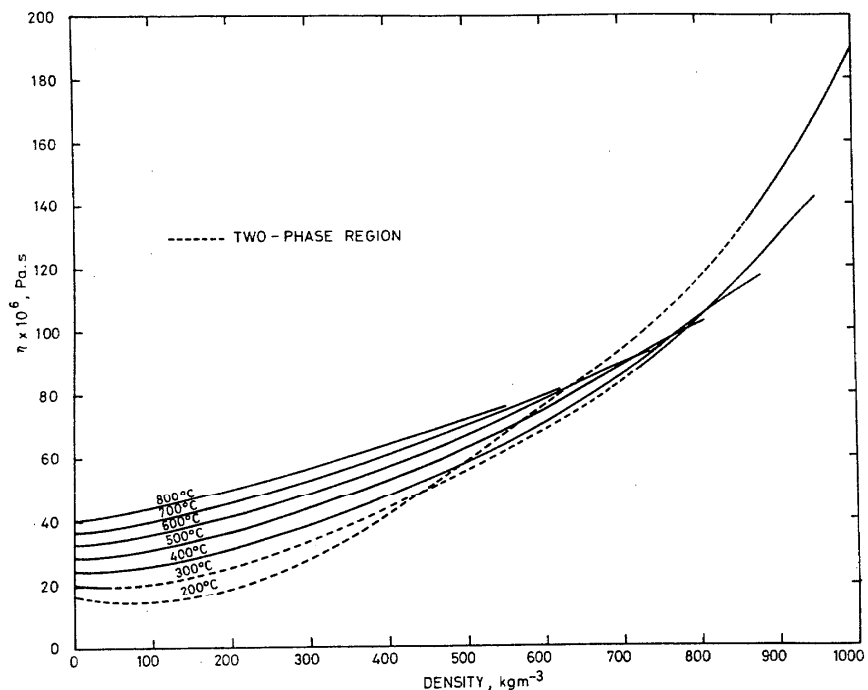
FIGURE 9.   Viscosity of water and steam as a function of density at selected temperatures between 200 °C and 800 °C.
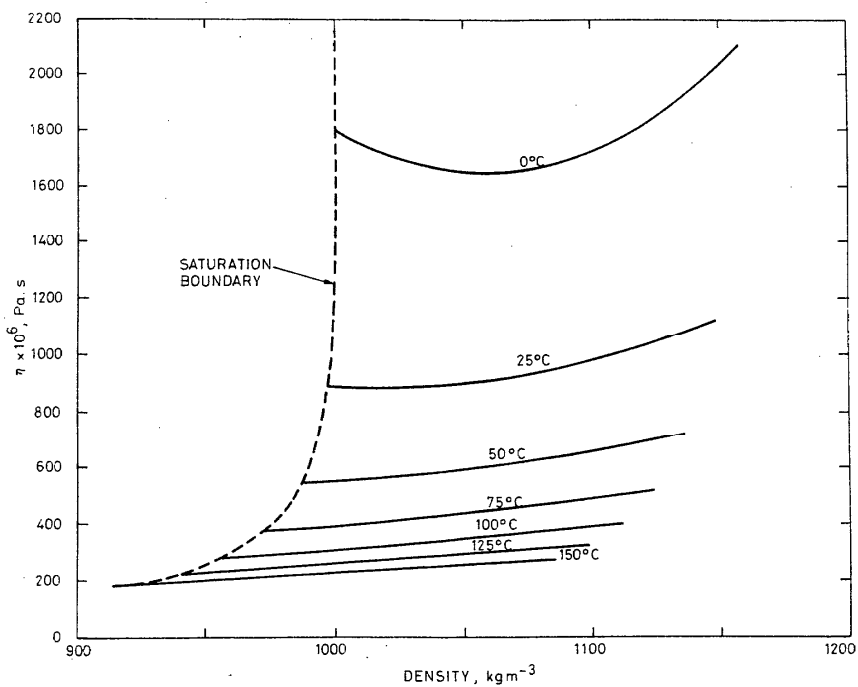

FIGURE 10. Viscosity of water as a function of density at selected temperatures between 0 °C and 150 °C.
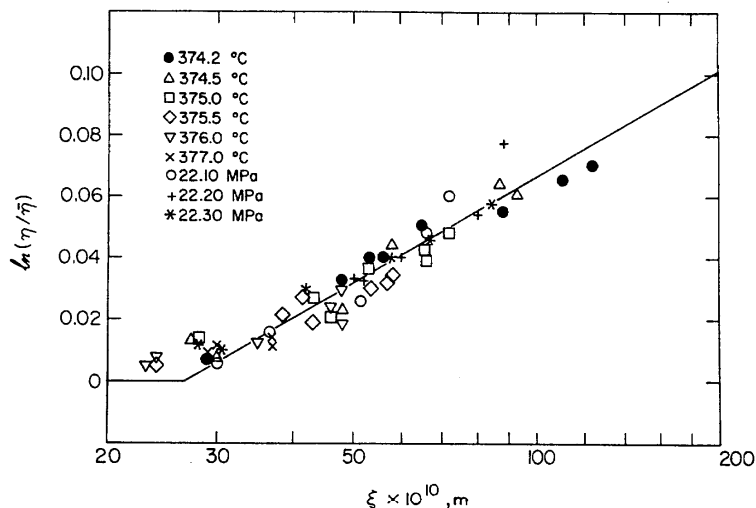
FIGURE 11. The logarithm of $\eta/\bar{\eta}$ as a function of the logarithm of ξ.

then increases with density. This anomalous behavior is well represented by our equation. The minima of the isotherms for the liquid state at low temperatures correspond closely to those found by Kestin et al. [K3] and by Cappi [C3]. Equation (4.7) implies that the initial slope of the isotherms in the liquid state changes from negative to positive values at $t = 34.5$ °C, which is 1 °C higher than reported in references [C3,K3].

4.3. Viscosity in the Critical Region

The enhancement of the viscosity in the vicinity of the critical point is represented by

$$\ln\left(\frac{\eta}{\bar{\eta}}\right) = \phi \ln(q\xi)\Theta(q\xi - 1). \tag{4.10}$$

Using the experimental data of Rivkin et al. as re-evaluated in section 2.3, we have plotted in figure 11 $\ln(\eta/\bar{\eta})$ as a function of ξ with $\bar{\eta}$ calculated from eq (4.7b). On a double logarithmic scale the data do approach a straight line. The slope of this line yields the exponent $\phi = 0.05$ and the intercept with the ξ-axis at $\ln(\eta/\bar{\eta}) = 0$ yields the constant $q^{-1} = 26.6 \times 10^{-10}$m. For a more detailed discussion of the range of the parameters ϕ and q^{-1} compatible with the experimental data the reader is referred to a separate publication [B3].

Equation (4.10) yields only contributions when $q\xi > 1$. The region of pressures and temperatures and the region of densities and temperatures where this condition is satisfied is shown in figures 12 and 13, respectively. This region is inside a range of densities and temperatures bounded by

$$372.15 \text{ °C} < t < 379.40 \text{ °C}, \ 240 \text{ kg/m}^3 < \rho < 410 \text{ kg/m}^3. \tag{4.11}$$

The temperature range in eq (4.11) is smaller than the temperature range corresponding to our original estimate quoted in eq (3.1). The reason is that even for $q\xi < 1$ a small trace of the anomaly could be seen as illustrated in figure 11. How-

ever, the effect is so small that it can be neglected compared to the absolute accuracy of the experimental data.

The behavior of the viscosity as a function of density was shown earlier in figures 4 and 5; the curves in these figures represent the values calculated from the interpolating equation (4.7). The percentage deviations of the experimental data in the critical region from the values calculated by the equation are shown in figure 14. The equation reproduces the experimental data with five of the 50 critical region points exceeding the evaluated precision.
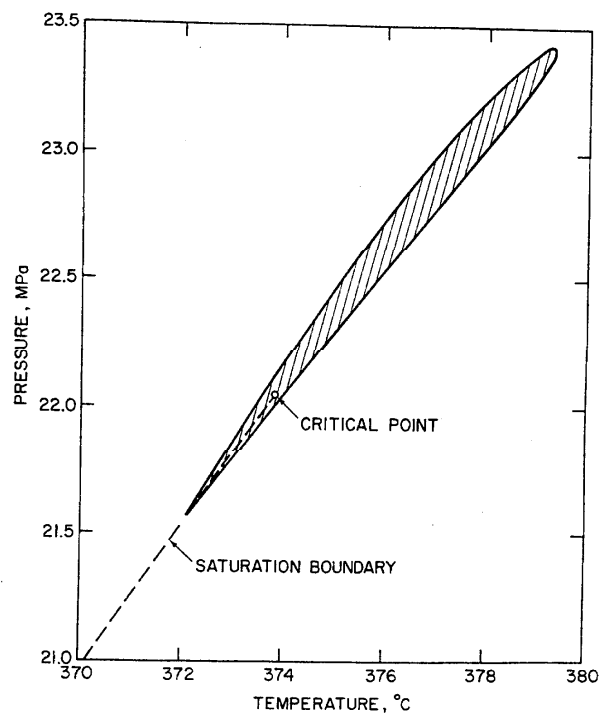
FIGURE 12. Region in the pressure-temperature plane with $q\xi \geq 1$.
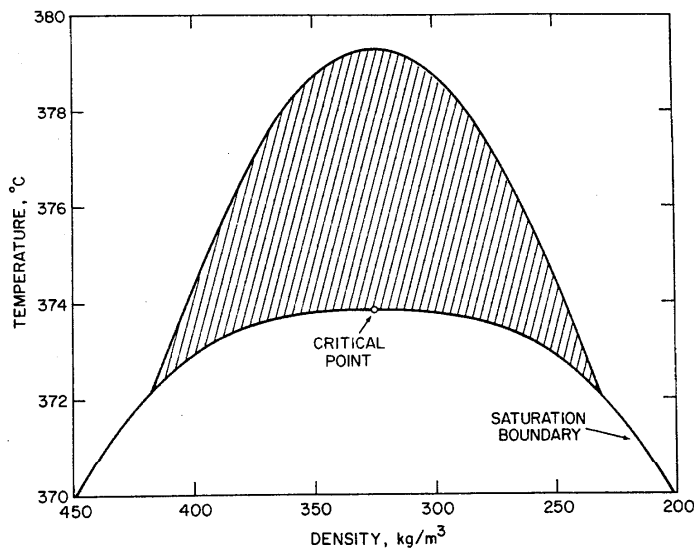
FIGURE 13. Region in the density-temperature plane with $q\xi \geqslant 1$.

4.4. Viscosity at the Saturation Boundary

Values for the viscosity in the vapor and liquid phase at saturation, calculated from our equation (4.7), are presented in table 6. It is often convenient to have a simple equation to calculate the viscosity at the phase boundary. Introducing a variable τ defined as[2]

$$\tau = \frac{T_c}{T} - 1, \qquad (4.12)$$

the following equations give a good representation of the tabulated values of the viscosity in the liquid and vapor phase at the saturation boundary between the triple point and the critical point. Indicating the viscosity of the liquid phase at saturation by η_ℓ and the viscosity of the vapor phase by η_v, we recommend

$$\ln\left(\frac{\eta_\ell}{\eta_v}\right) = c_1\tau^{1/3} + c_2\tau + c_3\tau^2 + c_4\tau^{7/3}$$
$$+ c_5\tau^{10} + 0.0319\ \Theta(\tau_0-\tau)\ln\left(\frac{\tau_0}{\tau}\right), \qquad (4.13)$$

[2] For recommended values of T_c see table B1 in Appendix B.

FIGURE 14. Comparison of the experimental viscosity in the critical region (see table 3) with the values calculated from the interpolating equation proposed in this paper.

Table 6. Viscosity of water substance at the saturation boundary.

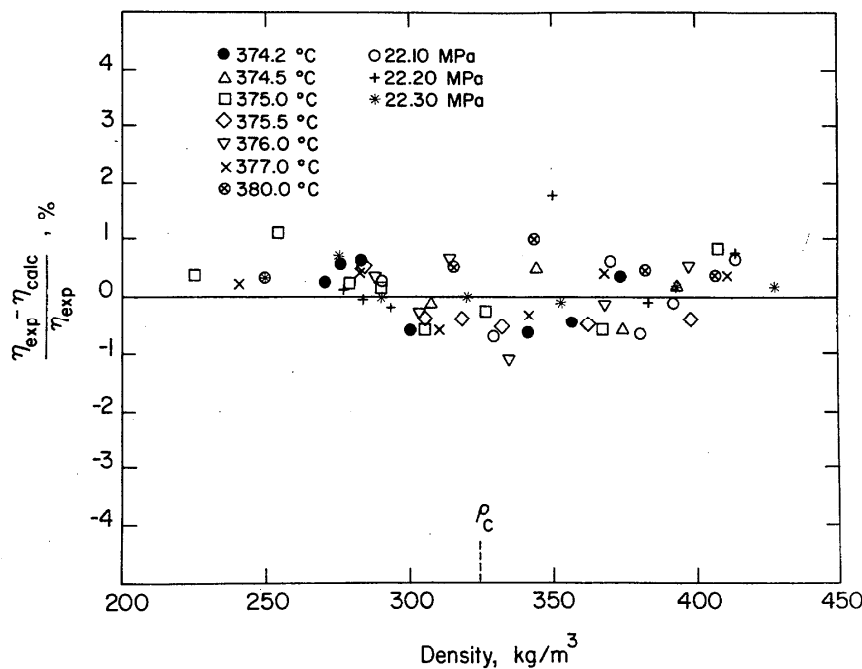| t, °C	$\eta_\ell \times 10^6$, Pa·s	$\eta_v \times 10^6$, Pa·s
0.01	1793.	9.22
10.	1307.	9.46
20.	1002.	9.73
30.	797.7	10.01
40.	653.2	10.31
50.	547.0	10.62
60.	466.5	10.93
70.	404.0	11.26
80.	354.4	11.59
90.	314.4	11.93
100.	281.7	12.27
110.	254.6	12.61
120.	232.0	12.96
130.	212.8	13.30
140.	196.4	13.65
150.	182.4	13.99
160.	170.2	14.34
170.	159.5	14.68
180.	150.1	15.02
190.	141.8	15.37
200.	134.3	15.71
210.	127.6	16.06
220.	121.6	16.41
230.	116.0	16.76
240.	110.9	17.12
250.	106.2	17.49
260.	101.8	17.87
270.	97.56	18.27
280.	93.56	18.70
290.	89.69	19.15
300.	85.91	19.65
310.	82.16	20.21
320.	78.38	20.85
330.	74.49	21.61
340.	70.36	22.55
350.	65.85	23.81
360.	60.41	25.70
370.	51.73	29.67
371.	50.38	30.44
372.	48.64	31.44
373.	47.67	34.24
373.5	47.02	36.90
373.846	∞	∞

$$\ln\left(\frac{\eta_v}{\eta_c}\right) = \sum_{i=1}^{5} d_i \tau^{i/3} + 0.0319\ \Theta(\tau_0 - \tau)\ln\left(\frac{\tau_0}{\tau}\right), \quad (4.14)$$

with

$$\overline{\eta}_c = 39.639 \times 10^{-6}\ \text{Pa·s}, \quad \tau_0 = 0.0027, \quad (4.15)$$

and where $\Theta(\tau_0 - \tau)$ is again the Heaviside step function introduced in section 4.1 such that $\Theta = 1$ when $0 \le \tau \le \tau_0$ and $\Theta = 0$ otherwise. The numerical values of the coefficients c_i and d_i are listed in table 7.

4.5. First Density Correction to the Viscosity in the Gas Phase

The initial density dependence of the viscosity in the gas phase may be represented by

$$\eta = \eta_0 + \eta_1 \rho, \quad (4.16)$$

where the coefficient η_1 of the first density correction is defined by

$$\eta_1 = \lim_{\rho \to 0} \left(\frac{\partial \eta}{\partial \rho}\right)_T. \quad (4.17)$$

The modern kinetic theory of gases predicts that the first derivative defined by eq (4.17) exists, but that the higher order derivatives with respect to the density diverge in the zero density limit. For a review of the issues involved, the reader is referred to a paper by Kan et al. [K14]. The coefficient η_1 has been investigated as a function of temperature for a number of fluids [H2,H9].

It follows from our equation (4.7) that the coefficient η_1 for water vapor and steam can be represented by

$$\eta_1 = \frac{\eta_0(T)}{\rho_r} \sum_{i=0}^{5} b_i (T^{*-1} - 1)^i, \quad (4.18)$$

where ρ_r is the reference density introduced in eq (2.5). The numerical values of the coefficients b_i are presented in table 8. In table 9 we show the values for η_1 between 0 °C and 1000 °C calculated from eq (4.18). For comparison we also show the values for η_1 calculated from the international viscosity equation [I1]. Equation (4.18) reproduces the coefficient η_1 within the tolerance limits proposed by Aleksandrov et al. [A2].

Table 7. Coefficients in the equations for the viscosity at the saturation boundary.

Coefficients c_i in equation (4.13)	Coefficients d_i in equation (4.14)
$c_1 = 1.5064$	$d_1 = -1.4122$
$c_2 = -0.11890$	$d_2 = -2.1384$
$c_3 = 1.7604$	$d_3 = 8.2674$
$c_4 = -0.53367$	$d_4 = -9.6091$
$c_5 = 4.9643 \times 10^{-3}$	$d_5 = 3.5757$

Table 8. Coefficients in equation (4.18) for n_1

b_o	=	$-0.203\ 356$
b_1	=	$-1.926\ 729$
b_2	=	$-2.738\ 265$
b_3	=	$-0.827\ 952$
b_4	=	$-0.778\ 257$
b_5	=	$+0.188\ 545$

5. Discussion

Using current theoretical concepts for the behavior of the viscosity near the critical point, combined with improved methods for the statistical analysis of data, we have remedied some shortcomings of the current international equation for the viscosity of water substance. The new equation was developed on the basis of the primary data set prepared by IAPS in 1974 [S15], critical region data of Rivkin et al. [R1,R2] and Oltermann [O1] and new experimental data reported by Kestin et al [K2,K3] and Nagashima [N2]. The equation consists basically of two parts: an expression for the normal viscosity similar to that adopted by IAPS [I1] but with a substantially smaller number of terms and a single multiplicative factor which relates the anomalous behavior of the viscosity in the critical region to the equation of state.

The resulting equation reproduces the international skeleton tables for the viscosity to well within their tolerances, accounts for the enhancement in viscosity near the critical point in a theoretically consistent way, and is in good agreement with the primary data set selected by IAPS as well as with new data sets not previously considered.

Further information for the potential users of the proposed equation is presented in Appendices A through D.

Table 9. The coefficient n_1 of the first density correction to the viscosity of water vapor and steam.

t °C	$n_1 \times 10^8$ Pa·s·m³/kg from eq (4.18)	$n_1 \times 10^8$ Pa·s·m³/kg from International Formulation [I1]
0	-34.6	-34.0
50	-21.2	-20.1
100	-14.0	-12.8
150	-9.71	-8.61
200	-6.82	-5.93
250	-4.75	-4.11
300	-3.20	-2.78
350	-1.99	-1.75
400	-1.03	-0.93
450	-0.25	-0.25
500	0.37	0.33
550	0.86	0.82
600	1.26	1.25
650	1.56	1.61
700	1.79	1.91
750	1.94	2.17
800	2.04	2.38
850	2.08	2.55
900	2.08	2.69
950	2.03	2.78
1000	1.94	2.85

Acknowledgments

Appendix A. Representative Equations for the Viscosity of Water and Steam

Dimensionless Variables[3]

$$T^* = \frac{T}{T_r}, \quad X = T^{*-1} - 1; \quad \rho^* = \frac{\rho}{\rho_r}, \quad Y = \rho^* - 1 \quad (A.1)$$

with

$$T_r = 647.27 \text{ K}, \quad \rho_r = 317.763 \text{ kg/m}^3. \quad (A.2)$$

T_r and ρ_r are reference values close to but not equal to the critical temperature and density.

Viscosity in the Low Density Limit

The viscosity in the low density limit $\eta_0 (T) \equiv \lim_{\rho \to 0} \eta(\rho,T)$ is represented by

$$\frac{\eta_0(T)}{10^{-6}\text{Pa·s}} = \sqrt{T^*} \left[\sum_{k=0}^{3} \left(\frac{a_k}{T^{*k}} \right) \right]^{-1}. \quad (A.3)$$

The coefficients a_k are given in table A1 and are the same as in the current international viscosity equation recommended by IAPS [I1].

Viscosity Outside the Critical Region

At temperatures and densities outside a region bounded by

$$645.30 \text{ K} < T < 652.55 \text{ K} ,$$
$$240 \text{ kg/m}^3 < \rho < 410 \text{ kg/m}^3, \quad (A.4)$$

the viscosity is represented by

$$\eta = \bar{\eta}(\rho,T) = \eta_0(T)\exp\left[\rho^* \left\{ \sum_{i=0}^{5} \sum_{j=0}^{6} a_{ij} X^i Y^j \right\} \right], \quad (A.5)$$

with $\eta_0(T)$ given by eq (A.3) and with 19 terms in the argument of the exponential function. The 19 non-zero coefficients a_{ij} are given in table A2.

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[3] Here we assume that the temperatures are expressed in terms of IPTS–48 for reasons discussed in section 2.4. For use of the equation with temperatures in terms of IPTS–68, see Appendix D.

Table A1. Coefficients a_k in equation (A.3) for $\eta_0(T)$

a_0	=	0.018 158 3
a_1	=	0.017 762 4
a_2	=	0.010 528 7
a_3	=	-0.003 674 4

Table A2. Coefficients a_{ij} in equation (A.5) for $\bar{\eta}(\rho,T)$

i	j	a_{ij}
0	0	a_{00} = 0.513 204 7
1	0	a_{10} = 0.320 565 6
4	0	a_{40} = -0.778 256 7
5	0	a_{50} = 0.188 544 7
0	1	a_{01} = 0.215 177 8
1	1	a_{11} = 0.731 788 3
2	1	a_{21} = 1.241 044
3	1	a_{31} = 1.476 783
0	2	a_{02} = -0.281 810 7
1	2	a_{12} = -1.070 786
2	2	a_{22} = -1.263 184
0	3	a_{03} = 0.177 806 4
1	3	a_{13} = 0.460 504 0
2	3	a_{23} = 0.234 037 9
3	3	a_{33} = -0.492 417 9
0	4	a_{04} = -0.041 766 10
3	4	a_{34} = 0.160 043 5
1	5	a_{15} = -0.015 783 86
3	6	a_{36} = -0.003 629 481

Viscosity in the Critical Region

At temperatures and densities inside a region defined by eq (A.4), the viscosity is represented by

$$\eta = \bar{\eta}(q\xi)^{0.05}, \text{ for } \xi > q^{-1} = 26.6 \times 10^{-10}\text{m}$$
$$\eta = \bar{\eta}, \qquad \text{for } \xi \leq q^{-1} = 26.6 \times 10^{-10}\text{m} \quad (A.6)$$

with $\bar{\eta} = \bar{\eta}(\rho,T)$ given by eq (A.5). A procedure for calculating the correlation length ξ as a function of density and temperature is presented in Appendix B. The region $\xi > q^{-1}$ was shown in figures 12 and 13 and can also be deduced from the calculated correlation length data in table B3.

Range and Accuracy of the Equation

The domain of validity of the viscosity equation is given by

$$0 \text{ °C} \leq t \leq 150 \text{ °C}, \; 0 \leq P \leq 500 \text{ MPa}$$
$$150 \text{ °C} \leq t \leq 600 \text{ °C}, \; 0 \leq P \leq 350 \text{ MPa} \quad (A.7)$$
$$600 \text{ °C} \leq t \leq 900 \text{ °C}, \; 0 \leq P \leq 300 \text{ MPa}.$$
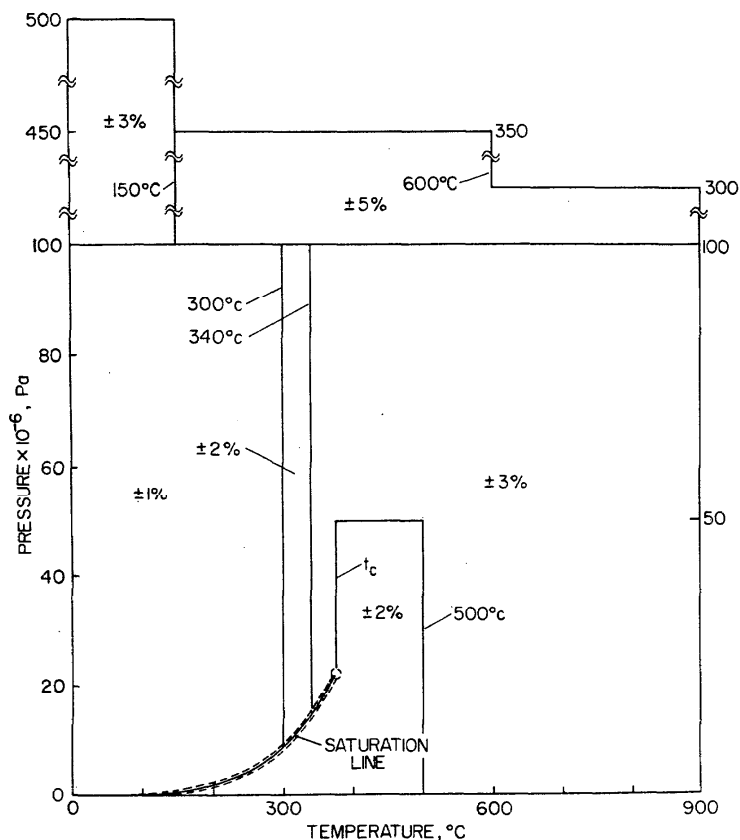
FIGURE 15. Estimated accuracy of proposed interpolating equation for the viscosity of water substance.

The representative equation yields the viscosity as a function of density and temperature. To calculate the viscosity as a function of pressure and temperature, the density should preferably be calculated using the 1968 IFC Formulation for Scientific and General Use [13] at pressures up to 100 MPa, except for a region around the critical point where use of a scaled equation of state is recommended as specified in Appendix B.

The estimated accuracy of the representative equation at various ranges of pressures and temperatures is indicated in

figure 15. In addition, we estimate the accuracy of the viscosity values calculated for the saturated liquid as $\pm 0.2\%$ at 0 °C$<t<40$ °C, $\pm 0.4\%$ at 40 °C $<t<150$ °C, $\pm 1\%$ at 150 °C$<t<300$ °C, $\pm 3\%$ at 300 °C$<t<t_c$, and for the saturated vapor as $\pm 4\%$ at 0 °C$<t<100$ °C, $\pm 2\%$ at 100 °C$<t<300$ °C, $\pm 3\%$ at 300 °C$<t<t_c$. These estimates are based on the tolerances proposed by Scheffler et al. [S2] and Kestin et al. [K13], but modified by an improved estimated accuracy near the critical temperature.

Appendix B. Equation of State and Correlation Length in the Critical Region

Equation of State in the Critical Region

It is recommended that in the critical region an equation of state be used that incorporates accurately the nonanalytic thermodynamic behavior of steam at the critical point. In this paper we have adopted a scaled parametric equation of state formulated by Murphy et al. [M4]. This equation of state is based on the so-called restricted linear model [L3,S8] and is defined by

$$\Delta \bar{T} = (T - T_c)/T_c = r(1 - b^2\theta^2), \qquad \text{(B.1a)}$$

$$\Delta \bar{\rho} = (\rho - \rho_c)/\rho_c = r^\beta k\theta, \qquad \text{(B.1b)}$$

$$\Delta \bar{P} = (P - P_c)/P_c = ar^{\beta\delta}\,\theta(1\text{-}\theta^2) + akr^{\beta(\delta+1)}\Bigg\{$$
$$\theta^2(1 - \theta^2) - f_0 - f_2\theta^2 - f_4\theta^4 \Bigg\} + \sum_{i=1}^{3} \tilde{P}_i\,(\Delta\bar{T})^i \quad \text{(B.1c)}$$

with

$$f_0 = -\frac{\beta(\delta-3) - b^2\alpha\gamma}{2b^4(2-\alpha)(1-\alpha)\alpha},$$

$$f_2 = +\frac{\beta(\delta-3) - b^2\alpha(1-2\beta)}{2b^2(1-\alpha)\alpha}, \qquad \text{(B.2)}$$

$$f_4 = -\frac{1-2\beta}{2\alpha}.$$

The equation of state (B.1) relates $\Delta\bar{P}$, $\Delta\bar{\rho}$ and $\Delta\bar{T}$ via two parametric variables r and θ defined by eqs (B.1a) and (B.1b). At the critical isochore $\theta = 0$, at the critical isotherm $\theta = \pm 1/b$ and at the two branches of the saturation boundary $\theta = \pm 1$. A computer program for calculating the parametric variables r and θ at given values of ρ and T can be found in references [M8, S17].

The values of the equation of state parameters, deduced by Murphy et al. [M4] from the equation of state data obtained

Table B1.   Parameters for critical region equation of state

Critical parameters	$T_c^{(48)}$	=	646.996 K
	$T_c^{(68)}$	=	647.073 K
	ρ_c	=	324.383 kg/m^3
	P_c	=	22.0477 MPa
Critical exponents	α	=	0.08712
	β	=	0.3505
	γ	=	1.2119
	δ	=	4.45757
Linear model parameters	a	=	24.0999
	k	=	1.6837
	b^2	=	1.40991
Coefficients $\tilde{P}_i$	$\tilde{P}_1$	=	7.87425
	$\tilde{P}_2$	=	−25.8448
	$\tilde{P}_3$	=	3.96522

Note: $\beta(\delta+1) = 2-\alpha$, $\beta(\delta-1) = \gamma$, $b^2 = (\delta-3)/(\delta-1)(1-2\beta)$

Table B2. Pressure at selected densities and temperatures near the critical point as calculated from the restricted linear model equation of state (P in MPa).

$\rho-\rho_c$, kg/m³ \ T-T$_c$,K	-1	-0.5	0	+0.5	+1	+5	+10	+20	+30
-80	21.779	21.892	22.005	22.118	22.230	23.117	24.212	26.378	28.521
-60	21.782	21.914	22.035	22.155	22.274	23.218	26.386	26.705	29.011
-40	21.782	21.914	22.046	22.172	22.297	23.292	24.529	27.002	29.474
-20	21.782	21.914	22.048	22.178	22.309	23.351	24.657	27.283	29.926
0	21.782	21.914	22.048	22.182	22.317	23.404	24.779	27.565	30.386
+20	21.782	21.914	22.048	22.186	22.326	23.461	24.910	27.865	30.875
+40	21.782	21.914	22.050	22.195	22.340	23.532	25.063	28.204	31.419
+60	21.782	21.914	22.065	22.218	22.372	23.634	25.260	28.610	32.051
+80	21.788	21.949	22.112	22.276	22.441	23.791	25.529	29.118	32.812

by Rivkin and coworkers [R3,R4] are given in table B1. These parameters satisfy the exponent relations

$$2-\alpha = \beta(\delta+1), \quad \gamma = \beta(\delta-1), \tag{B.3}$$

and the condition

$$b^2 = \frac{\delta-3}{(\delta-1)(1-2\beta)}. \tag{B.4}$$

This equation of state has been generalized by Levelt Sengers to a fundamental equation for the critical region of steam [L5]. We have used this equation of state in a range of temperatures and densities determined by (c.f., eq (2.12))

645.3 K < T < 680 K,
$$240 \text{ kg/m}^3 < \rho < 410 \text{ kg/m}^3. \tag{B.5}$$

Values of the pressure thus calculated at selected temperatures and densities in this region are presented in table B2.

Correlation Length

The correlation length ξ is calculated by relating it to the symmetrical compressibility $\chi_T = \rho(\partial\rho/\partial P)_T$, as discussed in a previous publication [S8]

$$\xi = \xi_0(\tilde{\chi}_T/\Gamma)^{\nu/\gamma}, \tag{B.6}$$

where ξ_0 and Γ are the amplitudes of the critical power laws for ξ and $\tilde{\chi}_T$, respectively, above T_c at $\rho = \rho_c$. In terms of the parametric equations, introduced above, we find $\Gamma = k/a$ and $\tilde{\chi}_T = \Gamma r^{-\gamma}[1 + \theta^2(2\beta\delta-3)/(1-2\beta)]^{-1}$, so that

$$\xi = \xi_0 r^{-\nu}\left(1 + \frac{2\beta\delta-3}{1-2\beta}\theta^2\right)^{-\nu/\gamma} \tag{B.7}$$

with

$$\xi_0 = 1.31 \times 10^{-10}\text{m}, \quad \nu = \beta(\delta+1)/3 = 0.6376. \tag{B.8}$$

Values of the correlation length thus calculated at selected temperatures and densities in the critical region are presented in table B3. It follows from eqs (B.7) and (B.8) that at the critical isochore above T_c, $\xi = \xi_0(\Delta\tilde{T})^{-\nu}$ and at the saturation boundary below T_c, $\xi = \xi_0'|\Delta\tilde{T}|^{-\nu}$ with $\xi_0' = 0.62 \times 10^{-10}\text{m}$.

Alternative Method for Calculating ξ

Equation (B.7) for the correlation length is obtained by expressing the compressibility $\tilde{\chi}_T$ in terms of the restricted linear model. An alternative method for calculating the correlation length is obtained by expressing the compressibility in terms of the so-called NBS or MLSG equation [L3,S8]. Introducing a scaling variable x as defined as

$$x = \Delta\tilde{T}/|\Delta\tilde{\rho}|^{1/\beta}. \tag{B.9}$$

Table B1   Calculated correlation length at selected densities and temperatures near the critical point (ξ in 10^{-10}m)

$\rho - \rho_c$, kg/m^3 \ $T - T_c$, K	-1	-0.5	0	+0.5	+1	+5	+10	+20	+30
-80	34.7	30.4	27.5	25.4	23.7	16.5	12.8	9.4	7.7
-60	-	-	46.5	39.2	34.6	20.5	14.9	10.4	8.3
-40	-	-	97.2	64.4	51.3	24.6	16.8	11.3	8.8
-20	-	-	343.0	103.0	71.1	27.8	18.2	11.8	9.2
0	-	-	∞	126.0	81.2	29.1	18.7	12.0	9.3
+20	-	-	343.0	103.0	71.1	27.8	18.2	11.8	9.2
+40	-	-	97.2	64.4	51.3	24.6	16.8	11.2	8.8
+60	-	-	46.5	39.2	34.6	20.5	14.9	10.4	8.3
+80	34.7	30.4	27.5	25.4	23.7	16.5	12.8	9.4	7.7

we then obtain [B2]

$$\xi = \frac{\xi_0 |\overline{\Delta\rho}|^{-\nu/\beta} E^{(\gamma-1)\nu/2\beta\gamma}}{x_0^\nu \left[1 + E\left(1+\frac{x}{x_0}\right)^{2\beta}\right]^{(\gamma-1)\nu/2\beta\gamma} \left[\delta + \frac{x}{x_0}\left\{\frac{\delta - \frac{1}{\beta} + E\left(1+\frac{x}{x_0}\right)^{2\beta}}{1 + E\left(1+\frac{x}{x_0}\right)^{2\beta}}\right\}\right]^{\nu/\gamma}}$$

(B.10)

with

$$x_0 = 0.0927, \quad E = 0.283.$$

(B.11)

The saturation boundary corresponds to $x = -x_0$, the critical isotherm to $x = 0$ and the critical isochore above T_c to $x = \infty$. At the critical isochore above T_c, (B.10) reduces again to $\xi = \xi_0(\Delta\tilde{T})^{-\nu}$ and at the saturation boundary below T_c to $\xi = \xi_0'|\Delta\tilde{T}|^{-\nu}$ with $\xi_0' = [\beta E^{(\gamma-1/2\beta)}]^{\nu/\gamma}$.

The corresponding expression for the compressibility cannot be integrated analytically to yield a closed form expression for the pressure as a function of density and temperature. Hence, this formalism is not well suited to calculate the density at a given pressure and temperature. However, once the density

is known, eq (B.10) enables one to calculate the correlation length without using the parametric variables r and θ.

It is possible to express the equation (A.6) for the viscosity in the critical region in terms of dimensionless quantities by introducing $\chi_T^* = \rho^*(\partial\rho^*/\partial P^*)_T$. Using the parameter values adopted in this paper, eq (A.6) may be written as

$$\frac{\eta}{\overline{\eta}} = 0.922\ \chi_T^{*0.0263}\ \text{for}\ \chi_T^* > \chi_0,$$

(B.12)

$$\frac{\eta}{\overline{\eta}} = 1 \qquad\qquad \text{for}\ \chi_T^* \leq \chi_0,$$

where the constant χ_0 is chosen such that

$$0.922\ \chi_0^{0.0263} = 1.$$

(B.13)

Appendix C. Tables of Calculated Viscosity Values

Table C1 gives the values of the viscosity at selected pressures and temperatures calculated from the interpolating equation specified in Appendix A. The pressures and temperatures correspond to the grid points of the skeleton tables of the international formulation for the viscosity of water substance [11]. In order to reproduce these values the densities are to be calculated with the 1968 IFC Formulation for Scientific and General Use [13][4]. Exceptions are the grid points at

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[4] A table of the reciprocal density values at the grid points is given in Appendix IV of reference [N1].

$$t = 375.0 \text{ °C}, P = 22.5 \text{ MPa}, \rho = 405.9 \text{ kg/m}^3;$$

$$t = 400.0 \text{ °C}, P = 30.0 \text{ MPa}, \rho = 357.1 \text{ kg/m}^3;$$

$$(C.1)$$

that are within the region defined by eq (B.5) and where the density has been calculated with the critical region equation of state presented in Appendix B.

In the critical region it is more illustrative to consider the viscosity as a function of density and temperature, because of the rapid variation of the density and the correlation length with pressure. In table C2 we give the values of the viscosity calculated from our interpolating equation at selected densities and temperatures in the critical region. The corresponding values of the density ρ and the correlation length ξ were given in tables B2 and B3.

Calculated viscosity values at the saturation boundary at selected temperatures were earlier presented in table 6.

Table C1. Calculated viscosity at selected pressures and temperatures corresponding to the grid points of the international skeleton tables (η in 10^{-6} Pa·s).

P, MPa \ t	0 °C	25 °C	50 °C	75 °C	100 °C	150 °C	200 °C	250 °C	300 °C	350 °C	375 °C
0.1	1793	890.4	547.1	377.8	12.27	14.18	16.18	18.22	20.29	22.37	23.41
0.5	1792	890.4	547.1	377.9	281.8	182.4	16.05	18.14	20.24	22.34	23.39
1.0	1791	890.2	547.2	378.0	281.9	182.5	15.89	18.04	20.18	22.31	23.37
2.5	1787	889.9	547.5	378.4	282.3	182.9	134.6	17.76	20.02	22.23	23.31
5.0	1781	889.4	547.9	379.1	283.0	183.6	135.2	106.5	19.80	22.12	23.25
7.5	1775	888.9	548.4	379.7	283.7	184.2	135.8	107.2	19.66	22.09	23.25
10.0	1769	888.4	548.8	380.4	284.4	184.8	136.5	107.9	86.48	22.15	23.33
12.5	1764	887.9	549.3	381.1	285.1	185.5	137.1	108.5	87.45	22.37	23.51
15.0	1758	887.5	549.8	381.7	285.8	186.1	137.7	109.2	88.37	22.93	23.86
17.5	1753	887.1	550.3	382.4	286.4	186.7	138.3	109.9	89.25	66.97	24.50
20.0	1748	886.8	550.8	383.1	287.1	187.4	138.9	110.5	90.10	69.26	25.90
22.5	1743	886.5	551.3	383.7	287.8	188.0	139.5	111.1	90.92	71.11	47.52
25.0	1738	886.2	551.8	384.4	288.5	188.6	140.1	111.8	91.71	72.69	58.31
27.5	1733	885.9	552.3	385.1	289.2	189.2	140.7	112.4	92.48	74.10	62.00
30.0	1729	885.7	552.9	385.7	289.8	189.9	141.2	113.0	93.23	75.38	64.57
35.0	1720	885.3	553.9	387.1	291.2	191.1	142.4	114.2	94.66	77.66	68.32
40.0	1712	885.1	555.1	388.5	292.5	192.3	143.5	115.3	96.01	79.67	71.20
45.0	1704	884.9	556.2	389.8	293.9	193.5	144.6	116.4	97.31	81.50	73.61
50.0	1697	884.9	557.4	391.2	295.2	194.7	145.7	117.5	98.55	83.17	75.71
55.0	1691	885.0	558.6	392.6	296.6	195.9	146.8	118.6	99.76	84.73	77.61
60.0	1685	885.1	559.8	394.0	297.9	197.1	147.9	119.7	100.9	86.20	79.34
65.0	1679	885.4	561.1	395.4	299.3	198.3	149.0	120.7	102.0	87.59	80.95
70.0	1674	885.8	562.4	396.8	300.6	199.4	150.0	121.7	103.1	88.90	82.46
75.0	1669	886.3	563.7	398.2	301.9	200.6	151.1	122.7	104.2	90.16	83.88
80.0	1665	886.8	565.1	399.6	303.3	201.7	152.1	123.7	105.2	91.37	85.22
85.0	1661	887.5	566.5	401.0	304.6	202.9	153.1	124.7	106.2	92.52	86.50
90.0	1658	888.3	567.9	402.5	306.0	204.0	154.1	125.6	107.2	93.65	87.72
95.0	1655	889.2	569.3	403.9	307.3	205.2	155.1	126.6	108.2	94.73	88.89
100.0	1653	890.1	570.8	405.3	308.6	206.3	156.1	127.5	109.1	95.78	90.02

Table C1. (Continued)

P, MPa (t)	400 °C	425 °C	450 °C	475 °C	500 °C	550 °C	600 °C	650 °C	700 °C	750 °C	800 °C
0.1	24.45	25.49	26.52	27.55	28.57	30.61	32.61	34.60	36.55	38.48	40.37
0.5	24.44	25.48	26.52	27.55	28.58	30.62	32.63	34.61	36.57	38.49	40.39
1.0	24.42	25.47	26.51	27.55	28.58	30.63	32.64	34.63	36.59	38.52	40.41
2.5	24.39	25.46	26.52	27.57	28.61	30.67	32.70	34.69	36.66	38.58	40.48
5.0	24.37	25.46	26.55	27.62	28.68	30.76	32.81	34.81	36.78	38.71	40.60
7.5	24.40	25.52	26.62	27.71	28.78	30.88	32.94	34.95	36.92	38.84	40.74
10.0	24.49	25.62	26.73	27.83	28.91	31.03	33.09	35.10	37.07	39.00	40.88
12.5	24.65	25.78	26.90	28.00	29.08	31.20	33.26	35.27	37.24	39.16	41.04
15.0	24.93	26.03	27.13	28.21	29.29	31.40	33.46	35.46	37.42	39.34	41.21
17.5	25.37	26.37	27.42	28.49	29.54	31.63	33.68	35.67	37.62	39.53	41.39
20.0	26.03	26.85	27.81	28.82	29.85	31.90	33.92	35.90	37.84	39.73	41.59
22.5	27.13	27.51	28.31	29.24	30.21	32.20	34.19	36.15	38.07	39.95	41.79
25.0	29.16	28.45	28.96	29.74	30.64	32.55	34.49	36.42	38.32	40.18	42.01
27.5	33.87	29.82	29.78	30.36	31.14	32.93	34.82	36.71	38.58	40.43	42.23
30.0	43.91	31.87	30.86	31.10	31.73	33.36	35.17	37.02	38.86	40.68	42.47
35.0	55.75	39.33	34.04	33.09	33.19	34.37	35.97	37.70	39.47	41.23	42.98
40.0	61.27	48.54	38.99	35.90	35.12	35.59	36.89	38.47	40.13	41.83	43.52
45.0	65.01	54.91	45.02	39.58	37.57	37.03	37.95	39.31	40.86	42.47	44.10
50.0	67.93	59.34	50.48	43.84	40.49	38.70	39.13	40.25	41.64	43.15	44.72
55.0	70.38	62.74	54.86	48.08	43.73	40.59	40.43	41.25	42.48	43.88	45.36
60.0	72.52	65.54	58.40	51.90	47.05	42.65	41.84	42.34	43.36	44.64	46.04
65.0	74.44	67.93	61.37	55.22	50.23	44.82	43.35	43.48	44.30	45.43	46.74
70.0	76.20	70.04	63.92	58.11	53.15	47.06	44.93	44.68	45.27	46.26	47.46
75.0	77.82	71.94	66.17	60.66	55.81	49.28	46.57	45.93	46.27	47.11	48.20
80.0	79.35	73.68	68.19	62.94	58.22	51.45	48.23	47.21	47.30	47.97	48.95
85.0	80.78	75.30	70.03	65.00	60.42	53.52	49.89	48.51	48.36	48.86	49.72
90.0	82.14	76.81	71.72	66.89	62.44	55.48	51.54	49.82	49.42	49.76	50.50
95.0	83.43	78.24	73.30	68.63	64.31	57.36	53.15	51.14	50.50	50.66	51.28
100.0	84.66	79.59	74.78	70.25	66.05	59.15	54.72	52.45	51.58	51.57	52.07

Table C2. Calculated viscosity at selected densities and temperatures near the critical point (η in 10^{-6} Pa·s)

$\rho-\rho_c$, kg/m³ \\ $T-T_c$, K	-1	-0.5	0	+0.5	+1	+5	+10	+20	+30
-80	33.3	33.1	33.0	33.0	33.0	33.2	33.5	34.0	34.6
-60	-	-	35.5	35.2	35.0	34.8	35.0	35.6	36.1
-40	-	-	38.6	37.8	37.4	36.4	36.7	37.2	37.7
-20	-	-	43.0	40.5	39.8	38.2	38.4	38.9	39.4
0	-	-	∞	42.9	42.0	40.1	40.1	40.6	41.1
+20	-	-	47.1	44.4	43.6	41.8	42.0	42.4	42.9
+40	-	-	46.3	45.4	44.9	43.6	43.8	44.3	44.8
+60	-	-	46.6	46.2	46.0	45.5	45.8	46.2	46.7
+80	47.9	47.6	47.4	47.3	47.4	47.5	47.8	48.2	48.6

Appendix D. Choice of Temperature Scale and Equation of State

Choice of Temperature Scale

The representative equation developed in this paper and summarized in Appendix A was developed with the temperatures expressed in terms of IPTS–48. This was done to retain consistency with the international formulation for the viscosity developed by IAPS. Should the user wish to use the equation with the more modern IPTS–68, then two possible courses of action are available.

The first and obvious approach, involving no loss of accuracy, is to convert the temperatures on IPTS–68 to temperatures on IPTS–48 prior to using the representative equations. Equations and tables for converting the temperature scale can be found in references [D4,R14]. This procedure is recommended if the viscosity equation is used in conjunction with the 1968 IFC Formulation which is also based on IPTS–48.

The second, and more direct approach, is to substitute into the viscosity equation the temperatures in terms of IPTS–68.

This procedure changes the calculated viscosity by at most 0.05%, which is well within the accuracy of the equation provided that the value of the critical temperature in the critical region equation defined in Appendix B is expressed in terms of IPTS–68. For this purpose we note that

$$T_c^{(68)} = T_c^{(48)} + 0.077 \text{ K}. \qquad (D.1)$$

The largest deviations in calculating the normal viscosity $\bar{\eta}$ are typically: $+0.02\%$ for saturated water at 30 °C, -0.03% for saturated water at 150 °C, $+0.02\%$ for steam at 600 °C and 0.1 MPa and $+0.05\%$ for steam at 800 °C and 0.1 MPa. Replacement of $T_c^{(48)}$ by $T_c^{(68)}$ in the critical region equations ensures that the values obtained for ξ and the critical enhancement factor $(q\xi)^\Phi$ remain the same.

Choice of Equation of State

Table D1 shows the percentage change in the normal viscosity $\bar{\eta}$ caused by a 1% increase in density. This table can

Table D1. Percentage change in the normal viscosity $\bar{\eta}$ due to a one percent increase in density.

P,MPa \ t,°C	0	100	200	300	350	400	500	600	700	800
0.1	-2.8	-*	–	–	–	–	–	–	–	–
5.0	-2.7	2.0	2.1	–	–	–	–	–	–	–
10.0	-2.6	2.0	2.2	1.5	–	–	–	–	–	–
15.0	-2.5	2.0	2.2	1.5	0.1	0.1	–	–	–	–
20.0	-2.4	2.0	2.2	1.6	1.2	0.1	0.1	0.1	–	–
25.0	-2.3	2.0	2.2	1.6	1.2	0.3	0.1	0.1	0.1	0.1
30.0	-2.2	2.1	2.2	1.6	1.3	0.8	0.2	0.1	0.1	0.1
35.0	-2.1	2.1	2.2	1.6	1.3	0.9	0.2	0.1	0.1	0.1
40.0	-2.0	2.1	2.3	1.6	1.3	1.0	0.3	0.2	0.1	0.1
45.0	-1.9	2.1	2.3	1.7	1.4	1.1	0.4	0.2	0.1	0.1
50.0	-1.8	2.1	2.3	1.7	1.4	1.1	0.5	0.2	0.2	0.1
60.0	-1.6	2.2	2.3	1.7	1.4	1.2	0.6	0.3	0.2	0.2
70.0	-1.4	2.2	2.3	1.8	1.5	1.2	0.7	0.4	0.3	0.2
80.0	-1.2	2.3	2.3	1.8	1.5	1.3	0.8	0.5	0.3	0.2
90.0	-0.9	2.3	2.4	1.8	1.5	1.3	0.9	0.5	0.4	0.3
100.0	-0.7	2.3	2.4	1.8	1.6	1.3	0.9	0.6	0.4	0.3

*indicates percentage change less than 0.05.

be used to predict the effect on $\bar{\eta}$ of using a density formulation different from the 1968 IFC formulation.

In the critical region the values of the density and the correlation length become very sensitive to pressure and temperature. To include the critical viscosity enhancement it is necessary to use an accurate equation of state equivalent to the critical region equation of state defined in Appendix B.

Viscosity Equation for Industrial Use

Calculations of the thermodynamic properties of steam for industrial purposes are often based on the 1967 IFC Formulation for Industrial Use [M1,M2,S1]. Should one want to use our interpolating equation in conjunction with the 1967 IFC formulation, it is recommended that the viscostity η be identified with the normal viscosity.

$$\eta = \bar{\eta} \qquad (D.2)$$

at all pressures and temperatures including the critical region with $\bar{\eta}$ still given by (A.5). The effects on the values calculated for $\bar{\eta}$ are the same as those given in Appendix VI of reference [N1]. The differences are typically of the order of $\pm 0.3\%$, but become somewhat larger in the critical region defined by (B.5). This procedure ignores the viscosity enhancement near the critical point which is unimportant for many practical applications and which is also ignored in the international formulation for the viscosity of water substance [I1]. The advantage of the procedure is that the calculated values are of the same quality as when the international viscosity formulation is used in conjunction with the 1967 IFC formulation but with an appreciably smaller number of terms in the viscosity equation.

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