A Primer on Multifactor Productivity: Description, Benefits, and Uses

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INTRODUCTION

This primer presents a description of multifactor productivity (MFP) and its calculation. Productivity is an important measure of the state of the economy at various levels: firm, industry, sectoral, and the macroeconomic. The method described is the one used by government agencies, such as the Bureau of Labor Statistics (BLS) and the Bureau of Transportation Statistics (BTS), as well as the Economic Research Service (U.S. Department of Agriculture). The method is also used outside of government, including academia and other private researchers. The estimation of multifactor productivity includes the measurement of output (of an industry, etc.) and the inputs used to create the output. It also includes a weighting of the inputs; the weights indicate the relative importance of the inputs in industry output.

Besides the description and measurement of multifactor productivity, the primer includes a discussion of the benefits of increasing MFP. These relate to increases in real income per capita in the country, which affects increases in the standard of living of the population. A rising standard of living has been a key goal of U.S. government policy as well as of other governments.

Lastly, the primer discusses the uses that can be made of MFP estimates. They are indications of production efficiency and can be used as proxies to indicate rates of return to capital and other resources used in production, in an industry—such as the various transportation industries. The MFP numbers provide knowledge that can help policymakers decide on the allocation of public funds (resources) to various sectors of the U.S. economy, including the transportation sector.

DESCRIPTION OF MULTIFACTOR PRODUCTIVITY

Defining Productivity

Productivity refers to the efficiency with which output—of a factory, company, industry, etc.—is produced with the available inputs used to produce the output—e.g., labor, capital, etc. Output, for example, can refer to the number of cars produced by an automobile factory in a year or the number of passengers transported by an airline company in a year (or some other time period). The inputs used in the making of a company's or industry's output typically are: labor (the workers employed by the company or industry), capital (e.g., machines used in production), land (on which production takes place—for a factory or airline terminal), and intermediate inputs (such as materials and energy purchased).

Productivity can be defined in terms of a single factor input, such as labor productivity, or in terms of many or all factor inputs, which is MFP. It can be estimated at various levels: industry, economic sector, or the economy.

In the basic growth-accounting methodology, used by R. Solow (1957), W. Kendrick (1973), and E. Dennison (1974), multifactor productivity (or "total factor productivity") is typically estimated as a growth rate. In the second main approach, the Tornqvist methodology, MFP is calculated as an index number (level), which is obtained by dividing the output index by a combined input index (BLS, 1983). Growth rates can be calculated from the index numbers. Appendixes A and B present the computational frameworks of the two approaches described above.

Labor productivity is defined as output per unit of labor, and is calculated by dividing output produced by a measure of the labor input used to produce the output (number of employees or labor hours). For example, suppose that a factory that produces automobiles makes 1,000 cars in a year and 50 people work in that factory. In that case, labor productivity is 20 cars per employee—obtained by 1,000/50. In the making of cars (the output), workers also use capital, in the form of machinery, and other inputs (e.g., metal parts). In a transportation example, the output of the airline industry is the delivery of a service—that is, the transportation of passengers (and freight). If the industry transports 100,000 passengers per year and employs 100 employees, labor productivity would be 1,000 passengers per employee.

Labor productivity can increase in the car factory or in the airline industry over time. Increases in labor productivity reflect the effects of two basic factors:

1. increased use of capital (e.g., machines) in production—which increases the amount of capital per worker, and
2. "technological progress" which can include a number of factors, such as improvements in labor or capital. Technological progress affects labor productivity and multifactor productivity, and it will be described at a later point.

Multifactor productivity refers to the productivity of all the inputs used in the production process. These include: labor, capital, land, and intermediate inputs (e.g., energy inputs and purchased services). Consequently, MFP measures output per unit of combined inputs, and indicates the overall production efficiency of an industry (sector or economy). In illustrating the two productivity measurements: labor productivity can be represented as 1,000 cars/50 workers; and MFP can be represented as 1,000/(Labor + Capital + Land + Intermediate Inputs). Thus, multifactor productivity is a more comprehensive measure of productivity than labor productivity or other single-factor productivity measures.

For estimating MFP at the industry level, there is need for data on the output and the inputs used to make that output. The output measure used is total output (rather than value added). The inputs used are: labor, capital, and intermediate inputs.

Output and inputs are measured in quantity terms or in constant dollars. The *output* measurement in constant dollars is obtained by deflating a current-dollar output measure by a price index (or indexes). The constant-dollar measure is equivalent to the quantity measure of output. Thusly, a percentage change (for a given time period) in the quantity measure should be similar to the percentage change in constant-dollar output.

For the transportation sector, output in quantity terms refers to measures such as passenger-miles (in air transportation) or ton-miles (in rail transportation or truck transportation). Output in constant dollars—e.g., for trucking—is derived by first obtaining data on output in current dollars. That relates to industry revenue, which is the product of the quantity of freight services sold times the price per service. Subsequently, the current-dollar output measure is deflated by a price index (which is meant to capture the increase in costs and prices over time). An index for measuring output in real terms is the Fisher Chain Quantity Index, used by the Bureau of Economic Analysis (BEA).

Output of an industry may change over time with regard to quality. That dimension should be incorporated into the output measure. A fair amount of writing has been devoted to this topic. In this primer, one notes that in the case of the constant-dollar output measure, the impact of quality changes can be incorporated in the price indexes used for deflation. In the quantity measure of output (e.g., ton miles) that measure (index) can distinguish freight of higher charge—of higher value freight though not necessarily of higher weight freight—by assigning weights to the various segments of freight carried by a transportation industry. These weights can relate to revenue shares of each product's segment in total industry revenue.

The inputs used in productivity analysis are also measured in real terms: in quantity terms or in constant dollars. Quantity measures relate to the measurement of the labor input by the number of employees or work hours. For capital and intermediate inputs, a measure in constant dollars involves a measure in current dollars deflated by an appropriate price index (or price indexes).

The *labor input*, as noted, is typically measured in either number of workers or number of work hours. If labor hours are obtained by multiplying the number of employees by an expected number of hours worked in a year, the two measures will be essentially the same. One also needs to address the measurement of part-time employees. In rail transportation, the labor input is measured by the Bureau of Labor Statistics in labor hours. In trucking, the Bureau of Economic Analysis data measures labor by the number of employees.

The quality of the labor input can be incorporated in the labor input index—by using numbers of workers in various skill categories weighted by the wages of each skill category. To the extent that that is not done in productivity measurement, the impact of quality changes will be included in the number for the MFP (residual). The estimation of multifactor productivity at the industry level by BLS and BTS does not make a distinction for different types of labor; that is, for hours of different types of employees in the measurement of the labor input (Bureau of Labor Statistics, 1983).

The *capital input* includes structures, equipment, inventories, and land (in a broad definition of capital). Intermediate inputs include purchased electricity, fuels, materials, and services. One can regard reproducible tangible capital—structures and equipment—and land as two distinct factors of production. For one, structures and equipment are man-made; they are the output of a production process. Land, on the other hand, is not man-made; it is a natural resource. Moreover, structures and equipment depreciate over time as they are used in production; land does not depreciate—at least for practical purposes.

The capital input is typically measured in constant dollars; that is a measurement of current dollars is deflated by a price index (or price indexes). In this measurement, improvements in the quality of the capital input could be measured by the price indexes used to deflate current-dollar capital data. However, such a measurement has been only partial in the case of air transportation (per communication with staff of the Bureau of Labor Statistics). Consequently, the impact of improvements in capital would be included in the number for MFP.

In estimating multifactor productivity, the inputs are weighted. The *weight* of each input is the share of the input in the total cost of production of the industry. These weights indicate the relative importance of the inputs in production and are used to estimate the contribution of each input to changes or increases in output.

The framework to examine multifactor productivity can be illustrated as follows:

Output = f (Labor, Capital, Land, Intermediate Inputs)

This equation states that output depends on (is a function of) the inputs of labor, capital, land, and intermediate inputs. That is, in order to transport passengers (the output), an airline company (or airline industry) needs:

1. Labor: pilots, cabin attendants, maintenance mechanics, ticket sales people, etc.;
2. Capital: airplanes, air terminals, etc.;
3. Land: on which to build the air terminals, runways, and maintenance facilities; and
4. Intermediate inputs: airplane fuel, etc.

Given the above production relationship, the output of the air transportation industry (number of passengers carried in a year) can increase through two basic ways: 1) By increasing the quantity of the inputs: that is, by using more airplanes and pilots, or 2) By increasing the productivity of the inputs. This would include the use of bigger airplanes (better-quality capital) that can carry more passengers (more output); the use of faster airplanes (better-quality capital) that can reach their destination faster (hence, more output); and the reduction in the number of pilots from three to two—made possible by technological advances of the airplane piloting system. This would increase the number of passengers (output) per pilot. Increases in the productivity of the factor inputs, such as those described above (under #2), would result in higher multifactor productivity.

Thus, a change or increase in the output of, for example, the airline industry is composed of two parts: the part that is affected by increases in the quantity of the inputs and the part that is affected by increases in the productivity of the inputs (MFP). This is the analytical framework used in the estimation of MFP.

Labor productivity is the most widely used measure of single-factor productivity. For this reason it seemed useful to compare estimates of labor productivity and multifactor productivity for transportation industries and the U.S. economy (the private business sector). Relevant data are presented in table 1. It can there be observed that for rail, truck transportation, and the U.S. economy, the overall increase of labor productivity, over the periods for which data are available, exceeds the increase of multifactor productivity. In air transportation, however, the increase of MFP exceeds that of labor productivity. The second thing to observe is that the increase in labor productivity, over the period, is rather close to the increase in MFP. The exception to this is rail transportation, where the increase in labor productivity exceeded substantially the increase in MFP.

MFP and labor productivity levels can be higher for one measure than for the other at various times. For example a chart of the two variables for the private domestic economy, over 1947 to 1967, shows that until 1960, MFP levels exceeded labor productivity; while from 1960, the situation reversed and labor productivity exceeded MFP (National Research Council, 1979, p. 40).

Factors to explain the degree to which one productivity number may exceed the other are not available. Such an explanation would relate to the mechanics of calculating the two numbers. When labor productivity exceeds MFP "substantially," it indicates that the efficiency of all the factors of production is increasing substantially slower than labor productivity.

Sources of Multifactor Productivity

At a given point in time, MFP is affected by the technology used by the firm or industry in making

TABLE 1 Data on Labor and Multifactor Productivity

	Air transportation NAICS 481		Rail transportation SIC 4011		Truck transportation NAICS 484		U.S. economy	
	Labor productivity (1987=100)	Multifactor productivity (1987=100)	Labor productivity (1987=100)	Multifactor productivity (1987=100)	Labor productivity (1987=100)	Multifactor productivity (1987=100)	Labor productivity (1987=100)	Multifactor productivity (1987=100)
1987	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1988	100.6	100.2	108.4	105.8	100.8	107.7	101.6	100.7
1989	97.1	97.9	114.7	109.8	98.8	101.1	101.0	100.3
1990	97.9	100.8	118.5	113.7	104.6	100.8	102.2	100.7
1991	100.9	99.9	127.8	117.5	103.9	104.2	101.7	99.3
1992	104.1	103.6	139.6	125.0	105.5	103.8	104.3	102.7
1993	104.1	97.0	145.4	129.0	98.0	100.8	100.5	100.2
1994	107.2	106.5	150.3	131.8	102.5	100.5	101.0	100.8
1995	105.0	104.0	156.2	139.6	97.5	97.7	100.2	99.7
1996	103.7	103.8	167.0	144.8	99.5	100.1	102.9	101.7
1997	101.2	101.1	169.7	144.9	104.9	99.8	101.9	100.9
1998	97.6	99.0	173.3	143.4	99.4	99.5	102.8	101.3
1999	100.6	101.8	182.5	147.9	99.7	98.7	103.1	101.3
2000	100.0	103.0			102.8	99.4	102.9	101.3
2001	93.6	95.9			101.2	97.5	102.8	100.2
2002					103.7	101.4	104.2	101.7
2003					103.5	100.7	103.9	102.7

Sources: For data on labor productivity:

1) Air transportation: ftp://ftp.bls.gov/pub/special.requests/opt/dipts/indmfp3.txt.
2) Rail transportation: ftp://ftp.bls.gov/pub/special.requests/opt/dipts/indmfp.txt.
3) Truck transportation: http://data.bls.gov/cgi-bin/dsrv
4) U.S. Economy: http://www.bls.gov/mfp/mprdload.htm

For data on multifactor productivity:

1) Air transportation: ftp://ftp.bls.gov/pub/special.requests/opt/dipts/indmfp3.txt.
2) Rail transportartation: ftp://ftp.bls.gov/pub/special.requests/opt/dipts/indmfp3.txt.
3) Truck transportation: http://www.bts.gov/programs/economics_and_finance.
4) U.S. Economy: http://www.bls.gov/mfp/mprdload.htm

Note: For the U.S. economy, labor productivity and multifactor productivity are measured by the "Private Business Sector" (excluding government enterprises).

the output (product or service). Over time, changes—hopefully, increases—in MFP are affected by improvements in technology. These concepts are described below.

Technology and Advances in Technology

Technology is the "recipe," the "know-how," that is used by producers in different industries in order to produce a product or deliver a service. The technology utilized should be the best available technology, in order to produce a product or service at the greatest possible level (and quality), given the available inputs or resources and their costs. The production of a product or service at the maximum level (given resources) also implies that it is produced at the lowest possible cost (cost per unit). If a firm or industry uses the various factor inputs—labor, capital, intermediate inputs, land—as efficiently as possible, the output produced would be the largest amount of output. In producing this output, there would be costs incurred (by the firm or industry) for using the factor inputs (wages, supplies, etc.), which would sum to the total costs of producing the output. In this case, since output is the greatest possible, the cost per unit of output (obtained by Total Costs/Output) would be the lowest possible.

Over time, changes (increases) in multifactor productivity can be affected by a number of factors, and these are typically classified under the term "technical progress" or "advances in technology." This category includes factors other than increases in the quantity of the inputs that affect increases in output. These factors include improvements in the quality of the inputs—labor, capital, etc. In the case of capital, quality improvements refer to the use of more efficient machines, such as computers of more recent vintage, with higher levels of technology incorporated in them. It can also include more efficient structures or buildings. In this regard, a study points out the technological progress in construction technology that has enabled the building of much higher buildings over time. The height of commercial buildings increased from 10 floors in 1885 (The Home Insurance Building in Chicago) to 110 stories in 1974 (Sears Tower in Chicago). In addition, technological progress has resulted in a reduction of the amount of space necessary to provide the expected level of comfort in buildings (air conditioning, etc.). This increases the value of the building because more floor space will be available to rent (Gort, Greenwood, and Rupert, 1999). Therefore, improvements in technology have led to quality improvements of the structures, which resulted in higher levels of output per structure and, thus, higher value per structure.

In air transportation, technical progress includes the use of advanced or improved equipment used in the inspection of cracks in airplane frames. This significantly reduced the amount of time needed to carry out airplane inspections and, consequently, the amount of an airplane's out-of-service time. This, in turn, leads to increases in its output—the number of passengers transported—which increases MFP. Factors, such as this, which affected increases in technical progress in air transportation are examined in a recent study (Apostolides, 2006-b).

Factors affecting "technical progress" also include changes or improvements in the arrangement of the production process so that more output is produced with the same inputs. This may include a rearrangement of workstations on a factory floor so that work flow proceeds more efficiently, with a corresponding increase in output.

Factors affecting MFP can also include changes in industry structure, brought about by mergers or acquisitions and bankruptcies, that result in increasing production efficiency of the industry. In the case of acquisitions, industry efficiency can increase when more profitable firms—presumably because they have higher efficiency or productivity—acquire less profitable, less efficient, firms. In the typical case after acquisition, there is a combining of certain central administrative functions of the two firms, such as personnel and payroll. This can lead to higher levels of output—of the two combined firms—but with the same number of labor (and perhaps capital or machines) input as was used by the acquiring firm. This leads to increases in firm and industry productivity.

In cases of bankruptcies, the firms going bankrupt and leaving the industry are typically unprofitable firms. They would generally be characterized by relatively low levels of efficiency or productivity. Consequently, when they leave the industry, there is expected to be an increase in the overall level of industry efficiency.

BENEFITS OF MULTIFACTOR PRODUCTIVITY INCREASES

Description of Benefits

Increases in multifactor productivity have crucial benefits for the economy and society. Productivity increases result in increases in output, on one side, and incomes of various economic groups, on the other side. The increase in output is a direct contribution to the country's economic growth. The increase in (real) incomes contributes to a rising living standard in the country. Real income of an individual refers to the purchasing power of his or her money income, that is, the amount of goods and services that a person's money income will buy. Consequently, real income can increase when there is an increase in a person's money income or when there is a decrease in the prices of goods or services. Economic growth and rising living standards have been basic and important economic goals of U.S. economic policy.

According to economic theory, increases in productivity in an industry (or firm) can affect profits, prices, and labor compensation. The basic benefit of increased productivity is that more output can be produced with the same quantity of inputs—while some inputs will be of improved quality or there can be changes in production arrangements. Alternatively, the same output can be produced with fewer resources. Therefore, other things being equal, increases in productivity result in a bigger difference between total revenues and total costs, and thus higher profits for the affected industry. This would be the initial and basic effect. The existence of higher profits can subsequently be followed by three effects:

1. The firms in the industry can keep a portion of the increased profits for internal use (additional investment, etc.), or distribute a part of it to their owners or shareholders;
2. The firms can decrease the prices of their service (or products) to the consumers, or—perhaps more likely—they may increase prices by less than they would have in the absence of productivity increases; and
3. The firms can provide higher compensation to their employees—in the form of higher wages and/or fringe benefits.

On the other hand, a decline in productivity would result in opposite effects of a productivity increase—and could lead to declines in labor wages and, in extreme cases, to bankruptcies of companies, accompanied by job losses.

As noted, the productivity increase benefits directly the affected industry, by increasing profits. Subsequently, the increase in profits can be followed by lower prices of the industry—particularly when there is competition among the producers of the industry. Competition is affected by the number of producers in the industry, among other things. The higher the number of producers in an industry, the more the expected competition in that industry.

All three impacts of a productivity increase can result in higher real incomes in the economy. In the case of the business enterprise, where productivity increases take place, there is a resultant increase in its profit or income. A portion of the higher profit can be kept by the company in the form of retained earnings—with which to finance future investment that can be lead to higher levels of productivity. If part of the increased profit goes to the stockholders of the firm in the form of higher dividends, their incomes would also increase. In the case of labor, there can be an increase in labor compensation of the workers working in the affected industry; this directly results in increased incomes. In the case of consumers or users of the industry's service (or product), there can be a decrease in the price of that service. Such a price decrease results in an increase of real incomes (the amount of goods and services which can be purchased with a given level of money or nominal income).

Productivity increases result in increases in output (industry to national) and rising incomes in the society—resulting in higher living standards. The rise in particular types of income depends on what companies do with the increased profits from productivity increases—as described above. Rising output and incomes contribute to, and are part of, the country's economic growth, which has been a most important objective of national economic policy. These are the basic benefits of productivity increases, and the reasons why productivity increases are desirable from the perspective of the company, industry, and the economy or society.

Productivity increases can also have second-round effects, as when labor uses its higher income to increase its consumption of various goods and services. This increased consumption stimulates sales of various products or services and subsequent production in other industries, with possible increases in employment and incomes there. Thus, the benefits of an initial productivity increase can have a ripple effect from the affected industry and influence positively other industries and the economy.

Application to Air Transportation

The economic framework described above has been used to analyze the impacts of productivity changes or increases in air transportation (Apostolides, 2006-a). The objective of that study was to assess the impacts of changes or increases in labor productivity and multifactor productivity of the air transportation subsector—in three economic areas: 1) industry profits, 2) consumers, and 3) airline employees.

The analysis resulted in several findings. First, there was a marked association between productivity changes (increases or decreases) in air transportation and industry profits. Second, the benefits of productivity increases in air transportation did not seem to transfer to consumers of air passenger services in the form of lower prices. On the other hand, users (shippers) of scheduled airfreight services benefited from lower rates of price increases. For these shippers, lower price rises restrained increases in their (distribution) costs, contributing to higher profit or income.

In addition, a portion of the benefit of productivity increases went to industry employees in the form of relatively high levels and increases of labor compensation. This resulted in an increase of their real incomes. Such increases in real incomes, to shippers and labor—as well as industry profits—are the important contributions of greater productivity. Increases in real incomes result in higher standards of living—a major objective of U.S. economic policy.

USES OF MULTIFACTOR PRODUCTIVITY DATA

Estimates of MFP can be calculated for the economy, for different economic sectors—e.g., manufacturing, transportation—and for individual industries such as air transportation and truck transportation.

Estimates of MFP can be used in several ways, described below:

MFP is an indication of the productivity, or efficiency, with which all factors of production are used in the creation of the output (of products or services)—of a firm, industry, etc. An increase in the MFP number indicates an increase in the productivity of utilizing the factor inputs. Such an increase in production efficiency means that the firm or industry is getting more benefit—in terms of output—from using the available inputs (resources). Thus, more output can be produced. And more people can, hopefully, share the benefits from higher levels of output.

The productivity of an industry affects the overall productivity of the U.S. economy. Thus, if MFP in a transportation industry grows faster than

MFP in the U.S. economy, that industry would contribute positively to the growth of productivity of the U.S. economy and to increases in the standard of living in the country. As an illustration, and with regard to the contribution of trucking to the economy's multifactor productivity, data show that over 1978-2000, MFP in truck transportation increased at a higher annual rate than MFP in the U.S. business sector—with particularly high rates during 1987-1995 (Apostolides, 2007). Consequently, during this period of time, trucking MFP contributed positively to the economy MFP.

Productivity increases in the U.S. economy over time have contributed significantly to economic growth and to improvements in the standard of living in the country. The goals of economic growth and rises in living standards—increases in real income per capita in the country—have been basic and important objectives of U.S. government policy. Consequently, measuring and assessing the productivity of the transportation sector is, or should be, a key priority. In addition, the enhancement of transportation productivity would be a relevant economic goal.

Multifactor productivity measures can be used as an indicator of the rate of return to all the resources used in an industry. This would be an indicator similar to the rate of interest or rate of return earned in the use of capital. For example, the rate of interest earned on Certificates of Deposit is an indicator of the rate of return earned by capital (financial capital, in this case). MFP numbers would indicate the benefit—increase in the output—that an industry, sector or an economy gets for investing its resources—labor, capital, land, and intermediate inputs, land—in a particular pattern. That pattern would relate to the types of inputs and the combination of those inputs used in production—such as the capital-intensity of techniques of production, the types of physical capital used, the labor-intensity of production, the types of labor used, etc.

If MFP figures are relatively high in transportation industries, compared to the economy (which includes other industries), these figures suggest that increased investment for transportation-related infrastructure and other support for transportation industries have a higher rate of return than for the economy as a whole.

QUALIFICATIONS TO MFP ESTIMATES: EXTERNAL COSTS

The price of a firm's or industry's service, or product, is expected to be sufficient to cover all costs of production (including a profit). Stated somewhat differently, all the costs of production are expected to be covered by the price of the service or product (total revenue = total costs, or price per unit = cost per unit). That is if the price of oranges is $2.00 per pound, that is expected to cover all the cost of producing and transporting that product to market (including, for example, $0.50 a pound for transportation costs). However, often the stated market price does not cover all the costs of production and distribution. This is due to the existence of costs that are incurred by the firm but are not included in the firm's price of the product or service. These costs are instead passed on from the firm to outside the firm or society; hence, they are called "external costs." These external costs typically refer to environmental pollution, such as air and water pollution (as discussed, for example, in Greene, et. al., 1997).

From the economic perspective, such effects typically result in lower stated production costs for the producer (of a service or product); however, they entail higher costs to the society. This happens when, for example, air, train, or truck transportation (or the manufacturing of soap) result in increased pollution of the air, and this pollution results in adverse effects on people's health, in both the short-run and the long-run.

Typically, the effects of external costs have not been measured in studies of productivity, which may relate to difficulties in their measurement. However, in a comprehensive measurement, they should be included in the analysis.

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APPENDIX A. BASIC GROWTH - ACCOUNTING METHODOLOGY

The empirical relationship used to estimate growth of multifactor productivity by the basic growth-accounting methodology is shown below:

$$\frac{\Delta T}{T} = \frac{\Delta Q}{Q} - \left[\left(\alpha * \frac{\Delta Labor}{Labor}\right) + \left(\beta * \frac{\Delta Capital}{Capital}\right) + \left(\gamma * \frac{\Delta Intermediate\ Inputs}{Intermediate\ Inputs}\right)\right]$$

Where:

$\frac{\Delta T}{T}$ = Growth of MFP

$\frac{\Delta Q}{Q}$ = Growth of gross output

$\frac{\Delta Labor}{Labor}$ = Growth of labor

$\frac{\Delta Capital}{Capital}$ = Growth of capital

$\frac{\Delta Intermediate\ Inputs}{Intermediate\ Inputs}$ = Growth of intermediate inputs

α = Share of labor cost in output

β = Share of capital cost in output

γ = Share of intermediate inputs cost in output.

APPENDIX B. METHODOLOGY WITH THE TORNQVIST INDEX

Multifactor productivity is the ratio of the output index to a weighted average of the input indexes. A Tornqvist formula expresses the change in multifactor productivity as the difference between the rate of change in output and the weighted average of the rates of change in the inputs. Let:

Ln = the natural logarithm of a variable

A = multifactor productivity

Q = output

I = combined input

K = capital input

L = labor input

M = intermediate input

W_k = the average share of capital cost in total cost in two adjacent periods

W_l = the average share of labor cost in total cost in two adjacent periods

W_m = the average share of intermediate input cost in total cost in two adjacent periods,

The change in the multifactor productivity is then:

$$(1) \quad \triangle LnA = Ln\left(\frac{A_t}{A_{t-1}}\right) = Ln\left(\frac{Q_t}{Q_{t-1}}\right) - \left[W_k\left(Ln\frac{K_t}{K_{t-1}}\right) + W_l\left(Ln\frac{L_t}{L_{t-1}}\right) + W_m\left(Ln\frac{M_t}{M_{t-1}}\right)\right]$$

Or

$$(2) \quad \triangle LnA = Ln\left(\frac{A_t}{A_{t-1}}\right) = Ln\left(\frac{Q_t}{Q_{t-1}}\right) - Ln\left(\frac{I_t}{I_{t-1}}\right).$$

A multifactor productivity index can be further developed by calculating the antilogs of ΔLnA, chaining up the resulting annual rates of change, and expressing the resulting series as a percentage of a selected base year. Equivalently, the change in the multifactor productivity can be directly expressed as $A_t/A_{t-1} = (Q_t/Q_{t-1}) / (I_t/I_{t-1})$. Again, A_t/A_{t-1} can be chained over time and converted into an index number.

All variables, except for cost shares, are in the form of a constant dollar quantity index. The output quantity index is usually derived by deflating the industry output in current dollars by an appropriate price index when the industry output is a single measure. When an industry produces multiple products and the output measure of each individual product is available, such individual outputs may be deflated separately by more detailed price indexes. In that case, the total output quantity index can be derived through a Tornqvist aggregation such as:

$$(3) \qquad \sum_{1}^{n} w_i \Delta LnQ_i \, ,$$

where Q_i is the output of the ith product, and

w_i is the average share of the ith product in the total output.