Economic Theory and BEA's Alternative Quantity and Price Indexes

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*I*N THIS issue, BEA is introducing new, alternative price and quantity indexes for the major components of the national income and product accounts (see "Alternative Measures of Change in Real Output and Prices" on page 32). This article describes the index number theory underlying these alternative indexes and discusses the interpretation of them.

Index number theory

Economic theory has long been used to specify the construction of price and quantity index numbers. The modern treatment originated in an article published in the 1920's by the Russian mathematician and economist A.A. Konüs.[1] Konüs analyzed the measurement of consumer prices, the theory of which he named the "true index of the cost of living." Cost-of-living index theory was developed independently by English-language economists in the early 1930's. The theory was summarized by Ragnar Frisch in 1936 in a famous review article on index numbers.[2]

The theory of the cost-of-living index applies directly to the measurement of consumption prices, such as the price index for the personal consumption expenditures (PCE) component of gross domestic product (GDP). This article will summarize the theory of the cost-of-living index, which is the best known and best developed part of the economic theory of index numbers; with suitable changes in language and notation and in some conditions and assumptions, the principles can be extended to investment goods as well.

Cost-of-living index number theory proceeds from the proposition that a consumption price index should measure the change in the cost of maintaining a fixed, or constant, standard of living. If the price index holds the standard of living constant, then any increase in per capita consumption expenditures that exceeds the increase in the price index can be interpreted as an increase in the standard of living. Conversely, if per capita consumption expenditures rise more slowly than the price index, the standard of living, or *real* per capita consumption, is falling. Real consumption, either per capita or in the aggregate, can be expressed as a quantity index, which is the counterpart of the consumption price index.

Thus, from the standard-of-living orientation, the price index measures the changing cost of a constant standard of living, and the quantity index measures increases or decreases in the standard of living. The same interpretation may also be given to conventional fixed-weighted indexes, such as the base-weighted indexes that traditionally have been employed in measuring real GDP. In the fixed-weighted PCE price index, one holds constant the collection of goods and services actually consumed in 1987, which is a way of holding constant the living standard that existed in 1987.

Cost-of-living index theory stresses, however, that consumers can reach the same standard of living in more than one way. Consumers may substitute between commodities that serve similar general purposes (for example, chicken or fish for beef) or even dissimilar ones (a new car for a vacation). Substitution implies that differing collections of goods and services may still represent equivalent standards of living.

Moreover, nationwide data indicate that consumers systematically substitute away from those goods and services whose prices rise the most rapidly and toward those goods and services whose prices rise less rapidly or decline. Commodities whose prices grow most rapidly show, on average, the slowest growth in consumption; commodities whose prices grow more slowly (or decline) show, on average, the most rapid growth in consumption. The same patterns also apply to many nonconsumption goods, such as investment or capital goods; for example, the prices of computer equipment have declined at

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1. A.A. Konüs, "The Problem of the True Index of the Cost of Living," *Econometrica* 7 (January 1939): 10–29.

2. Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica* 4 (January 1936): 1–38.

an extremely rapid rate over the past several decades, while the proportion of investment expenditures accounted for by computer equipment has increased dramatically.

Economic theory suggests that a consumption price index that truly tracks the cost of living should be based on the costs of collections of commodities that represent equivalent living standards and that this index should not, therefore, hold quantities fixed as consumers shift their expenditures. For example, when chicken is substituted for beef, one should look at meat consumption as a whole, rather than at fixed quantities of different kinds of meats, and perhaps one should even look at food consumption as a whole, rather than at fixed quantities of meat, vegetables, and so forth.

Economic theory also suggests that when consumers do substitute toward commodities whose prices rise less rapidly or decline, the cost of maintaining an equivalent standard of living rises less rapidly than the cost of the fixed basket of commodities that were consumed in a previous period, such as 1987. For example, when used to measure consumption prices between 1987 and 1992, a fixed basket of the commodities consumed in 1987 gives too much weight to the prices that rise rapidly over the timespan and too little weight to the prices that fall; as a result, using the 1987 fixed basket *overstates* the 1987–92 cost-of-living change. Conversely, because consumers substitute, a fixed basket of the commodities consumed in 1992 gives too much weight to the prices that have fallen over the timespan and too little to the prices that have risen; as a result, the 1992 fixed basket *understates* the 1987–92 cost-of-living change.

The difference between a fixed-weighted price index and a price index that accounts for substitution is often termed the "substitution bias" in fixed-weighted indexes.

Development of superlative indexes

The theoretical cost-of-living index was for many years regarded as purely an abstraction, an idea that could not be implemented in actual price index calculations. To compute a constant standard of living, one would have to know how much consumers substitute among commodities in response to relative price changes. In other words, one would have to be able to separate changes in consumption spending that raise (or lower) the standard of living from changes in spending that merely represent alternative ways of achieving the same living standard. Even with

econometric methods, which have been applied to the problem,[3] the research task is enormous, and the research results still leave a range of uncertainties.

In 1976, W. Erwin Diewert published an article that suggested a relatively simple way to approximate the theoretical cost-of-living index.[4] Abandoning the attempt to find a formula for the "exact" cost-of-living index, Diewert showed that a class of index numbers, which he named "superlative index numbers," would give good approximations to the "exact" formula. Some of these superlative index formulas turn out to be relatively simple to compute and use.

One of the most attractive of these superlative index numbers is the Fisher Ideal index, proposed by Irving Fisher in 1922. The Fisher Ideal index is simply the geometric mean of the fixed-weighted Paasche and Laspeyres indexes, the formulas for which have long been the primary ones used in constructing indexes for the U.S. national accounts.[5]

Another superlative index is the Tornqvist index, developed in the 1930's at the Bank of Finland. This index is a logarithmically defined index that employs an average of the weights for the two periods being considered.[6]

Diewert showed that the Fisher Ideal index and the Tornqvist index are theoretically better measures of the cost of living than the traditional fixed-weighted Paasche or Laspeyres indexes. The superlative indexes accommodate substitution in consumer spending while holding living standards constant, something the Paasche and Laspeyres indexes do not do. From the view of theory, the Fisher Ideal formula and the Tornqvist formula are equally good; therefore, one can choose between the two on pragmatic grounds.

The Fisher Ideal formula is somewhat easier to compute than the Tornqvist formula; modern

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3. The major studies are by Steven D. Braithwait, "The Substitution Bias of the Laspeyres Price Index: An Analysis Using Estimated Cost-of-Living Indexes," *American Economic Review* 70 (March 1980): 64–77; and Marilyn E. Manser and Richard J. McDonald, "An Analysis of Substitution Bias in Measuring Inflation, 1959–85," *Econometrica* 56 (July 1988): 909–930.

4. W. Erwin Diewert, "Exact and Superlative Index Numbers," *Journal of Econometrics* 46 (May 1976): 115–45.

5. Fisher Ideal quantity index =

$$\sqrt{\frac{\sum P_1 Q_2}{\sum P_1 Q_1} \times \frac{\sum P_2 Q_2}{\sum P_2 Q_1}}.$$

6. Logarithm of Tornqvist quantity index =

$$\sum ln\left(\frac{Q_2}{Q_1}\right)\frac{1}{2}\left(\frac{P_1 Q_1}{\sum P_1 Q_1} + \frac{P_2 Q_2}{\sum P_2 Q_2}\right).$$

computers make this only a marginal advantage. The Fisher Ideal index is also somewhat easier to interpret; a user can examine its component Laspeyres and Paasche indexes to gain a mechanical understanding of movements in the index, and such calculations assist in the analysis of price and quantity movements.

Finally, a major advantage of the Fisher Ideal formula is that it has a "dual" property that is not shared by the Tornqvist formula. A Fisher Ideal *price* index implies a Fisher Ideal *quantity* index, and the converse: That is, the product of a Fisher Ideal price index between two periods and a Fisher Ideal quantity index between the same two periods is equal to the total change in value (change in current-dollar expenditures) between those periods. In contrast, a Tornqvist price index multiplied by a Tornqvist quantity index does not equal the change in value between the two periods. In fact, the quantity index that corresponds to a Tornqvist price index does not have an explicit, algebraic formula (and likewise, the price index corresponding to a Tornqvist quantity index has no explicit formula).

Constructing time series with superlative indexes

Though economic theory indicates preferred index number formulas for making two-period comparisons, it gives less guidance on forming time series of index numbers covering three or more periods.

Consider the following table of annual price indexes that can be computed covering the years 1987–90:

Terminal year	Initial year			
	1987	1988	1989	1990
1987..................	$I_{87,87}$			
1988	$I_{87,88}$	$I_{88,88}$		
1989	$I_{87,89}$	$I_{88,89}$	$I_{89,89}$	
1990	$I_{87,90}$	$I_{88,90}$	$I_{89,90}$	$I_{90,90}$

Each entry in the table designates a superlative index (the Fisher Ideal, in these examples) that measures price change between 2 years with different quantity weights. For example, $I_{87,88}$ is a Fisher Ideal index number computed as the geometric mean of two indexes measuring price change between 1987 and 1988; the first uses weights from 1987 and the second, weights from 1988. Similarly, $I_{87,90}$ measures price change between 1987 and 1990 using a Fisher Ideal formula

that is the geometric mean of one index having 1987 weights and a second having 1990 weights.

Starting with the index for 1987 ($I_{87,87}$, which is, of course, equal to 1), there are two ways to measure price change between 1987 and 1990. One way is to use the "direct" index calculation procedure—that is, to go straight down the column labeled 1987 to compute the direct index number between 1987 and the year that is designated. The index $I_{87,88}$, for example, uses weights for 1987 and 1988; the index $I_{87,89}$ uses weights for 1987 and 1989 (ignoring 1988), and the index $I_{87,90}$ uses weights for 1987 and 1990. In this time series of index numbers, each entry measures price change from the base year of 1987 directly to the designated year, without considering either prices or quantities of intervening years. A statistical table would then record the results of the computations indicated in the column headed "1987" in the table.

The disadvantage of the direct index procedure is that some relevant index calculations are not in the 1987 column. Suppose one wants to know the price change between 1988 and 1989. For most purposes, it is reasonable to specify that the weights for such a price index should be taken from 1988 and 1989 (that is, the index $I_{88,89}$ from the second column of the table). This index is not, of course, present in the 1987 column. For some purposes, therefore, the direct index procedure does not give the "best," or most relevant, measure of *period-to-period* price change.

The second way to obtain price measures between 1987 and 1990 is to use the "chain" index calculation procedure. In terms of the table, the chain index uses the calculations that are indicated by the boldfaced diagonal; that is, starting with the $I_{87,88}$ index value, this value is multiplied by the indexes in the boldfaced diagonal, so that the chain index (1987–90) = $I_{87,87} \times I_{87,88} \times I_{88,89} \times I_{89,90}$. With the chain index procedure, the price index for every adjacent pair of years has weights from exactly those 2 years.

The disadvantage of the chain index procedure is that for price comparisons over a whole period, such as 1987–90, the chain index incorporates all the intervening shifting weights. Thus, if one wants to know the change in the cost of a constant standard of living between 1987 and 1990, the answer is given by the direct index $I_{87,90}$, which has weights only from 1987 and 1990.

It may be difficult to decide which calculation procedure to use. Neither one is best for all purposes. For some purposes, one wants a measure of the total change between 1987 and

1990; this will generally be given by the direct index between 1987 and 1990. However, for other purposes, one wants the best measure for, say, 1989–90, which is obtained from one of the links in the chain index. Because there are different uses for price measures—and also for quantity measures—it is generally advantageous for users to have access both to chain indexes, which are preferable for year-to-year or quarter-to-quarter comparisons, and to some form of direct index, which is preferable for longer term comparisons (1982 to 1987, or 1987 to 1990). To provide measures for different purposes, the new BEA alternative price and quantity measures include both a chain-type index (the annual weighted index) and a form of direct index (the benchmark-years-weighted index), both of which are based on the Fisher Ideal index number formula.

One qualification needs to be stated. For *very* long intervals, the assumptions necessary to produce direct indexes become insupportable. Suppose, for example, one wished to compare the change in a fixed standard of living between 1930 and 1990. Such a question becomes conceptually problematic because over an interval of 60 years, too many changes have occurred in the economy, in the way people live, and in tastes and customs. It might be reasonable to assume that economic conditions are sufficiently constant over, say, 5 years, so that a meaningful cost-of-living index can be computed. Computing one over 10 years poses perhaps a few more problems (for example, new goods are introduced or tastes change), but

the calculations may still be useful because the assumptions necessary to make such calculations are not sufficiently implausible as to render the interpretation of the numbers meaningless. The problematical parts become increasingly of concern as the interval lengthens to 15, 20, or 25 years. As one pushes these comparisons back further in time, any economic measurement becomes increasingly uncertain. Measuring the cost of a constant standard living over an interval as long as 500 years or more (which has been tried in some studies in economic history) involves a very large range of uncertainty that cannot be eliminated by any refinements in the formula used for calculating the price index.

The new BEA alternative price and quantity indexes provide direct indexes (in the form of the benchmark-years-weighted indexes) that cover the intervals between benchmarks, usually 5 years. Indexes for longer intervals (10 or 15 years or more) are produced by chaining these benchmark-years-weighted indexes together. Using this procedure does not necessarily imply that chain indexes are preferred for long-term comparisons. Rather, it recognizes that time series of index numbers will always require compromise, and the compromise adopted seems a useful one. The benchmark-years-weighted index procedure could readily be adapted to provide direct indexes covering longer intervals (for example, 1977–87, which encompasses two benchmark intervals), and such indexes might be of interest for some purposes.