ALGEBRA 2 STANDARDS

The DoDEA high school mathematics program centers around six courses which are grounded by rigorous standards. Two of the courses, AP Calculus and AP Statistics, are defined by a course syllabus that is reviewed and revised on an annual basis. The other 5 courses, Algebra 1, Algebra 2, Geometry, Discrete Mathematics and PreCalculus/Mathematical Analysis, have established standards designed to provide a sequence of offerings that will prepare students for their future goals. These standards serve as the foundation of a comprehensive effort to realize the vision for mathematics education of the students enrolled in DoDEA schools.

Vision: DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

Guiding Principals

Standards:
- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectation for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

Instruction:
- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

Assessment/Accountability
- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent, and standards-based instructional program that leads to high student achievement.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels

Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

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Strand: **A2.1 Students analyze complex numbers and perform basic operations with them.**

Students investigate the relationship between complex numbers and other real numbers. Students analyze quadratic relationships with complex solutions.

Standards: Students in Algebra 2 will:

A2.1.1 Define complex numbers and perform basic operations with them;

Example: Explain why the product of a complex number and its conjugate yields a real number without an imaginary part.

A2.1.2 Demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically;

Example: Plot the points (2-4i) and (7+3i) and illustrate the relationship of the resulting sum with the original addends.

A2.1.4 Determine rational and complex zeros for quadratic equations;

Example: Give an example of a quadratic equation in standard form ($ax^2 + bx + c = 0$) with that has a complex root.

A2.1.5 Determine and interpret maximum or minimum values for quadratic equations;

Example: Determine the maximum rectangular area that can be enclosed using 16 yards of fencing.

Strand: **A2.2 Sequences and Series**

Students define and use arithmetic and geometric sequences and series to solve problems.

Standards: Students in Algebra 2 will:

A2.2.1 Use recursion to describe a sequence;

Example: Write the first ten terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$.

Example: Every quarter (3 months) John replaces 600 ml. in his aquarium. He has discovered that 20% of the water has evaporated during the quarter. Write a recursive rule to illustrate the amount of water in the tank after the nth quarter. If the tank holds 5200 ml. and John does a cleaning when the water level is 75% of capacity, how often will he have to clean his aquarium?

Example: Find a sequence whose third term is a function of the two preceding terms. Write a recursive rule for the sequence and give the first 10 terms.

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A2.2.2   Determine the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series;

Example: Find the 20th term of the arithmetic sequence 4, 11, 18, 29 … . Find the sum of the first 20 terms.

Example: Find the sum of 1 + 1⁄2 + 1⁄4 + 1⁄8 + 1⁄16 + … .

Example: From 1950 to 1989, the resident population of the United States can be approximated by the model

$a_n = \sqrt{22{,}926 + 902.5n + 2.01n^2}$, where an is the population in millions and n represents the year, with n=0 corresponding to 1950. Find the population in 1989.

A2.2.3   Explain and use summation notation to model an arithmetic series and of both finite and infinite geometric series;

Example: Find the sum of the sequence described by the following:

$$\sum_{k=3} k^2 \qquad\qquad \sum_{i=1}^{12} (3 + 2i)$$

Example: An auditorium has 20 rows of seats. There are 20 seats in the first row, 2 in the second row, 22 in the third row, and so on. Create a summation model for this situation that uses the summation notation.

A2.2.4   Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series;

Example: Prove that the sum of an arithmetic series Sn =

$$n\left(\frac{a_1 + a_n}{2}\right).$$

Example: Find the sum of the infinite geometric series described with the following notation:

$$\sum_{n=1}^{\infty} 15 \left(\tfrac{2}{7}\right)^{n-1}$$

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Example: Prove that $a + ar + ar2 + ar3 + ar4 + \ldots = \dfrac{a}{1-r}$ for $|r| < 1$

A2.2.5 Explain and use the concept of limit of a sequence or function as the independent variable approaches infinity or a number;

Example: Explain what happens to the terms in the sequence $a^n = \dfrac{2n}{(2n)^2}$ when n approaches infinity.

Example: You put $100 in your bank account today, and then each day put half the amount of the previous day (always rounding to the nearest cent). Using summation notation, write a model for this situation. Will you ever have $250 in your account?

A2.2.6 Solve word problems involving applications of sequences and series;

Example: A theater has 20 seats in the first row and each successive row will contain 1 more than the previous row. Determine the number of seats in the theater if there are 26 rows.

Example: It took more than 200 years for the U.S. to accumulate $1 trillion debt. The federal debt during the decade of the 1980s is approximated by the model $a_n = 0.1\sqrt{82 + 9n}^2$, where a is the debt in trillions and n is the year, with n = 0 corresponding to 1980. Using this model, determine in what year the debt was expected to reach $5 trillion.

Strand: **A2.3 Exponential and Logarithmic Functions**
Students analyze the inverse relationship between exponents and logarithms.

Standards: Students in Algebra 2 will:

A2.3.1 Use the definition of logarithms to translate between logarithms in any base;

Example: Change $\log_2 5$ into both natural and common logarithms.

Example: Approximate each logarithm to four decimal places, by translating them into the log base *e* :
$\log_2 5$
$\log_5 2$

Example: Explain which of the following is larger: log 40 or ln 40.

A2.3.2   Explain and use basic properties of exponential and logarithmic functions and the inverse relationship between them to simplify expressions and solve problems;

Example: Simplify the expression $\log_2 100 - 2\log_2 5$.

Example: Given • $f(x) = 4^x$, write an equation for the inverse of this function. Graph the functions on the same coordinate grid.

Example: Derive the formulas:

$\log_b a \cdot \log_a b = 1$

$\log_a N = \log_b N \cdot \log_a b$

Example: Find the exact value of x :

$\log_x 16 = 34$

$\log_3 81 = x$

Example: Solve for y in terms of x:

$\log_a \dfrac{y}{x} = x$

$100 = x \cdot 10^y$

A2.3.3   Explain and use the laws of fractional and negative exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay;

Example: Evaluate the following:

$$(-27)^{\frac{2}{3}}$$

$$16^{-\frac{3}{4}}$$

Example: Use properties of exponents to simplify

$$\frac{\left(2\, x^{\frac{2}{5}}\, y^{-\frac{3}{4}}\right)^5}{x^2 y}$$

Example: In 2002 the amount of grape juice consumed increased 6% from the previous year. If the rate continues to increase at a rate of 6% per year, how long would it take for the amount of consumption to double from that recorded in 2002?

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A2.3.4   Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function;

Example: Find the equation for the inverse function of $y = 3^x$. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?

A2.3.5   Solve problems involving logarithmic and exponential equations and inequalities;

Example: If you fold a rectangular piece of paper in half, the fold divides the paper into two regions with each region half the original area. If you fold the paper again, you have four regions with each region a fourth of the original area. Write an equation that models the relationship between the number of folds and the number of regions formed; write an equation that models the relationship between the number of folds and the fractional size of the regions formed. Graph and compare the characteristics of each graph.

Strand: **A2.4 Conic Sections**
Students analyze equations and graphs for conic sections (circle, ellipse, parabola, and hyperbola).

Standards: Students in Algebra 2 will:

A2.4.1   Describe connections between the geometric definition and the algebraic equations of the conic sections (parabola, circle, ellipse, hyperbola);

Example: Describe the vertex, axis of symmetry, focus, and directrix for the parabola with equation $y + 11 = 2(x-3)^2$.

Example: Derive the general form for the equation of a parabola from its geometric definition.

A2.4.2   Identify specific characteristics (center, vertex, foci, directrix, asymptotes etc.) of conic sections from their equation or graph;

Example: Find the center, vertices, foci, and asymptotes for the hyperbola with equations $x^2 - 4y^2 -10x + 24y -15 =0$.

Example: Write an equation for a hyperbola with vertices at (0, 3) and (0, -3) and has asymptotes of $y = 3x$ and $y = -3x$.

A2.4.3   Use the techniques of translations and rotation of axis in the coordinate plane to graph conic sections;

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Example: Determine the equation that translates the graph of $y = x^2 - 4x + 2$ to a parabola with its vertex at the origin.

Example: Graph the equation $(y + 6)^2 - (x - 2)^2 = 1$.

Strand: **A2. 5 Functions and Relations**
Students analyze relations, functions and their graphs.

Standards: Students in Algebra 2 will:

A2.5.1 Determine whether a relationship is a function and identify independent and dependent variables, the domain, range, roots, asymptotes and any points of discontinuity of functions. (use paper and pencil methods and/or graphing calculators where appropriate);

Example: Generate a list of values for $y = x^2$. Using the list of values, determine the domain and range of $y = x$?

Example: Graph $y = \dfrac{2}{3 - x}$ - 2 and state the domain and range and determine the asymptotes, points of discontinuity and roots.

A2.5.2 Graph and describe the basic shape of the graphs and analyze the general form of the equations for the following families of functions: linear, quadratic, exponential, piece-wise, and absolute value (use technology when appropriate.);

Example: Determine the equation for the graph:
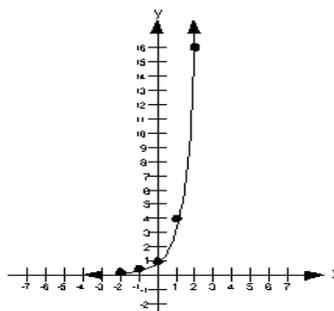
A2.5.3 Describe the translations and scale changes of a given function $f(x)$ resulting from parameter substitutions and describe the effect of such changes on linear, quadratic, and exponential functions;

Example: Describe the translation of the graph of $f(x) = 3^x$ to $f(x) = 3^{(x+2)}$.

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A2.5.4   Solve equations and inequalities involving absolute values of linear expressions;

Example: Solve for x. $|5x + 4| < 6$

A2.5.4   Solve systems of linear equations and inequalities in two variables by substitution, graphing, and use matrices with three variables;

Example: The Worldwide Widget Company is upgrading its product and has received two cost proposals. The first proposal indicates $125,000 for redesign and startup and will result in a product that costs $225 per unit to manufacture. The second proposal has $100,000 in startup and redesign but will result in a manufacturing cost of $275 per unit. You are charged with analyzing the proposals and briefing your boss. What are your conclusions?

A2.5.5   Determine the equation of a function as a variation or transformation of the general form of the equation for the basic family of the function;

Example: Write the equation of a parabola that translates the vertex of $f(x) = x^2$ to the point (3, -16) and has roots of 1 and 5.

A2.5.6   Describe the characteristics of a quadratic function (maximum, minimum, zero values, y-intercepts) and use them to solve real-world problems (use technology where appropriate);

Example: An object is tossed in a room that has a 15 ft. ceiling. Using the formula $H=-16t^2 + v_i t + h_i$ where H is the height of the object, t is the time since release of the object, v_i is the initial velocity of the object, and h_i is the height of the release, determine the maximum velocity which the object can be released from a height of 6 feet and ensure that the ball does not hit the ceiling.

A2.5.7   Determine the type of function that best fits the context of a basic application (e.g., linear to solve distance/time problems, quadratic to explain the motion of a falling object, exponential to model bacteria growth, piece-wise to model postage rates, or absolute value functions to represent distance from the mean);

Example: Determine what type of function best models the following situations: determining the amount in a savings account that issues simple interest; the diminishing return on the amount of

studying time with relationship to a grade attained on a test; the amount of lava flow from a volcano over a given period of eruption.

A2.5.8   Use tables, graphs, and equations to solve problems involving exponential growth and exponential decay, using technology where appropriate;

Example: A barometer measures air pressure. In general, higher altitudes have lower air pressure than lower altitudes. Here is a table giving average air pressure at different altitudes.

Altitude (thousand feet)	0	5	10	20	40	50	60	70	80	90	100
Air Pressure (inches of mercury)	29.92	24.9	20.58	13.76	5.56	3.44	2.14	1.32	0.82	0.51	0.33

Serena is a meteorologist, and she said that where she lives, the average air pressure is about 10 inches of mercury. Determine an equation that models the data and estimate the altitude where Serena lives?

A2.5.9   Use function notation to indicate operations on functions and use properties from number systems to justify steps in combining and simplifying functions;

Example: What properties of real numbers enable you to simplify

$f(x) = \dfrac{x^2 + 2x - 35}{x - 7}$ to $g(x) = x + 5$

Example: Given $f(x) = x^2 + 6x + 5$ and $g(x) = 2(x - 1)^2$. Identify the properties used in creating the equivalent form of $2 f(x) + g(x)$ (i.e. distributive property, addition property, and multiplicative property).

Explain where $\dfrac{f(x)}{g(x)}$ does not exist and why.

A2.5.10   Explain the meaning of composition of functions and combine functions by composition;

Example: For $f(x) = 6x^{-1}$ and $g(x) = 2x - 5$ determine the following and state the domain: $f(g(x)); \; g(f(x)); \; g(g(x)); \; f(f(x))$

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Example: An appliance store has a promotion program that allows a customer to select a scratch-off card that will reveal either a 20% discount, $100 dollars off, or both the discount and reduced price.

a) Using function notation, write a function D that models the 20% discount, a function R that reduces the price by $100.

b) If a customer is fortunate enough to reveal the combined promotions, determine when it would be advantageous to use D(R(x)) verses R(D(x).