ADVANCED FUNCTION STANDARDS

The DoDEA high school mathematics program is built on courses which are grounded by rigorous standards. The process and content of standards for mathematics courses offered in DoDEA schools prepares students to become *College and Career Ready* upon graduation. The traditional course sequence begins with Algebra 1 and culminates with Advanced Placement courses in Calculus and Statistics. In addition, a series of student interest courses provide students with elective options.

Vision: DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

Guiding Principals

Standards:

- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectation for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

Instruction:

- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

Assessment/Accountability

- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent, and standards-based instructional program that leads to high student achievement.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels

Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

Advanced Functions Standards
Guidance and Commentary

As you read through these standards we would like you to keep in mind the essence of the course as depicted in the course description. This course is designed to help students make connections between functions and real world applications through activities, modeling, and extensive conversations. The course should be taught with a decided emphasis on the application of the functions in real world contexts that would be of interest to a HS student. In light of 21st Century's Tenets of Learning that curriculum be interdisciplinary, project and problem based, and uses appropriate technology, the goal is to produce standards that will allow students to make those connections through modeling.

Rather than repeat the same strands for each of the identified functions, the standards have been grouped by the processes of interpreting and then building and modeling them. It is expected that each standard will be applied to all of the functions as identified in the strand.

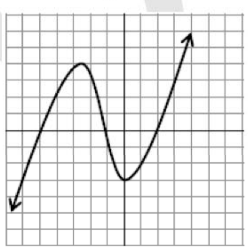
The course should provide the opportunity for the distributed practice of those function concepts previously learned in Algebras I and II. This should deepen their understanding of the concepts as well as enhance their problem solving skills and so prepare them for a college math course or provide them the necessary career ready skills.

Strand AF.1: Interpreting Functions
Students understand the concept of a function and use function notation for Linear, Quadratic, Polynomial, Exponential, Power, Recursively- defined, Trigonometric, and Logarithmic models.

AF.1.01 Identify and create models to apply the concepts of functions to real world models.

Example: The number of marbles that fit in a jar is modeled by the equation $N = 47 * 10-0.05x$, where x is the diameter in inches of one of the marbles. Graph this function and determine the size of the marble when the jar is filled with 26 of them.

Example: Find an equation for the graph:



AF.1.02 Describe characteristics of functions, and translate among verbal, numerical, graphical, and symbolic representations of functions, including linear, quadratic, polynomial, power, exponential, and logarithmic functions;

Example: Compare the graph of the exponential function $F(x) = e^x$ when x is positive with the graph when x is negative, when x is between 0 and 1.

Example: A college offers high-speed internet in dorm rooms. The monthly access fee in dollars is calculated using the function $A(m)=15+0.02m$

AF.1.03 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Example: A manufacturer of MP3 players has fixed daily costs of 15,700 Chinese Yuan, and it costs 178 Yuan to produce one MP3 player. If the manufacturer produces *x* players daily, express the daily costs C in Yuan as a function of *x*.

Example: Every quarter (3 months) John replaces 600 ml. in his aquarium. He has discovered that 20% of the water has evaporated during the quarter. Write a recursive rule to illustrate the amount of water in the tank after the nth quarter. If the tank holds 5200 ml. and John does a cleaning when the water level is 75% of capacity, how often will he have to clean his aquarium?

AF.1.04 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.

Example: Suppose that you are offered a job with a starting annual salary of $40,000 and annual increases of 4% of the current salary.

A. Write out the first six terms of a sequence a_n whose terms describe your salary in the first 6 years on this job.
B. Write the general term of the sequence in part A.
C. Find the value of the series $\sum_{n=1}^{6} a_n$. What does this number represent?

Example: Determine the type of function best models the following situations: determining the amount in a savings account that issues simple interest; the diminishing return on the amount of studying time with relationship to a grade attained on a test; the amount of lava flow from a volcano over a given period of eruption; the population growth of species in a given environment.

AF.1.05 Determine the type of function that best fits the context of a basic application (e.g., linear to solve distance/time problems, quadratic to explain the motion of a falling object, exponential to model bacteria growth, piece-wise to model postage rates, or absolute value functions to represent distance from the mean);

Example: Determine what type of function best models the following situations: determining the amount in a savings account that issues simple interest; the diminishing return on the amount of studying time with relationship to a grade attained on a test; the amount of lava flow from a volcano over a given period of eruption.

Example: Graph linear, quadratic, polynomial, exponential, power, and logarithmic functions and show intercepts, maxima, minima, zeros, asymptotes, and end behavior as applicable.

AF.1.06 Graph functions expressed symbolically and show key features of the graph. Graph linear, quadratic, polynomial, exponential, power, and logarithmic functions and show intercepts, maxima, minima, zeros, asymptotes, and end behavior as applicable.

Example: To convert a Fahrenheit temperature to a Celsius scale you can use the following formula: $T_c = 5/9\ (T_f - 32)$. Determine the inverse that would allow you convert from Celsius to Fahrenheit.

Example: The air pressure P at sea level is about 14.7 pounds per square inch. As the altitude h (in feet above sea level) increases, what happens to the air pressure? The relationship between air pressure and altitude can be modeled by the equations $P= 14.7\ e^{-.00004h}$. Graph the model and determine what the air pressure is at the top of a mountain that has a height of 18,000 feet.

AF.1.07 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Example: The heat generated (h_g) by a long distance runner is a function of height (H) of the runner, and the average speed of the runner (V). This relationship is modeled by the function $h_g = kH^3V^2$ where k is a constant. Create a form of the model that will allow you to determine the constant value k.

Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

AF.1.08 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

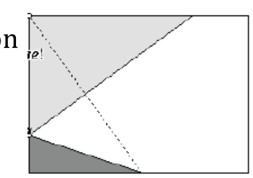
Example: Forest rangers estimate the height of a tree by measuring the tree's diameter at breast height (DBH) and then using a model constructed for a particular species. A model for sugar maples is $h = 2.9d + 30.2$ where d is the DBH in inches and h is the tree height in feet. Graph the function and answer the related questions below:

a. Interpret the slope of this model.
b. What is the effect of a 1 inch increase in DBH?
c. How tall is a sugar maple with a DBH of 3 inches? Round answer to the nearest foot.
d. What is the DBH of a sugar maple that is 45 feet tall? Round answer to the nearest inch.

Strand AF.2: Building Functions and Models
Students can identify and create functions and models that can apply the concepts of the functions to the solving of real world problems.

AF.2.01 Write an algebraic model to describe the relationship between two quantities in a given context.

Example: Take an 8 x 11.5 sheet of paper and lay it in landscape view. Select a point on the bottom edge of the paper and fold the top right corner to the point selected. You will notice a small triangle formed in the bottom left corner. Create a function model that will allow you to determine the area of the triangle based on the position of the point. Use your model to determine where the point should be located to produce the triangle with the largest area.



AF 2.02 Use characteristics such as domain, range, zeros, and inverse of a function to build a function.

Example: Graph the function $f(x)= 5^x$ and $g(x)=\log_b x$ to help describe characteristics of each function and the graphical relationship between these inverse functions.

Example: Determine an equation which models a rational function with a vertical asymptote x = -1; slant asymptote y = 2x – 2.

AF.2.03 Collect data, generate an equation, and use the equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models of real-world problems or applications.

Example: Rex collected the data in the table that represents the size of a bar of soap after daily use. Use this data to determine an algebraic model for that describes the way that the weight ofa bar of soap decreases over time.

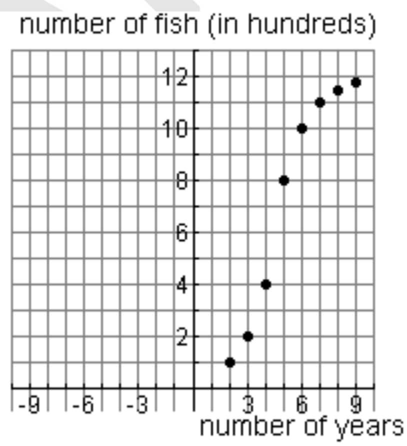
Date	Day	Weight
"30Aug99	0	124
"31Aug99"	1	121
"3Sept99"	4	103
"4Sept99"	5	96
"5Sept99"	6	90
"6Sept99"	7	84
"7Sept99"	8	78
"8Sept99"	9	71
"10Sept99"	11	58
"11Sept99"	12	50
"16Sept99"	17	27
"18Sept99"	19	16
"19Sept99"	20	12
"20Sept99"	21	8
"21Sept99"	22	6

Example: Have students take pictures of various running water fountains to analyze the path of the stream of water. Have them test a mathematical model that they generate on an untested fountain.

Example: The day length in daylight hours varies through the year in a sine wave. The longest day of of the year is on Day 172 and the shortest day is on Day 354. Find the number of hours for each of these days. Sketch a graph of this function, find its formula and determine which other day has the same length as July 4?

AF.2.04 Use knowledge of transformations to write an equation, given the graph of a function.

Example: The graph represents the population in a pond. Determine an algebraic model to represent the relationship between the population and the number of years since it was first stocked. Create new model (algebraically and graphically) that would represent to situation if the pond was stocked two years earlier and determine the population of the pond be today using the new model?



AF.2.05 Analyze the effect of introducing a parameter k to the function f(x) (i.e. f(x) + k, kf(x), f(kx), and f(x+k).

Example: Describe the transformation from $f(x) = x^3$ to $g(x)= 5 - 2(x^2-3)$

AF.2.06 Combine functions arithmetically and through composition to create new functions that model real world contexts.

Example: Use the graphs of $f(x)= x^2-2x+5$ and $g(x)= 1.5 - x^3$ to describe the graph of f(g(x)) in the first quadrant.

Example: An appliance store has a promotion program that allows a customer to select a scratch-off card that will reveal either a 20% discount, $100 dollars off, or both the discount and reduced price. a) Using function notation, write a function D that models the 20% discount, a function R that reduces the price by $100. b) If a customer is fortunate enough to reveal the combined promotions, determine when it would be advantageous to use D(R(x)) verses R(D(x).

AF2.07 Model different situations with a variety of functions (e.g. linear, quadratic, exponential, logarithmic, power, and piecewise) and determine the type of function that best fits the context.

Example: You put $100 in your bank account today, and then each day put half the amount of the previous day (always rounding to the nearest cent). Using summation notation, write a model for this situation. Will you ever have $250 in your account?

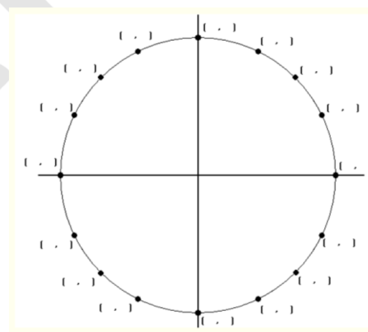
Trigonometric Functions
Build an understanding of periodic functions using trigonometric functions

AF.3.01 Use right triangles to geometrically determine values for sine, cosine, and tangent.

Example: An airplane is headed on a bearing of 120^0 at an airspeed of 400 km per hr. A 19 km per hr wind is blowing from a direction of 180^0. Find the ground speed of the airplane.

AF.3.02 Use the definition of radian measure of an angle as the length of the arc on the unit circle in a coordinate plane to to extend the understanding of trigonometric functions beyond right triangles.

Example: Fill in each of the point coordinates on the given standard s unit circle, and clearly label each point with the radian measure of the corresponding central angle.



AF.3.03 Use trigonometric functions to solve problems in a real world context.

Example: Determine a formula for the area of a triangle using the sine function when an two sides of the triangle and the angle between them are known.

Example: Suppose one gets two pairs of identical polarized sunglasses, and puts the left lens of one pair atop the right lens of the other, both aligned identically. If one pair is rotated, the amount of light that gets through is observed to decrease until the two lenses are at right angles to each other, when no light gets through. When the angle through which the one pair is rotated is θ the proportion of light that penetrates is $\cos^2 \theta$. Determine at what angle the lens is to be rotated to achieve 50% of the light penetration (25%?, 10%?).