Yves Thibaudeau

yves.thibaudeau@ccmail.census.gov

U.S. Bureau of the Census

Statistical Research Division

Research Report # RR-99-02

Model Explicit Item Imputation for Demographic Categories for Census 2000

Abstract

The paper shows results obtained when using a hierarchical log-linear model to produce item imputations based on the maximum likelihood estimator. We compare the results with those obtained using the sequential hot-deck imputation procedure. We apply the two procedures on the data collected in Sacramento for the 1998 dress rehearsal for Census 2000. To measure the relative differences between the two methodologies, we simulate the posterior and predictive distributions associated with the model. We run our simulation through data augmentation bayesian iterative proportional fitting (DABIPF). Gelman and Rubin (1991) first proposed a bayesian iterative proportional fitting (BIPF) to generate posterior conjugates for categorical log-linear models. Schafer (1997) proposes a variant of BIPF for direct application to hierarchical models. Schafer (1997) also extends the technique to DABIPF. In our situation Schafer's version of DABIPF yields: 1. An approximation for the posterior distribution of the inclusion probabilities. 2. An approximation for the predictive distribution of the population counts. The predictive distribution makes it possible to give a full inferential assessment of the unreported population counts, and to compare our item imputation procedure with the sequential hot-deck.

Yves Thibaudeau, Mathematical Statistician, Statistical Research Division,

US Bureau of the Census, Room 3000-4, Washington, DC 20233-9100

yves.thibaudeau@ccmail.census.gov

Disclaimer

This paper reports the results of research and analysis undertaken by Census Bureau staff. It has undergone a more limited review than official Census Bureau publications. This report is released to inform interested parties of research and to encourage discussion.

1 Introduction

In the paper we set to develop and evaluate a model-based production-oriented procedure for item imputation. We present our procedure as a competitor to the traditional hot-deck system at the Bureau of the Census. To support and evaluate our item imputation procedure, and to compare it with the hot-deck, we use a methodology developed in good part only in recent years. This methodology is powerful. It provides us with the tools we need for our evaluation. Because this evaluation methodology is relatively new, and some aspects of it were not entirely clear to us, we felt necessary to include proofs for the assumptions we make in the paper. We attempt to motivate our analysis in terms of its implications for the 1998 dress rehearsal in Sacramento, the example we use as a test bed for our item imputation procedure.

In section 2 we introduce the example we will exploit for this research, the item imputation for the 1998 dress rehearsal of Census 2000 in Sacramento, and we recall the current methodology used for this application. In subsection 2.1 we review a sample of the literature on the controversies associated with bayesian item imputation. In subsection 2.2, we introduce the 1998 dress rehearsal example. In subsection 2.3, we review the methodology of the Census Bureau to impute items unreported during the dress rehearsal. In section 3 we present our tools for evaluating an imputation methodology. Subsection 3.1 introduces the reference model and subsections 3.2 and 3.3 give a step by step description of the data augmentation bayesian iterative proportional fitting (DABIPF) procedure based on the model. DABIPF is the core of the evaluation machinery. Subsection 3.4 gives examples of posterior and predictive distributions produced with DABIPF. These distributions serve as gauges in the evaluation. In subsection 3.5 we propose an alternative procedure to produce item imputations. In section 4 we present results on the item imputation for the 1998 dress rehearsal of Census 2000 in Sacramento, and parallel results obtained with our alternative procedure. Subsection 4.1 gives interpretations for the predictive means and the predictive variances used to evaluate the results. Subsection 4.2 gives values for the imputed counts, along with predictive means and predictive standard deviations for several demographic categories. The results allow us in subsection 4.3 to evaluate the impact of the new Hispanic origin imputation methodology at the Census Bureau. In subsection 4.4, we evaluate the relative bias of hot-deck item imputation with respect to the model. In subsection 4.5, we analyze the performance of our alternative imputation procedure and we compare it with that of the hot-deck. In section 5 we draw appropriate conclusions.

2 Background

2.1 Why Use Models for Item Imputation?

We intend to show that bayesian model-based inference can improve the quality of the imputation of unreported items in censuses and demographic surveys. At the moment, the most common method for demographic (categorical) item imputation is the hot-deck, either fixed-cell, sequential, or nearest neighbor. It is particularly difficult to implement a hot-deck item imputation in a multivariate context, because multivariate item imputation requires a careful elicitation of the dependencies between items, and hot-deck specifications often fail to convey a complete representation of the associations between the items. At the same time, we are aware that pitfalls in the deployment of bayesian procedures involving item imputation have been reported. For instance, Fay (1996) gives an instance where the variance of a t statistic is severely overestimated when using a particular variance estimator based on bayesian multiple imputation. The estimator was proposed by Rubin (1978, p. 76). In the same situation, the frequentist jackknife variance estimator based on hot-deck imputation, which was proposed by Rao and Shao (1992), is nearly unbiased. Fay's example reports results for a situation where the bayesian estimator is used in a manner that is inconsistent with its design. As such, this example is artificial, but it points to a legitimate concern, namely the vulnerability of model-based methodologies to environments contrary to their underlying assumptions. Although there is no way to ensure full protection against model mispecification, we believe that the judgement of the analyst should ultimately help select a model adequately robust for the survey it is intended.

Depending on the judgement of the analyst to select a model seems to us no more risky than designing a multivariate hot-deck implemented through computer specifications. Analysts working with models benefit from powerful analysis tools: the posterior and predictive distributions. To an extent, we think that the separation between the professions of programmer and statistician, as opposed to a profile integrating both occupations, makes it difficult to move away from using computer specifications as the operational language in statistical institutions. We think computer specifications are insufficient to thoroughly implement the process of statistical inference.

2.2 The 1998 Short Form

In March 1998 the Census Bureau conducted a dress rehearsal for Census 2000. Three sites were targeted for the dress rehearsal: the city of Sacramento, a rural portion of South Carolina, and a Menominee Indian reservation. The Sacramento test site is more diverse than the others, in terms of race and Hispanic origin. We selected it to experiment with two item imputation procedures. Each housing unit in Sacramento was sent a census questionnaire requesting at least six demographic items for each occupant of the housing unit. The items were tenure, race, Hispanic origin, sex, and age. A mail reply is expected from the household in each housing unit. Units for whom no mail reply is received are the objects of a non-response follow-up operation, which included a sampling procedure. After the data collection operations are completed, a substantial proportion (about 13 %) of incomplete records remains. They correspond to units who did not provide all the demographic items requested. Those items must be imputed.

The paper focuses on imputation of the household items. We define four household items characterizing each household without ambiguity. The first household item is tenure, that is the home ownership status of the household. The three other household items are defined through the householder. They are race of the householder, Hispanic origin (origin) of the householder, and sex of the householder. There is exactly one householder per household. Therefore, these items are uniquely defined for each household. It is clear why tenure is considered a household item. There are operational reasons for treating race and origin as household items. When race or origin are unreported for an entire household, the values for the race or origin of the householder are imputed through statistical procedures. Then, the values of the imputations carry to the other members of the household. This procedure does not account for mixed households, but is a reasonable approximation of the reality. Note that when race or origin are reported for at least one member of the household, then the imputation is deterministic, in the sense that the unreported items are substituted with reported items according to a predetermined hierarchy (1. substitute from the brother, 2. from the mother, etc.). The sex of the householder seldom needs to be imputed. We include it among the household variables because it interacts with them. We refer to Williams (1998) for a model-based imputation procedure to impute age and to impute sex for household members other than the householder. In the next subsection we review the imputation methodology that was used for the dress rehearsal.

2.3 The Sequential Hot-Deck

The Census Bureau uses a sequential hot-deck (Kovar and Whitridge, 1995) to process the item-imputation for the decennial census. Treat (1994) summarizes the specifications for the 1990 specifications. The sequential hot-deck (SHD) is essentially a one-pass algorithm. Except for a preliminary initialization, the SHD processes the census records only once, and imputes any unreported item on the spot by substituting the last recorded value for the item. In general, the records are sorted according to geographical proximity, and thus the last record in the census file usually corresponds to a near-by neighbor. In the 1998 version, additional constraints are imposed on the SHD through the use of class variables (Treat, 1994). For example, race is a class variable for origin. Accordingly, when a household does not report the origin item, it is borrowed from the last household of the same race who reported origin. Subject matter experts are responsible for the selection of the class variables. This version of the SHD is analogous to an unidirectional nearest neighbor hot-deck. That is, the nearest neighbor always resides before the imputed record in the order of the file.

Fay and Town (1998) suggest a rationale for relying on nearest neighbor algorithms. They mention the concept of local exchangeability, which is akin to assumptions for some non-parametric procedures. The exchangeability assumption certainly holds when the demographic composition of a population is locally homogeneous with respect to geography. For example, we expect behavior with respect to home ownership to be fairly homogeneous among neighbors. Race and origin also exhibit a degree of local homogeneity. To the extent local homogeneity does not apply, the SHD continues to borrow information from “nearest” neighbors, even though the rationale to support the scheme has vanished. In the next section we propose an item imputation procedure based on information retrieved at multiple levels of geography.

3 A Population Process

3.1 A Model for Population Counts

In order to develop an alternative item imputation procedure and to evaluate it, we design a model representing a probabilistic population process at the level of a tract. A tract is a connected geography of approximately 1700 households. The model channels information at three levels of geography: the level of the household, the level of the neighbor, and the level of the tract. The household provides information through the reported items. Neighbor and tract level information is always available. Information about the neighbor and the tract is relevant for the purpose of item imputation, since it gives a snap shot of the immediate and extended neighborhood surrounding the household.

We establish the notation. For a given tract, let $N_{ijklmno}$ be the population count for the households with tenure *i*, race *j*, origin *k*, sex *l*, tenure of the neighbor *m*, race of the neighbor *n*, and origin of the neighbor *o*. The notation for tenure is *i* = 1 if the unit of the household is owned, and *i* = 2 if it is rented. The notation for race is *j* = 1 if the race of the householder is White, *j* = 2 if the race is Black, *j* = 3 if the race is Asian, and *j* = 4 if the race is Other. The notation for origin is *k* = 1 if the origin of the householder is non-Hispanic, and *k* = 2 if the origin is Hispanic. The notation for sex is *l* = 1 if the sex of the householder is male, and *l* = 2 if the sex is female. The indices *i, j, k, l* delineate 32 categories or cells for the households. We further characterize the households in terms of the demographics of the neighbor. The "neighbor" in this case is the household preceding the referenced household in the order of the census file. Census files are sorted by block. Blocks are smaller connected geographical units. In urban settings, blocks correspond to the concept they suggest. In general, consecutive blocks correspond to contiguous geographies. Thus the neighbor usually lives in the same, or a neighboring block. The tenure of the neighbor is represented by m = 1 if the neighbor owns, and m = 2 if the neighbor rents. The race of the neighbor is given by n = 1 if the neighbor (householder) is non-Black, and n = 2 if the neighbor is Black. The origin of the neighbor is defined by o = 1 if the neighbor is non-Hispanic, and o = 2 if the neighbor is Hispanic. The notation distinguishes between 256 types of household for each tract.

Assume that M, the size of the population of households, is known for a given tract. Let $p_{i\,j\,k\,l\,m\,n\,o}$ be the inclusion probability for type $i, j, k, l, m, n, o,$ that is the probability that an arbitrary household in the tract is of that type. Then, $\{N_{i\,j\,k\,l\,m\,n\,o}\}$, the set of counts for each household type, has a multinomial distribution with M repetitions, and with probabilities $\{p_{i\,j\,k\,l\,m\,n\,o}\}$. The likelihood function is

$$L\left(\{N_{i\,j\,k\,l\,m\,n\,o}\};\{p_{i\,j\,k\,l\,m\,n\,o}\}\right) = \prod_{i\,j\,k\,l\,m\,n\,o} \left(p_{i\,j\,k\,l\,m\,n\,o}\right)^{N_{i\,j\,k\,l\,m\,n\,o}} \tag{1}$$

Initially, there is only one constraint imposed over the 256 inclusion probabilities $\{p_{i\,j\,k\,l\,m\,n\,o}\}$. They must be greater than zero and add-up to one. Our goal is to construct a meaningful model based on a small number of key parameters. We want to keep the number of parameters small in part to minimize the computer resources, but also because it permits information borrowing amongst the estimates of the 256 inclusion probabilities. The following model integrates all two-way interactions between household items and between respective items of neighbors.

$$\begin{aligned}\log\left(p_{ijklmno}\right) \;=\;& \mu + T_i + R_j + H_k + S_l + \tau_m + \rho_n + \eta_o + (T*R)_{ij} + (T*H)_{ik} + (T*S)_{il} \\ & + (R*H)_{jk} + (R*S)_{jl} + (H*S)_{kl} + (T*\tau)_{im} + (R*\rho)_{jn} + (H*\eta)_{ko}\end{aligned} \qquad \textbf{(2)}$$

The log-linear parameters with one subscript represent the main effects corresponding to the items identified by the subscript. The log-linear parameters with two subscripts represent the interactions between the two items identified by the subscripts. To prevent redundancies, we set the following constraints:

$$\begin{aligned}&\sum_i T_i = \sum_j R_j = \sum_k H_k = \sum_l S_l = \sum_m \tau_m = \sum_n \rho_n = \sum_o \eta_o = 0 \\ &\sum_i (T*R)_{ij} = \sum_i (T*H)_{ik} = \sum_i (T*S)_{il} = \sum_j (R*H)_{jk} = \sum_j (R*S)_{jl} = \sum_k (H*S)_{kl} = 0 \\ &\sum_j (T*R)_{ij} = \sum_k (T*H)_{ik} = \sum_l (T*S)_{il} = \sum_k (R*H)_{jk} = \sum_l (R*S)_{jl} = \sum_l (H*S)_{kl} = 0 \\ &\sum_m (T*\tau)_{im} = \sum_n (R*\rho)_{jn} = \sum_o (H*\eta)_{ko} = \sum_i (T*\tau)_{im} = \sum_j (R*\rho)_{jn} = \sum_k (H*\eta)_{ko} = 0\end{aligned} \qquad \textbf{(3)}$$

We will represent the statement that the inclusion probabilities jointly satisfy the constraint $\sum_{i,j,k,l,m,n,o} p_{ijklmno} = 1$, the constraints in (2), and the constraints in (3) by

$$\left\{p_{ijklmno}\right\} \in \Theta \qquad \textbf{(4)}$$

The architecture of the log-linear model defined in (2) is inspired from the same concepts motivating the SHD, since the model includes interaction effects between the items of neighboring households.

3.2 Simulating the Parameters through Bayesian Iterative Proportional Fitting

The bayesian approach puts unreported items and parameters on a par. Both need to be simulated to produce measures that we can use to evaluate the SHD and prospective competing procedures. In this subsection we set-up the stochastic environment for the parameters. In the next subsection, we present the simulation of unreported items. To stochastically describe the parameters we identify a conjugate family of prior and posterior distributions. A natural choice for a conjugate family is to set the density of a member of the family to be of the same form as the likelihood. In that spirit, Schafer (1997, p. 306) defines the constrained dirichlet conjugate family. The corresponding density defines a probability distribution on the inclusion probabilities $\{p_{i\,j\,k\,l\,m\,n\,o}\}$. The density of a general member of the constrained dirichlet family is

$$\begin{aligned} f\left(\{p_{i\,j\,k\,l\,m\,n\,o}\}\,\middle|\,\{\alpha_{i\,j\,k\,l\,m\,n\,o}\}\right) \;&=\; C\left(\{\alpha_{i\,j\,k\,l\,m\,n\,o}\}\right)\prod_{i\,k\,l\,m\,n\,o}\;\left(p_{i\,j\,k\,l\,m\,n\,o}\right)^{\alpha_{i\,j\,k\,l\,m\,n\,o}\,-\,1} \\ \{p_{i\,j\,k\,l\,m\,n\,o}\} \;&\in\; \Theta \end{aligned} \qquad \textbf{(5)}$$

$C\left(\{\alpha_{i\,j\,k\,l\,m\,n\,o}\}\right)$ is the integration constant, and $\{\alpha_{i\,j\,k\,l\,m\,n\,o}\}$ are the characteristic hyperparameters of a member of the constrained dirichlet family. Θ is defined in (4). We have $\alpha_{i\,j\,k\,l\,m\,n\,o} > 0$, for all values of the subscripts. We state a fact that will give us a stochastic environment to simulate the inclusion probabilities $\{p_{i\,j\,k\,l\,m\,n\,o}\}$ when the density is of the form in (5). The proof follows directly from the definition of the likelihood in (1), and the density in (5) (DeGroot, 1970, p.174).

Fact 1: If $\{p_{ijklmno}\}$ has the prior density in (5), then, given the likelihood in (1) and (4), the posterior density of $\{p_{ijklmno}\}$ after observing $\{N_{ijklmno}\}$ is defined by replacing $\alpha_{ijklmno}$ with $\alpha^*_{ijklmno}$ in (5), where $\alpha^*_{ijklmno} = \alpha_{ijklmno} + N_{ijklmno}$.

Fact 1 gives a characterization of the posterior distribution in terms of the density in (5), and thus the constrained dirichlet is a genuine conjugate family. However, because of the constraint in (4), the constrained dirichlet is intractable, that is $C(\{\alpha_{ijklmno}\})$ in (5) is unavailable algebraicaly. Tierney and Kadane (1986) present applications of the method of Laplace to approximate the posterior moments of intractable distributions. Thibaudeau (1988) extends the method to multimodal posterior distributions. We expect approximations obtained with the method of Laplace to be accurate in the present situation, given the large sample size. The drawback is the amount of analytical work needed to apply the method for the model in (2). In this situation, the method involves the computation of the hessian for the twenty-six parameters in (2). An available alternative is to let the computer do all the work. Gelman and Rubin (1991) propose an algorithm, bayesian iterative proportional fitting (BIPF), to simulate the parameters of a log-linear model. The algorithm is based on a conjugate family associated with the product of gamma densities. Schafer (1997, p. 308) adapts BIPF to our situation, that is to simulate the parameters of the constrained dirichlet conjugate family. In our situation, Schafer’s version of BIPF involves nine simple parameterizations. They correspond to the following nine sets of joint marginal inclusion probabilities.

$$\begin{gathered}\{p_{ij+++++}\}, \{p_{i+k++++}\}, \{p_{i++l+++}\}, \{p_{+jk++++}\}, \{p_{+j+l+++}\}, \\ \{p_{++kl+++}\}, \{p_{i+++m++}\}, \{p_{+j+++n+}\}, \{p_{++k+++o}\},\end{gathered} \qquad \textbf{(6)}$$

To produce a valid simulation of the density in (5) using BIPF, we need two additional facts. These facts apply to the parameterizations in (6). We state them in terms of the first set in (6), corresponding to tenure and race, but analogous facts hold for the other sets. We include a proof of facts 2 and 3 in Appendix A.

Fact 2: If $\{p_{ijklmno}\}$ has the distribution given in (5), then, conditional on the set of conditional inclusion probabilities $\left\{ {p_{ijklmno}} \big/ {p_{ij+++++}} \right\}$, the distribution of $\{p_{ij+++++}\}$ is an unconstrained dirichlet with posterior parameters given by $\alpha^{*}_{ijklmno} = \alpha_{ijklmno} + N_{ijklmno}$, with the density

$$f\left(\{p_{ij+++++}\} \;\middle|\; \left\{ \frac{p_{ijklmno}}{p_{ij+++++}} \right\}, \{N_{ijklmno}\}, \{\alpha_{ijklmno}\} \right) \propto \prod_{\substack{i=1,2 \\ j=1,\ldots,4}} \left(p_{ij+++++} \right)^{\alpha^{*}_{ij+++++} - 1}$$

(7)

$$p_{ij+++++} > 0 \,; \quad i = 1, 2 \,; \quad j = 1, 2, 3, 4 \,; \quad \underset{\substack{i=1,2 \\ j=1,2,3,4}}{} \; p_{ij+++++} = 1$$

Fact 3: $\{p_{ij+++++}\}$ and $\left\{ {p_{ijklmno}} \big/ {p_{ij+++++}} \right\}$ are statistically independent.

Fact 3 follows directly from fact 2, since neither the functional on the RHS of (7), nor the range of $\{p_{i\,j+++++}\}$ in (7), involve $\left\{ p_{i\,j\,k\,l\,m\,n\,o} \big/ p_{i\,j+++++} \right\}$. The three facts in this subsection are sufficient to validate a simulation of (5) through BIPF. We detail one cycle of the BIPF algorithm. For now, we assume that the algorithm has been running, and has produced at least one value for $\{p_{i\,j\,k\,l\,m\,n\,o}\}$, according to (5). We discuss choosing a starting point in the next subsection. The steps below describe the cycle of BIPF involving the set of marginal inclusion probabilities $\{p_{i\,j+++++}\}$ corresponding to tenure and race. The cycles corresponding to the other sets in (6) are similar.

Step 1

Generate $\{p^{new}_{i\,j+++++}\}$, as follows:

- Compute $N_{i\,j+++++}$ the marginal counts corresponding to tenure and race.

- Generate eight gamma random variables G_{11}, G_{12}, …, G_{24} with scale parameters $N_{11+++++} + \alpha_{11+++++}$, …, $N_{24+++++} + \alpha_{24+++++}$, where $\{\alpha_{i\,j\,k\,l\,m\,n\,o}\}$ is the set of prior hyperparameters.

- $p^{new}_{i\,j+++++} = G_{i\,j} \big/ G_{++}$, yields the new simulation of $\{p_{i\,j+++++}\}$, which has the unconstrained dirichlet in (7).

Step 2

Update the inclusion probabilities through the formula:

$$p^{new}_{ijklmno} = p^{new}_{ij+++++}\left(\frac{p_{ijklmno}}{p_{ij+++++}}\right) \qquad \textbf{(8)}$$

BIPF then repeats steps 1 and 2 to update $\{p_{ijklmno}\}$ with respect to the other sets of joint marginals in (6). When all nine sets of joint marginals have been generated, repeat the nine types of cycle. BIPF simulates the density function given in (5). Schafer (1997, p. 315) gives a heuristic argument for the convergence of the BIPF algorithm described above, but does not mention fact 3, independence, which is necessary. We include a proof of convergence in appendix B.

3.3 Simulating the Parameters and the Unreported Items

Our ultimate goal is the evaluation of the SHD and of our competing item imputation procedure, which we introduce in subsection 3.5. The tool we need for this evaluation was partly defined in the preceding subsection. In this subsection we include material that allows us to define a fully functional evaluation tool. This tool is the predictive distribution connected to the model in (2). The predictive distribution will allow us to assess the compatibility of the model with an arbitrary item imputation procedure. To simulate the predictive distribution, Schafer (1997, p. 324) further extends BIPF to data augmentation iterative proportional fitting (DABIPF). DABIPF is a cross between BIPF, and the data augmentation algorithm of Tanner and Wong (1986). DABIPF is designed to generate a Markov chain Monte-Carlo (MCMC), which simulates both the inclusion probabilities $\{p_{ijklmno}\}$ and the unreported items of the population process. DABIPF consists of cycling through the following two steps:

I Step Conditional on the current inclusion probabilities $\{p_{ijklmno}\}$, impute the unreported items for each household and update the population counts $\{N_{ijklmno}\}$.

P Step Conditional on the population counts $\{N_{ijklmno}\}$, generate a new value for the inclusion probabilities $\{p_{ijklmno}\}$ through a cycle of BIPF.

At each cycle, the P step generates the values of a set of joint marginal inclusion probabilities in (6), as explained in the previous subsection for BIPF. It takes nine I steps, each followed by a P step, before the inclusion probabilities $\left\{p_{i\,j\,k\,l\,m\,n\,o}\right\}$ are updated with respect to each set of joint marginal probabilities in (6). Before explaining how DABIPF works, we take care of some unfinished business: the starting point. For both BIPF and DABIPF, an initial value for $\left\{p_{i\,j\,k\,l\,m\,n\,o}\right\}$ is needed to initiate the algorithm. This value must satisfy (4). A sure candidate for the initial value is obtained by setting the inclusion probabilities equal to each other. Given the initial value, the I step is carried by simulating a multinomial distribution based on the imputation probabilities. We illustrate the I step in the situation where only origin may be unreported. It is easily generalized to situations involving any number of unreported items. Let $\Lambda_{i\,j\,+\,l\,m\,n\,o}$ represents the observed count of households with known value for the items corresponding to *i, j, l, m, n, o,* but with unreported origin. To carry the I step we compute the imputation probabilities from the current inclusion probabilities with the standard formula

$$\Psi^{k}_{i\,j\,+\,l\,m\,n\,o} \;=\; \frac{p_{i\,j\,k\,l\,m\,n\,o}}{p_{i\,j\,1\,l\,m\,n\,o} \;+\; p_{i\,j\,2\,l\,m\,n\,o}} \quad ; \qquad k \;=\; 1,2 \qquad \textbf{(9)}$$

We impute origin for each of the $\Lambda_{i\,j\,+\,l\,m\,n\,o}$ households by flipping a coin with probability $\Psi^{1}_{i\,j\,+\,l\,m\,n\,o}$ of a tail landing, and probability $\Psi^{2}_{i\,j\,+\,l\,m\,n\,o}$ of a head landing. If the actual landing is a tail, we substitute “non-Hispanic” for the unreported origin of the household, if the landing is a head, we substitute “Hispanic” for the unreported origin.

After going through the I step, we have a full set of population counts defined by the observations and the imputations. Then, DABIPF updates $\{p_{ijklmno}\}$ through the P step, as described for BIPF. After a burn-in period, the DABIPF algorithm reaches its stationary distribution. The stationary posterior distribution is a finite mixture of constrained dirichlet distributions as given in (5). We give examples of posterior distributions in the next subsection.

The convergence of DABIPF to the posterior distribution in (5) follows from two facts: 1. The convergence of the data augmentation algorithm. 2. The convergence of BIPF. In appendix B we define the properties of invariance and irreducibility, which guarantee convergence. We can check that the invariance property holds for DABIPF, through the data augmentation equations (Tanner and Wong, 1986). Let $\{N^{O}_{ijk}\}$ represent the observed count for tenure, race, and origin *i, j, k*, where the subscripts take the value 0, when the corresponding item is misssing, and let *U* represent the information on the unreported items. The data augmentation equations are:

$$\begin{aligned} f\left(\{p_{ijk}\}\middle|\{N^{O}_{ijk}\}\right) &= f\left(\{p_{ijk}\}\middle|\{N^{O}_{ijk}\},U\right)\, f\left(U\middle|\{N^{O}_{ijk}\}\right) d(U) \\ f\left(U\middle|\{p_{ijk}\}\right) &= f\left(U\middle|\{p_{ijk}\},\{N^{O}_{ijk}\}\right)\, f\left(\{p_{ijk}\}\middle|\{N^{O}_{ijk}\}\right) d(\{p_{ijk}\}) \end{aligned} \qquad \textbf{(10)}$$

The first integral represents the P step: DABIPF generates a new value for $\{p_{ijk}\}$ from the posterior density $f\left(\{p_{ijk}\}\middle|\{N^{O}_{ijk}\},U\right)$, obtained by completing the observed data with a simulated value for *U*. The second integral represents the I step: DABIPF generates a new value for *U* from the predictive distribution $f\left(U \,\middle|\, \left\{p_{ijk}\right\}, \left\{N_{ijk}^{O}\right\}\right)$, obtained through a simulation of $\left\{p_{ijk}\right\}$. Since both the P and I steps draw simulated values from the correct posterior and predictive distributions, the MCMC remains consistent with the original posterior and predictive at each cycle. This implies that the MCMC is invariant. The irreducibility property follows from the irreducibililty of BIPF, and the nature of the multinomial distribution. In the next subsection we give simulation results based on the 1998 dress rehearsal.

3.4 The Posterior Distribution for Tract X

We give illustrations of the posterior distribution, as approximated through 5454 cycles of DABIPF for tract “X” from the Sacramento site. We deleted the initial 54 cyles, as DABIPF stabilizes during those initial cycles. The tract contains 1583 households and exhibits typical patterns for a mixed race and mixed origin neighborhood. The posterior distribution represents our knowledge of the parameters of the model in (2), after observing the data and helps us understand the interactions at work in the data.

Tract X has four non-Hispanic female renters who did not report their race and who have a non-Black non-Hispanic renter for neighbor. Figure 1A shows the approximation for the posterior distribution of $p_{2212211} \big/ p_{2+12211}$, which is the probability a householder with these characteristics is black. In tract X, there are also four householders who did not report race and are in the same situation as the previous four, except that their neighbor is Black. Figure 1B shows the approximation for the posterior of $p_{2212221} \big/ p_{2+12221}$, the probability that a householder is black given it exhibits the characteristics of the second group. The contrast between figures 1A and 1B reflects the effects of the race of the neighbor (non-Black, Black) on the race of a household, in probabilistic terms.

Figures 2A and 2B illustrate another contrast. Tract X has two male non-Hispanic Asian householders with non-Black non-Hispanic renters for neighbors, and two male non-Hispanic Black householders with non-Black non-Hispanic renters for neighbors. These four householders did not report tenure. Figures 2A and 2B show the posterior distributions of $p_{1211211} / p_{+211211}$ and $p_{1311211} / p_{+311211}$, the probabilities of owning a home, for a householder in the first and second pair respectively. The contrast between the two posterior distributions illustrates the effect of race on the probability of home ownership. It is clear that the probability of ownership is higher for Asians than it is for Blacks. But our knowledge for the Asians is less decisive than for the Blacks, that is the posterior in 2A is flatter. The contrast between 2A and 2B is an illustration of the interaction between tenure and race. We investigate the implications of this interaction in the next section. In particular we assess the discriminatory power of the SHD in reflecting this interaction through the imputed data.

The posterior distribution is a powerfull analytical instrument and can be used to elicit an appropriate model for item impuation. However, if we have sufficient confidence in the design of the model, we want to begin the production activity and we do not need analytical feed-back at every stage of the production process. In this context, we would like an efficient procedure, which produces item imputations consistent with the model. We present a procedure of that type in the next subsection.

Figure 1A
Posterior Distribution
Prob. Household is Black when Neighbor is Non Black

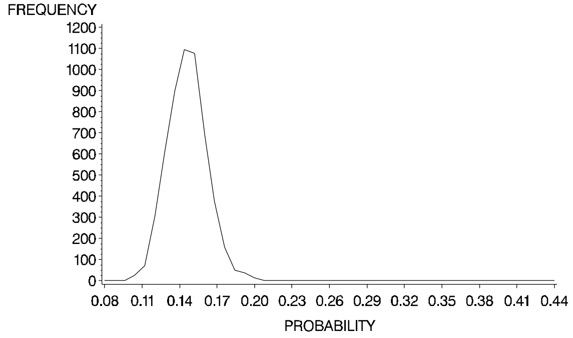


Figure 1B
Posterior Distribution
Prob. Household is Black when Neighbor is Black

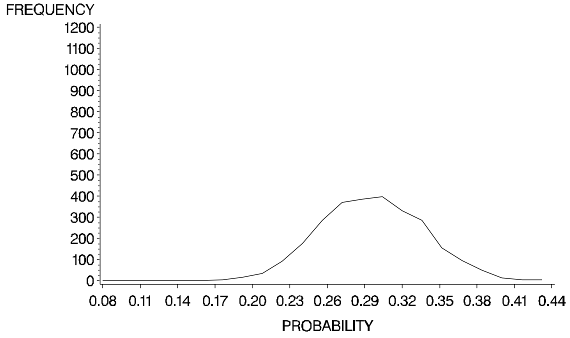


Figure 2A
Posterior Distribution
Prob. of Ownership when Household is Asian

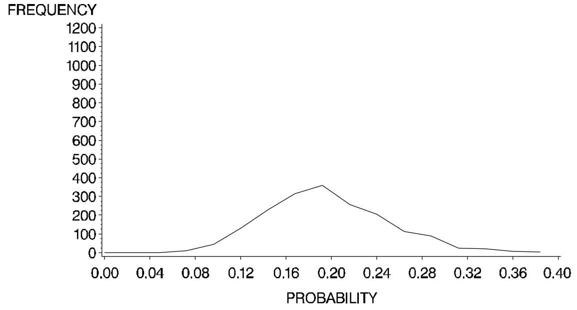
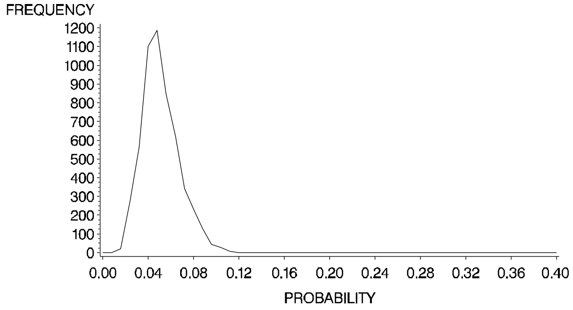


Figure 2B
Posterior Distribution
Prob. of Ownership when Household is Black



3.5 A Production-Oriented Imputation Procedure

In a production environment, if we assume that a reasonable model has been delineated, we can generate a single round of item imputation within a reasonably short period of time. We propose generating item imputations conditional on the maximum likelihood estimator (MLE) of the imputation probabilities. Thibaudeau et Al. (1997) give results using this procedure, but without error estimates. Zanutto and Zaslavsky (1997) use a similar technique, but they eliminate much of the noise of the simulation by contriving the imputed population counts to be equal to an estimate, rounded-off to the nearest integer. Reducing the synthetic noise leads to estimates that are near optimal. We suggest an alternative way to achieve that goal at the end of this subsection. However, for our present analysis, we do not attempt to control the noise associated with the simulation, because for this example, we think that it is negligible. We will see in the next section that the noise of the simulation and the variance are dwarfed by the size of some biases, at the level of Sacramento.

Our production-oriented item imputation procedure makes use of the EM algorithm (Dempster, Laird, and Rubin, 1977). The steps of our procedure are:

- For each tract, through the EM algorithm evaluate $\left\{\hat{p}_{ijklmno}\right\}$ the maximum likelihood estimator (MLE) of $\left\{p_{ijklmno}\right\}$ based on the model in (2).

- From $\left\{\hat{p}_{ijklmno}\right\}$, compute $\left\{\hat{\Psi}^{k}_{ij+lmno}\right\}$, the MLE's of the imputation probabilities $\left\{\Psi^{k}_{ij+lmno}\right\}$

computed in (9). When items other than origin are missing, the formula in (9) is adapted.

- Use the imputation probabilities with a multinomial random generator to produce an imputation for the unreported items.

This procedure is compatible with a production environment in terms of time and other resources. The M step of the EM algorithm can be implemented with a standard iterative proportional fitting routine, such as “CATMOD” in SAS. A multinomial random generator, such as “RANTBL” in SAS, is needed to generate the unreported items based on the MLE of the imputation probabilities. To control the simulation noise, a possible strategy is to repeat the last step of our procedure several times, and take the imputation yielding the population counts closest to the MLE, according to some metric. We give a full evaluation of the error associated with our procedure outlined in subsection 4.5.

4 Results for Model-Based Imputation and the Sequential Hot-Deck

In this section we present the reported results for the item imputation for the 1998 dress rehearsal of Census 2000 in Sacramento, along with an approximation of the predictive distribution obtained through DABIPF, and results for our production-oriented procedure of subsection 3.5. We had no input in the design or the implementation of the SHD used to produce the item imputations reported for the dress rehearsal. We conducted our research from files made available to us by the staff of the Decennial Management Division at the Census Bureau, after completion of the edit and imputation phase of the dress rehearsal, in January1999.

Our analysis is somewhat over-simplified as we exclude from it all the vacant housing-units. We also ignore the additional error created by the sampling for non-response operations that created virtual housing units amounting to about 2 % of all units in Sacramento. These sources of error can certainly be integrated in a more comprehensive model, but at this time we focus on evaluating the item imputation procedure while controlling for other effects.

4.1 Bayesian Measurements and Format of the Results

We begin this section with a review of our evaluation tool, the predictive distribution. We will use this tool to evaluate the counts produced with the SHD, and the counts obtained with our production-oriented procedure. For the remaining analysis, we drop the subscripts in the notation for the population count $N_{ijklmno}$, which we denote by *N*. A specific type of population will be defined explicitely if need be. We represent the entirety of the observations collected for the dress rehearsal by Ω. With that notation, the predictive mean and predictive variance of *N* are $E\left[N \mid \Omega\right]$ and $V\left[N \mid \Omega\right]$ respectively. Let $\left\{N^z\right\}$ represent the population counts generated at the *z*-th cycle of DABIPF. Then we can approximate the predicitive mean and predictive variance through

$$E\left[N \mid \Omega\right] \approx \frac{\sum_{z=1}^{Z} N^z}{Z} \quad ; \quad V\left[N \mid \Omega\right] \approx \frac{\sum_{z=1}^{Z} \left(N^z - E\left[N \mid \Omega\right]\right)^2}{Z} \tag{11}$$

These approximations are exact when *Z*, the number of DABIPF iterations, goes to infinity. Asymptotically in terms of sample size, or tract size here, the predictive mean equivalent to the maximum likelihood estimator (MLE) of the expectation of the population counts, conditional on the observations. Regardless of the convergence properties, the predictive mean is an optimal decision for the value of the population counts, under quadratic loss (DeGroot, 1970, p.228).

In Table 1 we give values obtained through DABIPF for the predictive means and predictive standard deviations of the population counts for 28 demographic categories. These values are approximations based on two implementations of DABIPF. We also give in table 1, two instances of imputations produced with our production-oriented imputation procedure of subsection 3.5, along with the corresponding reported counts (SHD). Each implementation of DABIPF generates 1800 cycles, for each of the 108 tracts in Sacramento. For each tract, the prior hyperparameters are set according to $\alpha_{i\,j\,k\,l\,m\,n\,o}$ = .01, in the prior distribution given by (5). Our intent is to have a posterior that is nearly identical to the likelihood. We discarded the generated counts and parameters for a burn-in period of 54 cycles for each implementation. We eyeballed through graphs of the population counts across cycles and the approximated predictive distribution appears stable. DABIPF stabilizes rapidly since it draws values from the exact posterior distribution, conditional on the last round of imputations, and less than 15% of the households are missing any item. In the tables, we used the formula in (11) to approximate the predictive mean. For the variance, to avoid induced correlations between the values of *N* in virtue of their contribution to the sample mean, we divided the data in ten groups of 180 realizations of the population counts. Each realization is separated from the next one in the same group by a lag of ten cycles of DABIPF. We took the sample variance for each group and then averaged the ten variances to get an estimate of the predictive variance. The standard deviations presented in the tables of this section are the square-root of those variances. It turns out that the results are almost identical to those we get if we blindly apply the variance formula in (11).

We programmed DABIPF in SAS, using the multinomial generator “RANTBL” and the gamma generator “RANGAM”. The original data file for Sacramento is about seventy megabytes. Each 1800 cycles implementation takes approximately twelve hours in CPU cycles on a SUN SPARC, generating about one-half gigabyte of data each. The output file in each case is a record of the household counts, for each pattern of reported and unreported items at each cycle. The counts are added over all tracts, for each pattern. We had SAS write all the intermediary files in a cache (/tmp), while the counts were accumulated on hard disk, at each cycle. This scheme resulted in a wall time practically equal to the CPU time.

Table 1 Sequential Hot-Deck vs. Quick Imputation and Two Predictive Distributions

Population	Reported Count: SHD And Dictionary	Production Oriented Imputation Adjusted for Diction. I^D	Production Oriented Imputation I	Predictive Mean Adjusted for the Dictionary $E\left[N^D \mid \Omega\right]$	Predictive S. D. Adjusted for the Dictionary $\sqrt{V\left[N^D \mid \Omega\right]}$	Predictive Mean $E\left[N \mid \Omega\right]$	Predictive S. D. $\sqrt{V\left[N \mid \Omega\right]}$
All	138271	138271	138271	138271	0	138271	0
White	89032	88890	88876	88927.1	36.3	88930.3	35.5
Black	19962	19937	19979	19957.0	15.6	19953.9	16.1
Asian	17405	17423	17439	17426.6	14.8	17427.1	15.2
Other	11872	12021	11977	11960.3	35.3	11959.8	32.8
Non-His.	117247	117221	117127	117232.3	10.3	117126.1	17.3
Hispanic	21024	21050	21144	21038.7	10.3	21144.9	17.3
White NH	79964	79936	79854	79934.8	15.6	79864.6	19.0
White His.	9068	8954	9022	8989.3	34.8	9065.6	35.9
Black NH	19357	19328	19344	19345.1	10.6	19335.0	11.8

Black His.	605	609	635	611.9	12.5	618.9	14.1
Asian NH.	16887	16911	16901	16909.8	10.1	16895.4	11.1
Asian His.	518	512	538	516.8	11.5	531.6	12.6
Other NH.	1039	1046	1028	1042.7	3.5	1031.1	3.9
Other His.	10833	10975	10949	10917.6	34.3	10928.7	33.0
Owner	70054	70064	70056	70021.7	42.6	70022.8	43.2
Renter	68217	68207	68215	68249.3	42.6	68248.2	43.2
White O.	47722	47776	47778	47770.8	40.6	47776.1	41.0
White R.	41310	41114	41098	41156.3	40.9	41154.1	42.1
Black O.	7661	7576	7543	7540.7	20.9	7538.7	20.2
Black R.	12301	12361	12436	12416.3	21.8	12415.2	22.3
Asian O.	9810	9848	9880	9874.1	19.0	9874.9	19.0
Asian R.	7595	7575	7559	7552.5	18.1	7552.2	19.0
Other O.	4861	4864	4855	4836.2	26.7	4833.1	26.7
Other R.	7011	7157	7122	7124.1	28.8	7126.7	29.1
NH Owner	60645	60676	60576	60615.4	38.7	60559.4	40.1
NH Renter	56602	56545	56551	56617.0	39.1	56566.7	40.3
His. Owner	9409	9388	9480	9406.3	20.3	9463.4	23.0
His. Renter	11615	11662	11664	11632.3	20.7	11681.4	23.3

4.2 Adjustment for the Dictionary of Hispanic Last Names

When setting-up our production-oriented procedure in subsection 3.5, we clearly intended it as a competitor for the SHD. To give a thorough comparative evaluation between our procedure and the SHD, in terms of population count, we must control for relative biases that are not artifacts of the methodological differences between the two procedures. In 1998, the Census Bureau developed a dictionary of last names to determine origin when unreported. The dictionary is site based. In Sacramento, out of 3934 households with unreported origin, 2604 had origin imputed through the dictionary. This means that, relative to the origin of last names in the Sacramento area, their last names had strong Hispanic, or strong non-Hispanic affiliations. The remaining 1330 households had their origin imputed with the SHD. To assess and adjust for the effect of the dictionary, we implemented two versions of DABIPF. For the first version, we ignore the imputations obtained through the dictionary and we assume that no information is available on the origin of the 3934 households who did not report it. Table 1 gives the predictive means and predictive standard deviations under this assumption. For the second version, we respect the imputation of origin based on the dictionary. In this case, we assume that only the 1330 households who were not assigned origin through the dictionary did not report it. Table 1 gives the predictive means and standard deviations under this assumption. Table 1 also gives population counts obtained through two runs of our production-oriented imputation procedure described in subsection 3.5, the first run ignoring the last name dictionary, and the second run respecting it. We give the results along with the reported counts for the 1998 dress rehearsal. The reported counts were obtained with the SHD and the dictionary. We refer to the values respecting the decision of the dictionary as being “adjusted for the dictionary”. In the next subsection we use the two versions of the predictive to assess the impact of the dictionary, and evaluate the SHD.

4.3 Imputation of Hispanic Origin with the Dictionary of Hispanic Last Names

We use the results from DABIPF to assess the magnitude of the effect of the dictionary of Hispanic last names, and account for this effect in our evaluation of the SHD. Ideally, we would use an evaluation methodology of the type discussed by Gelman and Meng (1996, p. 192). The scheme proposed by the authors is the progressive evaluation of a model through its expansion. This allows the statistician to formally update his/her posterior opinion after introducing a new variable, or a new interaction. Unfortunately, we face a "fait accompli". We do not have the information we need to enrich our model. We have an indicator corresponding to each household whose origin is imputed. The indicator simply records whether the SHD, or the dictionary of Hispanic last names is the source of the imputation for origin. We do not have information on the decision of the dictionary regarding the origin of households who reported it.

To assess the impact of the dictionary, and evaluate the SHD, we resort to a simplistic strategy. First we evaluate the relative distance between the reported population counts, and the predictive mean counts $E[N \mid \Omega]$, under the model in (2). Then we repeat the evaluation, but this time we compare the reported counts with $E[N^D \mid \Omega]$, the predictive mean counts adjusted for the dictionary. We assess the distance between the reported count and the predictive mean count in terms of the predictive standard deviation $\sqrt{V[N \mid \Omega]}$, and the predictive standard deviation adjusted for the dictionary $\sqrt{V[N^D \mid \Omega]}$, respectively. The contrast between the two distances determines our assessment of the impact of the dictionary.

Naturally, we first look at the population count for Hispanic households. We observe that the distance between the predictive mean count ($E\left[N \mid \Omega\right]$ = 21144.9) and the reported count of Hispanics (21024) is more than six standard deviations ($\sqrt{V\left[N \mid \Omega\right]}$ = 17.3), indicating a serious conflict between the model and the reported count. On the other hand, the distance between the adjusted predictive mean count ($E\left[N^D \mid \Omega\right]$ = 21038.7) and the reported count is moderate in terms of the standard deviation ($\sqrt{V\left[N^D \mid \Omega\right]}$ = 10.3). In this case the impact of the dictionary is very substantial. Table 1 reveals many conflicts between the reported count and the predictive mean for the categories involving origin. However, the adjustment for the dictionary resolves most of the conflicts. Because the adjustment lowers the predictive mean for Hispanics, the data suggests that the dictionary is deflecting a bias confounded with the value of the origin item. It appears that non-Hispanic households do not report origin in part because they are not Hispanic. Additional investigation is needed to confirm this hypothesis, but if it holds, the dictionary methodology is a breakthrough. The staff of the Population Division of the Census Bureau is responsible for the design of the dictionary of Hispanic last names. In the next subsection, we examine a situation where the reported count and the predictive mean are in conflict despite the adjustment for the dictionary.

4.4 Imputation of Tenure for Black Households

Table 1 reveals a sizable discrepancy between the predictive mean ($E\left[N \mid \Omega\right]$ = 7538.7) for the population count of Black owners, and the reported count (7661), in terms of the predictive standard deviation ($\sqrt{E\left[N \mid \Omega\right]}$ = 20.2). The situation is not alleviated by the adjustment for the dictionary of Hispanic last names. In an attempt to explain the conflict, we compare the rate of ownership for Blacks with the rate of ownership of their neighbors. Our rationale for this comparison comes from the design of the SHD. When tenure is unreported, the SHD borrows it from the neighbor, accounting for only one class variable, in this case, household size. We compute the means and variances for the rates of ownership for Blacks from the predictive distribution. Note that we need not be concerned about transforming the variables from counts to ratios.

In table 2 we observe that the reported rate of ownership for Blacks with imputed tenure (.436) is larger than the rate of ownership for Blacks with reported tenure (.379). At the same time, the rate of ownership for Blacks with imputed tenure is close to that of their neighbors (.419). This reflects the strategy of the SHD: replicate the tenure of the neighbor, borrowing the items from the nearest neighbor with a similar household size. This strategy does not adjust for race. On the other hand, the predictive mean of the rate of ownership for Blacks with imputed tenure (.375) is in agreement with the rate of ownership for Blacks with reported tenure. The conflict is serious. The SHD produced a rate of ownership more than five standards deviations (.0104) above the predictive mean rate for Blacks with imputed tenure.

The data imply that, overall, Black households own a home less frequently than their neighbors do. About 5% less often, relative to the size of the Black population. Imputation through the SHD blurs the interaction between race and tenure. We think that this discrepancy is typical of the SHD. One could argue that an additional class variable or a different sort order could solve the problem. But almost invariably, some multivariate dependencies get lost somewhere along the specification process.

Table 2 - Rates of Ownership for Black Households in Sacramento

Household Type	Number of Black Households	Ownership of the Neighbors	Ownership for the Households with Reported Tenure	Ownership for the Households with Imputed Tenure (SHD)	Predictive Mean Rate of Ownership	Predictive S.D. of the Rate of Ownership
Tenure Reported	18176	.428	.379	N /A	.378	.00035
Tenure Unreported	1786	.419	N /A	.436	.375	.010

4.5 Error Analysis for the production-oriented Imputation Procedure

In this subsection we give a full evaluation of the error associated with our production-oriented imputation procedure presented in subsection 3.5. A realization of the imputed population count *I*, simulated with our procedure based on model (2), is given in table1. A second realization, I^D, is given in table 1, and corresponds to an implementation of our procedure based on model (2) and adjusted for the dictionary of Hispanic last names. To evaluate the error involved when estimating the true population count *N* with *I*, we must account for two types of uncertainty. First, the uncertainty associated with the population process, as described by the model in (2). Second, the uncertainty associated with *I*, the outcome of our "coin flip" based on the value of $\{\hat{p}_{ijklmno}\}$, the MLE of the inclusion probabilities. Our knowledge and our level of uncertainty regarding *N* is represented by the predictive distribution approximated through DABIPF. The second uncertainty is synthetic. It is the noise associated with a multinomial random variable with constant probabilities $\{\hat{p}_{ijklmno}\}$. A measure of the total error associated with guessing *N* through *I* is the square root of the mean squared error:

$$\text{SQMSE} = \sqrt{E\left[(N-I)^2 \mid \Omega\right]} \quad = \quad \sqrt{\left(E\left[I \mid \Omega\right] - E\left[N \mid \Omega\right]\right)^2 + V\left[I \mid \Omega\right] + V\left[N \mid \Omega\right]}$$

$E\left[I \mid \Omega\right] - E\left[N \mid \Omega\right]$ is the estimation bias. It is the difference between the MLE of the population counts conditional on the observations and the predictive mean. $V\left[N \mid \Omega\right]$ is the predictive variance associated with the population process, and $V\left[I \mid \Omega\right]$ is the variance of the noise corresponding to our coin flip. Since *N* is the result of compounded uncertainties, while *I* is not, we would expect the inequality $V\left[I \mid \Omega\right] < V\left[N \mid \Omega\right]$ to hold. For evalution purposes, we generated 1800 rounds of item imputations for Sacramento with our production-oriented procedure. The results for nine categories representative of the the 28 categories of table 1 are given in table 3, along with predictive means and standard deviations obtained through DABIPF. These results correspond to estimated and imputed counts unadjusted for the dictionary in table 1. We observed that for the Hispanics, the inequality $V\left[I \mid \Omega\right] < V\left[N \mid \Omega\right]$ does not hold. Although we are not certain, this may reflect the approximative nature of our results. 1800 iterations of DABIPF may not yield enough discriminatory power when the variances are that close.

We'd like to point to the small values of the estimation bias. For all practical purposes, at the level of Sacramento, the predictive mean and the MLE of the population counts are the same. This has two implications. First, the population counts obtained with our production-oriented procedure will be centered around the predictive mean, with a variance slightly lower than the predictive variance. In other words, we are guaranteed to get imputations consistent with the model. Tables 1 and 3 are testimonies to that fact. Second, since the assymptotics are so close to reality here, we shall expect unbiased methodologies to yield nearly identical estimates. However, for the Black owners, the SHD (7661) departs severely from $E\left[N \mid \Omega\right]$ (= 7538.7) and $E\left[I \mid \Omega\right]$ (= 7539.3), while *I* (= 7543, table 1), our imputation through our production procedure, is compatible with the model. Thus either the SHD or our procedure is biased. We tend to think it is the former.

Table 3 Total Error Components for the Production-Oriented Imputations

Population Count	Predictive Mean $E[N \mid \Omega]$	Imputation Mean $E[I \mid \Omega]$	Estimation Bias	Predictive S. D. $\sqrt{V[N \mid \Omega]}$	Imputation S. D. $\sqrt{V[I \mid \Omega]}$.	Root Mean Squared Error
All	138271	138271	0	0	0	0
White	88930.3	88925.7	4.6	35.5	31.6	47.7
Black	19953.9	19955.5	-1.6	16.1	15.1	22.1
Hispanic	21144.9	21145.0	-.1	17.3	17.6	24.6
White His.	9065.6	9059.2	6.4	35.9	31.3	48.0
Black His.	618.9	621.7	-2.7	14.1	12.3	18.9
Owner	70022.8	70023.4	-.6	43.2	42.8	60.8
White O.	47776.1	47774.2	1.9	41.0	38.5	56.3
Black O.	7538.7	7539.3	-.6	20.2	19.4	28.0
His. Owner	9463.4	9461.7	1.7	23.0	21.7	31.7

5 Future Research and Conclusion

The model-based approach is very rich and we feel that we barely scratched the surface. In the context of the Decennial Census and demographic surveys, there remain several avenues of investigation that appear promising. For instance it is evident to us that the imputation of origin, assisted with the dictionary of Hispanic last names, is worth investigating. We would like to formally expand the model in (2), and evaluate the expansion, as suggested by Gelman and Meng (1996, p. 192). The expansion we have in mind is to add terms on the RHS of (2) accounting for the reliability of the dictionary. Let δ_p be the main discrimination effect of the dictionary on the population, and let $(H * \delta)_{k\,p}$ be the interaction effect between the origin of the last name and the origin of the household. We have *p* = 1, if the last name is not Hispanic, *p* = 2 if the name is Hispanic, and *p* = 3 if the origin of the name cannot be determined. *k* represents origin as before. This model directly accounts for cases where the origin of the last name does not correspond to the origin of the household.

In the paper we propose a production-oriented item imputation procedure. Our procedure has a measurable error. For instance, based on table 3, we can say that if the current demographic conditions hold, then the standard deviation of the error of our procedure for Census 2000, for the count of White households, is approximately 48. We mentioned that our current analysis is over simplified, since there are other factors affecting the error of the population counts, notably undercount and undercoverage problems. Nevertheless we feel that our work provides a rigorous basis for developing more accurate item imputation procedures with built-in self-diagnostic tools.

Appendix A

Proof of Facts 2 and 3

We prove fact 2 for a simpler log-linear model than (2), involving tenure, race, and origin only. The result generalizes for more elaborate hierarchical models. For this simpler model we have

$$\log\left(p_{ijk}\right) = \mu + T_i + R_j + H_k + (T*R)_{ij} + (T*H)_{ik} + (R*H)_{jk}$$

$$\sum_i T_i = \sum_j R_j = \sum_k H_k = \sum_i (T*R)_{ij} = \sum_j (T*R)_{ij} = 0 \qquad \textbf{(12)}$$

$$\sum_i (T*H)_{ik} = \sum_k (T*H)_{ik} = \sum_j (R*H)_{jk} = \sum_k (R*H)_{jk} = 0$$

The set of constraints on $\{p_{ijk}\}$ is the intersection of the constraints in in (12), and the constraint $\sum_{i,j,k} p_{ijk} = 1$. We express the statement that a value of $\{p_{ijk}\}$ statisfies all the constraints by

$$\{p_{ijk}\} \in \Theta^{TRH}. \qquad \textbf{(13)}$$

We show fact 2 for $\{p_{ij+}\}$, the marginal inclusion probabilities involving tenure and race. We reparametrize $\{p_{ijk}\}$ to $\{\{p_{ij+}\},\{\Phi_{ijk}\}\}$, where $\{\Phi_{ijk}\}$ is the set of conditional inclusion probabilities for origin, given tenure and race:

$$\Phi_{i\,j\,k} \;=\; \frac{p_{i\,j\,k}}{p_{i\,j\,+}} \tag{14}$$

From (12) we get

$$p_{i\,j\,+} \;=\; \left(\exp\left[\mu + T_i + R_j + (T*R)_{i\,j}\right]\right)\left(\sum_k \exp\left[H_k + (T*H)_{i\,k} + (R*H)_{j\,k}\right]\right) \tag{15}$$

$$\Phi_{i\,j\,k} \;=\; \frac{\exp\left[H_k + (T*H)_{i\,k} + (R*H)_{j\,k}\right]}{\sum_k \exp\left[H_k + (T*H)_{i\,k} + (R*H)_{j\,k}\right]} \tag{16}$$

To compute the distribution of $\{p_{i\,j\,+}\}$ conditional on $\{\Phi_{i\,j\,k}\}$, we first determine the conditional parameter space of $\{p_{i\,j\,+}\}$, given $\{\Phi_{i\,j\,k}\}$. For that purpose we reparametrize the set

$$U^1 \;=\; \left\{\{p_{i\,j\,k}\}\;\middle|\; \sum_{i,\,j,\,k} p_{i\,j\,k} \;=\; 1\right\} \tag{17}$$

to the set

$$U^2 \;=\; \left\{\{p_{i\,j\,+}\}, \{\Phi_{i\,j\,k}\}\;\middle|\; \sum_{\substack{i\,=\,1,\,2\\ j\,=\,1,\,\ldots,\,4}} p_{i\,j\,+} \;=\; 1\,;\; \Phi_{i\,j\,1} + \Phi_{i\,j\,2} = 1,\; \forall\, i,\, j\right\} \tag{18}$$

There is a one-to-one mapping between U^1 and U^2, and both sets. have 15 free parameters. Now we examine U^2 conditional on $\{\Phi_{ijk}\}$. This restricts the parameter space to a subset of

$$K = \left\{ \{p_{ij+}\}, \{\Phi_{ijk}\} \,\middle|\, {}_{\substack{i=1,2\\ j=1,\ldots,4}} p_{ijk} = 1\,;\ \Phi_{ij1} = K_{ij} \right\} \tag{19}$$

The K_{ij}'s are constants between 0 and 1 and are consistent with (15) and (16). Note that in the U^2 parametrization, the parameter space of $\{p_{ij+}\}$ conditional on $\{\Phi_{ijk}\}$ is $K \cap \Theta^{TRH}$. In addtion, note that $K \cap \Theta^{TRH} \subset K \cap U^2$. Therefore, since $K \cap U^2$ has exactly seven free parameters, $K \cap \Theta^{TRH}$ has at most seven free parameters. In other words, conditional on $\{\Phi_{ijk}\}$, Θ^{TRH} has at most seven free parameters. But, $\{T_1, R_1, R_2, R_3, (T*R)_{11}, (T*R)_{12}, (T*R)_{13}\}$ in (15) form a set of seven free parameters conditional on $\{\Phi_{ijk}\}$. These seven free parameters define the parameter space of $\{p_{ij+}\}$, given $\{\Phi_{ijk}\}$. Accordingly, the conditional parameter space of $\{p_{ij+}\}$ is defined by the constraints $p_{ij+} > 0$ for *i* = 1, 2, *j* = 1, … ,4, and the constraint ${}_{\substack{i=1,2\\ j=1,\ldots,4}} p_{ij+} = 1$.

Next we compute the joint density of $\{p_{ij+}\}$ and $\{\Phi_{ijk}\}$. Again we rely on the mapping between U^1 and U^2, as defined in (17) and (18). We compute the joint density in the U^2 parametrization. This density will give us an expression for the conditional density of $\{p_{ij+}\}$ given $\{\Phi_{ijk}\}$, up to a proportionality constant. First we substitute for $\{p_{ijk}\}$ in (5), in terms of $\{p_{ij+}\}$ and $\{\Phi_{ijk}\}$. Second, we divide by the Jacobian. Let $\alpha^{*}_{i j k} = \alpha_{i j k} + N_{i j k}$, where $\{\alpha_{i j k}\}$ is the set of prior hyperparmeters. We get

$$f\left(\{p_{i j+}\}\middle|\,\{\Phi_{i j k}\},\{N_{i j k}\},\{\alpha_{i j k}\}\right) \;\propto\; \prod_{\substack{i=1,2\\ j=1,\dots,4}} \left(\left(p_{i j+}\right)^{\alpha^{*}_{i j +}-1} \prod_{k=1,2} \left(\Phi_{i j k}\right)^{\alpha^{*}_{i j k}-1}\right) \qquad \textbf{(20)}$$

Since the parameter space of $\{p_{i j+}\}$ conditional on $\{\Phi_{i j k}\}$ does not depend on $\{\Phi_{i j k}\}$ we have

$$f\left(\{p_{i j+}\}\middle|\,\{\Phi_{i j k}\},\{N_{i j k}\},\{\alpha_{i j k}\}\right) \;\propto\; \prod_{\substack{i=1,2\\ j=1,\dots,4}} \left(p_{i j+}\right)^{\alpha^{*}_{i j +}-1} \qquad \textbf{(21)}$$

The constraints on $\{p_{i j+}\}$ are $p_{i j+} > 0$ for i = 1, 2, j = 1, … ,4, and $\ _{\substack{i=1,2\\ j=1,\dots,4}} p_{i j+} = 1$. When (13) holds the expression in (21) corresponds to the conditional distribution of $\{p_{i j+}\}$ given $\{\Phi_{i j k}\}$ and Θ^{TRH}. This shows fact 2. Moreover, since the RHS of (21) does not involve $\{\Phi_{i j k}\}$, we conclude that $\{p_{i j+}\}$ and $\{\Phi_{i j k}\}$ are statistically independent, and this shows fact 3. The decompostion in (15) and (16), and the factorization of the likelihood in (20) and (21) hold in general for any hierarchical log-linear model, in accordance with the order of the model, and thus facts 2 and 3 hold in those cases.

Appendix B

Convergence of BIPF

To show convergence we verify the convergence conditions given by Tierney (1995, p. 64): It is sufficient to show that the Markov chain Monte-Carlo (MCMC) produced by BIPF is 1. Invariant with respect to the distribution represented in (5). 2. Irreductible, that is any multivariate ball in the parameter space can be reached from any point in the parameter space. We alleviate the notation and we consider the case where the inclusion probabilities correspond to only three items. Let $\{p_{ijk}\}$ and $\{\Phi_{ijk}\}$ be as defined in (12) and (14).

Invariance: Suppose that BIPF is in a cycle updating the inclusion probabilities with respect to tenure and race. To show invariance, we assume that a realization of $\{\Phi_{ijk}\}$ through BIPF is a draw from its correct posterior distribution $f\left(\{\Phi_{ijk}\}\middle|\{N_{ijk}\}\right)$. As presented in subsection 3.2 and in virtue of fact 2, at step 1 BIPF draws a fresh value for $\{p_{ij+}\}$ from its correct posterior distribution $g\left(\{p_{ij+}\}\middle|\{N_{ijk}\}\right)$. Then, at step 2 of the cycle, BIPF produces the following tentative posterior distribution for the inclusion probabilities $\{p_{ijk}\}$:

$$h\left(\{p_{ijk}\}\middle|\{N_{ijk}\}\right) = {}_{Q}\; g\left(\{p_{ij+}\}\middle|\{N_{ijk}\}\right) f\left(\{\Phi_{ijk}\}\middle|\{N_{ijk}\}\right) \quad d\left(\{p_{ij+}\}\right) d\left(\{\Phi_{ijk}\}\right)$$
$$Q = \left\{ \{p_{ij+}\},\ \{\Phi_{ijk}\} \;\middle|\; (p_{ij+})(\Phi_{ijk}) = p_{ijk} \right\} \tag{22}$$

Now we argue that the RHS in (22) is in fact the correct posterior distribution for $\{p_{ijk}\}$. Indeed, by fact 3, $\{p_{ij+}\}$ and $\{\Phi_{ijk}\}$ are independent, and their joint posterior density factors out, as implied in the integrand. So each BIPF cycle reconstitutes the correct posterior distribution, and so the MCMC generated through BIPF is invariant under the distribution in (5).

Irreducibility: We consider the full set $\{\{p_{ijklmno}\}\}$. For any ball around $\{p^*_{ijklmno}\} \in \Theta$, we can form a chain of events, updating each of the nine set sets in (6) in turn through BIPF, that will reach the ball with a positive probability in at most nine cycles of BIPF. So the MCMC generated through BIPF is irreducible.

These results for invariance and irreducibility generalize for any hierarchichal log-linear model, and BIPF converges in those cases.

References

DeGroot, M. H. (1970). *Optimal Statistical Decisions,* McGraw-Hill

Dempster, A. P., Laird, N. M., Rubin, D. B. (1977). "Maximum Likelihood from Incomplete Data via the EM Algorithm," *Journal of the Royal Statistical Society,* Series B, Vol. 39, 1-22.

Fay, R. E., Town, M. K. (1998). "Variance Estimation for the 1998 Census Dress Rehearsal," *Proceedings of the Section on Survey Research Methods*, American Statistical Association.

Fay, R. E. (1996). "Alternative Paradigms for the Analysis of Imputed Survey Data," *Journal the American Association,* 91, 434.

Gelman, A., Meng, X. L. (1996). "Model Checking and Model Improvement," *Markov Chain Monte Carlo in Practice,* Gilks, Richardson, Spiegelhalter Ed., Chapman and Hall.

Gelman, A., Rubin, D. B. (1991). "Simulating the Posterior Distribution of Loglinear Contingency Table Models," Unpublished Technical Report, Harvard University.

Kovar, J. G., Whitridge, P. J. (1995). "Imputation of Business Survey Data," *Business Survey methods,* Cox, Binder, Chinnappa, Christianson, Colledge, Kott Ed., Wiley.

Rao, J. N. K., Shao, J. (1992). "Jackknife Variance Estimation with Survey Data under Hot-Deck Imputation," *Biometrika*, 79, 811-812.

Rubin, D. B. (1978), *Multiple Imputations for Nonresponse in Surveys*, Wiley.

Schafer, J. L. (1997). *Analysis of Incomplete Multivariate Data,* Chapman & Hall.

Tanner, M. A., Wong, W. H. (1986). “The Calculation of Posterior Distribution by Data Augmentation,” *Journal of the American Statistical Association,* 82, 528-540.

Thibaudeau, Y., Williams, T., Krenzke T. (1997). “Multivariate Item Imputation for the 2000 Census Short Form,” *Proceedings for the Section on Survey Research Methods,* American Statistical Association.

Thibaudeau, Y. (1988). *Approximating the Moments of A Multimodal Posterior Distribution with the Method of Laplace,* Department of Statistics, Carnegie Mellon University, Ph.D. Dissertation.

Tierney, L. (1996). “Introduction to General State-Space Markov Chain Theory*,” Markov Chain Monte Carlo in Practice*, Gilks, Richardson, Spiegelhalter Ed., Chapman and Hall.

Tierney, L., Kadane, J. B. (1986) “Accurate Approximations for Posterior Moments and Marginal Densities,” *Journal of the American Statistical Association*, 81, 82-86.

Treat, J. B. (1994). *Summary of the 1990 Census Imputation Procedures for the 100 % Population and Housing Items,* DSSD REX Memorandum Series BB-11, US Census Bureau.

Zanutto, E., Zaslavsky, A. M. (1997). “Modeling Census Mailback Questionnaires, Administrative Records, and Sampled Nonresponse Followup, to Impute Census Nonrespondents,” *Proceedings of the Section on Survey Research Methods,* American Statistical Association.

Williams, T. R. (1998). “Imputing Person Age for the 2000 Census short Form: A Model Based Approach”, Proceedings *of the Section on Survey Research Methods,* American Statistical Association.