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Short-Term Energy Outlook Supplement:
Energy Price Volatility and Forecast Uncertainty[1]

Summary

It is often noted that energy prices are quite volatile, reflecting market participants' adjustments to new information from physical energy markets and/or markets in energy-related financial derivatives. Price volatility is an indication of the level of uncertainty, or risk, in the market. This paper describes how markets price risk and how the market-clearing process for risk transfer can be used to generate "price bands" around observed futures prices for crude oil, natural gas, and other commodities. These bands provide a quantitative measure of uncertainty regarding the range in which markets expect prices to trade.

The Energy Information Administration's (EIA) monthly *Short-Term Energy Outlook* (*STEO*) publishes "base case" projections for a variety of energy prices that go out 12 to 24 months (every January the STEO forecast is extended through December of the following year). EIA has recognized that all price forecasts are highly uncertain and has described the uncertainty by identifying the market factors that may significantly move prices away from their expected paths, such as economic growth, Organization of Petroleum Exporting Countries (OPEC) behavior, geo-political events, and hurricanes. However, these descriptions do not provide a quantitative measure of the range of uncertainty regarding an expected future price. Nor do they indicate whether the uncertainty has increased or decreased since the last forecast was published.

Beginning with the October 2009 issue, the *STEO* will publish confidence intervals for crude oil and natural gas futures prices. A *confidence interval* is a range of prices between a low and a high price, i.e., the confidence limits. The range of the confidence interval is determined by the *confidence level*. The confidence level represents the probability that the final market price for a particular futures contract, e.g., December 2010 crude oil, will fall somewhere within the lower and upper range of prices. For

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example, if a confidence level of 95 percent is specified, then a range of prices can be estimated for any future month within which there is a 95-percent probability the price of the commodity in the expired contract's delivery month will fall within that range. The higher the specified confidence level, the wider the range between the lower and upper confidence limits.

Confidence intervals for expected prices can be calculated using a variety of alternative techniques, including estimates based on past price volatility, statistical analysis of past forecast errors, or estimates of parameter uncertainty in an econometric energy price forecasting equation. Such backward-looking approaches, notwithstanding their merits, cannot reflect changes in current market conditions and expectations that may lead to greater or lesser uncertainty about the future at any given time.

The *STEO* will instead focus on a measure of uncertainty derived from the New York Mercantile Exchange (NYMEX) light sweet crude oil options and natural gas options markets. EIA will derive confidence intervals around expected futures prices using the "implied volatilities" of these options. Implied volatility is nothing more than a standard deviation for expected returns embedded in the option's price. If an option's price is observed in the market, then a pricing model can be "run backwards" to calculate the volatility embedded in that price. This represents a market-cleared estimate of implied volatility, i.e., a buyer and seller have agreed on the value of an option. The advantage of this method is that it produces an assessment of future price uncertainty based directly on *current* market data and highly informed market participants' expectations. This approach is used by the U.S. Federal Reserve Board and the Bank of England to assess market uncertainty. Commercial banks also use implied volatilities to derive probability estimates that market participants assign to different price outcomes. As an estimate of risk, the use of implied volatility is well accepted in the financial literature.[1]

Based on our review of the relevant empirical literature, as summarized in the latter sections of this report, EIA has determined that implied volatilities currently provide the most useful estimate of the market's expectation for the range in which prices likely will trade. Therefore, EIA will use this method to generate its confidence intervals around the NYMEX futures prices.

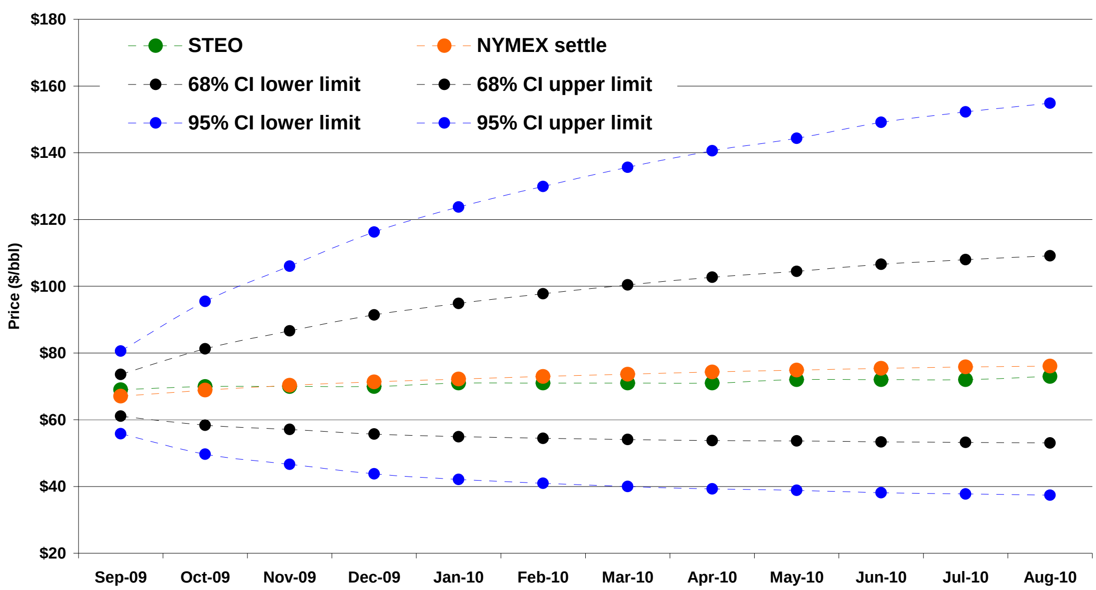
The markets represented by the NYMEX light, sweet crude oil and natural gas futures prices are directly related to the markets represented by the *STEO* West Texas Intermediate (WTI) crude oil and Henry Hub natural gas spot-price forecasts. However, while the NYMEX and *STEO* prices are expected values for physical commodities delivered to equivalent physical markets, and they generally are close, they are not identical. The NYMEX price is a firm price at which delivery is made in the month specified in the futures contract. The terminal NYMEX price at which all outstanding futures contracts go to physical delivery is determined on the last day of trading for a particular futures contract. Typically, this termination occurs in the calendar month preceding the delivery month (e.g., the terminal futures price for December 2010 WTI delivered to Cushing, OK, is determined on the last day of trading for that contract: November 19, 2010.) The *STEO* forecasts are average daily spot prices expected in the actual delivery month (e.g., during December 2010). These daily prices are reported by industry publications on a daily day-ahead basis during the delivery month (e.g., the average natural gas price for Henry Hub, LA, for next-day delivery *during* the month of December 2010). While both forecasts reflect delivered prices for the same commodity at the same location, they are measuring expected prices over different pricing intervals.

Because the implied volatilities and confidence intervals derived from the NYMEX options markets are derived from prices on the NYMEX futures and options markets, the confidence intervals are presented in relation to the NYMEX futures prices and not the STEO forecast price.

As shown in Figure 1, the implied volatility from options can imply a wide range of future price uncertainty. For example, as of July 27, 2009, the 95-percent confidence interval for the January 2010 WTI futures price ranged from $42 to $124 per barrel. At a lower confidence level, the price band narrows, but even a 68-percent confidence interval ranges from $55 to $95 per barrel. Moreover, confidence intervals typically widen as the length of the forecast horizon grows. By July 2010, the 95-percent confidence interval for the WTI price ranges from $38 to $152 per barrel.

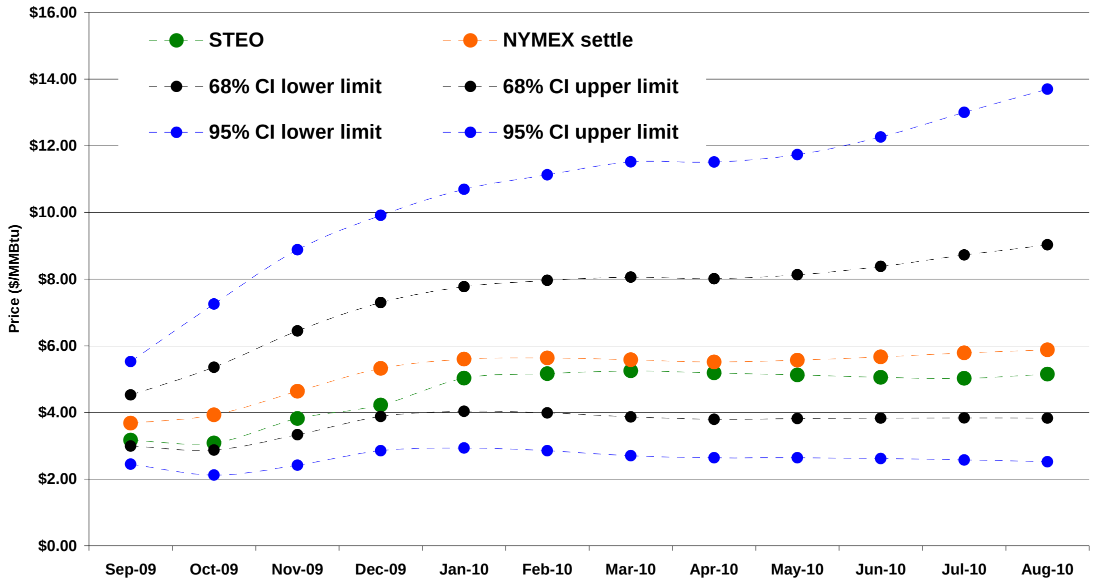
In Figure 2, the 95-percent confidence interval for the January 2010 NYMEX Henry Hub natural gas futures ranges from $2.93 and $10.69 per million Btu (MMBtu). The 68-percent confidence interval ranges from $4.03 to $7.78 per MMBtu. By July 2010, the 95-percent confidence interval for the natural gas futures price ranges from $2.60 to $13.00 per MMBtu.

Figure 1. NYMEX WTI oil futures price with two confidence intervals



Source: NYMEX.
Based on NYMEX average prices and implied volatilities, July 27-31, 2009.
Note: CI is abbreviation for Confidence Interval.

Figure 2. NYMEX natural gas futures price with two confidence intervals



Source: NYMEX.
Based on NYMEX average prices and implied volatilities, July 27-31, 2009.
Note: CI is abbreviation for Confidence Interval.

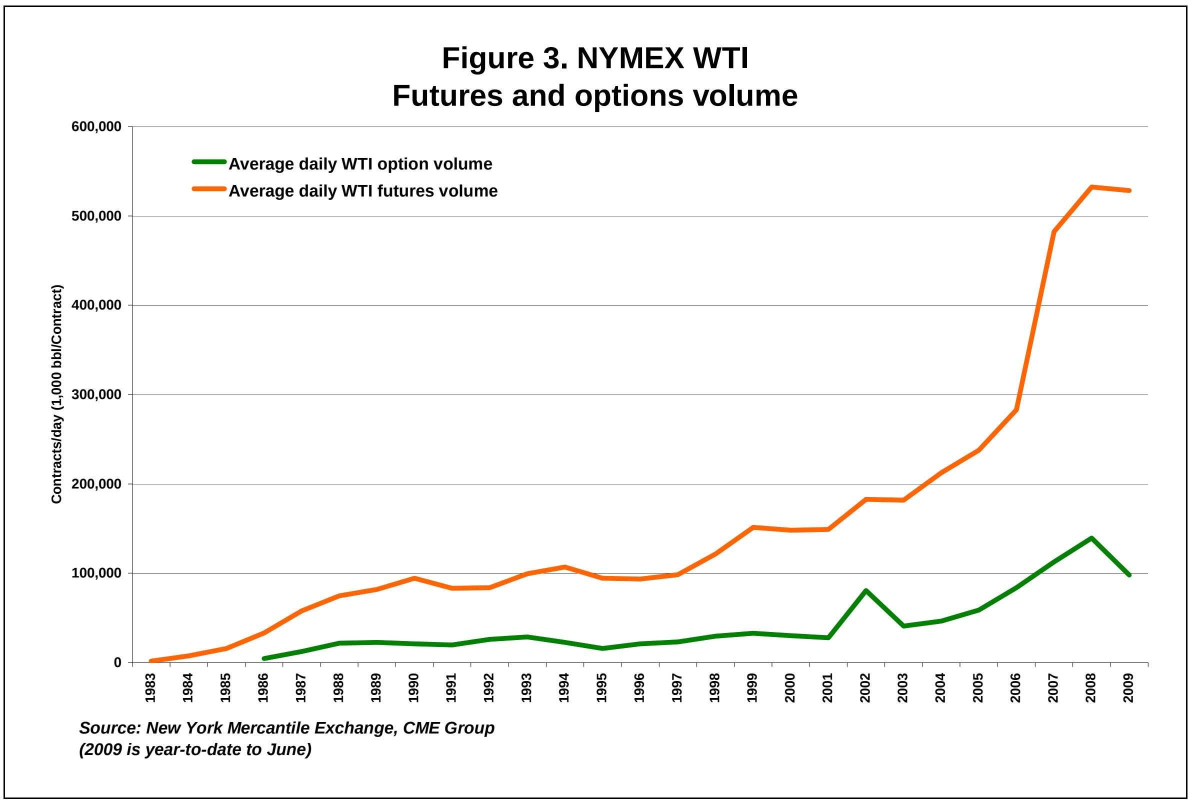
The 95-percent confidence intervals for future oil and natural gas prices derived from market data are quite wide, reflecting market participants' view that prices can change rapidly and cover a wide range in a short time interval. Additionally, realized prices at a future date can, and often do, diverge significantly from prices at which futures contracts for that delivery date are traded at earlier points in time. EIA believes that such confidence intervals provide important insight into the uncertainty inherent in price projections developed by EIA and other forecasting organizations, and provide a useful context for evaluating forecast performance.

Section 1 of this paper provides a general background on commodity futures and options markets, paying particular attention to the energy futures and options markets operated by the NYMEX. Section 2 describes institutional features of these markets necessary to understand the processes, procedures, and rules under which futures and options are traded. Section 3 lays out how market participants offset their risks or take on exposures via the trading process itself. Next, Section 4 examines the history of the analysis of randomness, focusing in particular on random price behavior, which is necessary for an understanding of the models employed to quantify the market's risk assessments. Section 5 shows how EIA and other analysts calculate confidence intervals for energy commodity prices, and then, Section 6 documents the parameters used to construct these measures. Section 7 summarizes daily procedures EIA will use to map confidence intervals for NYMEX energy futures. And, lastly, Section 8 outlines areas for future EIA research regarding confidence-interval estimation using market-derived parameters of expected price distributions.

1. Background on Commodity Futures and Options Markets

Futures and options markets evolved to manage the risks associated with commodity-price volatility. The earliest recorded instance of a formal futures market occurs in 1730 with the Dojima Rice Exchange in Osaka, Japan (Matao, 1999). These markets provide producers, consumers, merchants, and speculators a risk-transfer mechanism in the form of contracts for the future delivery of a physical commodity. Via the trading process, futures markets continually process information from these agents, and reflect it back as a price at which supply and demand clears the market.

The deepest, most liquid global markets for oil futures and options are the NYMEX light, sweet crude oil markets. These markets comprise what is known colloquially in the oil industry as the WTI market. Trading in WTI futures began on March 30, 1983, at NYMEX. Options on WTI futures began trading November 14, 1986. The NYMEX futures call for physical delivery of WTI in Cushing, Oklahoma. WTI is the benchmark crude oil for the Americas, meaning most spot-, forward- and over-the-counter-market transactions are priced on the basis of WTI prices discovered as a result of NYMEX trading.

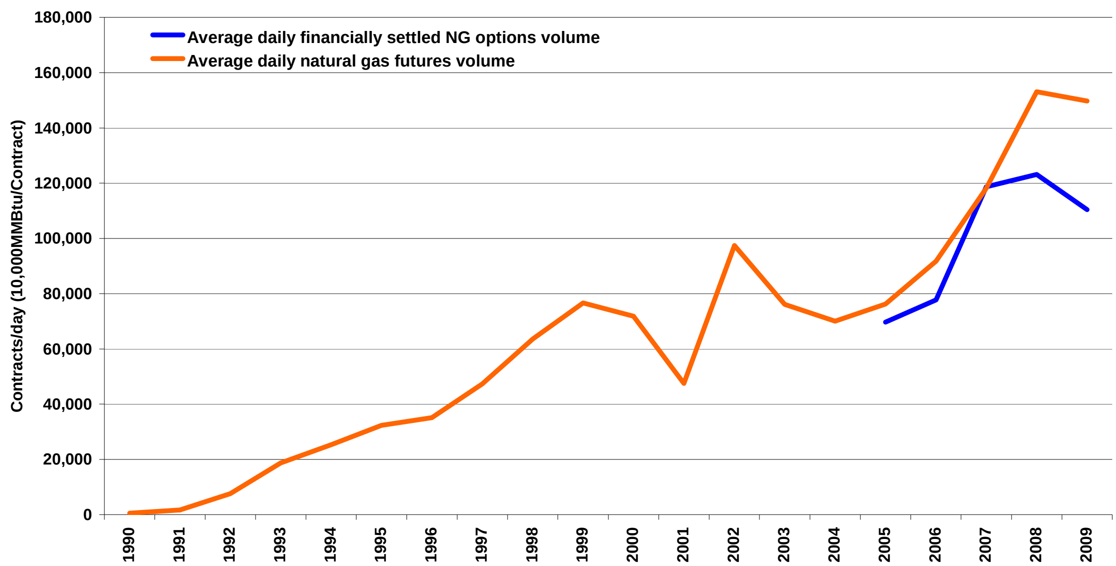


The WTI futures are the most actively traded physical commodity futures in the world. Figure 3 shows that trading in NYMEX WTI futures during 2008 averaged more than

500,000 contracts per day, which is equivalent to 500 million barrels of oil (each contract is for 1,000 barrels). NYMEX options trading volume approached almost 140,000 contracts per day in 2008. Average daily NYMEX WTI futures volume in the January-through-June 2009 period was 528,496 contracts. Options on WTI futures averaged 97,956 contracts per day over the same period.

NYMEX natural gas futures also are physical-delivery contracts, specifying delivery of 10,000 MMBtu of pipeline-quality natural gas per contract to the Henry Hub, Louisiana, pipeline system. Natural gas futures began trading on NYMEX April 3, 1990. Options on NYMEX natural gas futures began trading November 2, 1992. The NYMEX natural gas futures contract is the benchmark for North American physical hub trading. Contracts in the United States and Canada are traded on a "basis," or price differential, to NYMEX futures, with the basis depending on the location of a particular hub or trading location.

Figure 4. NYMEX natural gas (Henry Hub) Futures and options volume



Source: New York Mercantile Exchange, CME Group
(2009 is year-to-date to June)

In 2008 and the first half of 2009, trading in NYMEX natural gas futures averaged more than 151,000 contracts per day (Figure 4). NYMEX natural gas options trading volume averaged more than 6,500 contracts per day in 2008 and the first half of 2009, according to the CME Group, parent company of the NYMEX. In the January-through-June 2009 period, an average 149,781 NYMEX natural gas futures contracts traded, while an average 3,833 options that exercise into the underlying futures contract traded daily. The deeper NYMEX financially settled natural gas options, launched August 15, 2005, traded an average 110,365 contracts per day for the first six months of 2009 – almost 30 times the daily volume of the option settling into a futures contract.[2] The financially settled options' terminal values are calculated using the underlying futures price settlement for the business day prior to the futures final settlement date (i.e., penultimate settle).[3]

2. Institutional Features of Futures and Options Markets

The futures and options markets operated by the NYMEX are the most transparent and accessible of the major trading venues for crude oil in the world. Other markets, i.e., the over-the-counter financial markets, the spot and forward physical markets, while active, do not have the high visibility of the exchange-traded futures and options contracts.[4]

U.S. futures exchanges like the NYMEX are self-regulatory organizations (SROs), subject to Federal oversight by the Commodity Futures Trading Commission (CFTC). Trading can be conducted via open-outcry on a trading floor at the exchange (wherein buyers and sellers literally shout the prices at which they are willing to trade), or electronically on platforms operated by the exchange.

Commodity futures traded on the NYMEX are binding legal obligations to make or take delivery of a specific *physical* commodity at a particular date in the future. Every element of the deliverable commodity is specified in the contract: grade and quality (e.g., light, sweet crude oil or pipeline-quality natural gas); volume (1,000 barrels per contract for WTI; 10,000 MMBtu per contract for natural gas); timing and mode of delivery (via pipeline during the delivery month); delivery location (Cushing, OK, or Henry Hub, LA); *force majeure* events; pricing conventions (dollars and cents per barrel or per MMBtu); and other terms and conditions specified by the Exchange. As the delivery month approaches, the futures price and the spot-market price of the commodity converge. This is due to arbitrage between the physical and financial markets. If futures prices are above spot prices, market participants with access to physical supplies will buy oil to deliver against the futures obligation, thus raising spot prices relative to futures, and vice versa.[5]

All contract terms are standardized except the price at which delivery occurs. Price is determined via trading. After a deal is consummated, the exchange clearinghouse steps in to become seller to all buyers and buyer to all sellers. Since the exchange is now a party to the transaction, the clearinghouse requires collateral in the form of a performance bond on every open contract. Failure to maintain this collateral results in liquidation of the position.

Futures are "marked to market" daily, meaning the price of outstanding contracts at the close of today's trading session is compared to the previous session's close. Gains and losses are allocated by the clearinghouse among buyers and sellers, so that all collateral accounts for open positions have been credited or debited appropriately based on changes in position values that day. At the start of the next trading session, all gains and losses will have been realized from the previous day, and a new trading session will begin. This process will be repeated until positions are either liquidated (i.e., traded out of), the contracts go to delivery, or are financially settled.

Futures options also are legally binding contracts conferring the right, but not the obligation, to buy or sell a futures contract. The right to buy a futures contract is known as a "call option." The right to sell the underlying futures contract is known as a "put option." Because these contracts confer rights, not obligations, they are known as "contingent claims." The option buyer, also known as the "holder" of the contract, does not have to exercise his right to buy or sell the underlying futures contract if doing so causes him to incur a loss. Option sellers, on the other hand, known as "grantors," must stand ready to perform if the option they've sold is exercised by the holder. On the NYMEX, "American-style" options are traded, which allow holders to exercise any time prior to expiry; other types of options are "European-style," which permit exercise only at expiry, and "Asian-style" options, which reference an average price as the underlying variable against which the option settles.

Like futures, options contracts are completely specified by the exchange, including:

- the underlying contractual obligation that is to be delivered in the event of exercise, e.g., a specific futures contract such as December 2015 crude oil or natural gas futures contracts;
- the price at which a buyer can exercise the option into the underlying future, also known as a "strike price";
- the expiration date of the option, the "expiry"; and
- the deadline by which a buyer must convey intent to exercise to the clearinghouse.[6]

The only term not defined in the contract is the price of the option, known as the "premium," which is discovered via the trading process.

3. The "Long" and "Short" of Futures and Options Markets

Trading in futures and options markets occurs between exchange members. The public trades anonymously through member-brokers on the exchange's platforms. Market participants, hedgers or speculators, seeking to get "long," i.e., benefit from prices going up, can:

1. Buy a call. In return for paying the option premium, the buyer has the right to exercise into a "long" position, i.e., the option holder buys futures, in the underlying futures contract if its price exceeds the strike price of the call option, i.e., the call is "in the money". The maximum loss an option buyer faces is the premium paid for the call. If the option is "out of the money," i.e., futures prices are less than the strike price, on the expiration date the option is abandoned, and the premium is forgone.

2. Buy a futures contract. If the contract's price goes up, the position gains penny for penny with each tick above the price level at which the contract was purchased; if prices go down the position will lose penny for penny.

3. Sell a put. In return for granting the option, the put seller receives a premium. This is the maximum gain an option seller can realize. Upon exercise, the put seller is made long a futures contract by the clearinghouse if prices settle below the option's strike, i.e., if the put is in the money.

Those seeking to get "short," i.e., benefit from prices going down, can:

1. Buy a put. The put holder has the right to exercise into the underlying futures contract if prices fall below the strike price of the option, i.e., the put is "in the money". The premium paid for the option is the maximum loss the buyer can incur. If the futures price is above the strike, the put expires out of the money.

2. Sell a futures contract. If the contract's price goes down, the position gains penny for penny with each tick below the level at which the contract was sold; if prices go up, the position will lose one-for-one.

3. Sell a call. Again, the maximum gain to the grantor from selling an option is the premium. The call seller will be made short a futures contract upon exercise at the call's strike price, if the contract-month price settles above the strike price of the option granted.

4. The Behavior of Futures and Options Prices

> *"We would expect people in the market place, in pursuit of avid and intelligent self-interest, to take account of those elements of future events that in a probability sense may be discerned to be casting their shadows before them. (Because past events cast* their *shadows after them, future events can be said to cast their shadows before them.)"* Samuelson (1965, p. 44).

Commodity prices are volatile. Unexpected changes in weather, political regimes, global economic shocks, and countless other factors impact energy markets on a continual basis. News of such events arrives randomly to market participants—sometimes to all, sometimes to a few—and when it does, it causes current assessments of future prices and the range in which prices will trade to change. Sometimes the "news" correctly reflects a change in supply or demand, or both; sometimes it does not.

Understanding the random behavior of prices, commodity and otherwise, has occupied some of the greatest minds of the 20th century, beginning with Louis Bachelier, whose *theorie de la speculation*, published in 1900, ignited a revolution in the study of randomness in science and finance and marked the beginning of the study of Brownian-motion processes (Courtault, *et al*, 2000).[7]

In the 1930s and 1940s, Holbrook Working (1962) suggested an efficient-markets hypothesis by asserting that volatility in prices indicated futures markets were adjusting exactly as they should to the arrival of new information. "Pure random walk in a futures price is the price behavior that would result from perfect functioning of a futures market, the perfect futures market being defined as one in which the market price would constitute at all times the best estimate that could be made, from currently available information, of what the price would be at the delivery date of the futures contracts."[8] Nobel laureate Paul Samuelson (1965) formally demonstrated this proposition.[9]

The high-water mark of these investigations into risk and randomness in financial markets occurred in 1973 with the publication of two papers by Fischer Black, Myron Scholes (1973), and Robert Merton (1973), which presented a closed-form model for the valuation of stock options.[10] The Black-Scholes-Merton (B-S-M) model, as it's come to be known, was extended to commodities by Black (1976).

In Black's model, an option's value, i.e., its "premium," is determined by:

- the volatility of the underlying asset's price;
- its strike price;
- the price of the underlying asset itself, not its return;
- the risk-free interest rate; and
- the time to expiration of the option.

A call's value increases if the underlying futures price increases, volatility increases, or time to expiration increases, all else being equal. A call loses value if interest rates increase or if the strike price is increased, all else being equal. A put's value increases if futures prices decrease, volatility increases, or time to expiration increases, all else being equal. A put loses value if interest rates increase or futures prices increase, all else being equal, again.[11]

A particular type of random walk is assumed in the B-S-M and Black models, known as a geometric Wiener process.[12] In such a process, the likelihood of a 1-percent upward move in an asset's price is equal to the likelihood of a 1-percent downward move over a very small time increment. The most an asset can lose is 100% of its value (i.e., the price distribution is bounded at zero). This means returns would be normally distributed, with constant volatility, while absolute prices would be log-normally distributed at the option's expiry.[13]

The impact of these models on market functioning is significant. Stephen Figlewski (1989) notes, "Among all theories in finance, the Black-Scholes option pricing model has perhaps had the biggest impact on the real world of securities trading. Virtually all market participants are aware of the model and use it in their decision making. Academics regularly test the model's valuation on actual market prices and typically conclude that, while not every feature is accounted for, the model works very well in explaining observed option prices." This also is the case in the commodity option markets, particularly in the oil markets. Most commodity option models start with the Black model and build or modify from there.[14]

In Black's model, commodity futures prices are assumed to be log-normally distributed, so log returns are assumed to be normally distributed.[15] This can be represented in equation (1) below, in which the continuously compounded rate of return over some small period of time is equal to the average rate of return plus a stochastic term. In other words, commodity returns follow a random walk, which is assumed to be zero-drift, plus a random "shock" component.[16]

(1) $\ln\left(f_{(t+dt),k} / f_{t,k}\right) = \mu_k dt + \sigma_k z\sqrt{dt}$, where

$\ln[\bullet]$ = Napierian logarithm, or natural logarithm

$f_{t,k}$ = observed futures price at time = t for the kth-nearby contract

$f_{(t+dt),k}$ = futures price at $t + dt$ for the kth-nearby contract ($dt > 0$)

μ_k = mean logarithmic return

dt = infinitesimal change in time (Δt, as $\Delta t \rightarrow 0$)

$\mu_k dt$ = the "drift" term

σ_k = standard deviation of the kth-nearby contract's returns

z = standard normal random variable with mean = 0, var = 1

$\sigma_k z\sqrt{dt}$ = random-shock[17]

This diffusion process can be used to derive Black's commodity option pricing model under the risk-neutrality argument, as Yoshiki Ogawa (1988) demonstrates. Given these assumptions, we can derive the expected value of a futures price and then specify a confidence interval around this expected value, as is done below.

5. Methodology for Calculating Confidence Intervals

The expected values of the log returns and the futures price are shown in Appendix I to be:

(2) $\mathrm{E}\left[ln\left(f_{\tau,k} / f_{t,k}\right)\right] = \mathrm{E}\left(df_{\tau,k} / f_{t,k}\right) = \eta\tau$, and

(3) $\mathrm{E}\left(f_{\tau,k}\right) = f_{t,k}\, exp\left(\eta\tau\right)$, where

(4) $\eta = \left(\mu_k + \left(\sigma_k^2 / 2\right)\right)\tau$, with

τ = time to expiry (as a percent of a 252-day trading year).

In the standard formulation of the confidence interval (CI) for the returns, the expected value η is set to zero, consistent with the martingale assumption for futures,[18] so

(5) $\mu_k \tau = \left(-\sigma_k^2 / 2\right)\tau$, and

the confidence interval[19] around the expected value of the returns would be given by

(6) $\text{Prob}\left(\left(-\sigma_k^2 / 2\right)\tau - \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right) < \mu_k \tau < \left(-\sigma_k^2 / 2\right)\tau + \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right) = 1 - \alpha$, where

> $1-\alpha$ = confidence coefficient, or the degree of confidence; e.g., when α =0.05, the degree of confidence is 95 percent,[20] and
>
> $z_{\alpha/2}$ = Standardized normal value for α level of confidence.

This is a confidence interval for the normally distributed percent returns of the futures price. The mean and variance fully describe the expected distribution of returns.

In price terms, the confidence interval would be:

(7) $\text{E}\left(f_{\tau,k}\right) > f_{t,k} * exp\left(\left(-\sigma_k^2 / 2\right)\tau - \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right)$ for the lower limit,

and

(8) $\text{E}\left(f_{\tau,k}\right) < f_{t,k} * exp\left(\left(-\sigma_k^2 / 2\right)\tau + \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right)$ for the upper limit.

This is the standard confidence interval formulation for a lognormally distributed random variable. However, in the case of upper limit for the CI, this formulation produces inconsistent results for small confidence levels and for narrow CIs over longer time intervals, i.e., the upper limit of the CI could be less than the forward price anchoring the interval in such instances, depending on the confidence level specified.

A formulation for a confidence interval in which the upper limit is less than the forward curve used to compute the limit severely restricts the explanatory power regarding the range of price uncertainty. Therefore, we impose a correction equal to $\left(\sigma_k^2 / 2\right)\tau$ on either side of the confidence-interval calculation, which forces the upper confidence limit to converge on the forward price for small and narrow CI specifications, as the negative sigma-squared term's effect is negated. Thus the CI takes the form (for price):

(9) $\text{E}\left(f_{\tau,k}\right) > f_{t,k} * exp\left(-z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)$ for the lower limit,

and

(10) $E(f_{\tau,k}) < f_{t,k} * exp(z_{\alpha/2} * \sigma_k \sqrt{\tau})$ for the upper limit.

This is consistent with similar imposed corrections in the literature.[21]

The intuition of this confidence interval is consistent with the assumption of the geometric Wiener process. The expected futures price is dependent on a zero-drift term and a random-shock term. Per the assumptions and model, the only source of variation in the futures price between the time a price is observed and the as-yet-to-be-realized price at the expiry of the associated options contract are the random shocks resulting from the arrival of new information in the market. Thus, the zero-drift process is maintained while the futures price traverses a path consistent with the stochastic term, i.e., $\sigma * z\sqrt{dt}$, in this model.

6. Which σ to Use: Implied or Historical Volatility?

The above methodology for determining the confidence interval for the energy futures price requires an appropriate measure of variance for the price distribution.

The σ_k input to the Black pricing equation can be estimated in a variety of ways using historical futures price realizations. Typically, the historical volatility for the k^th-nearby futures contract ($\hat{\sigma}_k^h$) is computed using the maximum-likelihood estimator and daily historical futures price relatives:

$$\hat{\sigma}_k^h = \sqrt{(1/\text{n} - 1) * \sum_{i=1}^{n} [R_{i,k} - \overline{R}_k]^2}$$

$R_{i,k}$ = daily price relative = $ln[f_{t,k} / f_{t-1,k}]$, with i = t, t-1, …, t-n
Here, "n" is the number of days used to construct the historical volatility estimate, e.g., for 20 price observations, n = 20, we have 19 daily returns in the volatility estimate beginning with today's price relative at time t through to the price relative at time $t-19$ days ago.

$\overline{R}_k = \sum_{i=1}^{n} [R_{i,k}] / n$ = calculated average return of daily price relatives of the k^th nearby contract.

Alternatively, an historical volatility also can be estimated via econometric methods, such as autoregressive conditional heteroskedasticity (ARCH)-based models and regime-switching models, as was done by Duffie and Gray (1995).[22]

An entirely different tack can be taken by inverting the Black option pricing model.[23] All of the pricing inputs required to run Black's model are readily observable, with the exception of the volatility, i.e., the standard deviation of returns, or "σ_k", of the k^th futures contract. Consequently, given the underlying futures price, the strike price, interest rates, and time to expiry, the volatility for options written on the k^{th}-nearby futures contract that equates the Black model with the cleared option premium that trades in the market can be solved for by running Black's model "backwards." This is known as the market-based "Black implied volatility,"[24] denoted σ_k^i.

The Black implied volatility turns out to be a better estimate of realized volatility for commodities than the various historical volatilities, based on empirical tests of the markets. The implied volatility, also referred to as "the implied" by market participants, is a forward-looking estimate of the expected volatility for prices derived from the market-clearing process. Like the futures price, the implied is a cleared market-based parameter of an expected distribution. Hence, in terms of describing the range in which futures prices have the highest likelihood of trading, the implieds are expected to be more accurate than any of the historical measures of volatility.

Stein (1989) summarizes options-based estimates of volatility thusly: "Options can be thought of as reflecting a speculative market in volatility – the implied volatility on a given option (obtained by inverting a Black-Scholes-type formula) should equal the average volatility that is expected to prevail over the life of that option."

In an empirical analysis of crude oil, heating oil and natural gas trading markets, Duffie and Gray (1995) found that "Black-Scholes option-implied volatility, when available, provides a more reliable forecast of future volatility than either historical volatility, or than can be obtained from the standard Markovian models of volatility that we have examined; the latter included simple regime-switching models and ARCH-based models such as GARCH, EGARCH and multi-variate GARCH."[25]

Szakmary, Ors, Kim and Davidson (2003) assessed implied versus historical volatility and GARCH-based estimates in their analysis of 35 futures markets. Included in their study were crude oil, heating oil, gasoline, and natural gas futures. In particular, they find the implied volatilities of the energy options to be among the best predictors of realized volatility in the futures contracts they studied.

As a practical matter, financial markets in which futures and options trade collect the most current information on supply, demand and expectations vis-à-vis the future available. Bernanke (2004b) notes: "To assess recent developments in the oil market, it would be useful to know whether the high price of oil we observe today is a temporary spike or is instead the beginning of an era of higher prices. Although no one can know for sure how oil prices will evolve, financial markets are one useful place to learn about informed opinion. Contracts for future deliveries of oil, as for many other commodities, are traded continuously on an active market by people who have every incentive to monitor the energy situation quite closely. Derivative financial instruments, such as options to buy or sell oil at some future date, are also actively traded. The prices observed in these markets can be used to obtain useful information about what traders expect for the future course of oil prices, as well as the degree of uncertainty they feel in predicting the future."

These findings are consistent with other studies surveyed by Poon (2005), who notes, "the volatility forecasting contests show overwhelmingly that option implied volatility has superior forecasting capability, outperforming many historical price volatility models and matching the performance of forecasts generated from time series models that use a large amount of high-frequency data." A similar point was made by Engle (2002), citing research by Poon: "Do GARCH models out-forecast implied volatility models? The answer is complex depending upon the statistical approach to forecast evaluation, but generally it is found that implied volatilities are more accurate forecasts of future volatility than are GARCH models."[26]

Szakmary, *et al*, (2003) state, "Our findings … are consistent with the weak-form efficiency of futures options markets, in that the volatility information embedded in current option prices is a better predictor of future volatility than historical measures of volatility, regardless of how the latter are modeled."[27] They suggest institutional and structural effects, e.g., commodity options and futures typically trade in the same venue and have lower transactions costs versus equities and their associated options, partly explain these results.

Any test of a market's efficiency using a model is a joint test of the model used and the market's efficiency, as noted by Jorion (1995), Dimson and Mussavian (1998), Szakmary *et al* (2003) and Poon (2005).[28] Essentially, market participants trading the options are assumed to behave as if the model reflects reality, and the market is assumed to behave in a manner that would be consistent with the underlying assumptions of the model.

Additionally, implied volatility can be treated as a sufficient statistic and used to derive a confidence interval around the mean of the returns distribution. Because the returns are assumed to be normally distributed, their expected mean and variance fully characterize the expected distribution.[29]

7. Procedure Observed for Mapping NYMEX Futures Confidence Intervals

Using standard normal probability distribution tables found in most statistics textbooks, a price range, e.g., a 95-percent confidence interval, or multiple confidence intervals can be described.

The individual parameters used in mapping confidence intervals for WTI and Henry Hub natural gas futures are simple five-day averages computed from NYMEX settlement prices and the NYMEX at- and near-the-money implied volatilities published nightly by the Exchange.[30] These data are used by the Exchange to calculate margins for futures and options portfolios using the Exchange's Standard Portfolio Analysis of Risk (SPAN).[31] Averaging these observations reduces the likelihood a single observation will be overweighted by a large trade, or, at the other extreme, the likelihood a single day's trading is too sparse to produce prices with significant economic information. This averaging is a procedure referenced elsewhere in the literature.[32]

The graphical output produced using these procedures (Figure 1 and 2) shows

- The current *STEO* forecast
- The current NYMEX forward curve.
- The 68-percent (one standard-deviation) CI, and a 95-percent CI around the NYMEX forward curve are specific to each contract month, as the calculation relies on each contract month's futures price and implied volatility to form CIs.

The EIA confidence-interval model for energy futures deliberately chooses at- or near-the-money options to calculate the variance parameter, given the well-known "volatility smile" effects documented in numerous options markets – i.e., the tendency for deep-out- and deep-in-the-money options to have volatilities different from the at- and near-the-money options. This is done for two reasons: 1) The at- and near-the-money options typically are the most liquid options traded; and, 2) they are most sensitive to changes in information affecting the estimation of volatility.

Over the decades during which this phenomenon has been studied, sophisticated models designed to extract market participants' expectations from the "smile" have been developed to provide policy-makers real-time assessments of market uncertainty and the affects their innovations have on asset values, as Clews *et al* (2000), Melick and Thomas (1992), Jackwerth (2004), Bernanke (2004a), and Figlewski (2008) note.[33] There are numerous avenues for further research along these lines and elsewhere for energy markets, as we note below.

8. Areas for Future Research

The use of implied volatility as the best predictor of future realized volatility still is a source of debate. In addition, as one reviewer of this article noted, many of the academic studies cited herein were done for markets other than crude oil and natural gas futures. Results of any econometric test will be a function of the underlying financial variables and the time period covered in the tests. Therefore, results for energy markets will produce valid results for the time period studied. The reviewer also noted that futures have not been conclusively demonstrated to be superior predictors of realized future commodity values. Some studies indicate they are biased predictors of realized values.

Another reviewer noted the model assumed for EIA confidence intervals—geometric Brownian motion— does not explain the extraordinarily sharp price movements seen during the 2008-09 period, when WTI futures traded to more than $145/bbl and months later fell below $40/bbl, only to trade back up to around $70/bbl by mid-2009. He suggested testing for lognormality in prices to assess deviations from this assumption. In addition, he too questioned whether the lognormal price assumption underestimates the likelihood of extreme price realizations going forward (i.e., extreme outcomes are more likely than are implied by the distribution assumed after the Black (1976) model is inverted to recover the variance estimate).

The EIA model uses implied volatilities published by NYMEX, which, as mentioned above, inverts Black's model (i.e., a European-style option model) to calculate implied volatilities. This means the volatility parameter of the expected price distributions for

WTI crude oil and natural gas futures is based on the Black model's assumption the options can only be exercised at expiry. For natural gas, this is wholly consistent with the Exchange's methodology, since it inverts a Black model to solve for the volatility of its financially settled natural gas options, which are European-style options. Crude oil options, however, are American-style options, thus inverting a Black model to solve for implied volatility may underestimate volatility. As part of EIA's ongoing benchmarking of its model, we will test whether the volatility from an American-style model is significantly different from the European model inverted by NYMEX to solve for volatility. Two reviewers noted, the EIA model uses "the prices of American-style options, but their model is for European-style options. We did a quick comparison of the current prices of American and European-style options on WTI futures and found that the value of early exercise has little to no value. …"[34]

EIA will be conducting ongoing tests to benchmark its volatility model and its assumptions, as well as examining recently developed risk-neutral-density techniques to see if they offer better forecasts of realized volatility than the simple implied-volatility model using at- and near-the-money options presented herein. Lastly, EIA will be back-testing this model vis-à-vis confidence intervals developed using historical data and then checking to see if the model contain the realized prices predicted by this specification.

Appendix I: Derivation of the Confidence Interval for Futures Prices

To derive the confidence intervals for futures prices under the assumptions of the Black (1976) and Cox-Ross-Rubenstein (1979) models, we begin with

(1) $\ln\left(f_{(t+dt),k} / f_{t,k}\right) = \mu_k dt + \sigma_k z\sqrt{dt}$, with notation as before in Section 4.

First following Ogawa (1988) and Jarrow and Rudd (1983),[35] both sides of (1) are exponentiated, so

(2) $f_{(t+dt),k} / f_{t,k} = exp\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)$

This expression can be used to derive the expected value of the percent returns. Using Maclaurin's expansion for equation (2) gives

$$f_{(t+dt),k} / f_{t,k} = 1 + \left(\mu_k dt + \sigma_k z\sqrt{dt}\right) + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^2\right] / 2! + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^3\right] / 3! + \ldots$$

Let $f_{(t+dt),k} - f_{t,k} = df_{t,k}$, therefore, re-arranging,

$f_{(t+dt),k} = f_{t,k} + df_{t,k}$, and dividing both sides by $f_{t,k}$, gives

$f_{(t+dt),k} / f_{t,k} = \left(f_{t,k} + df_{t,k}\right) / f_{t,k}$, so

$$1 + df_{t,k} / f_{t,k} = 1 + \left(\mu_k dt + \sigma_k z\sqrt{dt}\right) + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^2\right] / 2! + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^3\right] / 3! + \ldots$$

Thus,

$$df_{t,k} / f_{t,k} = \left(\mu_k dt + \sigma_k z\sqrt{dt}\right) + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^2\right] / 2! + \left[\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)^3\right] / 3! + \ldots$$

Ignoring terms with order of $dt > 1$, since they get infinitesimally smaller as the order of dt increases, yields

(3) $df_{t,k} / f_{t,k} \cong \left(\left[\mu_k + \left(\sigma_k^2 / 2\right) z^2\right] dt\right) + \sigma_k z\sqrt{dt}$

Taking the expectation of (3) gives

(4) $\mathrm{E}\left(df_{t,k} / f_{t,k}\right) \cong \left(\mu_k + \sigma_k^2 / 2\right) dt + \ldots$, as $\mathrm{E}[z] = 0$, and $E\left[z^2\right] = 1$

For expositional convenience, this is treated as an equality. Formula (4) is the *expected value* of the percent returns. In Black's (1976) formulation, the return to a futures contract is zero. In section 2 of his paper, Black concludes: "For these commodities, neither those with long futures positions nor those with short futures positions have significantly positive expected dollar returns." This is consistent with Samuelson's proof (1965) and Ogawa's (1988) derivation, and recently was demonstrated by Hamilton (2009).[36] This is used to define a confidence interval for the return to holding a futures contract.

Let $\left(\mu_k + \sigma_k^2/2\right) = \eta$, so (4) becomes

(5) $$\mathrm{E}\left(df_{t,k} / f_{t,k}\right) = \eta dt$$

Jarrow and Rudd (1983) show the variance and standard deviations of the returns – i.e., $df_{t,k} / f_{t,k}$ – are

(6) $$\mathrm{Var}\left(df_{t,k} / f_{t,k}\right) = \sigma_k^2 dt,$$ therefore

(7) $$\mathrm{StDev}\left(df_{t,k} / f_{t,k}\right) = \sigma_k \sqrt{dt}$$

We also can derive an expression for the expected value of the futures price from the log-normal diffusion above. Taking the expectation in (2) above, gives

$$\mathrm{E}\left(f_{(t+dt),k} / f_{t,k}\right) = \mathrm{E}\left(exp\left(\mu_k dt + \sigma_k z\sqrt{dt}\right)\right)$$

$$\mathrm{E}\left(f_{(t+dt),k}\right) / f_{t,k} = \mathrm{E}\left(exp\left(\left(\mu_k dt\right) * exp\left(\sigma_k z\sqrt{dt}\right)\right)\right)$$

The second expression on the right-hand side in the exponent above, i.e., $\exp\left(\sigma_k z\sqrt{dt}\right)$, is the moment-generating function for a normal random variable.[37] Collecting terms yields:

$$\mathrm{E}\left(f_{(t+dt),k}\right) / f_{t,k} = exp\left(\left(\mu_k dt\right) * exp\left(\left(\sigma_k^2 / 2\right) dt\right)\right),$$ so

$$\mathrm{E}\left(f_{(t+dt),k}\right) = f_{t,k}\, exp\left(\left(\mu_k + \sigma_k^2 / 2\right) dt\right),$$ and, recalling $\left(\mu_k + \sigma_k^2/2\right) = \eta$, this becomes

(8) $$\mathrm{E}\left(f_{(t+dt),k}\right) = f_{t,k}\, exp\left(\eta dt\right)$$

Here it is seen that the expected futures price can be expressed as the current observed futures price times the expected return to holding the futures contract over an arbitrarily small time interval. The mean and variance of the price-relative scale proportionately with time in the geometric Wiener process assumed for futures prices,[38] thus:

$\mu_k dt \propto \mu_k \tau$, and

$\sigma_k^2 dt \propto \sigma_k^2 \tau$, so

$\sigma_k \sqrt{dt} \propto \sigma_k \sqrt{\tau}$

Here, τ = time to expiration (as a percent of a year) in the k^{th}-nearby option contract.

Given the mean and variance are linear in time, the following relationships hold:

$$(5^*) \quad \mathrm{E}\left[ln\left(f_{\tau,k} / f_{t,k}\right)\right] = \mathrm{E}\left(df_{\tau,k} / f_{t,k}\right) = \eta\tau \text{, and}$$

$$(8^*) \quad \mathrm{E}\left(f_{\tau,k}\right) = f_{t,k}\, exp\left(\eta\tau\right)$$

The equation numerals above are starred (*) to indicate these expressions are scaled-by-the-time-to-expiration versions of the original equations bearing those numerals.

In the standard formulation of the confidence interval for the returns, the expected value is

$$(9) \quad \left(\mu_k + \left(\sigma_k^2 / 2\right)\right)\tau = \eta$$

As shown above. Setting $\eta = 0$ above, per Ogawa (1988, p. 55) to be consistent with the martingale assumption, we see

$$(10) \quad \mu_k \tau = \left(-\sigma_k^2 / 2\right)\tau$$

and the confidence interval would be specified as

$$(11) \quad \mathrm{Prob}\left(\left(-\sigma_k^2 / 2\right)\tau - \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right) < \mu_k \tau < \left(-\sigma_k^2 / 2\right)\tau + \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right) = 1 - \alpha \text{,}$$

which, in price terms, would be:

$$(12) \quad \mathrm{E}\left(f_{\tau,k}\right) > f_{t,k} * exp\left(\left(-\sigma_k^2 / 2\right)\tau - \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right) \text{ for the lower limit,}$$

and

$$(13) \quad \mathrm{E}\left(f_{\tau,k}\right) < f_{t,k} * exp\left(\left(-\sigma_k^2 / 2\right)\tau + \left(z_{\alpha/2} * \sigma_k \sqrt{\tau}\right)\right) \text{ for the upper limit.}$$

This is the standard CI formulation. However, in the case of upper limit for for the CI, this formulation produces inconsistent results for small confidence levels and for narrow CIs over longer time intervals, i.e., the upper limit of the CI could be less than the forward price anchoring the interval in such instances, depending on the confidence level specified.

A formulation for a CI in which the upper limit is less than the forward curve used to compute the limit severely restricts the explanatory power regarding the range of price uncertainty. Therefore, we impose a correction equal to $(\sigma_k^2 / 2)\tau$ on either side of the confidence-interval calculation, which forces the upper confidence limit to converge on the forward price for small and narrow CI specifications, as the negative sigma-squared term's effect is negated. Thus the CI takes the form (for price):

(14) $E(f_{\tau,k}) > f_{t,k} * exp(-z_{\alpha/2} * \sigma_k \sqrt{\tau})$ for the lower limit,

and

(15) $E(f_{\tau,k}) < f_{t,k} * exp(z_{\alpha/2} * \sigma_k \sqrt{\tau})$ for the upper limit.

This is consistent with similar imposed corrections in the literature – see, e.g., Newell and Pizer (2003, p. 64).

Appendix II: Derivation of the Cumulative Normal Density for Futures Prices

In this appendix, the cumulative normal density function for commodity prices is derived, consistent with the Black commodity option pricing model.[39]

In Section 5 and Appendix I above, the confidence interval for expected futures prices was obtained, given an implied volatility. In this appendix, that derivation is expanded to show how the cumulative normal density function for commodity prices is derived within the Cox-Ross-Rubinstein (1979) risk-neutral framework.

In a risk-neutral economy, where utility preferences are linear, a hedge position can be constructed that earns the risk-free rate of return, and we can solve for the expected value of a call option.

The derivation for the value of a call option proceeds by solving for the net present value of the option on the kth nearby future at the call's expiration date at time = T. As before, the time to expiration of the kth-nearby future = τ_k = time to expiry as a percent of a year, which, for notational convenience, is written as τ. As before, a 252-day trading year is assumed, and business days between the current time (t) and expiry (T) are counted.

Let $C_{\tau,k}$ be the present value of the call, and $\mathrm{E}[C_{T,k}]$ be the expected value of the call at expiry. Under the risk-neutral assumptions, all assets return the risk-free rate = $exp(r_k * \tau)$, therefore, the expected value of the call in present-value terms would be

$$C_{\tau,k} = \mathrm{E}[C_{T,k}] * exp(-r_k \tau)$$

For expositional ease, let $\beta_{\tau,k} = exp(-r_k \tau)$, so the present value of the call would be

(1) $$C_{\tau,k} = \mathrm{E}[C_{T,k}] * \beta_{\tau,k}$$

The call has value if the futures price at expiry is greater than its strike price (x_k); otherwise, it will expire worthless.[40] This can be expressed as

(2) $$C_{T,k} = max(f_{T,k} - x_k, 0)$$, so

(3) $$C_{\tau,k} = \mathrm{E}[C_{T,k}] * \beta_{\tau,k} = \mathrm{E}[max(f_{T,k} - x_k, 0)] * \beta_{\tau,k}$$

Let $1 - p = \mathrm{Prob}[f_{T,k} < x_k]$, which would render $C_{\tau,k} = 0 * \beta_{\tau,k} = 0$

Let $p = \mathrm{Prob}[f_{T,k} > x_k]$, then $C_{\tau,k} = \mathrm{E}(f_{T,k} - x_k) * \beta_{\tau,k}$

Collecting the mutually exclusive probabilities, yields

$C_{\tau,k} = \{(1-p)(0) + (p)\mathrm{E}[(f_{T,k} \mid f_{T,k} > x_k) - x_k]\} * \beta_{\tau,k}$, which reduces to

(4) $$C_{\tau,k} = \{(p)\mathrm{E}[(f_{T,k} \mid f_{T,k} > x_k) - x_k]\} * \beta_{\tau,k}$$

Concentrating specifically on the probability $p = \mathrm{Prob}[f_{T,k} > x_k]$, the second term in brackets above is $p * x_k$, which is simply the strike price (x_k) times the probability the k[th]-nearby futures price is greater than the call option's strike price (a constant value) at the expiry of the option (ignoring the present-value discount factor). Given the log-normal price assumption, this is equal to

$x_k * p = x_k * \mathrm{Prob}[f_{T,k} > x_k] = \int_{x_k}^{\infty} x_k L(f_{T,k})\, df_{T,k} = x_k * \int_{x_k}^{\infty} L(f_{T,k})\, df_{T,k}$, where,
$L(f_{T,k})$ = log-normal probability density function.

Recall from the derivation above, that the mean and variance of the price-relative scale proportionately with time in the geometric Wiener process assumed for futures prices, so

$$\mathrm{E}(f_{T,k}) = f_{t,k} * exp(\mu_k \tau + \sigma_k z \sqrt{\tau})\text{, therefore,}$$

(5) $$p = \mathrm{Prob}\{f_{t,k} * exp(\mu_k \tau + \sigma_k z \sqrt{\tau}) > x_k\}$$

Going back to the "z" transformation, so as to work with a standard normal random variable,

$$p = \mathrm{Prob}\{z > -([ln(f_{t,k}/x_k) + \mu_k \tau] / \sigma_k \sqrt{\tau})\}\text{, and, this is equal to}$$

(6) $$p = \mathrm{Prob}\{z < ([ln(f_{t,k}/x_k) + \mu_k \tau] / \sigma_k \sqrt{\tau})\}$$

Recalling $(\mu_k + \sigma_k^2/2) = \eta = 0$ above, $\mu_k = -(\sigma_k^2/2)$, thus

(7) $$p = \mathrm{Prob}\{z < ([ln(f_{t,k}/x_k) - (\sigma_k^2/2)\tau] / \sigma_k \sqrt{\tau})\}\text{, so}$$

(8) $$x_k * \int_{x_k}^{\infty} L(f_{T,k})\, df_{T,k} = (x_k / \sqrt{2\pi}) * \int_{-\infty}^{[ln(f_{t,k}/x_k) - (\sigma_k^2/2)\tau] / \sigma_k \sqrt{\tau}} exp(-z^2/2)\, dz$$

To be consistent with Black's (1976) notation, let

(9) $$d_1 = ([ln(f_{t,k}/x_k) + (\sigma_k^2/2)\tau] / \sigma_k \sqrt{\tau})\text{, thus (7) can written as}$$

(10) $$p = \mathrm{Prob}\{z < (d_1 - \sigma_k \sqrt{\tau}) = d_2\}\text{, i.e.,}$$

(11) $$d_2 = ([ln(f_{t,k}/x_k) - (\sigma_k^2/2)\tau] / \sigma_k \sqrt{\tau})\text{, and equation (8) can be written as}$$

(12) $x_k * \int_{x_k}^{\infty} L(f_{T,k})\, df_{T,k} = x_k * \Phi[d_2]$, where

$\Phi[\cdot] =$ Cumulative normal distribution

From this derivation, we see that the probability the terminal futures price exceeds the strike price of a given option is equal to the cumulative normal density of the "d2" term above – i.e, $\text{Prob}[f_{T,k} > x_k] = \Phi[d_2]$.

This is an especially useful result: It allows us to specify the probability the underlying futures price against which a call option is written will exceed a given strike price, at the expiry of the option. This is useful in assessing the market's probability density functions for given prices.

For example, given the underlying futures price and the implied volatility for a given options contract, say, the at-the-money contract for December 2009 WTI futures, what is the probability this will exceed $100 per barrel?[3] On July 31, 2009, with the EIA-calculated December 2009 WTI five-day futures average at $71.36 per barrel and the calculated five-day implied volatility for December 2009 options at 45.33 percent, this likelihood was 33.67 percent.

For natural gas, the comparable statistics were: December 2009 EIA-calculated futures $5.32 per MMBtu for the Henry Hub futures average price and 56.40 percent for implied volatility. The likelihood on July 31, 2009, that the December 2009 natural gas futures would settle over $10.00 per MMBtu was 25.59 percent.[41]

This result for $\Phi[d_2]$ also is used in the derivation of the Black commodity option pricing model, which is given in Appendix III below.

Appendix III: Black's Commodity Option Formula

The Black commodity option pricing model for calls is

$$C_{\tau,k} = \beta_{\tau,k}\left[\left(f_{t,k} * \Phi[d_1]\right) - \left(x_k * \Phi[d_2]\right)\right]$$, where

$C_{\tau,k}$ = Present value of European Call written on futures contract with price $f_{t,k}$

$\Phi[\bullet]$ = Cumulative normal probability density function with

$[d_1 \text{ and } d_2]$ as before in Appendix II

$\beta_{\tau,k} = \exp(-r_k * \tau_k)$ = present-value discount factor, employing the risk-free interest rate

$f_{t,k}$ = observed k^th^-nearby futures contract's value at time *t*, k = 1, 2, … n

x_k = strike price corresponding to an option written on the k^th^-nearby futures contract

σ_k^2 = variance of the returns on the k^th^-nearby futures contract

σ_k = volatility

τ_k = time to expiration of the k^th^-nearby option contract (as a percent of a 252-day trading year)

Black's model for Puts ($P_{t,k}$) can be solved using the put-call parity relationship[42]

$$P_{\tau,k} = \beta_{\tau,k}\left[\left(x_k * \Phi[-d_2]\right) - \left(f_{t,k} * \Phi[-d_1]\right)\right].$$

End Notes

[1] See Hodge, *et al*, (2009, pp. 4 – 5). See also Clews, *et al,* (2000), and Bernanke (2004a and 2004b, p. 2).

[2] See endnote 30 and 34 below.

[3] Source: New York Mercantile Exchange. See NYMEX Light Sweet Crude Oil futures and NYMEX Natural Gas futures for additional detail on these contracts and markets. Physically and financially settled contracts are traded on the NYMEX. In NYMEX's natural gas markets, the financially settled options allowing exercise only upon expiry (i.e., the European-style options) have higher volume and open interest (i.e., contracts that have not been extinguished by delivery or offset) than the options settling into a physically delivered futures contract. NYMEX uses the financially settled European-style options as to determine the volatility of the less liquid American-style options that settle into the underlying futures contract for its daily margining purposes. Given the Exchange uses the deeper options markets to determine the volatility against which the financially settled and settlement-into-futures options to determine daily margins, the EIA model effectively uses the European-style option's volatility to estimate the volatility parameter of the expected natural gas price distribution. The model is developed beginning in Section 4. See NYMEX Monthly Volumes for data on energy futures trading volume. See Commodity Futures Trading Commission, Commitments of Traders reports, for detail on open interest in WTI and natural gas futures.

The IntercontinentalExchange(R), or ICE, is a competitor of the NYMEX, and operates the Brent Blend futures market, which is the benchmark for North Sea crude oil. ICE calculates its futures settlements based on the average price of trading in the 21-day North Sea Brent-Forties-Oseberg-Ekofisk market in the relevant delivery month as reported and confirmed by the industry media. See ICE OTC, *ICE Crude Oil*, for additional information. ICE also trades financial contracts settling against NYMEX WTI futures. In addition, the ICE operates natural gas futures and OTC markets referencing the NYMEX futures and other indices for settlement purposes. See ICE Homepage (https://www.theice.com/) for additional detail.

[4] Commodities trade in four distinct markets: futures, forwards, spot and over-the-counter derivatives markets. We discuss futures at length in this article. The other three markets are characterized by bilateral contracting and individual collateral requirements that are negotiated between counterparties – i.e., there is no clearinghouse interposition between buyer and seller in these bilateral markets. Buyers and sellers take clearinghouse credit risk trading futures; however, in the bilateral markets they take counterparty credit risk. In forward markets -- e.g., a long-dated natural gas physical sale – contracts are traded between unique counterparties and payment is made per contract terms after delivery occurs (there is no requirement contracts be marked to market, as is the case for futures, although there may be collateral requirements). These principal-to-principal contracts can incorporate standard terms and conditions, but also allow for customized terms. "Spot" contracts literally specify on-the-spot delivery – i.e., immediate or very-close-to-immediate – delivery of a commodity, for which payment is made shortly thereafter. Financially settled derivatives – e.g., swaps and options – typically are traded in the over-the-counter (OTC) markets. OTC contracts typically reference generally accepted pricing indices for oil or gas against which the derivatives settle. OTC markets also trade derivatives contracts that reference futures settlement prices. For discussions of spot, forward and futures markets, see Black (1976), Williams (1989), and Duffie (1989). See Hull (1997), Commodity Futures Trading Commission (September 2008), and Interagency Task Force on Commodity Markets (2008) for discussions of futures and OTC markets.

On March 16, 2009, the Commodity Futures Trading Commission (CFTC), the U.S. futures markets regulator, approved rules and amendments increasing oversight of so-called Exempt Commercial Markets (ECMs), on which principal-to-principal transactions occur via electronic trading platforms. These rules implemented provisions of the CFTC Reauthorization Act of 2008, creating a new regulatory category, ECMs with significant price discovery contracts (SPDCs). These electronic trading facilities are now subject to additional regulatory and reporting requirements.

The overall size of the bilateral OTC oil market is difficult to gauge; the Bank for International Settlement (BIS) estimates total notional OTC commodities contracts outstanding in 2008 stood at $4.4 trillion, down by 66.5 percent from 2007 levels. However, the BIS does not break energy contracts out separately. See Bank for International Settlements (May 2009, p.3), for additional information.

The U.S. Congress enacted the Food, Conservation, and Energy Act of 2008 on May 22, 2008, which reauthorized the CFTC until 2013, and gave it additional regulatory and enforcement tools to regulate the futures industry, particularly transactions in energy products. Hearings into energy futures market regulation were conducted in late July 2009 by the CFTC – see CFTC's Hearing on Speculative Position Limits in Energy Futures Markets, July 29, 2009, for additional information.

[5] For NYMEX's WTI crude oil futures and Henry Hub natural gas futures specifications, see Light Sweet Crude Oil Futures and Natural Gas Futures. See NYMEX Market Information for articles concerning NYMEX futures markets published by the Exchange.

[6] For complete specification of the NYMEX options, see Light Sweet Crude Oil options and Natural Gas options.

[7] See Connexions "Brownian Motion" module by Jason Holden and Kevin Kelly. The Brownian-motion process is named in honor of the Scottish botanist Robert Brown. Holden and Kelly note: "The first person to put forward an actual theory behind Brownian motion was Louis Bachelier, a French mathematician who proposed a model for Brownian motion as part of his PhD thesis in 1900." Brownian motion, Holden and Kelly note, is a mathematical description of "the random movements of minute particles upon immersion in fluids. As Brown once noted in his observations under a microscope, particulate matter such as, for example, pollen granules, appear to be in a constant state of agitation and also seem to demonstrate a vivid, oscillatory motion when suspended in a solution such as water." Bachelier arrived at his groundbreaking formulation studying price behavior in French financial markets.

[8] Working was referencing his 1949 article, "The Investigation of Economic Expectations." Working also published "A Random-Difference Series for Use in the Analysis of Time Series," in 1934, which showed that price randomness is to be expected if markets are efficient. See Dimson and Mussavian (1998, pp. 91-193), who note Working's findings and others related to random-walk theories were overlooked by economics researchers until the late 1950s. For a discussion of the evolution of the Efficient Markets Hypothesis to its modern appreciation see Lo (2007). Engle (2004) has an excellent development of how markets process "news" vis-à-vis volatility beginning on p. 407. This was Engle's Nobel lecture; he was awarded the Nobel Prize in 2003 for his contributions to understanding and modeling volatility, specifically via his development of the autoregressive conditional heteroskedasticity (ARCH) model and its progeny.

[9] Samuelson's proof showed that a futures contract's expected value is equal to the currently observed futures price. This is known as the "martingale" property and is used extensively in modern finance. Samuelson was instrumental in introducing the mathematics of random processes to the economics profession in the late 1950s, according to Dimson and Mussavian (1998).

[10] Merton and Scholes received the Nobel Prize in Economic Sciences in 1997 for their path-breaking work, which has been applied and extended throughout finance and economics. Black died in 1995; the award is not presented posthumously. See Merton (1998) and Scholes (1998) for their Nobel lectures.

[11] See Cox and Rubinstein (1985, Chapter 5, section 8, beginning on p. 215): "How changes in the variables affect Black-Scholes option values." In these models, there are no restrictions on short selling in the construction of risk-free portfolios; there are no taxes, transactions costs or arbitrage opportunities available to market participants. See Hull (1997, p. 236) for complete listing of assumptions in B-S-M. See Malone (2002), for a survey of studies where various assumptions in the B-S-M model are relaxed.

[12] The Wiener process is named after the American mathematician Norbert Wiener. See Jerison and Stroock (1995), and Mandrekar (1995). This is also known as a geometric Brownian motion (GBM).

[13] Under the assumptions of their model, B-S-M showed a risk-free portfolio could be constructed using continuously traded stocks and options, which would earn the riskless rate of return. See Hull (1997, p. 236 for list of assumptions.) B-S-M's arbitrage-based proof paved the way for further innovations in options theory (see, e.g., Margrabe, 1978).

[14] In the present note, for example, we employ, *inter alia*, Ogawa's derivation (1988), which follows the risk-neutral valuation methodology developed in Cox-Ross-Rubinstein (1979). The C-R-R article provides a more tractable alternative to deriving the B-S-M (1973) and Black (1976) results. C-R-R note in their article they use "elementary mathematics," i.e., a binomial tree model, to derive the option-pricing equation, which contains the B-S-M formula as a limiting case, whereas B-S-M and Black employ "quite advanced" stochastic calculus to derive their result.

Musiela and Rutkowski (2005) provide an in-depth development of the risk-neutral methodology, also known as the "martingale" method. See Melick and Thomas (1992 and 1998) for examples of how central banks expand the C-R-R and Cox-Ross (1976) techniques to recover market-derived probabilities vis-a-vis assessing monetary policy innovations. For a discussion of risk preference, see Fama and Miller (1972, particularly Chapter 5), and Musiela and Rutkowski (2005, Chapter 1).

[15] See Dimson and Mussavian (1999, p. 1762) for an excellent discussion of the development and evolution of asset-pricing models, including C-R-R's. They note: "The Cox-Ross-Rubinstein model has the advantage that it can easily be adjusted to price other derivatives, such as American puts, which are considerably harder to evaluate in the Black-Scholes framework. This approach is immensely popular, not only in the classroom buy also among practitioners."

[16] The lognormal assumption for asset prices, which translates into a normal assumption for log returns, has been a source of debate since at least the early 1960s. Numerous researchers have noted the distribution of daily returns of many assets is leptokurtic, i.e., "fat-tailed" and peaked. Jackwerth (2004, pp. 39 - 40) notes: "Two well-known reasons could be the cause. First, the return distribution may not be lognormal but leptokurtic. Second, a return distribution may be nonstationary over long periods. Returns are likely to be nonstationary if the economy fundamentally changes. … Over the 1928-96 period, the United States went through the depression, World War II, the postwar growth, the oil crises of the early 1970s, the advent of the computer revolution, and the modern service economy. For returns from each of these periods to look systematically different from the next period's returns and for their distributions to exhibit different parameter values would not be surprising. Nonstationarity introduces leptokurtosis even if the true underlying distribution is lognormal but with time-varying parameters." Jackwerth (2004) tested the lognormal assumption for S&P500 returns over the January 1928 – December 1996 period using a Kolmogorov-Smirnov test, a nonparametric test to measure whether an observed distribution conforms to an assumed distribution. He found support for the lognormal distribution hypothesis, which depended on the type and length of the sample (e.g., daily returns over three-month intervals). See Jackwerth (2004, pp. 40 – 43.) EIA will be conducting similar research on oil and natural gas prices and returns.

[17] See Cox and Ross (1976) for a description of the diffusion process. See also Ogawa (1988, p. 53), and Duffie (1989, Chapter 6), "Statistical behavior of futures prices" for a discussion of modeling stochastic processes.

[18] See Ogawa (1988, p. 55).

[19] See Freund and Walpole (1987, particularly Chapter 11).

[20] See Freund and Walpole (1987, p. 365).

[21] See, e.g., Newell and Pizer (2003, p. 64).

[22] See Duffie and Gray (1995), for a description and results of using such models.

[23] See Appendix III herein for specification of Black's (1976) model.

[24] See Musiela and Rutkowski (2005, p. 222). Duffie and Gray (1995) and others suggest using a Newton-Raphson search to determine the implied volatility for each option. EIA's model uses implied volatilities calculated by the New York Mercantile Exchange (NYMEX), which are used to compute margins for options under the Standard Portfolio Analysis of Risk (SPAN) program used by the CME Group, the parent company of the NYMEX. NYMEX inverts Black's model (1976) to calculate σ_k^i for calls and puts with open interest in each contract month. See CME SPAN for an overview of CME clearinghouse Standard Portfolio Analysis of Risk methodology.

[25] Duffie and Gray (1995, p. 52). GARCH stands for Generalized ARCH model; EGARCH stands for Exponential GARCH; see Engle (2004, p. 407). See also Green and Figlewski (1999), re GARCH modeling, who note: "… GARCH has some serious shortcomings as a forecasting tool. One is that the parameters must be estimated from past data, and this frequently requires quite a large data set."

[26] Engle (2002, p.3) cited a working paper by Poon and Granger that later was published as Poon, S.-H., and C.W.J. Granger (2003), "Forecasting financial market volatility: A review," in the *Journal of Economic Literature*, Vol. 41, 2, pp. 478-539. Poon cites this survey of the literature in the References of Poon (2005, p. 211).

[27] Szakmary, *et al* (2003, p, 2173). The regressions they estimated (p. 2158) generally follow the model specified by Canina and Figlewski (1993), and take the form:

$RV_t = \alpha + \beta[]V_t + e_{t,}$ where
RV_t = Realized volatility at time = t
α = constant
$\beta[]V_t$ = coefficient (β) times either Implied Volatility $[I]V_t$, or Historical Volatility $[H]V_t$
e_t = error term

$RV_t = \alpha + \beta IV_t + \beta' HV_t + e_t$ then is estimated to see if including HV adds explanatory power to the regression including IV. Later, GARCH estimates are substituted for HV. An unbiased predictor would have $\alpha = 0$, and β or $\beta' = 1$.

[28] Jorion (1995) found implied volatilities in foreign exchange (FX) options were superior estimators of realized volatility (i.e., had superior information content) to GARCH and historical volatility estimators, a finding cited by Szakmary, *et al* (2003) above, and confirmed for the 35 futures markets they examined. Among the most robust results they found were those for the oil and natural gas futures options.

Jorion's findings also were confirmed in Weinberg (2001). Weinberg's discussion is an excellent summary of the issues surrounding the use of implied volatilities as parameter estimators of the expected distribution for prices generally. According to Weinberg, Jorion's study was meant to test findings by Canina and Figlewski (1993), which showed implied volatility had no explanatory power vis-à-vis realized volatility in the S&P 100 index. The Canina-Figlewski results thus were not confirmed by either Jorion or Weinberg.

Of particular note, Weinberg finds Black-Scholes at-the-money implied volatility provided the best estimate of realized volatility among the models tested for FX options, including recently developed nonparametric techniques for estimating risk-neutral probability density functions for the underlying futures contracts. The two measures of realized volatility were highly correlated, however. This suggests the more parsimonious model, the B-S-M or Black implied volatility for at- and near-the-money options, will produce as good an estimate of realized volatility, hence the market's expectation of price ranges, as the more computationally intensive techniques for deep options markets. See also Figlewski (2008) for additional discussion of risk-neutral density estimation.

An area for future study is the exploration of whether these techniques, particularly the use of risk-neutral probability distributions recovered from options markets, improve the efficiency of confidence-interval estimation for energy markets**.** See, for example, Melick and Thomas (1998, 1992). For an excellent summary and literature survey regarding these techniques, see Jackwerth (2004).

[29] This would be consistent with *weak-form market efficiency* – i.e., "… prices fully reflect the information implicit in the sequence of past prices," according to Dimson and Mussavian (1998). Thus, including historical volatility in any estimate of future returns distributions would not add new information to the estimate; the implied volatility already subsumes all of the information contained in the historical data and statistics.

[30] The NYMEX uses its deeper financially settled natural gas options to determine volatility for daily margining purposes, which, as a result, are the volatility estimates EIA will use in its model. NYMEX's financially settled natural gas options allow exercise only upon expiry (i.e., the European-style options). These options have nearly 30 times the volume of the options exercisable into a futures contract. NYMEX uses the financially settled European-style options as to determine the volatility of the less liquid American-style options. For crude oil options, the NYMEX inverts a Black (1976) model to solve for the implied volatility, even though these options are American-style options allowing for exercise at any time during the options' life. EIA addresses this issue in endnote 34 below; further research is planned in this area as it pertains to estimating the parameters of price distributions.

[31] See footnote 24 above.

[32] See, for example, Figlewski, (2004, particularly p. 79 and 93), and Poon (2005, Chapter 10, p. 115, Section 10.1 and 10.2). Consistent with Szakmary, *et al* (2003), the EIA model herein requires an option to have more than 10 trading days remaining in its life to be used the implied volatility calculation for that option. See Szakmary, *et al* (2003, p. 2156 and p. 2159 footnote 11). At- and near-the-money options are also used in the EIA model, for the reasons explained by Szakmary, *et al* (2003, p. 2156-57): They "are least affected by the non-normal distribution of returns…." See Cox and Rubinstein (1985, Chapter 5, section 8) re option sensitivities to changes in pricing parameters; at- and near-the-money options have the highest "vega" – the first partial derivative of an option's premium with respect to change in volatility – making these the most sensitive options to volatility. See also Poon (2005, Chapter 10) for a discussion of most-sensitive-option techniques and various weighting schemes vis-à-vis proximity to at-the-money options employed to extract information from implied volatilities.

[33] See Poon (2005, Chapter 10), for a survey of the literature concerning the information content of different options (at- versus deep-out-of- and in-the-money options, e.g.). Considerable research over a period spanning more than 30 years into the so-called "volatility smile" – also known as the "skew" – suggests market participants price options to different volatilities to capture the leptokurtic, or "fat-tailed," price distribution that historically is observed in asset markets.

Recent research, particularly at central banks, makes use of the skew to estimate the distribution being discounted by the options market participants, so as to better gauge the markets' reactions to monetary policy innovations. See, e.g., Clews, e*t al*, (2000), for a discussion of central banks' use of risk-neutral probability techniques discussed above. See also Bernanke (2004a, particularly p. 14-5): "Financial data provide insight about market uncertainty that is obtainable nowhere else. For example, the Board's staff regularly analyzes the prices of options on eurodollar futures to estimate the degree of uncertainty that market participants have about monetary policy at different horizons. Indeed, by examining options with different strike prices, and under some reasonable additional assumptions, one can produce a full probability distribution of market expectations for the level of the federal funds rate at various dates in the future. Likewise, analysis of various types of options can generate distributions of expectations for economic and financial variables ranging from oil prices to the exchange value of the dollar to future stock prices." Bernanke was referring to the risk-neutral techniques, some of which were refined at the Fed, to render distributions with higher densities in the tails than would be produced in a constant-variance model. This technique was first applied to oil markets by Melick and Thomas (1992). This is a topic of future EIA research. See Jackwerth (2004) and Figlewski (2008) for in-depth discussions of these techniques re strengths and weaknesses.

[34] As an area of future research, EIA will, as part of its benchmarking process, explore whether, for the purposes of estimating the variance parameter of a price distribution in the EIA model, there is a meaningful difference between the volatility calculated using an American-style model versus a European-style model. The standard deviation (volatility) can be tested vis-à-vis its assumed distribution (i.e., it is assumed to be chi-square distributed per Freund and Walpole (1987, p. 379 - 380), so for the purposes of our model, we can test whether European-style volatility is statistically different from American-style volatility in a meaningful way when it comes to parameterizing the distribution. See Jorion (1995, pp. 512 – 514) for a discussion of this issue re using a European model to solve American-style implied volatilies for foreign exchange futures. Jorion notes the difference between the models is very small. EIA will test this for energy options in the future.

[35] See Jarrow and Rudd (1983, pp. 86 – 93). See also Ritchken (1987, particularly chapters 5, 6, and 8), re the risk-neutral economic argument. See also Hull (1997, particularly chapters 9, and 11), and Jackwerth (2004) for further discussion of risk-neutral techniques.

[36] The martingale property (or a close approximation thereof) for oil prices was demonstrated in Hamilton (2009a, pp. 179 – 206). Hamilton estimated several models for quarterly crude oil price changes and price levels over the period 1970 – 2008 and concludes, "… the broad inference with which we come away is that the real price of oil is not easy to forecast. To predict the price of oil one quarter, one year, or one decade ahead, it is not at all naïve to offer as a forecast whatever the price currently happens to be" (p. 181). Later, Hamilton (2009a, p. 184) observes, "It is sometimes argued that if economists really understand something, they should be able to predict what will happen next. But oil prices are an interesting example (stock prices are another) of an economic variable which, if our theory is correct, we should be completely unable to predict."

[37] See Jarrow and Rudd (1983, p. 89).

[38] See Ritchken (1987, Chapter 6, beginning on p. 118) re the representation of the geometric Wiener process and the linear scaling of the mean and variance.

[39] This derivation follows Ogawa (1988 pp. 54-55); Jarrow and Rudd (1983, pp. 90-92); and Ritchken (1987, pp. 173-74).

[40] This is a European-style option, i.e., it can be exercised only at expiry. The NYMEX trades American-style options for crude oil. However, Merton (1973) shows the European and American calls typically have the same value; since a call typically is worth more alive than dead it does not pay to early exercise.

[41] See Bentley Trading Room for tips on using Excel functionality to calculate the $\Phi[d_2]$ term, i.e., the cumulative normal density (normsdist) function.

[42] See Ogawa (1988, pp. 54-56), and Ritchken (1987, pp. 173-74) for derivations. See Hull (1997, Chapter 12, particularly pp. 273-80) re put-call parity.

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