Bureau of Labor Statistics
Multifactor Productivity
September 26, 2007

Technical Information About the BLS Multifactor Productivity Measures

Background

The multifactor productivity indexes for major sectors measure the value-added output per combined unit of labor and capital input in private business and private nonfarm business. Multifactor productivity indexes for aggregate manufacturing and for 18 manufacturing industries provide measures of sector output per combined unit of capital (K), labor (L), energy (E), materials (M), and purchased business services (S) input—KLEMS inputs.

Table 1 summarizes the availability of productivity measures for major sectors of the U.S. economy.

Table 1. Availability of productivity measures for major sectors and subsectors of the economy

Productivity measure	Output	Input(s)	Index available
Multifactor productivity			
Private business	Value added	Labor, capital	Annually
Private nonfarm business	Value added	Labor, capital	Annually
KLEMS[1] multifactor productivity:			
Manufacturing and 18 3-digit NAICS manufacturing industries services	Sectoral output	Labor, capital, energy, materials, services	Annually

Description of Measures

BLS publishes two sets of multifactor productivity measures for the major sectors and subsectors of the U.S. economy, each using a distinct methodology. The first set includes multifactor productivity for major sectors and the second set measures multifactor productivity for total manufacturing and 18 3-digit North American Industry Classification System manufacturing industries. Each set of measures involves a comparison of output and input measures.

The first set of measures covers multifactor productivity for major U.S. sectors.[2] In these measures, output, consisting of only goods and services sold to final consumers, is measured net of price changes and inter-industry transactions and the input measure is an aggregate of labor input and capital service flows. These measures have been developed in recognition of the role capital growth plays in output growth. They are updated annually.

Comparisons of output with a broader set of inputs constitute the second set of measures.[3] Because the scope of industries within manufacturing is narrower than that of the nonfarm business sector, output in manufacturing industries includes shipments to other industries as well as to final consumers.[4] Consistent with such an output concept is an input measure which includes intermediate inputs. Accordingly, input includes labor and capital, and also energy, nonenergy materials, and purchased business services. These measures are available for a comprehensive set of 18 manufacturing industries (roughly corresponding to the 3-digit North American Industry Classification System (NAICS) level) as well as for total manufacturing, durable goods manufacturing and nondurable goods manufacturing. As the focus narrows to more specific industries, intermediate inputs take on an increasingly important role in productivity measurement and analysis. This set of measures consists of annual data and is updated once a year.

Data Sources and Estimating Procedures

Private business and private nonfarm business sectors. The multifactor productivity indexes for these sectors measure output per combined unit of labor and capital input. Real gross domestic product in the business and nonfarm business sectors is the basis of the output components of the major sector labor productivity and multifactor productivity measures. The output measures for private business and private nonfarm business are similar to the Fisher-Ideal indexes of output for business and nonfarm business except that output of government enterprises is omitted. Estimates of the appropriate weights for labor and capital in government enterprises cannot be made because subsidies account for a substantial portion of capital income.

Labor input in the private business and private nonfarm business sectors is obtained by Tornqvist aggregation of the hours at work by all persons, classified by education, work experience, and gender with weights determined by their shares of labor compensation. Hours paid of employees are largely obtained from the BLS Current Employment Statistics (CES) program. These hours of employees are then converted to an at-work basis by using information from the Employment Cost Index (ECI) of the National Compensation Survey and from historical information collected by the Hours at Work Survey. Hours at work for nonproduction and supervisory workers are derived using data from the CPS, the CES, and the NCS. The hours at work of proprietors, unpaid family workers, and farm employees are derived from the Current Population Survey. The hours at work for each of 1,008 types of workers classified by their educational attainment, work experience and gender are aggregated using an annually chained (Tornqvist) index. The growth rate of the aggregate is therefore a weighted average of the growth rates of each type of worker where the weight assigned to a type of worker is its share of total labor compensation. The resulting aggregate measure of labor input accounts for both the increase in raw hours at work and changes in the skill composition (as measured by education and work experience) of the work force.[5]

Capital inputs for the multifactor productivity measures are computed in accordance with a service flow concept for physical capital assets—equipment, structures, inventories, and land. Capital inputs for major sectors are determined in three main steps: 1) A very detailed array of capital stocks is developed for various asset types in various industries; 2) asset-type capital stocks are aggregated for each industry to measure capital input for the industry; and 3) industry capital inputs are aggregated to measure sectoral level capital input.

The asset detail consists of 42 types of equipment and software, 21 types of nonresidential structures, 9 types of residential capital, inventories (manufacturing available for 3 stages of fabrication), and land. BLS measures of capital stocks for equipment and structures are prepared using NIPA data on real gross investment. Real stocks are constructed as vintage aggregates of historical investments (in real terms) in accordance with an "efficiency" or service flow concept (as distinct from a price or value concept). The efficiency of each asset is assumed to deteriorate only gradually during the early years of an asset's service life and then more quickly in its later life. These "age/efficiency" schedules are based, to the extent possible, on empirical evidence of capital deterioration. Inventory stocks are developed using data from the NIPA and IRS. Farm land input is based on data from the Economic Research Service of the U.S. Department of Agriculture. A benchmark for nonfarm land is estimated by applying a land-structure ratio based on unpublished estimates by the BLS to the value of structures. These ratios are based on data from 2001 for all counties in Ohio. [6] This benchmark is extrapolated using gross stocks of structures calculated from Bureau of Economic Analysis investment data. The resulting nonfarm land data series is allocated to industries based on Internal Revenue Service data on book values of land.[7]

For each industry (the BLS procedures are applied to 60 industries in the private business sector at approximately the 3-digit NAICS level), these measures of capital stocks are aggregated using a Tornqvist chain index procedure as described below. The weight for each asset type is based on the share of capital income estimated to accrue to that asset type in each industry, averaged over 2 years. Capital income in each industry is allocated to asset types by employing estimates of the "implicit rental prices" of each asset type.[8] The implicit rental price concept is based on the neoclassical theory of the firm and provides a framework for deriving weights for asset-type capital stocks. Because some asset types tend to deteriorate much more quickly than others and because of tax rules specific to asset types, the real economic cost of employing a dollar's worth of stock varies substantially by asset type.

At the sector level, aggregate capital input is obtained by chaining together (Tornqvist aggregating) all of the individual industry capital inputs, using each industry's two-period average share of total capital income as weights.

Once the sector's capital input is measured, total input is computed by aggregating capital and labor. For each input, the weight is the input's share of total costs and is derived from NIPA data on the components of nominal value added by industry. At both the sector and the industry levels, labor costs are measured as compensation to employees (wages, salaries, and supplements) plus a portion of noncorporate income.[9] Most other components of nominal value added are assigned to capital.[10] The exceptions are those taxes on production and imports which are not assigned either to capital or labor (notably sales and excise taxes). Thus total cost is less than value added by an amount equal to these taxes. Labor and capital shares in total cost are computed and then used in the aforementioned aggregation of capital and labor.[11] Finally, major sector multifactor productivity indexes are calculated as the ratio of output to input.[12]

Along with other factors, technological change plays an important role in multifactor productivity. For private nonfarm business, the BLS reports estimates of the impact of multifactor productivity growth from firms' research and development (R&D) efforts including benefits that spill over to other firms within the same industry. Because research and development spending and the resulting technological improvements are a source of productivity change, the BLS has not made any adjustments in multifactor productivity calculations to exclude their effects. Instead we measure the annual contribution of R&D to multifactor productivity growth for the private nonfarm business sector. This is measured as the product of an estimated rate of return to R&D and the stock of research and development per unit of output. The stock of research and development in private business is obtained by cumulating constant dollar measures of research and development expenditures net of depreciation. The BLS develops price deflators and also estimates the rate of depreciation of R&D.[13]

Preliminary Multifactor Productivity. A simplified methodology is used to make preliminary estimates of multifactor productivity change in the private business and private nonfarm business sectors. The simplified method uses preliminary data to obtain estimates in the year after the target year. The full method yields better estimates but takes longer, and is completed in the second year following the reference year. The simplified methodology involves making estimates of the growth rates of output, labor input and capital input, and of the shares of each input. Using the same basic structure and assumptions, the simplified methodology is designed to yield estimates of multifactor productivity approximating closely that calculated by the full methodology. The simplified methodology includes fewer categories of capital than the full methodology. The simplified measure is usually based on information from the full calculation from the previous year and on up-to-date information from NIPA and other sources available early in the year following the target year. The resulting simplified measure will later be supplanted by the full measure, when complete data become available. In practice, the revision of the simplified estimate to obtain a full estimate reflects both the difference in methodologies and concurrent revisions to the underlying source data that become available.[14]

Manufacturing industries. Multifactor productivity indexes for aggregate manufacturing, durable goods manufacturing, nondurable goods manufacturing and for 18 NAICS manufacturing industry groups also measure output per unit of input. In this case, input is a weighted aggregate of capital, labor, energy, nonenergy materials, and purchased business services inputs.[15]

For these multifactor productivity manufacturing measures, output is the deflated value of production shipped to purchasers outside of the domestic industry, not just production for final users as is used for the major sector multifactor productivity indexes. Because the major sector multifactor productivity indexes are aggregated over the entire domestic economy, they are based on sales of final goods and services. The manufacturing multifactor productivity indexes are based on sectoral output—sales to final demand plus the intermediate goods sent to other industries. Sectoral output is defined as gross output excluding intra-industry transactions. This measure defines output as deliveries to consumers outside the sector in an effort to avoid the problem of double-counting that occurs when one establishment provides materials used by other establishments in the same industry.[16]

Capital is measured as it is for the major sector multifactor productivity indexes; rental prices of capital are computed for each industry. However, labor is measured as a direct summation of hours at work. Thus manufacturing is not adjusted for the experience and education of the work force unlike the measures for the more-aggregate major sectors.

The inclusion in the manufacturing multifactor productivity measures of all intermediate inputs—energy, nonenergy materials, and purchased business services—is consistent with the use of total value of production as the output measure. Intermediate inputs (energy, materials, and purchased business services) are obtained from BEA's annual input-output tables. Tornqvist indexes of each of these three input classes are derived at the 3-digit NAICS level and then aggregated to total manufacturing. As with the sectoral output measures, materials inputs are adjusted to exclude transactions between establishments within the same sector. The five input indexes (capital services, hours, energy, materials, and purchased business services) are combined using Tornqvist aggregation, employing weights that represent each component's share of total costs. Total costs are defined as the value of manufacturing sectoral output. The index uses changing weights: The share in each year is averaged with the preceding year's share.

Total input is computed from components as a Tornqvist chain index number series. The weight for each input is its share in total input cost. Measures of multifactor productivity for 18 manufacturing industries are available for 1987 to the present.

Analysis and Presentation

Indexes of labor productivity show changes in the ratio of output to hours of labor input. Similarly, indexes of multifactor productivity show changes in the ratio of output to combined inputs. However, these indexes should not be interpreted as presenting the contribution of the particular input, or combination of inputs, to production. Rather, changes over time in the output, labor input, or combined input measures underlying these productivity indexes may reflect the influence of other factors including variations in the characteristics and efforts of the work-force, changes in managerial skills, changes in the organization of production, changes in the allocation of resources between sectors, the direct and indirect effects of R&D, and new technologies.

In aggregate sectors, productivity changes through time reflect movements within the various component industries as well as shifts in the relative importance of each of the industries. For example, changes in multifactor productivity are influenced by the relative shift of inputs (labor and capital) from low- to high-productivity industries and by productivity changes in the component subsectors.

Short-term movements in productivity can result from cyclical variations and may also reflect unusual events such as drought. These short-term movements are sometimes substantially greater or smaller than long-term averages of productivity movements. For example, productivity growth for 1 or 2 years can be substantially greater than the average for the business cycle that includes these years.

Multifactor productivity measures are announced each year in the news release, "Multifactor Productivity Trends." Included are annual indexes of multifactor productivity, capital inputs, and related measures for the private business and private nonfarm business sectors. Preliminary measures for the private business and the private nonfarm business sectors are announced separately in the news release "Preliminary Multifactor Productivity Trends." Manufacturing measures are announced separately on an annual basis in the news release, “Multifactor Productivity Trends in Manufacturing”, see http://www.bls.gov/news.release/prod5.toc.htm.

Indexes of multifactor productivity and related measures are available monthly in *Monthly Labor Review*, and can be accessed through the BLS home page at http://www.bls.gov on the Internet. This includes data from both the multifactor and KLEMS productivity measures.

Calculation Procedures

BLS aggregates inputs for its multifactor productivity measures using a Tornqvist chain index. Some of the basic properties of this index are: It is calculated as a weighted average of growth rates of the components; the weights are allowed to vary for each time period; and the weights are defined as the mean of the relative compensation shares of the components in two adjacent years. For example, the index of inputs (I) in each year, t, for major sectors is calculated by linking to the previous year's index:

$$\ln\left(\frac{I(t)}{I(t-1)}\right) = \left(\frac{S_k(t)+S_k(t-1)}{2}\right)\left(\ln\frac{K(t)}{K(t-1)}\right) + \left(\frac{S_l(t)+S_l(t-1)}{2}\right)\left(\ln\frac{L(t)}{L(t-1)}\right)$$

where $S_k(t)$ = capital costs(t) / total costs(t),
and $S_l(t)$ = labor costs(t) / total costs(t),

and the percentage change in I is given by[17] [exp(Δ ln I)-1] or [I(t)/I(t-1)] - 1.

The formula for input, I, also can be written as

$$\Delta \ln I = 1/2 * [S_k(t) + S_k(t-1)]\ \Delta \ln K + 1/2 * [S_l(t) + S_l(t-1)]\ \Delta \ln L$$

Similarly, both capital, K, and labor, L, are Tornqvist indexes. Each is a weighted average of the growth rates of detailed types of capital, k_i, and labor inputs, l_i, respectively.

$$\Delta \ln K = \sum_i\ 1/2 * [S_{ki}(t) + S_{ki}(t-1)]\ \Delta \ln k_i$$

where $S_{ki}(t) = C_{ki}(t) * k_i(t)$ / total capital costs
and where $C_{ki}(t)$ is the rental price for capital asset k_i.

$$\Delta \ln L = \sum_i\ 1/2 * [S_{li}(t) + S_{li}(t-1)]\ \Delta \ln l_i$$

where $S_{li}(t)= v_{li}(t) * l_i(t)$/ total labor costs
and $v_{li}(t)$ is the hourly compensation for worker group l_i.

Changes in the index of labor composition, LC, are defined as the difference between the changes in the aggregate labor input index, L, and the changes of the hours of all persons, H.

$$\Delta \ln LC = \Delta \ln L - \Delta \ln H$$

The Tornqvist index for major sector multifactor productivity growth, A, is:

$$\Delta \ln A = \Delta \ln Q - \Delta \ln I$$

where Q is the Fisher-Ideal index of sector output as measured by BLS.

For manufacturing and the 18 NAICS-based industries which comprise manufacturing, aggregate input has a conceptually similar definition except that there are 5 inputs rather than just the 2 used in the major sector measures.

$$\Delta \ln I = 1/2 * [S_k(t) + S_k(t-1)]\ \Delta \ln K + 1/2 * [S_l(t) + S_l(t-1)]\ \Delta \ln L$$
$$+ 1/2 * [S_e(t) + S_e(t-1)]\ \Delta \ln E + 1/2 * [S_m(t) + S_m(t-1)]\ \Delta \ln M$$
$$+ 1/2 * [S_s(t) + S_s(t-1)]\ \Delta \ln S$$

where L = total hours at work
$S_l(t)$ = labor costs(t)/total costs(t)
$S_k(t)$ = capital costs(t) / total costs(t)
$S_e(t)$ = energy costs(t)/total costs(t)
$S_m(t)$ = materials costs(t) / total costs(t)
$S_s(t)$= purchased business services costs(t) / total costs(t)

and total costs are the current dollar value of shipments adjusted for inventory change.

Using this definition for aggregate input, multifactor productivity for manufacturing or any of the 18 NAICS-based industries which comprise manufacturing is identically defined as above.

$$\Delta \ln A = \Delta \ln Q - \Delta \ln I$$

where Q is a Tornqvist output index developed by BLS.

For private business and private non-farm business, the relationship of aggregate multifactor productivity to aggregate labor productivity is given by the following equation:

$$d(\ln Y - \ln L) = d \ln A + w_k[d(\ln K - \ln L)] + w_l\,[d \ln LC]\ ,$$

where d denotes the derivative with respect to time,
w_i denotes the cost share weight of input i, (i = k, l)

This equation shows that labor productivity growth is decomposed into the contribution of multifactor productivity growth, the contribution resulting from K/L substitution (capital deepening) and the contribution of the labor composition effect.

Uses and Limitations

Measures of output per hour (labor productivity), output per unit of capital (capital productivity), and output per combined unit of multifactor input (multifactor productivity) and related measures of costs are designed for use in economic analysis and public and private policy planning. The data are used in forecasting and analysis of prices, wages, and technological change.

Certain characteristics of the productivity data should be recognized in order to apply them appropriately to specific situations. First, the data for aggregate sectors reflect changes within various constituent industries as well as shifts in the relative importance of these industries. Second, the relationships among variables are often difficult to identify over short time periods. Third, data and other resources available for their preparation somewhat limit the productivity, output, compensation, capital and labor composition and hours measures which can be constructed.

Technical References

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[1] Capital (K), labor (L), energy (E), materials (M), and purchased business services (S)

[2] *Trends in Multifactor Productivity, 1948-81*, Bulletin 2178 (Bureau of Labor Statistics, 1983).

[3] These measures were first introduced in William Gullickson and Michael J. Harper, "Multifactor Productivity in 20 U.S. Manufacturing Industries, 1949-83," *Monthly Labor Review*, October 1987, pp. 18-28

[4] In the more aggregate sectors, private business and private nonfarm business, the delivery of goods to final users closely corresponds to value added or gross product originating (GPO). In less aggregate economic sectors, such as manufacturing, where inter-industry transactions represent a smaller proportion of goods and services produced, deliveries include output which is not part of final demand and more closely approximates a gross output measure.

[5] See *Labor Composition and US Productivity Growth, 1948-90* for a complete description of Tornqvist aggregation of hours. See also *Changes in the Composition of Labor for BLS Multifactor Productivity Measures,*http://www.bls.gov/mfp/mprlabor.pdf

[6] Ohio uniquely had the taxable value of real property separated into classes: residential, agricultural, industrial, commercial, and mineral. The Ohio data also had separated land/structures values calculated for each class. Since this level of detail was only available for Ohio, it was decided that this data source was the best one for estimating the U.S. values as a whole and the U.S. land/structure ratios were derived using these data.

[7] These methods are described in detail in *Trends in Multifactor Productivity, 1948-81*, appendix C. See also *Overview of Capital Inputs for the BLS Multifactor Productivity Measures* at the BLS web site, http://www.bls.gov/mfp/mprcaptl.pdf.

[8] The rental price formula and related methodology and data sources are described in *Trends in Multifactor Productivity, 1948-81*, appendix C. The rental price formulas described in this publication have been modified to eliminate large fluctuations due to inflation in new goods prices. Research on this issue is reported by Michael J. Harper, Ernst R. Berndt and David O. Wood, "Rates of Return and Capital Aggregation Using Alternative Rental Prices," in Dale W. Jorgenson and Ralph Landau, *Technology and Capital Formation*, 1989, MIT Press, pp. 331-37.

[9] Noncorporate income is allocated to labor and capital costs in each year using the following assumption: Initially self-employed persons and unpaid family workers are assumed to receive the same hourly compensation as employees and the rate of return to non-corporate capital is assumed to be the same as in the corporate sector. Based on these assumptions, the resultant income of proprietors is adjusted to match actual proprietors income reported in the GPO data by scaling proportionately the hourly compensation of the self-employed and the noncorporate rate of return. This treats any apparent excess or deficiency in noncorporate income neutrally with respect to labor and capital.

[10] Capital costs are the sum of 1) the balance of noncorporate income, 2) corporate profits, 3) net interest, 4) adjusted capital consumption allowance, 5) inventory valuation adjustments, and 6) portions of taxes on production and imports assumed to be associated with capital (notably motor vehicle and property taxes), 7) business transfer payments.

[11] Excluding these indirect business taxes from the calculation of factor shares has the effect of assuming the incidence of these taxes are neutral with respect to capital and labor income

[12] BLS builds multifactor productivity measures from three-digit NAICS detail. However, most of the critical data used to calculate these measures prior to 1998 are available only on SIC rather than NAICS basis. For example, most critical income-component data used for the construction of capital income measures are not available on NAICS basis before 1998. Such SIC-based data are converted to NAICS basis by the use of 1997 SIC-to-NAICS bridge ratios. Corporate income components prior to 1998 are calculated by applying 1998 corporate conversion factors to income components. Land estimates from 1987 to 1997 are calculated using a combination of SIC–to-NAICS conversion factors and more detailed IRS data.
See BLS Economic News Release: Multifactor Productivity Trends at the BLS web site http://www.bls.gov/news.release/pdf/prod3.pdf **.**

[13] Current dollar expenditures for privately financed research and development are obtained from annual issues of Research and Development in Industry published by the National Science Foundation. Further description of these data and methods can be found in BLS Bulletin 2331 (September 1989), "The Impact of Research and Development on Productivity Growth."

[14] See "Preliminary Estimates of Multifactor Productivity Growth" by Peter B. Meyer and Michael J. Harper, *Monthly Labor Review*, June 2005, pp. 32-43, at the BLS web site http://www.bls.gov/opub/mlr/2005/06/art3full.pdf .

[15] An explanation of the methods and some results are presented in "Measurement of Productivity Growth in U.S. Manufacturing," by William Gullickson, *Monthly Labor Review*, July 1995, pp. 13-27, available at http://www.bls.gov/mfp/mprgul95.pdf .

[16] Further information on this concept of output is available in "Measurement of Productivity Growth in U.S. Manufacturing," by William Gullickson, *Monthly Labor Review*, July 1995, pp. 13-27, available at http://www.bls.gov/mfp/mprgul95.pdf .

[17] The triangle (delta) refers to discrete change with respect to time. For example, $\Delta \ln Z = \ln Z(t) - \ln Z(t-1) = \ln[Z(t) / Z(t-1)]$ for variable Z. The percentage change in Z is given by $[\exp(\Delta \ln Z)-1]$ or $[Z(t)/Z(t-1)] - 1$.