Antitrust Contests

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This work in progress is our own and does not necessarily reflect the views of the Federal Trade Commission or any of the individual Commissioners.

Motivation

- Way Cool Data
  - FTC wins and losses in antitrust cases
  - Various measures of “effort” by FTC and defendants to win antitrust contests
    - Expenditures on experts
    - Expenditures on staff
    - Number of staff
    - And more
  - Data on other variables potentially influencing wins and losses
    - Judge training, party, age, complexity of cases
    - HSR Filings, merger activity
    - And more
- Unique opportunity to structurally estimate Tullock model of an antitrust contest
- Work in progress, comments appreciated

Roadmap

- Theoretical foundations of structural estimation
- Overview of antitrust contests in the US
- Data
- Structural estimates
- Monte Carlo results: Reliability & bias
- Conclusions

Theoretical Foundations

- Two contestants
  - FTC (player 1)
  - Defendant (player 2)
- Potentially different values to each of winning and losing
- Contest Success Function:

Model 1 (Generalized Tullock)

$$
\begin{aligned}
\Pr\left(\textbf{FTC Wins}\middle| X\right) &= p\left(x_1, x_2\right) \\
&= \frac{\sigma x_1^r}{\sigma x_1^r + x_2^r} \\
&= \frac{1}{1 + \frac{1}{\sigma}\left(\frac{x_2}{x_1}\right)^r} \\
&\equiv \frac{1}{1 + \frac{1}{\sigma} z^r}
\end{aligned}
$$

Model 2 (Logistic)

$$\Pr\left(\textbf{FTC Wins}|X\right) = F\left(X\beta\right)$$

where *F is logistic:*

$$F\left(\omega\right) = \frac{\exp\left(\omega\right)}{1+\exp\left(\omega\right)}$$

Pros and Cons

- Advantage of Generalized Tullock
  - Well established theoretical literature (by all of you and others)
- Advantages of Logistic
  - Structural micro foundations (McFadden and others)
  - Empirically estimable using standard logit estimation rather than problematic binomial MLE methods

Key Result: Structural Equivalence

- Model 2 structurally equivalent to Model 1 when

$$X\beta = \ln \sigma - r \ln z$$

- Can use logit estimation to structurally estimate *r* and *σ*
  - Regress binary outcomes (*FTC win = 1*, *FTC loss = 0*) on *CONSTANT* and *ln(z)*
    - Recover *r* from the coefficient of *ln(z)*
    - Recover *σ* = *exp(CONSTANT)*
  - Even a theorist can run this using Stata:
    - `logit ftcwins lnz, robust`

Remark 1

- Can easily generalize to allow σ and r to depend on other explanatory variables

Remark 2

- Structural equivalence works for more general contest success functions, such as

$$\begin{aligned} p(x_1, x_2) &= \frac{\alpha_1 x_1^r}{\alpha_1 x_1^r + \alpha_2 x_2^r} \\ &= \frac{1}{1 + \frac{\alpha_2}{\alpha_1}\left(\frac{x_2}{x_1}\right)^r} \\ &\equiv \frac{1}{1 + \rho z^r} \end{aligned}$$

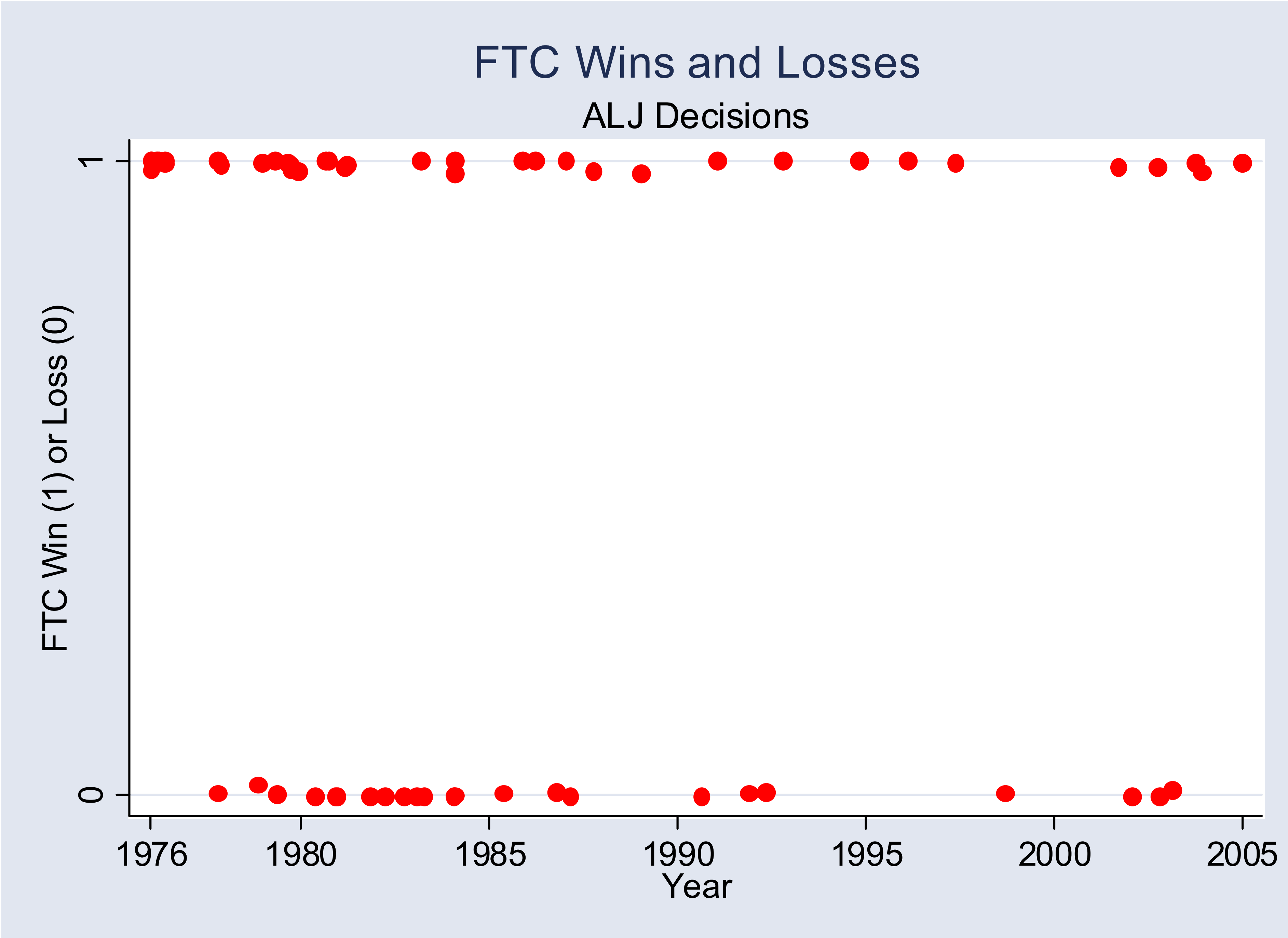
  - But cannot separately identify α's, only the ratio

Remark 3

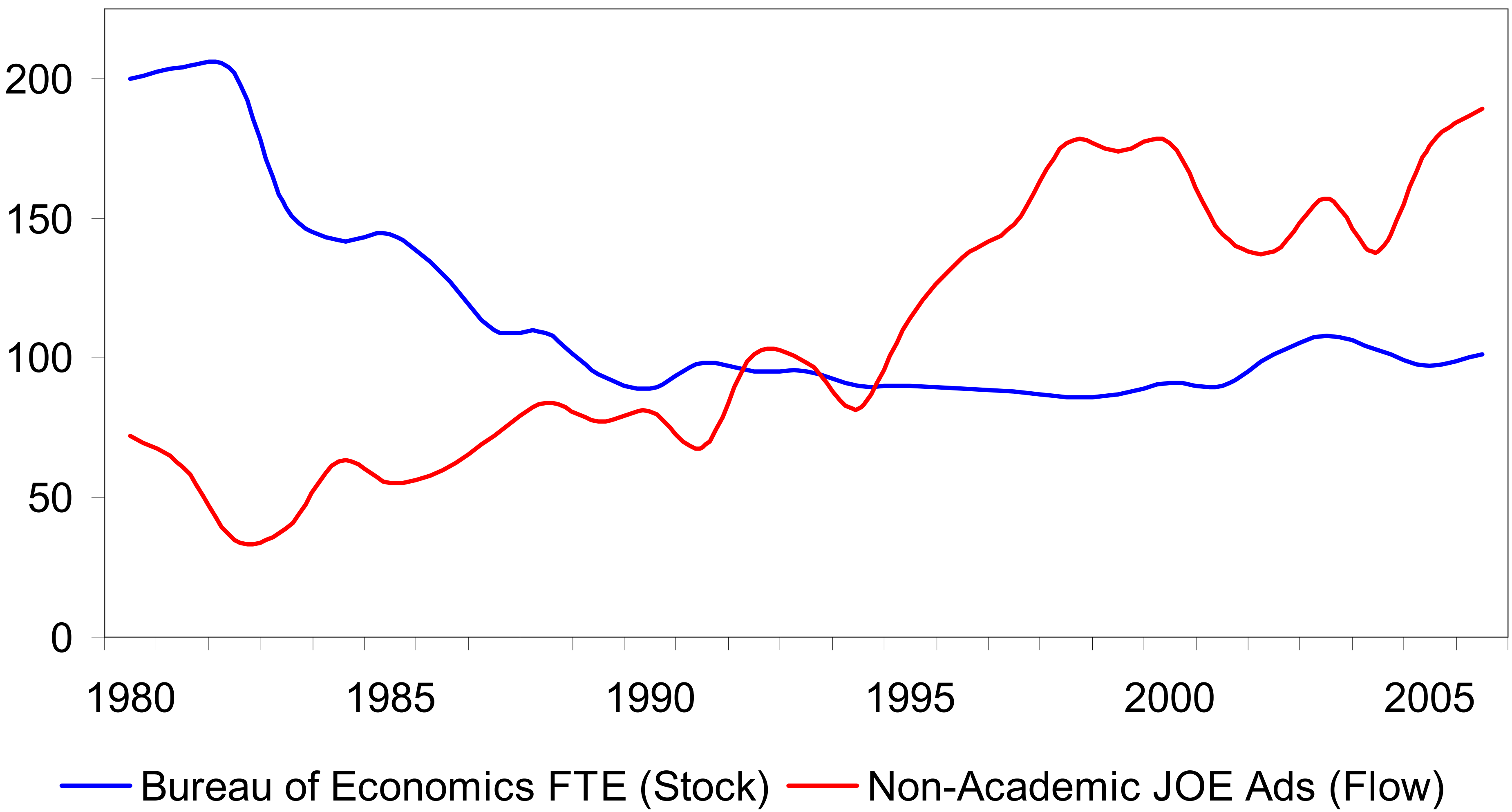
- Not generally feasible to exploit additional structure
  - For $\sigma > 0$, $r > 0$ equilibrium in mixed-strategies guaranteed, but structure of strategies generally unknown except for specific values of σ and r
  - For some parameter configurations, equilibrium is in pure strategies, but these regions depend on r, σ, as well as the *(unknown)* values of winning and losing

Antitrust Process & Data

- Merger cases (HSR filings)
- Non-merger investigations
- Settlement/Litigation
- Data overview
  - 60 cases litigated before an ALJ, 1976-2005
  - $Pr(FTC\ Wins) = .62$


FTC Wins and Losses
ALJ Decisions
FTC Win (1) or Loss (0)
1
0
1976
1980
1985
1990
1995
2000
2005
Year

Economic Labor Effort (FTC vs. Outside Parties)



Results: The Good

Table 2: Implied Structural Parameters						
	Baseline		Random Effects			
	(1) Baseline	(2) Controls for Type of Case	(3) Unobserved Case Heterogeneity	(4) Unobserved Case Heterogeneity	(5) Unobserved ALJ Heterogeneity	(6) Unobserved ALJ Heterogeneity
r	0.26	0.31	0.27	0.33	0.26	0.31
Sigma (Pooled)	0.76		0.74		0.75	
Sigma (Merger Case)		0.51		0.49		0.51
Sigma (Other Case)		0.79		0.78		0.79

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Sigma (Pooled)	0.76		0.74		0.75	
Sigma (Merger Case)		0.51		0.49		0.51
Sigma (Other Case)		0.79		0.78		0.79

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	(1) Baseline	**(2) Controls for Type of Case**	**(3) Unobserved Case Heterogeneity**	**(4) Unobserved Case Heterogeneity**	**(5) Unobserved ALJ Heterogeneity**	**(6) Unobserved ALJ Heterogeneity**
LN(z)	-0.256	-0.309	-0.269	-0.327	-0.258	-0.311
	(0.72)	(0.84)	(0.65)	(0.78)	(0.68)	(0.80)
MERGER DUMMY		-0.435		-0.458		-0.432
		(0.79)		(0.77)		(0.79)
CONSTANT	-0.281	-0.231	-0.297	-0.247	-0.287	-0.238
	(0.26)	(0.21)	(0.25)	(0.20)	(0.25)	(0.20)
Observations	60	60	60	60	60	60
Number of ALJ's					17	17
Number of Cases			60	60		
Note: Robust z statistics in parentheses						

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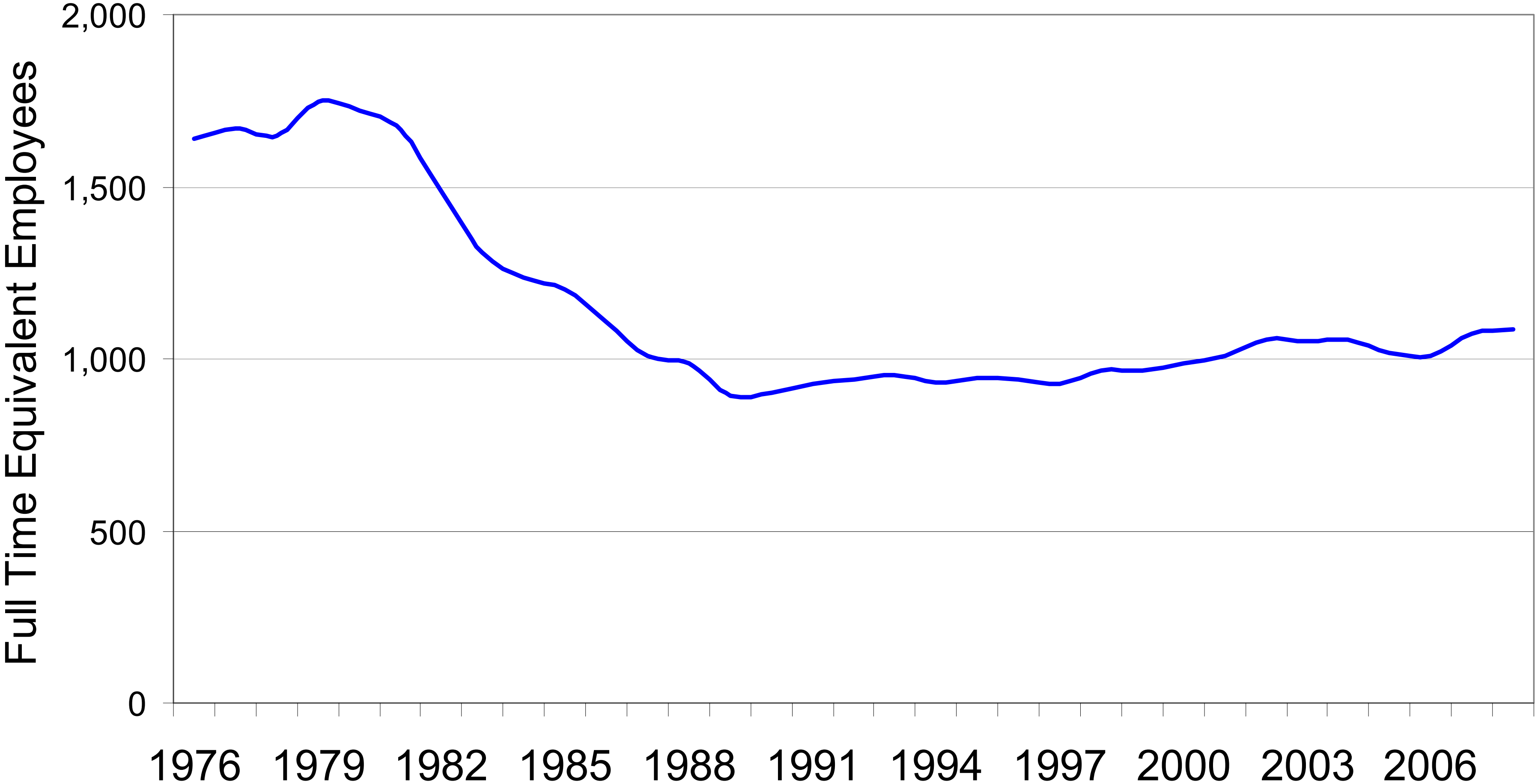
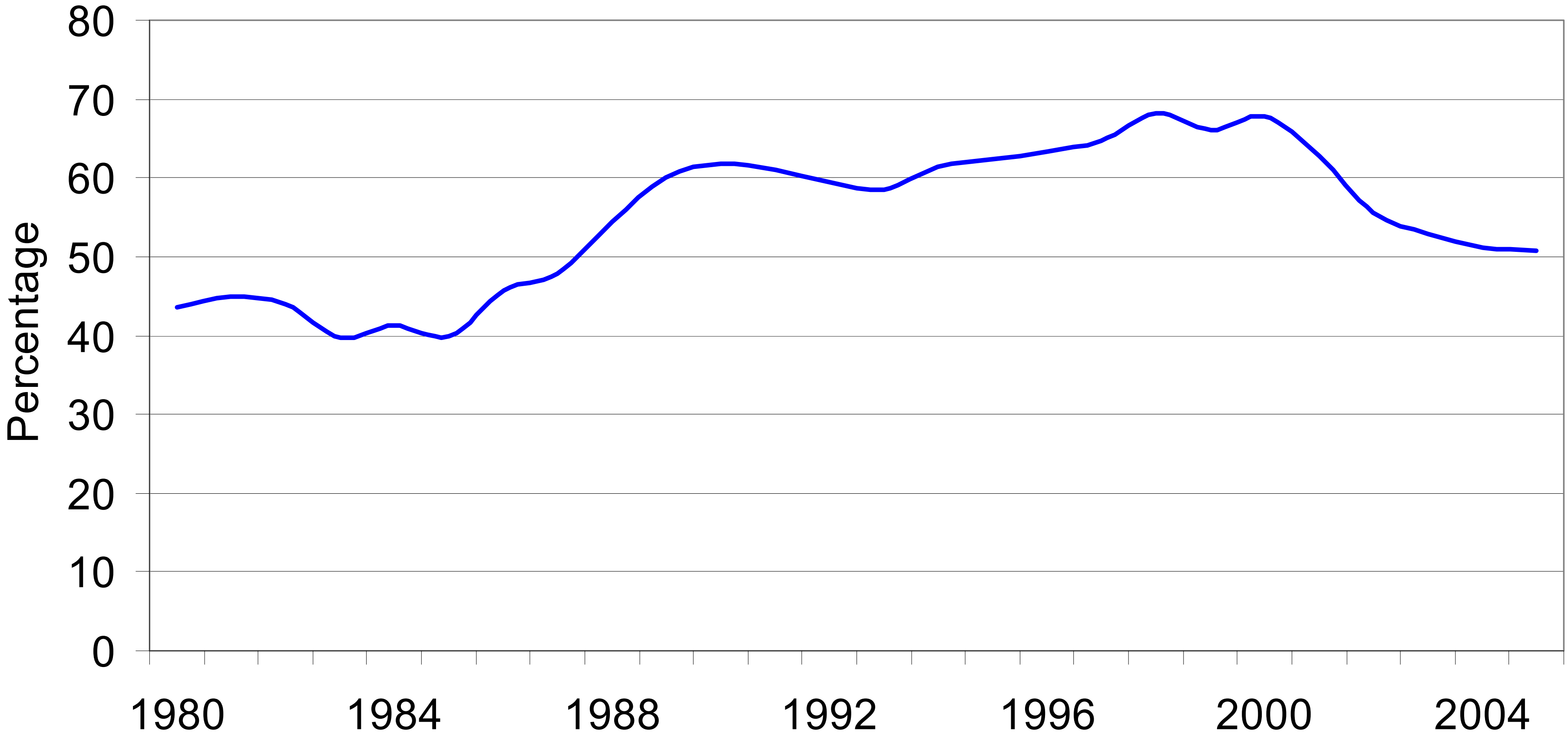
Better Data?

- Include other inputs (attorneys)
- Adjustments for time on antitrust versus other activities (consumer protection or advocacy)
- Expenditures on experts

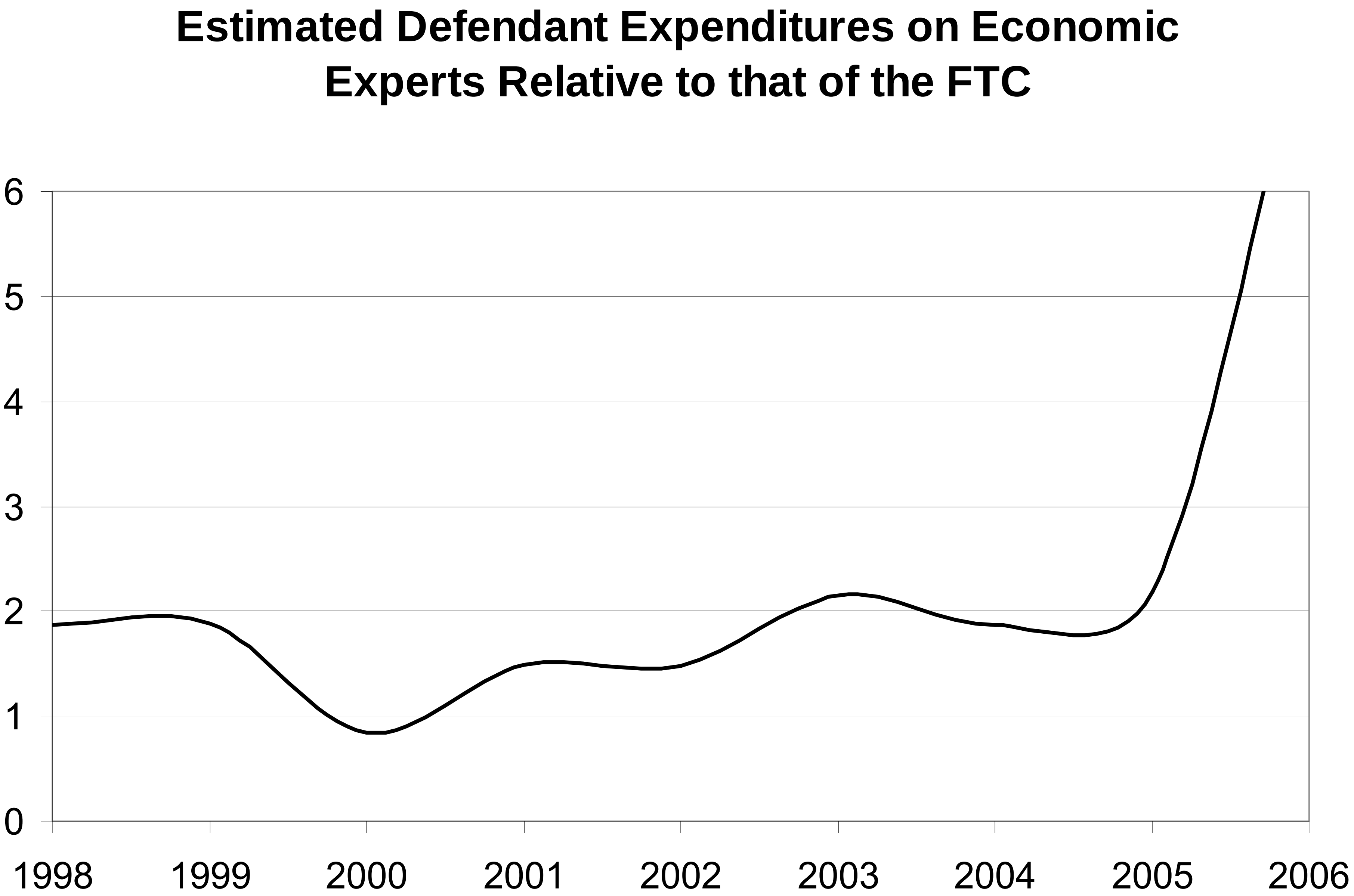
Total FTC Employees



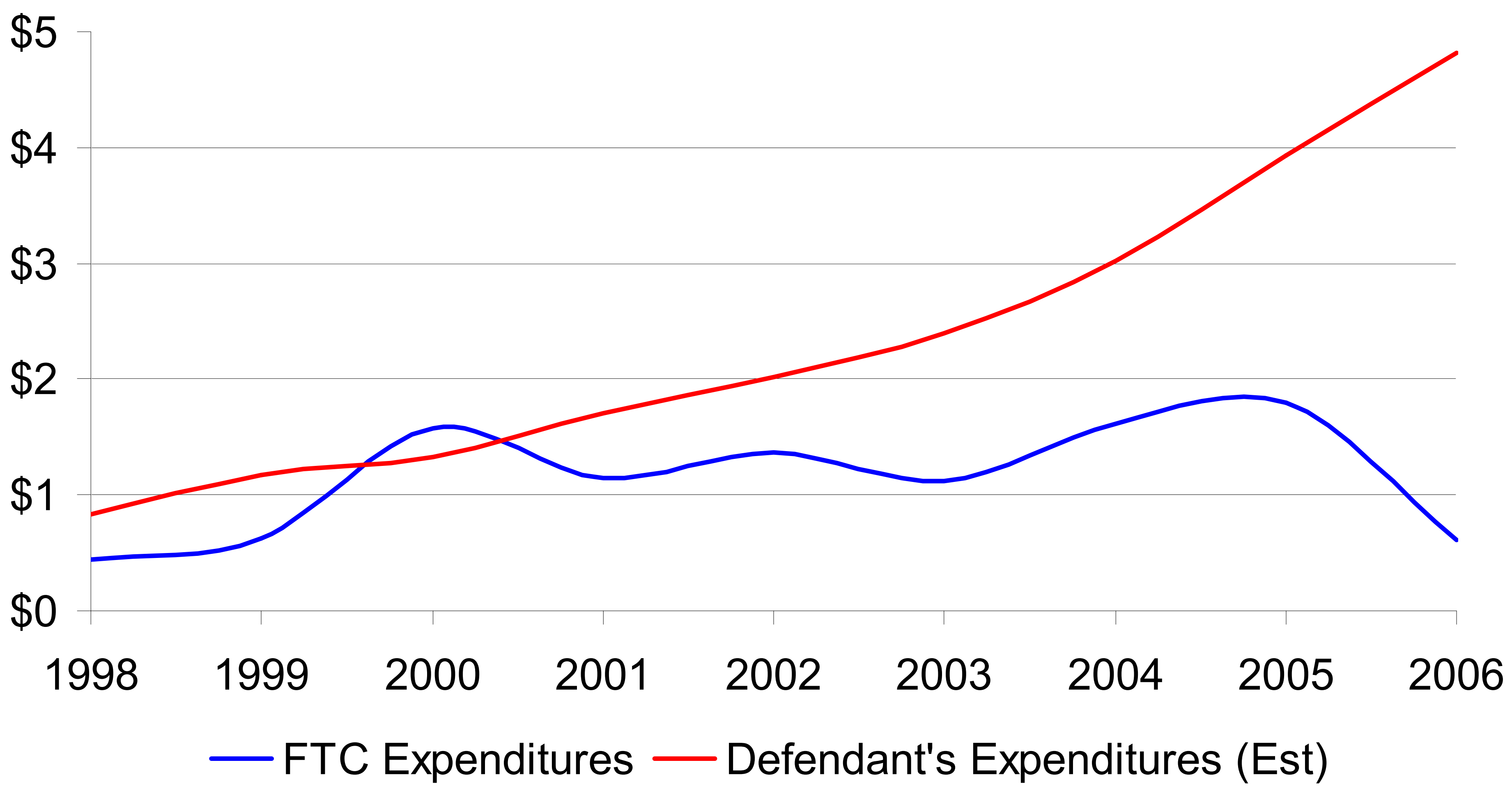
Percentage Allocation of Economist Time for Antitrust



Example of Alternative *z* Measure:



Estimates of FTC and Defendant's Expenditures on Economic Experts (Antitrust)



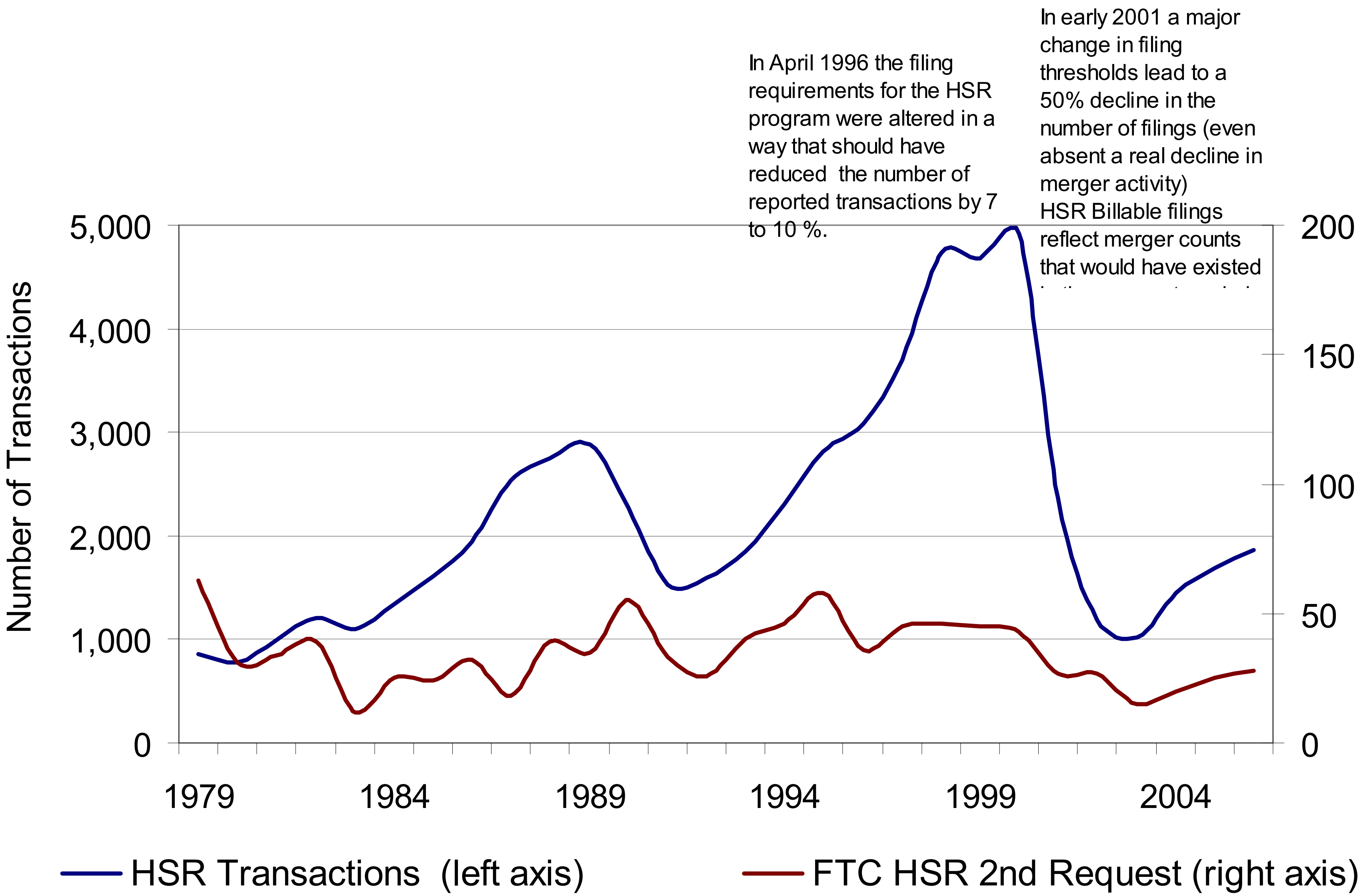
Results From These Data

- Similar sorts of estimates
- No more reliable

What About Endogeneity?

- Merger activity
- Selection issues
- Endogenous effort
  - Impose restrictions on z implied by PSNE and use proxies for values of winning

Hart-Scott-Rodino Transactions & Second Requests



Accounting for Endogeneity

- Doesn't help!
- What's going on?

Monte Carlo

- Generated data from a “true” model

$$r \in \{.25,\ 1,\ 1.5\}$$

$$\sigma \in \{.25,\ 1,\ 1.5\}$$

- Low, medium, high cross sectional variation in z (measured by coefficient of variation)
- 20 obs, 60 obs, 400 obs
- Replicated 10,000 times each

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60									
Parameter or Summary Statistic	**True Parameters**								
	σ = .25			***σ = 1***			***σ = 1.5***		
	r = .25	***r = 1***	***r = 1.5***	***r = .25***	***r = 1***	***r = 1.5***	***r = .25***	***r = 1***	***r = 1.5***
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

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	$\sigma = .25$			$\sigma = 1$			$\sigma = 1.5$		
	$r = .25$	$r = 1$	$r = 1.5$	$r = .25$	$r = 1$	$r = 1.5$	$r = .25$	$r = 1$	$r = 1.5$
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

Parameter or Summary Statistic	True Parameters								
	σ = .25			σ = 1			σ = 1.5		
	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60									
Parameter or Summary Statistic	True Parameters								
	σ = .25			σ = 1			σ = 1.5		
	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

- Punch Line for estimating *r* with 60 obs…
  - Small bias…
  - But unreliable estimates (high variance)

What About Estimates of σ?

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60									
Parameter or Summary Statistic	**True Parameters**								
	σ = .25			***σ = 1***			***σ = 1.5***		
	r = .25	***r = 1***	***r = 1.5***	***r = .25***	***r = 1***	***r = 1.5***	***r = .25***	***r = 1***	***r = 1.5***
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60									
Parameter or Summary Statistic	True Parameters								
	σ = .25			σ = 1			σ = 1.5		
	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5
r-hat	0.30	1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58
bias(r-hat)	0.05	0.09	0.07	0.00	0.07	0.11	0.05	0.05	0.08
sd(r-hat)	2.55	2.57	2.65	1.97	1.99	2.00	2.01	2.05	2.03
pseudo t-statistic	0.12	0.43	0.59	0.13	0.54	0.81	0.15	0.51	0.78
min(rhat)	-17.61	-11.52	-12.84	-7.48	-6.37	-6.13	-8.13	-7.86	-7.45
max(rhat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31	11.87
σ-hat	0.25	0.25	0.25	1.04	1.03	1.04	1.58	1.58	1.58
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
sd(σ-hat)	0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46
pseudo t-statistic	2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42
min(σ-hat)	0.01	0.02	0.01	0.35	0.33	0.35	0.61	0.51	0.55
max(σ-hat)	0.71	0.83	0.80	3.28	2.88	3.05	5.03	5.27	5.03

- Punch line for estimating *σ* with 60 obs: More reliable estimates, but critically depends on the presence of "good" data on effort
  - Scaling of x_i distorts interpretation of *σ*
  - Unreliable if true effort is $\theta_i x_i$ plus noise
  - Both are likely problems in real data

Tullock's r : The Ugly

- Unlikely to be able to obtain reliable structural estimates of r in antitrust contests
  - 400 obs not generally enough
  - Also a problem in other contest environments where sample sizes are not huge
- But…substantial variation in z can help
  - Value of r accentuates/dampens variation in z, making in easier/harder to reliably estimate r

Table 4b: Monte Carlo Results for Structural Estimation: r = .25, σ = 1

Parameter or Summary Statistic	Variation in Effort								
	CV(z) = 1.5%			CV(z) = 14.2%			CV(z)= 68%		
	20 Obs	60 Obs	400 Obs	20 Obs	60 Obs	400 Obs	20 Obs	60 Obs	400 Obs
r-hat	-0.38	0.74	0.21	0.32	0.25	0.25	0.31	0.26	0.25
bias(r-hat)	**-0.63**	**0.49**	**-0.04**	**0.07**	**0.00**	**0.00**	**0.06**	**0.01**	**0.00**
sd(r-hat)	43.25	19.82	7.15	4.27	1.97	0.70	0.99	0.44	0.16
pseudo t-statistic	**-0.01**	**0.04**	**0.03**	**0.08**	**0.13**	**0.36**	**0.32**	**0.60**	**1.60**
min(rhat)	-355.00	-82.36	-27.00	-30.38	-7.48	-2.27	-7.06	-1.63	-0.33
max(rhat)	361.65	92.71	25.31	59.31	12.15	2.99	15.66	2.17	0.86
σ-hat	1.15	1.04	1.01	1.16	1.04	1.00	1.16	1.04	1.00
bias(σ-hat)	0.15	0.04	0.01	0.16	0.04	0.00	0.16	0.04	0.00
sd(σ-hat)	0.78	0.29	0.10	0.81	0.28	0.10	0.94	0.29	0.10
pseudo t-statistic	1.48	3.63	9.91	1.43	3.66	10.02	1.24	3.57	9.87
min(σ-hat)	0.03	0.33	0.68	0.04	0.35	0.65	0.06	0.30	0.67
max(σ-hat)	26.53	3.04	1.45	35.18	3.28	1.48	53.63	3.04	1.52

Table 6b: Monte Carlo Results for Structural Estimation: r = 1.5, σ = 1									
Parameter or Summary Statistic	Variation in Effort								
	CV(z) = 1.5%			CV(z) = 14.2%			CV(z)= 68%		
	20 Obs	60 Obs	400 Obs	20 Obs	60 Obs	400 Obs	20 Obs	60 Obs	400 Obs
r-hat	1.77	1.92	1.46	1.95	1.61	1.52	2.02	1.62	1.51
bias(r-hat)	0.27	0.42	-0.04	0.45	0.11	0.02	0.52	0.12	0.01
sd(r-hat)	43.00	19.98	7.19	4.39	2.00	0.71	2.05	0.59	0.20
pseudo t-statistic	**0.04**	**0.10**	**0.20**	**0.44**	**0.81**	**2.12**	**0.99**	**2.75**	**7.55**
min(rhat)	-351.48	-98.98	-26.76	-18.70	-6.13	-1.59	-2.12	-0.11	0.80
max(rhat)	386.27	78.61	28.88	52.35	9.97	4.05	84.85	6.56	2.38
σ-hat	1.17	1.03	1.00	1.26	1.04	1.00	3.48E+06	1.05	1.01
bias(σ-hat)	0.17	0.03	0.00	0.26	0.04	0.00	3.48E+06	0.05	0.01
sd(σ-hat)	0.99	0.28	0.10	9.43	0.29	0.10	3.47E+08	0.32	0.11
pseudo t-statistic	1.17	3.66	9.87	0.13	3.57	9.82	0.01	3.24	8.94
min(σ-hat)	0.04	0.33	0.63	0.01	0.35	0.71	0.00	0.27	0.67
max(σ-hat)	43.79	2.99	1.44	936.14	3.05	1.47	3.47E+10	3.32	1.50

Concluding Remarks

- Structural estimation of Tullock's r problematic, unless:
  - Have **large sample,** true underlying model has **large** r and **large variation in** z
- Structural estimation of σ requires exceptionally good measures of effort
- Suggests utility of developing alternative contest models more amenable to structural estimation
- Monte Carlo tests of alternative existing models
- Tullock framework still potentially useful for testing predictions via reduced form estimation

Concluding Remarks (Continued)

- Best estimate of r in antitrust contests brought by the FTC between 1976-2005:
  - $r \approx 1/4$
- Monte Carlo simulations suggest estimate is unbiased, but unreliable (high variance)