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Detection and Identification of Structural Damage from Dynamic Response Measurements

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ABSTRACT

The random vibrational response of a structural system contains the characteristic signal of the structure. Using proper signal processing techniques, the characteristic signal can be retrieved from the random response. Structural damages can then be identified by studying the changes of the characteristic signal.

Two signal processing techniques are being used to retrieve the structural characteristic signal from the random responses. The first is in the frequency domain, using FFT technique to obtain the averaged frequency response of the system. Followed by a curve-fitting computer program, the system's eigenvalues and eigenvectors are resolved from the frequency response curves. The second is in the time domain, applying the random decrement technique to convert the random response to the random decrement free decay signature. Using an auto-regressive method, the system's eigenvalues and eigenvectors can be determined from the random decrement signature.

Cross random decrement signature between two positions correlates the random responses of the two. If an array of cross random decrement signature between a number of positions in the structure is evaluated, the location of the damage can be determined following proper system identification processes.

The system identification technique we developed uses state equation formulation. Once the system's eigenvalues and eigenvectors are determined, the system's mass, stiffness and damping matrices can be obtained. The changes of the matrix elements will provide the indication of the location and severeness of the structural damage.

These techniques have been verified with a number of theoretical and experimental tests. A result from 13.8 scale model offshore platform in soil including the effect of the piles is also presented.

1. INTRODUCTION

Complex structural systems such as flight vehicles, naval ships and offshore platforms are exposed to severe wind or wave loading which over an extended period can lead to fatigue failure. Initiation and propagation of cracks change the structural response of the system which manifests itself in a change in the dynamic equations of motion. In principle, if one can completely determine the system's parameters in the dynamic equations, then the nature as well as location of the damage occurring in a structural system are identified. For a structure the complete set of system parameters is the mass, stiffness and damping matrices.

The eigenvalues of a structure system are associated with the modal frequencies and damping, and the eigenvectors with the mode shapes. The information of the modal frequencies, damping and mode shapes are contained in the response signals. To identify the system's parameters from the response measurements, there are generally two approaches one can follow: (1) retrieve the modal frequencies, damping and model shape vectors from the measured reaponses first. Then, reconstruct the mass, stiffness and damping matrices from the retrieved eigenvalues and eigenvectors; and (2) resolve the mass, stiffness and damping matrices directly from the measured responses.

For complex systems, the number of vibration modes contained in the response signal is high. Direct, simultaneous resolution of the mass, stiffness and damping matrices from the measured response is likely to be inaccurate, especially in the presence of random noise. Since the damages are often too small to be detected, the accuracy of the calculated eigenvalues is of crucial importance to differentiate the changes. With proper filtering process, it is possible to accurately resolve the eigenvalues a few modes at a time. In present work, we adopt the first approach to identify the structural system's parameters for the purpose of damage detection.

In this paper, mathematical schemes as well as computer algorithms are developed and tested for a few simple systems from the measured responses. Two algorithms are used to retrieve the system's eigenvalues and eigenvectors from the random dynamic responses. One is in time domain, the other in frequency domain. After system's eigenvalues and eigenvectors are obtained either from time domain technique or from frequency domain technique, another algorithm is used to reconstruct the mass, stiffness and damping matrices from the eigenvalues and eigenvectors. The frequency domain technique uses an FFT spectrum analyzer to calculate the system's transfer function, which requires the measurements of both the output response and the input. When the system is randomly excited, the transfer function is averaged over many samples. A curve fitting algorithm is then used to determine the eigenvalues and eigenvectors from the transfer functions. In time domain technique, random decrement and cross random decrement method are used to retrieve the system's free decay responses from the random dynamic responses. The excitation forces are assumed to be purely random and have zero average values. Eigenvalues are calculated from the free decay responses using auto-regressive method and eigenvectors calculated using a linear least square curve fit method.

Numerous authors have studied the various techniques mentioned above: the frequency technique (1-3), time domain technique (4-5), random decrement technique (6-9), auto-regressive technique (10), as well as the system identification technique (11). Our primary goal here is to study the feasibility of detecting small damages in structures using all these computer algorithms. Some test results are presented which include a two-degree-of-freedom analog computer system, a cantilever beam system.

Response measurement has also been performed on a 1/13.8 scale model of an offshore platform. Time domain eigenvalue retrieval technique using random decrement has been applied to determine the modal frequencies and damping of this offshore platform structure.

2. MATHEMATICAL MODEL OF THE SYSTEM IDENTIFICATION TECHNIQUE

Consider a structural system which can generally be represented by an N degree-of-freedom linear system. The dynamics of the system is governed by its equation of motion

$$[M][\ddot{X}] + [C][\dot{X}] + [K][X] = [f],$$

where $[X]$, $[\dot{X}]$, $[\ddot{X}]$ are the displacement, velocity and acceleration column vector of degree N, respectively. $[f]$ is the force column of degree N. $[M]$, $[C]$, $[K]$ are the NxN mass, damping and stiffness matrices, respectively. The exercise of the system identification involves the identification of $[M]$, $[C]$, $[K]$ matrices from the known responses $[\ddot{X}]$, $[\dot{X}]$, $[X]$, and the known forcing function $[f]$.

Adding a trivial differential equation

$$[M][\dot{X}] - [M][\dot{X}] = 0$$

to the above equation, we obtain a set of state equations which still describe the motion of the structural system,

$$\begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix} \begin{bmatrix} \dot{X} \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$$

or

$$[D][\dot{q}] + [E][q] = [Q],$$

where

$$[D] = \begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix}$$

$$[E] = \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix}$$

$$[\dot{q}] = \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix}, \quad [q] = \begin{bmatrix} \dot{X} \\ X \end{bmatrix}, \quad [Q] = \begin{bmatrix} 0 \\ f \end{bmatrix}.$$

After Laplace transformation, we obtain

$$[B(s)] = [[D]s + [E]]$$

where

$$[B(s)][q(s)] = [Q(s)],$$

is the system matrix.

It can be proved that $[D]$ and $[E]$ can be represented by the eigenvalues, P_k, and eigenvectors, $[Y_k]$, of the system matrix which are determined by the homogeneous equation

$$[B(P_k)][y_k] = 0$$

When $[M]$, $[C]$, $[K]$ are all symmetric, the expressions are

$$[D] = [Y]^{-1T}[I][Y]^{-1}$$

$$[E] = [Y]^{-1T}[-P][Y]^{-1}$$

where $Y = [y_1, y_2, y_3, \ldots, y_N]$ is the eigenvector matrix

$$P = \begin{bmatrix} P_1 & & 0 \\ & P_2 & \\ 0 & & \ddots\ P_n \end{bmatrix}$$ is the eigenvalue matrix.

it can also be shown that the system's transfer function can be represented by the eigenvectors and

and eigenvalues,

$$[H(s)] = [Y](s-P)^{-1}[Y]^T$$

$$= \sum_{k=1}^{N} \left[\frac{y_k y_k^T}{s-p_k} + \frac{y_k^* y_k^{*T}}{s-p_k^*} \right] .$$

The above derivation states the fact that if one can determine the eigenvalues and eigenvectors of a system, the mass, stiffness and damping matrices of the system are simply the products of the eigenvalue and eigenvector matrices. The eigenvalues and eigenvectors can be obtained from the measured system response data for a known force input using frequency domain technique or a time domain technique.

3. SYSTEM IDENTIFICATION TECHNIQUE

Implementation of the Mathematical Model in the System Identification Technique is illustrated in Figure 1. The random vibrational response of a structural system contains the characteristic signal of the structure. Using proper signal processing techniques, the characteristic signal can be retrieved from the random response. Structural damages can then be identified by studying the changes of the characteristic signal.

Two signal processing techniques have been developed to retrieve the structural characteristic signal from the random responses. One technique analyzes the structure signal in the frequency domain, the other in the time domain.

3.1 FREQUENCY DOMAIN TECHNIQUE

Structural responses from a known random force input are collected into a Fast Fourier Transform (FFT) analyzer to obtain the frequency response in digital form. The digitized frequency responses are curve fitted with a computer program to yield the eigenvalues and eigenvectors. The frequency domain curve fitting program uses a linear-least square method to find the best fit of the collected system's transfer function to the following theoretical expression

$$H(s) = \sum_{k=1}^{N} \left(\frac{a_k}{s-p_k} + \frac{a_k^*}{s-p_k} \right)$$

where p_k and a_k are the poles and residues of the transfer function. The poles are system's eigenvalues and the residues related to the eigenvectors. After the eigenvalues and eigenvectors are found, the system identification techniques are applied to find the mass, stiffness and damping matrices.

3.2 TIME DOMAIN TECHNIQUES

The time responses of a structure system when excited by a random forcing function is digitized and processed with random decrement technique. The random decrement signature represents system's characteristics from which the system's modal frequencies and damping values can be determined.

The impulse response of a structure system contains the characteristic time function of the system. The frequencies and damping values of the impulse response are the eigenvalues of the system. And, the amplitudes of the impulse responses at different locations are the eigenvectors of the system. If one assumes that the random decrement signatures represent the impulse response functions of the system, then the frequencies, dampings and relative amplitudes of the random decrement signatures may be used to represent the eigenvalues and eigenvectors of the system.

To accurately retrieve the eigenvalues and the eigenvectors, a time domain curve fitting procedure is applied to the random decrement signatures of the structure. First, the frequency and damping value of a random decrement signature are determined using the auto-regression method. After the frequency and damping are determined, the random decrement signatures are curve fitted with the following expression to determine the amplitudes of the signatures.

$$x(t) = a_o + \sum_{i=1}^{M} a_i \, e^{-\omega_i \zeta_i t} \sin(\omega_i t + \phi_i)$$

where ω_i and ζ_i are natural frequency and damping ratio of the i-th mode of the system. α_i and ϕ_i are the amplitude and phase angle. Again the curve fitting procedure uses the least square method.

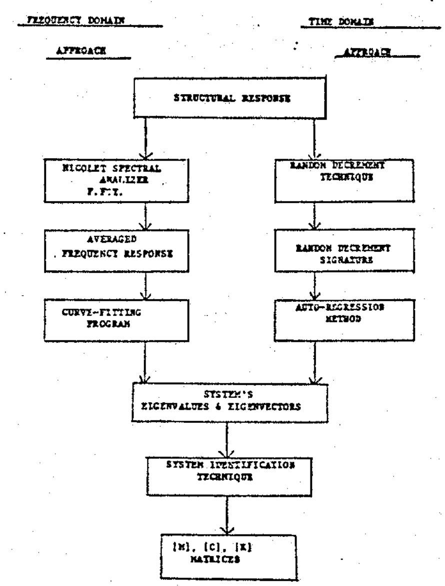


FIGURE 1 SYSTEM IDENTIFICATION TECHNIQUE

4. COMPUTER PROGRAM DEVELOPEMNT

A computer program was developed to implement the System Identification Technique illustrated in Figure 1. The Program includes a frequency domain curve fitting program, a random decrement program, a time domain curve fitting program and a system identification program. The random decrement program, written in Z80 Assembly language, is implemented in a CROMEMCO microcomputer. The resulting random decrement signature is transferred to the UNIVAC 1180 computer, where the other three programs are located, for subsequent processing.

The input to the frequency domain curve fitting program is the experimental transfer function which is collected using a NICOLET FFT Spectrum analyzer in which many instantaneous transfer functions are averaged. The averaged transfer function is transferred to the UNIVAC computer through an interface microcomputer system. When the frequency domain curve fitting program receives the experimental transfer function from the spectrum analyzer and the time domain curve fitting program receives random decrement signatures of a structure from the random decrement microcomputer, both programs reduce the input data to yield the system's eigenvalues and eigenvectors. Then, the system identification program will pick up these eigenvalues and eigenvectors and reduces them to the system's [M][C][K] matrices.

4.1 PERFORMANCE TEST OF THE TIME DOMAIN ALGORITHM

The time domain approach uses an auto-regressive curve fitting method to resolve frequencies and damping values of a multidegree-of-freedom signal simultaneously. It was successfully tested with theoretical data of the type:

$$f(t) = \sum_{k=M}^{M} A_k e^{-\gamma_k t} \cos\omega_k t + B_k e^{-\gamma_k t} \sin\omega_k t, \quad M = 3$$

the signal simulates the free decay response of a structure or the random decrement signature of the structural random response. Given various values of γ_k, ω_k, the time function f(t) is fed into the program. The program then resolves the frequencies and dampings from the time function f(t). The results are compared with the originally given values of frequencies and dampings. Many cases have been tested including different sampling rates, time length and modal separations. Due to the numerical truncation error plus the randomness of the real system, it is necessary to test the performance of the time domain eigenvalue retrieving algorithm under noisy condition. A few test cases have been conducted whose results are presented below.

A theoretical signal which consists of three sinusoidal waves with frequencies 1456.3, 2427.2, 3398.1 Hz and damping rations 1% for all three modes was convolved with a random white noise signal collected from an analog noise generator with a sampling rate of 9708.74 Hz. The result simulates the random response of a structure. After the auto-regression of the random decrement signature, the frequencies and damping ratios were resolved. The accuracies in the frequency calculation for all three modes are very good, all within 1%. However, the calculations of the damping ratios are less accurate. The error of the damping ratio of the first mode is 10%, second mode 15%, and third mode 85%. This is due to the fact that the added noise is not a white noise so that it can not be completely removed by the random decrement process. The modal separations of this signal are considered high. The three frequencies are at 15%, 25% and 35% of the sampling rate.

In the cases where many modes cannot be resolved simultaneously, filtering process helps reduce the number of modes and improve the accuracy of the eigenvalue retrieval. A signal of 291.26 Hz, 485.44 Hz, 679.61 Hz (3%, 5%, 7% of the sampling rate) and damping ratio 1% was convolved with the random analog noise and filtered with filter band pass from 4.5% to 5.5% of the sampling rate. The filtered signal contains the dominant mode of 485.44 Hz. Other spectral modes attribute to the filter and the noise. When the three mode auto-regression algorithm was applied to the filtered signature, the 485.44 Hz distinct mode was picked out and the other two modes were used as error compensation. Due to the presence of the noise, the frequency and damping ratio obtained by using auto-regression method depends on the sampling rate and the number of data points used. Based on previous experimental experience, optimum sampling rate was found to be 3.5 - 7 times the frequency of interest and optimum number of data points was near 128. Using a sampling rate 3.33 times the frequency, the number of data points 128, the above filtered signal was resolved by the three mode auto-regression algorithm. This resulted in a frequency 482.86 Hz and damping ratio of 1.025%. Compared to the theoretical value, the frequency has 1% accuracy and the damping ratio 2% accuracy. Hence we believe the time domain algorithm accurately resolves the frequency and damping values of a multidegree-of-freedom system.

4.2 PERFORMANCE TEST OF THE FREQUENCY DOMAIN ALGORITHM

The frequency domain curve fitting program has successfully tested with theoretical frequency response functions generated by the following formula:

$$F(s) = \sum_{k=-M}^{M} \frac{a_k}{s - p_k}$$

Initially, the eigenvalues p_k and the residues a_k were assigned. The generated frequency response functions were fed into the frequency domain curve fitting program. The program resolved the residues and poles a_k, p_k of each mode. These values when compared to the original assigned values are very accurate.

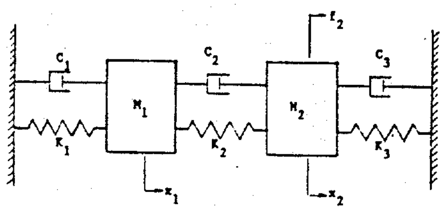
4.3 PERFORMANCE TEST OF THE SYSTEM IDENTIFICATION ALGORITHM

In order to demonstrate the ability of the computer program to retrieve the system identification parameters from theoretical response data, a simple computer experiment was conducted. Spring mass systems consisted of two or three degrees of freedom were analyzed. The theoretical response to a random loading was calculated for known values of mass, stiffness, and damping values. The system's eigenvalues and eigenvectors were then retrieved using the frequency domain technique with the theoretical response as input. Mass, stiffness, and damping matrices were determined using the System identification algorithm and compared with the original input values. Excellent agreement was obtained for all cases studied. This demonstrated that the technique can accurately retrieve the system identification parameter even for large variations in mass, stiffness and damping parameters.

5. ANALOG COMPUTER EXPERIMENT

In order to further demonstrate the feasibility of retrieving the System Parameters from response data, the analog computer was used to simulate response from real systems. An analog computer system was used to simulate the dynamic response of a two-degree-of-freedom spring-mass-dashpot system as shown in Figure

2. System input parameters can be easily adjusted by changing the resistor values of the potentiometers in the analog computer circuit. The response signals of the analog computer contains circuit noise which simulates the natural white noise contained in the dynamic responses of the real system.



Spring-mass system in the analog computer simulation

FIGURE 2

TABLE 1

Poles and residues of the velocity transfer functions

(1st stage damage)

		1st Mode	2nd Mode
$\dot{x}_1$	Frequency (rad/sec)	15.78	50.22
	Damping Ratio	0.1802	0.1551
	Residues	0.04227 + i0.006686	-0.04111 - i0.004233
$\dot{x}_2$	Frequency (rad/sec)	15.79	50.23
	Damping Ratio	0.1796	0.1541
	Residues	0.04426 + i0.008879	0.01880 - i0.000418

The dynamic equations of the spring-mass-dashpot system of Figure 2 are

$$\ddot{x}_1 = -\frac{c_1 + c_2}{m_1}\dot{x}_1 + \frac{c_2}{m_1}\dot{x}_2 - \frac{k_1 + k_2}{m_1}x_1 + \frac{k_2}{m_1}x_2$$

$$\ddot{x}_2 = \frac{c_2}{m_2}\dot{x}_1 - \frac{c_2 + c_3}{m_2}\dot{x}_2 + \frac{k_2}{m_2}x_1 - \frac{k_2 + k_3}{m_2}x_2 + \frac{f(t)}{m_2}$$

where m_1, m_2 are masses, c_1, c_2, c_3 damping constants, k_1, k_2 k_3 stiffness, f(t) is the input forcing function at mass 2. Using k_1 = 1500 lb/ft, k_2 = 6000 lb/ft, k_3 = 1500 lb/ft, m_1 = 4 slugs, m_2 = 8 slugs, c_1 = 10 lb-sec/ft, c_2 = 20 lb-sec/ft, c_3 = 30 lb-sec/ft, and applying a random input forcing function to mass 2, the displacement transfer functions at mass 1 and 2 were obtained as shown in Fig. 3 and Fig. 4 respectively. These transfer functions were fed into the frequency domain eigenvalue retrieving program from which the residues and poles of the system transfer functions were found as shown in Table 1. When the eigenvalues and eigenvectors were fed into the system identification program, the system's [M][C][K] matrices were identified, as shown below

$$M = \begin{bmatrix} 3.944 & -0.00492 \\ -0.000284 & 7.88 \end{bmatrix} \text{ slugs}$$

$$C = \begin{bmatrix} 28.4 & -19.6 \\ -19.6 & 50.8 \end{bmatrix} \text{ lb-sec/ft}$$

$$K = \begin{bmatrix} 7477 & -5933 \\ -5933 & 7452 \end{bmatrix} \text{ lb/ft}$$

The exact values of the system's [M][C][K] matrices from theoretical calculations are

$$M_{theo.} = \begin{bmatrix} 4.0 & 0 \\ 0 & 8.0 \end{bmatrix} \text{ slugs}$$

$$C_{theo.} = \begin{bmatrix} 30.0 & -20.0 \\ -20.0 & 50.0 \end{bmatrix} \text{ lb/sec/ft}$$

$$K_{theo.} = \begin{bmatrix} 7500 & -6000 \\ -6000 & 7500 \end{bmatrix} \text{ lb/ft}$$

The comparison between the identified and theoretical values of the [M][C][K] matrices are within 5%. This again demonstrated the ability of the System Identification Technique to accurately retrieve [M][C][K] matrices from the response data.

In order to demonstrate the ability to detect system changes from simulated damage, three stages of damage were simulated. Stage 1 was considered to have characteristics of the system just analyzed. Stages 2 and 3 were as given below:

Stage 2 Damage: m_1 = 4.0 slugs, m_2 = 8.0 slugs
c_1=40 lb-sec/ft, c_2=20.0 lb-sec/ft, c_3=30.0 lb-sec/ft
k_1=1500 lb/ft, k_2=3000 lb/ft, k_3=1500 lb/ft

Stage 3 Damage: m_1 = 4.0 slugs, m_2 = 8.0 slugs
c_1=10 lb-sec/ft, c_2=20.0 lb-sec/ft, c_3=30.0 lb-sec/ft
k_1=1500 lb/ft, k_2=3000 lb/ft, k_3=1500 lb/ft

The poles and residues of the velocity transfer functions for the 2nd and 3rd stages of damage are listed in Table 2 and 3. The corresponding [M][C][K] matrices are shown below. Again it is demonstrated that the System Identification Technique using the frequency domain algorithm can accurately retrieve the [M][C][K] matrices from response data. Moreover, changes simulating various stages of damage are detectable.

2nd stage damage:

$$M_{theo.} = \begin{bmatrix} 4.0 & 0 \\ 0 & 8.0 \end{bmatrix} \quad \text{slugs}$$

$$C_{theo.} = \begin{bmatrix} 60 & -20.0 \\ -20.0 & 50.0 \end{bmatrix} \quad \text{lb-sec/ft}$$

$$K_{theo.} \begin{bmatrix} 7500 & -6000 \\ -6000 & 7500 \end{bmatrix} \quad \text{lb/ft}$$

$$M = \begin{bmatrix} 3.944 & -0.001989 \\ -0.00334 & 7.934 \end{bmatrix} \quad \text{slugs}$$

$$C = \begin{bmatrix} 59.139 & -19.428 \\ -19.428 & 49.205 \end{bmatrix} \quad \text{lb-sec/ft}$$

$$K = \begin{bmatrix} 7494.0 & -5963.8 \\ -5963.8 & 7520.8 \end{bmatrix} \quad \text{lb/ft}$$

3rd stage damage:

$$M_{theo.} = \begin{bmatrix} 4.0 & 0 \\ 0 & 8.0 \end{bmatrix} \quad \text{slugs}$$

$$C_{theo.} = \begin{bmatrix} 30 & -20 \\ -20 & 50 \end{bmatrix} \quad \text{lb-sec/ft}$$

$$K_{theo.} = \begin{bmatrix} 4500 & -3000 \\ -3000 & 4500 \end{bmatrix} \quad \text{lb/ft}$$

$$M = \begin{bmatrix} 3.936 & -0.0117 \\ -0.0058 & 7.878 \end{bmatrix} \quad \text{slugs}$$

$$C = \begin{bmatrix} 28.74 & -19.52 \\ -19.52 & 49.90 \end{bmatrix} \quad \text{lb-sec/ft}$$

$$K = \begin{bmatrix} 4509 & -2970 \\ -2970 & 4511 \end{bmatrix} \quad \text{lb/ft}$$

TABLE 2

Poles and Residues of the Velocity Transfer Functions

(2nd Stage Damage)

		1st Mode	2nd Mode
$\dot{x}_1$	Frequency (rad/sec)	15.89	50.58
	Damping Ratio	0.1088	0.1020
	Residues	0.04094 + i0.004655	-0.04089 - i0.004923
$\dot{x}_2$	Frequency (rad/sec)	15.89	50.56
	Damping Ratio	0.1092	0.1020
	Residues	0.04474 - i0.01735	0.01863 - i0.003211

TABLE 3

Poles and Residues of the Velocity Transfer Functions

(3rd Stage Damage)

		1st Mode	2nd Mode
$\dot{x}_1$	Frequency (rad/sec)	15.85	37.88
	Damping Ratio	0.1109	0.1359
	Residues	0.03958 + i0.004973	0.03944 - i0.006832
$\dot{x}_2$	Frequency (rad/sec)	15.85	37.87
	Damping Ratio	0.1103	0.1349
	Residues	0.04697 + i0.004429	0.01645 - i0.004332

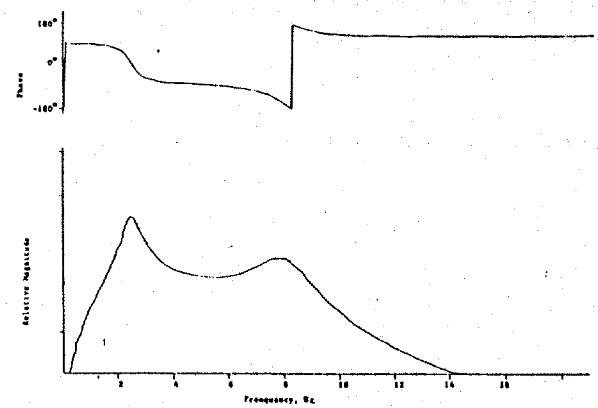


FIGURE 3

The displacement time responses of the analog computer system were obtained with initial conditions $x_1(o) = -0.80$ ft, $x_2(o) = 0.80$ ft. Two forcing conditions were studied: one with $f_1 = 611.6$ lb applied to mass 1, the other with $f_2 = 1222.4$ lb applied to mass 2. A typical response is shown in Fig. 5. Using the time domain eigenvalue retrieving program, the frequencies, dampings and amplitudes of the vibration modes contained in the time responses were calculated. The resolved values corresponding to each of the three damage stages are listed in tables 4, 5 and 6.

The resolved eigenvalues and eigenvectors were used to construct the normalized damping and stiffness matrices $[\tilde{C}]$, $[\tilde{K}]$, which are defined as the ratio of the actual damping and stiffness matrices to the mass matrices, i.e.

$$\tilde{C} = M^{-1}C, \; \tilde{K} = M^{-1}K.$$

The calculated $[\tilde{C}]$, $[\tilde{K}]$ and their corresponding theoretical values for the three stages of damage are listed below in Table 7.

The results demonstrate that the system identification technique using the time algorithm also accurately retrieves system's characteristic matrices, and changes simulating damage are detectable.

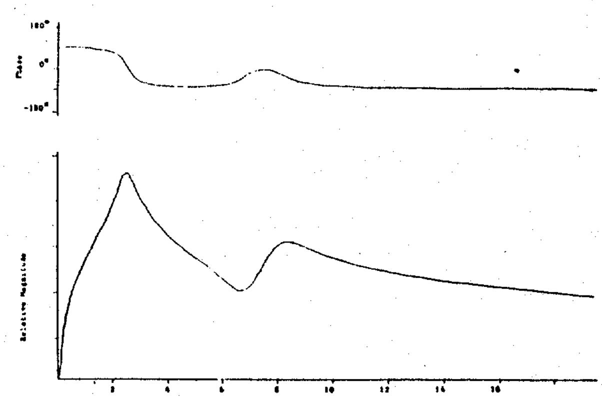


FIGURE 4

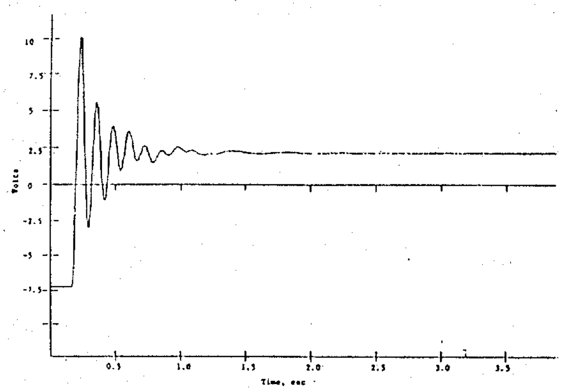


FIGURE 5 DISPLACEMENT TIME RESPONSE AT MASS 1, WITH CONSTANT FORCE AT MASS 1

TABLE 4

Frequencies, Dampings and Amplitudes of the Time Responses (1st Stage Damage)

Conditions	1st mode Freq. (Hz), Damping ratio, Amplitudes			2nd mode Freq. (Hz), Damping Ratio, Amplitudes		
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_1 = 611.6$ lb						
$x_1(t)$	2.52	0.1135	0.07164	8.04	0.1039	-1.0296
$x_2(t)$	2.49	0.1102	0.07285	7.99	0.1033	0.4597
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_2 = 1222.4$ lb						
$x_1(t)$	2.48	0.1118	-0.1370	8.00	0.1029	-0.9267
$x_2(t)$	2.48	0.1108	-0.1457	8.00	0.1031	0.4256

TABLE 5

Frequencies, Dampings and Amplitudes of the Time Response

(2nd Stage Damage)

Conditions	1st mode Freq. (Hz), Damping ratio, Amplitudes			2nd mode Freq. (Hz), Damping Ratio, Amplitudes		
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_1 = 611.6$ lb						
$x_1(t)$	2.46	0.1852	0.08358	7.93	0.1558	-1.0148
$x_2(t)$	2.46	0.1846	0.08270	7.95	0.1562	0.4461
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_2 = 1222.4$ lb						
$x_1(t)$	2.46	0.1827	-0.1293	7.94	0.1555	-0.9344
$x_2(t)$	2.45	0.1843	-0.1456	7.97	0.1531	0.4066

TABLE 6

Frequencies, Dampings and Amplitudes of the Time Responses

(3rd Stage Damage)

Conditions	1st mode Freq. (Hz), Damping ratio, Amplitudes			2nd mode Freq. (Hz), Damping Ratio, Amplitudes		
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_1 = 611.6$ lb						
$x_1(t)$	2.47	0.1135	0.1006	5.98	0.1364	-1.0381
$x_2(t)$	2.48	0.1111	0.11167	5.99	0.1349	0.4348
$x_1(0) = -0.80$ ft $x_2(0) = +0.80$ ft $f_2 = 1222.4$ lb						
$x_1(t)$	2.46	0.1137	-0.1086	5.98	0.1359	-0.90167
$x_2(t)$	2.46	0.1138	-0.1276	5.97	0.1372	0.3837

TABLE 7

Normalized damping and stiffness matrices

	$\tilde{C}$	$\tilde{K}$	$\tilde{C}_{theo.}$	$\tilde{K}_{theo.}$
1st stage damage	$\begin{bmatrix} 7.73 & -4.57 \\ -2.52 & 6.09 \end{bmatrix}$	$\begin{bmatrix} 1865 & -1524 \\ -732 & 933.8 \end{bmatrix}$	$\begin{bmatrix} 7.5 & -5 \\ -2.5 & 6.25 \end{bmatrix}$	$\begin{bmatrix} 1875 & -1500 \\ -750 & 937.5 \end{bmatrix}$
2nd Stage damage	$\begin{bmatrix} 15.6 & -3.98 \\ -2.53 & 5.66 \end{bmatrix}$	$\begin{bmatrix} 1873 & -1515 \\ -738 & 932.6 \end{bmatrix}$	$\begin{bmatrix} 15 & -5 \\ -2.5 & 6.25 \end{bmatrix}$	$\begin{bmatrix} 1875 & -1500 \\ -750 & 937.5 \end{bmatrix}$
3rd stage damage	$\begin{bmatrix} 7.41 & -5.16 \\ -2.42 & 6.36 \end{bmatrix}$	$\begin{bmatrix} 1119 & -756 \\ -369 & 560.2 \end{bmatrix}$	$\begin{bmatrix} 7.5 & -5 \\ -2.5 & 6.25 \end{bmatrix}$	$\begin{bmatrix} 1125 & -750 \\ -375 & 562.5 \end{bmatrix}$

6. THE CANTILEVER BEAM EXPERIMENT

A cantilever beam was tested to verify that the system identification technique described in the previous section is equally effective for a continuous system. The beam, as shown in Figure 6, was excited with single and random impact near the end. Six accelerometers were attached to the beam at six equally spaced positions. The transfer functions from the impact position to any accelerometer position was obtained by feeding the output acceleration signal and input forcing function into a spectrum analyzer: the Nicolet FFT analyzer. In the analyzer, the input and output signals were digitized and the Fast Fourier transform of the signals was performed. The instantaneous transfer functions were obtained by dividing the two spectra. Final transfer function was obtained by averaging over a series of instantaneous transfer functions. Damages were introduced to the system. The first damage scenario was a one-fourth depth saw cut into the edge of the beam. Transfer functions corresponding to the 3 damage scenario at a representative position are shown in Figure 7. Close examination of the results indicates that the model frequency shifts depend on the depth of the saw cut simulating a crack.

In future research these transfer functions will be used as inputs into the frequency domain curve fitting program from which eigenvalues and eigenvectors will be calculated. Then the system's [M][C][K] matrices will be reconstructed from these eigenvalues and eigenvectors using the developed system identification technique. It is expected that the results will shown that the crack depth has a detectable effect on the identified system parameters. Future researc will deal with correlating crackage and location with observed changes.

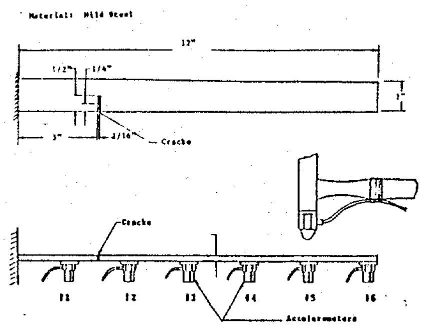


FIGURE 6 THE CANTILEVER BEAM SYSTEM

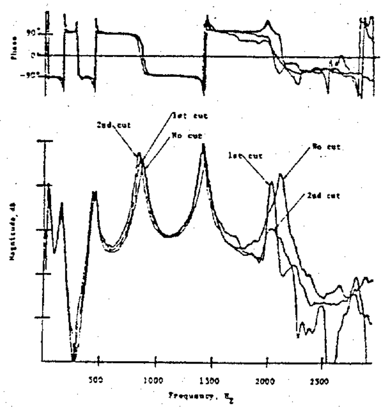
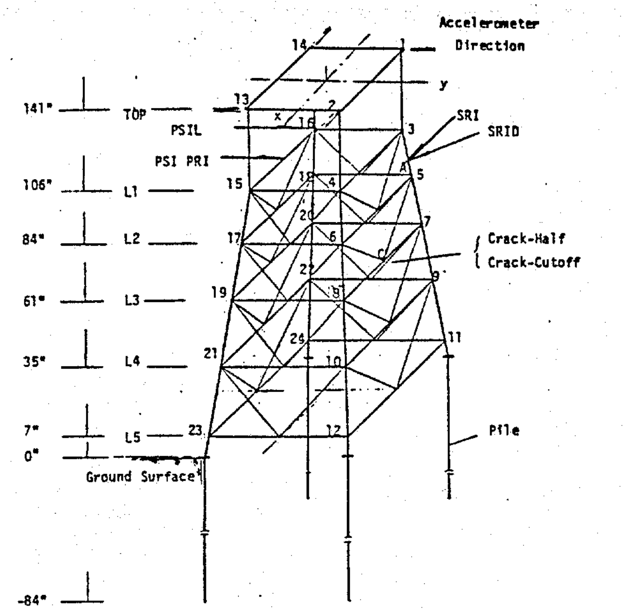


FIGURE 7 TRANSFER FUNCTIONS AT LOCATION 1, WITH FORCE AT LOCATION 5



PSIL—Pendulum Test, Single Impact, at L1, Left
PSI —Pendulum Test, Single Impact, at L1
PRI —Pendulum Test, Random Impact, at L1
SRI —Shaker Test, Random Impact, at A, X-Direction
SRID—Shaker Test, Random Impact, at A, Diagonal Direction
1,2,3........24 Accelcrometer, Measure Acceleration Response

FIGURE 8 OFFSHORE PLATFORM MODEL

7. STRUCTURAL RESPONSE MEASUREMENT OF AN OFFSHORE PLATFORM MODEL ON SOIL FOUNDATION.

In this section, we apply the eigenvalue retrieving algorithms developed in previous sections to determine the natural frequencies and damping of 1:13.8 scale model of an offshore platform. The model structure consists of four legs made of 2 inch diameter steel pipes, a 1.5 inch thick top plate and cross bars made of 3/4 inch steel pipes. It has six levels, labled as the top level and levels 1 through 5, with elevation of 141", 106", 84", 61", 35" and 7" respectively. The base has dimensions 57" x 57" and the top plate 38" x 38". The structure was mounted on a foundation consisting of four piles made of steel pipes, each seven feet long and embedded in the soil of the earth ground, as shown in Fig. 8.

A hammer was used to excite the structure with a single impact on one of the legs, along the axis of a horizontal connecting bar in level 1. At the other end of the connecting bar was attached an accelerometer which measured the response. The forcing function and the response were recorded and analyzed with the time domain eigenvalue retrieving technique. The frequencies and dampings of the lowest five dominant modes are shown in Table 8.

Modes	FFT	Random Decrement	
	Freq. (Hz)	Freq. (Hz)	Damping Ratio
1	11.875	11.896	0.002400
2	20.25	20.315	0.005972
3	30.50	30.706	0.009893
4	35.50	36.217	0.010822
5	43.75	44.134	0.007472

TABLF 8 IDENTIFIED MODAL FREQUENCIES AND DAMPINGS OF THE OFFSHORE PLATFORM MODEL

In order to acquire information concerning the pattern of the structural responses due to the type and severeness of damage as well as for the identification of damage locations, a series of systematic measurements and tests were conducted. Twenty-four accelerometers, arranged in four different directions, were attached to the four legs of the model. Dynamic responses at these 24 positions were tape recorded for various test conditions. The test conditions consisted of four different forcing functions and three damage scenarios. A hammer was set up that provided single and random impact forces at two positions along the cross bar between positions 15 and 16. An electro-mechanical shaker was also set up that provided random forcing functions in two different directions to the leg at the middle of positions 3 and 5. Two saw cuts were done at the middle of the cross bar between positions 7 and 8. The depth of the first cut was half of the diameter of the cross bar. The second cut completely separated the cross bar into two sections.

The recorded response signals will be extensively analyzed with the developed system identification technique in the continuing research effort.

8. CONCLUSIONS

The feasibility of using structural response data from a known random input to completely characterize the System Parameters, [M][C][K], has been demonstrated for discrete spring-mass-dashpot systems. Both the frequency domain curve fitting and the time domain curve fitting algorithms can give satisfactory eigenvalues and eigenvectors of the system. When the system's parameters are gradually changed the present identification technique is able to resolve the difference and thus show the feasibility of tracking progressive fracture of structural systems.

Preliminary research on a continuous system such as a cantilever beam with induced cracks (saw cuts) and excited by a random input, indicates that the crack size manifests itself by detecable changes in the transfer function at the higher frequency modes. Analyses of the response data by the System Identification Technique gives promise that the effects of the crack size can be detected by changes in the [M][C][K] matrices. There remains a question of how many degrees of freedom the [M][C][K] matrices should have to represent a continuous system. When the number of degrees of freedom is large, difficulties will arise concerning the computation accuracy.

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