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Considerably before the recent revival of interest in nuclear shell structure, Schmidt[1] had found a systematic variation with Z of the deviation of nuclear charge distributions from a sphere (which he derived from nuclear quadrupole moments) and pointed out minima of the magnitudes of nuclear quadrupole moments near the "magic numbers" Z=50 and 82. Schmidt's plot has been extended with more recent data and quadrupole moments correlated to some extent with nuclear shell structure.[3,4]

The following simple model, based on nuclear shell considerations, leads to the proper behavior of known nuclear quadrupole moments, although predictions of the magnitudes of some quadrupole moments are seriously in error.

1. Neutrons and protons fit into single particle levels in a scheme similar to those proposed for correlating spins, thus producing what may be called proton and neutron shells.
2. Proton and neutron shells tend to be oriented or polarized to allow maximum overlap between proton and neutron distributions.

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This model leads to the conclusions:

A. For an odd-proton nucleus, the quadrupole moment is primarily dependent on the number of protons P and can be written $Q_{podd} = Q_p(P)$, where Q_p is always positive immediately before, and always negative immediately after a shell is filled.

B. For an odd-neutron nucleus, the magnitude of the quadrupole moment depends on the number of protons, but its sign is determined by the number of neutrons N, being given by $\frac{Q_n(N)}{Q_n(N)}$. $Q_n(N)$ is the electric quadrupole moment which would be produced by the neutrons if they were charged, and Q_n is very nearly the same function as Q_p.

C. For odd-odd nuclei, estimation of quadrupole moments is more complex and depends on the way in which the mechanical moments of the odd neutron and odd proton add. If these moments are essentially parallel, the quadrupole moment should be of the same sign and approximately the same magnitude as for a similar odd-proton nucleus, if the neutron and proton mechanical moments are not essentially parallel, the quadrupole moment magnitude should be considerably reduced.

Known quadrupole moments appear to fit these expectations. Figure 1 shows nuclear quadrupole moments[5] as a function of number of nucleons.

The sign of Q near the closing of major shells is that expected in all cases except for Li^7. In view of uncertainties in the charge distributions in the molecules from which Q for Li^7 was derived, perhaps present indications that Q is positive should not be regarded as conclusive. The negative quadrupole moment of S^{33} and the reversal of sign to a positive moment for S^{35} is a striking illustration of the above rules since S^{35} has 19 neutrons, or one less than the filled shell of 20. This model would similarly predict that the quadrupole moments of K^{39} and K^{41} are positive, that of Sc^{45} and Zr^{91} negative.

Se^{77} seems from microwave measurements to have a quadrupole moment less than $0.001 \times 10^{-24} cm^2$, yet atomic spectra indicate a spin for this nucleus greater than 3/2. Both experiments may be correct, and the Se^{77} moment very small because of the spherical distribution of protons in this nucleus due to completion of the 4 f subshell. Ca^{43} is a similar case which should have a complete proton shell and small quadrupole moment.

The negative quadrupole moment of Ge^{73} with 41 neutrons suggests that a completed sub-shell occurs at 40 neutrons (pointed out as a possibility by Nordheim) corresponding to filling the 3 p levels in Nordheim's and Mayer's schemes.[6] If the neutron and proton shells fill in the same manner, the Cb^{93} quadrupole moment should also be negative and the Zr^{91} quadrupole moment small.

The odd-odd nuclei with known quadrupole moments consist of Li^6, B^{10}, N^{14}, Cl^{36}, and Lu^{176}. Their moments correspond to expectations from the model described above, and those cases for which the moments of neutron and proton shells are parallel (B^{10}, N^{14}, Lu^{176}) are plotted in Fig. 1.

Gordy's statement[3] that completion of a neutron shell tends to make quadrupole moments more negative does not follow from the above considerations. This is perhaps not serious because the evidence for this empirical observation is questionable. A number of quadrupole moments plotted by Gordy are values he predicts on the basis of an empirical relation between nuclear electric quadrupole moments and magnetic dipole moments. The most significant applications of this relation are to cases of nuclei of the same spin and differing by two protons. In the only two known nuclear pairs of this type, (Cu^{65} - Ga^{69}, Ga^{71} - As^{75}), Gordy's relation is in considerable error and in one case it gives the wrong sign for the quadrupole moment. Gordy has plotted the Cs^{133} quadrupole moment as $0.3 \times 10^{-24} cm^2$ and from this value derived other moments. However, known information on Cs^{133} allows only the conclusion that for this nucleus Q $0.3 \times 10^{-24} cm^2$. If the Cs^{133} moment is taken as zero or negative and if the predicted quadrupole moments are questioned, then no evidence appears to remain that completion of a neutron shell makes nuclear quadrupole moments more negative.

For the nuclei which contain closed proton shells plus or minus only one proton, the nuclear shell model gives not only a definite prediction of the sign of Q, but also, if the state of the odd proton is determined from the nuclear spin and magnetic moment, it gives a fairly definite value for the quadrupole moment magnitude. In the cases Li^7, N^{14} and Bi^{209}, the calculated magnitudes are in substantial agreement with those observed. This model also gives roughly the proper ratios between the quadrupole moments of In^{113}, In^{115}, Sb^{121}, and Sb^{123} (protons differing by one from a closed shell of 50). However, in spite of the rather extensive success of this model, it appears difficult to reconcile the magnitudes of the In^{113}, In^{115}, and Sb^{123} quadrupole moments with a nuclear shell model including a closed shell at 50 nucleons. They are larger by a factor of four than can reasonably be produced by a single particle. In addition the very large quadrupole moments in the vicinity of Z = 71 present difficulties to a nuclear shell model. To give Lu^{176} its quadrupole moment of $7 \times 10^{-24} cm^2$, approximately 35 protons in the most favorable orbits must contribute to Q. Since the first fifty protons are presumably in a closed shell and contribute nothing to Q, only 21 protons are available and even all of them could hardly be put in the few orbits which contribute most to a positive quadrupole moment.

Failure of the nuclear shell model to give correct quadrupole moments is in contrast to the situation with nuclear magnetic moments, which can all be accounted for by a suitable admixture of states of a single nucleon. In the shell model approximation, these large quadrupole moments must represent a considerable contribution from the protons in the closed shells. The polarization of this core would presumably require a sharing of angular momentum between the protons of the incomplete shell and those of the closed shells. The magnitude of the polarization, however, and the resulting large asymmetry of the nucleon distribution is hardly consistent with the single particle-central field quantization which is the basis of the shell structure model.

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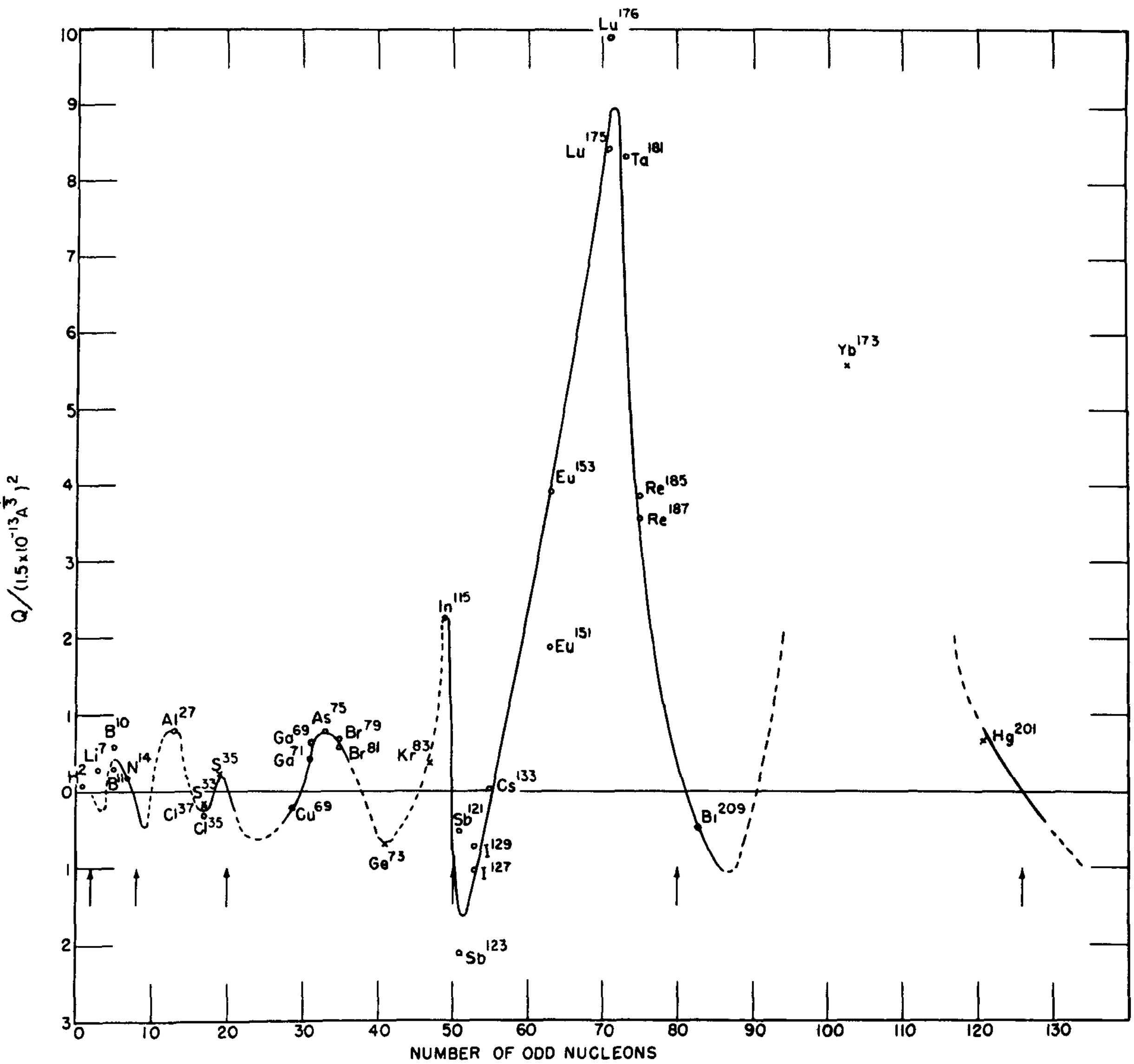

Fig. 1

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