Shaping of Filamentary Streamers by the Ambient Field

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ABSTRACT

We have simulated the fast streamer stage of liquid dielectric breakdown as stochastic growth of a branching fractal tree. Breakdown and threshold properties of the fluid are represented in the random filter procedure. A range of fractal densities, from sparse to bushy, is approximated by the choice of power-law (cube to linear). The choice of threshold (cutoff) voltage also significantly affects the growth form. These parameters combine with the shape and concentration of the electric field, to regulate the distribution and directedness of the local discharge growth pattern. A large grid (128 cubed) is used for the discretization. Diagonal growth paths to neighbor-vertices are included, increasing the choice of available directions for each discharge event. We use a combination of data-parallel programming and three-dimensional visualization. Complete growth histories, evolving from the voltage distribution, can be displayed in animation or in color banding against the "trials" variable, which simulates a time tick. Side views of the structures provide comparison against sub-microsecond snapshots from experiment. Results include sparse, directed trees evolving from a cube-law filter; also dense trees from a linear filter, whose conical upper-envelope boundary is strongly influenced by the choice of threshold (cutoff) potential.

INTRODUCTION

Filamentary streamers evolve rapidly in a surrounding voltage field, which influences their shape and density. We explore a simplified "stochastic Laplacian fractal" simulation for this phenomenon [1]. We do not include microscopic details of the physical processes. Our purpose is to capture the global features of the growth.

Elements of the algorithm are:

1. Assume the streamer tree is fully conductive, and attached to the starting electrode.
2. Solve Laplace's equation throughout the full interior region of a $128 \times 128 \times 128$ cube.
3. Examine neighbor sites one grid-step away from the tree. Where voltage at these sites exceeds a threshold level, then compare against random numbers by a method which yields a weighted distribution of survivors. Attach these survivors to the tree.
4. Cycle steps 2 and 3 until the counter-electrode is reached.

By varying the weighting power-law exponent from 3 to 1 we demonstrate that the increasing visual bushiness (sparse to dense) results from the spread of forward directional concentration, about the axes of the individual growth tips.

High geometric resolution, detailed three-dimensional calculation of the voltage field, and a plausible method for estimating time progression are essential. As Biller has noted [2], length, voltage, and time must be explicitly scaled in the discretized model to approximate a history of the physical growth, which is more than a geometrical abstraction.

Our simulation has several features in common with the recent work of Karpov and Kupershtokh [3], including simultaneous (concurrent) distributed growth, and growth along diagonal links to neighbors. They calculate a "stochastic time", whose intent is similar to our "Monte Carlo" time tick. In our case the time tick is directly counted unless the waiting time is very long, in which case it is estimated by a method [4] adapted from molecular mechanics.

Some differences include:

- They assume an ohmic potential drop from tips to base along the streamer tree.
- They assume a spherical outer boundary condition about a starting point, with the tree growing as a ball. They find the local fractal dimension declining as the tips reach larger radii. Our simulation, confined between electrode plates, shows some increase in local fractal density, as the tips progress.

- They assume a statistical spread of lower threshold (cutoff) values; whereas we take a sharp threshold [5].

The model is non-specific with regard to the microscopic mechanism of streamer growth. It does not treat the microscopic electric field strength at the tip surface. Neighbor-voltage values are taken at a common radius (one grid step length) from the tip; they are considered to be proportional to the vector electric field strength. (For statistical uniformity, each diagonal-step voltage is down-weighted by a 1/r factor.)

Clocking of Monte Carlo time is considered in this model; thus the distribution, directions, and rate of growth are followed together, as they are controlled by the evolving voltage field. In the examples we give, the "Monte Carlo" time tick is extremely short – nanoseconds or less – taking its scale from the overall duration of the discharge growth.

Display is carried out in three dimensions. On a work station, color banding can be used to mark the time history of growth; dynamic rotation of the image is extremely useful in distinguishing details of the growth form. Animation of frames against the time-tick variable is also readily achieved.

CUBE-LAW

Figure 1 is a representative case of streamer simulation, modeled with a probability filter which produces a cube-law survivor population. Note that the branches are sparse and forward-directed. The bare-tree appearance results from differential preference for tip extension versus lateral branching. Cube-law also tends to maximize the apparent growth speed in the short-gap stages of the path, so that the advance across the second half of the interval occurs in about one-eighth of the total elapsed time.

Experimental streamers, of similar sparse appearance, can be identified in recent results by Miyano *et al.* [6], which describe positive streamer propagation in perfluoro liquids.

SQUARE-LAW

Figure 2 is a representative growth obtained with a square-law weighting. In this example a relatively low threshold (cutoff) voltage has been assumed (0.0500), corresponding to a higher overvoltage in experiment. Noticeable features are the more pronounced major side-branching, and the wider envelope shape. The streamer tree has more branches and a generally more dense, bushy appearance, because the relative probability of lateral branching has increased. Individual branches are less tightly forward-directed by the voltage distribution.

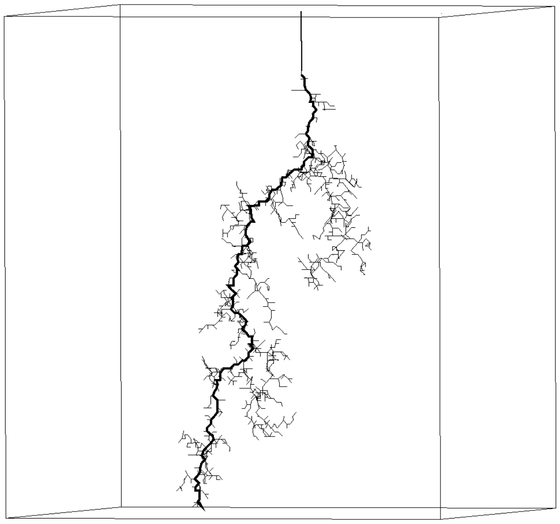

Figure 1. Cube-law simulation on a 128 × 128 × 128 grid. Top surface of the cube represents starting electrode, with an attached central needle of length fifteen grid steps. Lower surface is counter-electrode; side boundary conditions are periodic. The threshold (cutoff) voltage is set at 0.0700, just slightly below the largest voltage on a neighbor site to the needle. 2544 statistical tries have produced 1525 discharged links.

Experimental cases having a similar structure are found in results by Lesaint and Massala [7] on positive streamer propagation in large oil gaps. At various levels of overvoltage, they find streamer forms with a range of sparseness and branching patterns, having a range of growth rates.

LINEAR WEIGHTING

Figures 3 and 4 show the dense, widely-branched structure which a linear-weighted growth filter produces. By contrast with the cube- and square-law cases, these trees tend to divide immediately into several (3 or 4) large major branches, which spread densely through the volume but remain self-avoiding and singly connected. Figure 3 evolves from a history with threshold (cutoff) voltage of 0.1200, Figure 4 with 0.2000. The strong influence of this parameter on the conical upper envelope of the dense structure is noted. The bushiness (fractal density) increases in the later stages of growth, as higher val-

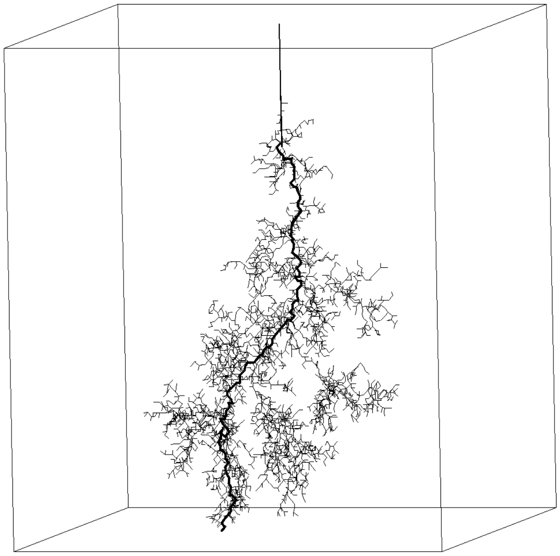

Figure 2. Square-law simulation. In these examples, the continuous "leader" path between electrodes is indicated by heavy line weight. 346 statistical tries have produced 4503 discharged links. The needle is 31 grid steps in length. Threshold voltage is set at 0.0500, well below the largest neighbor voltage.

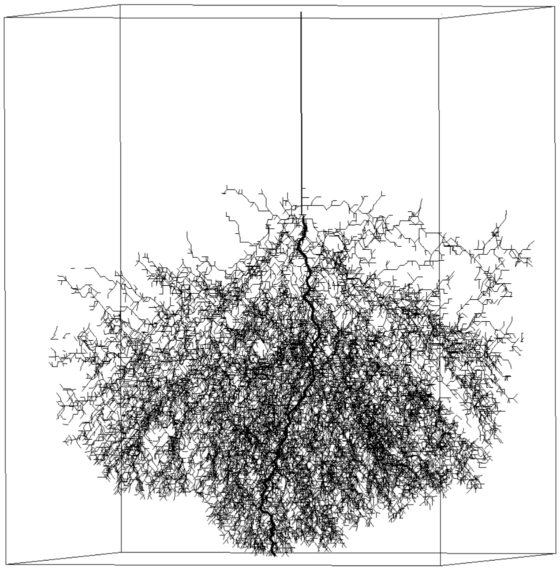

Figure 3. Linear filter. The needle has been lengthened to 47 grid steps, approximating the point-to-plane configuration. The threshold (cutoff) voltage is set at 0.120. The very dense, spheroidal cone of growth has produced 27837 discharged links, in 91 statistical tries. The upper envelope of growth is a flat cone, almost level with the tip of the needle.

ues of neighbor voltages occur in the reduced gap to the cathode. The effect of electrical screening is pronounced. The upper development of the tree, near the anode, is limited at a roughly conical envelope, where growth has ceased as the voltage field is screened back below the abrupt cutoff value.

A conical envelope, for dense positive streamers in n-hexane, has been noted experimentally by Stricklett em et al. [8]. Badent, Kist, and Schwab [9] have observed a constriction of the conical propagation sector, associated with increasing pressure, in experimental positive streamers in insulating oil. Earlier high-resolution images by Chadband [10] had demonstrated the fine filamentary detail of dense positive streamers.

SUMMARY

Thus, as its parameters are changed, the model simulates a range of effects which have experimental counterparts.

The power-law lines for different exponents are sketched in Figure 5. These correspond to nominal response curves for the fluid. Raising the exponent exerts an axial constriction on the local growth cone, concentrating growth ahead of the tip [11]. Lowering the exponent gives rise to a more dense, spread pattern of twig growth in a more rounded frontal zone. This effect is seen more clearly when diagonal growth paths are included.

Varying the threshold (cutoff) voltage parameter also has a noticeable effect on the overall density of discharged links. (The lines in Fig. 5 are limited by a drop to zero at their left end, which is positioned by this choice.) Choice of threshold value controls the conical sector angle into which the dense linear-weighted fractal tree can spread.

PROGRAMMING METHODS

Keeping track of the voltage field surrounding the streamer tree, which is a growing highly irregular object, poses the major difficulty. We re-evaluate the voltage field at each growth stage, throughout the full interior volume. The enormous flow of numerical activity is handled by parallel (SPMD) computing methods.

We have developed a scalable block-parallel program to realize this model. It exploits the power and local memory of multiprocessors to handle the bulky Laplacian calculation. Each process is assigned a

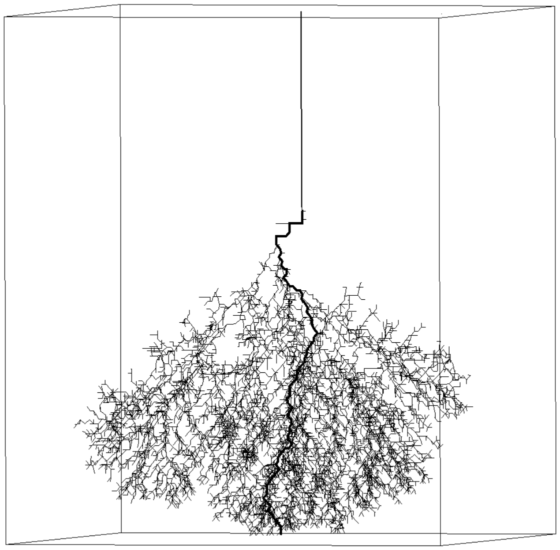

Figure 4. Linear filter. The threshold (cutoff) voltage has been raised to 0.200. Branches next to the narrower conical upper envelope show a more sparse appearance than the dense structure approaching the cathode. Total number of discharged links has been reduced to 10000, in 127 statistical tries. The cutoff from screening is having a pronounced narrowing effect on the conical sector of propagation.

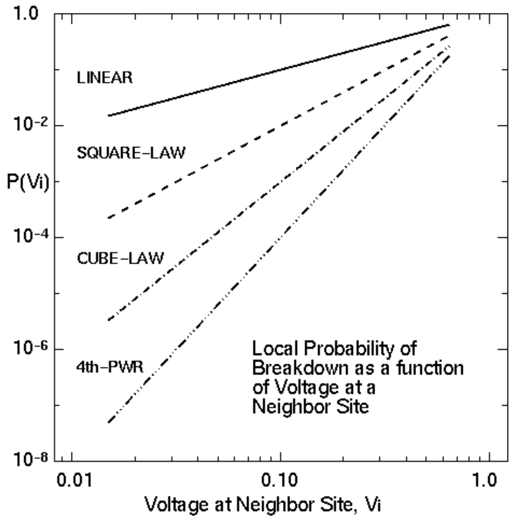

Figure 5. Local breakdown probability (for a link discharge), P(V_i) as a function of neighbor-site voltage. Scales are log-log. Choice of power-law exponent represents the nominal response of the fluid. Threshold (cutoff) parameter determines where the left end of each line drops to zero. Voltage distributions overlap this dropoff point.

rectangular slab subdivision of the cube volume. Data is passed through the face-planes between adjoining blocks. Details are given in an earlier article [12, 13].

The program CADMUS is written in a readable high-level language, which leaves the physics clear. Fortran 90 is used as the parallel language for the code. It is supplemented by NIST's DPARLIB, a set of subroutines which extend F90 instructions across block-process boundaries [14], in the environment of the Message Passing Interface (MPI) [15]. It has run on networks of workstations, in the LAM environment [16], but is faster on more tightly coupled multiprocessors, such as the IBM SP2 and SGI Origin 2000. We offer the source code for interested users, via Internet:

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http://www.itl.nist.gov/div895/sasg/
  dielectric/dielabs.html
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