Version 3 Tutorial

April 27, 2007

1 Checklist of input parameters

All of the information below must be specified in the various input files used in VISCO1D.

Earth model

• Viscoelastic stratification: density r, bulk modulus κ, shear modulus of Maxwell element μ_1, shear modulus of Kelvin element μ_2, Maxwellian viscosity η_1, Kelvin viscosity η_2, and long term strength μ' (eqn 2 of *Manual*) as a function of radius

• Radius of Earth

• Depth range over which Greens functions will be stored

• Minimum and maximum spherical harmonic degree of deformation field expansion

Fault model

• Number of fault planes

• Strike, dip, rake, slip, length, depths of upper and lower fault edges, latitude and longitude of one fault corner

Observation

• Observation depth

• Number of observation points

• Latitude, longitude of these observation points

• Deformation option: (1) Displacement and strain (2) Velocity and strain rate

• Time interval with respect to earthquake origin time (start, end times for deformation option #1; single observation time for deformation option #2).

2 Examples

The examples given here are abbreviated descriptions of how to compute post-earthquake deformation for single-plane ruptures. Before running the examples, compile the needed programs using the Makefile (i.e., 'make all')

2.1 Example 1

This example will do all of the calculations necessary to duplicate Figure 3 of Rundle (1982) – the $t = 5 \times \tau$ curve of gravitational-viscoelastic deformation. Rundle's definition of τ is $\tau = 2 \times \eta_1/\mu_1$. The calculation follows the approach of Pollitz (1997). In the viscoelastic structure (Figure 1), the elastic plate is underlain by a uniform viscoelastic sphere of Maxwellian viscosity $\eta_1 = 10^{19}$ Pa s and shear modulus $\mu_1 = 30$ GPa.

To run this example, run the command file 'go.xTHRUSTg', which contains the lines

```
cp earth.modelHOMO30 earth.model
nice decay4m <<! > /dev/null
2 1500
!
nice vsphm <<! > /dev/null
10.
!
nice decay <<! > /dev/null
2 1500
!
nice vtordep <<! > /dev/null
10.
!
nice strainA < strainx.inTHRUST > /dev/null
mv strainA.out strainA.outTHRUSTg
```

This sequence of commands is typical for calculations of postseismic deformation. The tasks it accomplishes are:

- Get the desired stratified viscoelastic model into 'earth.model'
- Determine characteristic inverse decay times s_j for spheroidal modes (eqn (24) of *Manual*). The minimum and maximum spherical harmonic degrees are set at 2 and 1500 km, respectively. The deformation field at higher spherical harmonic degrees is strongly attenuated by the 30-km thick up-

per elastic plate, which acts as a low-pass filter. A rule of thumb to use is $l_{\max} \sim 2\pi R/H_e$. With Earth radius $R = 6371$ km and $H_e = 30$ km we get $l_{\max} = 1334$, close to what is actually used. Some care should be taken if deformation at greater depth is desired. At a depth of 10 km, the minumum wavelength content of the deformation field is expected to be of the order of the distance to the base of the elastic plate = 20 km. This would demand using the larger value $l_{\max} \sim 2\pi R/(20\text{km}) = 2000$.

- Determine the spheroidal mode eigenfunctions y_1 and y_3, their radial derivatives and the associated ϵ_j (eqn (21) of *Manual*) in a form suitable for direct use in eqns (16) of *Manual*, to determine the source excitation functions. A subset of this information is re-written at a specified depth level (10 km in this case) in order to supply eigenvalue information for use in eqn (25) of *Manual*.
- Determine characteristic inverse decay times s_j for toroidal modes (eqn (39) of *Manual*).
- Determine the toroidal mode eigenfunctions y_1, its radial derivative and the associated ϵ_j (eqn (37) of *Manual*) in a form suitable for direct use in eqns (32) of *Manual*, to determine the source excitation functions. A subset of this information is re-written at a specified depth level (10 km in this case) in order to supply eigenvalue information for use in eqn (40) of *Manual*.
- Calculate the displacement vector and strain tensor at several observation points for a particular fault model, as specified in 'strainx.inTHRUST'.
- Re-name the output file created by **strainA** so that it will not be written over by subsequent runs of **strainA**. One may also re-name the output files'decay4.out', 'vsph.out', 'decay.out', and 'vtor.out ' . This set of four output files represents the viscoelastic response functions for the particular structure represented by 'earth.modelHOMO30', and they are all read in by **strainA**. The same set could be used repeatedly to generate postseismic deformation for other time intervals/fault geometries/observation points by re-running **strainA**.

Explanation of input files which appear in 'go.xTHRUSTg'

A) 'earth.modelHOMO30'

31 2 6371.000 0.568

[31 layers, the value 2 is not used, keep at that value. The radius of the earth is specified at 6371 km. Eigenfunctions are evaluated from the surface (depth 0) to depth 28× DEPFAC km, where here DEPFAC= 0.568. This is adequate because the intended fault will not extend below 16 km depth in this example. If we intended to specify a fault that penetrated the entire 30 km of the elastic plate, then it would be appropriate to use

DEPFAC=(30/28)=1.071]
5856.300 5887.500 2.800 5.000 3.000 0.100000E+02
[bottom radius of layer=5856.3 km, top radius of layer=5887.5 km, density=2.800 g-cm^{-3}, bulk modulus=5.0×10^{10} Pa, shear modulus=3.0×10^{10} Pa, viscosity=(10.) $\times 10^{18} = 10^{19}$ Pa s]
6341.000 6346.000 2.800 5.000 3.000 0.100000E+12
6365.000 6367.000 2.800 5.000 3.000 0.100000E+12
6367.000 6369.000 2.800 5.000 3.000 0.100000E+12
6369.000 6371.000 2.800 5.000 3.000 0.100000E+12
[bottom radius of layer=6369.0 km, top radius of layer=6371.0 km, density=2.800 g-cm^{-3}, bulk modulus=5.0×10^{10} Pa, shear modulus=3.0×10^{10} Pa]

B) 'strainx.inTHRUST'
15.00 0.0 30.
[Lower/upper fault edge depths are 15.00 and 0.0 km. Fault dip is 30°.]
1975. 1975. 2080.82 1.
[t_0, t_1, t_2, VMULT. $t_0 = 1975$ is the origin time of the synthetic earthquake. If ISRATE (the second-to-last line of this input file) = 0, then we evaluate cumulative postseismic displacement and strain from time t_1 to time t_2 from an earthquake occurring at time t_0. If ISRATE = 1, then we evaluate velocity and strain rate at time $0.5 \times (t_1 + t_2)$ from an earthquake occurring at time t_0. In this example, we will get the cumulative displacement and strains for the first 105.82 years following the synthetic earthquake (i.e., 5τ as defined by Rundle, 1982). VMULT is used if you want to scale all viscosities in the earth model up or down by a constant factor. The current earth.modelLP has viscosity below 30 km depth = 10^{19} Pa s. After the decay times and eigenfunctions have been calculated for this model, if you decide you want to have the deformation with viscosity $2x10^{19}$ Pa s then use VMULT=2.0. If we want to keep viscosity= 10^{19} Pa s then use VMULT=1.0, as in this input file. This option allows you to change viscosity without having to recompute the decay times and eigenfunctions all over again.]
1
[One fault plane, with additional parameters in the following line.]
0.899361 0.233661 200. 0. 90. 100.
[The first two numbers are latitude and longitude of the lowermost corner of the fault closest to the strike direction. The lower edge is at $x = 15$ km/tan(30°) = 25.98 km, corresponding to a longitude of 0.233661 deg.; the upper edge is at x=0 (longitude 0 deg.). The fault length is 200 km; the fault has strike=0, i.e., it runs north-south; rake = 90° (reverse slip); slip is 100 cm.]

101
[101 observation points as listed below]
0.000000 -0.809425
0.000000 -0.787841
0.000000 -0.766256
. . .
0.000000 1.34904
0
[ISRATE (see explanation above)]
0
[IOBS. If IOBS=0 then surface deformation is evaluated. If IOBS=1 then deformation is at specified depth (that used as input to **vtordep** and **vsphm**). Thus one could evaluate the deformation field at 10 km instead of 0 km depth, at the same 101 observation lat,lons, by changing this line to a 1]

Explanation of output files generated by 'go.xTHRUSTg'

The output files from **decay** and **decay4m** are decay.out and decay4.out, respectively. These contain pairs (l, s_j) for toroidal and spheroidal motion, respectively. These are plotted in Figure 2. The output files from **vtordep** and **vsphm** are 'vtor.out' and 'vsph.out', respectively. They are unformatted and contain excitation functions for toroidal and spheroidal motion from specific dislocation sources.

strainA.outTHRUSTg will have 101 lines of output, corresponding to deformation at the 101 observation points in the order in which they appeared in 'strainx.inTHRUST'

A single line of strainA.outTHRUSTg looks like
-115.995 -99.997 0.713065E+01-0.107873E-04 0.359200E+01 0.206685E+00-0.260879E-01-0.566978E-05-0.252745E-08 0.191121E-12-0.601988E-01-0.136269E-04
[Here, -115.995 -99.997 are approximate (x, y) Cartesian coordinates of the observation point relative to the reference point on the (first) input fault, 0.713065E+01-0.107873E-04 0.359200E+01 are the East -displacement u_x , North -displacement u_y , and Up -displacement u_z in cm, followed by the 6 nontrivial strain components e_{xx} , e_{yy} , e_{xy}, e_{xz}, e_{yz} , and e_{zz} in units of 10^{-6}, and the rotation $\omega_{xy} = \partial_y(u_x) - \partial_x(u_y)$ in units of 10^{-6}. The strain components $\dot{e}_{xz}$ and $\dot{e}_{yz}$ are theoretically zero at the surface.]
Note that if we had used ISRATE=1 instead of ISRATE=0, then the output would contain velocities in units of cm/yr and strain rates in units of 10^{-6}/yr.

The observation points are situated along a profile running East-West (along the x-axis) and bisecting the fault. The resulting vertical postseismic displacement at $t = 5\tau$ are shown in Figure 3 (red curve). It compares well with Figure 3 of Rundle (1982). The agreement is not perfect because Rundle (1982) uses a slightly different rheology: his $\lambda(s) = \kappa(s) - (2/3)\mu(s)$ is considered a constant function of s, i.e., non-relaxing; hence both μ and κ are non-constant functions of s. In our treatment, bulk modulus κ is considered non-relaxing, and μ and λ are generally non-constant functions of s.

2.2 Example 2

This example will do all of the calculations necessary to duplicate Figure 2 of Rundle (1982) – the $t = 5 \times \tau$ curve of viscoelastic deformation. This is the non-gravitational case, and the calculation procedes exactly as in *Example* 1 but with with the nongravitational programs **decay4** and **vsphdep** used in place of the gravitaitonal programs **decay4m** and **vsphm**.

The viscoelastic structure is again as shown in Figure 1.

To run this example, run the command file 'go.xTHRUST', which contains the lines

```
cp earth.modelHOMO30 earth.model
nice decay4 <<! > /dev/null
2 1500
!
nice vsphdep <<! > /dev/null
10.
!
nice decay <<! > /dev/null
2 1500
!
nice vtordep <<! > /dev/null
10.
!
nice strainA < strainx.inTHRUST > /dev/null
mv strainA.out strainA.outTHRUST
```

The observation points are situated along a profile running East-West (along the x-axis) and bisecting the fault. The dispersion of viscoelastic modes on the non-gravitational model are shown in Figure 4. The resulting vertical postseismic displacements at $t = 5\tau$ are shown in Figure 3 (green

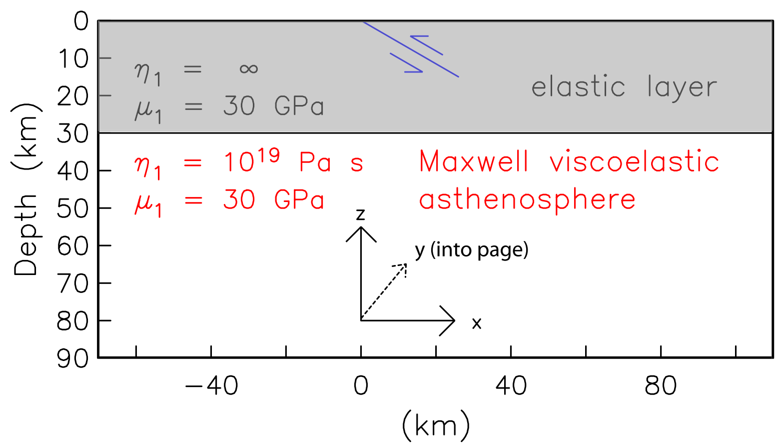

Figure 1: Viscoelastic stratification of *Examples* 1 and 2. The elastic plate of thickness 30 km is underlain by a homogeneous viscoelastic spherical shell with indicated Maxwell-body parameters. The thrust fault ruptures the upper 15 km of the elastic plate.

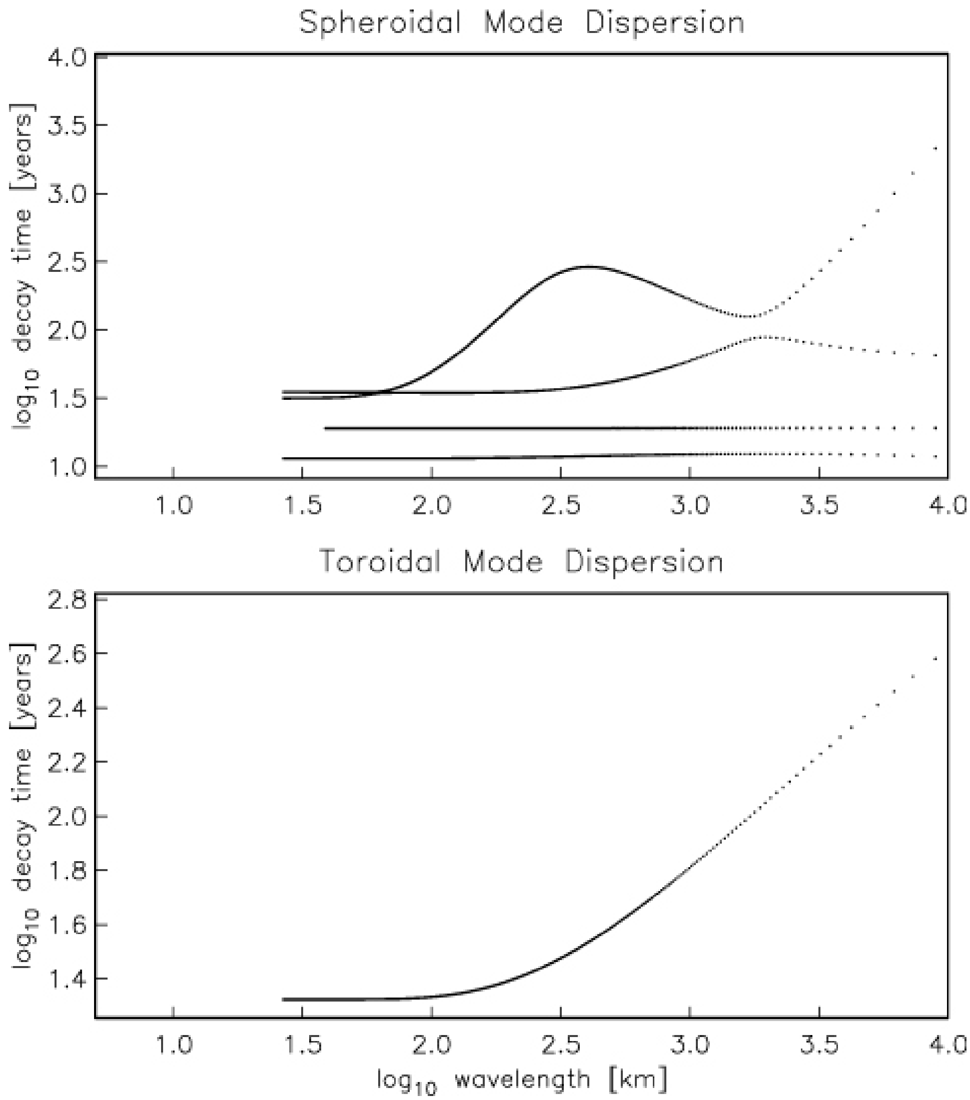

Figure 2: Dispersion on viscoelastic model 'earth.modelHOMO30' for the gravitational case.

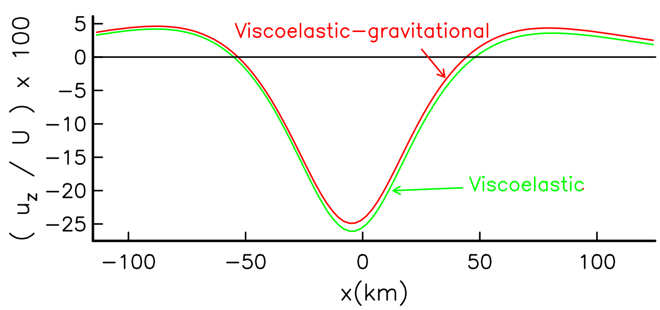

Figure 3: Surface vertical displacements from viscoelastic-gravitational relaxation (red curve) or viscoelastic relaxation (green curve) following a thrust event, normalized to the coseismic slip U on the fault.

curve). It compares well with Figure 2 of Rundle (1982). Again, the agreement is not perfect because Rundle (1982) uses a slightly different rheology.

2.3 Example 3

This example will compute the viscoelastic-gravitational postseismic velocity and strain rates from right-lateral strike slip on a vertical fault embedded in an elastic layer underlain by a Burgers-body viscoelastic structure. These rates are evaluated 0.5 years after the slip event.

For a Burgers body (Pollitz, 2003), the rheology is described by a Maxwell element with shear modulus μ_1 and viscosity η_1 and a Kelvin element with shear modulus μ_2 and viscosity η_2 (Figure 5). The s-dependent shear modulus is then given by

$$\mu(s) = \frac{\mu_1 s \left(s + \frac{\mu_2}{\eta_2} \right)}{\left[\left(s + \frac{\mu_2}{\eta_2} \right) \left(s + \frac{\mu_1}{\eta_1} \right) + s \frac{\mu_1}{\eta_2} \right]} \tag{1}$$

In the limit $\eta_1 \rightarrow \infty$, $\mu(s)$ reduces to a standard linear solid (eqn 3 of the *Manual*).

In the viscoelastic structure (Figure 6), the elastic plate is underlain by a uniform viscoelastic sphere of Maxwellian viscosity $\eta_1 = 10^{19}$ Pa s and shear modulus $\mu_1 = 30$ GPa, Kelvin viscosity $\eta_2 = 5 \times 10^{17}$ Pa s and shear modulus $\mu_2 = 30$ GPa.

To run this example, run the command file 'go.xSSg', which contains the lines

```
cp earth.modelBURG30 earth.model
nice decay4m <<! > /dev/null
2 1500
!
nice vsphm <<! > /dev/null
10.
!
nice decay <<! > /dev/null
2 1500
!
nice vtordep <<! > /dev/null
10.
!
nice strainA < strainx.inSS > /dev/null
```

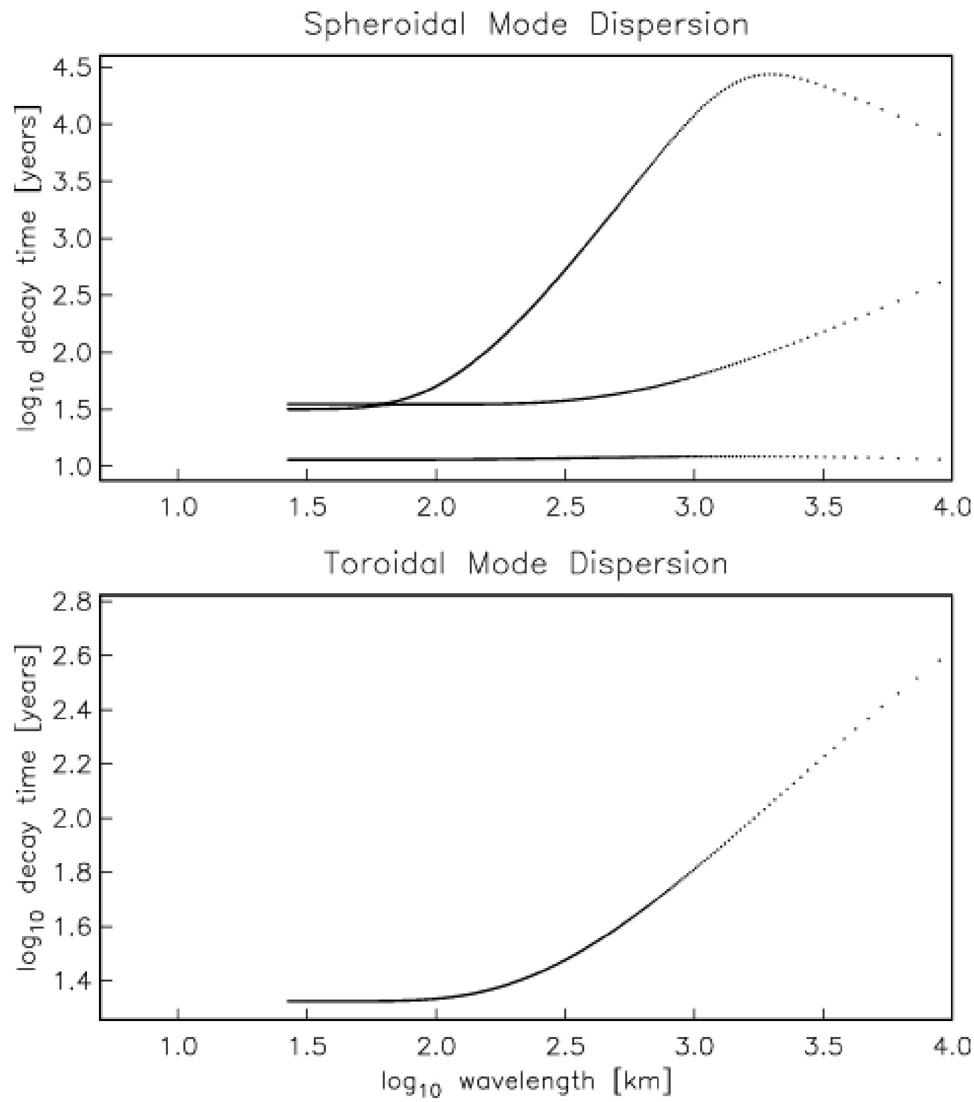

Figure 4: Dispersion on viscoelastic model 'earth.modelHOMO30' for the non-gravitational case.

mv strainA.out strainA.outSSg

Explanation of input files which appear in 'go.xSSg'

A) 'earth.modelBURG30'

The input file 'earth.modelBURG30' shares the same structure in the elastic layer (radius $\geq$ 6341 km) and specifies viscoelastic structure at radius $<$ 6341 km with lines such as

6338.000 6341.000 2.800 5.000 3.000 1.500 5.000000E-01 1.000000E+01

[bottom radius of layer=6338 km, top radius of layer=6341 km, density=2.800 g-cm^{-3}, bulk modulus=5.0 $\times$ 10^{10} Pa, Maxwellian shear modulus μ_1 = 3.0$\times$$10^{10}$ Pa, long term strength $\mu' = 1.5{\times}10^{10}$ Pa (eqn 2 of *Manual*), Kelvin viscosity $\eta_2 = 5 \times 10^{17}$ Pa s, Maxwellian viscosity $\eta_1 = (10.) \times 10^{18} = 10^{19}$ Pa s. Note that the Kelvin shear modulus $\mu_2 = \mu_1\mu'/(\mu_1 - \mu') = 3.0 \times 10^{10}$ Pa]

B) 'strainx.inSS'

15.00 0.0 90.

[Lower/upper fault edge depths are 15.00 and 0.0 km. Fault dip is 90°.]

1975. 1975.5 1975.5 1.

[t_0, t_1, t_2, VMULT, as in *Example* 1. When $t_1 = t_2$ and ISRATE=1, as in this example, the resulting deformation is postseismic velocity and strain rates at time t_1 from a source acting at time t_0.]

1

[One fault plane, with additional parameters in the following line.]

0.899361 0.0 200. 0. 180. 100.

[The first two numbers are latitude and longitude of the lowermost corner of the fault closest to the strike direction. The lower edge is at $x = 0$ km, corresponding to a longitude of 0.0 deg.; since the fault is vertical the upper edge is also at x=0. The fault length is 200 km; the fault has strike=0, i.e., it runs north-south; rake = 180° (right-lateral strike slip); slip is 100 cm.]

101

[101 observation points as listed below]

0.000000 -0.809425
0.000000 -0.787841
0.000000 -0.766256
. . .
0.000000 1.34904

1

[ISRATE (see explanation in *Example* 1)]

0

[IOBS. If IOBS=0 then surface deformation is evaluated. If IOBS=1 then

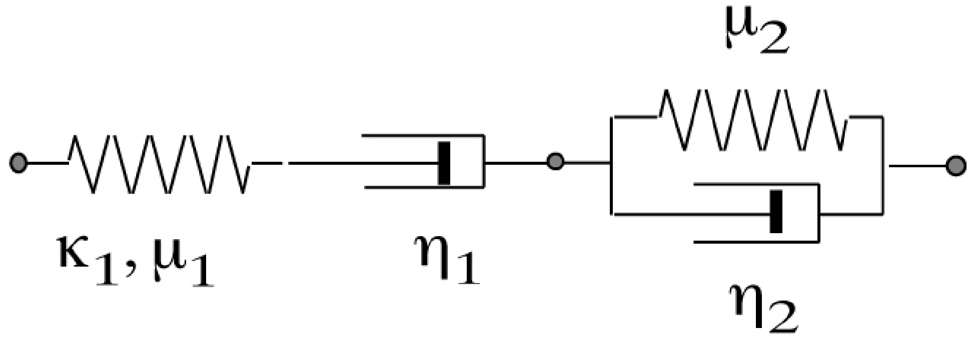

Figure 5: A transient rheology as represented by a Burghers body. It consists of a Maxwell element in series with a Kelvin element, which are characterized by steady state shear and bulk moduli μ_1 and κ_1, respectively, steady state viscosity η_1, transient viscosity η_2, and transient shear modulus μ_2. If $\eta_1 = \infty$, then the material behaves like a standard linear solid with relaxed shear modulus $\mu' = \mu_1\mu_2/(\mu_1 + \mu_2)$. If $\eta_2 = \infty$ or $\mu_2 = \infty$, then the material behavior reduces to a Maxwell rheology.

deformation is at specified depth (that used as input to **vtordep** and **vsphm**). Thus one could evaluate the deformation field at 10 km instead of 0 km depth, at the same 101 observation lat,lons, by changing this line to a 1]

The dispersion of the viscoelastic-gravitational modes on the Burgers body model is shown in Figure 7.

A single line of strainA.outTHRUSTg looks like
-68.409 -100.005 0.451815E-04 0.112624E+01 0.132138E-04-0.623650E-06 0.285701E-05 0.103401E-01-0.712755E-14-0.994359E-10-0.743708E-06-0.931525E-01
[Here, -68.409 -100.005 are approximate (x, y) Cartesian coordinates of the observation point relative to the reference point on the (first) input fault, 0.451815E-04 0.112624E+01 0.132138E-04 are the East -velocity v_x , North -velocity v_y , and Up -velocity v_z in cm/yr, followed by the 6 nontrivial strain rate components $\dot{e}_{xx}$, $\dot{e}_{yy}$, $\dot{e}_{xy}$,$\dot{e}_{xz}$, $\dot{e}_{yz}$, and $\dot{e}_{zz}$ in units of 10^{-6}/yr, and the rotation $\dot{\omega}_{xy} = \partial_y(v_x) - \partial_x(v_y)$ in units of 10^{-6}/yr. Because of the symmetry of the problem, we expect $v_y = v_z = 0$ and $\dot{e}_{xx} = \dot{e}_{yy} = \dot{e}_{zz} = 0$. The strain components $\dot{e}_{xz}$ and $\dot{e}_{yz}$ are theoretically zero at the surface.]

The resulting North-velocity v_y at time $t = 0.5$ years after the synthetic event is shown in Figure 8.

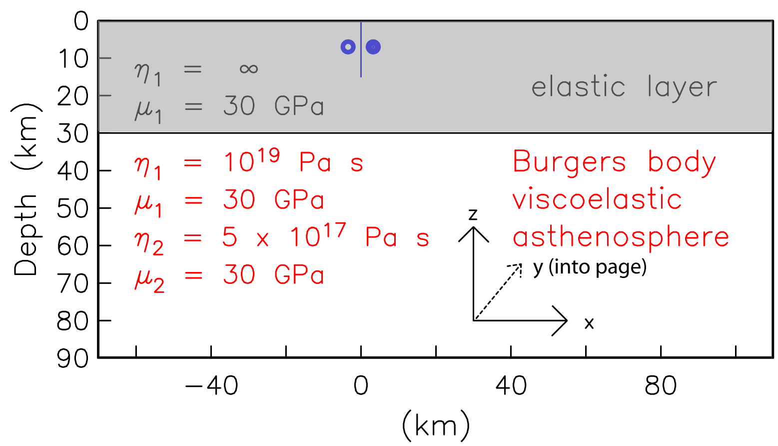

Figure 6: Viscoelastic stratification of Example 3. The elastic plate of thickness 30 km is underlain by a homogeneous viscoelastic spherical shell with indicated Burgers-body parameters. The strike-slip fault ruptures the upper 15 km of the elastic plate.

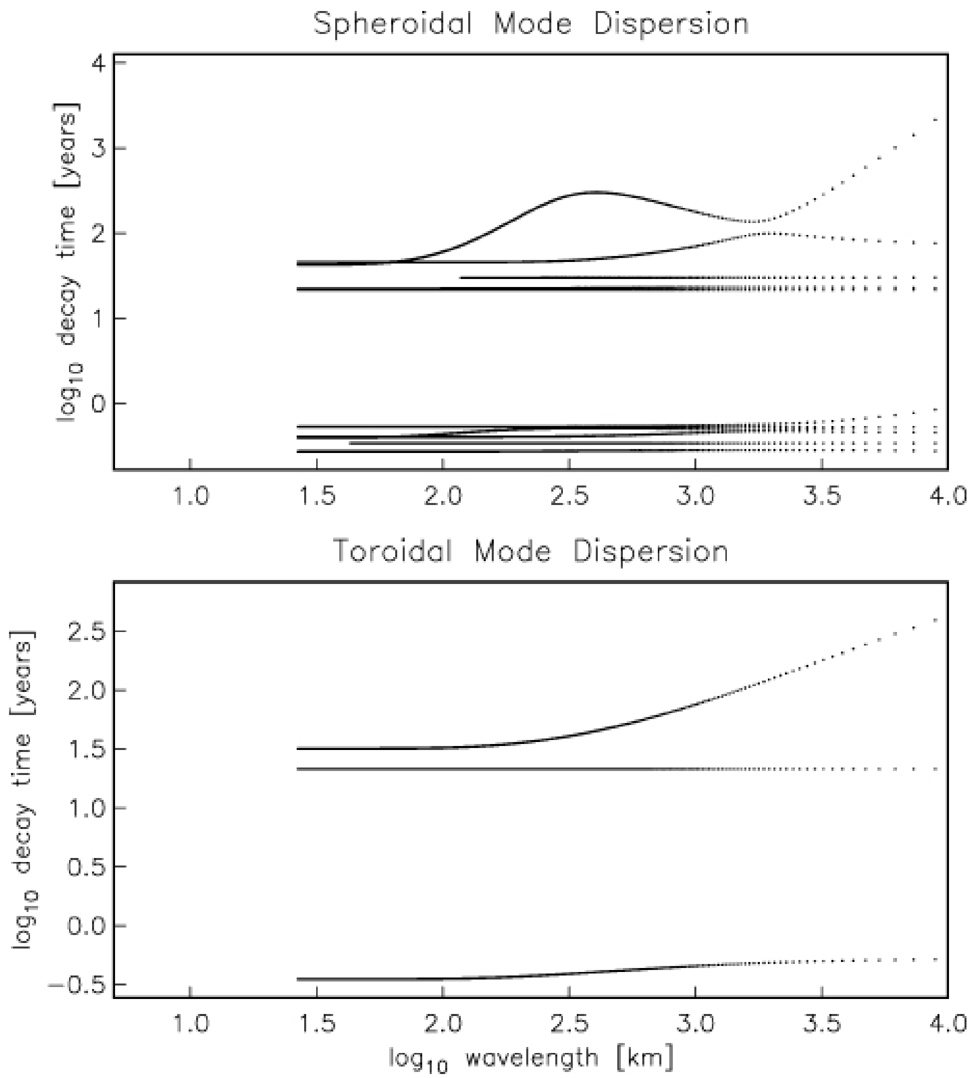

Figure 7: Dispersion on viscoelastic model 'earth.modelBURG30' (gravitational case).

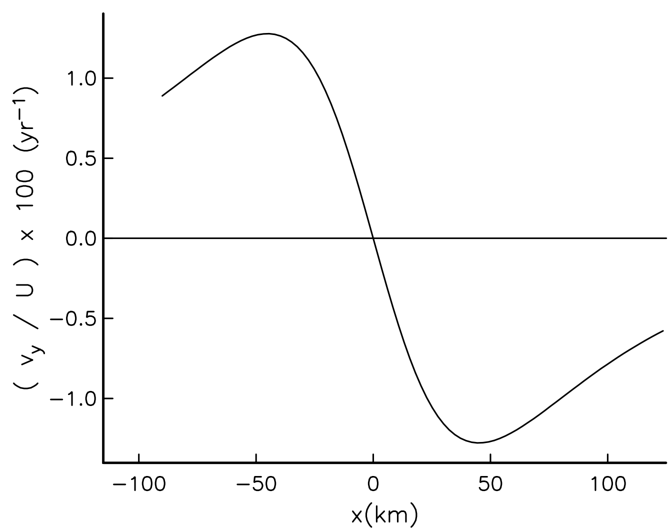

Figure 8: North-velocity at the surface from viscoelastic-gravitational relaxation following a strike-slip event, normalized to the coseismic slip U on the fault.

2.4 Example 4

This example will compute the viscoelastic-gravitational postseismic displacement and strains from the 2004 M=9.2 Sumatra-Andaman earthquake on a 2190 km long SW-NE vertical profile, at 11 depths between 0 and 100 km. The structure consists of a 62 km thick elastic layer underlain by a Burgers-body viscoelastic spherical shell from 62 to 220 km depth, and by a Maxwsell viscoelastic sphere below 220 km (Figure 9). The displacements are evaluated from 0 to 1 year after the slip earthquake.

There are nine coseismic rupture planes used to model the Sumatra rupture (Banerjee et al., 2007). Four input files are needed to represent the rupture: strainx.inSUM1.PROF (2 planes), strainx.inSUM2.PROF (4 planes), strainx.inSUM3.PROF (1 plane), and strainx.inSUM1.PROF (2 planes). Within a given input file, each fault plane shares the same lower edge depth, upper edge depth, and dip. (This information is given in the first line of each input file).

To run this example, run the command file 'go.xSUMATRAg', which contains the lines

```
cp earth.modelBURG-SUM earth.model
nice decay4m <<! > /dev/null
2 800
!
nice decay <<! > /dev/null
2 800
!
nice vsphm <<! > /dev/null
0.0
!
nice vtordep <<! > /dev/null
0.0
!
nice strainA < strainx.inSUM1.PROF > /dev/null
mv strainA.out strainA.outSUM1_0.0
nice strainA < strainx.inSUM2.PROF > /dev/null
mv strainA.out strainA.outSUM2_0.0
nice strainA < strainx.inSUM3.PROF > /dev/null
mv strainA.out strainA.outSUM3_0.0
nice strainA < strainx.inSUM4.PROF > /dev/null
mv strainA.out strainA.outSUM4_0.0
```

```
nice vsphm <<! > /dev/null
10.0
!
nice vtordep <<! > /dev/null
10.0
!
nice strainA < strainx.inSUM1.PROF > /dev/null
mv strainA.out strainA.outSUM1_10.0
nice strainA < strainx.inSUM2.PROF > /dev/null
mv strainA.out strainA.outSUM2_10.0
nice strainA < strainx.inSUM3.PROF > /dev/null
mv strainA.out strainA.outSUM3_10.0
nice strainA < strainx.inSUM4.PROF > /dev/null
mv strainA.out strainA.outSUM4_10.0

......

nice vsphm <<! > /dev/null
100.0
!
nice vtordep <<! > /dev/null
100.0
!
nice strainA < strainx.inSUM1.PROF > /dev/null
mv strainA.out strainA.outSUM1_100.0
nice strainA < strainx.inSUM2.PROF > /dev/null
mv strainA.out strainA.outSUM2_100.0
nice strainA < strainx.inSUM3.PROF > /dev/null
mv strainA.out strainA.outSUM3_100.0
nice strainA < strainx.inSUM4.PROF > /dev/null
mv strainA.out strainA.outSUM4_100.0
```

This command file performs similar tasks to those of *Examples 1-3*. In each input file 'strainx.inSUM[m].PROF', the last number is 1, i.e., IOBS=1, signifying that deformation is to be evaluated at the depth d specified in the input to **vsphm** and **vtordep**. For given d, the total deformation along the 51-point profile at that depth is the sum of deformation contained in the output files strainA.outSUM1_[d], strainA.outSUM2_[d], strainA.outSUM3_[d], and strainA.outSUM4_[d]. In this manner the 44 different output files are reduced to profiles of total deformation along the same spherical arc at 11

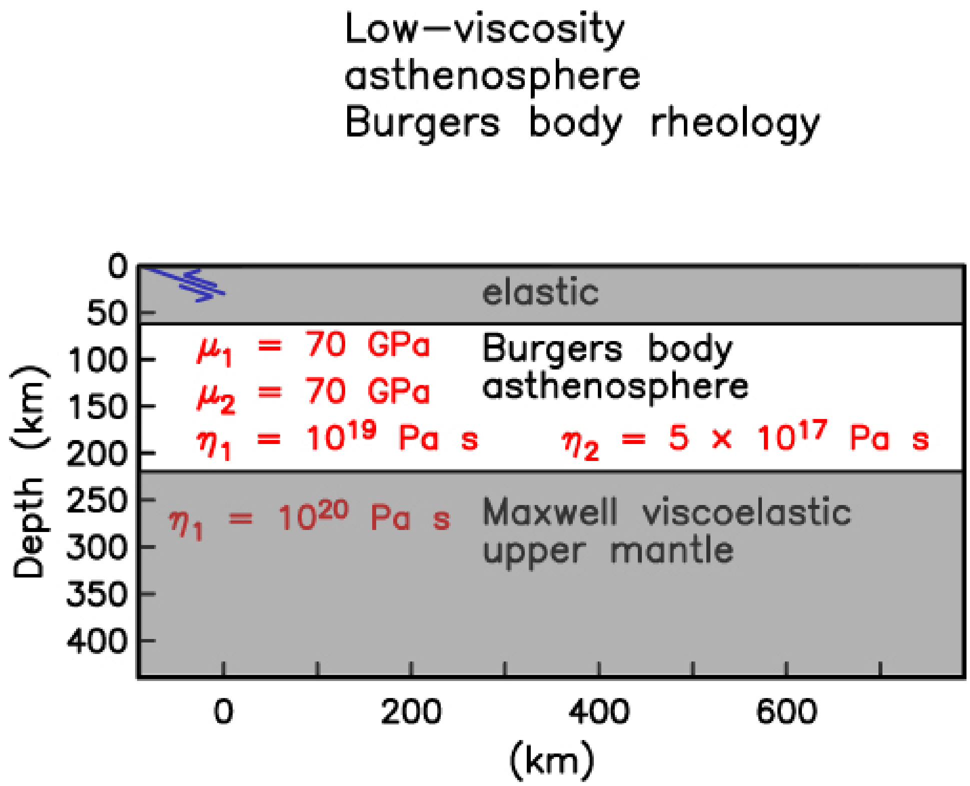

Figure 9: Viscoelastic stratification of Example 4. The elastic plate of thickness 62 km is underlain by a Burgers body asthenosphere from 62 to 220 km depth and a homogeneous Maxwell viscoelastic shere with indicated parameters.

different depths.

The toal postseismic displacement field resulting from this computation is shown in Figure 10.

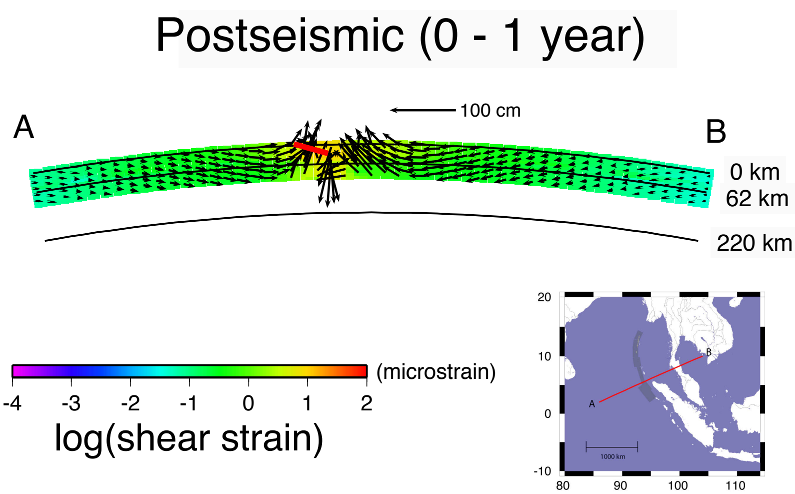

Figure 10: Vector postseismic displacement field resolved along the vertical profile indicated in the inset. Cumulative displacements are evaluated from the time of the December 2004 earthquake up to 1 year after the earthquake. The shear strain value is one-half the difference between the maximum and minimum principal strains. The red line segment is the trace of the Sumatra rupture at the profile location.

References

Banerjee, P., Pollitz, F., and Bürgmann, R. (2007). Coseismic slip distributions of the 26 December 2004 Sumatra-Andaman and 28 March 2005 Nias earthquakes from GPS static offsets. *Bull. Seismol. Soc. Am.*, 97.

Pollitz, F. F. (1997). Gravitational viscoelastic postseismic relaxation on a layered spherical earth. *J. Geophys. Res.*, 102:17921–17941.

Pollitz, F. F. (2003). Transient rheology of the uppermost mantle beneath the Mojave Desert, California. *Earth Planet. Sci. Lett.*, 215:89–104.