Detection of Damages in Structures by the Cross Random Decrement Method

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Abstract

The purpose of this paper is to develop a better understanding of the mechanism of failure of structures and to develop and test the new Cross Random Decrement method for the early detection, identification and location of damages. An experimental study has been conducted on a scale model offshore platform structure. The dynamic responses of the platform under single impact and random excitation were measured for three damage senarios. The response signals were first analyzed using an FFT spectrum analyzer which gives the transfer functions. The changes in the modal parameters such as frequencies and dampings indicate the severeness of the structural damage. For the case of random excitation, system's modal information were obtained from its time signature. Random decrement signal processing technique combined with proper filtering process was used. The changes in the time signatures were correlated with the progressive severeness of the structure damage.

A cross random decrement signal processing technique has also been investigated. The time signatures resulting from the cross random decrement process provide information concerning the time lag between the responses of the two different locations in the structure when it is randomly excited. This information was used in the determination of the location of the damage.

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Introduction

Offshore platform structures subjected random wave loading are susceptable to rosion and fatigue failure. To detect structural changes one often monitors eigenvalues of the structural system. envalues include natural frequencies dampings of the structural vibration. envalues alone only provide information cerning the overall change of the ucture. In order to identify the ation of the defect or damage in the ucture, the mode shapes of vibration, ch are the relative amplitudes and ses at various locations, have to be ermined.

Usually, the eigenvalues and mode pes of vibration are obtained from the tem's frequency responses or from the e decay vibration. The former requires measurements of the input excitation the output responses. The latter uires the measurements of the natural ration and the knowledge of initial ditions. In the case of distributed e loading, complete measurements of the out excitation are nearly impossible and continuous nature of the loading cludes the determination of the initial ditions. Statistical approach which rages many samples of the random ponses, eliminates the necessity of wing the initial conditions and random out excitation must be adopted. Random rement technique has been widely used this purpose (1)-(3).

In this paper, a digital signal cessing technique called cross random rement is used to analyze the vibration nals resulted from an experimental test a 1:13.8 scale model of an offshore tform structure. Two damage stages were roduced to the platform model. For each ge of damage and for the baseline, the celeration responses at two selected ations were measured. The two channels vibration signals are time correlated ing the cross random decrement hnique. If the random input averages to zero, the random decrement signatures present the free decay responses of the ucture. The frequencies and dampings then found from the free responses ing an autoregressive curve fitting thod (4). The relative amplitudes and ases of vibration modes between the two lected positions are also found from the ee decay responses.

The relative phase of vibration tween two points depends on the local ructural property. By varying the cation of the two selected points and nitoring the resulting changes in the lative phases. It is possible to rrelate the location of the damage to e changes in the relative phases.

2. Cross Random Decrement Technique

Free decay responses contain the charateristic information of a structure. Any change in the structural integrity should be reflected in the change of the free decay responses. For structures subjected to continuous random excitations, the characteristic signals are contained in the random dynamic responses of the structures. The random dynamic responses are actually results of the convolution between the free decay responses and the random input excitations. The random decrement process is a signal processing technique which extracts the free decay responses from the random dynamic responses by removing the contribution of the random input excitations.

Modal vectors of structural vibration are represented by the relative amplitudes and phases of the free decay responses between different positions in the structure. The relative amplitudes and phases can only be retrieved from the simultaneous measurement of the responses at relavent positions. The cross random decrement technique uses two channels of measurements each time, obtain the free decay responses and calculate the modal eigenvalues and relative amplitudes and phases between the two selected positions. By shifting the positions of the selected measurements around the structure, the complete modal vectors can be determined.

The cross random decrement technique is mathematically formulated below. Let $x_1(t)$, $x_2(t)$ be the structural random dynamic responses at positions 1 and 2 respectively. The cross random decrement signatures, which correspond to the free decay responses of these two positions, are given by

$$\begin{aligned} y_1(\tau) &= \frac{1}{N}\sum_{i=1}^{N} x_1(t_i+\tau) \\ y_2(\tau) &= \frac{1}{N}\sum_{i=1}^{N} x_2(t_i+\tau) \end{aligned} \tag{1}$$

where t_i is determined by the threshold condition $x_1(t_i) = x_0$. N is the number of averaged samples. The free decay responses contain many structural modes. The modal frequencies, dampings and the complex amplitudes are resolved by curve fitting the free decay curve by the following expression

$$\begin{aligned} y_j(\tau) &= \sum_{k=-M}^{M} A_{jk} e^{i\sigma_k \tau}, \quad j=1,2 \\ \sigma_k &= \omega_k + i\gamma_k \\ A_{jk} &= a_{jk} + ib_{jk} \end{aligned} \tag{2}$$

where ω_k is the frequency and γ_k the damping of the k-th mode. $\sqrt{a_{jk}^2 + b_{jk}^2}$ is the amplitude and $\arctan(b_{jk}/a_{jk})$ the phase of the k-th mode at j-th position. M is the number of structure modes.

The curve fitting procedure include two steps. The first step is to find the complex frequencies from the following polynomial equation

$$z^{2M} - \sum_{k=1}^{2M} c_k z^{2M-k} = \prod_{k=1}^{M} \{z-\exp(i\sigma_k)\}\{z-\exp(-i\sigma_k^*)\} \tag{3}$$

where the polynomial coefficients c_k are obtained using a 2M-th order auto-regression process of the free decay response $y(\tau)$.

$$y(\tau_i) = \sum_{k=1}^{2M} c_k y(\tau_{i-k}) \ , \tag{4}$$

$$\tau_i = (2M+1)\Delta t,\ (2M+2)\Delta t,\ \ldots,\ n\Delta t$$

The descritization time interval Δt and the number of sampled data points n are important factors affecting the numerical accuracy of the coefficients c_k .

The second step is to find the complex amplitudes A_{jk} using a linear least-square-fit method, which is straight forward by minimizing the difference between the right and left hand sides of Equation (2).

3. Experimental Results

An 1:13.8 scale model of an offshore platform structure was set up on the earth ground outside the laboratory for damage tests. The model structure consists of four legs made of 2 inch diameter steel pipes. It has six levels, labled as the top level and levels 1 through 5, with elevation of 141", 106", 84", 61", 35", and 7", respectively. The base has dimensions 57"x57" and the top plate 38"x38". The structure was mounted on a foundation consisting of four piles made of steel pipes, each seven feet long and embedded in the soil of the earth ground. Twenty four accelerometer positions were selected at each level of each leg, respectively. They are labled position 1 through 24, and arranged in four different directions. The configuration of the model structure and the positions of the accelerometers are depicted in Fig.1.

A pendulum was set up to provide random impact excitation at the middle point of the horizontal connecting beam between positions 15 and 16. The random responses at four pairs of accelerometer positions were monitored before and after damage was introduced. The four pairs are positions 5 vs. 3, positions 7 vs. 5, positions 9 vs. 7, and positions 11 vs. 9. Two stages of damage were introduced to the structure. The first stage damage was a saw cut at the middle point of the horizontal connecting beam between positions 7 and 8, halfway through the diameter of the beam. The second stage damage was a complete cutaway at the same location. Therefore, there are three damage senarios to be analyzed: baseline, first stage and second stage damage.

Free decay response at position 5 due to pendulum single impact were collected for the three damage senarios. Their frequency spectra as shown in Figs. 2 - 4 were calculated using Fast Fourier Transform. The spectra indicate there are distinct frequency shifts between 30 - 60 Hz before and after damage. The frequencies corresponding to the vibration modes at 35.7 Hz and 43.7 Hz show slight decreases after the first stage damage and significant increases after the second stage damage. The frequency of the vibration mode at 53.3 Hz shows significant decrease after the first stage damage and further decrease after the second damage. To determine more accurately the amount of frequency shifts and to correlate the damage location to the output responses, cross random decrement technique was applied to find the relative phases of each mode between selected pairs of positions, as well as the relative amplitudes, modal frequencies and dampings.

Pendulum random impacts were the applied to the structure. The random responses at positions 3 and 5 were tape recorded simultaneously. Simultaneous recordings are also repeated for positions 5 and 7, 7 and 9, 9 and 11, respectively. To analyze these recorded signals, a 32 - 60 Hz bandpass filter was used to pick up the three vibration modes at 35.7 Hz, 43.7 Hz and 53.3 Hz from the replayed signals. The filtered signals were then fed into a microcomputer where they were digitized with a 12-bit analog to digital convertor. The sampling frequency used in the digitization was selected at 250 Hz. The root mean square value of the digitized random response of the first channel was calculated and used as the threshold value. About 500 samples of the random responses, sampled according to the threshold criterion, were averaged to obtain the cross random decrement signatures of both channels 1 and 2. One set of typical cross random decrement signatures between positions 5 and 7, before and after damage, are demonstrated in Figs. 5 - 10.

All the random decrement signatures were curve fitted to resolve the frequencies, dampings, amplitudes and phases. Relative amplitudes of vibration modes between positons 3 and 5 were obtained by substrating the phases at position 3 from the corresponding phases at position 5. The same calculation procedures apply for the pair positions 5 and 7, 7 and 9, 9 and 11, respectively. The resolved frequencies, dampings, relative amplitudes and relative phases are listed in Tables 1, 2 and 3 for the vibration modes at 35.7 Hz, 43.7 Hz and 53.3 Hz respectively.

As indicated in the tables, the first stage damage induced about 0.3 Hz frequency decrease for the mode at 35.7 Hz, 0.2 Hz decrease for the mode at 43.7 Hz, and 1.5 Hz decrease for the mode at 53.3 Hz. The second stage damage induced about 1.5 Hz frequency increase for the mode at 35.7 Hz, 0.4 Hz increase for the mode at 43.7 Hz, and 0.4 Hz decrease for the mode at 53.3 Hz, apart from the first stage damage. These observations are in agreement with those from the spectra of free decay responses.

In principle, structural modal frequencies and dampings are independent of the position of measurement. If the location of the damage is to be correlated to the output response, it should be correlated to the mode vectors which consist of the amplitudes and phases of the vibration modes at various positions. Like the fact that the determined frequency values are more stable than the damping values in the random excitation environment since frequencies are not directly related to the energy dissipation mechanism, the determined phases are expected to be more stable than the values of amplitudes. Therefore, the phase changes at different positions should give reasonable indication to the location of damage occured in the structure.

Fig. 11 shows the changes in the relative phases between four pairs of accelerometer positions, produced by the first stage damage, for the three vibration modes monitored. Fig. 12 shows the corresponding changes in the relative phases produced by the second stage damage, apart from the first stage damage. Knowing the fact that the damages were made near position 7, the phase changes of the vibration modes at 35.7 Hz and 53.3 Hz do have the highest magnitudes near position 7, namely for the accelerometer pairs 5 - 7 and 7 - 9. And, these are true for both cases of the first and the second stage damages. For the 43.7 Hz mode, accelerometer pair 7 - 9 still shows the highest changes in relative phases but pair 5 - 7 does not.

The exact correlation between the phase changes at various positions and the location of the damage is complex. It depends on the geometrical constraints of the structure. For a fixed structural configuration, certain correlation pattern exists. Extensive investigations of the correlation mechanism are needed to precisely locate the damage in a structure.

4. Conclusions

Experimental tests have been performed on a 1:13.8 scale model of an offshore platform to investigate proper approaches to detect damages in structures. Cross random decrement signal processing technique has been applied to resolve the structural frequencies, dampings from the measured random response data. It also calculated the relative amplitudes and phases between two arbitrarily selected positions in the structure. The correlation between the changes of the relative phases at various positions and the location of the damage has demonstrated the feasibility of using the cross random decrement technique to determine the damage location. However, due to the complexity of the large structure configuration, the relationship between phase changes and the damage location still need more research effort to clarify.

References

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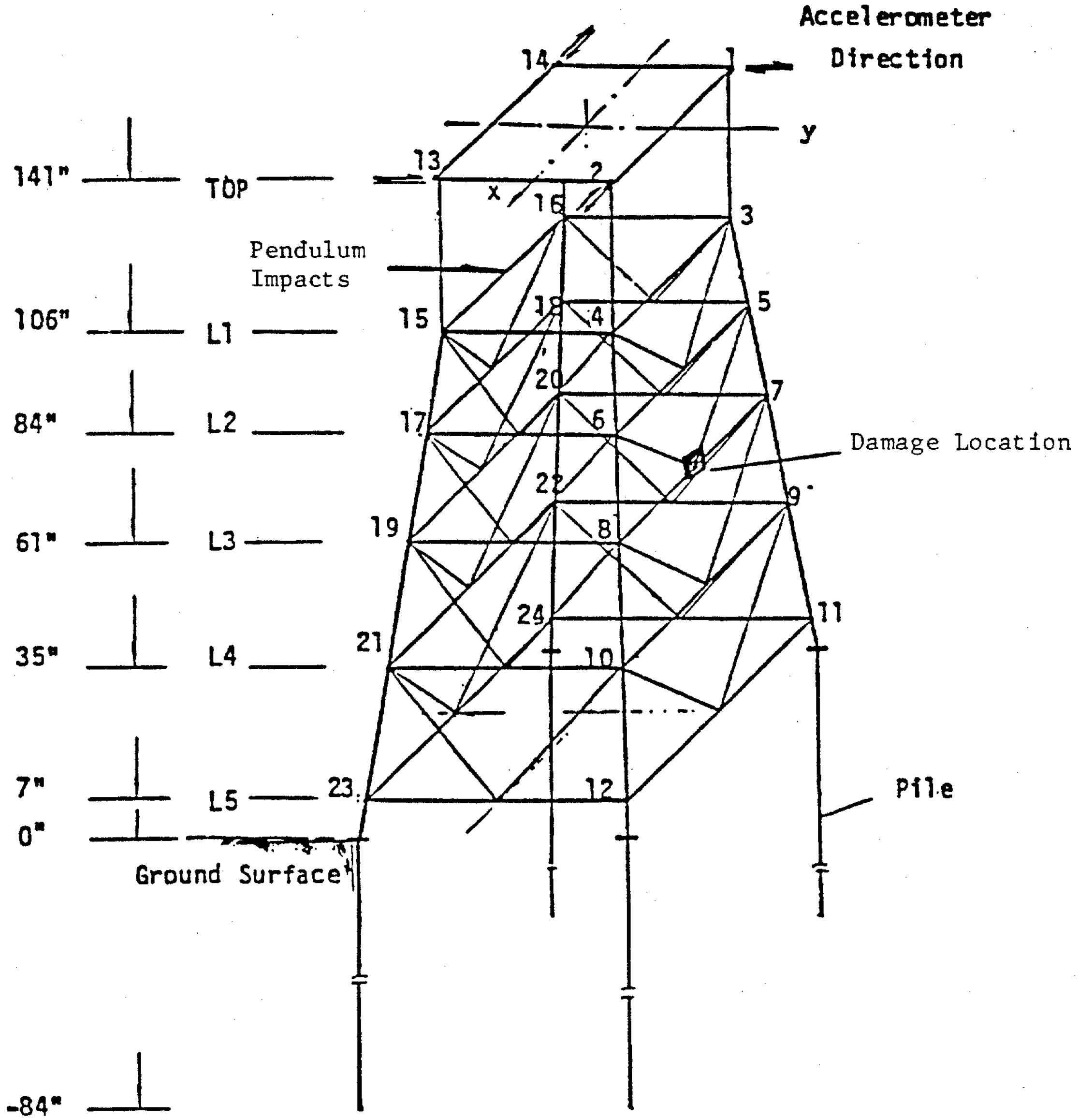

Fig. 1 Offshore Platform Model

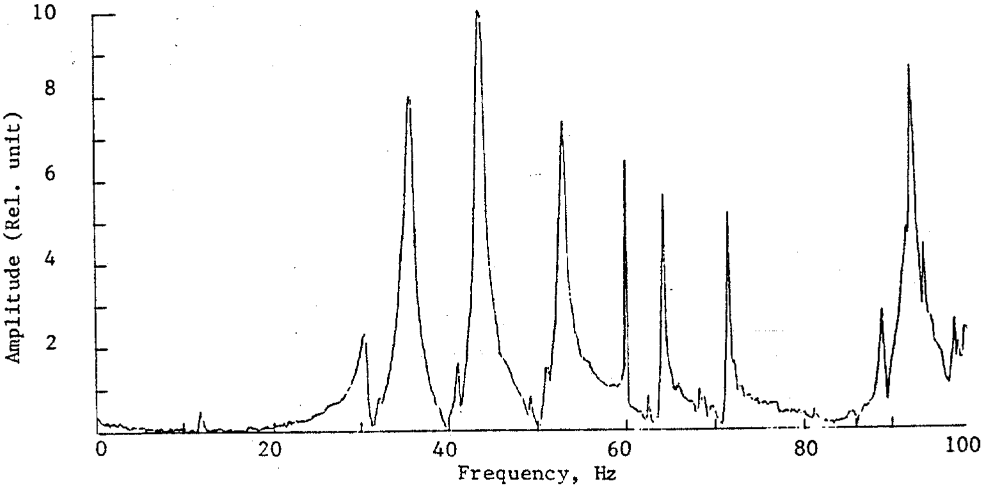

Fig. 2 Transfer function at position 5, Baseline

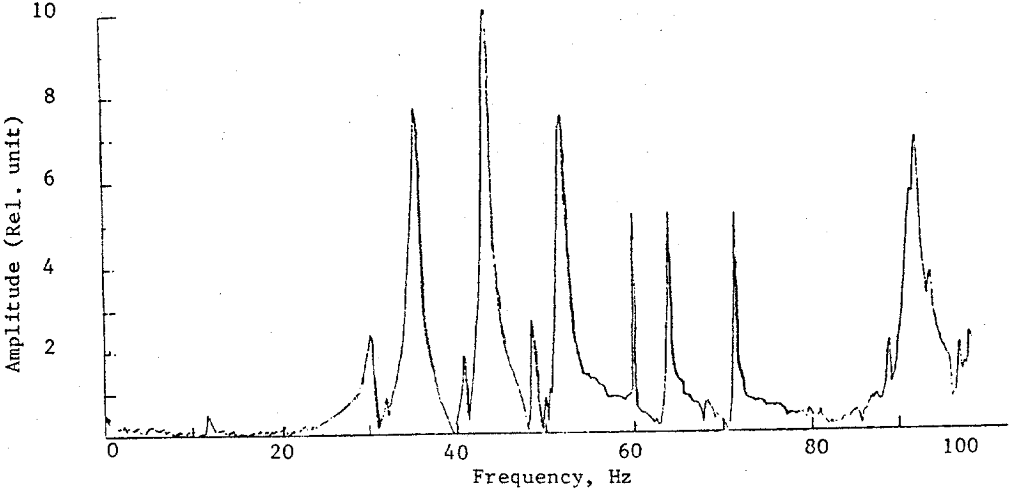

Fig. 3 Transfer function at position 5, 1st stage damage

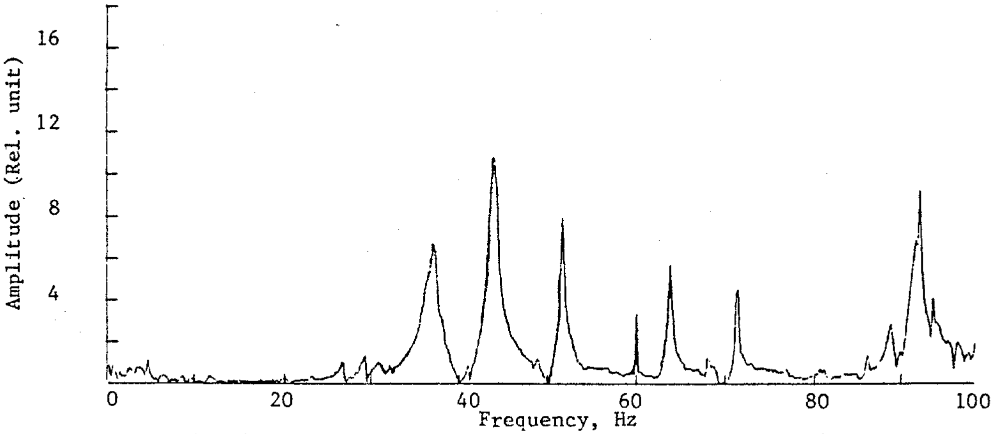

Fig. 4 Transfer function at position 5, 2nd stage damage

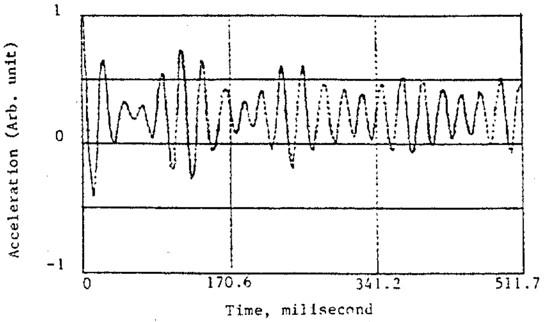

Fig. 5 Random decrement signature at position 5, 32-60 Hz, Baseline

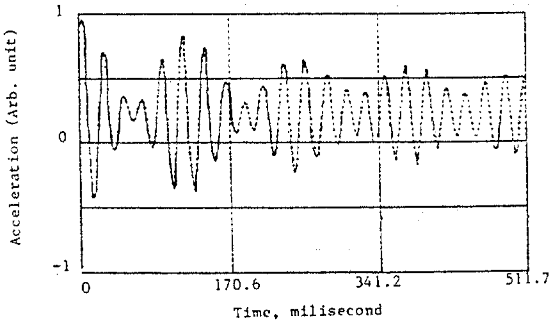

Fig. 6 Cross random decrement signature of position 7 vs. position 5, 32-60Hz, Baseline

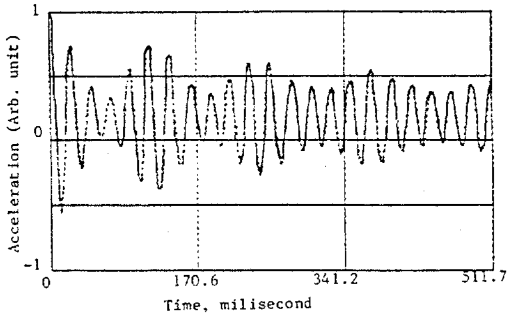

Fig. 7 Random decrement signature at position 5, 32 - 60 Hz, 1st stage damage

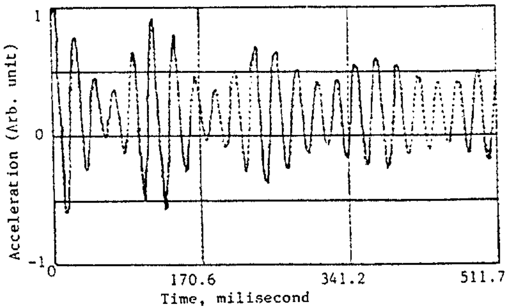

Fig. 8 Cross random decrement signature of position 7 vs. position 5, 32 - 60 Hz, 1st stage damage

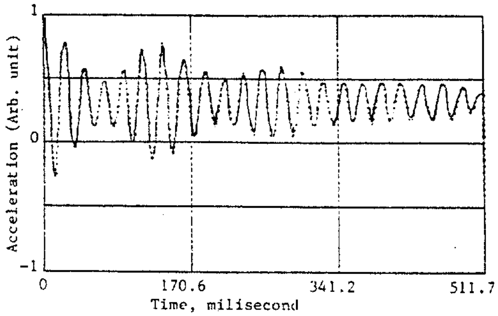

Fig. 9 Random decrement signature at position 5, 32 - 60 Hz, 2nd stage damage

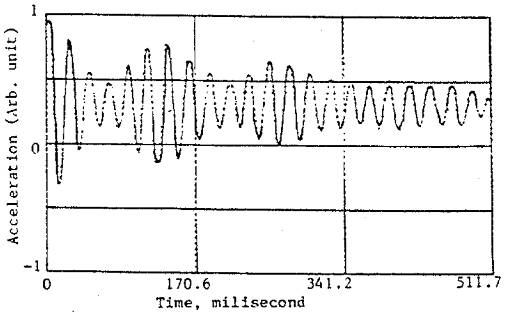

Fig. 10 Cross random decrement signature of position 7 vs. position 5, 32 - 60 Hz, 2nd stage damage

Table 1 Modal Parameters of the Mode at 35.7 Hz

		Baseline	First Stage Damage	Second Stage Damage
Accelerometer Pair 3 - 5	Frequency	35.67 Hz	35.42 Hz	36.75 Hz
	Damping Factor	0.0095	0.0048	0.0138
	Relative Amplitude	0.983	1.005	0.779
	Relative Phase	14.17°	22.80°	22.57°
Accelerometer Pair 5 - 7	Frequency	35.77 Hz	35.49 Hz	36.98 Hz
	damping Factor	0.0169	0.0161	0.0148
	Relative Amplitude	0.618	0.926	0.892
	Relative Phase	2.59°	19.55°	4.51°
Accelerometer Pair 7 - 9	Frequency	35.60 Hz	35.31 Hz	36.93 Hz
	Damping Factor	0.0139	0.0180	0.0116
	Relative Amplitude	1.168	1.593	0.886
	Relative Phase	5.41°	23.23°	0.24°
Accelerometer Pair 9 - 11	Frequency	35.57 Hz	35.55 Hz	36.92 Hz
	Damping Factor	0.0152	0.0153	0.0121
	Relative Amplitude	0.942	0.54	1.061
	Relative Phase	0.91°	14.62°	22.76°

Table 2 Modal Parameters of the Mode at 43.7 Hz

		Baseline	First Stage Damage	Second Stage Damage
Accelerometer Pair 3 - 5	Frequency	43.73 Hz	43.61 Hz	44.02 Hz
	Damping Factor	0.0083	0.0085	0.0081
	Relative Amplitude	1.121	1.067	1.205
	Relative Phase	5.93°	12.43°	15.5°
Accelerometer Pair 5 - 7	Frequency	43.75 Hz	43.50 Hz	43.89 Hz
	damping Factor	0.0071	0.0062	0.0093
	Relative Amplitude	1.055	1.112	1.118
	Relative Phase	13.06°	17.71°	19.89°
Accelerometer Pair 7 - 9	Frequency	43.76 Hz	43.60 Hz	43.88 Hz
	Damping Factor	0.0103	0.0114	0.0121
	Relative Amplitude	1.210	0.666	1.026
	Relative Phase	19.05°	29.15°	14.35°
Accelerometer Pair 9 - 11	Frequency	43.71 Hz	43.63 Hz	44.51 Hz
	Damping Factor	0.0124	0.0075	0.0120
	Relative Amplitude	0.712	0.774	1.86
	Relative Phase	29.95°	19.86°	-2.3°

Table 3 Modal Parameters of the Mode at 53.3 Hz

		Baseline	First Stage Damage	Second Stage Damage
Accelerometer Pair 3 - 5	Frequency	53.27 Hz	52.14 Hz	52.01 Hz
	Damping Factor	0.0092	0.0100	0.0186
	Relative Amplitude	1.415	0.947	1.747
	Relative Phase	21.65°	30.74°	25.73°
Accelerometer Pair 5 - 7	Frequency	53.38 Hz	52.12 Hz	51.89 Hz
	damping Factor	0.0177	0.0165	0.0135
	Relative Amplitude	0.620	0.182	1.149
	Relative Phase	80.26°	15.82°	-11.8°
Accelerometer Pair 7 - 9	Frequency	54.20 Hz	52.30 Hz	51.82 Hz
	Damping Factor	0.0473	0.0152	0.0059
	Relative Amplitude	112.1	3.565	1.080
	Relative Phase	-97.78°	29.36°	13.22°
Accelerometer Pair 9 - 11	Frequency	53.99 Hz	52.24 Hz	51.84 Hz
	Damping Factor	0.0325	0.0106	0.0080
	Relative Amplitude	0.0919	0.869	1.626
	Relative Phase	13.49°	9.61°	5.95°

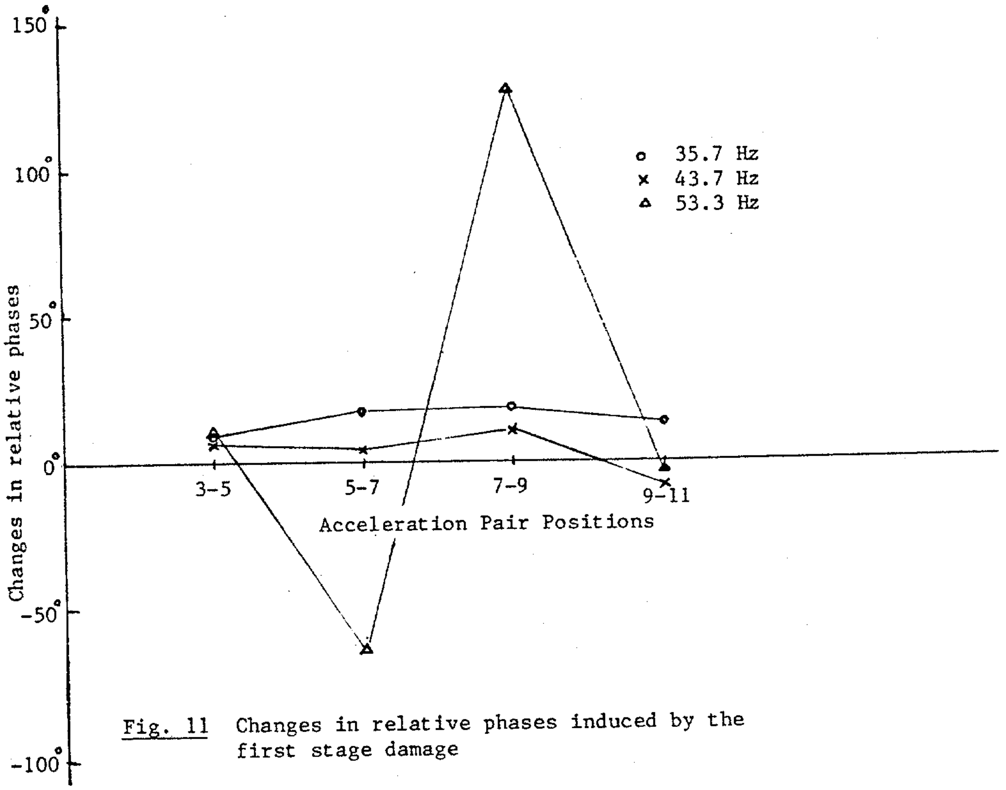

Fig. 11 Changes in relative phases induced by the first stage damage

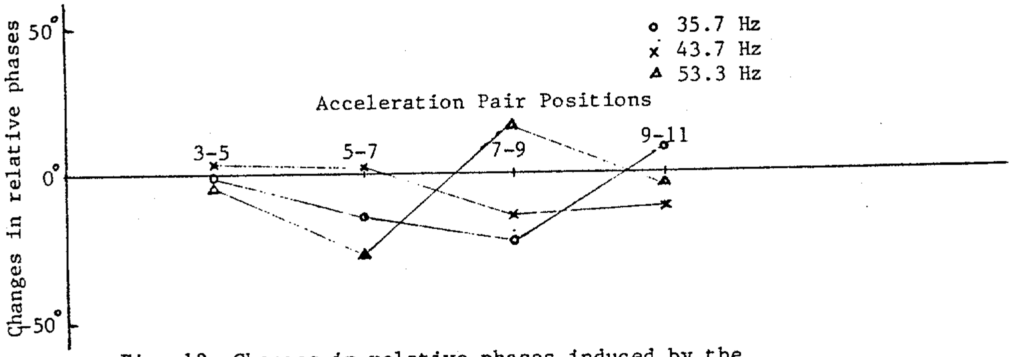

Fig. 12 Changes in relative phases induced by the second stage damage