Mathematics Standards for *Engineering Applications*

The DoDEA high school mathematics program is built on courses which are grounded by rigorous standards. The process and content of standards for mathematics courses offered in DoDEA schools prepares students to become *College and Career Ready* upon graduation. The traditional course sequence begins with Algebra 1 and culminates with Advanced Placement courses in Calculus and Statistics. In addition, a series of student interest courses provide students with elective options.

Vision: DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

Guiding Principles

Standards:
- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectation for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

Instruction:
- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

Assessment/Accountability
- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent, and standards-based instructional program that leads to high student achievement.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:**	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:**	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:**	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:**
• build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	• recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	• organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	• recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	• create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

Our Vision for Engineering Applications with Commentary

DODEA students will take challenging academic courses that will be hands-on and based in real world experience. Learning activities support innovation, collaboration, and problem solving. Students are working in a technology-rich that includes probes and probeware, graphing calculators, computers equipped with spreadsheet, word processing, and presentation software, measurement tools, and lab equipment and tools to build, measure and test equipment. The learning experiences will focus on active exploration of mathematical concepts that integrate concepts and skills from algebra, geometry, and statistics. The course is a rigorous, focused, and engaging study of relevant, real world applications. Activities are designed so that they are accessible to students who have completed Algebra 1 but promote rich learning experiences for students who have competed Algebra 2 or higher-level courses as well.

As you read through these standards we would like you to keep in mind the essence of the course as depicted in the course description. This course is not designed to be a "paper and pencil" math course in which students read about an application and then do "word problems" in those contexts. Students are expected to measure, predict, model with mathematics, and then test their models. Mathematics is used as a tool and as a way of understanding. The course should provide challenges where students make conjectures about mathematical ideas and how things work and then create test cases to explore those conjectures. Further, students should recognize that theoretical models do not always work. They must be tested in the lab or in the field and course activities mirror this actuality.

The standards for Engineering Applications are taken from the Common Core State Standards. Most are used verbatim. Some of the broader standards have been broken into subcomponents for clarity of expectations. As you explore the standards, you will notice that DC circuits, forces, and motion figure prominently in the examples. These examples are not meant to limit the engineering contexts used in the course, but are examples of topics where the mathematics used in the field is a good match with the targeted mathematics standards.

Strand 1: Number

Quantities

Reason quantitatively and use units to solve problems.

EA.1.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas;

Example: The acceleration of an object is the rate at which the velocity is changing and is measured in m/s^2.   The following graph models the velocity of a braking automobile. Explain how you would describe this unit of measurement in this context. Determine the acceleration for each of the intervals (0,2), (2,5), (5,8) and describe what you think may be occurring with the automobile during the 8 second interval.

Velocity (m/s)

Time (s)
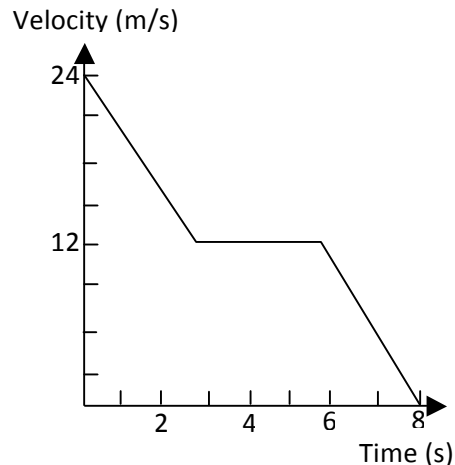
EA 1.2 Choose and interpret the scale and the origin in graphs and data displays.

Example: The equation $165 x^3 - 138x^2 + 432x + 10$ provides an algebraic model that describes the population of an ant colony where x is the number of months since the colony began. What are the window settings on your graphing calculator that would be appropriate to analyze the growth trends for the colony?

EA.1.3 Measure forces directly and compute with derived measurements.

Example: Design and conduct an experiment to verify that the equivalent stiffness K of two springs connected in series with stiffness of k_1 lb/in and k_2 lb/in is modeled in the formula $K = \frac{k_1 k_2}{k_1 + k_2}$.

Example: A car with mass 1200 kg is traveling at 10 meters per second and stops in 3 seconds. What is the braking force in Newtons? The same car, traveling at the same rate hits a brick wall and stops in 0.25 seconds. What is the force on the car?

Example: A cart weighs 250 grams. Three 1-kg weights are put in the cart. The cart is pulled up a ramp with a 10% grade. What force is needed to pull the cart discounting acceleration? Test your calculation in the lab. Why do you think there may be differences between your prediction and your result?

EA.1.4 Use dimensional analysis when organizing solutions to equations and making conversions among measurement units.

Example: The volume of a wooden block is 5.80 in^3. What is the volume of the block using cm^3?

Example: What is the density of mercury (13.6 g/cm^3) in units of kg/m^3?

The Complex Number System

For complex numbers the emphasis is on representations in the complex plane and for modeling vector quantities.

Perform arithmetic operations with complex numbers.

EA1.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

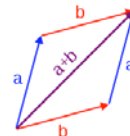Example: Show that the product of a complex number with its conjugate is a real number.

Represent complex numbers and their operations on the complex plane.

EA1.3. (+) Represent complex numbers on the complex plane in rectangular form.

Example: The complex conjugate of a complex number is a reflection about the real axis. Demonstrate this concept by graphing $3 + 2i$ and its complex conjugate in the complex plane.

EA1.4. (+) Represent addition, subtraction and multiplication of complex numbers geometrically on the complex plane; use properties of this representation for computation.

Example: The commutative property for addition of complex numbers is illustrated in the following vector representation a + b. Use a similar illustration to determine the validity of commutativitiy for the subtraction of two complex numbers.

EA.1.5 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoint.

Example: Show that the distance between a complex number and its reflection about the imaginary axis is the difference between the real components of the two complex numbers.
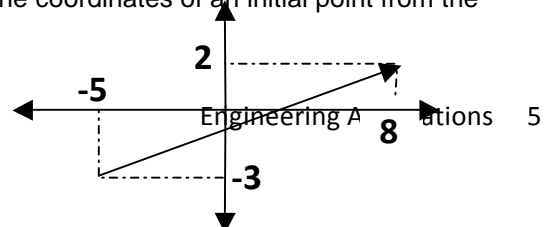
Vectors

Represent and model with vector quantities.

EA.1.6. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$, v).

Example: The following illustration provides a pictorial view of the four forces acting on a rock that is being dragged up a slope. The second diagram provides vector representation for three of the forces. What could be said about the fourth force (pull) if you are to move the rock up the slope?
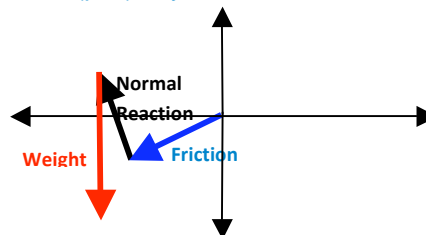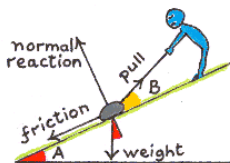
EA.1.7. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

Example: Determine the components of the vector shown.

EA.1.8. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Example: A plane is flying at an air speed of 100 mph at a heading 300° south of east. If there is a wind
blowing due west at 20 mph, what is the velocity of the plane with respect to the ground?

Example: Determine the components of the vector shown.

$v=50\ m/s$

60°
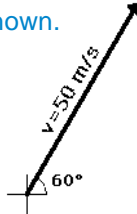
Perform operations on vectors.
EA.1.9. (+) Add and subtract vectors.
a. Add vectors end-to-end, component-wise, and by the parallelogram rule.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same
magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by
connecting the tips in the appropriate order, and perform vector subtraction component-wise.

Example: Three strings are attached to a ring. They are pulled on a flat surface (they are coplanar). One
string is pulled with a force of 50 N. Another string is pulled with a force of 20 N at an angle of 90 degrees
relative to the first string. What is the predicted direction and magnitude of a force needed to balance
these forces? Test your solution with string, three scales and a protractor.

Example: When a 50-pound weight hangs straight down on a cable, the force on the cable is 50 pounds.
A traffic light weighs 50 pounds and is suspended from a cable across the street as shown. What is the
magnitude of the force (also called the tension) on the cable in each direction? Is this what you expected?
(Big idea – The magnitude of the sum of vectors is rarely equal to the sum of their magnitudes.)

50 lbs.

50 lbs.

?   ?

7°   7°

50 lbs.
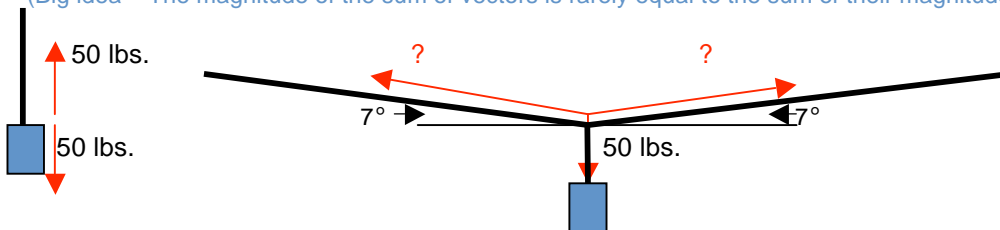
EA.1.10. (+) Multiply a vector by a scalar.
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction;
perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing
that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Example:

Velocity is a vector. Measure the average velocity of a robot at a power setting. Point the robot in a set
direction. Use vector notation and units to describe the velocity of the robot. Change the power setting
but keep the initial direction the same. What do you predict the vector value of the velocity will be? Test
it. Keep the initial direction the same. Go back to the original power setting but put the robot in reverse.
How do you predict the vector will be the same? How will the vector be different?

Strand 2 -ALGEBRA

Seeing Structure in Expressions
Interpret the structure of expressions
EA.2.1. Interpret expressions that represent a quantity in terms of its context. including parts of an expression, such as terms, factors, and coefficients including Interpreting complicated expressions by viewing one or more of their parts as a single entity.

Example: When designing a cylindrical can to hold a specific volume how can one compute the surface area of the can using the circumference of the can?

Example: The height of a sky diver is modeled by the expression $6000 + 2t - 16t^2$. Where t is time in seconds and distance units are in feet. What do the 6000, 2, and -16 represent?

EA.2.2. Use the structure of an expression to identify ways to rewrite it.

Example: The ideal mechanical advantage for a screw is defined as h where C is the circumference of the screw in inches. The pitch is defined as 1 where *TPI* represents threads per inch. Write an expression for the mechanical advantage of a screw using circumference and threads per inch.

Creating Equations
Create equations that describe numbers or relationships
EA.2.3. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Example: Based on your experiments with resistors in series and parallel, create equations that describe total resistance of a circuit as more identical resistors are added to the circuit.

EA.2.4. Create equations in two or more variables to represent relationships between quantities; and graph equations on coordinate axes with labels and scales.

Example: Use a rheostat to vary the resistance in a 6 volt circuit. Measure the current in the circuit for each resistance value. Plot the current in amps as a function of the resistance in ohms. Be sure to include units in your graph and choose a scale that best displays the trend of the data. What kind of relationship seems to be present? How would you write this relationship as an equation? Plot your equation on your graph? How well does it seem to match your data?

EA.2.5. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Example: Before you design your product, take market situations and production costs in mind. At what point will the number of units produced and the selling price maximize your profits? You will need equations or inequalities relating demand to the selling price, and equations predicting the cost per unit depending on the number of units produced.

EA.2.6. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Example: Students rearrange Ohm's law V = IR to highlight resistance R or students transform the formula for velocity to find the time given initial velocity, acceleration, and distance traveled and use the resulting formula in a spreadsheet to perform repeated calculations to predict the movement of a model car.

Reasoning with Equations and Inequalities
Solve equations and inequalities in one variable

EA.2.8. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Examples: In a DC circuit, what resistance is needed to create a current between 10 and 12 milliamps when the circuit is attached to a 9-volt battery? Use the formula $V = iR$

EA.2.9. Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Example: An iron ball with a mass of 2 kg is 20 meters off the ground. How fast will it be moving just before it hits the ground in meters per second? Use 9.8 m/sec^2 as the acceleration due to gravity and the formulas PE = mgh and KE = $12\ mv^2$ to guide your work.

Solve systems of equations
EA.2.10. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Example: A toaster with 2 resistors and 10 meters of coils has a total resistance of 30 ohms. There is a break in the coil and the shorter section is removed to that the toaster still has 2 resistors but only 8 meters of coils. The repaired toaster has a resistance of 25 ohms. What is the ohm rating of each resister and the resistance of the coil in ohms per meter?

EA.2.11. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Example: A straight truss is set at a 45 angle. It is under the vertex of a parabolic arch. How far off the ground will it intersect the arch if an equation for the arch is y = 0.5x2 + 24, where distances are in feet?

Strand 3 - FUNCTIONS

Construct and compare linear, quadratic, and exponential models and solve problems
EA.3.1. Distinguish between situations that can be modeled with linear functions and with other functions including recognizing situations in which one quantity changes at a constant rate per unit interval relative to another.

Example: As you investigate the motion of your car as it rolls down the ramp, what do you notice about the distance the car rolls as a function of time? What do you notice about the speed of your car as a function of time?

Example: As you pull a cart up a ramp, what happens to the force needed to pull the car as the height of the ramp changes? As you pull a cart up a ramp, what happens to the force needed to pull the car as the weight of car changes but the angle of the ramp is unchanged?

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Interpret expressions for functions in terms of the situation they model
EA.3.2. Interpret the parameters in linear and other function types in terms of a context.

Example: The height above ground level of two model rockets are determined by the equations

Model periodic phenomena with trigonometric functions

EA.3.3. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Example: Look at the sound wave samples captured using an electronic probe. Capture the points and then use technology to approximate the sound with a sine function by experimenting with the constant values of the sine function   = +   (+).

Strand 4 - GEOMETRY

Similarity, Right Triangles, and Trigonometry G-SRT
Prove theorems involving similarity
EA.4.1. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Examples:
A construction team is setting concrete forms for a rectangular pad. The dimensions of the pad are to be 30 feet by 20 feet. How can the construction foreman use a tape measure to be sure that the sides are the correct length and that they meet at right angles?

Julie wants to put a horizontal support in a triangular brace. The triangular brace is 6 feet long and sticks out 3 feet. The horizontal piece will be 15 inches long. How far up the brace will it be?
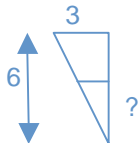
Define trigonometric ratios and solve problems involving right triangles
EA.4.2. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Examples:
This wedge is in the shape of an isosceles triangle.
Two sides are 8 inches in length and the base has a length of 2 inches.
What is the ideal mechanical advantage of the wedge?

EA.4.3. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Example: Use the Law of Sines and the Law of Cosines to find the magnitude and direction of the resultant force for these two vectors.

50 N   120°   30 N

Strand 5 - DATA

Summarize, represent, and interpret data on two categorical and quantitative variables

EA.5.1 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Example: Experiment with a first class lever. How does the force required to lift a weight change as the position of the fulcrum changes? How is the force required to lift an object related to the distance from the fulcrum to the object? Is this a linear relationship? Do another experiment where you change the length of the lever but maintain the distance from the fulcrum to the object lifted. How are the changes in the force required to lift the object similar? How are they different?

EA.5.2 Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and trigonometric models.

Example: Use a light intensity probe to measure the intensity of light projected on a surface changes as the wattage of the bulb increases. Use technology to find a function that matches your data. Explain the values of your function and the behavior of the function in terms of designing a lighting plan. Perform a second experiment where instead of changing the wattage of the bulb, you vary the distance between the bulb and the surface to be lighted. How does the function match the geometry of the situation? How can you use this information in a lighting plan?

EA.5.3 Informally assess the fit of a function by plotting and analyzing residuals.

Example: Plot the distance a robot travels when it is programmed to go for 1, 2, 3, 4, 5, 6, 7, 10, and 15 seconds as a function of the number of seconds it travels. Plot a best fit line for the function. Draw the residuals (the vertical segment from each point to the line). Can you change your line to make the sum of the residuals smaller? Which points have the greatest residual? Can you suggest an explanation for points that do not match the line well?

EA.5.4 Fit a linear function for a scatter plot that suggests a linear association.

Example: Plot the distance a robot travels when it is programmed to go for 1, 2, 3, 4, 5, 6, 7, 10, and 15 seconds as a function of the number of seconds it travels. Plot a best fit line for the function. Use your knowledge of linear functions to write the equation of your best fit line. What distance would your line predict for 8 seconds, 20 seconds and one minute?

Interpret linear models

EA.5.5 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Example: To predict the temperature of a room at a given time on Monday morning the building engineer uses the formula $T = 3.5h + 55$ where T is the temperature in degrees Fahrenheit and h is the time in hours. What does the 3.5 stand for? What does the 55 stand for? What time does the engineer need to start heating the building so that it is 70° by 7:45?

EA.5.6 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Example: Create a ramp that meets the angle requirements set by the ADA. Use a spring scale to measure the force needed to pull the cart up the ramp with different amounts of weight in the cart. Use Excel to plot the force needed as a function of the weight carried by the cart. Does the plot appear to be linear? If so, use Excel to calculate the correlation coefficient. What does this number tell you about the relation between the weight in the cart and the force needed to pull the cart up the ramp? Based on the correlation coefficient, how closely will an equation of a best fit line model the data?