ASTM Geotechnical Testing Journal

VOLUME 6, NUMBER 3 SEPTEMBER 1983 CODEN: GTJODJ

Published quarterly by ASTM
1916 Race St., Philadelphia, PA 19103

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Dynamic Shear Modulus and Damping Ratio from Random Loading Tests

REFERENCE: Al-Sanad, H., Aggour, M. S., and Yang, J. C. S., **"Dynamic Shear Modulus and Damping Ratio from Random Loading Tests,"** *Geotechnical Testing Journal*, GTJODJ, Vol. 6, No. 3, Sept. 1983, pp. 120–127.

ABSTRACT: Dynamic shear modulus and damping ratio determination from random loading tests on dry Monterey and Ottawa sands in a resonant column device, in addition to sinusoidal loading, are presented. In the random excitation test, white noise was used to excite the specimens, and the responses of the specimens were analyzed by the random decrement technique. Using a sine-wave generator, the dynamic soil properties were also determined by the conventional technique. The results obtained indicated a good agreement between the modulus and damping obtained from the application of sinusoidal or random loading. It was concluded that the relationship of shear modulus and damping ratio with shear strain amplitude is independent of random or sinusoidal forcing functions within the range of variables tested.

KEYWORDS: sands, cohesionless soils, damping, random loading, shear modulus, resonant column, random decrement technique, logarithmic decrement

The two primary dynamic soil properties for dynamic analysis are the shear modulus and the damping characteristics (damping ratio, logarithmic decrement, and so forth). Dynamic shear moduli are presently determined using laboratory techniques and in situ tests. The soil damping properties for use in response calculations are presently determined using only laboratory techniques, as there is as yet no test for determining usable data in situ. Since no field tests are readily available to determine damping, the correlation between in situ and laboratory values is one of the areas in which data are lacking.

A difficulty in the field testing of damping is in the extracting of damping from the response signal of soil deposits subjected to random loading. This difficulty in analyzing random response is overcome by the use of a method called "random decrement technique." In this technique, the random vibration responses are ensemble averaged to form a signature that is representative of the free-vibration decay curve from which damping and frequency can be identified. That is, damping values for random loading are calculated by the logarithmic decrement method as in the case of sinusoidal loading. Thus, the two primary dynamic soil properties could be determined by the random decrement technique either in the laboratory or by field testing.

The objective of this study is twofold: to determine the dynamic soil properties from both sinusoidal and random vibrations and to compare the results. This is important because almost all laboratory testing utilizes the type of force excitation, namely, sinusoidal loading, though loading such as earthquake loading is of a random nature. Thus, for the measured laboratory value of modulus and damping to be consistent with those occurring in the field during earthquakes, random loading should be used in laboratory testing. Secondly, the objective is to determine the feasibility of the use of the random decrement technique for the determination of the damping characteristics and the natural frequency of soil specimens in the laboratory. Such a determination will lead to a credible application of the random decrement technique for specimens in the field.

To accomplish the above objectives, a comprehensive testing program covering a wide range of variables was carried out. The damping and shear modulus were calculated for sinusoidal and random vibrations, and the results from both loadings were compared.

In this paper a description of the random decrement method and its verification utilizing the resonant column test for an aluminum rod is presented. The method of testing soil specimens subjected to random loading is presented and factors influencing the results, such as filtering, are discussed. Finally an example of the data obtained is included.

The first conclusion of this study is that the results of the experimental program showed a good agreement between the modulus and damping obtained from the application of sinusoidal or random loading. That is, the dynamic properties of dry cohesionless soil were not influenced by the type of loading applied within the range of variables tested. Second, the good agreement obtained leads to the conclusion that the random decrement method is a feasible technique for extracting dynamic properties of soils subjected to random loading in the laboratory or in the field.

Random Decrement Technique

The random decrement (randomdec) technique was developed by Henry Cole for the measurement of damping and for the detection of structural deterioration of airplane wings subjected to wind flutter excitation [*1,2*]. Other applications have been studied by various other authors [*3–7*].

The random decrement technique is a fast-converging method

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for extracting information from random data. In this process segments of the random vibration response of a transducer placed on an object subjected to random excitation are ensemble averaged to form a signature that is representative of the free-vibration decay curve of the structure. This signature can be used to measure damping or to detect incipient failures. The method is particularly useful in field measurements because excitation is provided naturally by such random inputs as acoustic noise, fluid flow, wind, and so forth.

The random decrement technique assumes that the random response of a damped structure is composed of two parts: a deterministic part (impulse or step function or both) and a random part. By averaging enough segments of the same random response, the random part will average out, leaving the deterministic part. It can be shown that by proper digital processing, the deterministic part that remains is the free-decay response from which the damping can be measured. Hence, the random decrement technique uses the free decay responses of a system under random loading to identify its vibration parameters, namely, frequencies and damping.

The following is a brief explanation of the principles of the random decrement technique. A more extensive mathematical derivation was developed by Reed [8].

The response $x(t)$ of a linear system is governed by the following basic equation

$$m\ddot{x}(t) + c\,\dot{x}(t) + k\,x(t) = f(t) \tag{1}$$

where

$x(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$ = the displacement, velocity, and acceleration, respectively,
m, c, and k = the mass, damping, and stiffness, respectively, and
$f(t)$ = the excitation force.

The solution of this differential equation depends on its initial conditions and the excitation force $f(t)$. Since for linear systems the superposition law applies, the response can be decomposed into three parts: response caused by initial displacement $x_d(t)$, response caused by initial velocity $x_v(t)$, and finally response caused by the forcing function $x_f(t)$.

The random decrement technique consists of dividing a record of length τ_l of the system response (this could be velocity, acceleration, or displacement, and so forth versus time) into N equal length segments τ possibly overlapping as shown in Fig. 1 in the following manner: the starting time t_i of each segment is selected such that each begins at a selected amplitude, that is, $x_i(t_i) = x_s$ = constant, and the slope $\dot{x}_i(t_i)$ alternates between positive and negative, that is, half the segments have an initial positive slope, and the other half have an initial negative slope. These segments are then ensemble averaged and represented mathematically as

$$\delta(\tau) = 1/N \sum_{i=1}^{N} x_i(t_i + \tau) \tag{2}$$

where

$x_i(t_i) = x_s$, $i = 1, 2, 3$, and so forth,
$\dot{x}_i(t_i) \geq 0$, $i = 1, 3, 5$, and so forth, and
$\dot{x}_i(t_i) \leq 0$, $i = 2, 4, 6$, and so forth.

The function $\delta(\tau)$ is called the random decrement signature and is defined only in time interval $0 \leq \tau \leq \tau_l$. The meaning of the random decrement signature can now be determined. As shown in Fig. 2, if the parts caused by initial velocity are averaged, they cancel out because the positive and negative initial slopes are taken alternately and their distribution is random. Furthermore, if the parts caused by the excitation are averaged, they also vanish because, by definition, the excitation is random. Finally, only the parts caused by initial displacement are left, and their average is the random decrement signature, which for a linear single degree of freedom system represents the free-vibration decay curve of the system caused by an initial displacement, which corresponds to the initial value (bias level) x_s.

The starting value x_s can be arbitrarily chosen. The freedom to select the initial value (bias level) is one of the main advantages of the method. If the record is nonstationary and contains periods of low-level vibration in which the signal to noise ratio is very low, the initial amplitude can be chosen above this level, and the low-amplitude portion of the record is disregarded.

In practice, the technique is implemented on a computer by converting each segment into digital form and adding it to the previous

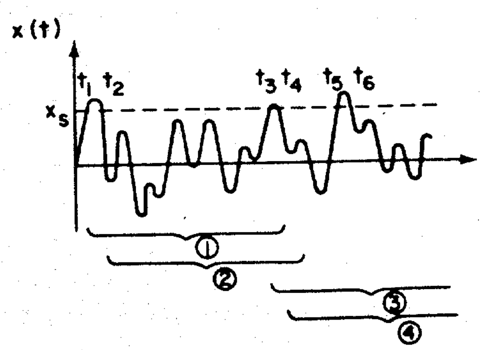

FIG. 1—*Extraction of the equal length segments.*

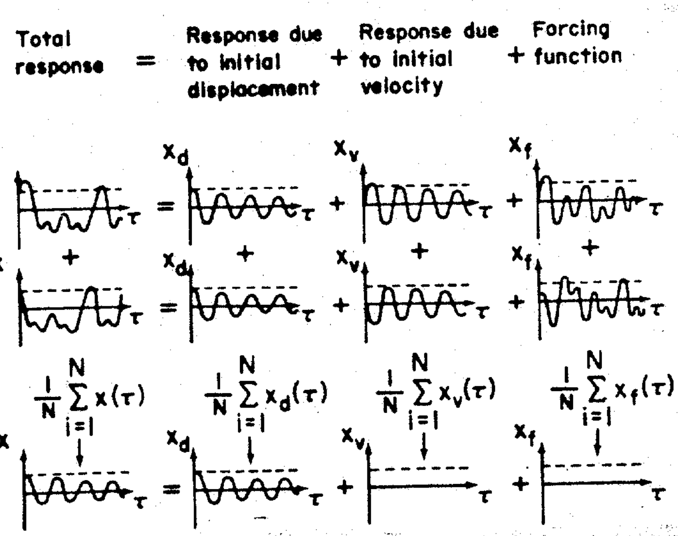

FIG. 2—*Principles of random decrement technique.*

segments; the average is then stored in the memory and can be displayed on a screen. The number of segments to be averaged for the random decrement signature depends on the signal shape; usually 400 to 500 averages are sufficient to produce a repeatable signature. An interesting characteristic of the random decrement technique is that it requires no knowledge of the excitation $f(t)$ as long as it is random.

In general, linear systems have many degrees of freedom, and the signature is a combination of modes, although it still represents the free vibration response to an initial displacement. In multimode problems, the response records are band-pass filtered to isolate different frequency bandwidths. Damping values are then calculated for each mode. For nonlinear systems, the signature represents the free-vibration response of a system in which the nonlinear properties are averaged to a value dependent on the amplitude.

Verification of the Random Decrement Technique

The accuracy of the random decrement technique was checked by Caldwell [9] for several linear single degree of freedom spring mass damper systems with different damping ratios that were modeled on an analog computer. In each case, the random decrement method produced signatures that closely resembled the actual system decay curves. He also analyzed several nonlinear single degree of freedom systems where damping forces were functions of the displacement and velocity. He found that the random decrement technique can be used to extract damping ratio values accurately for values of damping less than 20%. The theoretical validation of damping measurements was also checked for a two degree of freedom system.

To verify the applicability of the random decrement method to soils, the first step taken was to test an aluminum rod in the resonant column device. This rod was provided by the manufacturer for purposes of calibrating the resonant column device. In separate tests, the rod was excited by random vibrations from the white noise generator and by the sinusoidal wave generator. Both torsional and longitudinal excitations were applied, and the response was recorded. The results of these tests are summarized in Table 1 and show that the values of damping and frequencies of both excitation types showed strong agreement. It should be mentioned here that since the aluminum rod is a linear system in the range of testing, the values of damping did not change much with the changing excitation level, that is, the root mean square (RMS) of the signal.

Experimental Procedure

The applicability of the random decrement technique for the determination of dynamic characteristics of soils can be determined by a variety of laboratory testing equipment. The most widely used equipment is cyclic triaxial shear, cyclic simple shear, cyclic torsional shear, and the resonant column device. The resonant column method is a relatively nondestructive test. The primary advantage of using this testing method in this study is that at small shearing strains, less than 0.06%, damping values can be obtained for a soil specimen by both the free decay method and the random decrement method consecutively at different intervals of time without introducing the effect of previous measurements at that pressure. The resonant column device may also be used in the "destructive range" (up to 1.0% shear strain amplitude) to measure damping. The resonant column test procedure, apparatus description, and damping measurements can be found in Hardin [*10*] and Drnevich et al [*11*].

In this study, the logarithmic decrement method was used in evaluating damping for sinusoidal loading because it utilizes the free-vibration decay curve from which damping can be measured. The reason for using this method is that the random decrement method results in a signature that is the free decay curve from which damping is also calculated using the logarithmic decrement method.

For the sinusoidal loading test, the input signal is generated by a variable frequency sine-wave oscillator. For the random vibration test, the excitation was provided by a random wave generator. The output of the random wave generator (white noise generator) is passed through a band-pass filter, then connected to the drive coils via a power amplifier. Response was recorded on a magnetic tape with an FM tape recorder. A schematic of the Drnevich-type resonent column device used in this test program is shown in Fig. 3.

After construction of the specimen and assembly of the apparatus, the random vibration was first applied by connecting the white noise generator to the driving coils. The amplitude of the response vibration (RMS) was adjusted to a predetermined value of acceleration that was read on the multimeter. The predetermined response levels were approximately 50, 100, 200, 300, 400, and 500 mV. At each response level, the exciting force (RMS) was also measured on the multimeter in terms of millivolts. The response signals at each level were recorded on the magnetic tapes. The recorded response signal was passed through a band-pass filter, then through a signal amplifier before it was fed into a microcomputer for discretization and digitization.

TABLE 1—*Summary of aluminum rod results.*

Random Vibration				Sinusoidal Vibration			
Excitation (RMS), mV	Response (RMS), mV	Damping, %	Frequency, cycles/s	Excitation (RMS), mV	Response (RMS), mV	Damping, %	Frequency, cycles/s
			TORSIONAL MOTION				
660	330	0.25	50.0	6	213	0.24	48.5
375	200	0.23	50.5	12	580	0.19	49.3
460	250	0.24	49.5	6	210	0.19	48.5
700	320	0.24	49.8	13	600	0.24	50.0
430	210	0.20	50.5	20	1080	0.22	50.5
			LONGITUDINAL MOTION				
600	360	1.9	928	190	350	1.88	929

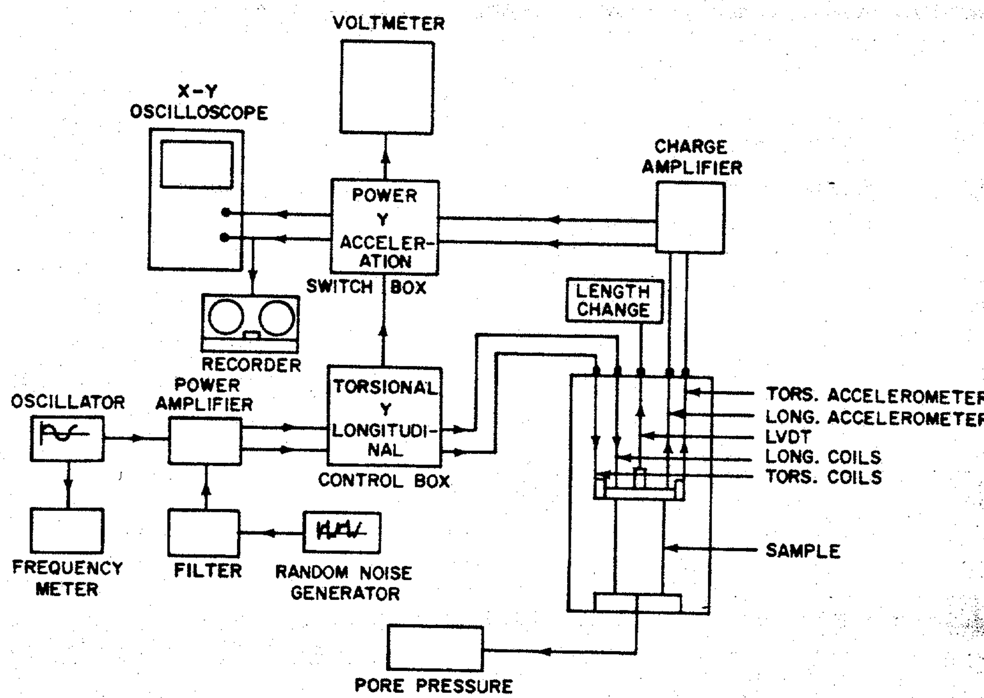

FIG. 3—*Schematic diagram of the resonant column electronics.*

Computer programs were set up such that the data could be stored and manipulated in various ways that included performing random decrement analysis and obtaining the power spectral density of the signal. The computer analysis was continued until 1000 segments or more had been ensemble averaged resulting in a random decrement signature. A graphic terminal was used in issuing commands to perform the different operations and in displaying the random decrement signature as well as power spectral density for examination. Hard copies of the graphs were obtained from a dot matrix printer. A schematic diagram of the computer setup is shown in Fig. 4.

In the next step, by disconnecting the random wave generator and connecting the sine-wave oscillator, sinusoidal torques were applied to the soil specimen. To accomplish response at the predetermined response levels of 50, 100, 200, 300, 400, and 500 mV, the following procedures were followed. The output level on the sine-wave generator was increased until a response of 10 mV was read on the multimeter. Then the frequency on the sine-wave generator was adjusted until the first mode natural frequency was obtained by observing the Lissajous figure formed on the oscilloscope. To accomplish resonance at the predetermined response levels, it was necessary to adjust the output level and frequency simultaneously. Both excitation input and response output were read on the voltmeter, and the resonant frequency was read on the frequency meter. Damping was determined by turning off the driving power at resonance and recording the decaying vibrations. The decayed wave was recorded on a magnetic tape, then fed into a microcomputer for discretization and digitization.

A computer program was written to calculate the amplitude at each cycle of the decayed curve to carry out the regression between number of cycles and the amplitude followed by the calculation of damping by the logarithmic decrement method. This procedure reduced the error from manual measurement of the amplitude and

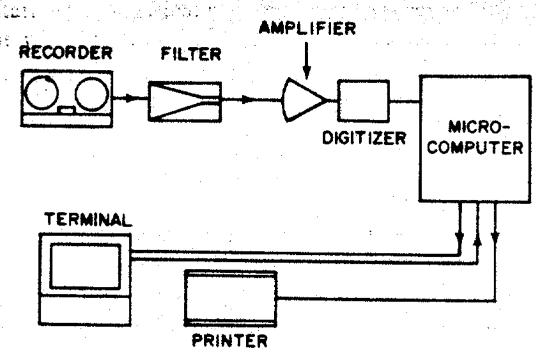

FIG. 4—*Schematic diagram of the computer setup.*

also reduced the time required for data reduction. The above procedures were repeated for all response levels required.

Effect of Filtering on the Analysis

A band-pass filter was used in the analysis of the data to effectively reduce the response to that of a single degree of freedom system. To set the low- and high-pass frequencies of the filter, the power spectral density was determined to define the system's resonant frequency, knowledge of which enabled one to choose the low- and high-pass frequencies. In this study, only low-pass frequency was used, that is, the high-pass frequency was zero. The low-pass frequency for the torsional vibration was set at 100 Hz. To study the effect of the filter width on the results, the low-pass frequency was varied from 100 to 2000 Hz. As shown in Fig. 5 for the same segment of recorded response, the random decrement signature and the power spectral density in both cases were identical, and the

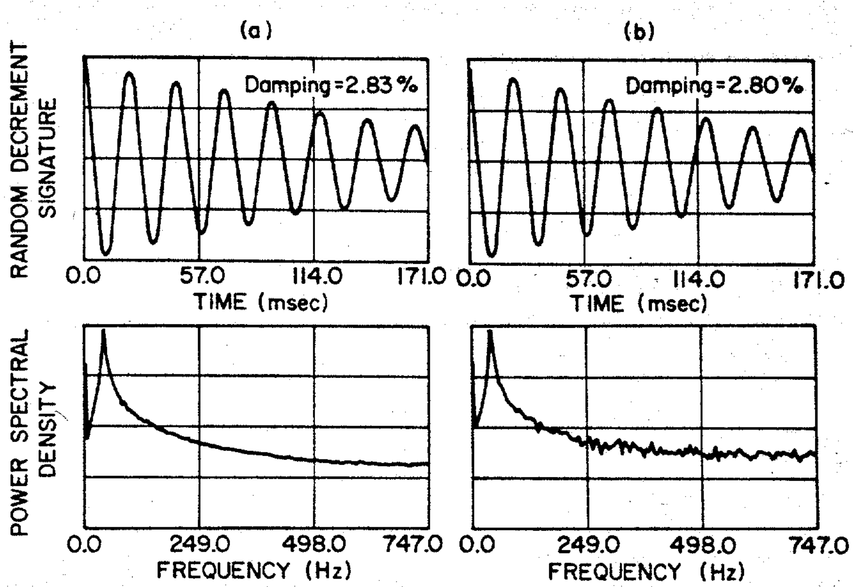

FIG. 5—*Random decrement signatures and power spectral density for Ottawa sand:* (a) *low-pass frequency at 100 Hz and* (b) *low-pass frequency at 2000 Hz (strain 0.014% and resonant frequency 43 Hz).*

damping and resonant frequency were 2.8% and 42 Hz, respectively. The wide range of the filter width used did not affect the results because the system mainly had a first mode of frequency in the range of 40 Hz while the second mode was in the range of 2000 Hz.

Strain Calculation

In the resonant column test, the strain amplitude is proportional to the acceleration of the top plate that could be controlled by the input levels of the sine-wave generator. The root mean square (RMS) of the specimen response is used in the calculation of the strain. The strain calculations for specimens subjected to sinusoidal loading followed the same procedure presented by Drnevich et al [11].

To compare the results obtained from the sinusoidal excitation with those from the random excitation test, it was necessary to first determine the strain induced in a soil specimen caused by applied random loading so that the dynamic properties could be compared at the same strain level. The RMS of the response to the random loading was adjusted to give the same value of the response as for the sinusoidal loading. When this was done, the ratio of the input excitation (RMS) of random loading to that of the sinusoidal loading was found to be always much greater than one; this was in agreement with what Robson [12] showed theoretically. Since the random decrement technique transforms the random response of a system into the system's free-vibration response and the RMS of the response of both vibration types were forced to be the same, we can use the same technique to approximately calculate the strain for the random loading. This strain was termed the equivalent strain.

A justification for using the RMS values in calculating the equivalent strain for the random loading could be explained as follows. Since the RMS of a signal represents a summation of the RMS's from all major modes (frequency content) in the system, if the system is clearly a one-mode system, the RMS is thus representative of the energy for that first harmonic. The RMS for the second, third harmonic, and so forth is thus very small and can be neglected. In comparing the spectrum of both the sinusoidal and random output as shown in Fig. 6, it can be seen that both are very similar and

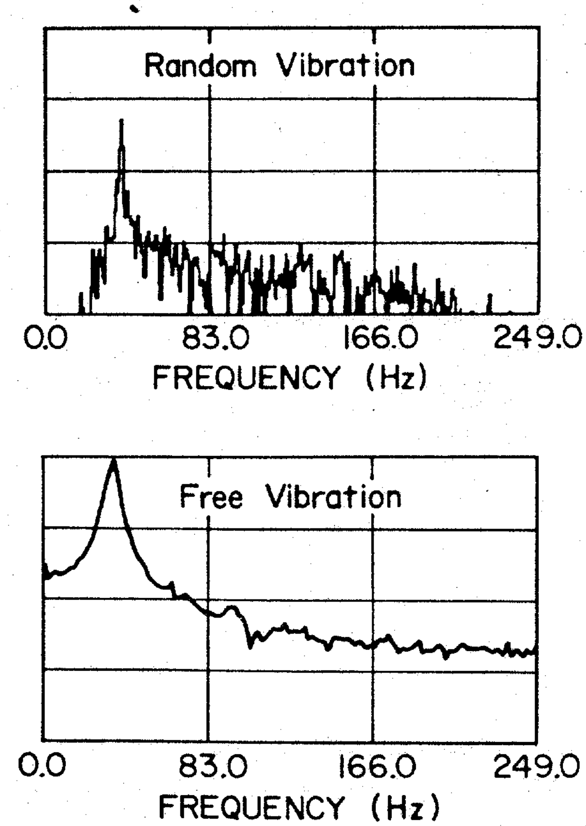

FIG. 6—*The power spectral density of the output for random and free vibration of an Ottawa sand specimen.*

contain only one predominant mode; thus the RMS represents the energy in that mode and, hence, if equal for both loadings, implies similar strains for both loadings. This also implies that for systems with more than one mode, the equivalent strain concept may not be valid.

Experimental Program

Two different sands were tested in the resonant column device in this investigation. These sands were chosen because their dynamic properties have been studied by other researchers, and there is data in the literature with which results can be verified and compared: Monterey #0 sand (used in the round robin testing program) and standard Ottawa sand (ASTM Test for Tensile Strength of Hydraulic Cement Mortars [C 190]).

The specimens were 3.53 cm in diameter and 8.0 cm long. Two methods were used for specimen preparation. For a dense condition the specimen was constructed by pouring the sand in layers and tamping. For a loose condition the specimen was obtained by pouring the sand through a funnel. The minimum and maximum void ratios that were obtained in this study were 0.59 and 0.69 for the Monterey #0 sand and 0.52 and 0.58 for the Ottawa sand, respectively.

Several series of dynamic tests were performed in this study. The confining pressure varied from 103 to 241 kPa, and the void ratio corresponded to both the dense and loose condition. In all of the test series, the magnitude of shear strain was varied from approximately $1.5 \times 10^{-3}\%$ to $4 \times 10^{-2}\%$ to study the effect of shear strain amplitude on the dynamic properties of the soil. Details of the testing program are presented in Ref *13*.

Example of Results

At each strain level damping was calculated by the random decrement method when the random excitation was applied and by the logarithmic decrement method when the sinusoidal excitation was applied. The capacity of the resonant column device and electronic equipment limited the RMS of the specimen response to between 30 and 70 mV for the random loading. In addition, to be able to test the same specimen for random and sinusoidal excitation without disturbing the specimen, the RMS of the response was limited to a maximum of about 600 mV.

For the purpose of limiting the size of this paper only an example of the data obtained will be presented. Figure 7 shows the white noise excitation and the response time history and power spectral density for an Ottawa sand specimen. Table 2 shows the type of data obtained for each test for both the random and sinusoidal loading. Figure 8 shows the variation of the shear modulus with the shear strain for the specimen of a loose Monterey #0 sand, and Fig. 9 shows the variation for the same sand in a dense condition.

Figure 10 shows the variation of the damping ratios with shear strain for loose Monterey #0 sand for a confining pressure of 103 kPa, and Fig. 11 shows the variation for loose Ottawa sand for a confining pressure of 172 kPa.

In all cases, damping values obtained from random loading using the random decrement technique were slightly higher than those obtained from sinusoidal loading using the free decay technique. This can be explained by the fact that since damping of soil is dependent on the strain level an experimentally determined free decay of the soil will develop decaying cycles that have decreasing amplitudes, whereas the free decay curve from the random decrement technique is a mathematical manipulation of some random

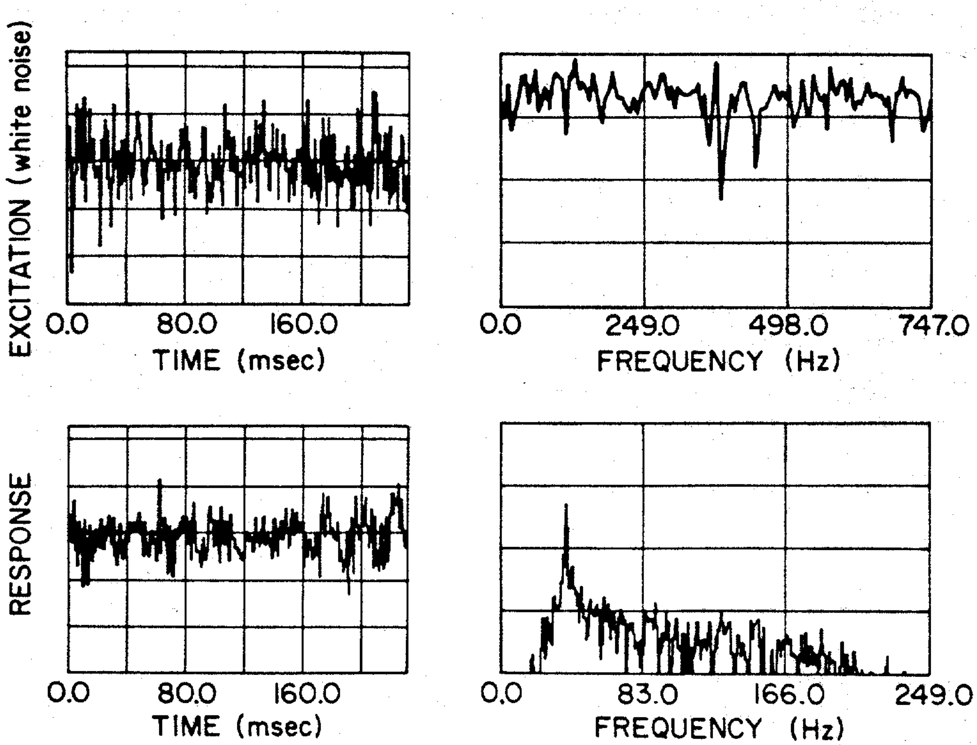

FIG. 7—*Excitation and response—time history and power spectral density, respectively, for Ottawa sand.*

TABLE 2—*Example of data obtained for each specimen.*[a]

Random Vibration					Sinusoidal Vibration				
Response (RMS), mV	Frequency, cycles/s	Damping, %	Modulus, MPa	Strain, %	Response (RMS), mV	Frequency, cycles/s	Damping, %	Modulus, MPa	Strain, %
106	38.3	1.12	95.0	4.9×10^{-3}	107	38.0	1.27	93.6	4.8×10^{-3}
202	37.5	2.40	91.0	9.3×10^{-3}	206	37.0	1.61	89.0	9.7×10^{-3}
304	36.5	2.80	86.0	1.5×10^{-2}	328	35.5	2.41	82.0	1.7×10^{-2}
388	35.6	3.90	82.0	2.0×10^{-2}	397	35.0	2.84	79.4	2.1×10^{-2}
519	35.4	4.14	81.0	2.7×10^{-2}	530	34.0	3.30	75.0	3.0×10^{-2}

[a] Monterey #0 sand, void ratio 0.69, and confining pressure 103.0 kPa.

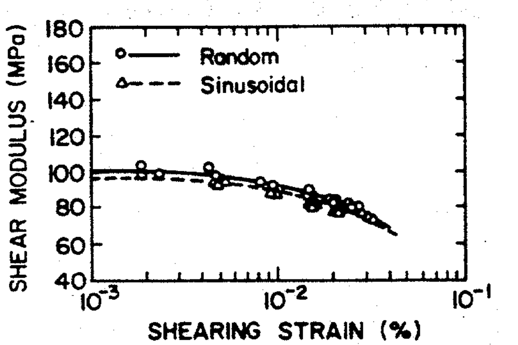

FIG. 8—*Effect of shear strain on shear modulus (Monterey sand, void ratio 0.69, and confining pressure 103 kPa).*

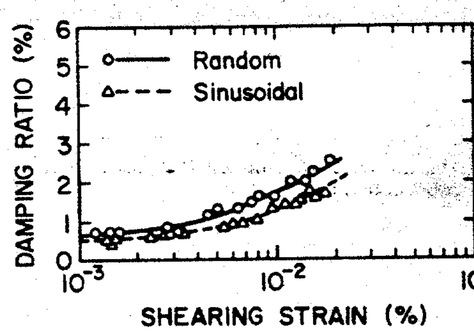

FIG. 11—*Effect of shear strain on damping (Ottawa sand, void ratio 0.57, and confining pressure 172 kPa).*

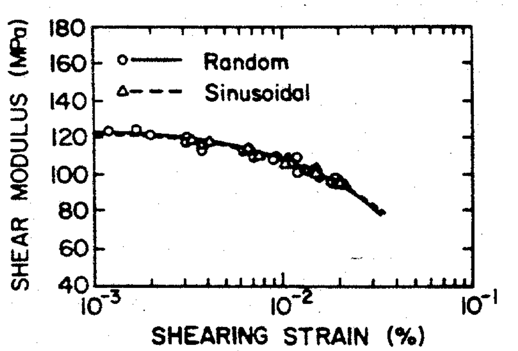

FIG. 9—*Effect of shear strain on shear modulus (Monterey sand, void ratio 0.59, and confining pressure 103 kPa).*

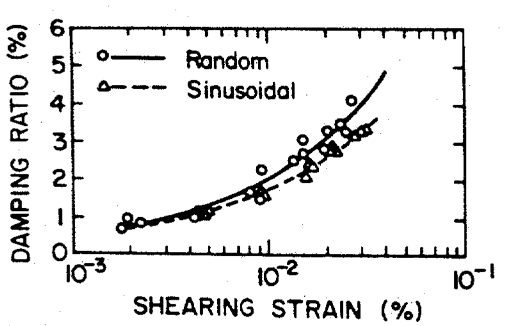

FIG. 10—*Effect of shear strain on damping (Monterey sand, void ratio 0.69, and confining pressure 103 kPa).*

response signal. Thus when constant RMS values are imposed on both the random response signal and the sinusoidal response signal, the strain level is slightly higher for the random signal, which would produce higher damping values. On the other hand, the shear modulus values obtained from random vibrations agree strongly with the values obtained for sinusoidal loading.

Summary and Discussion

In this study the dynamic properties of both Monterey #0 and Ottawa sand were studied using random loading in the resonant column device, and the results were compared with those properties obtained using sinusoidal loading. The difficulties with the analysis of the random response of a system were overcome by the use of a new technique called "random decrement."

The damping of the sinusoidal loading was calculated from the free-vibration decay using the logarithmic decrement method. Similarly the damping of the random loading was calculated from the random decrement signature using the logarithmic decrement method. The shear modulus was calculated using the resonant frequency from the resonance condition when a sinusoidal vibration was applied and from the power spectral density or random decrement signature when a random vibration was applied. These frequencies along with response (RMS) were used in computing strain for sinusoidal loading and equivalent strain for random loading. The concept of equivalent strain was justified by the similarity of the response power spectral density of both excitation types.

The values of the resonant frequency, damping ratio, and shear modulus calculated for the random loading showed good agreement with those values of the sinusoidal loading over a range of strains from 1.5×10^{-3} to 4×10^{-2}%.

The good agreement obtained leads to the conclusion that the random decrement method is a feasible technique for extracting dynamic properties of soils subjected to random loading. This will lead to the application of random decrement to determine in situ damping characteristics. The significance of field determination of damping is that existing information on damping is based on laboratory test results in the laboratory specimens, which are complicated by specimen disturbance that occurs during field sampling operations, the inability to reapply the natural in situ stress conditions to the specimen, and the inability to account for the cementation that exists in natural soil.

An advantage of the random decrement technique is that in comparison to the spectral power density method, where damping is measured by the half-power bandwidth method, it was found that the spectral power density method showed a large measurement variance, especially when the bandwidth was small, or the spectral peak was not well defined. This is especially true in field testing; as an example, Abdel-Ghaffar and Scott [14] in their full-scale dynamic testing on the Santa Felicia dam indicated that model damping values were determined by the logarithmic decrement method applied to recordings of the damped free vibration of the dam. This method was used because the width of the model peaks of the resonance curves were generally not suitable for the determination of model damping values by the half-power bandwidth method.

In addition, when two modes are close, the classical method cannot be applied [15]. In the random decrement technique, as part of the analysis, the records can be filtered to isolated modes if the modes are not too close together. When the modes are close together, a numerical curve fitting technique can be applied to the random decrement signature to separate the modes and extract the damping ratios. Also, the convergence is fast, which is important when analyzing records of very short duration. Experience with random decrement analysis has shown that records of the same location can be put end to end to increase their length if needed.

Conclusion

The main conclusion of this study is that the data from the experimental program showed a good agreement between the modulus and damping obtained from the application of either sinusoidal or random loading. That is, the dynamic properties of dry cohesionless soil were not influenced by the type of loading applied within the range of variables tested.

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