Mathematics Content Strands

M1 Numbers and Operations

Number pervades all areas of mathematics. The other four Content Standards as well as all five Process Standards are grounded in understanding number. Central to this standard is the development of number sense, which allows students to naturally combine or decompose numbers, solve problems using the relationships among operations and knowledge of the base-ten system, and make a reasonable estimate for the answer to a problem.

Computational fluency – having and using efficient and accurate methods for computing – is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms. All students should use calculators at appropriate times, setting the calculator aside when the instructional focus is on developing computational algorithms.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

M2 Algebra

The ideas of algebra are a major component of the school mathematics curriculum and help to unify it. Mathematical investigations and discussions of arithmetic and its properties frequently include aspects of algebraic reasoning. Such experiences present rich contexts and opportunities for enhancing mathematical understanding and are an important precursor to the more formalized study of algebra in the middle and secondary grades. A strong foundation in algebra should be in place by the end of the eighth grade, and all high school students should pursue ambitious goals in algebra.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

M3 Geometry

Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades.
Geometry should be learned using concrete models, drawings, and dynamic software. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

M4 Measurement

The study of measurement is crucial in the K-12 mathematics curriculum because of its practicality and pervasiveness in many aspects of everyday life. Measurement is possibly the area of mathematics that is most important when considering everyday applications of mathematics, and highlights connections between mathematics and areas outside of the school curriculum such as social studies, science, art, and physical education. The study of measurement helps students establish connections within mathematics and provides an opportunity for learning about and unifying ideas concerning number and operations, algebra, geometry, statistics, probability, and data analysis.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability

To analyze data and reason statistically are essential to be an informed citizen, employee, and consumer. The amount of statistical information available to help make decisions in business, politics, research, and everyday life is staggering. Through experiences with the collection and analysis of data, students can learn to make sense of and interpret information and allow them to make appropriate arguments and recognize inappropriate arguments as well.

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

Strand: **M1** **Numbers and Operations**

Standards: In Grade 3, all students should:

3.M.1a: read, write, and interpret the place value for each digit in whole numbers up to 10,000;

Example: Write 463 for the number stated verbally or written as “four hundred sixty-three.” Explain that the 6 in 4<u>6</u>3 represents 6 tens or 60.

3.M.1b: read, write, and interpret the place value for each digit in decimal numbers between 0 and 1 through hundredths;

Example: Write .23 for the number stated verbally or written as “twenty-three hundredths.” Explain that the 3 in .2<u>3</u> represents 3 hundredths.

3.M.1c: identify and generate equivalent forms of common fractions less than one whole, e.g., halves, thirds, quarters, fifths, and tenths; and explain their relationship to a whole;

Example: Draw pictures to show that 3/5, 6/10, and 9/15 are equivalent fractions.

3.M.1d: identify odd and even numbers up to 10,000, identify factors or multiples of a given number, and describe their characteristics;

Example: Find the even numbers: 37, 108, 253, 540, 739

3.M.1e: compare whole numbers, up to 10,000, using mathematical symbols (>, <, =) and words (greater than, less than, equal to) and arrange them in numerical order;

Example: What is the smallest whole number you can make using the digits 3, 9, 1? Use each digit exactly once.

3.M.1f: explain the relationship between multiplication and division as inverse operations;

Example: Use the inverse relationship of $3 \times 2 = 6$ to find other related facts:
Find __ × 3 = __, __ ÷ 3 = __, and __ ÷ __ = 3.

3.M.1g: explain and solve problems involving the sum or difference of two whole numbers up to 10,000;

Example: Subtract 236 from 632. Explain your method.

3.M.1h: explain and solve problems involving the multiplication of two whole numbers where factors are 99 or less;

Example: Multiply 36 times 52. Explain your method.

3.M.1i: demonstrate and explain multiplication and division through the use of representations and mathematical symbols where factors are 99 or less;

Example: Karen shared 10 cookies among 3 friends. Draw a picture to show how many cookies each friend receives. Karen eats 1 cookie for herself.

3.M.1j: develop and use strategies to estimate the results of whole number addition and subtraction and verify;

Example: Joan says that 72 – 29 = 57. Using estimation, explain why you think Joan's answer is right or wrong.

3.M.1k: solve multi-step problems for using mathematical tools and strategies involving addition, subtraction, and multiplication;

Example: You have $5.00. Can you buy two books that cost $2.15 each? What is the highest price of book for which three can be purchased? Explain your answer.

Strand: **M2** **Algebra**

Standards: In Grade 3, all students should:

3.M.2a: identify, describe using their own words, and extend the rules of repeating, number, and growing patterns;

Example: What is the next number: 4, 8, 16, 32, … ? Explain how you found your answer. What is the 8^{th} number in this pattern or sequence? Explain your answer.

3.M.2b: make predictions, identify relationships, and solve problems by using a relationship between two quantities;

Example: A small cup of ice cream costs 40 cents. Find the costs of 2, 3, 4, … cups of ice cream. What is the pattern? Continue the pattern to find the cost of ice cream 15 students.

3.M.2c: demonstrate mathematical relationships of quantity in the form of numeric expressions, equations, or inequalities with mathematical symbols;

Example: Joe's mom gave him money to buy four drinks that cost 65 cents each. He gave 40 cents change to his mom. Write an equation to find the amount of money Joe's mom originally gave him.

3.M.2d: solve open sentences by representing an expression in more than one way using the commutative and associative properties of multiplication;

Example: Multiply 8, 3, 6 in this order. Now multiply them in the order of 6, 3, 8. Which is easier and why?

3.M.2e: organize and order data in tables to discover patterns and make predictions;

Example: Keenan just started a jogging program. He jogs 4 miles the first week, 8 miles the second week, and 12 miles the third week. If he continues in the same pattern for two more weeks, how many miles will he jog the fifth week?

3.M.2f: identify and describe the difference between qualitative and quantitative changes;

Example: A survey asking 3rd graders' favorite ice cream flavors includes chocolate, vanilla, cookies and cream, and strawberry. Will the data collected in the survey be qualitative or quantitative? Explain your answer.

Strand: **M3** **Geometry**

Standard: In Grade 3, all students should:

3.M.3a: identify and compare two-dimensional shapes and three-dimensional objects using attributes and properties, e.g., sides and angles in two-dimensional shapes, faces and edges in three-dimensional objects;

Example: Draw a triangle on your paper. Describe using mathematical attributes and properties of two-dimensional shapes how you know it is a triangle.

3.M.3b: identify and describe the relative size of angles using right angles as a reference;

Example: In a collection of 3 different triangles, describe the size of angles of each triangle as *greater than, less than*, or *equal to* right angles.

3.M.3c: use coordinate systems to specify locations and describe paths;

Example: How do you use a coordinate grid to find a specific location on a map?

3.M.3d: verify if two-dimensional shapes have symmetry by drawing vertical, horizontal, and diagonal drawing lines;

Example: Write the capital letters of the alphabet and draw all the lines of symmetry that you see. Describe in your own words how you know these are lines of symmetry.

3.M.3e: build, draw, and analyze two-dimensional shapes;

Example: Create a parallelogram from triangle pattern blocks.

Strand: **M4** **Measurement**

Standards: In Grade 3, all students should:

3.M.4a: describe the relationship of units within the customary and metric measurement systems;

Example: How many minutes are in 2 hours?

3.M.4b: use accurate vocabulary to describe measurements of length, volume, and weight;

Example: Measure a table and record your answer using the correct label of measurement.

3.M.4c: select units, strategies, and tools to estimate and calculate perimeter and area of two-dimensional shapes;

Example: Find the perimeter of the top of your school desk. Describe your strategy for finding your answer.

3.M.4d: use measurement tools and techniques to construct a figure;

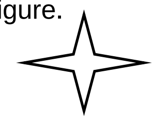
Example: Construct a cube. Describe the tools and methods you used.

3.M.4e: select standard units and tools to compare the attributes of two-dimensional shapes and three-dimensional objects;

Example: Compare a triangle and a pyramid.

3.M.4f: select units and develop strategies for estimating the perimeter of irregular two-dimensional shapes;

Example: Estimate the perimeter of this figure.



3.M.4g: measure temperature in Celsius and Fahrenheit;

Example: What is the temperature in the room?

3.M.4h: tell time to the nearest minute and find how much time has elapsed;

Example: You begin recess at 11:20 and finish recess at 11:43. How long is your recess?

Strand: **M5** **Data Analysis and Probability**

Standard: In Grade 3, all students should:

3.M.5a: develop and implement a plan to collect and organize data to address a given question;

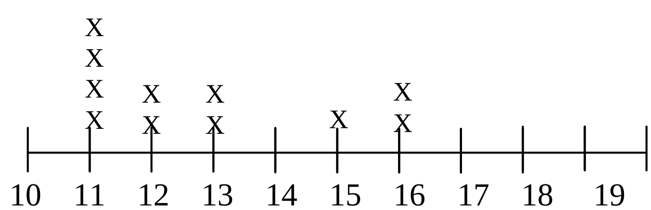
Example: Choose a question and develop a plan to collect data and analyze the results.

3.M.5b: translate information from one data representation to a graph or table, e.g., frequency table, bar graph, picture graph, line plot;

Example: Convert a tally chart into a bar graph.

3.M.5c: Analyze and interpret information by writing at least one statement to support a conclusion or prediction with evidence from data;

Example: Summarize the data from the following graph which represent third grade student times in seconds for a race.



3.M.5d: organize and graphically display data using categories and intervals;

Example: Using the rainfall data for the month, create a graphic display.

3.M.5e: describe characteristics of graphically represented data, e.g., median, mode, range;

Example: Using your rainfall graph, describe data characteristics you notice.

3.M.5f: examine graphs and tables that display the same set of data to identify what each representation contributes to the interpretation of data and conclusions drawn;

Example: Display the rainfall data in two additional ways and draw conclusions about the data.

3.M.5g: represent the possible outcomes for a simple probability situation;

Example: What is the probability of drawing a red marble from a bag containing four red marbles and three green marbles?