STRUCTURAL DAMAGE DETECTION
BY THE SYSTEM IDENTIFICATION TECHNIQUE

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Over an extended period of time, exposure to severe loading very often results in fracture or crack damage of structures which can ultimately lead to fatigue failure. The research described in this paper, concerns the development of techniques with the potential to detect and track progressive fracture by observing changes in the identified system parameters: mass, stiffness and damping matrix elements. The method, called the system identification technique, has two steps: a process of retrieving the eigenvalues and eigenvectors during a dynamic response phase and the determination of mass, stiffness and damping matrices from these values. The proposed technique was verified on cantilever beam continuous structure systems through finite element simulation and experimental studies. Results from both studies have indicated the feasibility of damage detection by identifying the structural system matrices. For a cantilever beam system, the location of crack type damage seems to be best identified by the flexibility matrix which is the inverse of the stiffness matrix.

1. INTRODUCTION

Many ships and offshore structures have a predicted design life which is generally based on conservative design criteria to compensate for uncertainties in the load environment and associated damage effects. Severe loading over an extended period of time, may lead to fatigue failures of exposed structures. Initiation and propagation of cracks change the structural response of the system which manifests in a change in the dynamic equations of motion. Therefore, the System Identification Technique, from which the dynamic equations of motion may be deduced from experimental data, offers the potential of being able to detect cracks, flaws and other features by observing changes of structural parameters such as mass, stiffness and damping elements of matrices.

The identification and modeling of multi-degree of freedom dynamic systems through the use of experimental approaches, is a problem of considerable importance in the area of system dynamics, automatic controls and structural analysis. Indication of the wide range of applicability of this subject is shown in the literature related to system parameters identification efforts (Refs. 1-11).

Purely mathematical model representation of the real problem may prove to be a very powerful tool for the analysis and design of complex structural systems. The mathematical model representation could, of course, be devised from a theoretical understanding of the system and its components, or from a finite element model in the case of purely structural systems. These techniques are inferior compared to one which is based on an actual experimental response approach. Furthermore, when the system becomes more complex and sophisticated, it becomes more difficult to understand its mechanisms, and, therefore, to develop an appropriate theoretical model, which will give a good prediction of its dynamical response.

For these reasons, the objective of this research is to develop a new and more accurate dynamic system identification technique for determination of dynamic equations of motion, from dynamic response data, of a system with high modal density. This project seeks to demonstrate that it is feasible to detect damage in structures due to existing cracks or flaws by observing the changes of structural parameters as elements of mass [M], stiffness [K] and damping [C] matrices, and also to observe changes in the power spectral density and resonant frequencies.

The ultimate objective of the subsequent

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research is to correlate the cracks, flaw sizes and their location with the obtained changes in system parameters.

2. MATHEMATICAL MODEL OF THE SYSTEM IDENTIFICATION TECHNIQUE

Let us begin by considering a structural system which can generally be represented by an N degree-of-freedom linear system. The dynamics of the system are governed by its equation of motion:

$$[M]\,[\ddot{X}] + [C]\,[\dot{X}] + [K]\,[X] = [f] \quad (1)$$

where $[X]$, $[\dot{X}]$, $[\ddot{X}]$ are the displacement, velocity, and acceleration column vectors of degree N, respectively. Force $[f]$ is also an N-column vector. The $[M]$, $[K]$, and $[C]$ are N x N mass, stiffness, and damping matrices, respectively.

The system identification technique involves the identification of $[M]$, $[K]$, and $[C]$ matrices of the system, from the known responses $[X]$, $[\dot{X}]$, $[\ddot{X}]$ and the known forcing function $[f]$.

Adding to equation (1) a trivial differential equation:

$$[M]\,[\dot{X}] - [M]\,[\dot{X}] = 0 \quad (2)$$

a set of equations which describe the motion of the same structural system are obtained:

$$\begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix} \begin{bmatrix} \dot{X} \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix} \quad (3)$$

or in the condensed form:

$$[D]\,[\dot{q}] + [E]\,[q] = [Q]$$

where the matrices are defined as:

$$[D] = \begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix}$$
$$[E] = \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix} \quad (4)$$
$$[\dot{q}] = \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix}, \quad [q] = \begin{bmatrix} \dot{X} \\ X \end{bmatrix}, \quad [Q] = \begin{bmatrix} 0 \\ f \end{bmatrix}$$

After performing the Laplace transformation, we obtain:

$$[B\,(s)]\,[q\,(s)] = [Q\,(s)] \quad (5)$$

where

$$[B\,(s)] = \left[[D]s + [E]\right]$$

is the system matrix. It can be proved that $[D]$ and $[E]$, which contain the system's $[M]$, $[C]$, $[K]$ matrices, can be represented by the eigenvalues P_k, and eigenvectors $[Y_k]$, produced from the system matrix and determined by the homogeneous equation (Ref. 12):

$$[B\,(P_k)]\,[Y_k] = 0 \quad (6)$$

When $[M]$, $[K]$, and $[C]$ are symmetric, the following expressions can be proved:

$$[D] = [Y]^{-1T}[I]\,[Y]^{-1}$$
$$[E] = [Y]^{-1T}[-P]\,[Y]^{-1} \quad (7)$$

where

$$Y = [y_1, y_2, y_3, \text{---}, y_N]$$

is an eigenvector matrix while the eigenvalues matrix is:

$$\begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ - & - & - \\ 0 & 0 & p_n \end{bmatrix}$$

It can be shown that the system's transfer function could be represented as a function of eigenvalues and eigenvectors, that is:

$$[H(s)] = [Y][s-P]^{-1}[Y]^T = \sum_{k=1}^{N} \left[\frac{y_k y_k^T}{s-p_k} + \frac{y_k^* y_k^{*T}}{s-p_k^*} \right] \quad (8)$$

or

$$[H(s)] = \sum_{k=1}^{2n} \frac{[a_k]}{s-p_k} \quad (9)$$

where

P_k = kth root of $\{\det\,(B(s)) = 0\}$

$[a_k]$ = residue matrix for the kth root

In general, the ij-th element of the residue matrix $[a_k]$ is written as:

$$a_{ij}(k) = y_{ik} y_{jk} \quad (10)$$

which provides the connection between residues and eigenvectors.

The transfer function H(s) is experimentally measurable. Using various curve-fitting procedures (Refs. 13-16), the eigenvalue and eigenvectors can be retrieved from the transfer function as indicated by Equations (9) and (10).

The proposed technique has been verified on a two-degree of freedom system simulated by analog computer circuits (Ref. 17). The results indicated that the system identification could accurately determine the mass, stiffness and damping matrices of a lumped spring-mass-dashpot system whose degree of freedom is low. The work described below includes the continuing verification of the stated technique on continuous structural systems. The physical system considered was a cantilever beam. The verification was conducted in two ways: NASTRAN finite element simulation and experimental measurement.

3. NUMERICAL APPROACH IN DETECTION OF DAMAGE OF A CANTILEVER BEAM

In the measurement of real structure response signals, error often exists. Such error can greatly affect the accuracy of the identified system matrices, especially when the degree of freedom of the structural system is high. At the initial stage of development of the system identification technique, it is desirable to generate structural signals as close to theoretical values as possible to be used as verification of the technique. Numerical approach was adopted in which a cantilever beam was modeled with the NASTRAN computer program to generate the numerical vibration signals.

The mesh configuration of the finite element model of the beam is shown in Fig. 1. The dimension of the beam is 1" wide, 12" long, and 1/8" thick. The model is composed of 200 CQUAD4 bending elements of MSC/NASTRAN version of the finite element method. The material of the beam is mild steel whose properties are:

Young's Modulus	$E = 3.0 \times 10^7$ lb/in^2
Poisson Ratio	$\nu = 0.33$
Mass Density	$\rho = 7.557 \times 10^{-4}$ slug/in^3

Six stations were chosen from which the frequency response functions were taken. These are labeled stations 1 through 6, located along the beam center line and separated 2 inches apart (Fig. 1). Dynamic forces were applied at station 2. Transfer functions at the six stations, which are defined as the ratio of the Fourier Transform of the dynamic responses at the six stations to that of the input force at station 2, were obtained using NASTRAN modal analysis method. Dampings were introduced into the system by adding artificial modal damping coefficient to each mode. The attained transfer functions containing no noise except the numerical inaccuracies were used as input data for theoretical verification of the identification technique.

The frequencies, dampings and the amplitudes of vibration at the six stations were obtained using a frequency domain curve fitting routine. This constituted the first phase of the signal processing which retrieved eigenvalues and eigenvectors from the system's dynamic responses. The second phase of the signal processing is to construct the [M], [C], [K] matrices from the eigenvalues and eigenvectors.

To demonstrate the capability of damage detection of the proposed technique, two grid points on each side of the beam, located 3 inches away from the clamped edge, were released by splitting each grid point into two (Fig. 2b). The splitting of the two grid points induced first stage damage to the structure. Again the computer programs were run to obtain the frequencies, dampings and the [M], [C], [K] matrices for the damaged structure.

Two more stages of damage were introduced and the same system identification procedure was carried out for all the damage cases. In the second damage stage, two grid points on the second rows from each side of the beam, located 3 inches from the clamped edge, were released (Fig. 2c). In the third damage stage, two additional grid points on the third rows were released (Fig. 2d).

The severeness of the damage induced by splitting the grid points can be demonstrated by the resulting frequency changes which, as seen from Table 1, are very small. The mass matrices obtained for the four damage cases are all close to diagonal with off-diagonal elements one or two order of magnitude smaller than the diagonal elements. The diagonal elements of the mass matrices are listed in Table 2, which show very small changes ($\leq 1\%$) for the damages produced by splitting the grid points. Because of the complex nature of the damping mechanism, the obtained damping matrices will not be correlated to their physical implications. For the obtained stiffness matrices, it was found that their inverses, the flexibility matrices, can provide better physical correlation for a cantilever beam system. The flexibility matrices are near diagonal, whose diagonal elements are listed in Table 3 for the four damage cases. It is found from these values that for response stationsbefore the damage location the flexibility does not change significantly, while for response stations after the damage locationthe flexibilities change progressively according to the severeness of damage and the distances from the damage location. This trend is illustrated by the graphical depiction of Fig. 3.

To investigate the correlation between the location of damage and the changes in the elements of the flexibility matrix, theoretical derivation can be conducted to obtain the analytical expression of the flexibility matrix. The expressions of the diagonal elements of the flexibility matrix of a cantilever beam are listed in the Appendix for the six response stations. As can be seen from the Appendix, the elements of the flexibility matrix are algebraic sums of terms inversely proportional to the local stiffness, $E_i I_i$. The progressive changes in the matrix elements due to the change in local stiffness at a particular station are clearly displayed in the analytical expressions.

TABLE 1

Natural Frequencies in Hz for the NASTRAN Simulated Responses of the Cantilever Beam with Four Damage Cases

Modes	No Damage	1st Stage Damage	2nd Stage Damage	3rd Stage Damage
1	24.991	24.970	24.828	24.499
2	157.67	157.66	157.62	157.53
3	443.82	443.53	441.70	437.55
4	873.60	872.83	868.36	858.68
5	1446.93	1446.46	1444.18	1439.11
6	2136.62	2135.90	2132.73	2125.30

TABLE 2

Diagonal Elements of the Mass Matrices (10^{-3} slugs)

Stations	No Damage	1st Stage Damage	2nd Stage Damage	3rd Stage Damage
1	0.1314	0.1313	0.1318	0.1312
2	0.1979	0.1987	0.1995	0.1989
3	0.2038	0.2028	0.2040	0.2041
4	0.2008	0.2004	0.2008	0.2010
5	0.2011	0.2017	0.2021	0.2035
6	0.1923	0.1927	0.1931	0.1943

TABLE 3

Diagonal Elements of the Flexibility Matrices (10^{-3} in/lb)

Stations	No Damage	1st Stage Damage	2nd Stage Damage	3rd Stage Damage
1	0.5033	0.5016	0.5031	0.5021
2	3.9860	3.9756	3.9976	4.0220
3	14.343	14.391	14.430	14.901
4	31.540	31.940	31.803	33.063
5	62.948	62.425	62.389	64.624
6	101.66	101.81	102.40	105.34

4. EXPERIMENTS WITH A CANTILEVER BEAM

In addition to the numerical verification of the system identification technique described in section 2 as applied to a continuous structural system, an experimental verification was also conducted. A cantilever beam having dimensions 19-1/2 inches long, 1 inch wide, and 1/4 inch thick was used in the experiment. The beam was made of aluminum, with Young's modulus 1.03×10^7 lb/in^2, Poisson's ratio $\nu = 0.33$ and mass density $\rho = 2.485 \times 10^{-4}$ slug/in^3. Six accelerometers were attached to the beam at six stations (Fig. 4). A hammer was set up to excite the aluminum cantilever beam with transient or random impact at station 5, as shown in Fig. 5a. The transfer functions from the impact station to any accelerometer station were obtained by feeding the output acceleration signal and input forcing function into a spectrum analyzer: the Nicolet 660B dual channel FFT analyzer supported by a Data General MP/200 computer (see Fig. 5b). In the analyzer, the input and output signals were digitized and the Fast Fourier Transform of the signal was performed. The instantaneous transfer functions were obtained by dividing the two spectra. The final transfer function was obtained by averaging a series of instantaneous transfer functions.

The obtained transfer functions were processed further, according to the mathematical procedure suggested by the proposed structure identification technique. The final results are represented in the form of structural matrices [M], [C], and [K]. It should be emphasized that the phase 1 (transfer function) was experimentally accomplished, in contrast to the finite element analysis described previously. As such, this is a totally experimental approach which will be an effective and useful technical approach for damage detection.

The saw cut on the cantilever beam, introduced between stations 2 and 3, represents the damages of the structure in the experiment (Fig. 4). The frequencies and damping values of the lowest vibration modes were obtained from the transfer functions for no cut case and the cut case (Table 4). Significant changes due to cut exist in the experimentally determined frequencies. Table 5 and 6 list the diagonal elements of the mass and flexibility matrices for the no cut and cut cases. It is also found that the damage introduced by the saw cut results with significant changes in the flexibility elements.

TABLE 4

Experimental Values of Frequencies and Damping Ratios of the Aluminum Cantilever Beam

Modes	NO CUT		CUT CASE	
	Natural Freq. (Hz)	Damp. Ratio (%)	Natural Freq. (Hz)	Damp. Ratio (%)
1	19.53	0.360	19.00	0.247
2	122.05	0.241	115.85	0.183
3	339.26	0.125	332.36	0.0788
4	661.73	0.0946	646.91	0.0805
5	1085.22	0.120	1037.46	0.0979
6	1594.59	0.0974	1591.36	0.0973

TABLE 5

Diagonal Elements of the Mass Matrices (10^{-6} slugs)

Stations	No Cut	Cut
1	2.9460	2.8029
2	6.7463	6.0645
3	6.9833	7.8842
4	7.0791	8.3550
5	7.2694	7.9536
6	6.4664	6.5813

TABLE 6

Diagonal Elements of the Flexibility Matrices (in/lb)

Stations	No Cut	Cut
1	0.3257	0.2526
2	1.8184	1.4985
3	7.5954	7.8514
4	18.594	21.003
5	33.817	51.553
6	66.075	87.246

Theoretical study, as illustrated by the results of NASTRAN simulation, indicates that the diagonal elements of the flexibility matrix [F] should deviate in an orderly fashion with respect to the location of the damage. Comparing the flexibility matrices of the cases cut and no cut (Fig. 6), this orderly deviation does exist in the diagonal elements and allows one to identify the location of the cut.

5. CONCLUSIONS AND DISCUSSIONS

The feasibility of using the system identification technique for a continuous structural system, such as a cantilever beam, has been demonstrated. Both the numerical simulation and experimental verification indicate that the technique is capable of identifying structural damages. Furthermore, for a cantilever beam, the location of the damage can be identified by observing the changes in the diagonal elements of the flexibility matrix.

However, to obtain useful results for more practical purposes, a number of improvements to the technique will be necessary. In the experiment conducted, the cut made to the cantilever beam was considered a very severe structural damage, thus resulting in significant changes of the system's matrices and made the system identification possible. For real applications damages of a precatastrophy type are usually very small. If the error during the signal processing is large enough to suppress the deviations in [M], [C], [K] matrices due to damages, then it is impossible to detect structural damages by observing changes in the identified [M], [C], [K] matrices. Therefore, the requirement of high accuracy signal processing is essential for practical purposes.

For the present system identification technique, the accuracy can be controlled in three steps: (1) the signal acquisition in vibration measurements; (2) retrieval of the system's eigenvalues and eigenvectors; and (3) conversion of eigenvalues and eigenvectors to the [M], [C], and [K] matrices. The first step requires careful calibration of the measurement transducers. The second step involves the accuracy of the analog to digital signal conversion and numerical accuracy in the proper eigenvalue retrieval algorithm. The third step is purely numerical and consists only of a series of matrix operations.

In our present research, an aluminum cantilever beam of sufficient length system has been used. This retained the system in lower vibration frequencies so that the lowest six modes were well within the accelerometer response characteristics. Attention has also been given to the structural symmetry so that unwanted vibrations, such as torsional modes, were eliminated. Efforts were directed to improve the measurement accuracy.

New mathematical approaches to convert the eigenvalues and eigenvectors to the [M], [C], and [K] matrices can be pursued to provide better accuracy. For example, one can use only matrices of dimension N x N for an N-degree of freedom system in the computation algorithms. As compared to the system matrices of dimension 2N x 2N, used in the present research, such approach contains four times less the number of unknown variables. It is expected that accuracy will be improved by the reduction of matrix dimensions in the numerical array operations.

For a continuous structural system, the number of degrees of freedom is infinity. If it is to be modeled with an N degree-of-freedom [M], [C], and [K] matrices, then the conditions under which the system identification procedure is proper should also be verified for practical application.

APPENDIX: Diagonal Elements of the Flexibility Matrix of a Cantilever Beam

$$f_{11} = 2G_1L_1^2$$

$$f_{22} = 2G_1(L_{12}^2 + L_{12}L_2 + L_2^2) + 2G_2L_2^2$$

$$f_{33} = 2G_1(L_{13}^2 + L_{13}L_{23} + L_{23}^2) + 2G_2(L_{23}^2 + L_3L_{23} + L_3^2) + 2G_3L_3^2$$

$$f_{44} = 2G_1(L_{14}^2 + L_{14}L_{24} + L_{24}^2) + 2G_2(L_{24}^2 + L_{24}L_{34} + L_{34}^2) + 2G_3(L_{34}^2 + L_{34}L_4 + L_4^2) + 2G_4L_4^2$$

$$f_{55} = 2G_1(L_{15}^2 + L_{15}L_{25} + L_{25}^2) + 2G_2(L_{25}^2 + L_{25}L_{35} + L_{35}^2) + 2G_3(L_{35}^2 + L_{35}L_{45} + L_{45}^2) + 2G_4(L_{45}^2 + L_{45}L_5 + L_5^2) + 2G_5L_5^2$$

$$f_{66} = 2G_1(L_{16}^2 + L_{16}L_{26} + L_{26}^2) + 2G_2(L_{26}^2 + L_{26}L_{36} + L_{36}^2) + 2G_3(L_{36}^2 + L_{36}L_{46} + L_{46}^2) + 2G_4(L_{46}^2 + L_{46}L_{56} + L_{56}^2) + 2G_5(L_{56}^2 + L_{56}L_6 + L_6^2) + 2G_6L_6^2$$

$$G_i = \frac{L_i}{6E_iI_i} \qquad L_{ij} = L_i + \ldots + L_j$$

where L_i is the distance between response stations

I_i the moment of inertia of the beam cross section

E_i the local Young's Modulus

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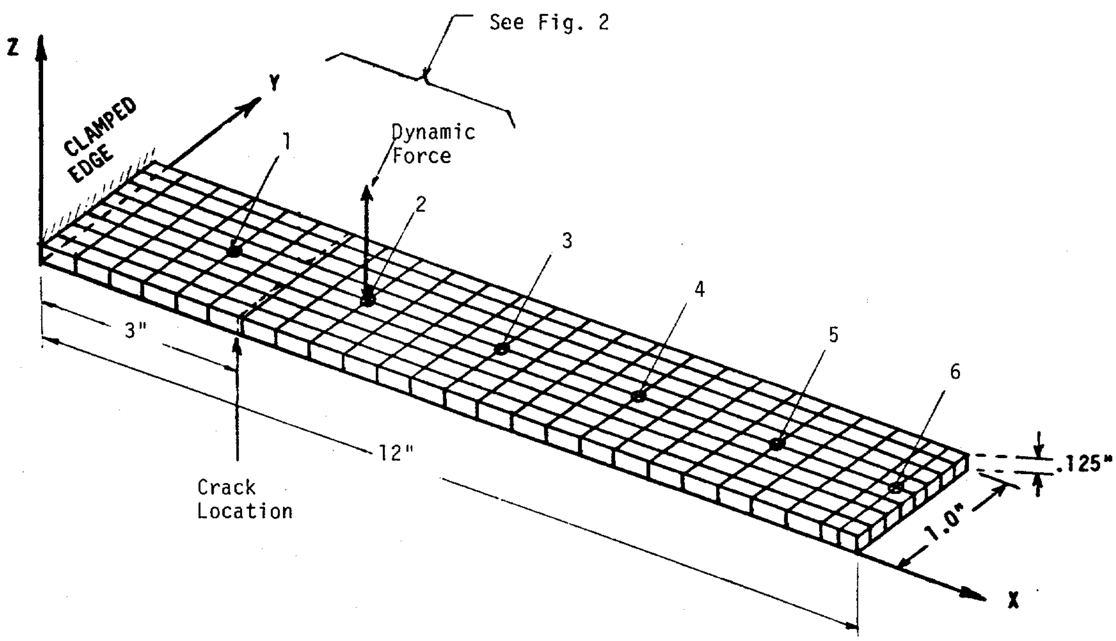


Fig. 1 Mesh Configuration, Location of Forcing, Crack and Stations of NASTRAN Simulated Steel Cantilever Beam

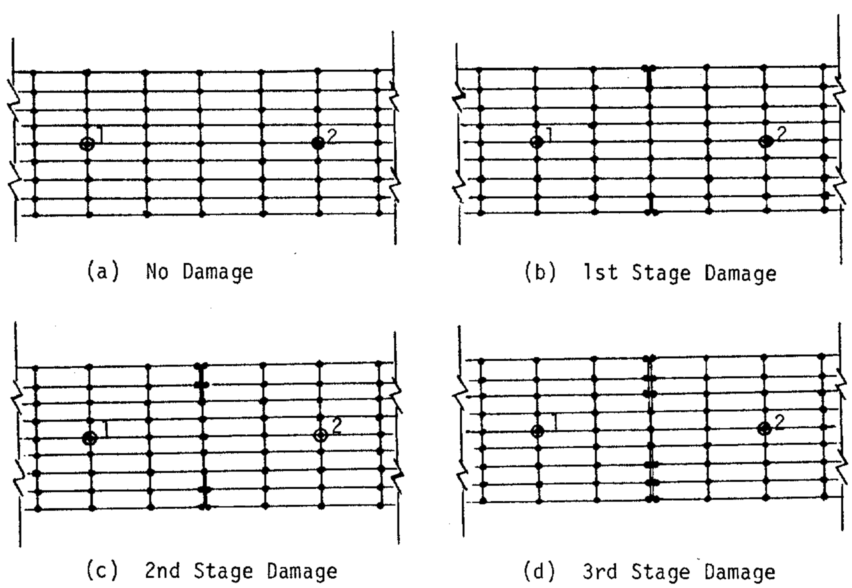


Fig. 2 NASTRAN Grid Points Arrangements for Simulated Damages

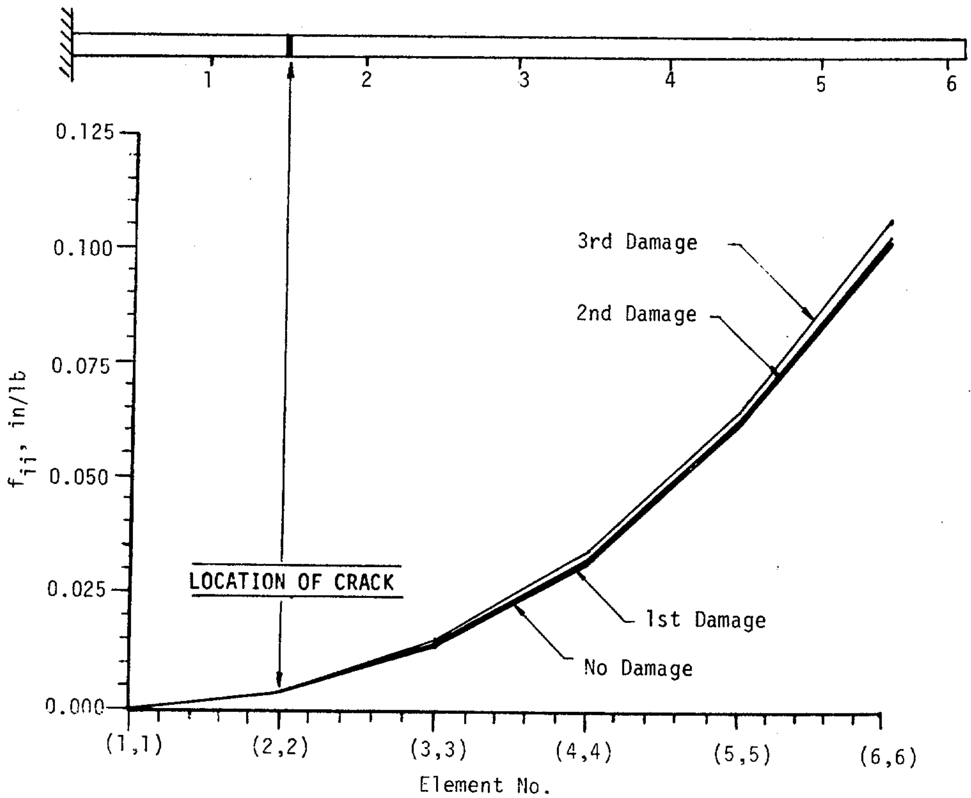


Fig. 3 Diagonal Elements of the Flexibility Matrix of the NASTRAN Simulated Steel Cantilever Beam

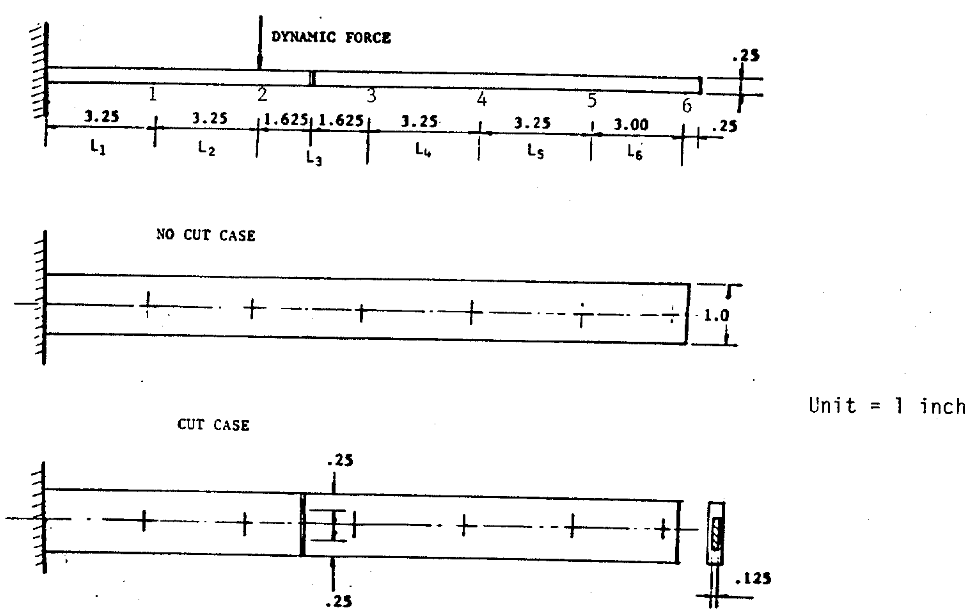
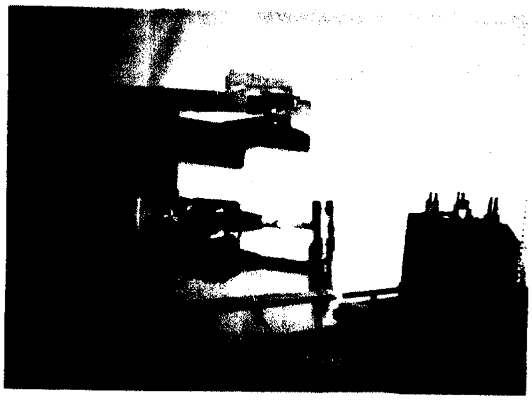
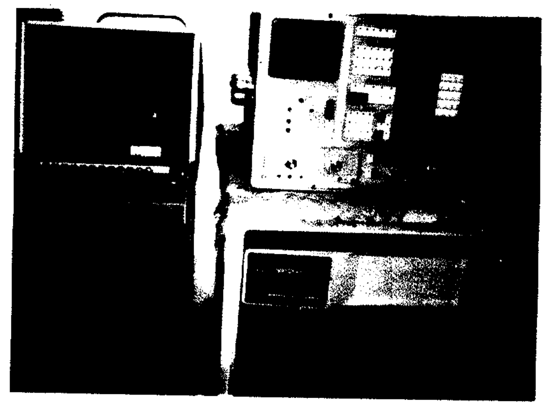


Fig. 4 Geometry, Location of Stations and Cut of the Aluminum Cantilever Beam



a. Set-up of cantilever beam with mechanism for application of excitation force



b. NICOLET 660B, dual channel FFT analyzer, supported by Data General MP/200 computer

Fig. 5 Set-Up of Cantilever Beam Experiment

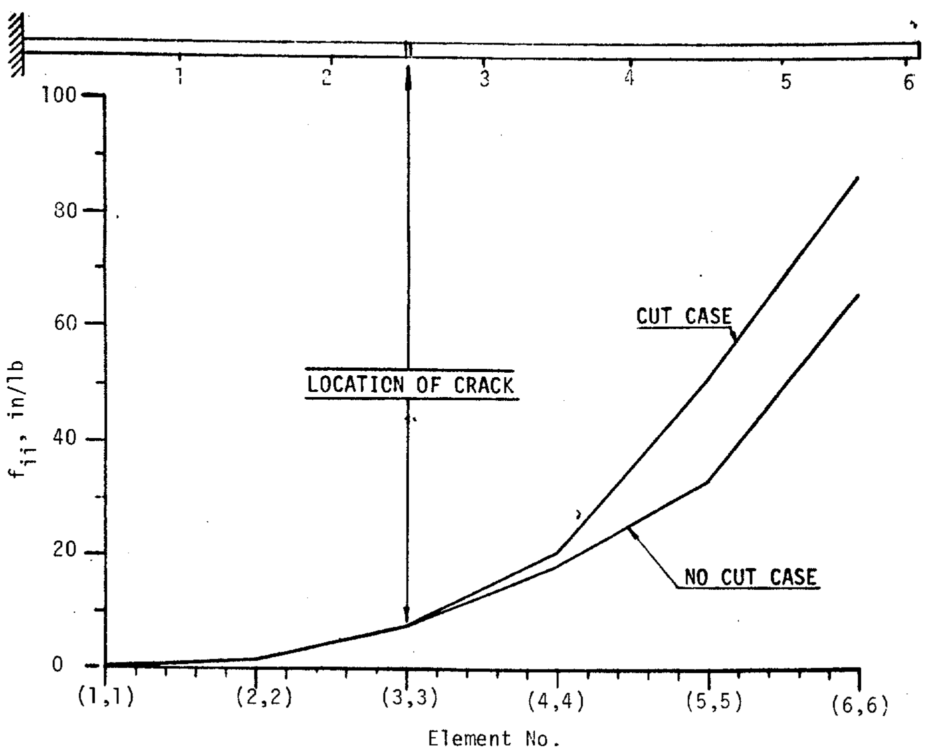


Fig. 6 Diagonal Elements of the Flexibility Matrix of the Aluminum Cantilever Beam