## Mathematics: Pre-Kindergarten Through Grade 8

## Mathematics Content Strands

## M1 Numbers and Operations

Number pervades all areas of mathematics. The other four Content Standards as well as all five Process Standards are grounded in understanding number. Central to this standard is the development of number sense, which allows students to naturally combine or decompose numbers, solve problems using the relationships among operations and knowledge of the base-ten system, and make a reasonable estimate for the answer to a problem.

Computational fluency - having and using efficient and accurate methods for computing - is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms. All students should use calculators at appropriate times, setting the calculator aside when the instructional focus is on developing computational algorithms.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.


## M2 Algebra

The ideas of algebra are a major component of the school mathematics curriculum and help to unify it. Mathematical investigations and discussions of arithmetic and its properties frequently include aspects of algebraic reasoning. Such experiences present rich contexts and opportunities for enhancing mathematical understanding and are an important precursor to the more formalized study of algebra in the middle and secondary grades. A strong foundation in algebra should be in place by the end of the eighth grade, and all high school students should pursue ambitious goals in algebra.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.


## M3 Geometry

Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades.
Geometry should be learned using concrete models, drawings, and dynamic software. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.


## M4 Measurement

The study of measurement is crucial in the K-12 mathematics curriculum because of its practicality and pervasiveness in many aspects of everyday life. Measurement is possibly the area of mathematics that is most important when considering everyday applications of mathematics, and highlights connections between mathematics and areas outside of the school curriculum such as social studies, science, art, and physical education. The study of measurement helps students establish connections within mathematics and provides an opportunity for learning about and unifying ideas concerning number and operations, algebra, geometry, statistics, probability, and data analysis

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.


## Data Analysis and Probability

To analyze data and reason statistically are essential to be an informed citizen, employee, and consumer. The amount of statistical information available to help make decisions in business, politics, research, and everyday life is staggering. Through experiences with the collection and analysis of data, students can learn to make sense of and interpret information and allow them to make appropriate arguments and recognize inappropriate arguments as well.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.


## Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

| Problem Solving | Reasoning and Proof | Communication | Connections | Representation |
| :---: | :---: | :---: | :---: | :---: |
| Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: <br> - build new mathematical knowledge through problem solving; <br> - solve problems that arise in mathematics and in other contexts; <br> - apply and adapt a variety of appropriate strategies to solve problems; <br> - monitor and reflect on the process of mathematical problem solving. | Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: <br> - recognize reasoning and proof as fundamental aspects of mathematics; <br> - make and investigate mathematical conjectures; <br> - develop and evaluate mathematical arguments and proofs; <br> - select and use various types of reasoning and methods of proof. | Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: <br> - organize and consolidate their mathematical thinking through communication; <br> - communicate their mathematical thinking coherently and clearly to peers, teachers, and others; <br> - analyze and evaluate the mathematical thinking and strategies of others; <br> - use the language of mathematics to express mathematical ideas precisely. | Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: <br> - recognize and use connections among mathematical ideas; <br> - understand how mathematical ideas interconnect and build on one another to produce a coherent whole; <br> - recognize and apply mathematics in contexts outside of mathematics. | Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: <br> - create and use representations to organize, record, and communicate mathematical ideas; <br> - select, apply, and translate among mathematical representations to solve problems; <br> - use representations to model and interpret physical, social, and mathematical phenomena. |

## DoDEA Mathematics Standards: Grade 4

Strand:

Standards: $\quad$ M1 $\quad$| Numbers and Operations |
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| In Grade 4, all students should: |

4.M.1a: identify verbally and in writing the place value for each digit in whole numbers up to 1,000,000 and decimals between 0 and 1, up to thousandths;
Example: Write 463,022 for the number stated verbally or written as "four hundred sixty-three thousand twenty-two." Explain that the 3 in 463,022 represents 3 thousand or 3,000.
4.M.1b: identify and generate equivalent representations for the same number by decomposing and composing the whole number up to $1,000,000$;

Example: Using expanded form, write an equivalent representation for 263,754.
4.M.1c: judge the size of fractions by using benchmarks, e.g., $0,1 / 2,1$; and use the terms greater than, less than, or equal to or appropriate mathematical symbols >, <, or = to compare a fraction to the benchmark;

Example: Mrs. Johnson has a number line from 0 to 1 that hangs across the blackboard. At lunch time, someone came into class and took all the fractions off the number line! Help Ms. Johnson by putting the fractions back on the number line.

4.M.1d: identify and describe using their own words whole numbers according to their characteristics including primes, composites, and perfect squares;
Example: Between 4 and 14 find a number that is a square number; a prime number; a composite number.
4.M.1e: use models and symbols to recognize and generate equivalent forms of fractions, mixed numbers, and decimals;

Example: Using variety of models, show the relationship between the fraction $1 / 4$ and its decimal equivalent 0.25 .
4.M.1f: use models to represent division problems as the inverse of multiplication, as partitioning, or as successive subtraction and describe the meaning of remainders;
Example: Fifteen pieces of candy are to be shared between 4 friends. Draw a picture to model this problem. What is the meaning of the remainder?
4.M.1g: use models and equivalence to add and subtract fractions with like denominators of 12 or less;

Example: Use fraction pieces to model ${ }^{2} / 3-1 / 6$.
4.M.1h: use models to add and subtract decimals through thousandths;

Example: Use coins to help you find \$0.72-\$0.67.
4.M.1i: divide two-digit whole numbers by one-digit divisors;

Example: Divide 24 crackers equally among 6 children. Divide 24 crackers equally to find out how many children receive 6 cookies.
4.M.1j: use the inverse relationships of addition and subtraction, and multiplication and division to solve problems and verify solutions;

Example: Bill added 14 baseball cards to his collection. If he now has 132 cards, how many cards were in the original collection?
4.M.1k: use estimation to make predictions and check the reasonableness of result;
Example: You buy 2 CDs for $\$ 14.95$ each. The cashier tells you that will be $\$ 49.90$. Does that surprise you? Why or why not?
4.M.11: identify, compare, and order the relative position of fractions and decimals on a number line;
Example: Draw a number line and label it with 0 and 5 . Find the position of $1 / 2,3 / 10,6 / 5,2.70$, and 0.60 and label these numbers on the number line.
4.M.1m: demonstrate mastery in sums to 20 and related subtraction facts and multiplication through 12X12 and related division facts;

Example: multiply 9 times 8 without pencil and paper.

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Strand: M2 Algebra
In Grade 4, all students should:
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4.M.2a: use models and words to describe, extend, and generalize repeating, number, and growing patterns and relationships;
Example: To make brownies you need 1 tablespoon of water for every 3 tablespoons of flour. Explain how you can find the number of tablespoons of water needed if you have 27 tablespoons of flour.
4.M.2b: represent and analyze repeating, number, and growing patterns using words, tables, and graphs;
Example: In a school food drive a local grocery store will donate 1 can for every 2 cans purchased. Create a table to show the first 5 donated cans and write an expression that generalizes for any number donated.
4.M.2c: represent and describe mathematical relationships using algebraic expressions, equations, or inequalities with mathematical symbols;

Example: A falcon flies 3 times as fast as a hummingbird. Write an equation that represents this relationship.
4.M.2d: apply order of operations and the commutative and associative properties of addition and multiplication to numeric expressions;

Example: Solve the number sentence $8 \times 6-4 \div 2=$ ?
4.M.2e: use and interpret variables, mathematical symbols, and properties of addition and multiplication (e.g., commutative, associative, and the distributive property) to write and simplify mathematical expressions and sentences;
Example: A cab charges $\$ 2.00$ plus $\$ 0.75$ per mile. Write an expression to represent these charges.
4.M.2f: write and solve algebraic equations or inequalities using variables that represent problem situations;
Example: The Arbor Club gives 7 small trees to every new member. How many trees are needed for 13 new members? Write and solve an equation that represents this problem.
4.M.2g: identify and describe patterns of change to make predictions that identify the relationship represented in a table or graph.
Example: Liz has recorded the number of laps she swam this week. Describe the pattern and predict how many laps she would swim for the rest of the week.

| Day | MON | TUE | WED | THUR | FRI |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of Laps | 10 | 12 | 14 |  |  |

Strand:
Standard:

## M3 Geometry

In Grade 4, all students should:
4.M.3a: identify, draw representations, and describe the relationships between and among points, lines, line segments, and rays using appropriate mathematical tools; e.g., intersecting, parallel, and perpendicular lines;
Example: Use the markings on a football field to identify two lines that are parallel. Place a rope across the parallel lines and identify any acute angles created by the rope and the parallel lines.
4.M.3b: identify and draw representations of right angles, obtuse angles, and acute angles using appropriate mathematical tools;
Example: Using a protractor, draw a $90^{\circ}, 125^{\circ}$, and $45^{\circ}$ angle.
4.M.3c: identify and draw congruent figures using appropriate mathematical tools;

Example: Using a ruler and pencil, draw a rectangle that is congruent to a given rectangle.
4.M.3d: describe the results of subdividing, combining, and transforming shapes;

Example: Describe how you can transform a parallelogram into a rectangle in order to determine its area.

4.M.3e: find the distance between locations of points along horizontal and vertical lines of a coordinate grid;
Example: The grid below show the location of 3 areas at the County Fair. Determine the number of yards between the Crafts area and the Food area.

4.M.3f: predict and describe transformations (i.e., translation, reflection, and rotation) to show that two shapes are congruent;

Example: John wants to place one of the pentagons on top of the other to show congruence. Describe the transformations necessary to get one of the pentagons on top of the other.

4.M.3g: identify and describe line and rotational symmetry in twodimensional shapes and designs;

Example: What kinds of symmetries have the letters O and S ?
4.M.3h: identify geometric solids which could be composed of other solids;
Example: Identify the figures used to construct the building.


## Strand: <br> Standards:

## M4 Measurement

In Grade 4, all students should:
4.M.4a: recognize and describe that measurements are approximations;

Example: You are buying a ground cloth to cover the floor of your living room before you paint the room. How accurate should you be: to the nearest inch, foot, or yard? Explain in your own words how you know your answer is correct.
4.M.4b: measure with accuracy using both customary and metric systems of measurement;

Example: Measure the items listed below and record the measurement in both metric and customary. Be sure to record the units that you used.

Metric

1. Height of a desk
2. Length of your foot
3. Width of the classroom
4. Length of a pencil
5. Width of the chalkboard
6. Length of your fingernail
7. $\qquad$
(item of your choice)
4.M.4c: recognize that the area is the measure of the space enclosed by a two-dimensional figure and that angles are figures made by two rays with the same endpoint;

Example: Name the rays that make up angle x.

4.M.4d: determine the possible dimensions of rectangles when the area is constant;
Example: Using graph paper, draw a rectangle of area 24 units and label its dimensions (length and width). Can you draw other rectangles with the same area? If so, draw at least two other rectangles with different dimensions, and label the dimensions for each rectangle.
4.M.4e: estimate measurements of perimeter, area, and angle size;

Example: Given the rectangle with only one dimension, estimate the perimeter and area.

4.M.4f: use standard tools and units to include measure of perimeter and area of two-dimensional figures;
Example: Measure the sides of the rectangle and determine the perimeter and area.

4.M.4g: describe strategies to determine the perimeter and area of right triangles;

Example: Explain one way to determine the area of the triangle.

4.M.4h: describe strategies for estimating the area of irregular shapes;

Example: If the square has sides of 4 cm , explain how you could estimate the area of the irregular figure.

4.M.4i: solve problems involving perimeter and areas of rectangles;

Example: Bob has bought a new rug for his room. The rug is 3 ft . wide and 5 ft . long. Find the area of the rug.
4.M.4j: know the process for counting coins and bills using standards monetary notations;

Example: Mario has two rabbits. He buys food for $\$ 7.67$ and gives the clerk $\$ 10.00$. How much change should Mario receive? How many and what kinds of bills and coins should be handed to him?

## Strand: M5 Data Analysis and Probability <br> In Grade 4, all students should:

4.M.5a: describe how data collection methods affect the information that is gathered to address a question;

Example: If you were investigating the cost of airline fares, explain why you would want to examine more than one airline. How many airlines would you think appropriate?
4.M.5b: identify the median of a data set and describe what it indicates about the data set;

Example: The students in Ms. Janssen's class held a race yesterday to see who the fastest runner in the class was. Ms. Janssen kept track of everybody's time as they ran the length of the soccer field. When the race was over, the class made a graph of the results:
What is the median time and what does it indicate about the class times?

4.M.5c: use the median, mode, and range to compare and contrast the characteristics of related data sets;
Example: The students in Mr. Kleiman's class ran the same race. Their results are below. Compare the data with Ms. Janssen's class whose median time was 12 seconds and the range was 11-16 seconds. Which class was faster? Explain your reasoning.

| Student | John | Meg | Derek | Jess | Carl | Buzz | Beca | Jo | Ana | Julia | Desh |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Time | 12 | 13 | 10 | 12 | 12 | 14 | 18 | 14 | 10 | 13 | 14 |
|  | sec. | sec. | sec. | sec. | sec. | sec. | sec. | sec. | sec. | sec. | sec. |

4.M.5e: select the appropriate data representation form for a diverse set of investigations and justify the choice in each case;

Example: Conduct an experiment to find the heights of individual bean plants growing for 3 weeks for each student in your class. Conduct a survey to find the favorite summer activity for each student in your class. Decide whether to use a bar, line, or pictograph to display each set of data. Explain in your own words how you selected a representation for each data set.
4.M.5f: identify the likelihood of an event occurring as impossible, equally likely, and certain. Recognize the numerical values of 0 (impossible) and 1 (certain);

Example: What is the probability of rolling an 8 on a standard die?
4.M.5g: conduct experiments to determine experimental probability of an event occurring for a given number of trials (no more than 12 trials), using models;

Example: Determine the probability of the event that when the names of 12 classmates are put in a shoebox, a name that begins with $R$ will be drawn.
4.M.5h: list and count all possible combinations using one member from each of several sets;

Example: Ben wants to by some ice cream. He can choose from chocolate, vanilla, or strawberry and can have it in a cup, wafer cone, or waffle cone. List all the possible combinations from which he has to choose.

