MATHEMATICS ANALYSIS STANDARDS

The DoDEA high school mathematics program centers around six courses which are grounded by rigorous standards. Two of the courses, AP Calculus and AP Statistics, are defined by a course syllabus that is reviewed and revised on an annual basis. The other 5 courses, Algebra 1, Algebra 2, Geometry, Discrete Mathematics and Pre-Calculus/Mathematical Analysis, have established standards designed to provide a sequence of offerings that will prepare students for their future goals. These standards serve as the foundation of a comprehensive effort to realize the vision for mathematics education of the students enrolled in DoDEA schools.

Vision: DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

Guiding Principals

Standards:

- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectation for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

Instruction:

- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

Assessment/Accountability

- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent, and standards-based instructional program that leads to high student achievement.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

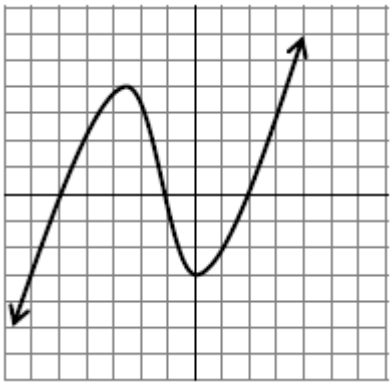
MATHEMATICS ANALYSIS/PRECALCULUS

Strand: **MA.1 Students analyze relations, functions and their graphs.**
Standards: Students in the Mathematics analysis/Precalculus course will:

MA.1.1 Describe characteristics of functions, and translate among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power, trigonometric, exponential, logarithmic, and piece-wise functions;

Example: The number of marbles that fit in a jar is modeled by the equation $N = 47 * 10^{-0.05x}$, where x is the diameter in inches of one of the marbles. Graph this function and determine the size of the marble when the jar is filled with 26 of them.

Example: Find an equation for the graph:



MA.1.2 Describe the behavior of a function including domain, range, intercepts, zeros, boundedness and unboundedness, extrema, intervals of increasing and decreasing values, and discontinuity and connect these concepts to functions represented graphically and numerically;

Example: Graph the function, $Y = \frac{1}{\sqrt{x}}$ **,** state the domain, range and equation of the asymptotes.

MA.1.3 Explain the fundamental theorem of algebra and determine solutions for a polynomial equation;

Example: Find the quartic equation that has solutions $5 - i\sqrt{3}$ and i .

Example: Determine the roots of the polynomial equation $y= 3x^3 + 5x^2 – x – 2$.

MA.1.4 Determine if a function is odd, even, or neither and describe its symmetry;

Example: Use the functions $f(x) = x^2$ and $g(x) = x^3$ to make an conjecture about the composition ($f \circ g(x)$)of an even function and an odd function.

MA.1.5 Write a rational function from its written description;

Example: Determine an equation which models a rational function with a vertical asymptote x = -1; slant asymptote y = 2x – 2.

MA.1.6 Solve rational equations and inequalities and verify the solutions to be certain there are no extraneous ones;

Example: The red wolf has been reintroduced into newly acquired park lands. The population is modeled by the equation $p(x) = (\frac{10(5+3t)}{1+0.04t})$, where t is the number of years since the introduction. Determine how much time it will take for the population will double.

MA.1.7 Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems;

Example: Determine the rate of change for the equation $x^2 + y^2 = 25$ at the points where x = -5, -2, 0, 2, and 5.

MA.1.8 Determine the inverse of a given function and describe its graph;

Example: To convert a Fahrenheit temperature to a Celsius scale you can use the following formula: $T_c = 5/9 (T_f - 32)$. Determine the inverse that would allow you convert from Celsius to Fahrenheit.

MA.1.9 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

Example: Describe convergence in the following sequence

$$A_n = 1 + \frac{(-1)^n}{n}$$

MATHEMATICS ANALYSIS/PRECALCULUS

Strand: **2.1** Students analyze Trigonometric functions.
Standards: Students in the Mathematics analysis/Precalculus course will:

MA.2.1 Define angles, describe their units of measure (both degrees and radians) and convert between degrees and radians;

Example: Convert 90º, 45º, and 30º to radians.

MA.2.2 Define trigonometric ratios using the unit circle with both degree and radian scales;

Example: Find an acute angle A for which sin $^{5\pi}/_{6}$ = sin A.

Example: Find the acute angle *A* for which sin 150º = sin *A*.

MA.2.3 Use the unit circle to determine exact sine, cosine, and tangent values for 0, $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$, $\frac{\pi}{6}$, and multiples of π. Use those values to find other trigonometric values;

Example: Find the values of cos $\frac{\pi}{2}$, tan $\frac{9\pi}{4}$, csc $\frac{2\pi}{3}$, $\sin^{-1}$ - $\frac{\sqrt{3}}{2}$, and sin 3π.

MA.2.4 Graph trigonometric functions and describe domain, range, intercepts, periods, amplitudes, and asymptotes of trigonometric functions. (i.e., sine, cosine, tangent, cotangent, secant, cosecant);

Example: Give the domain and asymptotes for tan x.

MA.2.5 Describe, interpret, and predict the effects of the parameters a, b, and c have on period, amplitude, and phase shift for the graph of y = a sin(b(x - c)); similarly for the cosine and tangent;.

Example: Describe the effect of y = 3sin θ – $\frac{\pi}{3}$ with respect to y = sin θ.

Example: Draw the graph of $y = 5 + \sin\left(x - \frac{\pi}{3}\right)$.

MA.2.6 Define and graph inverse trigonometric functions;

Example: Graph $\sin^{-1} x$.

Example: Find $\tan^{-1} = \sqrt{3}$

MA.2.7 Solve problems involving applications of trigonometric functions including those that require the use of angle sum and difference formulas, half-angle formulas and double-angle formulas for sines, cosines, and tangents;

Example: Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and use it to find a formula for $\sin 2x$..

Example: Prove that $\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$.

Example: A kickoff is executed from a team's own 35 yard line. If the path of the football can be modeled with the equation $D = {}^{1}/_{32} (v^2) \sin 2\theta$ where D is the distance and v is the initial velocity. Determine the angle that will result in the ball being kicked into the end zone.

Example: You want to find the width of a river that you cannot cross. You decide to use a tall tree on the other bank as a landmark. From a position directly opposite the tree, you measure 50 m along the bank. From that point, the tree is in a direction at 37º to your 50 m line. How wide is the river?

Example: In Virginia, the day length in daylight hours varies through the year in a sine wave. The longest day of 15 hours is on Day 172 and the shortest day of 11 hours is on Day 354. Sketch a graph of this function, find its formula and determine which other day has the same length as July 4?

MA.2.8 Apply the laws of sines and cosines to solving problems;

Example: Calculate the area of a triangle with sides of length 8 cm and 6 cm enclosing an angle of 60º.

Example: You want to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78º. From the second position, the angle between the mountain and the first position is 53º. How far is the mountain from each position?

Example: For a standard baseball field (square), the pitcher's mound is 46 feet from home plate and the distance between bases is 60 feet. Find the distance from the pitcher's mound to first base.

MA.2.9 Explain the basic trigonometric identity $\cos^2 x + \sin^2 x = 1$ and prove that it is equivalent to the Pythagorean Theorem;

Example: Use a right triangle to show that $\cos^2 x + \sin^2 x = 1$.

MA.2.10 Use basic trigonometric identities to verify other identities and simplify expressions;

Example: Simplify the expression $\cos\left(\frac{\pi}{3} - x\right) \cot x$

Example: Show that $\frac{\tan^2 x}{1 - \tan^2 x} = \sin^2 x$.

MA.2.11 Solve trigonometric equations.

Example: Solve $3\sin 2x = 1$ for x between 0 and 2π.