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ISSUES INVOLVED WITH THE SEASONAL ADJUSTMENT OF ECONOMIC TIME SERIES

by

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Abstract:

In Part I of this paper we briefly review the history of seasonal adjustment and statistical time series analysis in order to understand why seasonal adjustment methods have evolved into their present form. This provides insight into some of the problems that must be addressed by seasonal adjustment procedures, and points out that advances in modern time series analysis raise the question of why seasonal adjustment should be performed at all. This leads to a discussion in Part II of issues involved in seasonal adjustment. We state our own opinions about the issues raised and review some of the work of other authors. First, we comment on reasons that have been given for seasonal adjustment and suggest a new possible jusitification. Then we emphasize the need to precisely define the seasonal and nonseasonal components and offer our own definitions. Finally we discuss criteria for evaluating seasonal adjustments. We contend proposed criteria based upon empirical comparisons of estimated components are of little value, and suggest that seasonal adjustment methods can be evaluated based upon whether or not they are consistent with the information in the observed data. This idea is illustrated with an example.

Keywords: seasonal adjustment, model-based seasonal adjustment, seasonality, signal extraction, time series, Census X-11

When most consumers of seasonally adjusted data — and that includes nearly every economically literate person — are confronted by the question of why they prefer such a series to the original, the most common and natural reaction is that the answer is obvious. Yet on further reflection the basis for such a preference becomes less clear, and those who give the matter extensive thought often finish by becoming hopelessly confused. Grether and Nerlove (1970 p.685)

Introduction

The impact of seasonally adjusted data upon modern U.S. society is pervasive. The Federal Reserve Board sets monetary policies based in part upon seasonally adjusted data, presidential and congressional economic policies are influenced by seasonally adjusted economic indicators, and seasonally adjusted values are routinely reported by the news media. While unadjusted figures are also published, they do not receive the attention of the adjusted data. Thus, society is conditioned to expect and even demand seasonally adjusted data.

While the public appears for the most part to be comfortable with seasonally adjusted data, we doubt that many users of this data understand the methods by which it is produced. It may be too much to expect the statistically unsophisticated person to understand the procedures underlying seasonal adjustment, but even statistical experts are often mystified by these procedures, including the most widely used method, Census X-11. It uses a set of moving averages in producing seasonally adjusted data, the basic idea of which is simple enough, but the method in which they are applied in the X-11 program is extremely complex. Also, the theoretical statistical underpinnings of X-11 and many other seasonal adjustment methods are not understood by many users. Thus, many users of adjusted data merely trust that the adjustment procedure is providing useful data, while critics have advocated the abolishment of seasonal adjustment.

The purposes of this paper are to express some of our ideas about seasonal adjustment, to attempt to clarify certain aspects of the subject, and to stimulate discussion in areas we feel need more attention. Our thinking on seasonal adjustment has been structured around three questions:

1. Why has seasonal adjustment been done in the past, and why have the current procedures evolved into their present forms?
2. Why should one do seasonal adjustment?
3. Given that seasonal adjustment is desirable, how should it be done?

In Part I of this paper we will attempt to answer the first question by giving a historical overview of developments in seasonal adjustment, and by relating these to developments in time series analysis. We shall see that seasonal adjustment was initially developed in the 1920's and 1930's as a tool for the analysis of seasonal economic time series in the absence of suitable statistical models for such series. The methods were developed empirically using tools such as moving averages. Adequate models for seasonal series were not used until the 1950's, and did not come into widespread use until after the publicaton of the time series book by Box and Jenkins in 1970, and the subsequent development of computer software for time series modeling.

In the 1950's, Julius Shiskin started doing seasonal adjustments on electronic computers at the Census Bureau, which permitted the adjustment of large numbers of time series. This also marked a transition for seasonal adjustment from a tool used by analyzers of data to a requirement of data publishers.

As time series models and related computer software have become widely used in recent years, seasonal adjusters have looked to time series modeling to solve some of the problems in seasonal adjustment. This has led to approaches such as the X-11 ARIMA method and various model-based methods that have been developed. However, considering that seasonal adjustment developed as an analysis tool in the absence of suitable models for seasonal time series, and that it is now possible to adequately model many seasonal time series, then it is not clear what is gained in general by seasonal adjustment. The use of models in connection with seasonal adjustment raises questions about whether seasonal adjustment should be done at all.

This leads us in Part II to investigate the reasons for seasonal adjustment. In our view, reasons that have been given in the past for seasonal adjustment have tended to be too vague. We suggest that consumers of adjusted data should be concerned that simplifications resulting from seasonal adjustment should not be at the expense of a significant loss of information. Seasonally adjusted data is useful to the statistically unsophisticated user only if information loss is small. We review the literature related to information loss in the seasonal adjustment process and contend that the results to date are inconclusive, and that more research into this area is desirable.

Since the question of whether or not to do seasonal adjustment is a difficult one, and since seasonal adjustment is presently a requirement of data publishers, we also consider how one should do seasonal adjustment given that it is desirable. Methods of seasonal adjustment are determined by the assumptions made, explicitly or implicitly, about the components. We thus argue that it is essential to rigorously define the components being estimated. This has not been done in the past. We present an approach to defining the components and attempt to justify our definitions. A rigorous definition of the components makes it possible to critically examine the assumptions underlying an adjustment method, and to compare the differences in assumptions for different methods.

Finally, we discuss the evaluation of seasonal adjustment procedures. Reviewing approaches that have been suggested, we argue that empirical comparisons based on criteria for a "good" adjustment are for the most part useless in evaluating competing methods. We recommend examining the assumptions underlying adjustment methods, which must remain subjective to an extent, but which can be partially checked against the data. We therefore believe the most important criterion is that a seasonal adjustment method be consistent with the information about seasonality present in the data being adjusted. We present an approach to assessing whether or not this is the case.

We emphasize to the reader that we will not attempt to answer all the questions involved with seasonal adjustment. Many of the issues involved are complex, some are nonstatistical, and there will always remain some arbitrary elements. However, we do feel that insufficient attention has been paid to several of these issues. We hope to shed new light on some of them, and perhaps most importantly, to stimulate further discussion and research ultimately leading to a better understanding of seasonal adjustment.

Preliminaries

Seasonal adjustment involves the decomposition of an observed time series, Z_t, into unobserved seasonal and nonseasonal components, S_t and N_t. The underlying decomposition is usually viewed as either additive, $Z_t = S_t + N_t$ or multiplicative, $Z_t = S_t \cdot N_t$. By taking logarithms the multiplicative decomposition becomes additive, thus for the purpose of analysis, we shall use the additive decomposition. The nonseasonal component can be further decomposed into trend and irregular components if desired, however we shall not consider this decomposition for reasons of simplicity.

Many approaches to seasonal adjustment use symmetric moving averages in estimating S_t and N_t. A symmetric moving average of Z_t (of length 2M+1) is $(2M+1)^{-1} \sum_{j=-M}^{M} Z_{t+j}$, or more generally $\sum_{j=-M}^{M} \alpha_j Z_{t+j}$ (sometimes called a weighted symmetric moving average), where $\alpha_j = \alpha_{-j}$ and $\sum_{j=-M}^{M} \alpha_j = 1$. In estimating S_t it is relevant to use seasonal moving averages which use only values of a time series for the same calander month. For t near enough to the end of the observed data so that not all of Z_{t+j} in $\sum_{j=-M}^{M} \alpha_j Z_{t+j}$ are available, either an asymmetric $(\alpha_j \neq \alpha_{-j})$ moving average is used, or the data are augmented with forecasts so that the symmetric moving average may be used.

We shall use the seasonal autoregressive-integrated-moving average (ARIMA) time series model (Box and Jenkins 1970)

$$(1-\Phi_1 B^s-\ldots-\Phi_P B^{sP})\ (1-\phi_1 B-\ldots-\phi_p B^p)\ (1-B^d)\ \ (1-B^s)^D\ Z_t =$$

$$(1-\theta_1 B-\ldots-\theta_q B^q)\ \ (1-\Theta_1 B^s-\ldots-\Theta_Q B^{sQ})\ a_t$$

$$\text{or} \qquad \Phi(B^s)\ \phi(B)\ (1-B)^d\ (1-B^s)^D\ Z_t = \theta(B)\ \Theta(B^s)\ a_t.$$

Here B is the backshift operator $(B\ Z_t = Z_{t-1})$, the seasonal and nonseasonal AR operators, $\Phi(B^s)$ and $\phi(B)$, have zeroes outside the unit circle, the seasonal and nonseasonal MA operators, $\Theta(B^s)$ and $\theta(B)$, have zeroes outside or on the unit circle, and the a_t's are independent and normally distributed with zero mean and variance σ_a^2. For short, we will write this as $\phi^*(B)\ Z_t = \theta^*(B)\ a_t$, where $\phi^*(B) = \Phi(B^s)\ \phi(B)\ (1-B)^d\ (1-B^s)^D$, $\theta^*(B) = \theta(B)\ \Theta(B^s)$. We will assume that we are dealing with monthly time series so that $s = 12$; however, our remarks apply equally well to other seasonal periods such as quarterly $(s=4)$.

When Z_t follows the ARIMA(p,d,q)x$(P,D,Q)_{12}$ model given above, its spectral density, $f_Z(\lambda)$, is given by

$$f_Z(\lambda) = \frac{\sigma_a^2 \; \theta^*(e^{i\lambda}) \; \theta^*(e^{-i\lambda})}{2\pi \; \phi^*(e^{i\lambda}) \; \phi^*(e^{-i\lambda})} \quad \lambda \; \varepsilon \; [-\pi,\pi]$$

$$= \frac{\sigma_a^2}{2\pi \; \Pi(e^{i\lambda})\Pi(e^{-i\lambda})}$$

where

$$\pi(B) = \phi^*(B)/\theta^*(B).$$

The model $\Pi(B)Z_t = \sum_{j=0}^{\infty} \Pi_j Z_{t-j} = a_t$ is the infinite autoregressive form of the ARIMA model. Strictly speaking, $f_Z(\lambda)$ above is not correct when $d > 0$ or $D > 0$, since then Z_t is nonstationary and does not have a spectral density. However, $f_Z(\lambda)$ as defined above is still useful in theoretical manipulations if one is careful to make sure the end results are correct. In particular, spectral densities defined in this way are useful in doing signal extraction, which is used in model-based seasonal adjustment. Bell (1984) discusses the assumptions under which such results are correct.

Notice that $f_Z(\lambda)$ given above is well-defined (even when $d > 0$ or $D > 0$) for all $\lambda \; \varepsilon \; [-\pi,\pi]$ except for $\lambda = 0$, and for the seasonal frequencies $\lambda = k\pi/6$ $k = \pm 1,\ldots,\pm 6$. The denominator in $f_Z(\lambda)$ is zero for these λ, and at these values we will define $f_Z(\lambda)$ to be $+\infty$.

Our use of ARIMA models in this discussion of seasonal adjustment does not imply that we could not have used other types of time series models. ARIMA models are widely used and are convenient for our purposes, but our comments would generally apply with other types of time series models. We are more interested in drawing distinctions between time series modelling and seasonal adjustment than between different approaches to time series modelling.

PART I: Historical Perspectives

To investigate the first question posed earlier, it is useful to examine the historical development of both seasonal adjustment and of time series analysis. By comparing the development of both, we can see how seasonal adjustment and time series analysis dealt with various problems presented by economic time series, and why historically seasonal adjustment might have been preferred to other methods of analysis. We shall also review model-based adjustment methods to see why empirical methods of adjustment may have been preferred to these, and to understand what recently proposed model-based methods may have to say about seasonal adjustment today.

In considering the historical development of seasonal adjustment, we must admit that tradition doubtless played an important role. Many seasonal adjusters, even to present times, may have studied unobserved components in time series because this was the traditional approach, and may not have worried about whether techniques other than seasonal adjustment might better serve their ultimate objectives. To shed some light on issues surrounding seasonal adjustment today, we will examine what options were available to early seasonal adjusters and ask how the choices made among available methodologies could have been justified, though these alternatives may not have been seriously considered by some people.

1. Historical Development of Seasonal Adjustment

This discussion of developments in seasonal adjustment concentrates on work done in the United States. This is partly justified by the fact that the Census X-11 seasonal adjustment method is today the most widely used method; therefore, it is relevant to look at the progression of events leading up to X-11, most of which took place in the U.S. BarOn (1973) and Burman (1979) discuss seasonal adjustment methods used in other countries, and Dagum (1978) and Nerlove, Grether, and Carvalho (1979) give historical discussions of seasonal adjustment from somewhat different points of view than the one given here. Pierce (1980a) discusses recent work in seasonal adjustment.

Nerlove, Grether, and Carvalho (1979) point out that the idea that an observed time series comes from several unobserved components is an old one that came originally from astronomy and meteorology and became popular in economics in England during the period 1825-1875. They also give an extensive discussion of the work of Dutch meteorologist Buys Ballot (1847), who is frequently cited as an early seasonal adjustment reference. For our purpose, it is appropriate to begin our survey somewhat later.

1920's and 1930's

There was a substantial amount of work on seasonal adjustment in the 1920's and early 1930's, much of it inspired by the work of Persons (1919). He viewed time series as being composed of (i) a long-time tendency or secular trend, (ii) wave-like or cyclical movements, (iii) seasonal movements, and (iv) residual variation. He presented a method, called the link-relative method, for isolating the components, and used detrended and seasonally adjusted data to construct business indices. For a concise description of Person's method see Persons (1923, p. 714-716). Persons was not the first to do seasonal adjustment or to specify the four basic components;[1] however, he may have been the first to come up with a method that people felt could adequately decompose economic series. At any rate his work led to an explosion of interest in seasonal adjustment.

Several important concepts regarding seasonal components and adjustment became fixed in the 1920's and early 1930's. These included (i) the idea that seasonality changes over time, (ii) the need to account for trends and cycles when estimating the seasonal component, (iii) the impossibility of describing trends and cycles by explicit mathematical formulas, and (iv) the need to deal with extreme observations.

Changing seasonality was noted as early as 1852 by Gilbart (1852, quoted by Kuznets 1933), who found it in the circulation of bank notes. Persons (1919, p. 19), observed that, "Although we wish to ascertain if a systematic variation exists it is not accurate to think of seasonal variation (or, for that matter, the other types of fluctuations) as being exactly the same year after year." However, Persons used fixed seasonal factors when adjusting, probably because he did not see a convenient way to produce varying seasonal factors. According to King (1924), the first to adjust data with varying seasonal factors were

Sydenstricker and Britten of the U.S. Public Health Service, while investigating causes of influenza. Their graphical method is briefly described in Britten and Sydenstricker (1922). King (1924) modified Sydenstricker and Britten's method, retaining some graphical elements, but also using moving medians (taking the median of successive sets of 2M+1 data points) and reemphasized the need to account for changing seasonality. Snow (1923) suggested fitting straight lines to each quarter (or month) separately, and checking for varying seasonality by examining the lines to see if they were parallel. Crum (1925) gave a general discussion of varying seasonality and modified Person's link relative method to handle changing seasonality. Other methods of dealing with changing seasonality were suggested by Hall (1924), Gressens (1925), Clendenin (1927), and Joy and Thomas (1928). Kuznets (1932) suggested a method to detect and adjust for changes in seasonal amplitude from year to year assuming the seasonal pattern remained constant. Mendershausen (1937) reviewed efforts made to that time to deal with changing seasonality.

The early writers discovered it was necessary to adjust data for the effects of trend before, or at the same time as, estimating the seasonal.[2] We will refer to this problem as nonseasonal nonstationarity. Several different approaches to this problem were used. Some authors made simple transformations of the data to remove trend, then obtained seasonal estimates and converted these to estimates of seasonal effects in the original series. In this group Persons (1919) took the ratio of each monthly value to the preceeding value ("link relatives"), and Robb (1929) took second differences of the original data. Other authors estimated trend first and then removed it, usually by division, i.e., $Z_t/\hat{T}_t$ - the "ratio to trend" approach. Here Falkner (1924) used a straight line trend, King (1924) a trend curve drawn freehand, and Joy and Thomas (1928) and Macauley (1931) used moving average trend estimates ("ratio to moving average" method). Carmichael (1927) suggested a hybrid approach, taking first or second differences of the ratio of the data to a trend estimate. Finally, some authors (Snow 1923, Clendenin 1927) estimated the trend separately for each series of values for a particular calendar month to simultaneously get at both trend and seasonality.

Although there was initially some use of specific trend functions such as the linear trends of Snow (1923) and Falkner (1924) mentioned above, by the 1930's it was generally felt that one should not specify a functional form for the trend. The prevailing attitude was reflected by Macauley (1931 p. 38): "The type of smooth curve which might be expected to appear in any particular time series if the series were unaffected by the minor or temporary factors which give rise to seasonal and erratic fluctuations is not necessarily representable throughout its length by any simple mathematical equation." Thus, it was natural for Macauley and others to consider using moving averages and actuarial graduation formulas to obtain trends, rather than using explicit functions of time.

Finally, there was concern about the influence of extreme observations. For example, Falkner (1924, pp. 168-169) objected to the use of monthly means in seasonal adjustment primarily for this reason, stating that, "The arithmetic average is peculiarly subject to extreme items, and it is for that reason that a monthly seasonal index obtained by this method may be governed more by an exceptional irregular deviation than by the systematic seasonal movement." Concern about the effects of outliers led Persons (1919) and others to use medians instead of means in deriving seasonal factors (some replaced moving averages by moving medians). Crum (1923a) suggested using medians or trimmed means, and Falkner (1924) and Joy and Thomas (1928) also advocated the use of trimmed means. These trimmed means involved considerable trimming, the mean being computed using as few as two or three observations. Although the need to deal with extreme observations was established early, the problem of how to do it has continued to the present day.

Impact of Computers on Seasonal Adjustment

The next major development in seasonal adjustment did not come until 1954 when Julius Shiskin started doing seasonal adjustments (Method I) on the Univac 1 computer at the Census Bureau (see Shiskin 1957 and 1978). Method II was introduced in 1955, with successive variants continuing through the development of X-11 in 1965 (Shiskin, Young, and Musgrave 1967). Soon after Shiskin's efforts in 1954, other organizations in the U.S. and abroad began using the Census method or developing their own computer methods. As a result of the interest in doing seasonal adjustment on electronic computers, in 1960 a conference on the subject was held in Paris (O.E.C.D. 1960).

One of the objectives in doing seasonal adjustment on computers was to increase the number of series that could be adjusted. Shiskin (1957, p. 245) states that in 1954, "Principal users of current economic series -- for example, the chairman of the Council of Economic Advisers and the chief economist of the National Industrial Conference Board — complained that many of the monthly series published by the government were not adjusted for seasonal variations at all; that many others were adjusted by crude methods; and that for still others the seasonal adjustments did not reflect the most recent experience." He further notes that this was, ". . .attributable primarily to the huge amount of computation required and to the large costs involved," and that, "The large-scale digital electronic computer has brought an end to this situation." With electronic computers literally thousands of time series could be seasonally adjusted by government agencies. This had important implications for the procedures that were developed. The calculations required could now be complicated, but the amount of time that could be spent in determining how best to adjust each particular series was reduced. This was something of a reversal of the situation prior to computers. The adjustment methods that were developed (including X-11) were basically complex modifications of previously used methods that attempted to incorporate automatically, at least to a degree, the professional judgment that was previously required. This helped lend an air of objectivity to the seasonal adjustment process, so that seasonal adjusters would not be accused of tampering with the data, a consideration that has become even more important in recent years. In this respect, the situation today is much different from the 1920's when some people advocated free hand smoothing (e.g., King 1924) as part of their adjustment method.

Another important development in seasonal adjustment methodology made easy by computers was the use of regression techniques to account for trading day variation. Important work on this was done by Eisenpress (1956), Marris (1960), and Young (1965), whose approach was incorporated into the X-11 program. Before this work adjustments for trading day effects were generally based on a priori evidence or opinions about the proportion of activity occurring on each day of the week. Young (1965) discusses some of the difficulties with such an approach. Holiday effects are important in some series and have been considered for many years: see, for example, Joy and Thomas (1927) and Homan (1933). However, even today adjustments for holiday effects tend to be made on an ad-hoc basis, although recently Hillmer, Bell, and Tiao (1983) have suggested a modeling approach to dealing with holiday effects in seasonal adjustment.

Recent Developments

In recent years there have been many attempts to improve the seasonal adjustment process. The most important recent development is the X-11 ARIMA method of Dagum (1975), which involves forecasting the data one year ahead using an ARIMA model. The forecasted values are used as if they were actual data so that the filters used in adjusting current data are closer to the symmetric filter that will eventually be used when more data are available. Similar approaches using autoregressive models have been investigated by Geweke (1978a) and by Kenny and Durbin (1982). The idea of forecasting the series for this purpose is not new; it was recommended by Macauley (1931, p. 95-96). Statistics Canada and the U.S. Federal Reserve Board use X-11 ARIMA. Also, in the U.S. the Bureau of Labor Statistics uses it on many of their series and the Bureau of Economic Analysis on some of their series. It remains to be seen what action the Census Bureau will take. Eventually (typically after 3 years) the X-11 ARIMA adjustments converge to the X-11 adjustments, so that discussion of the characteristics of X-11 is relevant to X-11 ARIMA as well.

2. Historical Development of Time Series Analysis and its Relation to Seasonal Adjustment

In considering historical developments in time series we are interested in the question of why people used seasonal adjustment as an analysis technique rather than other time series methods. The developments mentioned here were chosen with this in mind. We concentrate on relevant developments in time series modeling, but also mention some important developments in spectral analysis and signal extraction. In reviewing the history of time series analysis it is useful to keep in mind the following essential problems presented by data being seasonally adjusted: (i) changing seasonality, (ii) nonseasonal nonstationarity (trends and cycles), (iii) the impossibility of describing seasonality, trends, and cycles by simple mathematical functions of time, and (iv) outliers.

Time Series Modeling

The first important developments in time series modeling were the introduction in 1927 of autoregressive models by Yule and of moving average models by Slutsky. Yule (1927) discussed properties of autoregressive models, introduced partial autocorrelations, and fit low order models to Wolfer's annual sunspot series by least squares. In his 1927 paper, Slutsky (see 1937 translation) introduced moving average models and investigated how these models could lead to cyclical series. Wold (1938) was the first to fit moving average models to data. He also introduced the important innovations representation for stationary series and solved the prediction problem.

During the 1940's progress was made in the area of inference for time series models. Mann and Wald (1943) derived asympotic theory for parameter estimation in autoregressive models. Champernowne (1948) suggested the use of least squares estimates for autoregressive models and autoregressive models with regression terms, although he did not derive properties of the estimators. Cochrane and Orcutt (1949) suggested autoregressive filtering or differencing of the dependent and independent variables when using a regression model with autocorrelated errors. The asymptotic theory for sample autocorrelations was developed by Bartlett (1946) and Moran (1947).

Whittle (1952) seems to have been the first to use high lags in time series models to account for seasonality. Using a model discrimination procedure he arrived at a model of the form $Z_t - \phi_1 Z_{t-1} - \phi_2 Z_{t-2} - \phi_8 Z_{t-8} = a_t$ for the Beveridge wheat price series. In Whittle

(1953a, 1954a) he used the model $Z_t - \phi_1 Z_{t-1} - \phi_{22} Z_{t-22}$ for 6 month sunspot data with a cycle of 22 periods. About the same time, Whittle (1953a, 1953b) derived properties of approximate maximum likelihood parameter estimates for a general model that includes autoregressive-moving average models as a special case. He then (Whittle 1954b) obtained results for simultaneous estimation of regression and time series parameters. In an effort to find simpler procedures than Whittle's, Durbin suggested another approach and obtained results for moving average models (1959), models with regression terms and autoregressive errors (1960a), and mixed autoregressive-moving average models (1960b). Walker suggested still another approach and obtained results for moving average (1961) and autoregressive-moving average (1962) models.

More recently, the publication of the book by Box and Jenkins (1970) and the development of suitable computer software has led to a growing popularity and widespread use of ARIMA models in the analysis of time series data. ARIMA models use nonseasonal and seasonal differencing to deal with nonseasonal and seasonal nonstationarity. While differencing had been suggested many years before in other contexts by Carmichael (1927), Robb (1929), and many others (e.g., the literature on the "variate-difference method," see Tintner 1940), and seasonal differencing was even considered by Yule (1926), Box and Jenkins popularized it as part of a modeling procedure for nonstationary series. Also, for ARIMA models to be useful in the analysis of seasonal time series, lags as high as the seasonal period are needed. Other than Whittle's attempts in the 1950's, this type of model was not widely used. Box and

Jenkins' introduction of the multiplicative seasonal model was important because it provided a representation involving relatively few parameters which was a good approximation for many seasonal time series.

Finally, approaches to handling outliers when modeling time series have been presented by Fox (1972), Abraham and Box (1979), Denby and Martin (1979), Martin (1980), and Chang (1982) (see also Hillmer, Bell, and Tiao 1983). For outliers with an assignable cause the intervention analysis of Box and Tiao (1975) is relevant. Historically, outliers would have presented more of an obstacle to time series modeling than they do today, although for series with no serious outliers this would not have been a problem. Still, more work needs to be done on outliers both for time series modeling and seasonal adjustment.

Spectral Analysis

Spectral analysis actually became available before time series modeling and the work on seasonal adjustment discussed earlier, with the introduction of the periodogram by Schuster (1898). Since spectral analysis can be used to look for periodic components in time series it would seem to be useful to investigators of economic cycles. Beveridge (1921, 1922) in fact used the periodogram to look for cycles in a de-trended series of wheat prices. Fisher (1929) suggested a significance test for detecting periodicity in a time series. Daniell (1946), Bart-lett (1950), and Tukey (1950) suggested smoothed periodogram spectral estimators, and many other spectral estimators have been developed since then. Also, spectral analysis has become more practical in recent years with the advent of electronic computers and improved computational techniques, especially the Fast Fourier Transform (Cooley and Tukey, 1965). Spectral analysis for nonstationary time series has been investigated by Priestly (1965) and Hatanaka and Suzuki (1967). For a more extensive historical survey of spectral analysis see Robinson (1982).

Despite Beveridge's work, spectral analysis was not widely used on economic time series in the early days of seasonal adjustment. One problem, as noted by Kendall (1945), was that people used the periodogram to look for exact periodicities but economic cycles are not exactly periodic. This problem was overcome with the development of improved spectral estimators and a better understanding of spectral analysis. A more permanent problem was identified by Crum (1923b) who criticized use of the periodogram on economic series saying that seasonality influences the appearance of cycles in the periodogram making them more difficult to detect (Crum advocated seasonal adjustment). In modern terms this is known as "leakage." A typical approach today to doing spectral analysis with seasonal series is to remove or reduce the seasonal effects by prefiltering the data, which leads right back to seasonal adjustment.

Signal Extraction

The signal extraction problem is to estimate the signal S_t in $Z_t = S_t + N_t$ when the observations Z_t contain "noise" N_t. Kolmogorov (1939, 1941) and Wiener (1949) independently solved this problem for stationary time series, obtaining $\hat{S}_t$ to minimize $E[(S_t - \hat{S}_t)^2]$ for any linear function, $\hat{S}_t$, of the observations Z_t. Hannan (1967), Sobel (1967), Cleveland and Tiao (1976), and Bell (1984) have extended this result to nonstationary time series. Identifying S_t and N_t as the seasonal and nonseasonal components, signal extraction can be used, in conjunction with suitable models for Z_t, S_t, and N_t, to do seasonal adjustment. This approach has been taken in recent years by a number of authors, whom we discuss in the next section.

3. Model-Based Seasonal Adjustment

Some early authors criticized the popular empirical approaches to seasonal adjustment. For example, Snow (1923, p.334) criticized the approach of Persons by saying, "The method of allowing for seasonal variations seems cumbersome and the logic of it is not clear." Also, Fisher (1937, p.179) said "To the student of mathematics it appears strange that economists and statisticians have adopted such rather primitive methods in measuring seasonal variations when, as a matter of fact, more elegant and also more practical mathematical tools, requiring a far smaller amount of tedious arithmetical calculations than the methods of the gifted academic schoolmen, have been available for more than half a century." (The more elegant and more practical tools he refers to are the orthogonal polynomials of J.P. Gram and the quasi-systematic error theory of T.N. Thiele.) This dissatisfaction with the empirical nature of many seasonal adjustment methods led these and later authors to investigate the use of time series models to do seasonal adjustment. We shall refer to such methods of seasonal adjustment as model-based methods.

Model-based methods of seasonal adjustment generally use an additive decomposition, $Z_t = S_t + N_t$, or an additive decomposition for some transformation of Z_t (such as $\ln Z_t$), and use explicit statistical models (or spectral densities) for Z_t, S_t, and N_t. The model for Z_t can be estimated from observed data, but since S_t and N_t cannot be observed their models depend on more or less arbitrary assumptions (see Part II of this paper). The various methods differ in the type of model fit to the observed Z_t's and in the assumptions used in specifying models for S_t and N_t. S_t and N_t are estimated either directly when fitting the model for Z_t (as in regression methods), or after fitting the model for Z_t using signal extraction theory.

Regression methods provided the first model-based approaches to seasonal adjustment. The basic approach consists of specifying functional forms for the trend and seasonal components which depend linearly on some parameters, estimating the parameters by least squares, and subtracting out the estimated seasonal component. The most popular specifications use polynomials in time for the trend component and seasonal means for a stable seasonal component (with modifications to handle changing seasonality). The error terms are generally assumed to be white noise, although Rosenblatt (1965) points out that the regression residuals tend to be autocorrelated and this should be allowed for.

Regression methods of seasonal adjustment have been proposed by Hart (1922), Snow (1923), Fisher (1937), Mendershausen (1939), Cowden (1942), Jones (1943), Hald (1948), Eisenpress (1956), Hannan (1960, 1963), Lovell (1963, 1966), Jorgenson (1964, 1967), Rosenblatt (1965), Henshaw (1966), and Stephenson and Farr (1972). These efforts seem to have had little effect on the way U.S. government agencies do seasonal adjustment. It may be that the regression approach was doomed from the start since it requires explicit specification of the mathematical forms of the trend and seasonal components. We have indicated that as early as the 1930's seasonal adjusters felt that this could not be done effectively.

Recently there has been considerable interest in using either stochastic models or spectral estimates to do seasonal adjustment by signal extraction. The first such model-based approach to seasonal adjustment was that of Hannan (1964), who filtered the data to remove trends and chose a model for the seasonal component consisting of trigonometric terms at the seasonal frequencies multipled by independent time series following first order autoregressive models. These models were stationary, but the approach was extended to nonstationary (random walk) models by Hannan (1967) and Hannan, Terrell, and Tuckwell (1970), where the approach is described in detail (see also Sobel (1971)). The method required ad-hoc specification of the relative magnitude of the seasonal and nonseasonal spectral densities near the seasonal frequencies.

Methods based on spectral estimation have been suggested by Melnick and Moussourakis (1974) and Geweke (1978b). Melnick and Moussourakis estimated the spectrum of the data after detrending it with a least squares straight line, and then empirically determined neighborhoods of the seasonal frequencies that they assumed contained all the seasonal power. They used spectral ordinates outside these neighborhoods in estimating the (detrended) nonseasonal spectrum within the neighborhoods, and thus obtained their seasonal adjustment filter. Geweke estimated the spectrum of the original data at the seasonal frequencies by the periodogram ordinates, and at other frequencies by smoothing the periodogram while leaving out the seasonal ordinates. The spectrum of the nonseasonal was estimated by smoothing the periodogram with ordinates at and near the seasonal frequencies left out. He also used this approach with spectral density matrices to do multivariate seasonal adjustment via multivariate signal extraction - simultaneously seasonally adjusting several time series.

Several authors have suggested seasonal adjustment methods which involve fitting an ARIMA model (possibly with deterministic terms) to Z_t, and using this along with some assumptions to determine models for S_t and N_t. Pierce (1978) suggested using ARIMA models and deterministic terms to allow for both stochastic and deterministic trends and seasonality. After estimating and removing the deterministic effects, he filtered the resulting series (e.g., by differencing) to remove stochastic trends and specified a seasonal ARMA (1,1) model for the filtered stochastic seasonal component when stochastic seasonality was present. This model was identified using assumptions including one that the variance of the seasonal be the minimum value consistent with the model. Wecker (1978) suggested an extension to Pierce's approach. Box, Hillmer, and Tiao (1978) started with the model $(1-B)(1-B^{12})Z_t = (1-\theta_1 B)(1-\theta_{12} B^{12})a_t$ and derived models for the seasonal, trend, and irregular components consistent with this overall model, using certain assumptions including one that the variance of the irregular component should be maximized, which minimizes the variance of both the seasonal and the trend. This approach was later extended to more general ARIMA models by Burman (1980) and by Hillmer and Tiao (1982), who discuss some properties of the approach (see also Hillmer, Bell, and Tiao (1983)). Cleveland (1979) fit ARIMA models to the observed data after removing seasonal means and used simple ARIMA models for the components. He chose their moving average parameters to try to make these models approximately consistent with the model for the original series (the autoregressive parameters are determined by assumptions).

The preceeding methods all involved determining ARIMA models for the components and then using signal extraction theory to estimate them. Brewer, Hagan, and Perazelli (1975) took a different approach, fitting an ARIMA model to Z_t, and then decomposing interpolated values of Z_t (estimates of Z_t using the data other than the observation at t) into seasonal and trend-cycle components. This was done by considering a seasonal-trend-cycle-irregular decomposition of the filter that produces one-step-ahead forecasts. A modification of this approach was later suggested by Brewer (1979). Roberts (1978a) suggested a related method where part of the fitted ARIMA model is identified as a seasonal adjustment filter.

The final model-based approach we shall mention involves specifying parametric models for the components, which leads to a model for Z_t subject to constraints. Estimating the model for Z_t subject to the constraints also yields models for S_t and N_t, which can then be used to do seasonal adjustment by signal extraction. Engle (1978) used ARIMA models for the components, but found estimation of the model for Z_t subject to the constraints to be computationally burdensome, so he relaxed some of them. Others used models for the components that made the constrained estimation somewhat simpler. Abrahams and Dempster (1979) used fractional Brownian motion for the trend component and a modification of this for the seasonal component. Fractional processes generalize the idea of differencing a time series to stationarity, thus providing a generalization of ARIMA models - see Granger and Joyeux (1980) for a discussion. Akaike (1981) took a smoothness priors approach (related to that of Schlicht (1981)) which led to ARIMA type models for the components, and used an information criterion to select from among alternative models. Kitagawa and Gersch (1983) further developed this approach, extending it to allow a wider variety of ARIMA type component models.

4. Summary and Conclusions

Seasonal adjustment originally developed in the early part of this century out of a tradition of looking for unobserved components in time series. Early seasonal adjusters found that their time series contained nonstationary trends and changing seasonality, and that this behavior could not be described by explicit mathematical functions of time. They empirically developed seasonal adjustment methods using such tools as moving averages to deal with these problems. Some early authors criticized the empirical nature of the early adjustment methods. However, time series models capable of dealing with the series being adjusted were not available at that time, thus, early attempts at modeling and model-based adjustment failed.

In the 1950's Whittle began using models suitable for the sort of time series being seasonally adjusted. Widespread use of such models followed the publication of the book by Box and Jenkins in 1970. While these models were being developed, government agencies started using electronic computers to seasonally adjust large numbers of time series. This made model-based methods impractical by comparison, at least until the recent development of computer software for use in modeling time series.

Whereas seasonal adjustment was originally done as part of the analysis of time series data by statisticians and economists, computerized seasonal adjustment has come to serve the needs of political officials, business managers, and journalists - largely a statistically unsophisticated group with little interest in time series modelling. Also, the responsibility for performing seasonal adjustments has shifted from the analyzers of the data to the publishers of the data.

In recent years with the further development of time series models and associated computer software, seasonal adjusters have looked to time series models to improve seasonal adjustment methods. Examples are the X-11 ARIMA method, which is now being used by several government agencies, and the recently proposed stochastic model-based methods. However, we shall see in the next section that if one can model a time series, then it is not clear what is gained by arbitrarily decomposing the series into seasonal and nonseasonal components. Thus, the use of modeling in connection with seasonal adjustment raises the basic question of why seasonal adjustment should be done at all.

Part II Current Issues in Seasonal Adjustment.

While seasonal adjustment has become a well-established practice for historical reasons discussed in part I, we feel it is time to take a fresh look at seasonal adjustment and seasonal adjustment methods. Thus, in part II, we will address the second and third questions listed in the Introduction - those regarding the why and how of seasonal adjustment today. We will not dwell on technical details but rather hope to stimulate discussion about some of the broader issues. We will express some of our own opinions about the issues raised and attempt to provide the reasoning which shaped our opinions. Our hope is not that everyone will agree with our opinions, but rather that readers will see that there are a number of important issues which require extensive thought and discussion before they can be satisfactorily resolved.

5. Reasons for Seasonal Adjustment

In Part I, we noted that seasonal adjustment was developed in the 1920's and 1930's as a tool for analyzing seasonal economic time series in the absence of suitable statistical and economic models for such series. In recent years, as new modeling procedures have become available, the reasons for doing seasonal adjustment have become less clear. Reasons that have been given for seasonal adjustment have typically been rather vague, but seem to follow three main themes: (i) to aid in doing short term forecasting, (ii) to aid in relating a time series to other series, external events, or policy variables, and (iii) to achieve comparability in the series values from month to month.

Shiskin (1957, p. 222) argues that adjusted data are useful in short term forecasting when he says

> A principal purpose of studying economic indicators is to determine the stage of the business cycle at which the economy stands. Such knowledge helps in forecasting subsequent cyclical movements and provides a factual basis for taking steps to moderate the amplitude and scope of the business cycle.

He goes on to say that knowledge of the seasonal pattern in sales of products ". . . is needed by all companies to determine the level of production that is most efficient . . ." and suggests forecasts of a series can be obtained by taking forecasts of annual totals and allocating these to months in proportion to the seasonal factors. Burman (1980) says that the most common purpose of seasonal adjustment ". . . is to provide an estimate of the current trend so that judgmental short-term forecasts can be made."

Several authors have argued that seasonal adjustment is useful because seasonality in a series can obscure the relationships between the time series and other series, external events, or policy variables. It is hoped seasonal adjustment will make these relationships easier to investigate, and in the case of relationships with policy variables, make them easier to exploit. With regard to using adjusted data in relating several series, Burman (1980) says that seasonal adjustment, ". . . may be applied to a large number of series which enter an economic model, as it has been found impracticable to use unadjusted data with seasonal dummies in all but the smallest models . . ." Also, Granger (1978a) sees a possible advantage in that, "By using adjusted series, one possible source of spurious relationship is removed." An example of the use of seasonally adjusted data to examine the effect of external events on a series is provided by BarOn (1978), who relates several seasonally adjusted economic series to unusual external events. Finally, governments use seasonally adjusted data in setting policy variables designed to control various aspects of their economies. According to Dagum (1978, p. 10), "The main causes of seasonality, the climatic and institutional factors, are exogenous to the economic system and cannot be controlled or modified by the decision makers in the short run." Thus, the nonseasonal component may be what can be controlled, to some degree, by government intervention, and so seasonally adjusted data are useful because they ". . .provide the basis for decision making to control the level of the economic activities," (Dagum, 1978, p. 14). However, note that for some series, seasonality may also be controllable. For example, the Federal Reserve Board has effectively removed seasonality from interest rates through monetary policy.

The third reason given for seasonally adjusting data is that it makes values comparable from month to month. This may be true, but do we really want comparability, or should observations for different months be regarded differently? For instance, atmospheric temperature data are highly seasonal, but people seem comfortable with the original data. We suspect the desire for comparability has something to do with the two points discussed above — forecasting series, and relating series to other series, external events, or policy variables.

Justification for Signal Extraction

Seasonal adjustment may be viewed as a signal extraction problem. In both cases we observe $Z_t = S_t + N_t$, where S_t and N_t are unobserved components we wish to estimate using the observed series Z_t. In signal extraction S_t and N_t are "signal" and "noise", while in seasonal adjustment they are "seasonal" and "nonseasonal." Z_t can be a transformation of the original series, such as the logarithm, in which case we can view the decomposition as multiplicative. To put the issues regarding justification of seasonal adjustment in perspective, let us consider how one might justify doing signal extraction in general. That is, if we observe Z_t, why should we try to estimate S_t and N_t? To answer this in any given situation, we must consider three basic questions:

(1) Is there reason to believe the observed data Z_t are generated as $Z_t = S_t + N_t$?

(2) Given $Z_t = S_t + N_t$, are we really interested in S_t and N_t, rather than Z_t or something else related to Z_t?

(3) Given that we are interested in S_t and N_t, how can we estimate them?

For signal extraction to be appropriate, we must be able to adequately answer these three questions. In conection with the third question, it should be noted that standard signal extraction results on estimating the components require that the models for S_t and N_t be known.

Much of the original motivation for studying signal extraction came from problems in the field of communications engineering. In this field there are physical reasons which imply $Z_t = S_t + N_t$ (see for example, Section 7 of Chapter 13 in Blanc-Lapierre and Fortet, 1965). Here S_t is an emitted signal, and the received signal, Z_t, is corrupted by noise, N_t. The problem is to produce an estimate, $\hat{S}_t$, as close as possible to the emitted signal S_t by attempting to remove the noise N_t. It is obvious that in communications engineering (1) the decomposition $Z_t = S_t + N_t$ makes sense and (2) the interest is in the signal S_t rather than the observed data Z_t. Furthermore, Yaglom (1962, p. 127) notes

i. the model for Z_t is calculated (or estimated) from observed data

ii. the model for N_t ". . . can be determined by using the same measuring device and the same observer . . . to make a series of measurements of any quantity whose value is known precisely, e.g., which equals zero because of the conditions of the experiment."

Thus, the models for Z_t and N_t, and hence for $S_t = Z_t - N_t$, can be obtained, so that standard signal extraction results can be used to estimate the components. Therefore, the use of signal extraction methods in communications engineering is sensible.

Consider now how seasonal adjustment of economic time series fits into the framework of the three questions. Question (1) can always be answered affirmatively, in that mathematically the decomposition $Z_t = S_t + N_t$ is always possible. Whether or not S_t was physically generated this way, by certain economic forces generating S_t and N_t separately and then combining them (additively or otherwise) to get Z_t, is another question. Some writers have regarded the seasonal, trend, and irregular components as arising from different economic factors. In particular, Mendershausen (1937, 1939) advocated this point of view and attempted to model seasonality in terms of meteorological and social variables. Factors generating seasonality in financial data are discussed in Board of Governors of the Federal Reserve System (1981). Trading day adjustments, as done today, provide a causal explanation for some of the seasonality in economic series. Also, the idea that the nonseasonal component is subject to control through manipulation of policy variables while the seasonal component is not, relates to the idea of S_t and N_t being generated separately. However, today little emphasis is placed on physical causes when adjustment is actually done, so without physical justification we view the decomposition as a mathematical one.

For seasonal adjustment, the answer to question (2) depends upon what the components will ultimately be used for and on our ability to precisely define the components. For instance, if the purpose is short term forecasting of Z_t, then S_t and N_t are not of direct interest, and some would argue that seasonal adjustment is unnecessary. We shall argue in section 6 that the components have not been precisely defined. Until a more rigorous definition of the components is provided it is difficult to justify the proposition that the components are of interest as ends in themselves.

Question (3) is not difficult to answer if the components can be precisely defined. If not, it is difficult to construct estimators since we don't know what is being estimated. With precisely defined components, it seems logical to use signal extraction theory to estimate them.

Justification of Seasonal Adjustment

We favor modeling series in terms of the original data, accounting for seasonality in the model, rather than using adjusted data. Others have voiced similar opinions. For example, Watts (1978) states ". . . I have yet to be convinced that seasonal adjustment is the best thing to do to a series. I believe, rather, that the aim of time series model building should be to develop forecasting models that yield white-noise residuals." Also, Roberts (1978b) says that ". . . it appears to me that seasonal adjustments can be only a source of trouble to a statistician interested in forecasting unadjusted values . . ." and ". . . surely the route to better scientific understanding is to incorporate the seasonality directly into multivariate models that are formulated in terms of unadjusted data so that the source, transmission, and effects of seasonal variations can be better understood." Some econometricions, see Crutchfield and Zellner (1963), Plosser (1978), and Wallis (1978), have argued that knowledge of a series' underlying economic structure can provide an understanding of the nature of seasonality in specific time series. This can permit the incorporation of seasonality directly into an economic model, eliminating the need to work with seasonally adjusted data. In fact, Plosser (1978) argues that use of adjusted data could lead one to misspecified models, misleading inferences about parameters, and poor forecasts.

In light of these remarks and the previous discussion, it is relevant to ask whether seasonal adjustment can be justified and, if so, how? It is important to remember that the primary consumers of seasonally adjusted data are not necessarily statisticians and economists, who could most likely use the unadjusted data, but people such as politicians, business managers, and journalists, who often have little or no statistical training. We thus offer the following possible justification for seasonally adjusting time series.

> Seasonal adjustment is done to simplify data so they may be more easily interpreted by statistically unsophisticated users, without a significant loss of information.

We say "possible" justification because we believe its validity has not yet been established. The key phrase is "without a significant loss of information." Obviously, many people have found seasonally adjusted data to be simpler to use than unadjusted data, but to establish that the above justification is valid, we need to know that the amount of information lost in adjusting is not excessive in some appropriate sense. We believe that in general there will be some information loss from seasonal adjustment, even when an adjustment method appropriate for the data being adjusted can be found. The situation will be worse when the seasonal adjustment is based on incorrect assumptions. If people will often be misled by using seasonally adjusted data, then, in our opinion, their use cannot be justified.

Loss of Information from Seasonal Adjustment

There has been some work on the consequences of using seasonally adjusted data. It has concentrated on how seasonal adjustment affects (i) forecast accuracy and (ii) relating one series to another.

Makridakis and Hibon (1979) forecast 111 time series by various methods and compared the overall accuracy of the forecasts produced by different methods. They used methods which handled seasonal series directly (such as ARIMA modeling), and nonseasonal methods applied to seasonally adjusted data. With these latter methods, forecasts were reseasonalized by applying seasonal factors. They used their own method of seasonal adjustment which produced fixed seasonal factors. Their results do not permit direct assessment of the effects of seasonal adjustment on forecast accuracy because (i) the forecast results for seasonal and nonseasonal series are not separated, and (ii) most of the methods used directly on the seasonal series were not used in nonseasonal form with the adjusted data. Still, they found the methods which used seasonally adjusted data did somewhat better than the methods which handled seasonality directly — including forecasting with ARIMA models. Their results may be influenced by their use of constant seasonal factors, and by their use of measures of forecast accuracy that aggregate over series that differ in forecastibility (thus giving undue influence to series that are inherently difficult to forecast).

Plosser (1979) forecast five economic time series with seasonal ARIMA models, and forecast the X-11 adjusted series with nonseasonal ARIMA models. Instead of reseasonalizng the forecasts of the adjusted data, he converted the monthly forecasts to annual totals to compare forecast accuracy. He used fully revised seasonally adjusted values, which could favor the use of the adjusted data since they are obtained using future values of the series. He found the seasonal ARIMA models performed substantially better on two series, slightly better on two series, and slightly worse on one series. These results seem to be inconclusive, since direct comparisons were not made in the Makridakis and Hibon paper and Plosser examined only five series.

However, there is an important aspect of forecasting not considered in these two studies. This is the estimation of forecast error variances and the subsequent provision of confidence intervals for the future observations. There are well-established procedures for estimating forecast error variances and getting forecast intervals when using ARIMA or other time series models (Box and Jenkins 1970, chapter 5). However, use of seasonally adjusted data in forecasting, whether the forecasting is done formally through a model or informally, would seem to preclude estimation of forecast error variances and production of forecast intervals. This is obviously true for forecasting the unadjusted data, but it is also true if one wishes to forecast the adjusted data (though we question why anyone would want to do this). Future adjusted values depend on future seasonals through the future unadjusted data, hence forecast error variances for adjusted data should allow for errors in forecasting the seasonal, and there is no way to get at this with adjusted data. These problems will not be solved if, as has been recommended, government agencies start publishing standard errors for seasonally adjusted data, because it is not clear how to get from these to forecast error variances.

Seasonally adjusted data have been used in relating time series, as in econometric modeling, presumably on the assumption that their use would eliminate the need to deal explicitly with seasonality in the model, without altering the relationships between the series. We now survey some of the work that has been done on the consequences of using adjusted data for this purpose. A more detailed discussion of some of this work is given by Nerlove, Grether, and Carvalho (1979, p.162-171).

Lovell (1963, 1966) and Jorgenson (1964, 1967) investigated regression approaches to seasonal adjustment and the appropriateness of seasonally adjusting time series before subsequently using them in a regression analysis. Lovell (1963) showed that prior adjustment by regressing the dependent and explanatory variables on seasonal dummy variables can be appropriate in that this gives the same results as including the seasonal dummy variables in a regression with the unadjusted data. He also noted that adjusting effectively uses up some degrees of freedom and that results with the adjusted data should be modified accordingly. Jorgenson (1964) discussed optimal (minimum mean squared error) estimation of the seasonal component (in a regression model). Their subsequent papers (Lovell, 1966; Jorgenson, 1967) point out the interesting result that the optimal estimate of the seasonal component is not generally appropriate for adjusting series prior to relating them in a regression model.

Sims (1974) considered the estimation of a distributed lag relation between the nonseasonal components of two time series y and x, when they are observed with seasonal noise added. He observed that the estimated lag distribution can be biased (especially if a smooth, one-sided, rather than long, two-sided lag distibution is estimated) and that seasonal adjustment of both y and x by a linear filter that removes seasonality in x can reduce the bias. He constructed adjustment filters for this purpose, noting that official procedures (or seasonal differencing or removal of seasonal means) may not be suitable. He found if y and x are adjusted with different filters, then the bias may be reduced, but it may be made much worse, so it is usually safer to use the same filter on y and x. The exception to the rule occurs when the seasonal components of y and x are unrelated, in which case optimal (minimum mean squared error) adjustment of x alone will remove the bias.

Wallis (1974) also observed that adjusting y and x with different filters can distort the lag relationships between them so that using the same filter is safer. He further observed that using the filter which reduces the residuals in the distributed lag regression of y on x to white noise will produce efficient estimates, since this is the same as doing generalized least squares. He then used simulated time series to verify his conclusions regarding the effect of seasonal adjustment on estimated relations between series, and also to check that a linear filter approximation to X-11 that he devised behaved similarly in this respect to X-11 itself.

An additional point of interest was made by Granger (1978a). He showed that if the seasonal components of two series are correlated while the nonseasonal components are independent, then if both series are adjusted separately with linear filters, the adjusted series will be correlated. Thus, the adjusted series will exhibit a relationship even though the nonseasonal components are unrelated.

Newbold (1980) illustrated some problems that can arise when relating one adjusted series to another through a transfer function (distributed lag) model. For his example, nonseasonal models were inadequate for his adjusted series and led to distortions in the estimated transfer function and noise models. He remedied these problems by putting "antiseasonal" terms (leading to negative correlations at seasonal lags) in his model to correct for this. His example illustrates that it is dangerous to assume, at least without checking, that nonseasonal models will be appropriate for seasonally adjusted data, and he shows how one might proceed when a nonseasonal model is inappropriate.

From these studies we might conclude that it is hard to say what effect using seasonally adjusted data has on forecast accuracy. However, seasonally adjusted data has a severe disadvantage in forecasting in that its use prevents estimation of forecast error variances and production of forecast intervals, something which can be done with the unadjusted data. Adjusted data can be useful in relating series; here it is usually safer to use the same adjustment filter on all series, unless the seasonal components of the series are known not to be related. Sims (1974) and Wallis (1974) offer guidance here, the latter pointing out in a footnote that using X-11 (with standard options) on all series is close to using the same linear filter on all of them. However, it should be kept in mind that the simplicity of using adjusted data is bought at some risk of biased or inefficient estimation of relationships between series, that degrees of freedom need to be modified if adjusted data are used, and that, as illustrated by Newbold (1980), even the simplicity of adjusted data is sometimes illusory.

6. Defining the Components

It is surprising to us that so many people have provided estimates of seasonal, trend, cycle, and irregular components without bothering to define what it was they were estimating. Statements that have been made as to what the components are have tended to be vague — really being descriptions rather than definitions. For example, Falkner (1924) said, "Seasonal variation is that part of the fluctuation due to the persistent tendency for certain months of each year to be regularly higher than certain other months of the year . . ." and "Secular trend is the long-time tendency of the items of the series to grow or decline . . ." Shiskin, Young, and Musgrave (1967) state that, "The seasonal component is defined as the intrayear pattern of variation which is repeated constantly or in an evolving fashion from year to year." Although few would argue with these statements, they are certainly not enough to define what is being estimated.

In recent years, there have been efforts in the direction of more mathematically precise definitions of the seasonal component based on spectral considerations. The first of these was by Nerlove (1964) who defined seasonality as "that characteristic of a time series that gives rise to spectral peaks at seasonal frequencies." Granger (1978a) gave a reasonably precise definition of when a series is seasonal and when it is strongly seasonal. He suggested taking intervals of width δ for some small $\delta > 0$) about the seasonal frequencies $\frac{2\pi k}{12}$ $k = 1, \ldots, 6$ and defined a time series to be seasonal when its spectral density has peaks somewhere in these intervals, and strongly seasonal when the spectral density integrated over all these intervals almost equals the integral of the spectral density over $[0, \pi]$. The problem is that these definitions only tell us when a series has a seasonal component, not what the seasonal component is.

In our opinion, it is essential that the component models be precisely specified, for otherwise it is not known what is being estimated in seasonal adjustment. We now present an approach to defining the seasonal and nonseasonal components for the additive decomposition $Z_t = S_t + N_t$. Z_t may, of course be transformed data. We assume that trading day and other deterministic effects have been removed from Z_t. The definitions of the components are based upon the following assumptions, grouped for purposes of discussion.

Basic Assumptions

1. $Z_t = S_t + N_t$
2. $\{S_t\}$, $\{N_t\}$ are independent of each other[3]

Harmless Assumptions

3. Z_t follows a known ARIMA model $\phi^*(B)Z_t = \theta^*(B)a_t$

4. S_t follows an unknown ARIMA model $\phi_S(B)S_t = \theta_S(B)b_t$

5. N_t follows an unknown ARIMA model $\phi_N(B)N_t = \theta_N(B)c_t$

6. $\phi_S(B)$ and $\phi_N(B)$ have no common zeroes

Arbitrary Assumptions

7. $\phi_S(B) = 1 + B + \ldots + B^{11}$

8. the order of $\theta_S(B) \leq 11$

9. $\sigma_b^2 = \mathrm{Var}(b_t)$ is as small as possible consistent with assumptions 1-8.

Under these assumptions, the results of Hillmer and Tiao (1982) can be used to show that the models for S_t and N_t are uniquely determined.[4] We then define the components S_t and N_t to be the unobserved time series satisfying these assumptions. This definition does not allow us to exactly calculate S_t and N_t from Z_t, nor should it, but it does tell us what models they follow, which allows us to use signal extraction theory to estimate them. We believe it is vital to discuss why it might be reasonable to make the above assumptions.

The basic assumptions, 1 and 2, define the problem. Someone who does not want to make these assumptions is working on a different problem. In 3, it is assumed that an ARIMA model can be built from the observed data to adequately approximate the covariance structure of Z_t. This allows us to handle a wide range of time series since data that are seasonally adjusted can often be modeled with ARIMA models. A larger class of models than pure ARIMA models is actually allowed, since it is assumed that deterministic effects, such as trading day variation, have been subtracted out. With regard to assumptions 4 and 5, if Z_t follows an ARIMA model, it seems harmless to assume that S_t and N_t also follow ARIMA models. For all the ARIMA models here, we assume the autoregressive and moving average polynomials for a given model have no common zeroes and the white noise series (a_t, b_t, c_t) have zero mean and constant variance. If 6 does not hold, then the spectral densities of S_t and N_t will have peaks of similar intensity at the same frequency, which in our opinion seems unreasonable.

Based on our experience with series that are seasonally adjusted, appropriate models for these series typically have

$$\phi^*(B) = \phi(B)(1 - B)^d(1 - B^{12}) = \phi(B)(1 - B)^{d+1}(1 + B + \dots + B^{11})$$

where $d \geq 0$ and $\phi(B)$ is of low order in B (say ≤ 3). Given assumptions 1 through 6, Findley (1982) has shown that $\phi^*(B) = \phi_S(B)\ \phi_N(B)$, so that for the above $\phi^*(B)$ we let

$$\phi_S(B) = 1 + B + \dots + B^{11} \qquad \phi_N(B) = \phi(B)(1 - B)^{d+1}$$

which leads to assumption 7.

Hillmer and Tiao (1982) show that our choice for $\phi_S(B)$ leads to a spectral density for the seasonal having infinite peaks at the seasonal frequencies and relative minima between them. Also, 7 implies that summing S_t over 12 consecutive months produces a stationary series with mean zero, which is consistent with the general belief (as in X-11) that (in an additive decomposition) the seasonal component should sum to something near zero over a year. The nonseasonal component will be nonstationary and its spectral density will have an infinite peak at zero frequency. Thus, in our view, assumption 7 leads to reasonable seasonal and nonseasonal component models.

One issue that should be addressed in relation to the choice of autoregressive operators in assumption 7, is what to do with seasonal autoregressive operators. For models including a factor $1 - B^{12}$, we invariably find seasonal moving average terms to be more appropriate than seasonal autoregressive terms. We have modeled a few series without a $1 - B^{12}$, but with a seasonal autoregressive operator, $1 - \phi_{12}B^{12}$, where ϕ_{12} is not near 1. We have chosen not to adjust such series because the seasonal pattern of the data tends to change very quickly — the highest month could become the lowest month after four or five years. A similar choice was made by Hannan (1964). It is possible to make a different choice of $\phi_S(B)$ and still use the above framework if a set of rules replacing 7 for specifying $\phi_S(B)$ and $\phi_N(B)$ given $\phi^*(B)$ is provided.

Hillmer, Bell, and Tiao (1983) note that 8 implies that the forecast function in the model for S_t follows a fixed annual pattern that sums to zero over 12 consecutive months. In contrast, if the order of $\theta_S(B)$ exceeds 11, then the forecast function for the seasonal component will change its annual pattern. We believe the forecastable change in the seasonal pattern should be part of the trend, and hence in N_t.

Given assumptions 1-8, Hillmer and Tiao (1982) show that σ_b^2 must lie in some known range $[\underline{\sigma}_b^2, \bar{\sigma}_b^2]$, and that the models for S_t and N_t are uniquely determined once a choice of σ_b^2 is made. They call the decomposition corresponding to a σ_b^2 in $[\underline{\sigma}_b^2, \bar{\sigma}_b^2]$ an admissible decomposition, with corresponding admissible seasonal and nonseasonal components, and they call the decomposition corresponding to the choice $\sigma_b^2 = \bar{\sigma}_b^2$ the <u>canonical decomposition</u>. Thus, we have defined the seasonal component to be the <u>canonical seasonal</u>, $\bar{S}_t$, corresponding to the choice $\sigma_b^2 = \bar{\sigma}_b^2$. The <u>canonical nonseasonal</u>, $\bar{N}_t$, is then $Z_t - \bar{S}_t$. Hillmer and Tiao (1982) show that choosing $\sigma_b^2 = \bar{\sigma}_b^2$ minimizes $Var[(1 + B + ... + B^{11})S_t]$, making the seasonal pattern as stable as possible. In addition, they show that for any other choice of σ_b^2, the corresponding seasonal component, S_t', can be written

$$S_t' = \bar{S}_t + e_t$$

where e_t is white noise. Thus, any admissible seasonal is the sum of the canonical seasonal, which follows as stable a pattern as possible and is as predictable as possible, and white noise, which is totally unpredictable and nonseasonal. We see no reason to add white noise to $\bar{S}_t$ when defining the seasonal component.

Assumptions 1-9 lead to precise definitions of the seasonal and nonseasonal components. If we have built a model for the observed data Z_t, then once assumptions 1-9 are made, we know the models for $\bar{S}_t$ and $\bar{N}_t$ and can use signal extraction theory to estimate these components. This is the approach to seasonal adjustment taken in Burman (1980), Hillmer and Tiao (1982), and Hillmer, Bell, and Tiao (1983). Of course, assumptions other than 1-9 can be made about S_t and N_t, even while remaining consistent with the model for Z_t; in particular, a choice of σ_b^2 in $[\bar{\sigma}_b^2, \tilde{\sigma}_b^2]$ other than $\bar{\sigma}_b^2$ could be used. Different assumptions will lead to different definitions and models for the components which, when used in signal extraction theory, will lead to different methods of seasonal adjustment.

This discussion points out the arbitrariness inherent in seasonal adjustment. Different methods produce different adjustments because they are making different assumptions about the components and, hence, are estimating different things. This applies even to methods (such as X-11) which do not make their assumptions explicit, since they must implicitly make the same sort of assumptions as we have discussed here (the assumptions implicit in additive X-11 with standard options are investigated by Cleveland and Tiao (1976) and in Section 7.4). Unfortunately, there is not enough information in the data to define the components, so these types of arbitrary choices must be made. We have tried to justify our assumptions but do not expect that everyone will agree with them. However, if anyone wants to do seasonal adjustment, but does not want to make these assumptions, we urge them to make clear what assumptions they wish to make. Then the appropriateness of the various assumptions can be debated. In our opinion this dialogue would be more productive than the current ongoing one regarding what seasonal adjustment procedure should be used, in which no one bothers to specify what is being estimated. Thus, if debate can be centered upon what it is we want to estimate in doing seasonal adjustment, then there may be no dispute about how to estimate it.

7. Evaluating Seasonal Adjustments and Seasonal Adjustment Methods

Given the arbitrary nature of seasonal adjustment, people have found it difficult to decide when a "good" adjustment has been done, or when one method is "better" than another. In this section we discuss the problems with approaches that have been used to evaluate adjustments and adjustment methods, including criteria for evaluating adjustments, simulation studies, and revisions comparisons. Finally, we make some suggestions as to how this subject might be approached.

7.1 Criteria for Evaluating Seasonal Adjustments

Various criteria have been proposed for assessing the adequacy of a seasonal adjustment and for deciding when one method does a better job adjusting a series than another. Attempts at designing such criteria have failed so that today there are no accepted standards by which adjustments can be judged.

Criteria proposed for evaluating seasonal adjustments have generally reflected properties that were thought desirable for nonseasonal components. These have been phrased in both spectral and time domain terms. It was thought that a method performed adequately if the adjusted series exhibited properties similar to those of the "true" nonseasonal component, and the performance of different adjustment methods has been compared based upon this belief. Unfortunately, although the suggested criteria may reflect desirable properties for the nonseasonal component of a series, this does not mean that they reflect desirable properties for the adjusted series, which is an estimate of the nonseasonal component. Anderson (1927) emphasized long ago that the estimated components are not the same as the true components. Furthermore, even if models for Z_t, S_t, and N_t are known the true underlying components cannot be calculated, and the best estimates of the components will behave differently enough from the true components so as to make the criteria that have been proposed of little or no value in evaluating seasonal adjustments. To substantiate this, we cite two examples.

Nerlove (1964) suggested various spectral criteria that a "good" adjustment should satisfy, including (i) high coherence between original and adjusted series, except at seasonal frequencies, (ii) minimal phase shifts in the cross spectrum between the original and adjusted series, and (iii) removal of peaks at the seasonal frequencies in the spectral density of the original series, without producing dips at these frequencies or greatly affecting the spectral density at other frequencies. Subsequently, Grether and Nerlove (1970) investigated empirically (by simulating series from known component models) and theoretically the performance of the optimal (minimum mean squared error linear) method of adjustment. They discovered that the optimal method did not look good in terms of these criteria. It reproduced all the undesirable features that Nerlove (1964) noted for X-11. Since the minimum mean squared error linear estimator is a reasonable choice if it is available, they concluded the criteria in Nerlove (1964) left much to be desired.

As a second example, Granger (1978a) reviewed some criteria which could be used for evaluating adjustments, including (i) and (iii) of Nerlove (1964) which he referred to as "highly desirable." In their discussions of Granger's paper, both Sims (1978) and Tukey (1978) show that the spectral properties he suggested have unreasonable parallels in other situations and that the minimum mean squared error linear adjustment need not satisfy these properties.[5] Granger (1978b) then responded, "The criteria I suggested have been shown to be impossible to achieve in practice, and thus, should be replaced by achievable criteria. However, I am at a loss to know what these criteria should be."

We believe that empirical studies comparing the performance of different adjustment methods on various sets of data using the previously proposed criteria are of little value in determining which methods of adjustment are "better" than others. We doubt that useful criteria which are functions only of the adjusted data can be found. However, there may be a role for the previously mentioned criteria. Since these criteria are reasonable when applied to the true nonseasonal component, they may be useful in evaluating the assumptions made about the components by adjustment methods. Thus, in our approach to defining the components discussed in Section 6, we used some criteria to evaluate the properties of the assumed underlying component models. These and other criteria might be applied to the assumptions underlying other seasonal adjustment methods. We believe that efforts would be better spent evaluating the assumptions underlying adjustment methods, rather than trying to evaluate methods by looking at adjusted data.

7.2 Simulation Studies

Another approach that has been suggested for evaluating seasonal adjustment methods is to check their performance on simulated series. The S_t and N_t components are generated and an adjustment method applied to $Z_t = S_t + N_t$ to see how accurately the method estimates the components. We think little will be learned in general from such studies.

The basic problem with this approach is that the results depend heavily on what the adjustment methods being considered are actually estimating. This can vary considerably from method to method. If method I makes assumptions about S_t and N_t which are similar to those used in generating them, while method II makes different assumptions, then method I will estimate the components more accurately than method II. This phenomenon is reflected in the results of Godfrey and Karreman (1967). Comparing different methods on simulated data will merely verify that the methods make different assumptions.

To illustrate the above remarks, we generated S_t and N_t series of length 900 from each of the following two models, the rationale for which will become apparent.

1. Min Seasonal Model

$$(1 + B + \ldots + B^{11})\, S_{1t} = (1 + 1.45B + 1.50B^2 + 1.44B^3 + 1.24B^4 + .99B^5 + .72B^6 + .45B^7 + .23B^8 + .002B^9 - .11B^{10} - .43B^{11})b_{1t}$$

$$b_{1t} \text{ i.i.d. } N(0, .0107)$$

$$(1-B)^2 N_{1t} = (1 - 1.38B + .39B^2)d_{1t} \qquad d_{1t} \text{ i.i.d. } N(0, .8223)$$

2. Max Seasonal Model

$$(1 + B + \dots + B^{11})\, S_{2t} = (1 + 1.10B + 1.10B^2 + 1.05B^3 + .99B^4 + .96B^5 + .94B^6 + .94B^7 + .86B^8 + .80B^9 + .83B^{10} + .87B^{11})b_{2t}$$

$$b_{2t} \text{ i.i.d. } N(0, .4422)$$

$$(1 - B)^2 N_{2t} = (1 + .01B - .98B^2)d_{2t} \quad d_{2t} \text{ i.i.d. } N(0, .0740)$$

For both of these models the resulting model for the sum $Z_{it} = S_{it} + N_{it}$ is the same, and is given by

$$(1 - B)(1 - B^{12})Z_{it} = (1 - .4B)(1 - .8B^{12})a_{it} \quad a_{it} \text{ i.i.d. } N(0, 1).$$

Actually, the Min Seasonal Model corresponds to making assumptions 1-9 given in Section 6 (lowest possible σ_b^2), while the Max Seasonal Model makes assumptions 1-8 and then chooses the maximum possible σ_b^2. The series were generated in such a way that in fact the same series was obtained from both models, that is, we have here $Z_t = S_{it} + N_{it}$ $i = 1,2$. The following model was identified and estimated for the observed data Z_t:

$$(1 - B)(1 - B^{12})Z_t = (1 - .41B)(1 - .85B^{12})a_t \qquad \sigma_a = .967$$

Using signal extraction theory and the estimated model for Z_t, S_{it} and N_{it} $(i=1,2)$ were estimated from Z_t under two assumptions:

(i) that the true model for S_{it} had minimum σ_b^2, and

(ii) that the true model for S_{it} had maximum σ_b^2.

Thus, there are four cases:

Case	Model Used to Generate Data	Model Assumed in Constructing $\hat{S}_{it}$, $\hat{N}_{it}$
A	Min Seasonal	Min Seasonal
B	Min Seasonal	Max Seasonal
C	Max Seasonal	Max Seasonal
D	Max Seasonal	Min Seasonal

In cases A and C, the correct models for S_{it} and N_{it} (within parameter estimation error) have been used, and in cases B and D incorrect models have been used. The error series $e_{it} = S_{it} - \hat{S}_{it} = \hat{N}_{it} - N_{it}$ were computed in each of the four cases, and the e_{it}'s were standardized by dividing them by their standard deviation (from signal extraction theory) when the correct model is used. The results are shown in Figures 7.1 through 7.4 for the middle 100 observations.

When the correct model is used, as in Figures 7.1 and 7.3, the standardized e_{it}'s vary about zero reasonably within ±2 limits. However, when the incorrect model is used, as in Figures 7.2 and 7.4, the e_{it}'s are considerably larger. This does not tell us that either the Min Seasonal or Max Seasonal method of adjustment is better, it merely illustrates how the accuracy of the estimator depends heavily on what is being estimated.

7.3 Revisions

Most seasonal adjustment methods are based on symmetric two-sided filters. When the observation for the current time period is adjusted, future observations are not available, thus near the end of the series one-sided filters must be used. As more observations become available, one can come closer to using the symmetric filter. This results in changes in the seasonally adjusted values as additional observations are added which are called revisions.

Many researchers who have conducted empirical studies of seasonal adjustment methods have used measures of the magnitude of revisions as one criteria for evaluating the different methods. This makes sense when comparing adjustment methods that give the same final adjustment, such as X-11 and X-11 ARIMA, or X-11 in year-ahead and concurrent modes. In this case the different methods are all shooting at the same target value, the final X-11 adjustment. Comparisons of the magnitudes of total revisions (changes from the initial to the final adjustment) reflect how close the initial adjustments come to the target. Since, presumably, the final adjustment is better than the earlier adjustments (or we would not bother to revise as additional data became available), lower total revisions are better. Studies comparing total revisions for X-11 and modifications to X-11 still yielding the same final adjustment have been done by Dagum (1978), Geweke (1978a), Kenny and Durbin (1982), and McKenzie (1984).

However, there is a fundamental problem with using revisions as a standard of comparison when the methods being compared produce different final adjustments, and are estimating different nonseasonal components. In this case the magnitude of revisions can be greatly affected by the choice of the nonseasonal components. This choice should be based on information in the data and beliefs about seasonality (see sections 6 and 7.4), not on the magnitude of revisions. In the extreme one could use a method based exclusively on one-sided filters, which leads to no revisions - an approach that has seldom been adopted.

To illustrate the dependence of the size of revisions on the final adjustment used, we shall consider additive X-11 with standard options, the model-based method of Section 6 (min seasonal), and the max seasonal variant of this discussed in Section 7.2. Suppose these methods are used by applying their symmetric filters to data extended with minimum mean squared error forecasts and backcasts. This minimizes the mean squared revisions (MSR) (Geweke 1978a, Pierce 1980b), so that differences in MSR between the methods for a given model for Z_t are due only to the different final adjustment targets. Using results of Pierce (1980b) we computed mean squared total revisions for the particular case where Z_t follows the model

$$(1-B)(1-B^{12})Z_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t$$

for various values of θ_1 and θ_{12}, with $\sigma_a^2 = 1$. Table 7.1 presents some illustrative results. Notice that the magnitude of revisions for a given model depends dramatically on the relevant final adjustment. It also depends on the characteristics of the data, so the relation in MSR for different adjustment methods can be different for different series. These results point out the inappropriateness of using total revisions to evaluate seasonal adjustment methods giving different final answers.

We might also consider how the behavior of yearly revisions, the changes in the adjusted values as each additional year of data is added, depend on the final adjustment underlying a method. It might be argued that first year revisions are relevant since some users will not be concerned about revisions more than a year or two after the initial adjustment, and thus will not be concerned with the actual final adjustment.[6] Under the assumptions and model given above, Hillmer, Bell, and Tiao (1983) found theoretical first year MSR are smaller for the min seasonal model-based method when $\theta_{12} > .4$ and smaller for X-11 when $\theta_{12} < .4$, with the difference being more pronounced the further θ_{12} is from .4 (the max seasonal method was not considered). These theoretical calculations were confirmed empirically by studying the first year revisions of 76 times series which were modeled and adjusted by both approaches. They found that the model-based approach gave substantially lower first year revisions, and argued this was because the estimated values of the seasonal moving average parameter, θ_{12}, for the 76 series was almost always substantially larger than .4. Since the model-based method effectively uses longer filters than X-11 when $\theta_{12} > .4$ and shorter filters when $\theta_{12} < .4$, this leads to the conclusion that longer filters lead to smaller first year revisions. The filters used to adjust the most recent observations for both methods are modifications of the symmetric filter used for the final adjustment, and the lengths of the filter used for recent data correspond to the lengths of the filters used for the final adjustments. Thus, the final adjustment underlying a method has a profound effect on first year revisions.

To reemphasize our point, these results illustrate that it is inappropriate to use measures of revisions to judge the relative merits of seasonal adjustment methods giving different final adjustments. The decision as to what final adjustment is appropriate should be based on information in the data, beliefs about seasonality, and, when possible, on the objectives of the seasonal adjustment. Therefore, in choosing a seasonal adjustment method it is important that attention be concentrated upon what is being estimated, the target, rather than upon revisions. Using revisions to evaluate seasonal adjustment methods giving different final adjustments is like judging a parameter estimator by how rapidly it converges as the sample size increases, even if it converges to the wrong value.

7.4 Consistency with the Data

Consider the ideal situation where we know the spectral densities for Z_t, S_t, and N_t ($f_Z(\lambda)$, $f_S(\lambda)$, and $f_N(\lambda)$ respectively). From $Z_t = S_t + N_t$ the spectral densities (and hence the models) are constrained by the relation

$$f_Z(\lambda) = f_S(\lambda) + f_N(\lambda). \tag{7.1}$$

The minimum mean squared error estimator, $\hat{N}_t$, of the nonseasonal component is obtained by applying a symmetric linear filter, $W_N(B)$, to the observed data:[7]

$$\hat{N}_t = W_N(B)Z_t \tag{7.2}$$

where

$$W_N(e^{-i\lambda}) = \sum_{-\infty}^{\infty} W_{N,k}\, e^{-i\lambda k} = f_N(\lambda)/f_Z(\lambda) = 1 - f_S(\lambda)/f_Z(\lambda).$$

Notice that any two of $f_Z(\lambda)$, $f_S(\lambda)$, $f_N(\lambda)$, and $W_N(B)$ are sufficient to determine the other two using (7.1) and (7.2), but no one of them is sufficient to determine the other three.

In practice, while we will not know $f_Z(\lambda)$, we can at least approximate it by modeling Z_t. Let $\hat{f}_Z(\lambda)$ be our estimate of $f_Z(\lambda)$. Now suppose that we have a linear filter, $W_N(B)$, to be used in adjusting Z_t. From (7.2), the implied spectral densities for S_t and N_t are

$$\hat{f}_S(\lambda) = \hat{f}_Z(\lambda)[1-W_N(e^{-i\lambda})] \quad \hat{f}_N(\lambda) = \hat{f}_Z(\lambda)W_N(e^{-i\lambda}). \tag{7.3}$$

By examining $\hat{f}_S(\lambda)$ and $\hat{f}_N(\lambda)$ we can investigate the assumptions that are implicit when Z_t is adjusted with $W_N(B)$.

Suppose $W_N(B)$ results from signal extraction theory for some set of models for Z_t, S_t, and N_t, which we assume are expressed in infinite autoregressive form as

$$\Pi_Z(B)Z_t = a_t \quad \Pi_S(B)S_t = b_t \quad \Pi_N(B)N_t = c_t. \tag{7.4}$$

Then, $W_N(B)$ satisfies

$$W_N(e^{-i\lambda}) = f_N(\lambda)/f_Z(\lambda) = \frac{\sigma_c^2/\Pi_N(e^{i\lambda})\Pi_N(e^{-i\lambda})}{\sigma_a^2/\Pi_Z(e^{i\lambda})\Pi_Z(e^{-i\lambda})} .$$

We cannot say adjustment with $W_N(B)$ implies the models in (7.4), since if all the models in (7.4) are replaced by the models

$$\alpha(B)\Pi_Z(B)Z_t = a_t, \quad \alpha(B)\Pi_S(B)S_t = b_t \quad \alpha(B)\Pi_N(B)N_t = c_t \qquad (7.5)$$

where $\alpha(B) = \sum_0^\infty \alpha_j B^j$ has all its zeroes on or outside the unit circle, the adjustment filter is

$$\frac{f_N(\lambda)/\alpha(e^{i\lambda})\alpha(e^{-i\lambda})}{f_Z(\lambda)/\alpha(e^{i\lambda})\alpha(e^{-i\lambda})} = W_N(e^{-i\lambda})$$

so the adjustment filter stays the same. This reflects the fact that $W_N(B)$ alone cannot determine the models for Z_t, S_t, and N_t. However, if we have an estimated model $\hat{\Pi}_Z(B)Z_t = a_t$, then setting $\alpha(B) = \hat{\Pi}_Z(B)/\Pi_Z(B)$ in (7.5) leads to implied models for S_t and N_t:

$$\begin{aligned} &\hat{\Pi}_S(B)S_t = b_t && \hat{\Pi}_N(B) = c_t \\ &\hat{\Pi}_S(B) = \hat{\Pi}_Z(B)\Pi_S(B)/\Pi_Z(B) && \hat{\Pi}_N(B) = \hat{\Pi}_Z(B)\Pi_N(B)/\Pi_Z(B) \end{aligned} \qquad (7.6)$$

Of course, if (7.4) uses the estimated model for Z_t, then $\Pi_Z(B) = \hat{\Pi}_Z(B)$ and the models in (7.6) are the same as those in (7.4). The implied spectral densities for S_t and N_t are obtained from the relations

$$\hat{f}_S(\lambda) = \sigma_b^2/2\pi\hat{\Pi}_S(e^{i\lambda})\hat{\Pi}_S(e^{-i\lambda}) \quad \hat{f}_N(\lambda) = \sigma_c^2/2\pi\hat{\Pi}_N(e^{i\lambda})\hat{\Pi}_N(e^{-i\lambda}). \qquad (7.7)$$

The above suggests an approach to evaluating the suitability of any linear adjustment method for a particular time series. In our opinion the overriding consideration is that any method of seasonal adjustment should be consistent with the information in the data, which is summarized, at least approximately, by the estimated model (spectral density) for Z_t. If the implied models (spectral densities) for S_t and N_t in (7.3), (7.6) and (7.7) are then unreasonable, such as if the model for N_t is seasonal, we would conclude that seasonal adjustment using $W_N(B)$ is inconsistent with the information in the data. If the implied models (spectral densities) appear reasonable, we would say seasonal adjustment with $W_N(B)$ is consistent with the information in the data. This leads us to propose the following criterion for evaluating a method of seasonal adjustment with respect to a given set of data:

> A method of seasonal adjustment should be consistent with an adequate model for the observed data.

This condition is not sufficient for a "good" seasonal adjustment in the sense that just because a method satisfies the condition for a given set of data it does not follow that the resulting seasonal adjustment is "good." Since many different seasonal adjustments can be consistent with an adequate model for the data (see section 7.2), judgments about whether a method that is consistent with the data is "good" must either be made subjectively, or will require additional information, such as the use to be made of the adjusted data. However, we feel the criterion is necessary for a good seasonal adjustment, in that we would say any method not consistent with the information contained in an adequate model for a given set of data is certainly "bad" for that set of data. Application of the criterion depends on arbitrary judgments regarding the adequacy of the fitted model for Z_t and the reasonableness of the implied models (spectral densities) for S_t and N_t. Even with these difficulties we feel application of the criterion can be informative, and sometimes the conclusions will be obvious, as we shall illustrate with an example.

We should point out that (7.3) may not be defined at $\lambda = k\pi/6$ $k=0,\pm1,\ldots,\pm6$, since $\hat{f}_Z(\lambda)$ may well be $+\infty$ at these frequencies, while $W_N(e^{-i\lambda})$ or $1 - W_N(e^{-i\lambda})$ may be zero at any given one of these frequencies. Depending on $\hat{f}_Z(\lambda)$ and $W_N(B)$, it may be sensible to set $\hat{f}_S(\lambda) = +\infty$ at the seasonal frequencies and $\hat{f}_N(\lambda) = +\infty$ at $\lambda = 0$. This problem does not arise if $W_N(B)$ corresponding to models (7.4) and (7.6) is used. We present our criterion as a general approach to evaluating the consistency of a seasonal adjustment method with a model for the data, and hope to investigate the computational considerations further.

<u>Example</u>

We evaluate the use of the X-11 method (additive version with standard options) on the series Z_t = employed nonagricultural males, 20 and older (from the Bureau of Labor Statistics) from January 1965 through August 1979. Young (1968) found a linear filter which approximates additive X-11. Cleveland and Tiao (1976) then found approximately the same filter results from signal extraction theory using the following models for S_t and N_t:[8]

$$(1 - B^{12})S_t = (1 + .64B^{12} + .83B^{24})b_t$$

$$(1 - B)^2N_t = (1 - 1.252B + .4385B^2)c_t \tag{7.8}$$

$$\sigma_c^2/\sigma_b^2 = 24.5$$

These lead to a model for Z_t:

$$\begin{aligned}(1 - B)(1 - B^{12})Z_t &= (1 - .337B + .144B^2 + .141B^3 + .139B^4 \\ &\quad + .136B^5 + .131B^6 + .125B^7 + .117B^8 + \\ &\quad .106B^9 + .093B^{10} + .077B^{11} - .417B^{12} + \\ &\quad .232B^{13} - .001B^{20} - .003B^{21} - .004B^{22} - \\ &\quad .006B^{23} + .035B^{24} - .021B^{25})a_t \\ &= \eta(B)a_t. \qquad \sigma_a^2/\sigma_b^2 = 43.1\end{aligned} \tag{7.9}$$

For the employment series we obtained the model

$$\frac{(1-.26B)(1-B)(1-B^{12})}{1-.88B^{12}} Z_t = a_t \qquad \hat{\sigma}_a^2 = 16150 \tag{7.10}$$

The sample autocorrelations of the residuals, $\hat{a}_t$, from (7.11) are reported in Table 7.2. The statistics (Ljung and Box 1978)

$$Q_L = n(n+2) \sum_{k=1}^{L} r_k(\hat{a})^2/(n-k)$$

are approximately distributed as X^2_{L-2} if the model is adequate. For this example none of the $r_k(\hat{a})$'s is larger in magnitude than two standard errors (.16), and $Q_{12} = 10.2$, $Q_{24} = 20.6$, and $Q_{36} = 33.9$ are all insignificant. We proceed with the estimated model (7.10).

The logarithm of $\hat{f}_Z(\lambda)$, the estimated spectral density corresponding to (7.10) is plotted in Figure 7.5. It has infinite peaks (truncated at 20 for the graph) at $\lambda = 0$ and at the seasonal frequencies $\lambda = \pi k/6$ k=1,2,...,6. From (7.6) and (7.8)-(7.10) the implied models for S_t and N_t are

$$\frac{(1-.26B)(1-B^{12})\eta(B)}{(1-.88B^{12})(1+.64B^{12}+.83B^{24})} S_t = b_t \qquad \hat{\sigma}_b^2 = 374.7 \tag{7.11}$$

$$\frac{(1-.26B)(1-B)^2\eta(B)}{(1-.88B^{12})(1-1.252B+.4385B^2)} N_t = c_t \qquad \hat{\sigma}_c^2 = 9176.1 \tag{7.12}$$

The implied spectral densities, $\hat{f}_S(\lambda)$ and $\hat{f}_N(\lambda)$, were obtained[9] and their logarithms plotted in Figures 7.6 and 7.7. $\hat{f}_S(\lambda)$ has infinite peaks at the seasonal frequencies[10] and may appear reasonable. However, $\hat{f}_N(\lambda)$ has (finite) dips at the seasonal frequencies which is unreasonable. This is <u>not</u> the same as the overadjustment phenomenon referred to in Section 7.1, which has to do with dips in the spectral density of the adjusted data. Here we have dips in the implied spectral density for the underlying nonseasonal component, which is unreasonable. Thus, we conclude that X-11 is inconsistent with the information in the data for this series.

$$(1+B+\ldots+B^{11})S_t = (1 + 2.093B + 2.722B^2 + 2.977B^3 + 2.869B^4 + 2.581B^5 + 2.169B^6 + 1.670B^7 + 1.206B^8 + .745B^9 + .411B^{10} - .007B^{11})b_t, \quad \sigma_b^2 = 82.11 \text{ and}$$

$$(1 - .26B)(1 - B)^2N_t = (1 - .990B + .001B^2)c_t \qquad (7.13)$$

$$\sigma_c^2 = 14412.$$

The logarithms of the implied spectral densities for S_t and N_t are plotted in Figures 7.9 and 7.10. $\hat{f}_S(\lambda)$ has infinite peaks at the seasonal frequencies and minima in between, as was noted in general in Section 6. $\hat{f}_N(\lambda)$ has an infinite peak at $\lambda = 0$, and decreases smoothly after that, with no dips or peaks at the seasonal frequencies. This is reasonable behavior for the implied spectral density of an underlying N_t series. Thus, the canonical adjustment appears reasonable in this case while additive X-11 with standard options does not.

7.5 Classification of Linear Seasonal Adjustment Methods

As a general aid to comparing linear methods of seasonal adjustment and assessing their consistency with observed data, we present a scheme for classifying them. Since model-based approaches are linear and Young (1968) and Wallis (1974) have shown X-11 (and, hence, X-11 ARIMA) to be approximately linear, this scheme covers a large number of proposed adjustment methods. What counts in a linear adjustment method is the

To see why the dips arose in $\hat{f}_N(\lambda)$ notice that, to a rough approximation, $\eta(B)$ in (7.9) is

$$\eta(B) \doteq (1-.35B)(1-.4B^{12})$$

so that (7.12) becomes

$$\frac{(1-.26B)(1-.35B)(1-B)^2(1-.4B^{12})}{(1-1.252B+.4385B^2)(1-.88B^{12})} N_t \doteq c_t.$$

Thus, $\hat{f}_N(\lambda)$ contains

$$\frac{(1-.88e^{12i\lambda})\ (1-.88e^{-12i\lambda})}{(1-.4e^{12i\lambda})\ (1-.4e^{-12i\lambda})} = \frac{1.7744[1-.992\ \cos(12\lambda)]}{1.16[1-.690\ \cos(12\lambda)]}.$$

This function is plotted in Figure 7.8. It is near zero at the seasonal frequencies since the 1-.992 cos(12λ) in the numerator is quite small, while the 1-.690 cos(12λ) in the denominator is at least .31 at all frequencies. The end result is dips at the seasonal frequencies in $\hat{f}_N(\lambda)$. All this is due to the fact that the estimate (.88) of the seasonal moving average parameter in (7.10) is considerably larger than .4, the value implicitly used by X-11. Thus, this behavior can be expected whenever the estimate of θ_{12} is much greater than .4, which seems to be the case is most of the time (see Section 7.3).

In contrast, we examine the canonical decomposition. For the model (7.10), the component models turn out to be linear filter used, so our classifications are made according to how the linear filter is arrived at. Our scheme and some of the methods that fall in each group are as follows.

I. Methods which choose filters directly

(i) X-11
(ii) X-11 ARIMA
(iii) SABL

II. Methods which directly choose models for the components S_t and N_t

(i) Hannan, Terrell, and Tuckwell (1970)
(ii) Engle (1978)
(iii) Abrahams and Dempster (1979)
(iv) Cleveland (1979)
(v) Akaike (1980)
(vi) Kitagawa and Gersch (1983)

III. Methods which model the observed data and deduce models for the components from that model

(i) Melnick and Moussourakis (1974)
(ii) Brewer, Hagan, and Perazelli (1975)
(iii) Geweke (1978b)
(iv) Pierce (1978)
(v) Cleveland (1979)
(vi) Burman (1980)
(vii) Hillmer and Tiao (1982)

Actually, SABL is not really linear since it uses moving M-estimates instead of moving averages. However, ". . . the philosophy of its overall approach is exactly the same as that used in the X-11 procedure. . ." (Cleveland, Dunn, and Terpenning 1978), so it can be viewed as a robustification of a linear method, and the considerations we will discuss here should apply to it. Cleveland (1979) uses elements of both II and III since his approach is to fit a model to Z_t, directly choose component models, and then set the parameters in the component models to approximate the overall model for Z_t.

It should be obvious that for methods in group I, one would have to be extremely lucky to make a choice of filter which is consistent with an adequate model for Z_t in that it implied reasonable component models. The example in the last subsection illustrates this point for X-11. Use of nonstandard options in X-11 or other methods in group I may increase the chances that an adjustment method will be consistent with the data, but the number of available options in such methods is necessarily limited and options are generally selected subjectively, not objectively based on a model for the data. Thus, methods in group I are at a disadvantage when it comes to being consistent with the data.

The methods in group II afford the opportunity to begin with reasonable component models. Because the model for Z_t is determined (up to parameter estimates) by the specified component models, it is important when using these approaches to perform diagnostic checks upon the adequacy of the resulting model for Z_t. Even when the originally specified component models appear reasonable, if the model for Z_t is deficient in some way then these component models may not be consistent with an adequate model for Z_t. To determine if the resulting seasonal adjustment is consistent with the data, one would first have to find an adequate model for Z_t, and then proceed in the manner discussed in Section 7.4.

Another point to consider about methods in group II is that the overall model for Z_t should be estimated subject to the constraints imposed by the component models. Depending on the complexity of the component models, this may be a difficult task -- Engle (1978) was unable to estimate his model for Z_t subject to all the constraints of his component models, while Akaike (1980), using simpler component models, was able to do this.

In striving for consistency with the data, methods in group III have a potential advantage in that they begin with a model for the observed data. However, this advantage will be completely lost if the starting point is an inadequate model for Z_t; hence, diagnostic checking of the model is important here, too. The reasonableness of the assumptions leading from the model for Z_t to the component models should also be considered. Usually these assumptions are spelled out explicitly for these methods, which allows them to be readily evaluated.

In Section 7.4 we saw that a seasonal adjustment filter does not completely determine models for the components and Z_t. This makes it somewhat difficult to evaluate the assumptions being made about the components for methods in group I, requiring an analysis like that of Section 7.4 for each series. Typically methods in group I are applied without knowing what is being assumed. Regarding methods in groups II and III, there generally exist multiple sets of component models leading to the same model for Z_t. To avoid this identification problem, a particular choice must be made. In our opinion, problems arise when this process is given insufficient attention and the choice is not justified — this is why we attempt to justify our choices in Section 6. Again, methods in group III have a potential advantage here in that this approach forces consideration of the range of possible component models consistent with the model for Z_t. Methods in group II often choose component models based on considerations other than the suitability of their expression of beliefs about seasonality - considerations such as simplicity of the resulting estimation of the model for Z_t.

In conclusion, we favor adjustment methods in group III because we believe the model for Z_t is a logical starting point in developing an adjustment method that will be consistent with the data, and because we feel that acceptable assumptions, such as those offerred in Section 6, can be made leading from the model for Z_t to component models.

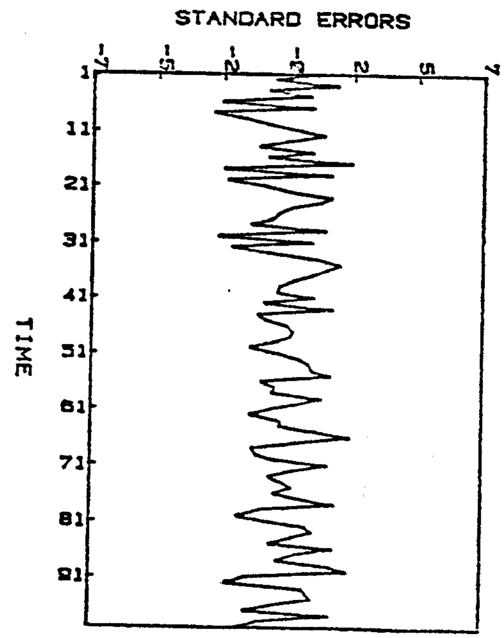

Figure 7.1
CASE A

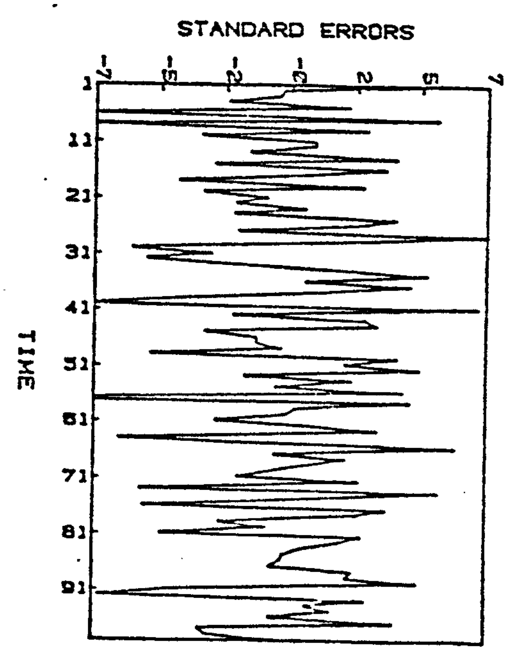

Figure 7.2
CASE B

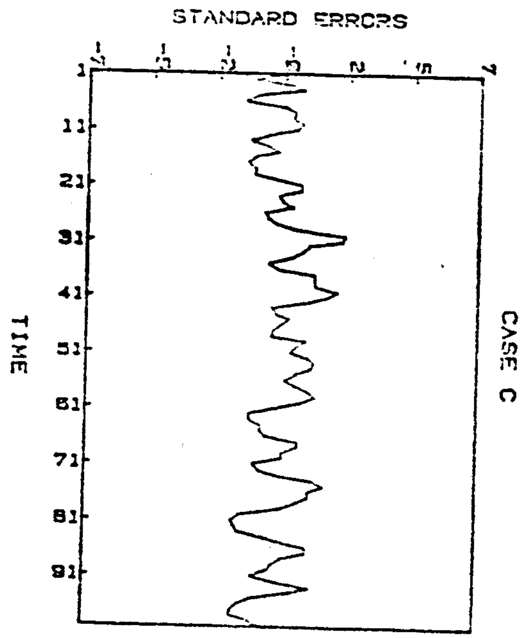

Figure 7.3
CASE C

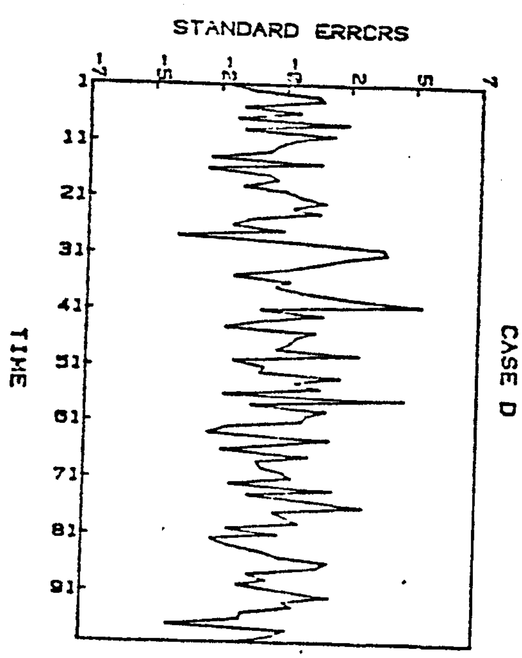

Figure 7.4
CASE D

Figure 7.5

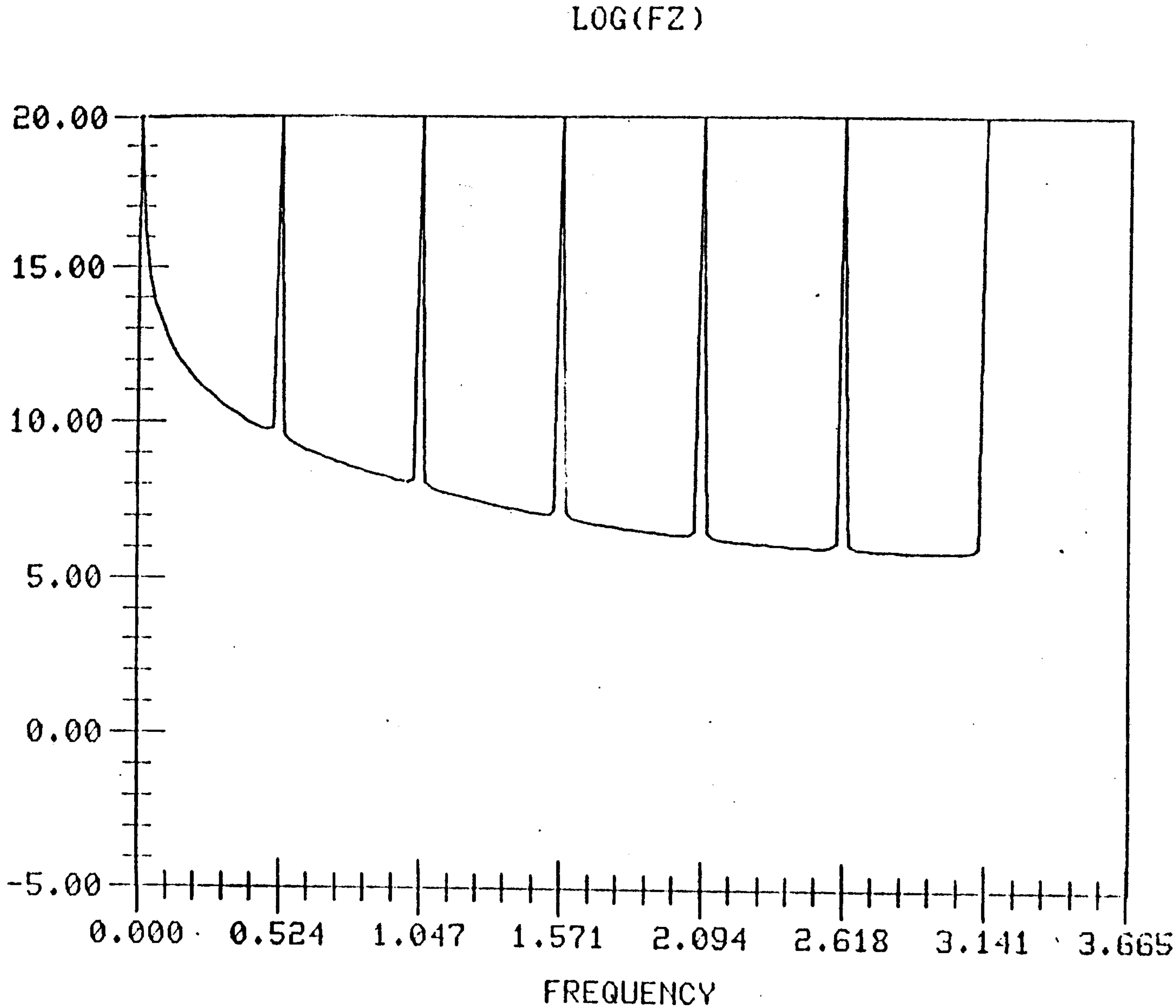
LOG(FZ)
20.00
15.00
10.00
5.00
0.00
-5.00
0.000
0.524
1.047
1.571
2.094
2.618
3.141
3.665
FREQUENCY

Figure 7.6

LOG(FS) -- X11 IMPLIED

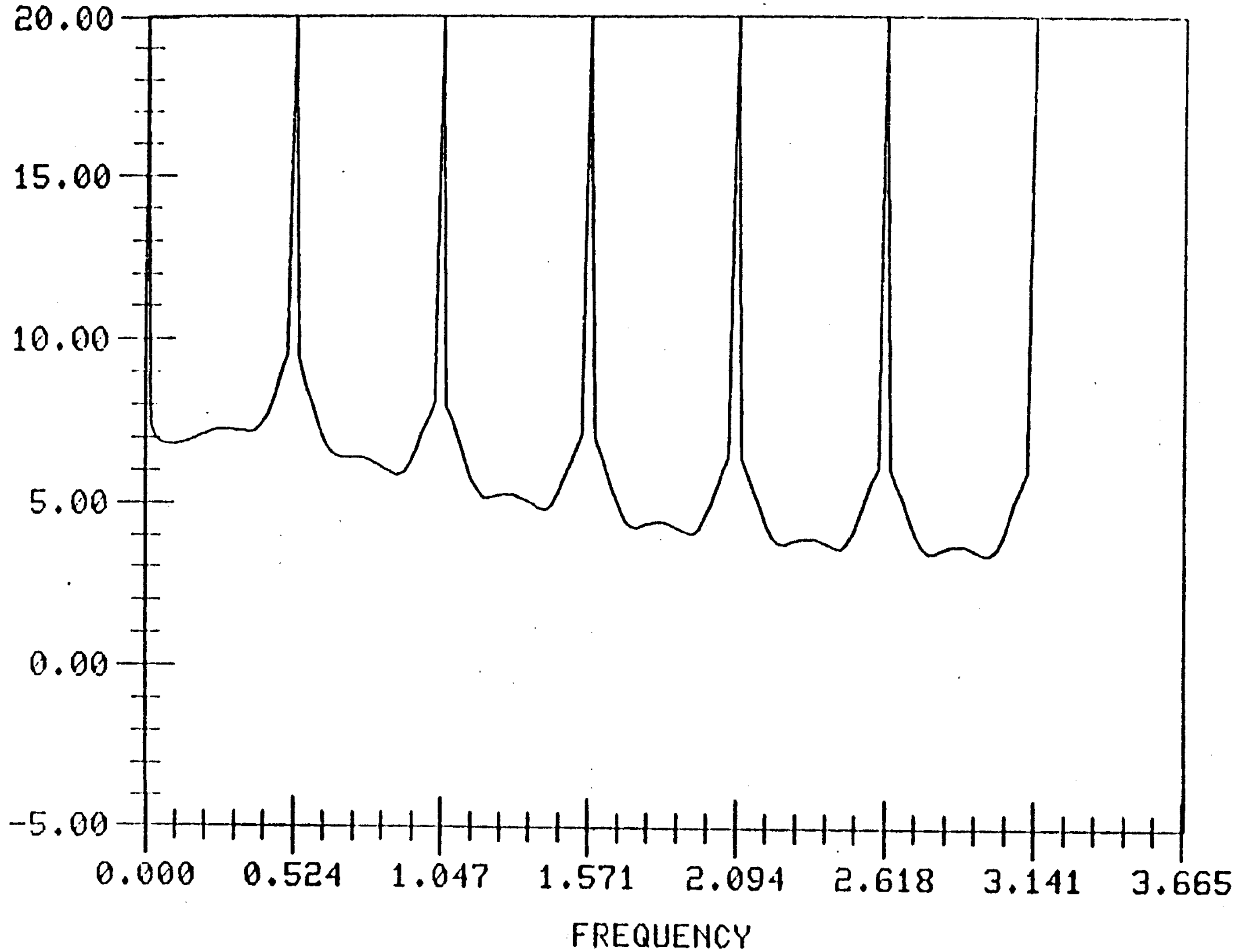

Figure 7.7

LOG(FN) -- X11 IMPLIED

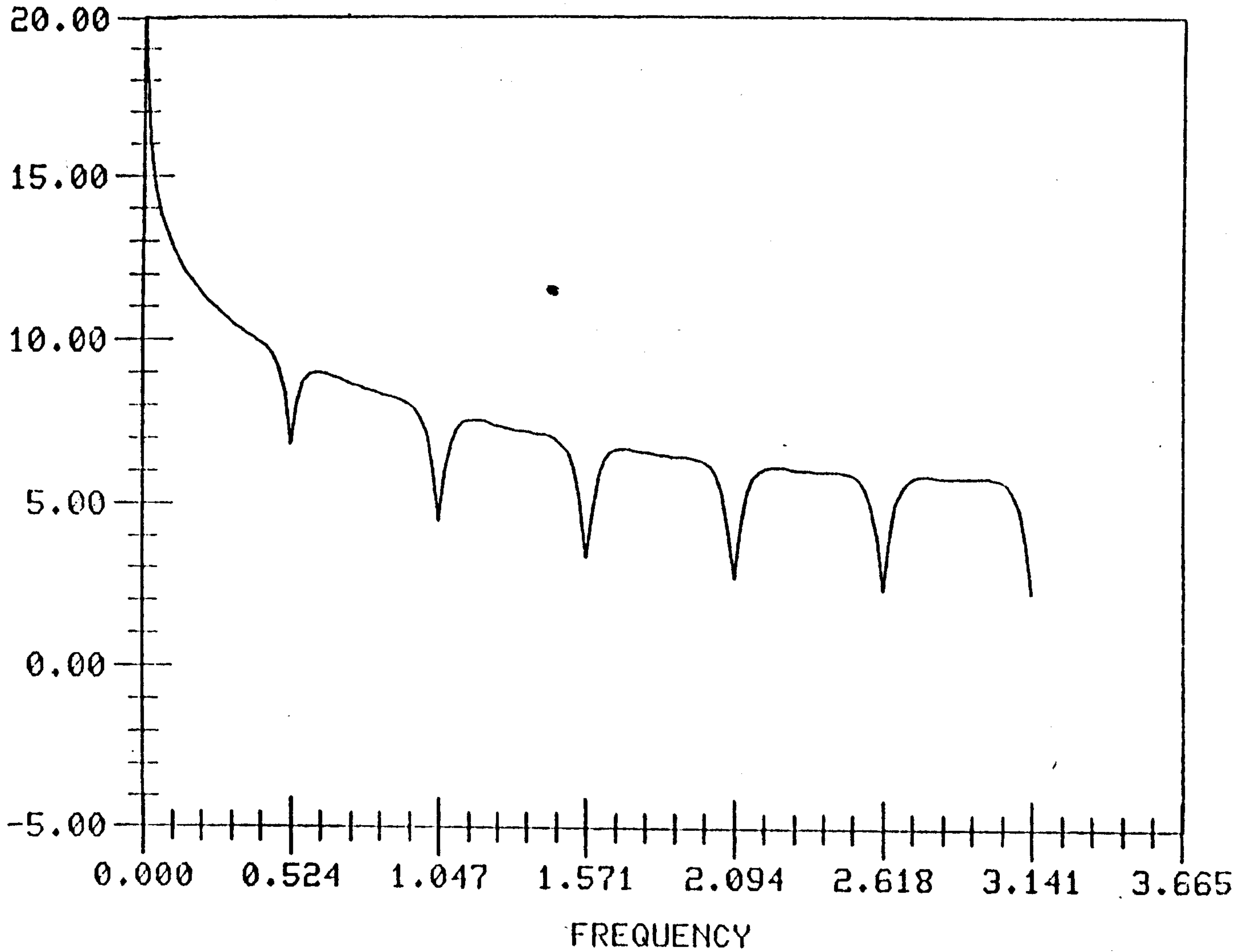

Figure 7.8

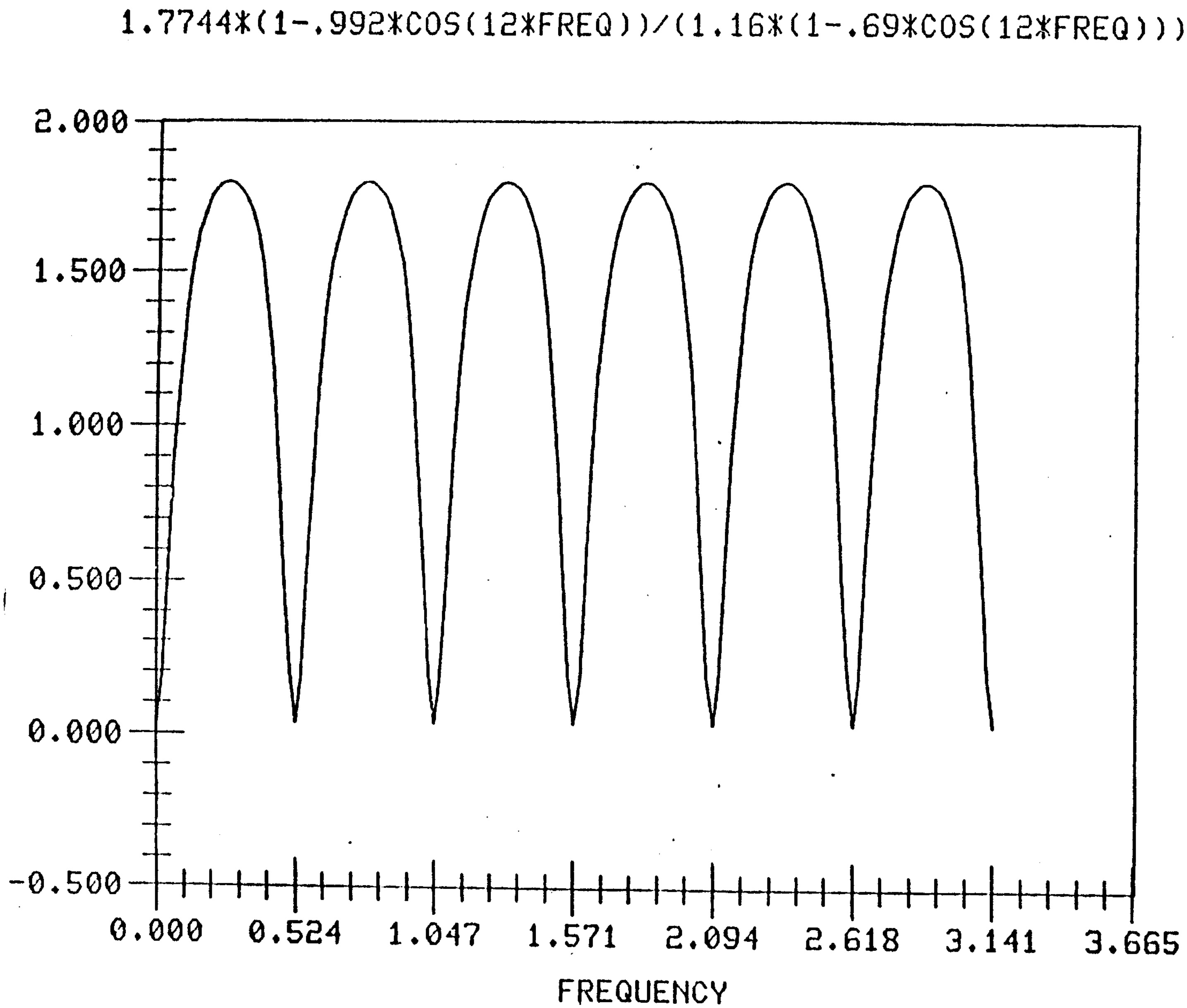
1.7744*(1-.992*COS(12*FREQ))/(1.16*(1-.69*COS(12*FREQ)))
2.000
1.500
1.000
0.500
0.000
-0.500
0.000
0.524
1.047
1.571
2.094
2.618
3.141
3.665
FREQUENCY

Figure 7.9

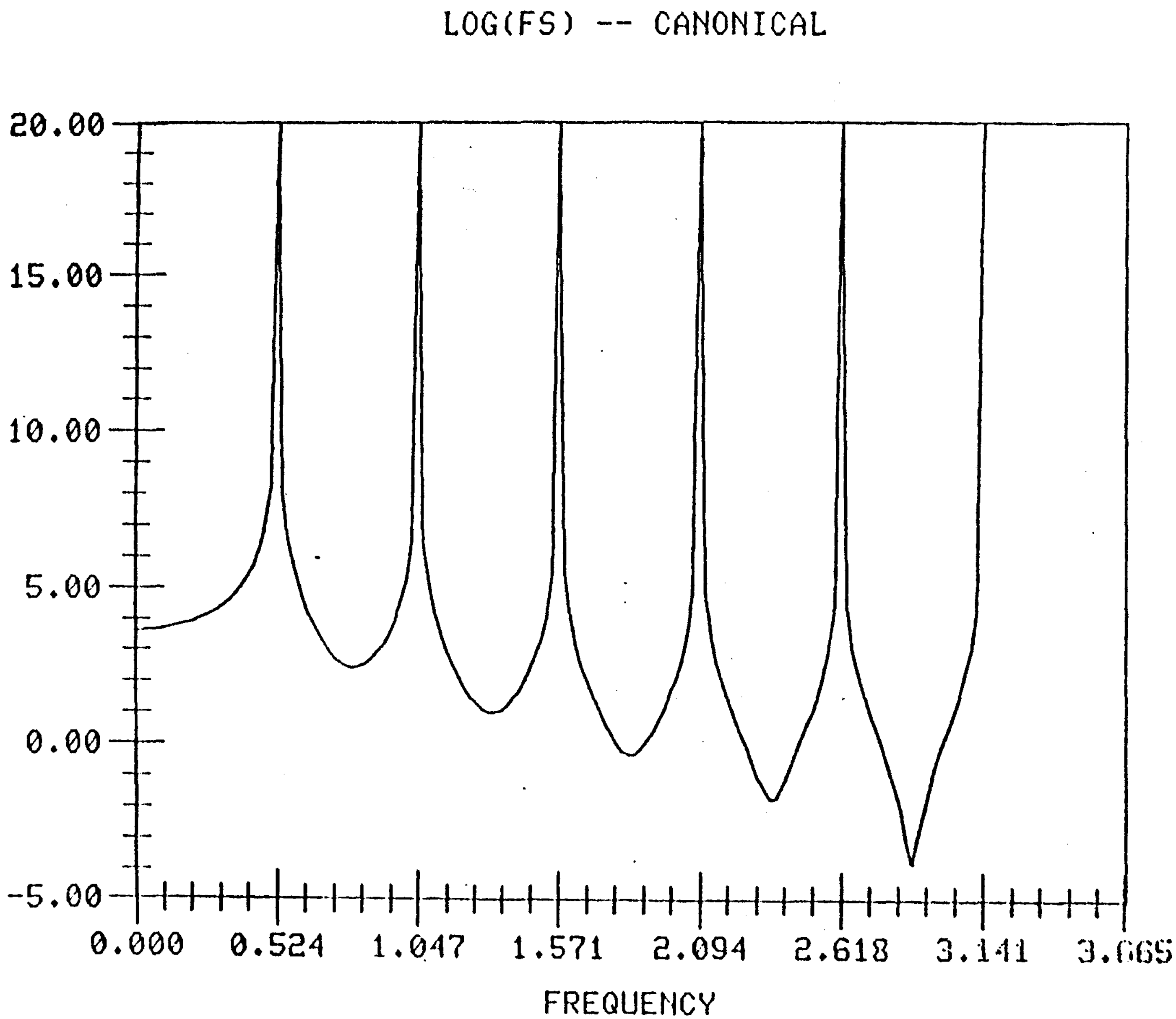
LOG(FS) -- CANONICAL
20.00
15.00
10.00
5.00
0.00
-5.00
0.000
0.524
1.047
1.571
2.094
2.618
3.141
3.665
FREQUENCY

Figure 7.10

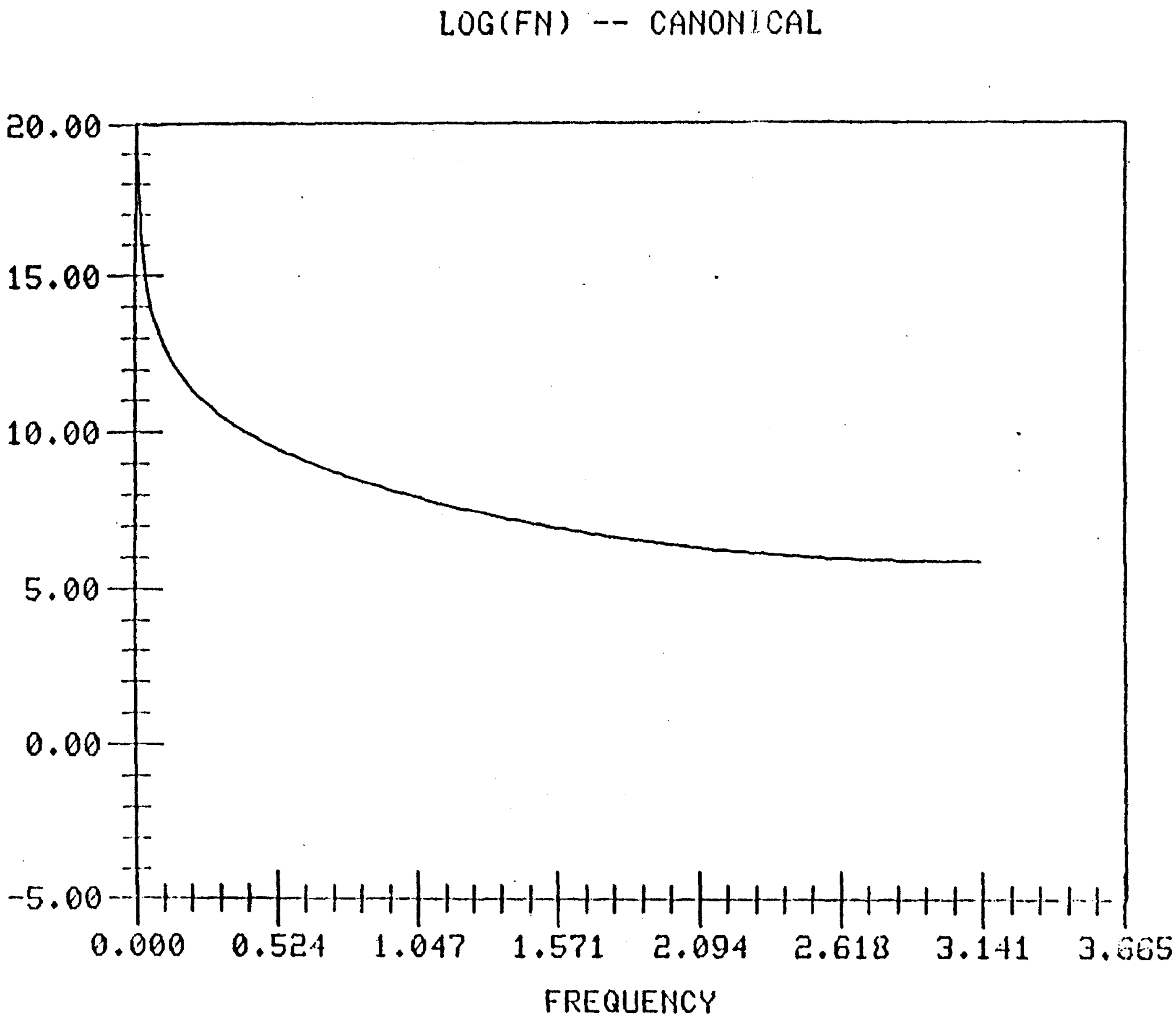
LOG(FN) -- CANONICAL
20.00
15.00
10.00
5.00
0.00
-5.00
0.000
0.524
1.047
1.571
2.094
2.618
3.141
3.665
FREQUENCY

Table 7.1

Mean Squared Total Revisions When

$$(1-B)(1-B^{12})Z_t = (1-\theta_1 B)(1-\theta_{12} B^{12})a_t \quad (\sigma_a^2 = 1)$$

θ_1	θ_{12}	X-11	Model-Based (min seasonal)	Model-Based (max seasonal)
.3	.5	.123*	.133	.177
.5	.9	.059	.032*	.114
.9	.7	.130	.079	.049*

*minimum across the row

Table 7.2

k	1	2	3	4	5	6	7	8	9	10	11	12
$r_k(\hat{a})$	.00	.00	.11	-.06	.02	.13	-.07	.07	.02	-.02	.06	.09

k	13	14	15	16	17	18	19	20	21	22	23	24
$r_k(\hat{a})$	-.03	.04	-.01	-.13	-.08	-.02	.10	-.08	-.08	-.01	-.03	-.05

k	25	26	27	28	29	30	31	32	33	34	35	36
$r_k(\hat{a})$	-.02	-.01	-.01	.01	-.12	-.02	.00	-.14	-.02	-.03	-.14	-.07

Footnotes

[1]Persons himself refers to a 1910 study by E.W. Kemmerer in which seasonal adjustment was done. Yule (1921) says that the four components were fixed by 1914, and quotes March (1905) as saying that one must distinguish ". . . des changements annuels, des changements polyannuels (décennaux par exemple), des changements seculáires, sans parler des périodes plus courtes qu' une anncé," which roughly translate to the seasonal, cyclical, secular trend, and residual components.

[2]For example, direct estimation of seasonal effects using complete calendar year data with an upward trend will result in seasonal factors that are too low in January and too high in December. Also, seasonality in a series makes direct estimation of trend difficult. This dilemma eventually led to iteration between trend and seasonal estimation - something currently done in X-11.

[3]Since S_t and N_t will be nonstationary, they will require starting values. We may want to allow these starting values to be correlated, and only assume b_t and c_t are independent.

[4]It is mathematically possible for the model for Z_t to be such that a decomposition according to these assumptions does not exist; however, we have rarely found this to happen in practice.

[5]Wecker (1978) makes similar comments about why "overadjustment," the production of dips at seasonal frequencies in the spectrum of the adjusted series, should not be regarded as a problem.

[6]For X-11 with standard options the final adjustment is effectively obtained three years after the initial adjustment, Young (1968). For the model based methods considered (min and max seasonal) the filters can be quite long so that the final adjustment comes much later or is effectively never achieved.

[7]Bell (1984) discusses the assumptions under which (7.4) provides the minimum mean squared error (linear) estimator of N_t when S_t or N_t or both is nonstationary.

[8]They actually give models for the trend (T_t) and irregular (I_t). The model for $N_t = T_t + I_t$ can be obtained from these.

[9]$\hat{f}_N(\lambda)$ was computed directly using (7.12), but $\hat{f}_S(\lambda)$ was obtained as $\hat{f}_Z(\lambda) - \hat{f}_N(\lambda)$ to satisfy (7.1). Due to the small number of significant digits provided by Cleveland and Tiao (1976), computing $\hat{f}_S(\lambda)$ directly from (7.11) would not have satisfied (7.1).

[10] There is also an infinite peak at $\lambda = 0$ due to the (1-B) factor implied by the $1-B^{12}$ in (7.9). It would not necessarily appear if another set of models, e.g. Cleveland's (1972), were used to approximate X-11.

References

Abraham, B., and Box, G. E. P. (1979), "Bayesian Analysis of Some Outlier Problems in Time Series," Biometrika, 66, 229-236.

Abrahams, D. M., and Dempster, A. P. (1979), "Research on Seasonal Analysis," ASA/Census Research Project Final Report, Statistical Research Division, Bureau of the Census.

Akaike, H. (1980), "Seasonal Adjustment by a Bayesian Modeling," Journal of Time Series Analysis, 1, 1-13.

Anderson, O. (1927), "On the Logic of the Decomposition of Statistical Series Into Separate Components," Journal of the Royal Statistical Society Ser. A, 90, 548-569.

BarOn, R. R. V. (1973), "Analysis of Seasonality and Trends in Statistical Series, Vol. 1, Methodology, Causes and Effects of Seasonality," Technical Publication No. 39, Jerusalem: Israel Central Bureau of Statistics.

_____ (1978), "The Analysis of Single and Related Time Series into Components: Proposals for Improving X-11," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 107-158.

Bartlett, M. S. (1946), "On the Theoretical Specification and Sampling Properties of Autocorrelated Time Series," Journal of the Royal Statistical Society Ser. B, 8, 27-41.

_____ (1950), "Periodogram Analysis and Continuous Spectra," Biometrika, 37, 1-16.

Bell, W. R. (1984), "Signal Extraction for Nonstationary Time Series," Annals of Statistics, 12, to appear.

Beveridge, W. H. (1921), "Weather and Harvest Cycles," Economic Journal, 31, 429-452.

_____ (1922), "Wheat Prices and Rainfall in Western Europe," Journal of the Royal Statistical Society, 85, 412-459.

Blanc-Lapierre, A., and Fortet, R. (1965), Theory of Random Functions, New York: Gordon and Breach.

Board of Governors of the Federal Reserve System (1981), "Seasonal Adjustment of the Monetary Aggregates: Report of the Committee of Experts on Seasonal Adjustment Techniques," Washington, D.C.: Federal Reserve Board.

Box, G. E. P., Hillmer, S. C., and Tiao, G. C. (1978), "Analysis and Modeling of Seasonal Time Series," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 309-334.

Box, G. E. P., and Jenkins, G. M. (1970), Time Series Analysis: Forecasting and Control, San Francisco: Holden Day.

Box, G. E. P., and Tiao, G. C. (1975), "Intervention Analysis with Applications to Economic and Environmental Problems," Journal of the American Statistical Association, 70, 70-79.

Brewer, K. R. W. (1979), "Seasonal Adjustment of ARIMA Series," Économie Appliquée, 1, 7-22.

Brewer, K. R. W., Hagan, P. J., and Perazzeli, P. (1975), "Seasonal Adjustment Using Box-Jenkins Models," Proceedings of the 40th Session of the International Statistical Institute, Warsaw, 133-139.

Britten, R. H., and Sydenstricker, E. (1922), "Mortality from Pulmonary Tuberculosis in Recent Years," Public Health Reports, 37, U.S. Public Health Service, 2843-2858.

Burman, J. P. (1979), "Seasonal Adjustment - A Survey," in Forecasting, ed. S. Makridakis and S. C. Wheelwright, TIMS Studies in the Management Sciences, 12, Amsterdam: North Holland, 45-57.

_____ (1980), "Seasonal Adjustment by Signal Extraction," Journal of the Royal Statistical Society Ser. A, 143, 321-337.

Buys Ballot, C. H. D. (1847), Les Changements Périodiques de Température, Utrecht: Kemink et Fils.

Carmichael, F. L. (1927), "Methods of Computing Seasonal Indexes: Constant and Progressive," Journal of the American Statistical Association, 22, 339-354.

Champernowne, D. G. (1948), "Sampling Theory Applied to Autoregressive Sequences," Journal of the Royal Statistical Society Ser. B, 10, 204-231.

Chang, I. (1982), "Outliers in Time Series," Ph.D. Thesis, University of Wisconsin-Madison.

Clendenin, J. C. (1927), "Measurement of Variations in Seasonal Distribution," Journal of the American Statistical Association, 22, 213-220.

Cleveland, W. P. (1972), "Analysis and Forecasting of Seasonal Time Series," Ph. D. Thesis, University of Wisconsin-Madison.

_____ (1979), "Report to the Review Committee of the ASA/Census Project on Time Series," ASA/Census Research Project Final Report, Statistical Research Division, Bureau of the Census.

Cleveland, W. P., and Tiao, G. C. (1976), "Decomposition of Seasonal Time Series: A Model for the X-11 Program," Journal of the American Statistical Association, 71, 581-587.

Cleveland, W. S., Dunn, D. M., and Terpenning, I. J. (1978), "SABL: A Resistant Seasonal Adjustment Procedure With Graphical Methods for Interpretation and Diagnosis," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 201-231.

Cochrane, D., and Orcutt, G. H. (1949), "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," Journal of the American Statistical Association, 44, 32-61.

Cooley, J. W., and Tukey, J. W. (1965), "An Algorithm for the Machine Calculation of Complex Fourier Series," Mathematics of Computation, 19, 297-301.

Cowden, D. J. (1942), "Moving Seasonal Indexes," Journal of the American Statistical Association, 37, 523-524.

Crum, W. L. (1923a), "The Use of the Median in Determining Seasonal Variation," Journal of the American Statistical Association, 18, 607-614.

_____ (1923b), "Cycles of Rates on Commercial Paper," Review of Economics and Statistics, 5, 17-29.

_____ (1925), "Progressive Variation in Seasonality," Journal of the American Statistical Association, 20, 48-64.

Crutchfield, J. and Zellner, A. (1963), Economic Aspects of the Pacific Halibut Fishery, U.S. Department of the Interior, Washington D.C., Government Printing Office.

Dagum, E. B. (1975), "Seasonal Factor Forecasts from ARIMA Models," Proceedings of the 40th Session of the International Statistical Institute, Warsaw, 206-219.

_____ (1978), "A Comparison and Assessment of Seasonal Adjustment Methods for Employment and Unemployment Statistics," Background Paper No. 5, National Commission on Employment and Unemployment Statistics, U.S. Government Printing Office, 1-94.

Daniell, P. J. (1946), "Discussion of Paper by M. S. Bartlett," Journal of the Royal Statistical Society, Suppl., 8, 27.

Denby, L., and Martin, R. D. (1979), "Robust Estimation of the First Order Autoregressive Parameter," Journal of the American Statistical Association, 74, 140-146.

Durbin, J. (1959), "Efficient Estimation of Parameters in Moving Average Models," Biometrika, 46, 306-316.

_____ (1960a), "Estimation of Parameters in Time Series Regression Models," Journal of the Royal Statistical Society Ser. B, 22, 139-153.

_____ (1960b), "The Fitting of Time-Series Models," Review of the International Statistical Institute, 28, 233-243.

Eisenpress, H. (1959), "Regression Techniques Applied to Seasonal Corrections and Adjustments for Calendar Shifts," Journal of the American Statistical Association, 51, 615-620.

Engle, R. F. (1978), "Estimating Structural Models of Seasonality," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 281-297.

Falkner, H. D. (1924), "The Measurement of Seasonal Variation," Journal of the American Statistical Association, 19, 167-179.

Findley, D. F. (1982), "On Backshift-Operator Polynomial Transformations to Stationarity," unpublished manuscript.

Fisher, A. (1937), "A Brief Note on Seasonal Variation," Journal of Accountancy, 64, 174-199.

Fisher, R. A. (1929), "Tests of Significance in Harmonic Analysis," Proceedings of the Royal Society of London Ser. A, 125, 54-59.

Fox, A. J. (1972), "Outliers in Time Series," Journal of the Royal Statistical Society Ser. B, 43, 350-363.

Geweke, J. (1978a), "Revision of Seasonally Adjusted Time Series," SSRI Report No. 7822, Department of Economics, University of Wisconsin.

_____ (1978b), "The Temporal and Sectoral Aggregation of Seasonally Adjusted Time Series," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 411-427.

Gilbart, J. W. (1852), "On the Laws of the Currency in Ireland, as exemplified in the changes that have taken place in the Amount of Bank Notes in Circulation in Ireland, since the passing of the Acts of 1845," Journal of the Statistical Society of London, 15, 307-325.

Godfrey, M. D., and Karreman, H. (1967), "A Spectrum Analysis of Seasonal Adjustment," in Essays in Mathematical Economics in Honor of Oskar Morgenstern, ed. M. Shubik, Princeton: Princeton University Press, 367-421.

Granger, C. W. J. (1978a), "Seasonality: Causation, Interpretation, and Implications," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 33-46.

_____ (1978b), "Response to Discussants," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 55.

_____, and Joyeux, R. (1980), "Introduction to Long-Memory Time Series Models and Fractional Differencing," Journal of Time Series Analysis, 1, 15-30.

Gressens, O. (1925), "On the Measurement of Seasonal Variation," Journal of the American Statistical Association, 20, 203-209.

Grether, D. M., and Nerlove, M. (1970), "Some Properties of 'Optimal' Seasonal Adjustment," Econometrica, 38, 682-703.

Hald, A. (1948), The Decomposition of a Series of Observations Composed of a Trend, a Periodic Movement, and a Stochastic Variable, Copenhagen: G. E. C. Gads Forlag.

Hall, L. W. (1924), "Seasonal Variation as a Relative of Secular Trend," Journal of the American Statistical Association, 19, 156-166.

Hannan, E. J. (1960), "The Estimation of Seasonal Variation," Australian Journal of Statistics, 2, 1-15.

_____ (1963), "The Estimation of Seasonal Variation in Economic Time Series," Journal of the American Statistical Association, 58, 31-44.

_____ (1964), "The Estimation of a Changing Seasonal Pattern," Journal of the American Statistical Association, 59, 1063-1077.

_____ (1967), "Measurement of a Wandering Signal Amid Noise," Jouranl of Applied Probability, 4, 90-102.

Hannan, E. J., Terrell, R. D., and Tuckwell, N. (1970), "The Seasonal Adjustment of Economic Time Series," International Economic Review, 11, 24-52.

Hart, W. L. (1922), "The Method of Monthly Means for Determination of a Seasonal Variation," Journal of the American Statistical Association, 18, 341-349.

Hatanaka, M. and Suzuki, M. (1967), "A Theory of the Pseudospectrum and Its Application to Nonstationary Dynamic Econometric Models," in Essays in Mathematical Economics in Honor of Oskar Morgenstern, ed. Martin Shubik, Princeton: Princeton University Press, 443-466.

Henshaw, R. C. (1966), "Application of the General Linear Model to Seasonal Adjustment of Economic Time Series," Econometrica, 34, 381-395.

Hillmer, S. C., Bell, W. R., and Tiao, G. C. (1983), "Modeling Considerations in the Seasonal Adjustment of Economic Time Series," to appear in the Proceedings of the Conference on Applied Time Series Analysis of Economic Data, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census.

Hillmer, S. C., and Tiao, G. C. (1982), "An ARIMA-Model-Based Approach to Seasonal Adjustment," Journal of the American Statistical Association, 77, 63-70.

Homan, J. (1933), "Adjusting for the Changing Date of Easter in Economic Series," Journal of the American Statistical Association, 28, 328-332.

Jones, H. L. (1943), "Fitting Polynomial Trends to Seasonal Data by the Method of Least Squares," Journal of the American Statistical Association, 38, 453-465.

Jorgenson, D. W. (1964), "Minimum Variance, Linear, Unbiased Seasonal Adjustment of Economic Time Series," Journal of the American Statistical Association, 59, 681-724.

_____ (1967), "Seasonal Adjustment of Data for Econometric Analysis," Journal of the American Statistical Association, 62, 137-140.

Joy, A., and Thomas, W. (1927), "Adjustments for the Influence of Easter in Department Stores Sales," Journal of the American Statistical Association, 22, 493-496.

_____ (1928), "The Use [illegible] Averages in the Measurement of Seasonal Variations," Journal of the American Statistical Association, 23, 241-252.

Kendall, M. G. (1945), "On the Analysis of Oscillatory Time-Series," Journal of the Royal Statistical Society Ser. B, 108, 93-129.

Kenny, P., and Durbin, J. (1982), "Local Trend Estimation and Seasonal Adjustment of Eocnomic and Social Time Series," Journal of the Royal Statistical Society Ser. A, 145, 1-28.

King, W. W. I. (1924), "An Improved Method for Measuring the Seasonal Factor," Journal of the American Statistical Association, 19, 301-313.

Kitagawa, G., and Gersch, W. (1983), "A Smoothness Priors - State Space Modeling of Time Series with Trend and Seasonality" to appear in Journal of the American Statistical Association.

Kolmogorov, A. N. (1939), "Sur L'interpolation et Extrapolation des Suites Stationnaires," C.R. Acad. Sci. Paris, 208, 2043-2045.

_____ (1941), "Interpolation und Extrapolation von stationären zufälligen Folgen," Bull. Acad. Sci. U.R.S.S. Ser. Math., 5, 3-14.

Kuznets, S. (1932), "Seasonal Pattern and Seasonal Amplitude: Measurement of Their Short-Time Variations," Journal of the American Statistical Association, 27, 9-20.

_____ (1933), Seasonal Variations in Industry and Trade, New York: National Bureau of Economic Research.

Ljung, G. M., and Box, G. E. P. (1978), "On a Measure of Lack of Fit in Time Series Models," Biometrika, 65, 297-303.

Lovell, M. C. (1963), "Seasonal Adjustment of Economic Time Series and Multiple Regression Analysis," Journal of the American Statistical Association, 58, 993-1010.

_____ (1966), "Alternative Axiomatizations of Seasonal Adjustment," Journal of the American Statistical Association, 61, 800-802.

Macauley, F. R. (1931), The Smoothing of Time Series, New York: National Bureau of Economic Research.

Makridakis, S., and Hibon, M. (1979), "Accuracy of Forecasting: An Empirical Investigation," Journal of the Royal Statistical Society Ser A, 142, 97-125.

Mann, H. B., and Wald, A. (1943), "On the Statistical Treatment of Linear Stochastic Difference Equations," Econometrica, 11, 173-220.

March, L. (1905), "Comparaison Numérique de Courbes Statistiques," Jl. de la Société de Statistique de Paris.

Marris, S. N. (1960), "The Measurement of Calendar Variation," in Seasonal Adjustment on Electronic Computers, Organization for Economic Cooperation and Development, 345-360.

Martin, R. D. (1980), "Robust Estimation of Autoregressive Models," in Directions in Time Series, eds. D. R. Brillinger and G. C. Tiao, Institute of Mathematical Statistics, 228-254.

McKenzie, S. (1984), "Concurrent Seasonal Adjustment with Census X-11," Journal of Business and Economic Statistics, 2, to appear.

Melnick, E. L., and Moussourakis, J. (1974), "Filter Design for the Seasonal Adjustment of a Time Series," Communications in Statistics, 3, 1171-1186.

Mendershausen, H. (1937), "Annual Survey of Statistical Technique: Methods of Computing and Eliminating Changing Seasonal Fluctuations," Econometrica, 5, 234-262.

_____ (1939), "Eliminating Changing Seasonals by Multiple Regression Analysis," Review of Economics and Statistics, 21, 171-177.

Moran, P. A. P (1947), "Some Theorems on Time Series," Biometrika, 34, 281-291.

Nerlove, M. (1964), "Spectral Analysis of Seasonal Adjustment Procedures," Econometrica, 32, 241-286.

Nerlove, M., Grether, D. M., and Carvalho, J. L. (1979), Analysis of Economic Time Series: A Synthesis, New York: Academic Press.

Newbold, P. (1980), "A Note on Relations Between Seasonally Adjusted Variables," Journal of Time Series Analysis, 1, 31-35.

Organization for Economic Co-operation and Development (1960), Seasonal Adjustment on Electronic Computers, Paris: OECD.

Persons, W. M. (1919), "Indices of Business Conditions," Review of Economics and Statistics, 1, 5-107.

_____ (1923), "Correlation of Time Series," Journal of the American Statistical Association, 18, 713-726.

Pierce, D. A. (1978), "Seasonal Adjustment When Both Deterministic and Stochastic Seasonality Are Present," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 242-269.

_____ (1980a), "A Survey of Recent Developments in Seasonal Adjustment," The American Statistician, 34, 125-134.

_____ (1980b), "Data Revisions With Moving Average Seasonal Adjustment Procedures," Journal of Econometrics, 14, 95-114.

Plosser, C. I. (1978), "A Time Series Analysis of Seasonality in Econometric Models," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 365-397.

_____ (1979), "Short-term Forecasting and Seasonal Adjustment," Journal of the American Statistical Association, 74, 15-24.

Priestly, M. B. (1965), "Evolutionary Spectra and Non-Stationary Processes," Journal of the Royal Statistical Society Ser. B, 27, 204-237.

Robb, R. A. (1929), "The Variate Difference Method of Seasonal Variation," Journal of the American Statistical Association, 24, 250-257.

Roberts, H. V. (1978a), "Discussion of Paper by Estela Bee Dagum," in "A Comparison and Assessment of Seasonal Adjustment Methods for Employment and Unemployment Statistics," National Commission on Employment and Unemployment Statistics, 116-122.

_____ (1978b), "Comments on 'The Analysis of Single and Related Time Series into Components: Proposals for Improving X-11' by Raphael Raymond V. BarOn," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 161-170.

Robinson, E. A. (1982), "A Historical Perspective of Spectrum Estimation," Proceedings of the IEEE (Special Issue on Spectral Estimation), 70, 885-907.

Rosenblatt, H. M. (1965), "Spectral Analysis and Parametric Methods for Seasonal Adjustment of Economic Time Series," Bureau of the Census Working Paper No. 23.

Schlicht, E. (1981), "A Seasonal Adjustment Principle and a Seasonal Adjustment Method Derived From This Principle," Journal of the American Statistical Association, 76, 374-378.

Schuster, A. (1898), "On the Investigation of Hidden Periodicities with Application to a Supposed 26-day Period of Meteorological Phenomena," Terrestrial Magnetism and Atmospheric Electricity, 3, 13-41.

Shiskin, J. (1957), "Electronic Computers and Business Indicators," Occasional Paper 57, National Bureau of Economic Research.

_____ (1978), "Seasonal Adjustment of Sensitive Indicators," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 97-103.

Shiskin, J., Young, A. H., and Musgrave, J. C. (1967), "The X-11 Variant of the Census Method II Seasonal Adjustment Program," Technical Paper No. 15, U.S. Department of Commerce, Bureau of Economic Analysis.

Sims, C. A. (1974), "Seasonality in Regression," Journal of the American Statistical Association, 69, 618-626.

_____ (1978) "Comments on 'Seasonality: Causation, Interpretation, and Implications' by Clive W. J. Granger," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 47-49.

Slutsky, E. (1937), "The Summation of Random Causes as the Source of Cyclic Processes," Econometrica, 5, 105-146.

Snow, E. C. (1923), "Trade Forecasting and Prices," Journal of the Royal Statistical Society, 86, 332-376.

Sobel, E. L. (1967), "Prediction of a Noise-distorted, Multivariate, Non-stationary Signal," Journal of Applied Probability, 4, 330-342.

_____ (1971), "Approximate Best Linear Prediction of a Certain Class of Stationary and Nonstationary Noise-Distorted Signals," Journal of the American Statistical Association, 66, 363-372.

Stephenson, J. A., and Farr, H. T. (1972), "Seasonal Adjustment of Economic Data by Application of the General Linear Statistical Model," Journal of the American Statistical Association, 67, 37-45.

Tintner, G. (1940), The Variate Difference Method, Bloomington: Principia Press.

Tukey, J. W. (1950), "The Sampling Theory of Power Spectrum Estimates," Proceedings of the Symposium on Applications of Autocorrelation Analysis to Physical Problems, Office of Naval Research, 47-67.

_____ (1978), "Comments on 'Seasonality: Causation, Interpretation, and Implications' by Clive W. J. Granger," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 50-54.

Walker, A. M. (1961), "Large Sample Estimation of Parameters for Moving-Average Models," Biometrika, 48, 343-357.

_____ (1962), "Large Sample Estimation of Parameters for Autoregressive Processes with Moving-Average Residuals," Biometrika, 49, 117-131.

Wallis, K. F. (1974), "Seasonal Adjustment and Relations Between Variables," Journal of the American Statistical Association, 69, 18-32.

_____ (1978), "Seasonal Adjustment and Multiple Time Series Analysis," in Seasonal Analysis of Eocnomic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 347-357.

Watts, D. G. (1978), "Comments on 'Estimating Structural Models of Seasonality' by Robert F. Engle," in Seasonal Analysis of Economic Time Series, U.S. Department of Commerce, Bureau of the Census, 303-307.

Wecker, W. E. (1978), "Discussion of 'Seasonal Adjustment When Both Deterministic and Stochastic Seasonality are Present' by David A. Pierce," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, U.S. Department of Commerce, Bureau of the Census, 274-280.

Whittle, P. (1952), "Tests of Fit in Time Series," Biometrika, 39, 309-318.

_____ (1953a), "The Analysis of Multiple Stationary Time Series," Journal of the Royal Statistical Society Ser. B, 15, 125-139.

_____ (1953b), "Estimation and Information in Stationary Time Series," Arkiv für Matematik, 2, 423-434.

_____ (1954a), "A Statistical Investigation of Sunspot Observations with Special Reference to H. Alfven's Sunspot Model," The Astrophysical Journal, 120, 251-260.

_____ (1954b), "Some Recent Contributions to the Theory of Stationary Processes," appendix 2 to A Study in the Analysis of Stationary Time Series (2nd ed.) by Herman Wold, Uppsala: Almqvist and Wicksells, 196-233.

_____ (1963), Prediction and Regulation by Linear Least-Square Methods, Princeton: Van Nostrand.

Wiener, N. (1949), The Extrapolation, Interpolation and Smoothing of Stationary Time Series with Engineering Applications, New York: Wiley.

Wold, H. (1938), A Study in the Analysis of Stationary Time Series, Uppsala: Almqvist and Wicksells.

Yaglom, A. M. (1962), An Introduction to the Theory of Stationary Random Functions, Englewood Cliffs: Prentice-Hall.

Young, A. H. (1965), "Estimating Trading-Day Variation in Monthly Economic Time Series," Technical Paper 12, Bureau of the Census.

_____ (1968), "Linear Approximations of the Census and BLS Seasonal Adjustment Methods," Journal of the American Statistical Association, 63, 445-471.

Yule, G. U. (1921), "On the Time Correlation Problem, With Especial Reference to the Variate-Difference Correlation Method," Journal of the Royal Statistical Society, 84, 497-526.

_____ (1926), "Why Do We Sometimes Get Nonsense-Correlations Between Time-Series?," Journal of the Royal Statistical Society, 89, 1-64.

_____ (1927), "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," Philosophical Transactions of the Royal Society A, 226, 267-298.