Mathematics Content Strands

M1 Numbers and Operations

Number pervades all areas of mathematics. The other four Content Standards as well as all five Process Standards are grounded in understanding number. Central to this standard is the development of number sense, which allows students to naturally combine or decompose numbers, solve problems using the relationships among operations and knowledge of the base-ten system, and make a reasonable estimate for the answer to a problem.

Computational fluency – having and using efficient and accurate methods for computing – is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms. All students should use calculators at appropriate times, setting the calculator aside when the instructional focus is on developing computational algorithms.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

M2 Algebra

The ideas of algebra are a major component of the school mathematics curriculum and help to unify it. Mathematical investigations and discussions of arithmetic and its properties frequently include aspects of algebraic reasoning. Such experiences present rich contexts and opportunities for enhancing mathematical understanding and are an important precursor to the more formalized study of algebra in the middle and secondary grades. A strong foundation in algebra should be in place by the end of the eighth grade, and all high school students should pursue ambitious goals in algebra.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

M3 Geometry

Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades.
Geometry should be learned using concrete models, drawings, and dynamic software. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

M4 Measurement

The study of measurement is crucial in the K-12 mathematics curriculum because of its practicality and pervasiveness in many aspects of everyday life. Measurement is possibly the area of mathematics that is most important when considering everyday applications of mathematics, and highlights connections between mathematics and areas outside of the school curriculum such as social studies, science, art, and physical education. The study of measurement helps students establish connections within mathematics and provides an opportunity for learning about and unifying ideas concerning number and operations, algebra, geometry, statistics, probability, and data analysis.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability

To analyze data and reason statistically are essential to be an informed citizen, employee, and consumer. The amount of statistical information available to help make decisions in business, politics, research, and everyday life is staggering. Through experiences with the collection and analysis of data, students can learn to make sense of and interpret information and allow them to make appropriate arguments and recognize inappropriate arguments as well.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

Problem Solving	**Reasoning and Proof**	**Communication**	**Connections**	**Representation**
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

Strand: **M1** **Numbers and Operations**

Standards: In Grade 7, all students should:

7.M.1a: use, interpret, and compare numbers in several equivalent forms such as integers, fractions, decimals, and percents;

Example: Represent the number of Red stripes found on the U.S. flag as a fraction, ratio, decimal, and percent of the total number of stripes.

7.M.1b: solve problems that involve percent greater than 100 or less than 1;

Example: If you invest money in a savings account at 3% annually, determine the relationship of the amount in the account at the end of one year to the amount or the original deposit.

7.M.1c: identify and use ratio and proportion to represent quantitative relationships;

Example: "They measured my right thumb, and desired no more; for by a mathematical computation, that twice round the thumb is once round the wrist; and so on to the neck and the waist ..." Jonathan Swift's *Gulliver's Travels* includes some ratios that the Lilliputians used to estimate sizes in order to construct clothes for Gulliver. Determine the accuracy of the Lilliputian's measurements by comparing them to your personal measurements.

7.M.1d: describe the difference between rational and irrational numbers;

Example: For the numbers $\sqrt{2}$ and $\sqrt{9}$ explain which one is rational and which is irrational and why?

7.M.1e: calculate and find approximations of square roots;

Example: Which is the best approximation of $\sqrt{45}$; 6.2, 6.7, 7.1, or 7.5?

7.M.1f: explain the relationship, meaning, and effects of arithmetic operations with the set of integers;

Example: Explain the validity of the following statement: If you add any number to a positive integer, the sum is always positive.

7.M.1g: use order of operations and properties to simplify numerical expressions involving integers, fractions, decimals and exponents;

Example: Find the value of $(-2)^2 + 4\,(½ + 3)$.

7.M.1h: simplify numerical expressions and solve real-life problems using the set of integers;

Example: If the temperature in Anchorage is -12° on March 1 and follows the pattern of increasing 5° for 3 days then dropping 2° on the 4th day, what will be the temperature on March 31?

7.M.1i: estimate and solve problems including ratios, proportions and percents, and justify reasoning;

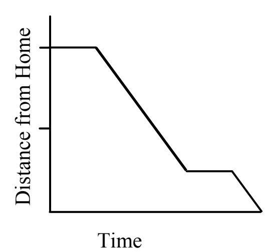
Example: If a store advertizes a sale of 50% off for a jacket on Monday and then reduces the price another 50% on Saturday, is the jacket now free? Why or why not?

Strand: **M2** **Algebra**

Standards: In Grade 7, all students should:

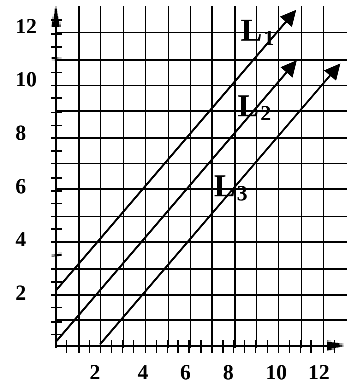
7.M.2a: represent and analyze relations and functions with tables, graphs, words, algebraic expressions, and equations;

Example: Write a story involving someone on a trip returning to their home that matches the information in the graph that is illustrated to the right.



7.M.2b: explain relationships between graphs of lines and the corresponding equation;

Example: Which of the following lines represents the equation Y Y= X + 2 ?



7.M.2c: generate equivalent forms of algebraic expressions by combining like terms;

Example: Show that 6x + 2 is equivalent to (7x -5) - (x + 3).

7.M.2d: use variables and operations to write an expression, equation, or inequality that represents a verbal description;

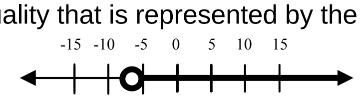
Example: Jerry noticed that to find the sum of the first 10 whole numbers he could add them in the following pairs: 1 + 10, 2 +9, 3 + 8, 4 + 7, 5 + 6 which is 5 groups of 11 to yield 55. Write an expression that will yield the sum of the first N whole numbers when N is an even number.

7.M.2e: model and solve equations using inverse operations;

Example: Solve the following equation for x: 4 x - 7 = 12

7.M.2f: represent linear equations and inequalities by plotting points;

Example: Write an inequality that is represented by the following set:



7.M.2g: analyze functional relationships to explain how a change in one quantity results in a change in the other;

Example: From the following table, determine the cost of 22 pencils.

Number of Pencils	Cost
5	$ 1.55
12	$ 3.72
19	$ 5.89
22	

7.M.2h: identify and explain the use of variables;

Example: Provide an example of how a variable can be used to represent the rule for a general arithmetic pattern.

Strand: **M3** **Geometry**

Standards: In Grade 7, all students should:

7.M.3a: identify and apply conditions that show two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures;

Example: If you cut out two congruent scalene triangles and flip one over, do they remain congruent? Explain your answer.

7.M.3b: use proportional reasoning to describe and express relationships between similar and congruent figures;

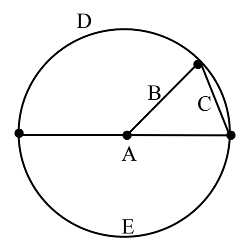
Example: Using square pattern blocks, Bill has created a 3X3 model of a rectangle. If Bob wants to make a similar model that is 4 times as large, how many squares will he need?

7.M.3c: classify and identify triangles by side and angle measurement and polygons as regular or irregular and/or by the number of sides;

Example: Explain why you can classify an equilateral triangle as an isosceles triangle but not as a scalene triangle.

7.M.3d: recognize and explain the following attributes of a circle, i.e., radius, diameter, arc, chord, semicircle, and central angle;

Example: Name each of the labeled parts for the circle in the figure.



7.M.3d: use coordinate geometry to represent special geometric shapes, such as regular polygons and polygons with pairs of parallel or perpendicular sides;

Example: Determine what shape if formed using the vertices (-2,2), (-2,-2), (2,-2), (2,2).

7.M.3e: determine the length of a side of a figure drawn on a coordinate plane with vertices having the same x or y coordinates;

Example: Find the area of a rectangle whose vertices are located at (-2,2), (-2,5), (2,-2), (2,5).

7.M.3f: demonstrate that congruence, similarity, and line or rotational symmetry of an object are retained in figures resulting from transformations.

Example: If a square has vertices of (-2,2), (-2,-2), (2,-2), (2,2) the x-axis and y-axis provide lines of symmetry. If the square is rotated 45^0 clockwise, describe the corresponding lines of symmetry.

Strand: **M4** **Measurement**

Standards: In Grade 7, all students should:

7.M.4a: select and use appropriate tools and units of measure when measuring and calculating angles, surface areas, and volumes of rectangular prisms;

Example: Measure the dimensions of a box and calculate the surface area and volume.

7.M.4b: analyze the structure and uniformity of the metric system and contrast with the customary system;

Example: List one advantage the metric system has over the customary system.

7.M.4c: identify and apply formulas to determine the surface area and volume of rectangular prisms;

Example: Determine how many centimeter cubes would fill a bracelet box that measures 5cm X 20cm X 4cm.

7.M.4e Recognize and differentiate between surface area and volume, and demonstrate that two objects may have the same surface area, but different volumes—or may have the same volume, but different surface areas;

Example: Find the dimensions of a rectangular prism that has the same volume as a 12 inch cube and compare their surface areas.

7.M.4f: use ratios and proportions to solve problems involving scale factors;

Example: Determine the number of square feet of carpet that will be needed to cover the floor of a room, if a blueprint uses the dimensions 4 in X 10 in for a room that is actually 10 feet wide.

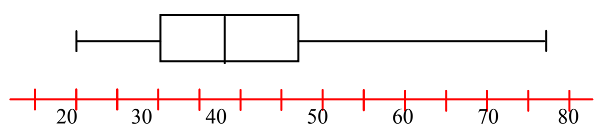
Strand: **M5**
Standards:

Data Analysis and Probability

In Grade 7, all students should:

7.M.5a create and interpret box and whisker plots, stem and leaf plots, scatter plots, and other appropriate types of graphs;

Example: The following Box and Whiskers plot provides the data for age ranges within a company. Determine the quartile an employee of age 35 would fall.



7.M.5b: analyze the effect of graphing decisions on graphical representation, e.g., scaling, types of graphs;

Example: Joy created a bar graph to represent the types of music preferred by her classmates. She decided that the bar graph did not allow her to compare a given genre to the rest of the data as a whole. What advantages are there to Joy using a pie chart?

7.M.5c: find, interpret, and appropriately use quartile, interquartile range, and outliers;

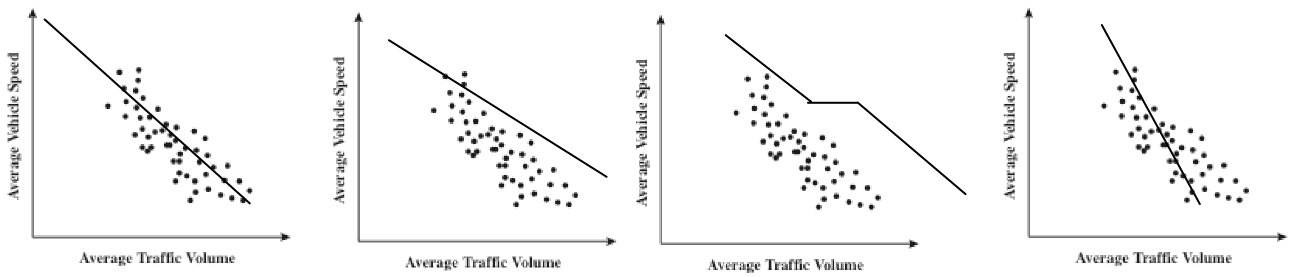
Example: What inferences can be made between two similar data sets if the interquartile range on the first set of data is nearly twice that of the second set of data?

7.M.5d: explain how measures of central tendency are affected by extremes;

Example: In trying to analyze which college majors would earn the most salary a researcher noticed that the Sociology, which included Michael Jordan whose earns multimillion dollars per year, seemed to be higher than some other professional degrees. What causes this skewing and how can it be accommodated in the data?

7.M.5e: find and make predictions based on the line of best fit;

Example: The data for the average traffic volume and average vehicle speed on a certain freeway for 50 days in 1999 has been put in a scatter plot. Which of the following shows the best line of fit?



7.M.5f: identify possible misuses of measures of central tendency;

Example: The weekly salaries of six employees at McDonalds are $165, $220, $100, $190, $100, $195. Which measure of central tendency would be a misrepresentation of the data?

7.M.5g: use proportionality and probability to make and test conjectures about the results of experiments and simulations;

Example: Which vowel do you think is used most often? Take a page of your history book and count the number of times each vowel is used to test your conjecture.

7.M.5h: describe multiple outcomes of compound independent events, e.g., using tree diagrams and organized lists;

Example: Determine all the possible outcomes if you spin each of the following spinners once.