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THE $K^o\bar{K}^o$ SYSTEM

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PREFACE

These notes are part of a series of lectures delivered at Madras in December 1966 and January 1967.

Chapter I, by G. Charpak, is an introduction to the most important facts in this very rich field of the neutral kaon physics. These notes do not pretend to be a comprehensive study of the neutral kaon physics. They overlook some of the very elegant and important experiments which started this field, and only the most recent ones are usually considered.

The notes in Chapter I are an introduction to the second part, treated by M. Gourdin, where the theoretical significance of the experiments and the results are discussed in detail in Chapters II to IV.

The authors are indebted to Profs. J.S. Bell and J. Prentki for many clarifying discussions.

They also wish to express their special gratitude to Prof. A. Ramakrishnan for the warm hospitality he extended to them at the Matscience Institute, and to the colleagues of the Institute for the stimulating atmosphere of the lectures and discussions. One of the authors (M.G.) is indebted to Dr. D. Schiff for her help in the numerical computations performed on the UNIVAC 1107 of the Faculté des Sciences d'Orsay.

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CONTENTS

Page

CHAPTER IV

INTRODUCTION

In 1957 there was established the non-invariance of weak interactions under parity transformation. It was the observation of the K^+ decay into two channels of opposite parity (the θ-τ puzzle) which, in mid-1956, led Yang and Lee to question the validity of the parity conservation law. Ever since then, a great deal of attention has been paid to checking the conservation laws admitted in physics as simplifying logical hypothesis.

It rapidly appeared that the weak interactions also violate C-conjugation invariance. It was admitted as a law of nature that for each particle described by the quantum numbers *), Q the charge, B the baryon number, L_e, L_μ the lepton numbers, Y the hypercharge, and I_z the third component of isotopic spin, there exists an antiparticle for which these numbers are of opposite sign with the same mass and lifetime, enjoying the same interactions. For the fermions, in addition, the parity is also opposite, while all other quantum numbers are the same.

It is amusing to note that in 1932, when Oppenheimer suggested that the negative energy states of Dirac might be antielectrons, Pauli wrote[1]: "This explanation appears to be unsatisfactory because the laws of nature in this theory are exactly symmetric with respect to electrons and antielectrons .... Thus we do not believe this explanation can be considered seriously".

The same year, Anderson's discovery of the positron gave the first experimental proof of this symmetry law which remained unquestioned until 1957, since it was based on the existence of the C-conjugate mirror pairs: K^+K^-, $\mu^+\mu^-$ $\pi^+\pi^-$ $p\,\bar{p}$, $n\,\bar{n}$, etc.

The violation of C by weak interaction is illustrated, for instance, by the decay of the pion: the C-conjugate state of $\pi^+ \to \mu^+ + \nu$ is $\pi^- \to \mu^- + \bar{\nu}$. It was observed that the neutrinos emitted in the first reactions are left-handed. The C-conjugate reaction should correspond to antineutrinos that are also left-handed. Experiment shows that such a state never occurs in nature. The helicity states of the neutrinos are known from the helicity of the muons.

As soon as P and C conservation appeared to be violated, Landau[2], and independently Lee and Yang[3], put forward the hypothesis that all the laws of nature, including those governing weak interactions, were invariant under the combined operation PC. This restored the right-left symmetry of the universe. Since it is impossible to distinguish in a non-arbitrary way between matter and antimatter, the violation of parity does not provide us with a means of telling, in an absolute way, right from left.

The validity of the PC invariance was best illustrated by the interdiction of the decay into two pions for the long-lived neutral kaon, to the accuracy level of 1% established in 1958[4], and 0.3% in 1961[5].

From Lorentz invariance, the hypothesis of local interaction, and the causality principle in field theory, it was established[6-8] that all the laws of nature are invariant under the transformation L = PCT. This law connects together the PC invariance and the invariance under time reversal.

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*) These numbers are arbitrarily set to zero whenever they cannot be defined.

T invariance in weak interactions was checked in several experiments. The validity of this law seems to be established to the level of accuracy so far attained by difficult experiments.

The $K_1^0 K_2^0$ system was known prior to the discovery of parity violation. The hypothesis of two neutral kaons with short and long lifetimes was advanced in 1955 by Gell-Mann and Pais[9]. After the formulation of the strangeness scheme of Gell-Mann and Nishijima in 1953, Fermi is quoted as having said to Gell-Mann[*]: "I won't believe in your scheme until you have a way of telling K^0 from $\bar{K}^0$ ".

The K_1^0 was identified in 1955[11]. Until 1957, the K_1^0 and K_2^0 were defined as eigenstates of the C conjugation and, after 1958, as eigenstates of the PC conjugation. In 1958, the K_2^0 were discovered by Bardon et al.[4]. The study of the decay states strongly supported the hypothesis that they were eigenstates of PC, both P and C appearing to be violated in the decay.

In 1964, Christenson et al.[12] showed that a small fraction of the K_2^0 can decay in a channel forbidden by PC conservation $K_2^0 \to \pi^+\pi^-$.

Several hypotheses were put forward to save the PC conservation law. They were all discarded by experiment, which also established with great accuracy the amount of PC violating decay and the mass difference between the long-lived and the short-lived kaons.

Finally, in the fall of 1966, the measurements done at CERN[13] and at Princeton[14] of the decay of K_2^0 into two neutral pions overthrew the superweak theory of PC violation, which had to admit the validity of the $|\Delta I| = 1/2$ rule in the interaction responsible for the PC violating decays of the K_2^0.

In giving our lectures at this stage of the evolution, it appeared to us useful to make a more general analysis than the one used in some of the most frequently quoted papers on this subject in which, for instance, the hypotheses of PCT invariance or the $\Delta I = 1/2$ law are introduced early in the analysis. Michel Gourdin has made this theoretical analysis which he will present in this same series of lectures, and we shall discuss the theoretical implications of the observed facts.

The following subjects will be treated in these lectures:

- The $K_1^0 K_2^0$ system under the hypothesis of PC conservation. The $\Delta I = 1/2$ rule in the decay of the kaons.
- Properties of neutral kaons. Interaction with matter. Measurement of the mass difference and of the sign of the mass difference. Measurement of the parameters of the PC violating decays.
- General analysis of the $K^0\bar{K}^0$ system. Discussion of the observable consequences of the different conservation laws or selection rules.
- Theoretical implications.

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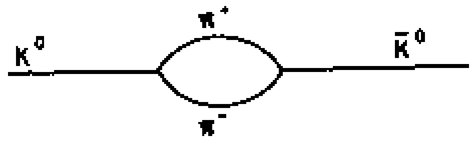
*) Quoted by J.J. Sakurai[10].

CHAPTER I

1. THE $K^0\bar{K}^0$ SYSTEM UNDER THE ASSUMPTION OF PC INVARIANCE

K^0 and $\bar{K}^0$ have a definite hypercharge (+1 and −1). This is the only quantum number which distinguishes them. Weak interactions do not conserve hypercharge. There can be transitions between K^0 and $\bar{K}^0$, for instance through diagrams of the type.



This situation is unique. It cannot occur for charged kaons, because of conservation of charge. It cannot occur for neutral baryons or leptons, because of the conservation of the baryonic or leptonic numbers.

1.1 The K_1^0 and K_2^0 states

From the point of view of weak interactions, K^0 or $\bar{K}^0$ cannot be considered as eigenstates of the Hamiltonian. Let us assume that PC is a good quantum number for the weak interactions. It is easy to construct two eigenstates of PC

$$K_1^0 = \frac{1}{\sqrt{2}}\,(K^0 + PC\,K^0) \qquad PC = +1$$

and (I.1)

$$K_2^0 = \frac{1}{\sqrt{2}}\,(K^0 - PC\,K^0) \qquad PC = -1 \ .$$

We can define $\bar{K}^0 = +PC\,\bar{K}^0$ or $\bar{K}^0 = -PC\,K^0$.

The sign is a matter of definition. Strong and electromagnetic interactions conserve hypercharge, as well as P and C, and the matrix elements of these interactions between K^0 and $\bar{K}^0$ vanish. This makes the relative phase of K^0 and $\bar{K}^0$ arbitrary. In these lectures we are going to use as convention $\bar{K}^0 = PC\,K^0$.

Energetically, K^0 or $\bar{K}^0$ (mass ~ 500 MeV) can decay into two or into three pions. Because of the constant reference in our discussion to the P, C, and PC eigenvalues of such states, the properties of these pion states (globally neutral) of which we make use are established in Appendix A and summarized in Table 1.

Table 1

Some properties of 2π and 3π states, globally neutral.

	$\pi^0\pi^0$	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0$	$\pi^0\pi^0\pi^0$
Parity P	+1	+1	−1	−1
Charge conjugation C	+1	+1	$(-1)^\ell$, $(-1)^{I+1}$ (ℓ: relative orbital momentum of $\pi^+\pi^-$ I: total isospin)	+1
Combined parity PC	+1	+1	$(-1)^{\ell+1}$, $(-1)^I$	−1

We see from this table that the PC = -1 state:

$$K_2^0 = \frac{1}{\sqrt{2}} \;(K^0 - \bar{K}^0)$$

can never reach the $\pi^+\pi^-$ state if PC is a good quantum number, while the

$$K_1^0 = \frac{1}{\sqrt{2}} \;(K^0 + \bar{K}^0)$$

can never reach a $\pi^0\pi^0\pi^0$ state In a strong interaction we produce the eigenstates of Y: K^0 and $\bar{K}^0$. They can be considered as superpositions of K_1^0 and K_2^0.

$$\begin{aligned} K^0 &= \frac{1}{\sqrt{2}} \;(K_1^0 + K_2^0) \\ \bar{K}^0 &= \frac{1}{\sqrt{2}} \;(K_1^0 - K_2^0) \,. \end{aligned} \tag{I.2}$$

Different channels are available for K_1^0 and K_2^0, and there is no reason for them to have the same lifetime. The decay into 2π being favoured over the three-body decay by the available phase space volume, one expects a shorter lifetime for the K_1^0.

Experimentally, one observes two neutral kaons with respective lifetimes $\tau(K_1) = (0.843 \pm 0.013)10^{-10}$ sec and $\tau(K_2) = (5.15 \pm 0.44)10^{-8}$ sec, the first one decaying mainly by emission of two pions.

1.2 The time propagation of the neutral kaon states

We have not proved at any stage that the PC = ±1 superpositions of K^0 and $\bar{K}^0$ have a definite mass and a definite lifetime. The theoretical proof relies on the treatment of Wigner-Weisskopf and is described in several papers to which we refer our readers[15,16]. Since, until 1964, experiments were showing the existence of a long-lived and a short-lived component decaying into two channels with opposite PC parity, it was a natural step to identify these particles with the PC eigenstates defined by our relations.

The Wigner-Weisskopf method introduces a complex mass matrix. For a stable stationary state the solutions of the wave equations contain the phase factor $e^{-im_0 t}$, in the rest frame. For an unstable particle decaying with a lifetime $1/\Gamma$, the phase factor becomes e^{-iMt} where $M = m_0 - i/2\,\Gamma$. This introduces in the norm of the states the factor $e^{-\Gamma t}$ describing the exponential decay.

The next step is to consider a kaon as a superposition of the K^0 and $\bar{K}^0$ just in analogy with the two spin states of a spin ½ particle.

Such a state $\psi = (K^0\;\bar{K}^0)$ propagates according to a generalized two-component Schrödinger equation:

$$i\,\frac{d}{dt}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

If K and K were not coupled, this is equivalent to two independent Schrödinger equations, with $\mathcal{M}_{12} = \mathcal{M}_{21} = 0$. The coupling of the two states via the weak interactions leads to the coupling of the two equations.

A rotation of the basis in the $K^0\bar{K}^0$ space allows the definition of two new states K_1^0 and K_2^0 which obey a Schrödinger equation with a diagonal M matrix. The diagonalization is done explicitly in Chapter II in the most general case. In other words, these states propagate with time according to the phase factors $e^{-iM_1\tau}$ and $e^{-iM_2\tau}$, where M_1 and M_2 are the complex numbers $M_1 = m_1 - i/2\,\Gamma_1$ and $M_2 = m_2 - i/2\,\Gamma_2$, m_1 and Γ_1 being the rest mass and the decay width of the short-lived component, while m_2 and Γ_2 refer to the long-lived component, τ being the proper time*). These states are the PC = +1 and PC = −1 eigenstates we have already defined in the case where PC is not violated in the decay.

As will appear clearly after the first lecture by Professor Gourdin, the mass matrix is simply

$$\begin{vmatrix} M & \frac{\Delta M}{2} \\ \frac{\Delta M}{2} & M \end{vmatrix}$$

when PCT and PC invariance holds, where ΔM = complex mass difference between K_1^0 and K_2^0, and $M = (M_1 + M_2)/2$.

After a proper time τ, a K^0 state will have the form

$$K^0(\tau) = \frac{1}{\sqrt{2}}\left(K_1^0\, e^{-iM_1\tau} + K_2^0\, e^{-iM_2\tau}\right).$$

If we square matrix elements obtained from such an expression, we are left with two factors $e^{-\Gamma_1\tau}$ and $e^{-\Gamma_2\tau}$ for each of the two components, reflecting the exponential decay of the states.

If the two states K_1^0 and K_2^0 are written in terms of vectors

we see from the above formulae that with time the length of the K_1^0 shortens because $\Gamma_1 \gg \Gamma_2$ and its phase changes relative to K_2^0 by an angle $\varphi = \Delta m \tau$, where Δm is the mass difference.

This is the basis of all the interference effects and of the measurement of the mass difference. Whenever we produce a mixture of K_1^0 and K_2^0 we can look on any physical effect whose amplitude can be contributed by both K_1^0 and K_2^0. The rotation of phase between the components contributions as a function of time will give rise to the interference effects dependent on Δm.

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*) The fact that a difference in the lifetimes should correspond to $m_1 \neq m_2$ was first pointed out by Pais and Piccioni[17].

1.3 The origin of the mass difference between K_1^0 and K_2^0

What can we expect a priori for Δm?

The virtual states corresponding to the rest masses of K_1^0 and K_2^0 are different and it is natural that their masses should be different. It is interesting to note that if the pions would have been of a mass higher than that of the kaons, thus forbidding the decay into real 2π or 3π states, the masses of the K_1^0 and of the K_2^0 would still have been different, although the experimental evidence for the two states would have been rather hopeless. If the mass of the pion would be such as to forbid the non-leptonic decays but allow the semi-leptonic ones, we would never have had such clear evidence about the PC properties, but the lifetimes would have been different and the semi-leptonic decays would have allowed a precise determination of the mass difference.

The contribution of the above matrix elements to the mass must be of the order of G^2 (G is the weak interaction constant $\sim 10^{-5}/m_p^2$, $\Delta m = \alpha\, G^2$); α must have the dimensions m^5. Since we are unable to calculate strong virtual states we have to make a guess and write $\alpha = m_\pi^5$. Then

$$\Delta m = 10^{-10}\ \frac{m_\pi^5}{m_p^4} \sim 10^{-5}\ \mathrm{eV} \sim 10^{+10}\ \mathrm{sec}^{-1}\ .$$

The value of α depends on what is the energy of the virtual states playing a role in the self-mass diagram. If these energies do not go beyond 1 GeV we have here a correct estimate of the order of magnitude. We are going to see that precisely $\Delta m \sim 10^{-5}$ eV.

If we had direct transitions through $\Delta S = 2$ currents from K^0 to $\bar{K}^0$, then $\Delta m = \alpha\, G$, $\alpha \sim m_\pi^3$, and $\Delta m \sim 10^{17}$ sec^{-1} which is seven orders of magnitude away from the preceding value. We can thus understand the importance that was attached to a measurement of Δm, even as an order of magnitude, as the best evidence for or against the existence of $\Delta S = 2$ transitions in weak interactions.

2. DECAY MODES AND SELECTION RULES IN THE KAON DECAY

The decays of the charged and neutral kaons are connected together by some selection rules governing the weak interactions. Because of the importance of these selection rules in the discussion of the PC violation in K_2^0 decay, let us first examine this connection between the decays of the charged and neutral kaons.

In Table 2 are listed the decays of the kaons, in such a way as to illustrate the relative values of the decay rates. The errors are not quoted in this table, since only the orders of magnitude are important in this discussion. However, the accuracy in the determination of some of the decay rates plays a most important role in the final analysis of the PC violating decays. For this reason, we give in Appendix B the table of Trilling[18)], where the errors are given. Some discrepancies among these values will give us the limits within which we have to accept certain conclusions about the selection rules.

<u>Table 2</u>

Table of the decay modes of charged and neutral kaons *).
Decay rates relative to the short-lived neutral kaon.
Total rate: $(1.18 \pm 0.02) \times 10^{10}$ sec $^{-1}$.

$K^{\pm}$	K_S	K_L	Scale with respect to Γ_S
$m_K = 493.82 \pm 0.11$ τ $(1.23 \pm 0.01)10^{-8}$ sec	$m_{KS} = 497.87 \pm 0.16$ $(0.843 \pm 0.013)10^{-10}$ sec	$m_{KL} = 497.87 \pm 0.16$ $(5.15 \pm 0.14)10^{-8}$ sec	
	$\pi^+\pi^-$ 7×10^{-1} $\pi^0\pi^0$ 3×10^{-1}		10^{-1}
			10^{-2}
$\mu^{\pm}\nu$ 3.8×10^{-3} $\pi^{\pm}\pi^0$ 1.46×10^{-3} (θ mode)			10^{-3}
$\pi^{\pm}\pi^+\pi^-$ 3.6×10^{-4} (τ mode) $e^{\pm}\pi^0\nu$ 2.8×10^{-4} $\mu^{\pm}\pi^0\nu$ 2.2×10^{-4} $\pi^{\pm}\pi^0\pi^0$ 1.1×10^{-4}	Leptonic decay modes not measured. Identical to K_L if PC good <u>or</u> PCT and $\Delta Q = \Delta S$ good	$\pi^{\pm}\mu^{\mp}\nu$ 5.3×10^{-4} $\pi^{\pm}e^{\mp}\nu$ 3.7×10^{-4} $\pi^0\pi^0\pi^0$ 4.2×10^{-4} $\pi^+\pi^-\pi^0$ 2.1×10^{-4}	10^{-4}
	$\pi^+\pi^-\pi^0 < 9 \times 10^{-5}$		10^{-5}
$\pi^{\pm}\pi^0\gamma$ 1.4×10^{-6}		$\pi^+\pi^-\gamma < 4.5 \times 10^{-6}$ $\pi^0\pi^0$ 5.6×10^{-6} $\pi^+\pi^-$ 2.8×10^{-6}	10^{-6}
$\pi^+\pi^+\mu^-\nu < 2 \times 10^{-7}$ $e^{\pm}\nu$ 1.4×10^{-7}		$\gamma + \gamma$ 1.1×10^{-6}	10^{-7}
$\pi^+\pi^-\mu^+\nu$ 5×10^{-8} $\pi^+\pi^+e^-\nu$ 1.4×10^{-8}		$e^+ + e^-$ $\mu^+ + \mu^-$ $< 5.5 \times 10^{-9}$	10^{-8}

*) This table is organized in such a way as to present the relative rate of the different decay modes with respect to the K_S total decay rate. Some values are approximate, since strong discrepancies exist between the published values. In Appendix B are presented the experimental values compiled by Trilling[18].

Table 2 calls for several comments.

i) <u>The physical long-lived neutral kaon is not an eigenstate of PC</u>

$\pi^+\pi^-$ and $\pi^0\pi^0$ states have been observed in the decay of K_2^0. This particle can then no longer be considered as the eigenstate of PC. For this reason, we keep the denomination K_2^0 and K_1^0 for the PC eigenstates defined by the relations (I.1), but in the following use K_L and K_S for the long-lived and short-lived physically observed particles. They will be defined in a general manner in the first chapter treated by Professor Gourdin.

ii) <u>The $\Delta I = \frac{1}{2}$ rule</u>

The decay mode $K^\pm \to \pi^\pm + \pi^0$ is 690 times slower than the two-pion decay of K_S. This points towards the existence of a selection rule governing the interactions responsible for the decay.

The total wave function of the two pions has to be completely symmetrical with respect to the interchange of the two pions. The orbital wave function is symmetrical, since the kaons have zero spin; but what about the isospin part of the wave function? Two pions can be in the states $I = 0, 1, 2$. Only 2 and 0 are symmetrical.

In the decay $K^\pm \to \pi^\pm + \pi^0$, the final state must be $I = 1$ or 2, since $I_z = \pm 1$. $I = 1$ is excluded because it is the antisymmetric combination $[\vec{\pi}(1) \times \vec{\pi}(2)]$. Thus only the $I = 2$ state is available. Since the initial isospin is $I = \frac{1}{2}$, only a transition $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ can lead to the decay of charged kaons into two pions.

The K^0 or $\bar{K}^0$ having a zero charge, their decay product $\pi^+\pi^-$ or $\pi^0\pi^0$ can be in a state $I = 0$. In this case, a transition $\Delta I = \frac{1}{2}$ is permitted. This conclusion also holds for K_1^0, since

$$< \pi\pi|H|K_1^0 > = \frac{1}{\sqrt{2}}\left\{< \pi\pi|H|K^0 > + < \pi\pi|H\ CP|K^0 >\right\} .$$

Assuming for the moment PC is conserved in the interaction $PC^{-1} H\ PC = H$, this expression equals

$$\frac{1}{\sqrt{2}}\left\{< \pi\pi|H|K^0 > + < \pi\pi|CP\ H|K^0 >\right\} = \sqrt{2}\left\{< \pi\pi|H|K^0 >\right\} ,$$

as

$$CP|\pi\pi > = |\pi\pi > \quad \text{(Table 1)}.$$

If the weak interactions are governed by a selection rule $\Delta I = \frac{1}{2}$, we can understand why the $K^\pm$ decay is strongly inhibited with respect to the neutral component K_1^0.

This rule has a further consequence in the decay of K_1^0. The operator in isospin space corresponding to $I = 0$ is

$$\vec{\pi}(1) \cdot \vec{\pi}(2) = \pi^0(1)\ \pi^0(2) + \pi^+(1)\ \pi^-(2) + \pi^-(1)\ \pi^+(2) \quad .$$

So one state, out of three occurring with equal probability, corresponds to $\pi^0\pi^0$. One thus expects a branching ratio:

$$u = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-) + \Gamma(K_S \to \pi^0\pi^0)} = \frac{1}{3}$$

A small phase space correction taking into account the $(\pi^{\pm}, \pi^{0})$ mass difference leads to a value: $u = 0.337$. The importance of the $\Delta I = \frac{1}{2}$ rule has led to a series of measurements of which the most precise give:

$u = 0.335 \pm 0.014$ (xenon bubble chamber, measurement of both $\pi^{+}\pi^{-}$ and $\pi^{0}\pi^{0}$ modes);

$u = 0.260 \pm 0.024$ (methyl-iodide chamber, measurement of $\pi^{0}\pi^{0}$);

$u = 0.288 \pm 0.021$ (hydrogen bubble chamber, measurement of $\pi^{+}\pi^{-}$).

Most of the recent papers quote the result of the compilation by Trilling[18]:

$$u = 0.309 \pm 0.02.$$

Owing to the slight inconsistencies between the measurements, it is worth keeping in mind that the experimental data still open a not too narrow door for some admixture of $\Delta I = \frac{3}{2}$. Even if the $\Delta I = \frac{3}{2}$ transitions would be completely forbidden in weak interactions, electromagnetic radiative corrections may introduce $\Delta I = \frac{3}{2}$ admixture. These corrections cannot be calculated with accuracy, and simple application of perturbation theory leads to smaller decay rates for $K^{\pm}$ than those observed.

If one assumes that the decay in $K^{\pm}$ is due to an admixture of $\Delta I = \frac{3}{2}$, which is also present in the K_1^0 decay, and neglecting any possible contribution from $\Delta I = \frac{5}{2}$, one finds [see Källen[19]), p. 446] that the theoretical branching ratio $\frac{1}{3}$ is changed into

$$0.29 \leq u \leq 0.38$$

which contains the experimental values. The importance of these considerations lies in the fact, established in the fall of 1966, that the branching ratio of the K_L into the two possible modes $\pi^{+}\pi^{-}$ and $\pi^{0}\pi^{0}$ is very different from that of the K_S, showing that the PC violating interaction responsible for these decays badly violates the $\Delta I = \frac{1}{2}$ selection rule, and excluding some of the theories put forward to deal with the PC violation.

The $\Delta I = \frac{1}{2}$ rule was so far admitted to hold in weak interactions and was in rather good shape at the time of the 1966 Berkeley Conference. In Appendix C we review briefly the supporting arguments for this law, together with its bearing on the kaon decays.

3. INTERACTION OF NEUTRAL K MESONS WITH MATTER*)

3.1 The different types of interactions

K^0 and $\bar{K}^0$ have different properties in nuclear matter. Many more channels are available for $\bar{K}^0$, which is of the same strangeness as Λ or Σ, than for the K^0.

The absorption of K_1 and K_2 beams because of nuclear interactions is the same, since the K^0 and $\bar{K}^0$ are absorbed independently and their norm is the same in K_1^0 and K_2^0. But the different elastic cross-sections for K^0 and $\bar{K}^0$ will change the phase relations existing between the K^0 and $\bar{K}^0$.

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*) We refer the reader to several articles where this problem is discussed[17,20-23].

After a scattering, the K_2^0 wave function is modified:

$$\psi = \frac{1}{\sqrt{2}} [f|K^0> - \bar{f}|\bar{K}^0>] ,$$

where f and $\bar{f}$ are complex numbers. This can be written as

$$\psi = \frac{f - \bar{f}}{2} |K_1^0> - \frac{f + \bar{f}}{2} |K_2^0> .$$

If $f \neq \bar{f}$ we have a finite probability of again having K_1^0 after passage through matter. Three mechanisms of regeneration can be considered:

i) interaction of the kaon with the individual nucleons from the nuclei;

ii) interaction of the kaons with all the nucleons of the nuclei, acting coherently in the forward direction: this is called the diffraction regeneration;

iii) interaction of the kaons with all the nuclei of the medium acting coherently in the forward direction: this is the coherent transmission regeneration which is the dominant effect with thick regenerators.

3.2 The coherent transmission regeneration

Let us consider a pure K_2^0 beam impinging upon a slab of material of length L. Such beams are easily obtained from many accelerators.

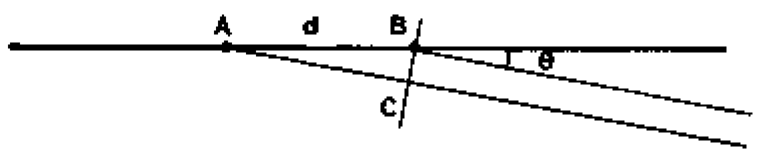
After a short time corresponding to a few centimetres from the point of production, the neutral K beams all consist of pure K_2^0.

Typically, an accelerator like the Princeton-Pennsylvania one (3 GeV) gives intensities of 5×10^4 K_2^0/sec at a mean momentum of 250 MeV/c, at 90° from the proton beam; while the neutron beam, which is the main contamination in such beams, is 5×10^7 at a momentum of 400 MeV/c. The γ rays are filtered by lead collimators, and sweeping magnets eliminate the charged particles.

Consider a plane wave traversing a material medium. If we consider the scattering of the wave by a single atom we have

$$\psi_{out} = e^{ikz} + \frac{1}{r} e^{ikr} f(\theta).$$

To the incoming plane wave is superimposed a spherical outgoing wave. The question then arises whether we can add the contributions from the different scattering centres:

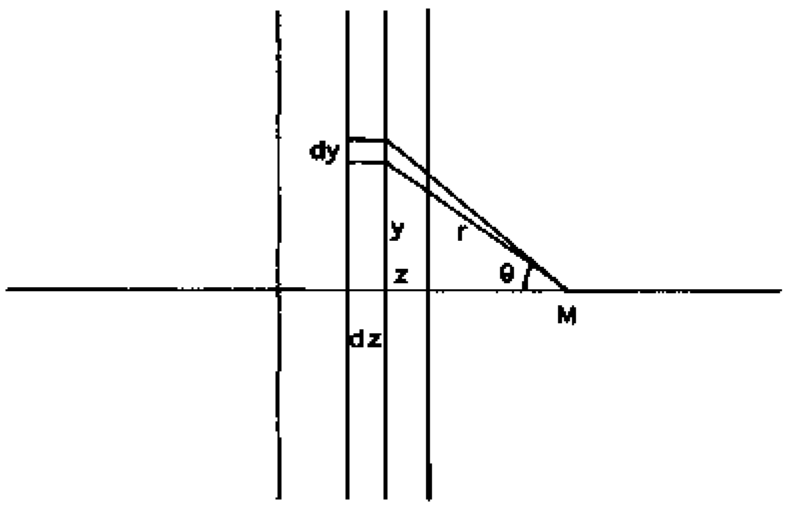


Consider two centres having a distance d along the path of K_2^0, d being typical of interatomic distances ~ 1 Å = 10^{-8} cm. The path difference for two waves scattered at the angle θ is AC - AB = $d(\theta^2/2)$. The two waves add coherently only if this difference is very small compared to the wavelength of the incident particle.

Typically, at 1 GeV/c, $\lambda(K^0) = 1.2 \times 10^{-13}$ cm and this condition means:

$$d\,\frac{\theta^2}{2} \ll 1.2 \times 10^{-13}\ \text{cm} \qquad \theta^2 \ll 2.4 \times 10^{-13}$$
$$\theta \ll 5\ \text{mrad}.$$

We see that it is only in a narrow forward cone that the coherent conditions hold for the adjacent atoms along the path. If we consider the coherent contribution from atoms all along the path in the material, this condition is even more stringent and only at zero angle can the contributions add. Consider a sheet dz, at z, inside the slab. We take as origin the point M where we measure the scattered wave



The contribution, at M, to the scattered wave by the atoms placed at a constant distance r from M is:

$$N\,dz\,2\pi y dy\,f(\theta)\,\frac{e^{ikr}}{r}\ ,$$

where N is the number of atoms per cubic centimetre.

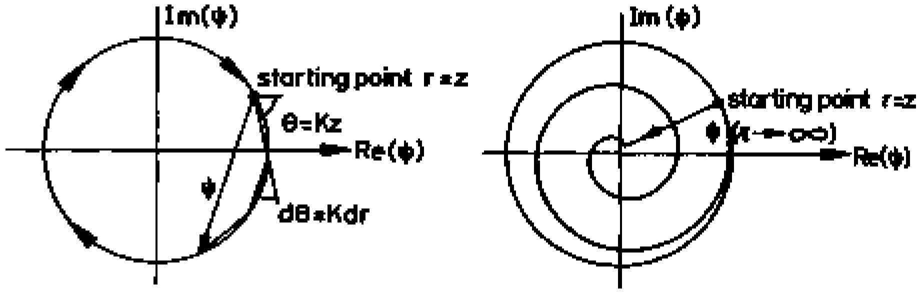
We have seen that only very small angles are considered in this process, and to an excellent approximation $f(\theta) = f(0)$. We call f and $\bar{f}$ the forward scattering amplitudes for K^0 and $\bar{K}^0$. We have the relations $y^2 + z^2 = r^2$, and for constant z, $ydy = rdr$. We have to integrate the expression $e^{ikr}\,dr$ from z to ∞, and we shall have a diffraction pattern. At $r' = r + \pi/k$ (limit of the first Fresnel zone) the contributions are in opposite phase to the one coming from the axis region. The integration leads to the expression

$$\frac{1}{ki}\left[e^{ik\infty} - e^{ikz}\right]$$

in which the first term is an indefinite quantity. It is a classical problem in optics.

$$\psi = \int_{r=z}^{r=\infty} e^{ikr}\, dr$$

can be considered as an infinite sum of very small vectors in the complex plane (Fig. a).



(a)

(b)

As we continue to add vectors we shall come back to the starting point and go along the circle for ever.

However, the physical situation is different. The regenerating plate does not extend to infinity. If we imagine that it decreases to zero progressively, or that some process such as absorption or decay reduces the contribution when r increases, then we add vectors of decreasing length whilst turning by the same angle (Fig. b). We thus spiral progressively towards the centre and for $r = \infty$ the integral is just e^{ikz}/k. The increment in amplitude $d\psi$ produced by the larger dz is then

$$\boxed{d\psi = 2\pi\, i\, N\, dz\, f(0)\, \frac{e^{ikz}}{k}} \qquad (I.3)$$

As can be checked easily, it is half the contribution from the first Fresnel zone. For a slab of finite length L the contribution will add linearity, and we have only to integrate this expression over the cell thickness L. A beam consisting of a superposition of K_1^0 and K_2^0 will have a time propagation determined by the nuclear interactions in the medium and by the propagation in vacuum, defined in Section 1.2.

$$\frac{d\psi}{dt} = \left(\frac{d\psi}{dt}\right)_{\text{vacuum}} + \left(\frac{d\psi}{dt}\right)_{\text{matter}} .$$

Taking into account that $dz = v\,\gamma\, d\tau = (k/m)d\tau$,

$$\frac{d}{d\tau}\begin{pmatrix} a \\ \bar{a} \end{pmatrix} = \frac{2\pi\, i\, N}{m}\begin{pmatrix} f & 0 \\ 0 & \bar{f} \end{pmatrix}\begin{pmatrix} a \\ \bar{a} \end{pmatrix}$$

where a and $\bar{a}$ are the amplitudes of a state in the $(K^0\ \bar{K}^0)$ basis, m being the mass of the neutral kaons.

Combining this variation with the one in vacuum we obtain

$$i\,\frac{d}{d\tau}\begin{pmatrix} a \\ \bar{a} \end{pmatrix} = \begin{pmatrix} M & \frac{\Delta M}{2} \\ \frac{\Delta M}{2} & \bar{M} \end{pmatrix}\begin{pmatrix} a \\ \bar{a} \end{pmatrix} - \begin{pmatrix} \frac{2\pi N}{m} f & 0 \\ 0 & \frac{2\pi N}{m}\bar{f} \end{pmatrix}\begin{pmatrix} a \\ \bar{a} \end{pmatrix}$$

$$= \begin{pmatrix} M - \frac{2\pi N}{m} f & \frac{\Delta M}{2} \\ \frac{\Delta M}{2} & \bar{M} - \frac{2\pi N}{m}\bar{f} \end{pmatrix}\begin{pmatrix} a \\ \bar{a} \end{pmatrix} .$$

If we start with a basis $(K_1^0\ K_2^0)$ we obtain

$$\frac{d}{d\tau}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\text{matter}} = \frac{\pi\, i N}{m}\begin{pmatrix} f + \bar{f} & f - \bar{f} \\ f - \bar{f} & f + \bar{f} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

leading to a total mass matrix

$$M_{\text{matter}} + M_{\text{vacuum}} = \begin{pmatrix} M_2 - \frac{\pi N}{m}(f + \bar{f}) & f - \bar{f} \\ f - \bar{f} & M_1 - \frac{\pi N}{m}(f + \bar{f}) \end{pmatrix}$$

where $M_2 = m_2 - i\,\Gamma_2/2$; $M_1 = m_1 - i\,\Gamma_1/2$. K_1 and K_2 are no longer eigenstates of this operator. It is necessary to diagonalize the mass matrix, and this leads to the approximate values for the eigenstates [see Bell and Steinberger[22]]:

$$|K_2^0>_{matt} = |K_2^0> + \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\;|K_1^0>$$

$$|K_1^0>_{matt} = |K_1^0> - \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\;|K_2^0> \quad , \tag{I.4}$$

with the complex masses

$$M_2' = M_2 - \frac{\pi N}{m}(f+\bar{f})$$
$$M_1' = M_1 - \frac{\pi N}{m}(f+\bar{f}) \tag{I.5}$$

neglecting terms of the order of $\left(\frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\right)^2$ *). Consider a K_2^0 incident on a piece of material. At the surface it is convenient to describe $|K_2^0>$ in terms of the diagonal states in the material since we know how they propagate.

From the relations (I.4)

$$|K_2^0> = |K_2^0>_{matt} - \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\left[|K_1^0> + \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}|K_2^0>\right] \quad ,$$

and neglecting terms in second order in $f-\bar{f}$,

$$|K_2^0> = |K_2^0>_{matt} - \frac{\pi N}{m}\frac{f-\bar{f}}{M_1-M_2}|K_1^0> \quad .$$

If it takes a proper time τ to travel in the matter, after this time we have in the matter

$$\phi = e^{-iM_2'\tau}|K_2^0>_{matt} - \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\,e^{-iM_1'\tau}|K_1^0>_{matt}$$

or, in terms of the eigenstates K_1^0 and K_2^0 in vacuum

$$\phi = e^{-iM_2'\tau}|K_2^0> + e^{-iM_2'\tau}\frac{\pi N}{m}\frac{f-\bar{f}}{M_1-M_2}\;|K_1^0>$$
$$- e^{-iM_1'\tau}\frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\;|K_1^0>$$

$$\phi = e^{-iM_2'\tau}\left[K_2^0 + \frac{\pi N}{m}\frac{f-\bar{f}}{M_2-M_1}\left(1-e^{+i\,\tau(M_2-M_1)}\right)K_1^0\right]$$

since $M_2'-M_1' = M_2-M_1$.

---

*) In Appendix D we give the derivation of this formula.

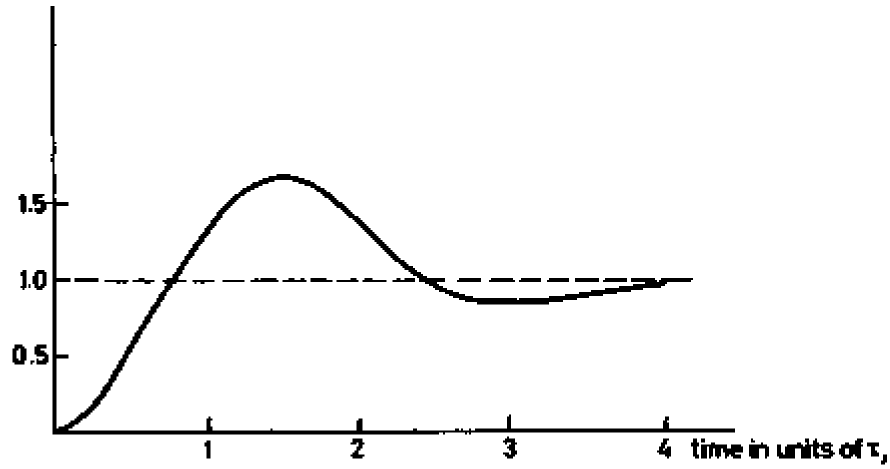
There is regeneration of the state K_1^0, the amplitude of regeneration being

$$\boxed{\rho = \frac{\pi N}{m} i\,[f - \bar{f}]\left[\frac{1 - e^{i\Delta M\tau}}{-i\Delta M}\right]}$$

where ΔM is the complex mass difference $\Delta m - 1/2\,\Gamma_1$ neglecting Γ_2 with respect to Γ_1.

This formula calls for the following remarks:

i) The intensity of a K_1^0 beam after traversal of a slab is found by squaring this expression. It is, in the case where $\Delta m = 1/\Gamma_1$,



proportional to $1 + e^{-\Gamma_1\tau} - 2e^{-\Gamma_1\tau/2}\cos\Delta m\tau$. We see that if $\Delta m >> 1/\tau$, in a time of the order of $1/\Gamma_1$ the $\cos\Delta m\tau$ term will oscillate very strongly during a lifetime. In the case envisaged above of $\Delta m = 10^7\,\Gamma_1$, this oscillation would prevent any measurement as a function of time, while if $\Delta m \sim \Gamma_1$ an oscillation in the appearance of K_1^0 can be observed.

ii) The forward amplitudes f and $\bar{f}$ are related to the total cross-section σ and $\bar{\sigma}$ by the optical theorem

$$\sigma_{\text{total}} = \frac{4\pi}{P_K}\,\text{Im}\,f\ .$$

If one knows the cross-sections for K^+ and K^- in the material, they are the same for K^0 and $\bar{K}^0$ by isospin symmetry in strong interactions. For a very thin regenerator,

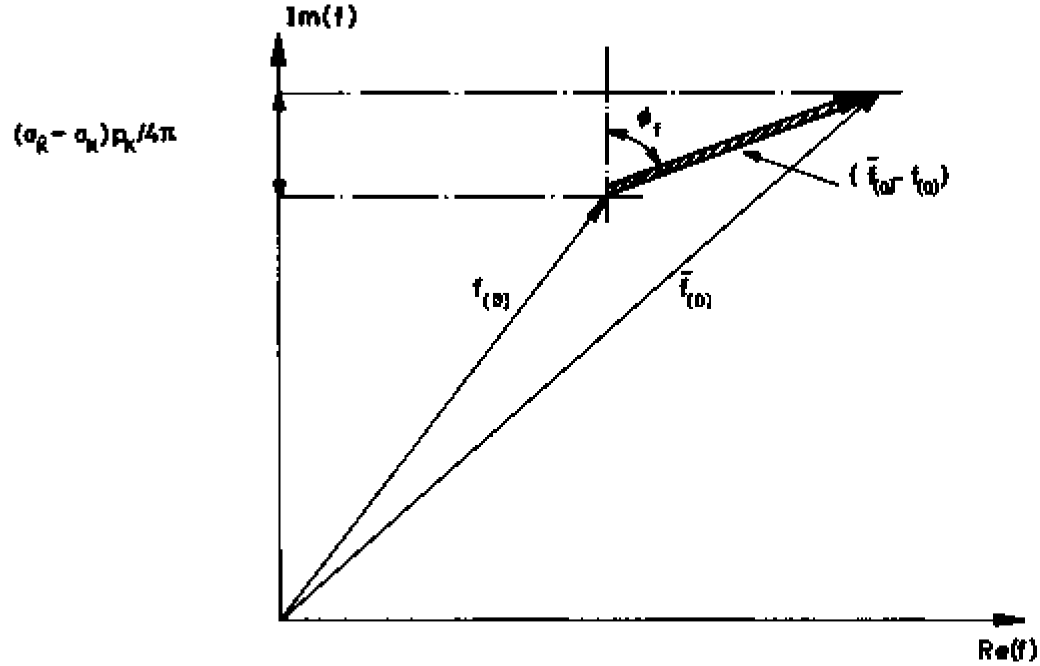
$$\varphi_f = -\arg i(f - \bar{f})\ ;$$

for a thick one

$$\varphi_\rho = \varphi_f + \arg\left(\frac{1 - e^{iM\Delta\tau}}{i\Delta M}\right) .$$

The second term can be calculated if one knows ΔM. The module of $f - \bar{f}$ is measured from the measurement of the regeneration amplitude $(f - \bar{f})$. One can thus obtain the absolute value of the phase:

$$\varphi_f = \cos^{-1} \frac{\mathrm{Im}\,[\bar{f} - f]}{[f - \bar{f}]}$$



This relation has been used by Rubbia and Steinberger[24] and Mischke et al.[25], to derive φ_ρ from the existing experiments on K^+ and K^- scattering.

iii) The cross-section for coherent transmission regeneration will be proportional to N^2 or L^2, where L is the length of the slab, in the region where absorption processes are negligible.

3.3 The diffraction regeneration

If the conditions for additive effect in the forward direction are not fulfilled, and if the K^0 and $\bar{K}^0$ undergo elastic collisions on nuclei in the forward direction, there will still be regeneration of K_1^0. But at a given point m along the axis we cannot add the amplitudes from different points. We have first to square the amplitude for each individual element of volume and then integrate. The cross-section for the production of a K_1^0 is proportional to $|f - \bar{f}|^2$. After scattering in the forward direction we have

$$\frac{f|K^0 > - \bar{f}|\bar{K}^0 >}{\sqrt{2}}$$

$$\frac{f - \bar{f}}{\sqrt{2}} |K_1^0 > + \frac{f + \bar{f}}{\sqrt{2}} |K_2^0 > .$$

Two cases have to be considered:

i) transmission regeneration: one adds the amplitudes;

ii) diffraction regeneration: one adds the cross-sections.

The number of K_1^0 generated in a layer dz and traversing the thickness z in the direction of the incident beam is:

$$\frac{d}{dz}\left(\frac{d\sigma}{d\Omega}\right)_0 = |f - \bar{f}|^2\ N\ e^{-\Gamma_1 \tau}$$

where

$$\tau = \frac{t}{\gamma}\ ,\quad t = \frac{z}{v}\ ,\ \tau = \frac{z}{v\gamma}$$

$$\frac{d}{dz}\left(\frac{d\sigma}{d\Omega}\right)_0 = N|f - \bar{f}|^2\ e^{-\Gamma_1 (z/v\gamma)}\ .$$

If we integrate for a slab of length L we find

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = |f - \bar{f}|^2\ N\left[1 - e^{-\Gamma_1 \tau}\right]\ .$$

where τ is the time taken by the K^0 to traverse the slab $\tau = L/v\gamma$. Comparing this with the previous result, one finds:

$$R = \frac{\sigma^2}{d\sigma/d\Omega} = \frac{A\lambda^2}{\delta^2 + 1/4}\quad \frac{1 + e^{-\ell} - 2\ e^{-\ell/2}\cos\delta}{1 - e^{-\ell}}$$

where

$\ell = \Gamma_1 \tau$,

$\delta = \Delta m/\Gamma_1$,

$\lambda = 2\pi/k$,

A = decay length = $\beta\gamma/\Gamma_1 = v\gamma\tau$,

$\frac{A}{v}$ = lab time t.

This ratio is independent of the scattering amplitudes. We see that the thicker the slab the more important is the coherent regeneration, up to the point where absorption processes are negligible.

Typically, in iron,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{inel}} = 37\ \mathrm{mb/sr}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{el}} = 280\ \mathrm{mb/sr}\ .$$

4. DISCUSSION OF SOME RECENT IMPORTANT EXPERIMENTS IN NEUTRAL KAON PHYSICS

4.1 The study of regeneration, by Christenson et al.[23,24]

For 30 GeV protons bombarding a 0.5 mm beryllium wire target, a beam was chosen at 30° to the incident proton defined by a lead collimator at 4.3 metres from the target (Fig. 1). The angular divergence accepted is 4 mrad. After the collimator, a bending magnet sweeps the charged particles, and the γ rays are filtered by passing the beam through 4 cm of lead, upstream from the collimator. The K_2^0 beam passes through the materials in which the regeneration was studied at 16 metres from the target. The positions and momenta of the decay pions coming from a regenerated K_1^0 are measured in a pair of magnet and spark chamber spectrometers, thus determining all kinematic properties of the K_1^0 and its decay.

An event is accepted whenever there is a coincidence between the two spectrometers, anti-coincidence counters and Čerenkov counters giving the additional guarantee that the event is produced by a neutral particle decaying into two fast particles ($v/c > 3/4$). The main source of background in such beams comes from the neutrons, and they were used to monitor the K_2^0 intensity. By placing a regenerator and calculating the mass of the K_1^0 from the kinematics, it is found that

$$m_K = 498.10 \pm 0.52 \text{ MeV}$$

in good agreement with the accepted value $m_K = 497.8 \pm 0.6$.

4.1.1 The regeneration. Figure 2 shows a typical distribution of $\cos\theta_K$, where θ_K is the angle between the line-of-flight of the parent particle and the beam axis for these events of mass in the interval 483-513 MeV.

The narrow forward peak contains the regenerated K_1^0, the forward incoherent diffraction peak, and some background.

If one reduces the density of the regenerator by one-half, the transmission regeneration should decrease by a factor of 4. This was done by comparing regeneration from 7.5 cm of copper with that from a stack of 12×3 mm plates separated by 3 mm air gaps. It is an ideal half-density regenerator in the limit where the gaps are small compared to the wavelength of the mass difference oscillation. In this case it was 11.5 cm. The observed ratio in the coherent peak should be 3.88, taking into account the small correction from the air gap.

For the incoherent peak, the attenuation should be 1.95 (instead of 2 for the same correction).

The observed values are:

$$S_{exp} = 4.05 \pm 0.48$$

$$\Delta\,(\cos\theta_K) = 0.00004 \quad .$$

This establishes firmly the correct interpretation of the K_1 intensity. To give an idea of the orders of magnitude, let us list some total forward regeneration cross-sections

Material	σ_T(mb)
Carbon	273
Iron	970
Copper	1080

We see that they are very high. They agree very well with optical model calculations. These calculations are based on the fact that the forward scattering cross-sections are related to the total K-nucleon cross-section, while charge independence relates K^+n to K^0p and K^+p to K^0n. One calculates independently the scattering amplitude for K^0 and $\bar{K}^0$, the K_1^0 amplitude being half their sum.

4.2 <u>The 2π decay of the long-lived neutral kaon</u>

4.2.1 <u>The decay into charged pions.</u> It is in the course of this last experiment on the regeneration of K_1^0 from K_2^0 beams that the doubly charged pion decay mode was found[12]. Even without any regeneration, a small proportion of 2π decay was found in the beam. The very narrow angular acceptance of the K_2^0 beam, as well as the high accuracy in the measurements of direction and momenta of the pions, make it possible to determine with accuracy the angle between the incoming K_2^0 and the outgoing K_1^0 and also to determine the invariant mass. This is necessary because of the very large background from the semi-leptonic decays of K_2^0.

Figure 3 shows the evidence obtained by these authors for the existence in helium gas of kaons decaying into two pions, identical both in mass and in angular spread to those obtained from coherent regeneration. The helium gas has such a low density that it can be safely considered as vacuum for this experiment.

Their value

$$R = \frac{K_2^0 \to \pi^+ + \pi^-}{K_2^0 \to \text{all charged modes}} = (2.0 \pm 0.4)10^{-3}$$

has been confirmed by all succeeding measurements, which are presented in Table 3.

<u>Table 3</u>

$(K_2^0 \to \pi^+ + \pi^- / K_2^0 \to \text{all charged modes})$

Authors	
Christenson et al.[26]	$(1.87 \pm 0.18) \times 10^{-3}$
Abashian et al.[27]	$(2 - 3) \times 10^{-3}$
Galbraith et al.[28]	$(2.08 \pm 0.35) \times 10^{-3}$
de Bouard et al.[29]	$(2.24 \pm 0.23) \times 10^{-3}$
Rubbia et al.[30]	$(1.96 \pm 0.35) \times 10^{-3}$

In discussing the methods used for the measurement of the mass difference Δm we will say a word about how the most accurate results are obtained.

4.2.2 <u>Measurements of the decay rate $K_2^0 \to \pi^0 + \pi^0$.</u> This effect is very difficult to measure: a neutral particle goes in and two neutral particles come out. To measure this rate in an absolute way, one needs to have an absolute monitor for the K_2^0 and absolute measurements for the π^0, which decay essentially into 2γ. These difficult tasks have recently been fulfilled by two groups, one at Princeton [Cronin et al.[14]] and another at CERN [Gaillard et al.[15]].

The two approaches have been very different. In the CERN experiments, the four γ rays are detected by conversion into thick spark chambers. The absolute calibration is done by comparing the number of events with those produced by K_1 from a carbon regenerator.

At Princeton they use a particular feature of the accelerator. The beam is split into short bursts so that the time of detection of an event gives its time-of-flight. This knowledge of the momenta of each kaon allows the use of only one γ ray from the π^0. The spectrum of the π^0 in the centre of mass is so different in the $2\pi^0$ and $3\pi^0$ decays, that one can very well separate the γ rays in a given geometry*). The energy of the γ rays is measured by a spark chamber magnetic spectrometer. The same group also made accurate measurements of the decay mode $K_L \to \gamma + \gamma$.

The results are:

$$|\eta_{00}| = \frac{\text{Rate } K_2 \to \pi^0 + \pi^0}{\text{Rate } K_1 \to \pi^0 + \pi^0} = (4.9 \pm 0.5)10^{-3} \quad \text{(Princeton)}$$

and

$$|\eta_{00}| = \left(4.3 \begin{array}{l} +1.1 \\ -0.8 \end{array}\right) 10^{-3} \qquad \text{(CERN)}$$

These values differ conclusively from

$$|\eta_{+-}| = (1.94 \pm 0.09)10^{-3} \qquad \text{(world average).}$$

Such a result also discards the theories attributing the $K_L \to 2\pi$ decay to a regeneration of K_S by any type of unknown interaction.

The Princeton group also finds

$$\frac{\text{Rate}(K_2^0 \to \gamma + \gamma)}{\text{Rate}(K_2^0 \to \text{all modes})} = (7.4 \pm 1.6)10^{-4}$$

in strong disagreement with the value of Criegee et al.[31] who found $(1.3 \pm 0.6)10^{-4}$. This disagreement is worth noting, and shows how difficult are the experiments in which decay rates of neutral particles into neutral particles are measured.

4.2.3 <u>The measurement of Δm.</u> The mass difference, mass (K_L^0) - mass (K_S^0), has been measured by a great variety of methods. They are described in several review articles[32].

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*) The knowledge of the precise momentum of the K^0 is essential in order to do the transformation to the centre of mass.

Table 4 gives a list of methods and results from a report by Myron L. Good given at the Argonne Conference on Weak Interactions (1965)[33].

Table 4

Experimental determinations of Δm
Mass difference between K_1 and K_2 in units of Γ_S.

Strangeness oscillations	
1) 1.9 ± 0.33	Fitch, Perkins and Piroué, Nuovo Cimento 22, 1160 (1961).
2) 1.5 ± 0.2	Camerini, Fry, Gaidos, Huizita, Natale, Willman, Birge, Ely, Powell and White, Phys.Rev. 128, 352 (1962).
3) $0.62 {+0.33 \atop -0.27}$	Meissner, Crawford, Crawford and Golden.
Coherent regeneration (compared to incoherent)	
4) $0.84 + 0.29$	R. Good, Matsen, Muller, Piccioni, Powell, White, Fowler and Birge, Phys.Rev.Letters 124, 1223 (1961).
Thickness dependence of coherent regeneration	
5) 0.72 ± 0.15	Fuji, Jovanovitch, Turkot and Zorn. [Earlier report in Phys.Rev.Letters 13, 253 (1964).]
"Gap" method	
6) 0.55 ± 0.13	Christenson, Cronin, Fitch and Turlay, Int.Conf. on Weak Interactions, Brookhaven (1963) p. 74; corrected for CP violation.
Leptonic decay charge ratio versus time	
7) 0.47 ± 0.20	Ecole Polytechnique
8) $0.15 {+0.35 \atop -0.50}$	Padua — J. Steinberger's talk, Oxford Conf., 1965

It would be impossible, within the frame of these lectures, to describe all these methods. As a function of time the values are stabilized around the value 0.5. Because of the systematic trend shown by some methods to give higher values, one first thought that there might be an anomalous strong mass difference introduced by absorbers. They were repeated recently, and the values reported at the Berkeley Conference in 1966 were all around 0.5; this can be seen from Table 5. This table includes values obtained by a new method making use of the 2π decay of K_L^o in interference experiments.

Table 5

Values of $\Delta m = |m_{K_2^0} - m_{K_1^0}|$

Author	Δm (τ_1^{-1})	Technique
CERN (Bott-Bodenhousen et al.)	0.480 ± 0.024	Counter
" (Alff-Steinberger et al.)	0.445 ± 0.034	"
La Jolla	0.44 ± 0.06	"
BNL (Fujii et al.)	0.35 ± 0.15	Counter
Carnegie (Canter et al.)	0.55 ± 0.15	B. Ch.
" (Hill et al.)	0.63 ± 0.16	B. Ch.

All that has been said in the preceding lines makes it easy to understand the principles underlying all the methods using regeneration.

Let me just sketch in a few words some of the methods and describe more fully some experiments because they illustrate at best the reason for the recent progress.

4.2.4 Strangeness oscillations. One starts from a beam containing a given relative proportion of K^0 and $\bar{K}^0$, for instance by reactions such as

$$K^+ + n \to K^0 + p$$

$$\bar{p} + p \to K^0 + \bar{K} \ .$$

In the case of the K^0 beam we have at time 0

$$K^0 = \frac{1}{\sqrt{2}} (K_1^0 + K_2^0) \ .$$

At a time t, the state will propagate according to

$$\phi = \frac{1}{\sqrt{2}} (K_1^0 \ e^{-iM_1\tau} + K_2^0 \ e^{-iM_2\tau})$$

$$\phi = \frac{1}{2} [K^0 (e^{-iM_1\tau} + e^{-iM_2\tau}) - \bar{K}^0 (e^{-iM_1\tau} - e^{-iM_2\tau})] \ .$$

The intensity of a given component K^0 or $\bar{K}^0$ varies like

$$\left(e^{-iM_1\tau} \pm e^{-iM_2\tau} \right)^2 = e^{-\Gamma_1\tau} + e^{-\Gamma_2\tau} \pm 2 \cos \Delta m\tau \times e^{-(\Gamma_1 + \Gamma_2)\tau/2} \ .$$

For times small compared to $1/\Gamma_2$, the intensity of any component varies as

$$1 + e^{-\Gamma_1\tau} \pm 2\cos\Delta m\tau\, e^{-\Gamma_1\tau/2} \quad .$$

One determines the intensity as a function of time of any of the components by requiring a signature of K^0 or $\bar{K}^0$ in a strong interaction. This is a popular method in bubble chambers where one can easily observe the K_1^0 close to the production point. The $\bar{K}^0$ are detected by looking at hyperons produced by the neutral kaon.

4.2.5 <u>The leptonic decay charge ratio versus time.</u> This method has some analogy to the preceding. In so far as the $\Delta Q = \Delta S$ rule is verified, the K^0 and $\bar{K}^0$ states are identified by their decays

$$K^0 \to \ell^+ + \nu + \pi^-$$

$$\bar{K}^0 \to \ell^- + \bar{\nu} + \pi^+$$

since the decays with the opposite charges are forbidden. In this case, we obtain the same formulae as the preceding one for the intensity of the positive or the negative components. However, it is more as a check of the rule $\Delta Q = \Delta S$ that these decays have been studied, and the interpretation of the results is rather complex[34].

4.2.6 <u>Methods based on regeneration.</u> There are several ways to make use of the regeneration mechanism in order to determine the amplitude and the sign of the mass difference. As an example, let us mention the method suggested by Fitch.

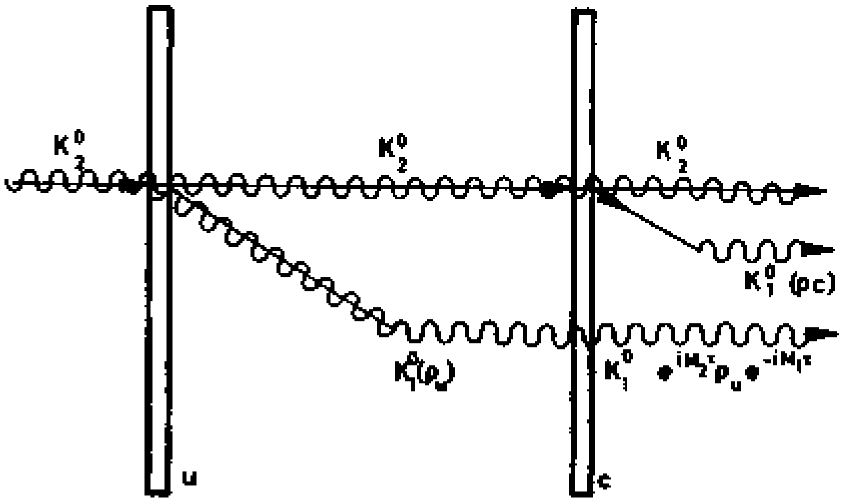
A regenerator consisting of two pieces of copper separated by a variable gap is used, keeping the total amount of material in the beam constant. As one piece is moved upstream along the beam, the resultant K_1^0 amplitude is the sum of the contributions from the two pieces. The amplitude of the K_1^0 pair regenerated in the first slab is proportional to $e^{-M_1\tau}$, where τ is the proper time taken from the end of the first slab to the exit of the second, while the contribution from the regeneration in the second slab is proportional to $e^{-M_2\tau}$, which is the amplitude of the K_2^0 at this position. This gives rise to the interference term in the amplitude of K_1^0. Figure 4 shows the variation of the coherent regeneration with the gap length, from which the authors[24] conclude that

$$\Delta m = \left(0.47 \begin{array}{l} + 0.11 \\ - 0.13 \end{array} \right) \Gamma_S \quad .$$

However, it should be pointed out that a zero mass difference has a probability of 32%. What such an experiment proved is, in fact, the unacceptability of the early high values for Δm. Since more precise determinations of Δm have been made, which we shall mention later, we do not want to discuss the matter here and refer the reader to the article of Good[33] where these methods are analysed at length.

5. <u>SIGN OF Δm</u>

Several ideas have been proposed to measure the sign. Four independent experiments have now demonstrated that $M(K_L) > M(K_S)$[35-38]. As example, let us give the method of Kobsarev and Okun[39].



Consider two slabs of different regenerators u and c. Take as origin of time the exit time from c.

The amplitude of K_S after c is the sum of two contributions, neglecting the absorption:

$$e^{+iM_2\tau}\ \rho_u\ e^{-iM_1\tau} + \rho_c\ ,$$

where ρ_u and ρ_c are the regeneration amplitudes in u and c. The intensity of the K_S component is:

$$\rho_u^2\ e^{(\Gamma_1 - \Gamma_2)\tau} + \rho_c^2 + |\rho_u|\ |\rho_c|\ \cos\left[\Delta m\tau + \varphi_u - \varphi_c\right]\ .$$

If we interchange u and c we have

$$\rho_c^2\ e^{(\Gamma_1 - \Gamma_2)\tau} + \rho^2 + |\rho_u|\ |\rho_c|\ \cos\left[\Delta m\tau - (\varphi_u - \varphi_c)\right]\ .$$

The difference between the two expressions gives rise to a term

$$\sin \Delta m\tau \sin\,(\varphi_u - \varphi_c)\ .$$

If the sign of $\varphi_u - \varphi_c$ is known, the variation of the intensity as a function of τ gives the sign of Δm. The sign of $\varphi_u - \varphi_c$ is calculated from optical model theory. The experiment has been performed recently by Jovanovitch et al.[36]. Figure 5 shows the variation of the K_S intensity as a function of the distance between the two absorbers. The positive sign of $K_L - K_S$ is clearly demonstrated.

All the other methods rely on some different techniques to have a sin Δm term appearing in the amplitude of the component being measured[33].

6. INTERFERENCE EXPERIMENTS IN THE 2π DECAY OF K_S AND K_L

The fact that K_S and K_L can both decay in the common 2π channel has opened the way to a series of important experiments based on the interference between the two components when they are mixed coherently.

To have K_S and K_L mixed coherently, the most straightforward way is to use a K^0 or $\bar{K}^0$ beam. After some metres only K_L are left, and such experiments are restricted to a region close to the target where the neutral kaons are produced. For this reason the counter experiments have been slow in this approach, since usually the targets are inside the accelerator, and large backgrounds prevent working so close to the machine. In bubble chambers, however, the target is in the sensitive volume itself, and small intensities of secondary beams are sufficient to produce enough neutral kaons, the decay region of which is inside the sensitive volume of the chamber. This is why the first interference experiments were done with bubble chambers, using the leptonic decay as a channel common to both components.

The availability of proton beams extracted from the accelerator and impinging on external targets has now opened the way to counter techniques (also for experiments close to the target), and at CERN, Rubbia and Steinberger have put forward an important experiment along these lines.

Another way to produce a coherent mixture of K_S and K_L is to regenerate K_S by coherent transmission through matter of a K_L beam. Such beams are easily produced by simply having a hole in the shielding walls of the accelerator viewing an internal target.

We have seen that the coherent regeneration gives rise to a K_S component, the amplitude of which is given by the parameter of regeneration

$$\rho = \frac{\pi N}{M} i \, (f - \bar{f}) \, \frac{1 - e^{-i\Delta m\tau}}{-i\Delta m} \, .$$

A proper time τ after the traversal of the matter, the particle beam can be described by the amplitude

$$e^{-iM_L\tau} K_L + \rho \, e^{-iM_S\tau} .$$

If we define

$$\eta_{+-} = \frac{a^L_{+-}}{a^S_{+-}} = \frac{\text{amplitude for } K_L \to \pi^+ + \pi^-}{\text{amplitude for } K_S \to \pi^+ + \pi^-} ,$$

the $\pi^+\pi^-$ decay rate per unit time can be written in terms of the K_S decay width $\Gamma_{S,+-}$

$$\frac{dN_{+-}}{dt} = \Gamma_{S,+-} \left[\rho\, e^{-iM_S \tau} + \eta_{+-}\, e^{-iM_L \tau} \right]^2$$

$$= \Gamma_{S,+-} \Big[|\rho|^2 e^{-\Gamma_S \tau} + |\eta_{+-}|^2 e^{-\Gamma_L \tau} \tag{I.6}$$

$$+ 2|\rho||\eta_+|\, e^{-(\Gamma_S + \Gamma_L)\tau/2} \cos(\varphi_\rho - \varphi_\rho - \Delta m \tau) \Big] ;$$

φ_ρ and φ_η are the phases of the complex numbers ρ and η.

Fitch et al.[40]) gave evidence for strong constructive interference between K_L and K_S by comparing the 2π intensity from two regenerators of different densities.

The most accurate recent results are obtained by measuring the intensity of the $\pi^+\pi^-$ decay as a function of the time travelled by the beam after the regenerator.

The amplitude of the interference oscillations depends on Δm while the phase of the oscillation depends on $\varphi_\rho - \varphi_\eta$.

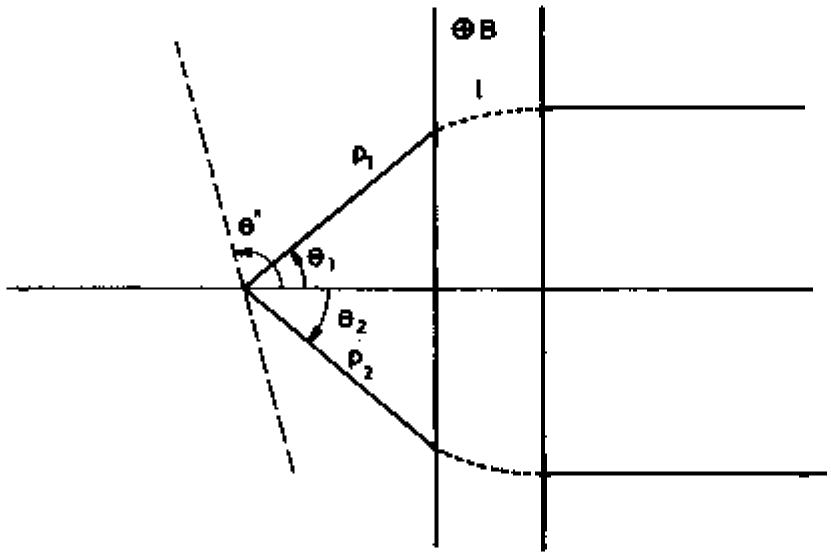
Several experiments have been performed giving the very accurate values for the mass differences reported in Table 5.

The large improvement in the accuracy comes from the fact that a clever use of the kinematical properties of the 2π decay allows a considerable increase of the acceptance of the detecting system for 2π decay, with respect to the leptonic decay.

As an example, let us consider the system put forward by Rubbia and Steinberger. If a K^0 decays into two pions, their transverse momenta in the laboratories are equal:

$$p_1 \sin\theta_1 = p_2 \sin\theta_2 = p^* \sin\theta^* \quad , \tag{I.7}$$

where p^* and θ^* are the momenta and the angle of emission in the c.m. system.



If the two pions pass through a uniform magnetic field B, with a length of trajectory ℓ, the momenta are rotated by an amount

$$\varphi = \frac{\ell}{\text{radius}} = \frac{p\ \ell}{e/c\ H} \ .$$

Let us consider only small curvatures so that the length of the track ℓ = width L of the magnet. Suppose that we choose H and L such that one of the momenta θ_1 becomes parallel to the initial beam. For small curvatures we have

$$\varphi_1 = \theta_1 = \frac{p_1\ L}{e/c\ H} \ .$$

The second pion will rotate by

$$\varphi_2 = p_2 \frac{L}{e/c\ H} = p_2 \frac{\theta_1}{p_1} \ . \qquad (I.8)$$

Now, since we are dealing with pions of several GeV/c while the transverse momentum is at most 210 MeV/c, the angles θ_1 and θ_2 are small so that $\sin\theta \sim \theta$ and relation (I.8) shows that $\varphi_2 = \theta_2$. In other words, the second pion is also parallel to the beam.

If the triggering system is such as to select events giving rise to two parallel pions, it will be highly selective for 2π decay, since the relation (I.7) does not hold for two charged particles emitted in the leptonic decays of a kaon. Such a system has the advantage that the relation of parallelism holds, irrespective of the decay point along the path, thus permitting a large acceptance.

The field parameters are chosen so as to hold for those pions emitted at 90° in the c.m. system. Because of the very small transverse momentum, the relation (I.7) still holds for pions emitted around 90° within a large solid angle.

Using this technique, 6% of the triggers correspond to 2π decays, while the branching ratio is only 2×10^{-3}. The efficiency for detecting a 2π decay is 50%.

The delicate point of these methods is that the acceptance of the system is a function of the position, and Monte Carlo calculations are necessary to determine the efficiency of the system along the path. Figure 6 shows the experimental results and the relative importance of this efficiency correction.

We have analysed the principle of this experiment (although so far it is not the one which has given the most accurate value for Δm), because of its simplicity and elegance. It is continuing at CERN with the K^0 beam from an external target with the ambition of achieving an accurate measurement of Δm and φ_{+-}.

Figures 6 and 7 show the results of the interference experiments done simultaneously at CERN. We see in both cases the importance of the efficiency corrections. The strong differences in the triggering systems reinforce our confidence in the two accurate values of Δm, since the systematic errors have little reason to be equal.

From the formula (I.6) we see that these experiments require a knowledge of the regeneration phase φ_ρ to extract $\varphi_{\eta_{+-}}$, which is a most important parameter to know.

We have seen that arg φ_ρ can be computed from arg $i(f - \bar{f})$, where f and $\bar{f}$ are the forward scattering amplitudes of K and $\bar{K}$. Using the arguments given on page 18 of this text and relying on the data for K^+n, K^-n, K^+p and K^-p scattering, Mishke et al.[23] obtain

$$\varphi_{\eta_{+-}} = + 0.44 \pm 0.44 \text{ rad.}$$

The contribution to the error from the uncertainty in φ_ρ is considerable (0.28 rad).

This result takes into account the independent determination of the sign of Δm[28-30] to which these interference experiments are not sensitive. Using the same arguments, Rubbia and Steinberger[24] obtained $\varphi_{(\eta_{+-})} = 0.60 \pm 0.23$. After the appearance of more accurate data of kaon scattering this result is modified to $\varphi_{\eta_{+-}} = 1.41 \pm 0.34$, while the interpretation of the other experiment done at CERN by Bott-Bodenhausen et al.[41] along the same lines gives $\varphi_{\eta_{+-}} = 1.22 \pm 0.36$.

This shows that a direct measurement of the phase $\varphi_{\eta_{+-}}$ would be gratifying.

One experiment is under way at CERN to get $\varphi_{\eta_{+-}}$ directly. Rubbia and Steinberger thought to study the 2π interference effect in a pure K^0 beam. After 10 to 14 lifetimes, the two-pion intensity from the K_S is reduced to the same level as the one coming from the K_L^0, and interference effects can be studied. The extracted proton beam impinges on a target, and at 6° from it the neutral kaon beam is used. There still remain serious problems of background. The initial kaon beam does not consist of pure K^0, but the cross-section for K^0 is seven times larger than the one for $\bar{K}^0$. The uncertainty about the exact initial composition does not directly affect the accuracy of the measurement, since the K^0 or $\bar{K}^0$ are initially not coherent in phase; it merely reduces the amplitude of the observed effect. However, because the interference is observed between 10 and 14 half-lives from the target, it is necessary to have a very high accuracy in the term $\Delta M\tau$ to have a good precision on $\varphi_{\eta_{+-}}$.

Δm will certainly be determined with the utmost accuracy by this experiment, but whether or not it will match the required accuracy is one of the unknowns of this experiment, the results of which are likely to appear this year.

7. CONCLUSION

We may conclude this incomplete and rather arbitrary choice of topic in the neutral kaon physics by stating some of the important results recently obtained by the experimentalists:

- The long-lived neutral kaon can decay into two charged pions with an amplitude η_{+-} relative to the decay of the short-lived kaon, where

$$|\eta_{+-}| = (1.94 \pm 0.09)10^{-3} \ ,$$

$$\varphi_{(\eta_{+-})} = 1.41 \pm 0.34 \text{ and } 1.22 \pm 0.36 \quad \text{(indirect measurements).}$$

- Interference is observed between K_L and regenerated K_S in the 2π decay.
- The long-lived kaon is heavier than the short-lived one.
- The most accurate determination of the mass difference is $K_L - K_S = (0.480 \pm 0.024)/\Gamma_S$.
- Two measurements give the modules of the amplitude of the decay of the long-lived kaon into two neutral pions:

$$|\eta_{oo}| = (4.3 {}^{+1.1}_{-0.8})10^{-3} \quad \text{and} \quad (4.9 \pm 0.5)10^{-3} \ .$$

- The phase $\varphi_{(\eta_{oo})}$ is not known but its measurement is under way.

* * *

CHAPTER II

1. STRONG AND ELECTROMAGNETIC INTERACTIONS

1) In strong and electromagnetic interactions the strangeness (or equivalently the hypercharge Y) is conserved. The mesons K^+ and K^0 have a hypercharge Y = 1; the mesons K^- and $\bar{K}^0$ have a hypercharge Y = -1

$$< K^0|Y|K^0 > = 1$$
$$< \bar{K}^0|Y|\bar{K}^0 > = -1 \; .$$

For strong and electromagnetic interactions we have a superselection rule due to the hypercharge conservation. All the matrix elements of observable quantities between a K^0 state and a $\bar{K}^0$ state vanish, and all the physical states are eigenstates of hypercharge. In particular, the relative phase between the states $|K^0>$ and $|\bar{K}^0>$ is completely arbitrary.

2) Let us define as H_0 the total Hamiltonian for strong and electromagnetic interactions. The mass of a stable particle with respect to strong and electromagnetic interactions can be defined as the eigenvalue of H_0 for a one-particle state at rest

$$< K^0|H_0|K^0 > = m_0$$
$$< \bar{K}^0|H_0|\bar{K}^0 > = \bar{m}_0 \; .$$

Because of the superselection rule for hypercharge

$$< K^0|H_0|\bar{K}^0 > = 0$$
$$< \bar{K}^0|H_0|K^0 > = 0 \; .$$

On the basis of $|K^0>$ $|\bar{K}^0>$ it is then possible to define a mass matrix M_0 which is diagonal and real

$$M_0 = \begin{vmatrix} m_0 & 0 \\ 0 & \bar{m}_0 \end{vmatrix} \; . \qquad \text{(II.1)}$$

3) The equality of the masses m_0 and $\bar{m}_0$ is obtained as a consequence of some discrete symmetries:

i) particle-antiparticle conjugation C;
ii) CP, where P is the space reflection;
iii) TCP, where T is the time reflection.

2. WEAK INTERACTIONS

1) The weak interactions are responsible for the decay of the K mesons. The various modes of decay can be classified as follows:

a) Hadronic decays: hypercharge violating in π mesons
   i) (2π) observed $\pi^+\pi^-$ and $\pi^0\pi^0$
   ii) (3π) observed $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$.

b) <u>Semi-leptonic decays</u>: charged π mesons and leptons

i) K_{ℓ_3} type in $\pi - \ell - \nu_\ell$

ii) K_{ℓ_4} type in $2\pi - \ell - \nu_\ell$.

c) <u>Leptonic decays</u>

Whereas the charged K mesons can decay into leptons

$$K^+ \to \mu^+ + \nu_\mu \; ; \qquad K^+ \to e^+ + \nu_e \; ,$$

corresponding decay modes for the neutral K mesons have not been observed. This experimental fact is understood as a suppression, at first order in weak interactions, of a neutral leptonic current

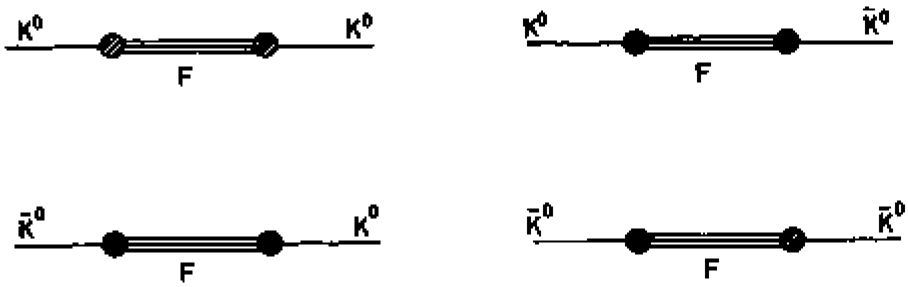
$$K^0 \not\to \mu^+\mu^- \; , \qquad K^0 \not\to e^+e^- \; , \qquad K^0 \not\to \nu_\ell \bar{\nu}_\ell \; .$$

d) <u>Radiative decays</u>

The 2γ mode has been detected.

2) <u>Self-energy</u>

The self-energy of the neutral K mesons is modified because of the presence of weak interactions. Let us call F a physical final state observed in the decay of neutral K mesons. We now have four types of transitions with <u>real</u> intermediate states as shown in the graph below.



These transitions occur at least at second order in weak interactions. Because of the existence of real intermediate states, the self-energy becomes complex and the imaginary part is related, by unitarity, to the decay amplitudes. As a second consequence there now exist non-diagonal transitions.

3) <u>Basis $K^0\bar{K}^0$</u>

In the absence of weak interactions, the states $K^0(t)$ and $\bar{K}^0(t)$ satisfy separately a time-dependent Schrödinger equation

$$i \frac{d}{dt} \begin{vmatrix} K^0(t) \\ \bar{K}^0(t) \end{vmatrix} = M_0 \begin{vmatrix} K^0(t) \\ \bar{K}^0(t) \end{vmatrix} \; . \qquad \text{(II.2)}$$

The introduction of new channels F, now open through weak interactions, completely modifies this simple situation, and for the amplitudes $K^0(t)$ and $\bar{K}^0(t)$ gives rise to a complicated system of coupled equations.

The two basic ingredients used to formulate the problem are:

i) the superposition principle in the $K^0\ \bar{K}^0$ space;

ii) the time-dependent Schrödinger equation.

A solution of the system of coupled equations has been given by Weisskopf and Wigner. We do not discuss here the details of the calculations, and instead of Eq. (II.2) we simply write a matrix equation

$$i\,\frac{d}{dt}\begin{vmatrix} K^0(t) \\ \bar{K}^0(t) \end{vmatrix} = \mathcal{M}\begin{vmatrix} K^0(t) \\ \bar{K}^0(t) \end{vmatrix} \qquad \text{(II.3)}$$

where $\mathcal{M}$ is independent of time.

The Hermitian part of $\mathcal{M}$ is the mass matrix M, and the skew Hermitian part of $\mathcal{M}$ is the decay matrix Γ, as will be shown in Section 4 of this chapter:

$$\mathcal{M} = M - i\,\frac{\Gamma}{2}\,. \qquad \text{(II.4)}$$

The matrices M and Γ are both Hermitian. The matrix Γ and the difference $M - M_0$ are due to weak interactions.

In the following, the indices 1 and 2 will be used for the states K^0 and $\bar{K}^0$

$$\mathcal{M}_{11} = \langle K^0|\mathcal{M}|K^0\rangle \qquad \mathcal{M}_{12} = \langle K^0|\mathcal{M}|\bar{K}^0\rangle$$

$$\mathcal{M}_{21} = \langle \bar{K}^0|\mathcal{M}|K^0\rangle \qquad \mathcal{M}_{22} = \langle \bar{K}^0|\mathcal{M}|\bar{K}^0\rangle$$

4) Physical states for weak decay

The operator $\mathcal{M}$ can be represented by a diagonal matrix after a linear transformation in the two-dimensional space $K^0\ \bar{K}^0$. The eigenvectors of $\mathcal{M}$ are the observed decaying states which, as usual, we call K_S and K_L.

a) The new basis is defined from the original one by a complex 2×2 regular matrix C

$$C = \begin{vmatrix} p & -q \\ r & s \end{vmatrix}$$

with $\det C = ps + qr \neq 0$.

We then have

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

$$|K_S\rangle = r|K^0\rangle + s|\bar{K}^0\rangle\,.$$

b) In its diagonal form the operator $\mathcal{M}$ is simply represented by

$$\mathcal{M} = \begin{vmatrix} M_L & 0 \\ 0 & M_S \end{vmatrix} ,$$

where M_L and M_S are two complex numbers directly related to measured quantities.

The relations between the matrix elements of $\mathcal{M}$ in the two bases are easily obtained using the transformation C

$$\begin{vmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{vmatrix} = C^{-1} \begin{vmatrix} M_L & 0 \\ 0 & M_S \end{vmatrix} C .$$

This equality contains four relations

$$\begin{aligned}
\mathcal{M}_{11} &= \frac{1}{2}(M_L + M_S) + \frac{1}{2}(M_L - M_S)\frac{ps - qr}{ps + qr} \\
\mathcal{M}_{22} &= \frac{1}{2}(M_L + M_S) - \frac{1}{2}(M_L - M_S)\frac{ps - qr}{ps + qr} \\
\mathcal{M}_{12} &= (M_S - M_L)\frac{qs}{ps + qr} \\
\mathcal{M}_{21} &= (M_S - M_L)\frac{pr}{ps + qr} .
\end{aligned} \qquad \text{(II.5)}$$

We can easily check the conservation of Tr $\mathcal{M}$ and det $\mathcal{M}$:

$$\mathrm{Tr}\ \mathcal{M} = \mathcal{M}_{11} + \mathcal{M}_{22} = M_L + M_S$$

$$\det \mathcal{M} = \mathcal{M}_{11}\mathcal{M}_{22} - \mathcal{M}_{12}\mathcal{M}_{21} = M_L M_S$$

which are the two conditions sufficient to obtain the eigenvalues of $\mathcal{M}$ in terms of the $\mathcal{M}_{ij}$'s.

The two other relations give the constraints on C to put $\mathcal{M}$ in its diagonal form

$$\frac{\mathcal{M}_{12}}{qs} = \frac{\mathcal{M}_{21}}{pr} = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{qr - ps} .$$

c) <u>Normalization conditions</u>

The states K^0 and $\bar{K}^0$ are normalized to unity

$$< K^0|K^0 > = 1; \qquad < \bar{K}^0|\bar{K}^0 > = 1$$

and are orthogonal because of the strangeness quantum number

$$< K^0|\bar{K}^0 > = 0 = < \bar{K}^0|K^0 > .$$

By assumption we normalize the physical states $|K_L>$ and $|K_S>$ to unity

$$< K_S|K_S > = 1; \qquad < K_L|K_L > = 1 .$$

The corresponding conditions on the C-matrix elements are

$$|p|^2 + |q|^2 = 1; \qquad |r|^2 + |s|^2 = 1.$$

In general $|K_L>$ and $|K_S>$ are not orthogonal and the scalar product is given by

$$< K_S|K_L > = r^*p - s^*q .$$

3. DISCRETE SYMMETRIES

1) Strong interactions

For the strong interactions of hadrons it is possible to define three discrete symmetries commuting with the strong interaction Hamiltonian:

i) space reflection P
ii) time reflection T
iii) particle-antiparticle conjugation C.

The same situation does not hold in weak interactions where at least P and C do not commute with the weak interaction Hamiltonian.

2) TCP theorem

Let us call L = TCP the product of the three discrete symmetries. The transformation L seems to be valid for all the types of interactions. The theoretical basis of such a statement is the TCP theorem.

In quantum field theory, the TCP theorem has been proved independently by Lüders and Pauli. The L transformation is equivalent to the product of a strong reflection by a Hermitic conjugation provided the following assumptions are satisfied in a <u>local</u> field theory:

i) invariance under the proper Lorentz group $L_+^\uparrow$;
ii) connection between spin and statistics;
iii) commutation or anticommutation of the kinematically independent fields depending on the statistics.

In the Fock space, L acts in the following way:

i) energy-momentum four-vector invariants;
ii) spin-direction reversed;
iii) all the charges: electric, baryonic, leptonic, change of sign.

Moreover, the transformation L is antilinear and the c numbers are changed in their complex conjugate.

We have a reciprocity relation for the S-matrix elements

$$< f|S|i > = < i_L|S|f_L > \qquad \text{(II.6)}$$

where the index L indicates that in the states $|i>$ and $|f>$ the transformation L has been applied changing spins and charges in their opposite.

3) L symmetry in the K^0 $\bar{K}^0$ complex

The particles K^0 and $\bar{K}^0$ are exchanged in an L transformation. Because of the reciprocity relation (II.6) the diagonal transitions K^0-K^0 and $\bar{K}^0$-$\bar{K}^0$ of $\mathcal{M}$ are equal:

$$\mathcal{M}_{11} = \mathcal{M}_{22} \ . \tag{II.7}$$

Such a result is obviously independent of any choice of phase for K^0 and $\bar{K}^0$.
From Eq. (II.5) it follows that

$$ps = qr \ . \tag{II.8}$$

It is now convenient to define the phases of K_L and K_S with respect to K^0, such that p and r are real and positive.

Using the normalization conditions

$$p^2 + |q|^2 = 1; \qquad r^2 + |s|^2 = 1 \ ,$$

Eq. (II.8) can be immediately solved

$$r = p; \qquad s = q$$

and the matrix C takes the form

$$C = \begin{vmatrix} p & -q \\ p & q \end{vmatrix} \text{ with } p^2 + |q|^2 = 1 \ .$$

Equation (II.5) reduces simply to

$$\mathcal{M}_{11} = \mathcal{M}_{22} = \frac{1}{2}(M_L + M_S)$$

$$\frac{\mathcal{M}_{12}}{q^2} = \frac{\mathcal{M}_{21}}{p^2} = \frac{M_S - M_L}{2pq} \ .$$

The scalar product of $|K_L>$ and $|K_S>$ is real

$$< K_S|K_L > = p^2 - |q|^2 \ .$$

Let us notice that the relative $\bar{K}^0$-K^0 phase remains arbitrary, giving us the possibility to absorb the phase of q in the definition of the $\bar{K}^0$ state. In fact the phase of q cannot be experimentally measured.

4) Time reversal invariance

Let us now study the consequences of the time reversal invariance. The reciprocity theorem gives us an equality between S matrix elements analogous to Eq. (II.6)

$$< f|T|i > = < i_T|S|f_T > \tag{II.9}$$

where the index T indicates that in the states $|i>$ and $|f>$ the time reversal transformation has been applied, thus changing the spins and the momenta to their opposite.

As a consequence, the non-diagonal $K^0 \bar{K}^0$ and $\bar{K}^0 K^0$ matrix elements of $\mathcal{M}$ are equal up to a phase. We define the relative phase of $\bar{K}^0$ with respect to K^0 such that

$$\mathcal{M}_{12} = \mathcal{M}_{21} \ . \tag{II.10}$$

From Eq. (II.5) we immediately deduce

$$pr = qs \ . \tag{II.11}$$

We now define the relative phases of K_S, K_L, $\bar{K}^0$ such that p and s are real and positive. The normalization conditions are simply written as

$$p^2 + |q|^2 = 1 \ ; \qquad |r|^2 + s^2 = 1 \ .$$

The solution of Eq. (II.11) is given by

$$p = s \ ; \qquad q = r \ .$$

The C-matrix takes the form

$$C = \begin{vmatrix} p & -q \\ q & p \end{vmatrix} \quad \text{with } p^2 + |q|^2 = 1 \ .$$

Equation (II.5) takes the simple form

$$\mathcal{M}_{11} = \frac{p^2 M_L + q^2 M_S}{p^2 + q^2}$$

$$\mathcal{M}_{22} = \frac{q^2 M_L + p^2 M_S}{p^2 + q^2}$$

$$\frac{\mathcal{M}_{12}}{pq^*} = \frac{\mathcal{M}_{21}}{pq} = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{q^2 - p^2} \ .$$

The scalar product of $|K_L>$ and $|K_S>$ is purely imaginary

$$< K_S|K_L > = p(\bar{q} - q) \ .$$

5) PC invariance

Even if we know that the PC invariance is not valid in the neutral K-meson decay, it is useful to study the consequences of the PC invariance as giving an approximate description of the physical situation.

Because of the strangeness superselection rule, the relative phase between K^0 and $\bar{K}^0$ is not determined by the strong interaction. We choose this phase such that

$$PC|K^0 > = |\bar{K}^0 > \ ; \qquad PC|\bar{K}^0 > = |K^0 > \ .$$

We now define two eigenstates of PC by

$$PC|K_1^0> = |K_1^0> ; \qquad PC|K_2^0> = -|K_2^0> .$$

If PC invariance holds that the decaying states K_S and K_L are eigenstates of PC, we identify with K_1^0 and K_2^0 as follows:

$$|K_S> \to |K_1^0> ; \qquad |K_L> \to |K_2^0> .$$

We then obtain

$$|K_L> = \frac{\eta_L}{\sqrt{2}} \{ |K^0> - |\bar{K}^0> \}$$

$$|K_S> = \frac{\eta_S}{\sqrt{2}} \{ |K^0> + |\bar{K}^0> \} ,$$

where η_L and η_S are two arbitrary phases we can incorporate in the definition of K_S and K_L. The matrix C takes then the well-known form

$$C = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} .$$

The TCP invariance condition $\mathcal{M}_{11} = \mathcal{M}_{22}$ and the time reversal invariance condition $\mathcal{M}_{12} = \mathcal{M}_{21}$ are then automatically fulfilled:

$$\mathcal{M}_{11} = \mathcal{M}_{22} = \frac{1}{2} (M_L + M_S)$$

$$\mathcal{M}_{12} = \mathcal{M}_{21} = \frac{1}{2} (M_S - M_L) .$$

6) The violation of PC invariance in neutral K-meson decay has been observed experimentally to be very small. It is then convenient to define the matrix elements of C referring to a PC invariant situation.

Let us study the cases of L or T invariance. We introduce the two PC violating parameters λ and α:

$$p = \cos\left(\frac{\pi}{4} - \lambda\right) = \frac{1}{\sqrt{2}} (\cos\lambda + \sin\lambda)$$

$$q = \sin\left(\frac{\pi}{4} - \lambda\right) e^{-2i\alpha} = \frac{1}{\sqrt{2}} (\cos\lambda - \sin\lambda)\, e^{-2i\alpha} .$$

a) <u>With L invariance</u>

$$\det C = \cos 2\lambda \; e^{-2i\alpha}$$

$$< K_S|K_L > = \sin 2\lambda .$$

b) With T invariance

$$\det G = [\cos 2\alpha + i \sin \alpha \sin 2\lambda]\, e^{-2i\alpha}$$

$$< K_S | K_L > = i \sin 2\alpha \cos 2\lambda \ .$$

4. UNITARITY CONDITIONS

1) The unitarity properties of the S matrix or, equivalently the conservation of probabilities, will give the relation between the skew Hermitian part of $\mathcal{M}$ and the transition amplitudes for decay.

2) The transition matrix T is defined on the mass shell from the S matrix by:

$$< f|S|i > = < f|i > + i\,(2\pi)^4\, \delta_4(P_f - P_i) < f|T|i >$$

where P_f and P_i are the energy-momentum four vectors for the final and the initial states.

As previously, we call $|F>$ an arbitrary physical state occurring in the decay of neutral K mesons.

We introduce $|\phi(t)>$ as an arbitrary mixture of the two basic states $|K^0(t)>$ and $|\bar{K}^0(t)>$.

The conservation of the probabilities, at a given time t, is simply written as a compensation between the decreasing of the norm of ϕ and the transition probability of decay of ϕ into the states F:

$$\frac{d}{dt} < \phi(t)|\phi(t) > + \sum_F | < F|T|\phi(t) > |^2 = 0 \qquad \text{(II.12)}$$

where

$$\sum_F \to \sum_{\substack{\text{spins}\\ F}} \int d\rho_F\, (2\pi)^4\, \delta_4(P_F - P) \ .$$

Here $d\rho_F$ is the density of final state F, P_F the total energy momentum of F, and P the initial energy-momentum.

3) Equation (II.12) must hold for an arbitrary initial state $\phi(t)$ and is then equivalent to a set of four relations; one immediately writes, using Eqs. (II.3) and (II.4):

$$\begin{aligned}
i(\mathcal{M}_{11} - \mathcal{M}_{11}^*) &= \sum_F | < F|T|K > |^2 = \Gamma_{11} \\
i(\mathcal{M}_{22} - \mathcal{M}_{22}^*) &= \sum_F | < F|T|\bar{K} > |^2 = \Gamma_{22} \\
i(\mathcal{M}_{12} - \mathcal{M}_{21}^*) &= \sum_F < F|T|\bar{K} >^* < F|T|K > = \Gamma_{12} = \Gamma_{21}^* \\
i(\mathcal{M}_{21} - \mathcal{M}_{12}^*) &= \sum_F < F|T|K >^* < F|T|\bar{K} > = \Gamma_{21} = \Gamma_{12}^* \ .
\end{aligned} \qquad \text{(II.13)}$$

These relations show why the Hermitian matrix Γ is called the decay matrix and they can be directly obtained with the Weisskopf-Wigner formalism.

4) The unitary conditions can be written in the physical basis K_S, K_L using the transformation C and Eq. (II.5). For $\psi(t)$ it is equivalent to use an arbitrary mixture of $K_S(t)$ and $K_L(t)$ and to apply, as previously, the conservation of probabilities. We then obtain four equations equivalent to (II.11):

$$\begin{aligned} i(M_S - M_S^*) &= \sum_F |<F|T|K_S>|^2 = \Gamma_S \\ i(M_L - M_L^*) &= \sum_F |<F|T|K_L>|^2 = \Gamma_L \\ i(M_S - M_L^*) <K_L|K_S> &= \sum_F <F|T|K_L>^* <F|T|K_S> \\ i(M_L - M_S^*) <K_S|K_L> &= \sum_F <F|T|K_S>^* <F|T|K_L> . \end{aligned} \tag{II.14}$$

In order to satisfy the two equations, we define the real parts and the imaginary parts of M_S and M_L as follows:

$$M_{S,L} = m_{S,L} - i\,\frac{\Gamma_{S,L}}{2} . \tag{II.15}$$

The last two equations, which are complex conjugate to each other, can be transformed into

$$\begin{aligned} \left[\frac{\Gamma_S + \Gamma_L}{2} - i(m_L - m_S)\right] <K_L|K_S> &= \sum_F <F|T|K_L>^* <F|T|K_S> \\ \left[\frac{\Gamma_S + \Gamma_L}{2} + i(m_L - m_S)\right] <K_S|K_L> &= \sum_F <F|T|K_S>^* <F|T|K_L> . \end{aligned} \tag{II.16}$$

5) The Schwartz inequality gives an upper bound for the right-hand side of equations (II.16):

$$\left|\sum_F <F|T|K_S>^* <F|T|K_L>\right|^2 \leq \left[\sum_F |<F|T|K_L>|^2\right]\left[\sum_F |<F|T|K_S>|^2\right] .$$

We then obtain

$$\left[\left(\frac{\Gamma_S + \Gamma_L}{2}\right)^2 + (m_L - m_S)^2\right] |<K_S|K_L>|^2 \leq \Gamma_S\,\Gamma_L .$$

Experimentally:

$$\frac{\Gamma_L}{\Gamma_S} \simeq (1.56 \pm 0.16)\ 10^{-3}$$

$$\frac{m_L - m_S}{\Gamma_S} \simeq 0.47 \pm 0.02\ .$$

It follows

$$| < K_S | K_L > | \leq 6 \times 10^{-2}\ .$$

The states $|K_L>$ and $|K_S>$ are nearly orthogonal.

* * *

CHAPTER III

1. THE VIOLATION OF PC INVARIANCE

1) A 2π-meson state has a well-defined value of PC equal to PC = +1. If PC is a good symmetry of weak interactions, the long lifetime component K_L is identified with the eigenstate of PC, K_2^0, and then cannot decay into 2π.

2) The experimental observation of the two decays:

$$K_L \to \pi^+\pi^- \qquad K_L \to \pi^0\pi^0$$

is evidence against the conservation of PC in weak decays. Moreover, it is still the only unambiguous evidence because of the existence of a selection rule if PC invariance holds.

3) It is now convenient and usual to define the violation of PC in weak interactions for $K \to 2\pi$ decay in terms of two measurable complex parameters

$$\eta_{+-} = \frac{< \pi^+\pi^-|T|K_L >}{< \pi^+\pi^-|T|K_S >} = |\eta_{+-}|\ e^{i\varphi_{+-}} \qquad (III.1)$$

$$\eta_{oo} = \frac{< \pi^0\pi^0|T|K_L >}{< \pi^0\pi^0|T|K_S >} = |\eta_{oo}|\ e^{i\varphi_{oo}} \ . \qquad (III.2)$$

Experimentally we have information about three of these quantities, and the last one will soon be available:

$$|\eta_{+-}| = (1.98 \pm 0.06)\ 10^{-3}$$

$$|\eta_{oo}| = (4.9 \pm 0.5)\ 10^{-3}$$

$$\varphi_{+-} = 80° \pm 20° \ .$$

As a first conclusion the violation of PC in $K \to 2\pi$ decay is small compared to the separate violations of P and C in weak interactions.

4) A complete analysis of the $K \to 2\pi$ decay can be done introducing a new parameter connected only with the K_S component:

$$R = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)} \ . \qquad (III.3)$$

If the $|\Delta \vec{I}| = 1/2$ rule holds, $R = 1/2$ up to phase space corrections due to the $\pi^{\pm}$-π^0 mass difference. The experimental situation for R is not absolutely clean (see page 9)

$$R = 0.447 \pm 0.045 \ .$$

2. UNITARITY CONDITION

1) Let us now study in some detail the unitarity condition written in Eq. (II.16) of the previous chapter:

$$\left[\frac{\Gamma_S + \Gamma_L}{2} + i(m_L - m_S)\right] < K|K_L > = \sum_F < F|T|K_S >^* < F|T|K_L > . \qquad \text{(III.4)}$$

The sum $\sum\limits_F$ can be split into the various types of decay modes

$$\sum_F = \sum_{2\pi} + \sum_{3\pi} + \sum_{\text{lept}} + \sum_{\text{rad}} .$$

2) For each particular decay mode $G \subset F$, we have a Schwartz inequality

$$\left| \sum_G < F|T|K_S >^* < F|T|K_L > \right|^2 \leq \Gamma(K_S \to G)\, \Gamma(K_L \to G)$$

giving an upper bound for $|\sum\limits_G|$.

The sum $\sum\limits_G$ is obviously zero in a PC invariant theory where K_L and K_S cannot decay in the same state G. Each sum is a measure of the PC violation in the corresponding $K \to G$ decay.

3) The first term is easily studied using the definition of η_{+-}, η_{oo} and R as given in the previous section. We simply obtain

$$\sum_{2\pi} < F|T|K_S >^* < F|T|K_L > = \frac{\Gamma(K_S \to 2\pi)}{1 + R}\,[\eta_{+-} + R\eta_{oo}] . \qquad \text{(III.5)}$$

4) The experimental data for the 3π-meson decay mode are the following:

$$\frac{\Gamma(K_L \to 3\pi)}{\Gamma_S} \simeq 0.65 \times 10^{-3} \qquad \frac{\Gamma(K_S \to 3\pi)}{\Gamma_S} < 10^{-4} .$$

An upper bound of $|\sum\limits_{3\pi}|$ is obtained using the Schwartz inequality

$$\frac{1}{\Gamma_S} \left| \sum_{3\pi} \right| < 2.6 \times 10^{-4} .$$

Detailed analyses of charged and neutral K meson decay into three pions have been performed. They do not exhibit a strong PC violation and the $|\Delta \vec{I}| = 1/2$ rule seems to be satisfied. Therefore it is very unlikely that $|\sum\limits_{3\pi}|$ can reach its upper bound.

5) For the semi-leptonic decay mode the experimental branching ratios are the following:

$$\frac{\Gamma(K_L \to \text{lept})}{\Gamma_S} \simeq \frac{\Gamma(K_S \to \text{lept})}{\Gamma_S} \simeq 10^{-3} .$$

The upper bound of $\left|\sum_{\text{lept}}\right|$ as deduced from the Schwartz inequality is relatively important:

$$\frac{1}{\Gamma_S}\left|\sum_{\text{lept}}\right| \leq 10^{-3} .$$

Careful analyses of K_{ℓ_3} decay for charged and neutral K mesons have been performed in order to estimate the violation of the PC invariance and to test the validity of the $\Delta Y = \Delta Q$ rule. In both cases there is no proof of violation.

In the framework of TCP invariance, the $\Delta Y = \Delta Q$ rule gives the following predictions:

$$\Gamma(K_L \to \text{lept}) = \Gamma(K_S \to \text{lept}) = \Gamma_{\text{lept}}$$

$$\sum_{\text{lept}} \langle F|T|K_S\rangle^* \langle F|T|K_L\rangle = \langle K_S|K_L\rangle\, \Gamma_{\text{lept}} .$$

In this case the $\sum_{\text{lept}}$ contribution is very small compared to the left-hand side of Eq. (III.4).

It is then reasonable, on this basis, to expect the $\sum_{\text{lept}}$ sum to be very far from its upper bound.

It should be noted that the $\Delta Y = \Delta Q$ rule in the framework of TCP invariance gives another useful result. The scalar product $\langle K_S|K_L\rangle$ can be directly obtained measuring the asymmetry in K_L (or K_S) leptonic decay

$$\frac{\Gamma(K_L \to \pi^- + \ell^+ + \nu_\ell) - \Gamma(K_L \to \pi^+ + \ell^- + \bar{\nu}_\ell)}{\Gamma(K_L \to \pi^- + \ell^+ + \nu_\ell) + \Gamma(K_L \to \pi^+ + \ell^- + \bar{\nu}_\ell)} = p^2 - |q|^2 = \langle K_S|K_L\rangle .$$

A good determination of $\langle K_S|K_L\rangle$ necessitates, of course, very accurate experiments and high statistics.

5) The 2γ radiative decay mode has been recently observed and we have an upper limit for the $\pi^+\pi\gamma$ branching ratio. We deduce

$$\frac{\Gamma(K_L \to \text{rad})}{\Gamma_S} < 5 \times 10^{-6} .$$

Assuming a comparable result for $\Gamma(\Gamma_S \to \text{rad})$ -- non-experimentally observed -- we obtain as an upper bound

$$\frac{1}{\Gamma_S}\left|\sum_{\text{rad}}\right| < 5 \times 10^{-6}$$

which allows us to neglect $\sum_{\text{rad}}$ in the discussion of $\sum_F$.

7) Summarizing the discussion of the experimental measurements of the various decay modes, the sum $\sum_F$ can be written as

$$\sum_F = \sum_{2\pi} + \gamma\, \Gamma_S .$$

A reasonable estimate of $|\gamma|$ is then

$$|\gamma| \ll 10^{-3} .$$

8) We now assume in the following of this section, that the (2π) contributions dominate the unitarity relation (III.4). Using Eq. (III.5) the scalar product $< K_S|K_L >$ can be determined from the experimental measurements of η_{+-}, η_{oo}, and R

$$< K_S|K_L > = \frac{\Gamma(K_S \to 2\pi)}{\Gamma_S} \frac{2}{1 + R} \left\{ \frac{\eta_{+-} + R\,\eta_{oo}}{1 + i\Delta} \right\} \tag{III.6}$$

where

$$\Delta = \frac{2(m_L - m_S)}{\Gamma_S}$$

$$\Delta_{exp} \simeq 0.94 \pm 0.04 .$$

Inserting the experimental information on $|\eta_{+-}|$, $|\eta_{oo}|$, and R, we deduce

$$| < K_S|K_L > | \leq 4 \times 10^{-3} .$$

9) Let us consider the case of TCP invariance. As has been explained in the previous chapter, the scalar product $< K_S|K_L >$ is real:

$$< K_S|K_L > = \sin 2\lambda \simeq 2\lambda .$$

Equation (III.7) gives two relations:

$$\mathrm{tg}\ \delta = \frac{\sin \varphi_{+-} + r \sin \varphi_{oo}}{\cos \varphi_{+-} + r \cos \varphi_{oo}} \tag{III.7}$$

$$\lambda = \frac{|\eta_{+-}|}{1 + R} \left\{ \cos \varphi_{+-} + r \cos \varphi_{oo} \right\} \tag{III.8}$$

where

$$\delta = \mathrm{arc\ tg}\ \Delta \qquad r = R \left| \frac{\eta_{oo}}{\eta_{+-}} \right|$$

$$\delta_{exp} \simeq 44° \pm 1° \qquad r_{exp} \simeq 1.1 \pm 0.22 .$$

Experimentally R, η_{+-}, and $|\eta_{oo}|$ have been measured. Equation (III.7) gives a prediction for the phase φ_{oo} in the framework of TCP invariance, and a direct experimental measurement of φ_{oo} is then highly crucial. Equation (III.7) has two solutions:

$$\varphi_{oo}^{(1)} = \delta + \mathrm{arc\ sin} \left[\frac{1}{r} \sin (\delta - \varphi_{+-}) \right]$$

$$\varphi_{oo}^{(2)} = \pi + \delta - \mathrm{arc\ sin} \left[\frac{1}{r} \sin (\delta - \varphi_{+-}) \right] . \tag{III.9}$$

Inserting the experimental numbers with their large errors:

$$- 25^\circ \leq \varphi_{oo}^{(1)} \leq +32^\circ$$

$$-124^\circ \leq \varphi_{oo}^{(2)} \leq -67 \quad .$$

The parameter λ is then obtained using Eq. (III.8):

$$0.88 \times 10^{-3} \leq \lambda^{(1)} \leq 2.2 \times 10^{-3}$$

$$-0.46 \times 10^{-3} \leq \lambda^{(2)} \leq 0.25 \times 10^{-3} \quad .$$

A graphical solution of Eqs. (III.7) and (III.8) is given in Fig. 8. The angle φ_{+-} is taken as 80° and the angle δ as 44°.

Because of the large uncertainties on φ_{+-} and r, it is possible to obtain approximate solutions of Eqs. (III.9), replacing r by unity:

$$\begin{aligned} \varphi_{oo}^{(1)} &\simeq 2\delta - \varphi_{+-} \\ \varphi_{oo}^{(2)} &\simeq \varphi_{+-} + \pi \quad . \end{aligned} \qquad \text{(III.10)}$$

10) We can study, in an identical way, the implications of time reversal invariance. As has been explained in the previous chapter, the scalar product $< K_S|K_L >$ is now purely imaginary:

$$< K_S|K_L > = i \sin 2\alpha \cos 2\lambda \simeq 2i\,\alpha \quad .$$

Equation (III.6) gives again two relations

$$-\mathrm{ctg}\ \delta = \frac{\sin \varphi_{+-} + r \sin \varphi_{oo}}{\cos \varphi_{+-} + r \cos \varphi_{oo}} \qquad \text{(III.11)}$$

$$\alpha = \frac{|\eta_{+-}|}{1 + R} \{\sin \varphi_{+-} + r \sin \varphi_{oo}\} \quad . \qquad \text{(III.12)}$$

The two solutions of Eq. (III.11) are given by

$$\begin{aligned} \varphi_{oo}^{(1)} &= \pi + \delta + \mathrm{arc}\cos\left[\frac{1}{r}\cos(\delta - \varphi_{+-})\right] \\ \varphi_{oo}^{(2)} &= \pi + \delta - \mathrm{arc}\cos\left[\frac{1}{r}\cos(\delta - \varphi_{+-})\right] \quad . \end{aligned} \qquad \text{(III.13)}$$

Inserting the experimental numbers with their errors the two solutions cannot be separated, and we obtain

$$-201^\circ \leq \varphi_{oo} \leq -71^\circ \quad .$$

The parameter α is then estimated using Eq. (III.12):

$$-0.6 \times 10^{-3} \leq \alpha \leq 1.96 \times 10^{-3} \quad .$$

A graphical solution of Eqs. (III.10) and (III.11) is given in Fig. 9. The angle φ_{+-} is taken as 80° and the angle δ as 44°.

For the same reasons as given previously, we replace r by unity to obtain a good approximation of Eq. (III.13):

$$\begin{aligned} \varphi_{oo}^{(1)} &= 2\delta - \pi - \varphi_{+-} \\ \varphi_{oo}^{(2)} &= \pi + \varphi_{+-} \ . \end{aligned} \qquad \text{(III.14)}$$

11) In Figs. 10 and 11 we have represented the variation of φ_{oo} and $\varphi_{oo} - \varphi_{+-}$ as a function of φ_{+-} for $\delta = 44°$ and $r = 1.1 \pm 0.22$. The connection between L = TCP invariance and time reversal invariance is given by

$$\varphi^{(L)} - \varphi^{(T)} = \frac{\pi}{2} \ .$$

The approximate solutions [Eqs. (III.10) and (III.14)] correspond to straight lines.

With the present accuracy of experiments, the solutions $\varphi_{oo}^{(2)}$ obtained in the framework of TCP invariance lie in the range of values predicted by time reversal invariance. More generally, from Eqs. (III.10) and (III.14) the solutions $\varphi_{oo}^{(2)}$ deduced with TCP invariance and T invariance almost coincide. If, experimentally, φ_{oo} is close to $\pi + \varphi_{+-}$ it will not be possible to reach any conclusion and to choose between these two discrete symmetries.

3. <u>ISOTOPIC SPIN ANALYSIS</u>

1) A 2π-meson state of angular momentum $J = 0$ can only have a total isotopic spin $I = 0$ and $I = 2$ because of the generalized Pauli principle. We then introduce four amplitudes to describe the decay of neutral K mesons into 2π

$$< 0|T|K_S > , \quad < 2|T|K_S > , \quad < 0|T|K_L > , \quad < 2|T|K_L > ,$$

where the final state is characterized by its isotopic spin. If the $|\Delta \vec{I}| = 1/2$ rule holds, only the final state with $I = 0$ can be reached at first order.

It is then convenient to define, as usual, three complex parameters:

$$\boxed{\begin{aligned} \epsilon &= \frac{< 0|T|K_L >}{< 0|T|K_S >} \\ \epsilon' &= \frac{< 2|T|K_L >}{< 0|T|K_S >} \\ \omega &= \frac{< 2|T|K_S >}{< 0|T|K_S >} \end{aligned}} \qquad \text{(III.15)}$$

The quantities ϵ and ϵ'/ω measure the violation of PC invariance. The ratio ϵ'/ϵ and the quantity ω measure the violation of the $|\Delta \vec{I}| = 1/2$ rule in K_L and K_S decays.

2) Using the Clebsch-Gordan coefficients to project the physical states $\pi^+\pi^-$ and $\pi^0\pi^0$ on the eigenstates of isotopic spin I = 0 and I = 2, it is easy to express η_{+-}, η_{00}, and R in terms of ϵ, ϵ', and ω.

$$\boxed{\begin{aligned} \eta_{+-} &= \frac{\epsilon + \frac{1}{\sqrt{2}}\epsilon'}{1 + \frac{1}{\sqrt{2}}\omega} \\ \eta_{00} &= \frac{\epsilon - \sqrt{2}\,\epsilon'}{1 - \sqrt{2}\,\omega} \\ R &= \frac{1}{2}\left|\frac{1 - \sqrt{2}\,\omega}{1 + \frac{1}{\sqrt{2}}\omega}\right|^2 . \end{aligned}} \qquad \text{(III.16)}$$

The experimental determination of η_{+-}, η_{00}, and R gives five real numbers. This is not sufficient in order to know the three complex parameters ϵ, ϵ', and ω. From the system of Eq. (III.16) we can only obtain a one-parameter set of solutions. Supplementary assumptions are needed to fix the remaining free parameter as, for instance, the existence of discrete symmetries.

3) The quantity ω measures the violation of the $|\Delta\vec{I}| = 1/2$ rule in $K_S \to 2\pi$ decay. Defining $\rho = \sqrt{2R}$, the equation

$$\rho = \left|\frac{1 - \sqrt{2}\,\omega}{1 + \frac{1}{\sqrt{2}}\omega}\right|$$

is easily resolved. In the complex plane of ω and for a fixed value of ρ, ω is located on a circle

$$\omega = \frac{\sqrt{2}}{4 - \rho^2}\left[2 + \rho^2 - 3\rho\, e^{-i\xi}\right] . \qquad \text{(III.17)}$$

The experimental value of ρ is

$$\rho_{\exp} \simeq 0.947 \pm 0.045 \qquad \text{(III.18)}$$

and the allowed values of ω are shown in Fig. 12.

4) The transition from a K-meson state to a 2π-meson state of total isotopic spin I = 2 can occur with a change in isotopic spin $|\Delta\vec{I}| = 3/2$ or $|\Delta\vec{I}| = 5/2$. In general there is no relation between the amplitudes $< 2|T|K^+ >$ and $< 2|T|K^0 >$, and the experimental data on $K^+ \to \pi^+\pi^0$ cannot help the present analysis.

In order to obtain information about the phase parameter ξ introduced in Eq. (III.17), we have to make a new assumption. Experimental data on non-leptonic decay (except the pathologic case of $K_L \to 2\pi$ decay) are in agreement with the prediction of the $|\Delta \vec{I}| = 1/2$ rule with a good accuracy. It is generally believed that such an agreement is not the result of accidental cancellations in each particular situation, but is due to the dominance of the $|\Delta \vec{I}| = 1/2$ transitions with respect to the other possible ones. If such a point of view is valid for $K_S \to 2\pi$ decay, the order of magnitude of the $< 2|T|K^0 >$ amplitude is given by the $K^+ \to \pi^+\pi$ width and found to be very small with respect to the $< 0|T|K^0 >$ amplitude. It is the reason why all the present analysis of neutral K-meson decay assume

$$|\omega|^2 << 1 \ ,$$

or in the ξ language

$$\xi^2 << 1 \ .$$

As an illustration of the previous considerations, a quantitative approach giving $|\omega|^2$ and ξ^2 can be developed assuming that the amplitudes $< 2|T|K^+ >$ and $< 2|T|K^0 >$ are both dominated by a $|\Delta \vec{I}| = 3/2$ transition. We use the TCP relation for the matrix element $< 2|T|\bar{K}^0 >$ and an approximate PC invariance for K_S ($p \simeq q \simeq 1/\sqrt{2}$) and we obtain

$$|\omega|^2 \simeq \frac{4}{3}\,\frac{\Gamma(K_S^+ \to \pi^+\pi^0)}{\Gamma_S} \simeq (1.95 \pm 0.05)\ 10^{-3} \ . \qquad \text{(III.19)}$$

On the other hand Eq. (III.13) gives

$$4 \sin^2 \frac{\xi}{2} = \frac{(4-\rho)^2}{3\rho(2+\rho^3)} \left\{ \frac{|\omega|^2}{2} - \left(\frac{1-\rho}{2+\rho} \right)^2 \right\} . \qquad \text{(III.20)}$$

The modulus $|\omega|$ is fixed by Eq. (III.19) and we have a second circle in the complex plane of ω. The previous assumption is physically acceptable if and only if the two circles intersect or, equivalently, if $\sin^2 (\xi/2) \gtrsim 0$. Equation (III.20) requires

$$|\omega|^2 \gtrsim 2\left(\frac{1-\rho}{2+\rho} \right)^2 \ .$$

Using the evaluation (III.19) of $|\omega|^2$ we deduce

$$\rho \gtrsim 0.91$$

which is consistent with the experimental data quoted in Eq. (III.18).

The angle ξ turns out to be very small; using Eqs. (III.18), (III.19), and (III.20), we obtain an upper limit for ξ^2:

$$\xi^2 \lesssim 10^{-3} \ .$$

The phase of ω, φ_ω, can also, in principle, be calculated from Eqs. (III.17) and (III.20):

$$\sin^2 \varphi_\omega = \frac{3\rho}{2+\rho^2} \left\{ 1 - \frac{2}{|\omega|^2} \left(\frac{1-\rho}{2+\rho} \right)^2 \right\} .$$

Unfortunately, because of the large experimental uncertainty on ρ, φ_ω is consistent with all the values between $-\pi/2$ and $\pi/2$.

5) Equations (III.16) and (III.18) allow a one-parameter determination of ϵ and ϵ':

$$\epsilon = \frac{1}{4-\rho^2} \left\{ (2 - \rho\, e^{-i\xi}) 2\eta_{+-} + (2e^{-i\xi} - \rho)\rho\eta_{00} \right\}$$

$$\epsilon' = \frac{\sqrt{2}}{4-\rho^2} \left\{ (2 - \rho\, e^{-i\xi}) \eta_{+-} - (2e^{-i\xi} - \rho)\rho\eta_{00} \right\} \tag{III.21}$$

If, now, the parameter ξ is very small (as discussed in the previous section) the quantities ϵ, ϵ', and Re ω are essentially independent of ξ and we obtain, instead of Eq. (III.21), more simple expressions:

$$\epsilon = \frac{1}{2+\rho} \left\{ 2\eta_{+-} + \rho\eta_{00} \right\}$$

$$\epsilon' = \frac{\sqrt{2}}{2+\rho} \left\{ \eta_{+-} - \rho\eta_{00} \right\} \tag{III.22}$$

$$\mathrm{Re}\,\omega = \frac{\sqrt{2}}{2+\rho} (1 - \rho) .$$

The ξ dependence is exhibited only in Im ω:

$$\mathrm{Im}\,\omega \simeq \frac{3\sqrt{2}\,\rho}{4-\rho^2}\,\xi .$$

6) Let us now study the problem of the $|\Delta \vec{I}| = 1/2$ rule in $K_L \to 2\pi$ decay in a way independent of any assumption about ω. Using Eqs. (III.15) and (III.16) we obtain:

$$\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)} = \frac{1}{R} \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = \frac{1}{2} \left| \frac{1 + \sqrt{2}\,\frac{\epsilon}{\epsilon'}}{1 - \frac{1}{\sqrt{2}}\,\frac{\epsilon}{\epsilon'}} \right|^2 = \frac{1}{2}\sigma^2 . \tag{III.23}$$

We then have to resolve the equation

$$\sigma = \left| \frac{1 + \sqrt{2}\,\frac{\epsilon}{\epsilon'}}{1 - \sqrt{2}\,\frac{\epsilon}{\epsilon'}} \right| .$$

Again in the complex plane of ϵ/ϵ', for a fixed value of σ, ϵ/ϵ' is located on a circle:

$$\frac{\epsilon}{\epsilon'} = \frac{\sqrt{2}}{4 - \sigma^2}\left\{3\sigma\, e^{i\varphi} - 2 - \sigma^2\right\} . \tag{III.24}$$

The experimental value of σ is ($\sigma = \rho/\tau$):

$$\sigma_{exp} \cong 0.86 \pm 0.13 . \tag{III.25}$$

The value $\sigma = 2$ predicted by the $|\Delta\vec{I}| = 1/2$ rule is excluded by the present experiments. As a trivial consequence ϵ' cannot be zero.

* * *

CHAPTER IV

1. THE TCP INVARIANCE

1) In order to study the implications of discrete symmetries such as TCP, T or PC, it is convenient to work in the $K^0\bar{K}^0$ basis and to define four amplitudes:

$$< 0|T|K^0 > , \quad < 0|T|\bar{K}^0 > , \quad < 2|T|K^0 > , \quad < 2|T|\bar{K}^0 > .$$

We then have the obvious relations

$$< I|T|K_L > = p < I|T|K^0 > - q < I|T|\bar{K}^0 >$$

$$< I|T|K_S > = r < I|T|K^0 > + s < I|T|\bar{K}^0 > .$$

2) The constraints due to PC invariance are simply obtained by observing that a 2π-meson state is an eigenstate of PC with PC = +1. With the phase assumption made in Chapter II we have

$$< I|T|K^0 > - < I|T|\bar{K}^0 > = 0$$

or, equivalently

$$< I|T|K_2^0 > = 0$$

$$< I|T|K_1^0 > = \sqrt{2} < I|T|K^0 > .$$

3) The L = TCP invariance implies a reciprocity relation

$$< f|T|i > = < i_L|T|f_L > . \qquad \text{(IV.1)}$$

The states $|i>$ and $|i_L>$ are one-particle states, and Eq. (IV.1) is useful if and only if T possesses some properties.

In order to take into account the strong interactions occurring in the final state, we must distinguish outgoing and ingoing states. From the S-matrix definition

$$< _{in}|S = < _{out}| \; ; \qquad S^*|_{in} > = |_{out} > .$$

Equation (IV.1) must be written as

$$< f_{out}|i_{in} > = < f_{L\ in}|i_{L\ out} >^* . \qquad \text{(IV.2)}$$

For the one-particle states $|i>$ and $|i_L>$ there is no difference between ingoing and outgoing states

$$|i_{in} > = |i > \; ; \qquad |i_{L\ out} > = |i_L > ,$$

and Eq. (IV.2) is simply written as

$$< f_{out}|i > = < f_{L\ in}|i_L >^* . \qquad \text{(IV.3)}$$

In order to interpret the right-hand side of Eq. (IV.3) as a decay amplitude we must relate the states $< f_{L\ in}|$ and $< f_{L\ out}|$. To do that, we must remember that the decay amplitudes we are considering can be evaluated at <u>first order in weak interactions</u> and we obtain a factorization of Eq. (IV.3) following

$$< f_{out}|i> = < f_{L\ out}|i_L>^* < f_{L\ out}|f_{L\ in}> . \qquad \text{(IV.4)}$$

The extra factor $< f_{L\ out}|f_{L\ in}>$ describes the final-state interaction and is the S-matrix element for the transition $f_L \to f_L$. In our case $|f>$ is a 2π-meson state and we introduce the phase shift δ_I for π-π scattering in an S-state at the energy of the K-meson mass, the total isotopic spin being I

$$< I_{out}|I_{in}> = e^{2i\delta_I} ;$$

The states $|K^\circ>$ and $|\bar{K}^\circ>$ are exchanged in a TCP operation

$$TCP|K^\circ> = |\bar{K}^\circ> ; \qquad TCP|\bar{K}^\circ> = |K^\circ> .$$

Equation (IV.4) takes two equivalent forms

$$\begin{aligned} < I|T|K^\circ> &= e^{2i\delta_I} < I|T|\bar{K}^\circ>^* \\ e^{-i\delta_I} < I|T|K^\circ> &= [e^{-i\delta_I} < I|T|\bar{K}^\circ>]^* \end{aligned} \qquad \text{(IV.5)}$$

It is then convenient to define four reduced amplitudes:

$$\begin{aligned} A_I &= e^{-i\delta_I} < I|T|K^\circ> \\ \bar{A}_I &= e^{-i\delta_I} < I|T|\bar{K}^\circ> \end{aligned} \qquad I = 0,2$$

The TCP invariance condition (IV.5) is then simply written as:

$$\bar{A}_I = A_I^* . \qquad \text{(IV.6)}$$

4) In the framework of the TCP invariance, the transformation C has been written as:

$$p = r = \frac{1}{\sqrt{2}} (\cos\lambda + \sin\lambda)$$

$$q = s = \frac{1}{\sqrt{2}} (\cos\lambda - \sin\lambda)\, e^{-2i\alpha} .$$

The reduced amplitudes A_I and $\bar{A}_I$ are related by Eq. (IV.6). We define two parameters ϑ_0' and ϑ_2' for the PC violation:

$$A_I = |A_I|\, e^{i\vartheta_I'} ; \qquad \bar{A}_I = |A_I|\, e^{-i\vartheta_I'} .$$

Only the sum

$$\vartheta_I = \vartheta_I' + \alpha$$

is measurable, and the computation of ϵ, ϵ', and ω is straightforward

$$\epsilon = \frac{\sin 2\lambda + i \cos 2\lambda \sin 2\theta_0}{1 + \cos 2\lambda \cos 2\theta_0}$$

$$\epsilon' = \frac{\sin 2\lambda \cos (\theta_2 - \theta_0) + i \left[\sin (\theta_2 - \theta_0) + \cos 2\lambda \sin (\theta_2 + \theta_0)\right]}{1 + \cos 2\lambda \cos 2\theta_0} \left| \frac{A_2}{A_0} \right| e^{i(\delta_2 - \delta_0)}$$

$$\omega = \frac{\cos (\theta_2 - \theta_0) + \cos 2\lambda \cos (\theta_2 + \theta_0) + i \sin 2\lambda \sin (\theta_2 - \theta_0)}{1 + \cos 2\lambda \cos 2\theta_0} \left| \frac{A_2}{A_0} \right| e^{i(\delta_2 - \delta_0)}$$

(IV.7)

The three complex quantities ϵ, ϵ', and ω are given in terms of five real parameters:

$$\lambda\ , \quad \theta_0\ , \quad \theta_2\ , \quad \left| \frac{A_2}{A_0} \right|\ , \quad \delta_2 - \delta_0\ .$$

The TCP invariance implies one relation between ϵ, ϵ', and ω. In order to explain such a relation it is convenient to use the ratio ϵ'/ω very similar to ϵ

$$\frac{\epsilon'}{\omega} = \frac{< 2|T|K_L >}{< 2|T|K_S >} = \frac{\sin 2\lambda + i \cos 2\lambda \sin 2\theta_2}{1 + \cos 2\lambda \cos 2\theta_2}\ . \qquad \text{(IV.8)}$$

The compatibility relation is then easily obtained as:

$$\frac{1}{2} < K_S|K_L > = \frac{1}{2} \sin 2\lambda = \frac{\mathrm{Re}\ \epsilon}{1 + |\epsilon|^2} = \frac{\mathrm{Re}\ \epsilon'/\omega}{1 + \left|\frac{\epsilon'}{\omega}\right|^2} \qquad \text{(IV.9)}$$

and the other two parameters of PC violation are given by

$$\tan 2\theta_0 = \frac{2\ \mathrm{Im}\ \epsilon}{1 - |\epsilon|^2} \qquad \text{(IV.10)}$$

$$\tan 2\theta_2 = \frac{2\ \mathrm{Im}\ (\epsilon'/\omega)}{1 - \left|\frac{\epsilon'}{\omega}\right|^2}\ . \qquad \text{(IV.11)}$$

Relation (IV.8) determines the parameter ξ introduced in Chapter III as a function of the experimental quantities η_{+-}, η_{00}, and R. In a TCP invariant situation the problem is then completely determined, and as a consequence the quantity ω can be obtained from neutral K-meson decay only without any reference to K^+-meson decay.

In fact the only test of TCP is the existence on a real angle ξ such that the compatibility relation (IV.9) is satisfied. We will see later that this problem has always two solutions.

The ratio ϵ is a linear combination of η_{+-} and η_{00}. It follows that Re ϵ and Im ϵ are both of the order 10^{-3}. We then have

$$|\epsilon|^2 << 1$$

and a first order calculation in λ and ϑ_0 is obviously sufficient

$$\epsilon \simeq \lambda + i\,\vartheta_0 \ . \qquad (IV.12)$$

Equation (IV.9) and (IV.10) are then simplified into

$$\lambda = \mathrm{Re}\,\epsilon = \frac{\mathrm{Re}\,\epsilon'/\omega}{1 + \left|\frac{\epsilon'}{\omega}\right|^2} \qquad (IV.13)$$

$$\vartheta_0 = \mathrm{Im}\,\epsilon \ . \qquad (IV.14)$$

5) The 2π contributions to the unitary condition (III.5) can be expressed in terms of ϵ, ϵ', and ω using the identity

$$\frac{\eta_{+-} + R\,\eta_{00}}{1 + R} = \frac{\epsilon + \epsilon'\,\bar{\omega}}{1 + |\omega|^2} \ , \qquad (IV.15)$$

If the unitarity relation is saturated by the 2π contributions--as expected from a previous discussion--we can write Eq. (III.6) as

$$\lambda(1 + i\,\Delta) = \frac{\epsilon + \epsilon'\,\bar{\omega}}{1 + |\omega|^2} \qquad (IV.16)$$

and the real part of this equation gives a new expression for λ

$$\lambda = \frac{\mathrm{Re}\,\epsilon + \mathrm{Re}\,\epsilon'\,\bar{\omega}}{1 + |\omega|^2} \ . \qquad (IV.17)$$

On the other hand, we have obtained $\lambda = \mathrm{Re}\,\epsilon$ in Eq. (IV.12) and, as a consequence of the unitarity condition we deduce a second compatibility relation

$$\lambda = \mathrm{Re}\,\epsilon = \mathrm{Re}\,\epsilon'/\omega \ . \qquad (IV.18)$$

Equations (IV.13) and (IV.18) agree if and only if

$$\left|\frac{\epsilon'}{\omega}\right|^2 \ll 1 \ , \qquad (IV.19)$$

e.g. if the PC violation angle ϑ_2 is also small

$$\vartheta_2^2 \ll 1$$

but not necessarily of the same order of magnitude as λ and ϑ_0.

We now assume, in the following, the condition (IV.12) to be satisfied. Equation (IV.17) becomes a consequence of the compatibility relation (IV.18), but we do not use the imaginary part of the unitarity relation for the moment. The expressions of ϵ' and ω reduce to

$$\begin{aligned} \epsilon' &= (\lambda + i\,\vartheta_2)\left|\frac{A_2}{A_0}\right| e^{i(\delta_2 - \delta_0)} \\ \omega &= \left|\frac{A_2}{A_0}\right| e^{i(\delta_2 - \delta_0)} \ . \end{aligned} \qquad (IV.20)$$

From Eqs. (IV.15) and (IV.18) the compatibility condition takes the very simple form

$$\mathrm{Re}\ \epsilon = \mathrm{Re}\ \frac{\eta_{+-} + R\ \eta_{00}}{1 + R} \ . \tag{IV.21}$$

We introduce the solution (III.18) for ϵ, and the equation determining ξ as a function of η_{+-}, η_{00}, and R is

$$\mathrm{Re}\ (\eta_{+-} - \eta_{00}) \cos \xi + \mathrm{Im}\ (\eta_{+-} - \eta_{00}) \sin \xi = \frac{3\rho}{2 + \rho^2}\ \mathrm{Re}\ (\eta_{+-} - \eta_{00}) \ . \tag{IV.22}$$

It is convenient to define an auxiliary parameter Φ measurable experimentally

$$\Phi = \arg\ (\eta_{+-} - \eta_{00}) \ .$$

As an equation in ξ, the compatibility relation (IV.22) has always two solutions because of the experimental unequality $\rho \leq 1$. We immediately obtain

$$\xi^{\pm} = \Phi + \mathrm{arc\ cos} \left[\frac{3\rho}{2 + \rho^2} \cos \Phi \right] \tag{IV.23}$$

where we have chosen for arc cos $\{[3\rho/(2+\rho^2)] \cos \Phi\}$ the determination close to Φ. In the extreme case $\rho = 1$, the solutions (IV.23) reduce to

$$\xi^{+} = 2\Phi \ ; \qquad \xi^{-} = 0 \ .$$

The physical situation is $0.91 \leq \rho \leq 1$, and the solution ξ^- remains always small and the solution ξ^+ lies around 2Φ.

In Fig. 13 we have represented ξ^+ and ξ^- as a function of Φ. We notice the following symmetry properties

$$\xi^-(\pi + \Phi) = \xi^-(\Phi)$$
$$\xi^+(\pi + \Phi) = 2\pi + \xi^+(\Phi)$$

$$\xi^-(-\Phi) \quad = \xi^-(\Phi)$$
$$\xi^+(-\Phi) \quad = \xi^+(\Phi) \ .$$

To the two values ξ^+ and ξ^- correspond two sets of solutions for ϵ, ϵ', and ω. It is easy to prove the interesting relation independant of Φ:

$$\omega^+ \omega^- = 2\ \frac{1 - \rho^2}{4 - \rho^2} \ .$$

The phases of ω^+ and ω^- are opposite. If we have some information about the sign of $\delta_2 - \delta_0$ it is then possible to <u>choose</u> between ω^+ and ω^-. In the complex plane of ω the situation is described in Fig. 14.

6) If the 2π contributions saturate the imaginary part of the unitarity relation, the phases of η_{+-} and η_{00} are not independent and satisfy the relations (III.19).

The argument Φ becomes a function of φ_{+-}. The knowledge of $|\eta_{+-}|$, $|\eta_{oo}|$, R, and φ_{+-} is now sufficient to solve the problem. In the framework of TCP invariance we obtain four solutions, two associated to ξ^{+} and called large ω solutions, and two associated to ξ^{-} and called small ω solutions. Table 7 gives the numerical results for:

$$R = 0.447 \ , \ |\eta_{+-}| = 1.98 \times 10^{-3} \ , \quad |\eta_{oo}| = 4.9 \times 10^{-3} \ , \quad \varphi_{+-} = 80° \ .$$

Table 7

φ_{+-}	80°			
φ_{oo}	12°		- 103°	
$10^3\ \lambda$	1.7		- 0.11	
$10^3\ \theta_0$	1.5	3.1	- 0.2	8
$10^3\ \theta_2$	22	- 1.4	119	- 1.4
$\left\lvert\frac{A_2}{A_0}\right\rvert$	0.10	0.66	0.027	2.55
$\delta_2 - \delta_0$	73°	- 73°	- 12°	12°
$10^3\ \lvert\epsilon\rvert$	2.3	3.5	0.23	8
φ_{ϵ}	41°	61°	- 120°	91°
$10^3\ \lvert\epsilon'\rvert$	2.3	1.5	3.2	3.5
$\varphi_{\epsilon'}$	158°	- 113°	78°	- 82°
	small $\lvert\omega\rvert$	large $\lvert\omega\rvert$	small $\lvert\omega\rvert$	large $\lvert\omega\rvert$
	Solution I		Solution II	

The constraints (III.9) between the phases of η_{+-} and η_{oo} imply a relation between the real parameters of this TCP invariant analysis. Such a relation expresses the PC violating parameter of the transformation matrix λ in terms of the two other PC (or T) violating phases θ_0 and θ_2. Using Eqs. (IV.12), (IV.16) and (IV.20) we easily obtain

$$\lambda\Delta = \frac{\theta_0 + \theta_2 \left|\frac{A_2}{A_0}\right|^2}{1 + \left|\frac{A_2}{A_0}\right|^2} \quad ,$$

and we have only two independent parameters for the PC violation associated to the isotopic spin states I = 0 and I = 2.

2. TIME REVERSAL INVARIANCE

1) If the TCP invariance is broken we have, as an alternative, the possibility of a time reversal invariance. We now perform the analysis of a neutral K-meson decay under such an assumption. The method of calculation is identical, and we given only the results.

2) We first write the reciprocity relation

$$< f|T|i > = < i_T|T|f_T > , \tag{IV.24}$$

and we introduce the final-state interaction as in Section 1 of this chapter. We obtain

$$\begin{aligned} < I|T|K^\circ > &= e^{2i\delta_I} < I|T|K^\circ >^* \\ < I|T|\bar{K}^\circ > &= e^{2i\delta_I} < I|T|\bar{K}^\circ >^* . \end{aligned} \tag{IV.25}$$

As is well known, the reduced amplitudes A_I and $\bar{A}_I$ are real. The PC violation is exhibited in the fact that $\bar{A}_I \neq A_I$.

3) We follow the phase assumptions of Chapter II where p is real and q complex

$$|K_L > = p|K^\circ > - q|\bar{K}^\circ >$$

$$|K_S > = q|K^\circ > + p|\bar{K}^\circ > .$$

The parameters ϵ and ϵ'/ω are given by

$$\epsilon = \frac{p\, A_0 - q\, \bar{A}_0}{q\, A_0 + p\, \bar{A}_0} \tag{IV.26}$$

$$\frac{\epsilon'}{\omega} = \frac{p\, A_2 - q\, \bar{A}_2}{q\, A_2 + p\, \bar{A}_2} . \tag{IV.27}$$

We have a compatibility condition due to time reversal invariance

$$\frac{1}{2\,i} < K_S|K_L > = \frac{1}{2\,i}\, p\, (\bar{q} - q) = \frac{\mathrm{Im}\, \epsilon}{1 + |\epsilon|^2} = \frac{\mathrm{Im}\, \frac{\epsilon'}{\omega}}{1 + \left|\frac{\epsilon'}{\omega}\right|^2} . \tag{IV.28}$$

Equation (IV.28) determines the parameter ξ of Chapter III as a function of η_{+-}, η_{00}, and R. Again, in the framework of time reversal invariance ϵ, ϵ', and ω can be computed from neutral K-meson experiments.

A first order calculation with respect to the PC violation parameters is sufficient for ϵ, and Eq. (IV.28) can be simplified into

$$\frac{1}{2\,i}\, p\, (\bar{q} - q) = \mathrm{Im}\, \epsilon = \frac{\mathrm{Im}\, \frac{\epsilon'}{\omega}}{1 + \left|\frac{\epsilon'}{\omega}\right|^2} . \tag{IV.29}$$

4) The imaginary part of the unitarity relation is saturated by the 2π contributions if and only if

$$\left|\frac{\epsilon'}{\omega}\right|^2 << 1 . \tag{IV.30}$$

Using now Eq. (IV.15) we finally obtain a very simple form for the compatibility relation

$$\mathrm{Im}\,\epsilon = \mathrm{Im}\,\frac{\eta_{+-} + R\,\eta_{00}}{1 + R} \quad . \tag{IV.31}$$

The equation determining ξ as a function of η_{+-}, η_{00}, and R is

$$\mathrm{Im}\,(\eta_{+-} - \eta_{00})\cos\xi - \mathrm{Re}\,(\eta_{+-} - \eta_{00})\sin\xi = \frac{3\rho}{2+\rho^2}\,\mathrm{Im}\,(\eta_{+-} - \eta_{00}) \; . \tag{IV.32}$$

With the auxiliary parameter Φ defined in the previous section, the two solutions of Eq. (IV.32) can be written as

$$\begin{aligned} \xi^+ &= \pi + \Phi + \arcsin\left[\frac{3\rho}{2+\rho^2}\sin\Phi\right] \\ \xi^- &= \Phi - \arcsin\left[\frac{3\rho}{2+\rho^2}\sin\Phi\right] \end{aligned} \tag{IV.33}$$

where we have chosen, for arc sin $\{[3\rho/(2+\rho^2)]\sin\Phi\}$ the determination close to Φ.

In the extreme case $\rho = 1$, solutions (IV.33) reduce to

$$\xi^+ = \pi + 2\Phi \; ; \qquad \xi^- = 0 \; .$$

The physical situation is $0.91 \leqslant \rho \leqslant 1$, and the solution ξ^- remains always small and the solution ξ^+ lies around $\pi + 2\Phi$.

In Fig. 15 we have represented ξ^+ and ξ^- as a function of Φ. We notice the following symmetry properties

$$\begin{aligned} \xi^-(\pi + \Phi) &= \xi^-(\Phi) \\ \xi^+(\pi + \Phi) &= \xi^+(\Phi) + \pi \\ \\ \xi^-(-\Phi) &= -\xi^-(\Phi) \\ \xi^+(-\Phi) &= 2\pi - \xi^+(\Phi) \; . \end{aligned}$$

As in the previous case the phases of ω^+ and ω^-, for a given value of Φ, are opposite.

In general the predictions of time reversal invariance and those of L = TCP invariance are connected by the following relations:

$$\begin{aligned} \varphi^{(L)} - \varphi^{(T)} &= \frac{\pi}{2} \\ \xi_T(\varphi^{(T)}) &= \xi_L(\varphi^{(L)}) \\ \omega_T(\varphi^{(T)}) &= \omega_L(\varphi^{(L)}) \\ \epsilon_T(\varphi^{(T)}) &= \frac{1}{i}\,\epsilon_L(\varphi^{(L)}) \\ \epsilon'_T(\varphi^{(T)}) &= \frac{1}{i}\,\epsilon'_L(\varphi^{(L)}) \; . \end{aligned} \tag{IV.34}$$

5) We perform the calculation of ϵ, ϵ', and ω at first order with respect to the PC violating parameters

$$\epsilon \simeq \frac{p - q}{p + q} + \frac{A_0 - \bar{A}_0}{A_0 + \bar{A}_0}$$

$$\frac{\epsilon'}{\omega} \simeq \frac{p - q}{p + q} + \frac{A_2 - \bar{A}_2}{A_2 + \bar{A}_2} \tag{IV.35}$$

$$\omega \simeq \frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0} e^{i(\delta_2 - \delta_0)} .$$

In a first order calculation the deviation of $p/|q|$ and $\bar{A}_I/A_I$ from unity are not separated. Let us define

$$a_I = \frac{A_I - \bar{A}_I}{A_I + \bar{A}_I} + \lambda ; \quad I = 0,2 \tag{IV.36}$$

and we obtain for ϵ, ϵ', and ω the following expressions

$$\epsilon \simeq a_0 + i\,\alpha$$

$$\frac{\epsilon'}{\omega} \simeq a_2 + i\,\alpha$$

$$\omega \simeq \frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0} e^{i(\delta_2 - \delta_0)} . \tag{IV.37}$$

If the 2π contributions saturate the real part of the unitarity condition, the phases of η_{+-} and η_{00} satisfy one of the equalities (III.13). In the framework of time reversal invariance we obtain four solutions. Table 8 gives the numerical results for

$$R = 0.447 , \quad |\eta_{+-}| = 1.98 \times 10^{-3} , \quad |\eta_{00}| = 4.9 \times 10^{-3} , \quad \varphi_{+-} = 80^\circ .$$

The constraints (III.13) between the phases of η_{+-} and η_{00} imply a relation between the real parameters of this time reversal invariant analysis. Such a relation expresses the only measurable PC violating parameter α of the transformation matrix in terms of the two other PC violating parameters a_0 and a_2. Using Eq. (IV.37) we easily deduce

$$-\alpha\Delta = \frac{a_0 + a_2 \left| \frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0} \right|^2}{1 + \left| \frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0} \right|^2} .$$

Relations (IV.34) take now the form

$$\begin{aligned}
\alpha(\varphi^{(T)}) &= -\lambda(\varphi^{(L)}) \\
a_I(\varphi^{(T)}) &= \theta_I(\varphi^{(L)}) \qquad\qquad I = 0,2 \\
\left(\frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0}\right)_T (\varphi^{(T)}) &= \left|\frac{A_2}{A_0}\right|_L (\varphi^{(L)}) \\
(\delta_2 - \delta_0)_T(\varphi^{(T)}) &= (\delta_2 - \delta_0)_L(\varphi^{(L)}) .
\end{aligned}$$

New calculations are not needed to obtain the variation of these parameters with respect to φ_{+-}.

Table 8

φ_{+-}	80°			
φ_{00}	181°		- 94°	
$10^3\ \alpha$	1.32		- 0.15	
$10^3\ a_0$	- 1.34	5.06	- 0.38	1.46
$10^3\ a_2$	91.4	- 2.40	19.36	- 7.6
$\frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0}$	0.028	2.42	0.166	0.412
$\delta_2 - \delta_0$	21°	- 21°	77°	- 77°
$10^3\ \lvert\epsilon\rvert$	1.88	5.22	0.38	1.46
φ_ϵ	- 224°	15°	- 159°	- 6°
$10^3\ \lvert\epsilon'\rvert$	2.58	6.64	3.20	3.12
$\varphi_{\epsilon'}$	22°	130°	77°	104°
	small $\lvert\omega\rvert$	large $\lvert\omega\rvert$	small $\lvert\omega\rvert$	large $\lvert\omega\rvert$
	Solution I		Solution II	

APPENDIX A

SOME PROPERTIES OF THE 2π AND 3π FINAL STATES FROM THE DECAY OF A KAON

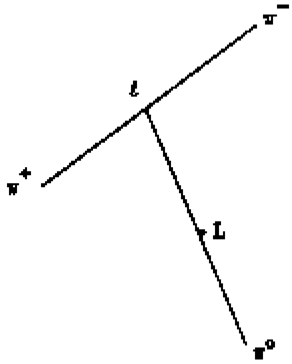
1. $C(\pi^+\pi^-) = (\pi^-\pi^+)$

Changing the sign of the pion is equivalent to a reflection with respect to the centre of mass, so

$$C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^\ell = +1 ,$$

where ℓ is the relative angular momentum of one pion with respect to the other (ℓ = kaon spin = 0). Before 1957, K_1^0 was defined as the eigenstate of $C = 1$, $K_1^0 = (1/\sqrt{2})(K^0 + C\,\bar{K}^0)$, while $K_2^0 = (1/\sqrt{2})(K^0 - C\,\bar{K}^0)$. The weak interactions were supposed to conserve C, and K_1^0 was identified with the short-lived component decaying to two pions, while the K_2^0 could not decay to this state which is the most favoured from the point of view of phase-space volume.

2. Parity of three-pion states



Call L the relative angular momentum of the dipion system $(\pi^+\pi^-)$ relative to π^0, and ℓ the relative angular momentum of one of the charged pions relative to the others.

In the rest system of the kaon, the three momenta are coplanar and the angular momenta are all parallel, perpendicular to the plane, and independent of a translation of the reference system. The total orbital momentum in the kaon rest system is thus $L + \ell$. The parity is $(-1)^3(-1)^{\ell + L} = -1$ since $\ell + L = 0$. This was the basis of the θ-τ puzzle, since the 2π system has $P = +1$, the 3π system has $P = -1$, and the decay of the charged kaon into these two channels was observed.

3. $C(\pi^+\pi^-\pi^0) = (-1)^\ell$

C interchanges the positions of the two charged pions and is thus equivalent to the parity operation in the dipion system. The eigenvalue is dependent upon the angular momentum of the three pions. It does not have a well-defined C-parity. However, because of centrifugal barriers, one can expect $\ell = 0$ to be favoured. In this case $C = +1$, as for the two-pion system. The observation in 1958, by Lederman and co-workers, of the abundant decay mode $\pi^+\pi^-\pi^0$ for the long-lived component was thus a further indication of C-violation in the decay.

4. $PC(\pi^+\pi^-) = (\pi^+\pi^-)$

PC is equivalent to changing the charge and space coordinates of the pions. Since they obey Bose statistics, PC = +1 independently of the angular momentum states.

5. $PC(\pi^0\pi^0) = \pi^0\pi^0$

6. $\underline{PC(\pi^+\pi^-\pi^0) = (-1)^{\ell+1}\,(\pi^+\pi^-\pi^0)}$

We have seen that $P(\pi^+\pi^-\pi^0) = -1$. C interchanges π^+ and π^- and is equivalent to a parity operation in this system, thus resulting in a factor $(-1)^{\ell}$.

7. Some relations between the C-parity and the G-parity of pion systems:

$$C(\pi^{\pm}) = \pi^{\mp}$$

$$C(\pi^0) = \pi^0 \; .$$

The charged states are not eigenstates of C.

Consider the vector isospin space subtended by $\pi_1\ \pi_2\ \pi_3$ where the physical states are defined by

$$\pi_+ = \frac{\pi_1 + i\pi_2}{\sqrt{2}}$$

$$\pi_- = \frac{\pi_1 - i\pi_2}{\sqrt{2}} \; .$$

Applying C gives

$$\pi_- = C\,\pi_+$$

$$\pi_+ = C\,\pi_-$$

$$\pi_o = C\,\pi_o \; .$$

A rotation of 180° around the y-axis, $\sim$ to the operation $e^{i\pi I_2}$ on the isospin wave function gives

$$e^{i\pi I_2}\,\pi_+ \rightarrow \frac{-\pi_1 + i\pi_2}{\sqrt{2}} = -\pi_-$$

$$e^{i\pi I_2}\,\pi_- \rightarrow \frac{-\pi_1 - i\pi_2}{\sqrt{2}} = -\pi_+$$

$$e^{i\pi I_2}\,\pi_o \rightarrow -\pi^0 \; .$$

In other words, all the charged and non-charged pion states are eigenstates of $G = C\,e^{i\pi I_2}$ with the eigenvalue -1. In all interactions conserving isospin and charge conjugation, the G-parity is a good quantum number. It is a multiplicative quantum number. The G-parity of n pions is $(-1)^n$.

8. C-parity of three pions in a neutral global state as a function of the total isospin

Since the charge of the system is zero, the isospin vector lies in the x-y plane. A rotation around I_2 can be replaced by a rotation around any vector in that plane, for instance I: $e^{i\pi I_2} = e^{i\pi I}$. Since $G = -1$ we have

$$\boxed{C = (-1)^{I+1}}$$

The three-pion system with zero charge can be in states of isospin value 0, 1, 2, 3.

* * *

APPENDIX B

Tables from Trilling's report UCRL-16473, 1965.

Table 1

K^+ rates

Mode	Branching ratio	Rate (sec^{-1})	Remarks
All modes [a]		$(8.045 \pm 0.027) \times 10^7$	
$K^+ \to \mu^+ + \nu$ [b]	$63.5 \pm 0.7\%$	$(5.11 \pm 0.06) \times 10^7$	
$K^+ \to \pi^+ + \pi^0$ [a]	$21.6 \pm 0.6\%$	$(1.74 \pm 0.05) \times 10^7$	
$K^+ \to e^+ + \pi^0 + \nu$ [c]	$4.49 \pm 0.25\%$	$(3.61 \pm 0.20) \times 10^6$	
$K^+ \to \mu^+ + \pi^0 + \nu$ [d]	$3.17 \pm 0.35\%$	$(2.55 \pm 0.28) \times 10^6$	
$K^+ \to \pi^+ + \pi^+ + \pi^-$ [e]	$5.59 \pm 0.11\%$	$(4.50 \pm 0.09) \times 10^6$	
$K^+ \to \pi^+ + \pi^0 + \pi^0$ [f]	$1.68 \pm 0.06\%$	$(1.35 \pm 0.05) \times 10^6$	
$K^+ \to e^+ + \nu$ [a]	$\sim 1.6 \times 10^{-5}$	$\sim 1.3 \times 10^3$	
$K^+ \to \pi^+ + \pi^- + e^+ + \nu$ [a]	$(3.6 \pm 0.8) \times 10^{-5}$	$(2.9 \pm 0.6) \times 10^3$	
$K^+ \to \pi^+ + \pi^- + \mu^+ + \nu$ [a]	$(7.7 \pm 5.2) \times 10^{-6}$	$(6.2 \pm 4.2) \times 10^2$	
$K^+ \to \pi^+ + \pi^0 + \gamma$ [a]	$(2.2 \pm 0.7) \times 10^{-4}$	$(1.8 \pm 0.6) \times 10^4$	55 MeV $< T_{\pi^+} < 80$ MeV
$K^+ \to \pi^+ + \pi^+ + \pi^- + \gamma$ [a]	$(1.0 \pm 0.4) \times 10^{-4}$	$(8.0 \pm 3.2) \times 10^3$	$E_\gamma > 10$ MeV
$K^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}$ [a]	$< 2 \times 10^{-6}$	$< 1.6 \times 10^2$	$\Delta S/\Delta Q = -1$ transition
$K^+ \to \pi^+ + \pi^+ + \mu^- + \nu$ [a]	$< 3 \times 10^{-6}$	$< 2.4 \times 10^2$	
$K^+ \to \pi^+ + e^+ + e^-$ [a]	$< 1.1 \times 10^{-6}$	$< 0.8 \times 10^2$	Involves neutral lepton currents
$K^+ \to \pi^+ + \mu^+ + \mu^-$ [a]	$< 3 \times 10^{-6}$	$< 2.4 \times 10^2$	

a) See text for discussion.

b) Calculated from 1 - sum (other branching ratios).

c) Input data on branching ratio:

$4.7 \pm 0.3\%$ (Ref. 5) $5.12 \pm 0.36\%$ (Ref. 14)

$5.0 \pm 0.5\%$ (Ref. 4) $4.04 \pm 0.24\%$ (Ref. 15)

Values measured relative to the τ mode have been renormalized to the τ rate quoted in the table.

d) Input data:
$3.0 \pm 0.5\%$ (Ref. 5)
$3.52 \pm 0.20\%$ (Ref. 62)
$2.82 \pm 0.19\%$ (Ref. 15)

e) Input data:
$5.54 \pm 0.12\%$ (Ref. 17)
$5.71 \pm 0.15\%$ (Ref. 18)
$5.10 \pm 0.2\%$ (Ref. 5)
$5.7 \pm 0.3\%$ (Ref. 4)
$5.2 \pm 0.3\%$ (Ref. 19)

f) Input data:
$1.8 \pm 0.2\%$ (Ref. 5)
$1.5 \pm 0.2\%$ (Ref. 19)
$1.7 \pm 0.2\%$ (Ref. 4)
$1.71 \pm 0.07\%$ (Ref. 20).

<u>NOTE</u>: The reference number corresponds to Trilling's article.

Table 2

Input data for K_2^0 rate determinations

Γ_{total}	=	$(1.85 \pm 0.18) \times 10^7$ sec^{-1} a)
$\Gamma_{charged}$	=	$(1.47 \pm 0.18) \times 10^7$ sec^{-1} b)
Γ_e	=	$(0.81 \pm 0.10) \times 10^7$ sec^{-1} c)
$\Gamma_e + \Gamma_\mu$	=	$(0.94 \pm 0.13) \times 10^7$ sec^{-1} d)
$\Gamma_\pi(+-0)$	=	$(0.254 \pm 0.025) \times 10^7$ sec^{-1} e)
$\Gamma_\pi(000)$	=	$(0.53 \pm 0.09) \times 10^7$ sec^{-1} f)
Γ_μ/Γ_e	=	0.70 ± 0.05 g)
$\dfrac{\Gamma_\pi(+-0)}{\Gamma_{charged}}$	=	0.152 ± 0.005 h)
$\dfrac{\Gamma_\pi(000)}{\Gamma_{charged}}$	=	0.25 ± 0.06 j)

a) Input data on mean life: $(5.3 \pm 0.6) \times 10^{-8}$ sec (Ref. 23)
$(6.1\ ^{+1.5}_{-1.2}) \times 10^{-8}$ sec (Ref. 24)

b) Ref. 22 with correction due to the new value of the K_1^0 mean life.

c) Ref. 25.

d) Ref. 21.

e) Input data: $(1.4 \pm 0.4) \times 10^6$ sec^{-1} (Ref. 21)
$(3.26 \pm 0.77) \times 10^6$ sec^{-1} (Ref. 26)
$(2.57 \pm 0.30) \times 10^6$ sec^{-1} (Ref. 27).

f) Ref. 27.

g) Input data: 0.73 ± 0.15 (Ref. 28)
0.81 ± 0.19 (Ref. 29)
0.85 ± 0.18 (Ref. 30)
0.680 ± 0.053 (Ref. 65).

h) Input data: 0.157 ± 0.03 (Ref. 28)
0.151 ± 0.02 (Ref. 29)
$0.15\ ^{+0.03}_{-0.04}$ (Ref. 30)
0.159 ± 0.015 (Ref. 31)
0.144 ± 0.006 (Ref. 32)
0.178 ± 0.017 (Ref. 33).

j) Input data: 0.24 ± 0.08 (Ref. 34)
0.25 ± 0.08 (Ref. 35).

Table 3

K^0 rates

Mode	Branching ratio	Rate (sec^{-1})	Comments
All K_1^0 modes [a]		$(1.155 \pm 0.019) \times 10^{10}$	
$K_1^0 \to \pi^0 + \pi^0$ [b]	$30.9 \pm 2.2\%$	$(0.357 \pm 0.025) \times 10^{10}$	
$K_1^0 \to \pi^+ + \pi^-$ [b]	$69.1 \pm 2.2\%$	$(0.798 \pm 0.025) \times 10^{10}$	
All K_2^0 modes [c]		$(19.9 \pm 1.0) \times 10^6$	
$K_2^0 \to \pi^\pm + e^\mp + \nu$ [c]	$38.4 \pm 1.4\%$	$(7.64 \pm 0.44) \times 10^6$	Normalized so that total branching ratio for these modes = 100%
$K_2^0 \to \pi^\pm + \mu^\mp + \nu$ [c]	$26.6 \pm 1.3\%$	$(5.30 \pm 0.38) \times 10^6$	
$K_2^0 \to \pi^+ + \pi^- + \pi^0$ [c]	$11.8 \pm 0.5\%$	$(2.34 \pm 0.13) \times 10^6$	
$K_2^0 \to \pi^0 + \pi^0 + \pi^0$ [c]	$23.2 \pm 2.0\%$	$(4.60 \pm 0.50) \times 10^6$	
$K_2^0 \to \pi^+ + \pi^- + \gamma$ [d]	$< 0.3\%$	$< 5 \times 10^4$	
$K_2^0 \to \pi^+ + \pi^-$ [e]	$(1.58 \pm 0.12) \times 10^{-3}$	$(3.15 \pm 0.17) \times 10^4$	CP violating
$K_2^0 \to \mu^+ + \mu^-$, $K_2^0 \to e^+ + e^-$, $K_2^0 \to e^\pm + \mu^\mp$ [f]	$< 10^{-4}$	$< 2 \times 10^3$	Involves neutral lepton currents
$K_2^0 \to 2\gamma$ [g]	$< 10^{-3}$	$< 2 \times 10^4$	

a) Input data on lifetimes:
$(0.90 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.94 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.885 \pm 0.025) \times 10^{-10}$ sec (Ref. 36)
$(0.85 \pm 0.04) \times 10^{-10}$ sec (Ref. 36)
$(0.87 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.86 \pm 0.04) \times 10^{-10}$ sec (Ref. 36)
$(0.848 \pm 0.014) \times 10^{-10}$ sec (Ref. 21).

b) Input data for $[\Gamma(2\pi^0)]/[\Gamma(2\pi)]$:
$33.5 \pm 1.4\%$ (Ref. 66)
$28.8 \pm 2.1\%$ (Ref. 67)
$26.0 \pm 2.4\%$ (Ref. 68).

c) From fit of data in Table 2.

d) Ref. 63.

e) Compilation by J. Cronin, presented at Argonne Weak Interactions Conference.

f) Refs. 40 and 61.

g) Ref. 64.

Table 4

Rate comparisons for $K \to 3\pi$ modes

Mode	Phase-space factor, Φ	γ = Rate/Φ (sec^{-1})
$K^+ \to \pi^+ + \pi^+ + \pi^-$	1.00	$(4.50 \pm 0.09) \times 10^6$
$K^+ \to \pi^+ + \pi^0 + \pi^0$	1.24	$(1.09 \pm 0.04) \times 10^6$
$K_2^0 \to \pi^+ + \pi^- + \pi^0$	1.22	$(1.92 \pm 0.11) \times 10^6$
$K_2^0 \to \pi^0 + \pi^0 + \pi^0$	1.49	$(3.09 \pm 0.34) \times 10^6$

Tests of $|\Delta \vec{I}| = \frac{1}{2}$

Mode	Experimental	Predicted
$\dfrac{\gamma\,(\pi^+\pi^-\pi^0)}{2\gamma\,(\pi^+\pi^0\pi^0)}$	0.88 ± 0.07	1.00
$\dfrac{\gamma\,(\pi^0\pi^0\pi^0)}{\gamma(\pi^+\pi^+\pi^-) - \gamma\,(\pi^+\pi^0\pi^0)}$	0.91 ± 0.12	1.00
$\dfrac{\gamma\,(\pi^+\pi^+\pi^-)}{4\gamma\,(\pi^+\pi^0\pi^0)}$	1.03 ± 0.04	1.00
$\dfrac{\gamma\,(\pi^0\pi^0\pi^0)}{\frac{3\gamma}{2}\,(\pi^+\pi^-\pi^0)}$	1.07 ± 0.12	1.00

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APPENDIX C

THE $\Delta I = \frac{1}{2}$ RULE IN THE DECAY OF K MESONS

In the decay of a hadron (strongly interacting particle), the hadrons in the initial and final states can be characterized by the quantum numbers: Y the hypercharge, Q the charge, and I the isospin. Calling ΔY, ΔQ, ΔI_3 the difference between the final and initial states of these quantum numbers, a relation between these quantities is introduced by the Gell-Mann - Nishijima formula:

$$\Delta Q = \Delta I_3 + \frac{\Delta Y}{2} .$$

The decay of the hadrons to purely hadronic final modes, or hadron + lepton final modes (semi-leptonic decays), or pure lepton modes shows up some selection rules.

1. $\Delta Q \neq 0$ in the semi-leptonic decays

This expresses the absence of "neutral currents" in weak interactions. A decay to a state involving leptons is always accompanied by a change of charge of the hadron state: one has never observed $K^+ \to \pi^+ + \nu\bar{\nu}$, or $K^+ \to \pi^+ e^+ e^-$, or $K_L \to \mu^+ \mu^-$.

2. $|\Delta Y| < 2$ in the non-leptonic decays

If $\Delta Y = 2$ would be allowed, direct transitions from K^0 to $\bar{K}^0$ would be allowed to first order, while in the absence of transitions with $\Delta Y > 1$ a second-order transition is required. The two hypotheses differ by a factor of 10^7 in the evaluation of the mass difference between K_S^0 and K_L^0, and the determination of this mass difference, which we have discussed at length in Chapter I, Section 4, eliminates the $\Delta S = 2$ transition. This rule is also borne out by the fact that transitions $\Xi^- \to n + \pi^-$ are not observed.

3. $\Delta Q = \Delta S$ in semi-leptonic or leptonic transitions

This selection rule allows

$$\left.\begin{array}{l} K^0 \to \pi^- \ell^+ \nu, \text{ amplitude } f \\ \bar{K}^0 \to \pi^+ \ell^- \bar{\nu}, \text{ amplitude } f' \end{array}\right. \qquad \Delta Y = \Delta Q$$

but forbids

$$\left.\begin{array}{l} \bar{K}^0 \to \pi^- \ell^+ \nu, \text{ amplitude } g \\ K^0 \to \pi^+ \ell^- \bar{\nu}, \text{ amplitude } g' \end{array}\right. \qquad \Delta Y = -\Delta Q$$

If one admits PCT invariance

$$f' = f^* , \quad g' = g^* ,$$

and if PC invariance is admitted, these coefficients are real, since

$$f' = f , \quad g' = g .$$

Since only K_1^0 and K_2^0 are physical observable states in the decay, one has to look into the effect of the admixture of the f' and g' terms to the leptonic decay. They appear clearly in a rather heavy expression. In this section it is sufficient for us to say that the violation of the $\Delta Q/\Delta S$ rule is expressed in terms of the complex parameter.

$$X = g/f = X\, e^{i\phi} ,$$

and that Fig. 8 shows the distribution of the values so far for X and ϕ. It is clear that although these experiments are not in contradiction with $X = 0$, their spread forbids one to draw strong conclusions from them. The importance of this check is that $\Delta Q = \Delta Y$ leads to $\Delta I_3 = \pm 1/2$ from Gell-Mann - Nishijima formula, but not $\Delta I = 1/2$. If, however, it appeared to be violated it would kill the $|\Delta I| = 1/2$ rule, since this rule leads to $\Delta I_3 = 1/2$ and $\Delta Q = \Delta Y$.

For the purpose of our discussion it is sufficient to say that the experimental study of the leptonic decay as a function of time of a pure K^0 or $\bar{K}^0$ state, leads to the fact that the order of magnitude of the leptonic decay rate of the K_S^0 is the same as the one of K_L^0.

On the other hand, the $\Delta Y = \Delta Q$ rule is checked in other weak decay. For instance, out of 208 events

$$K^+ \to \pi^+ + \pi^- + e^+ + \nu ,$$

no event

$$K^- \to \pi^+ + \pi^+ + e^- + \bar{\nu} \quad (\Delta Y = -\Delta Q)$$

has been found.

If the $\Delta Y = \Delta Q$ law holds as well as PC invariance, we have exactly $\Gamma(K_S^0 \to \pi \ell \nu) = \Gamma(K_L^0 \to \pi \ell \nu)$, where ℓ stands for lepton; if, in addition, $|\Delta I| = 1/2$ we have

$$\Gamma(K_S^0 \to \pi \ell \nu) = \Gamma(K_L^0 \to \pi \ell \nu) = 2\Gamma(K^+ \to \pi \ell \nu)$$

to be compared with the experimental data:

$$\Gamma(K_L^0 \to \pi \ell \nu) = (12.94 \pm 0.60)\ 10^6\ \text{sec}^{-1}$$
$$2\ (K^+ \to \pi \ell \nu) = 12.32 \pm 0.68 .$$

In the decays of the Σ the following limit has been obtained

$$\frac{\text{Rate } (\Sigma^+ \to n + e^+ + \nu)}{\text{Rate } (\Sigma^- \to n + e^- + \bar{\nu})} < 0.05 .$$

4. <u>$|\Delta I| = 1/2$ law in non-leptonic modes</u>

For $K \to$ (pions) we have always $\Delta Q = 0$, $\Delta Y = \pm 1$, and $|\Delta I_3| = 1/2$. The more general rule $|\Delta I| = 1/2$ that has been proposed explains, as we have seen in Chapter I, Section 2, the inhibition of the $K^\pm$ decays into two pions, as compared to the K_S^0 decay mode and the observed branching ratio of K_S^0 into $\pi^+\pi^-$ and $\pi^0\pi^0$. In the decay of kaons into three pions the implications of the $|\Delta I| = 1/2$ rule are more complicated and, referring the reader to detailed calculation in various books[19,42], I will merely quote the results of the analysis and the experimental checks.

Table 6

Some predictions of the $|\Delta I| = \frac{1}{2}$ rule in the decay of kaons into three pions

	Predictions		
	No phase space corrections	Phase space corrections	Experiment
$K^+ \rightarrow \frac{\pi^+\pi^0\pi^0}{\pi^+\pi^+\pi^-}$	0.25	0.311	0.300 ± 0.013
$K_L^0 \rightarrow \frac{\pi^0\pi^0\pi^0}{\pi^0\pi^+\pi^-}$	1.5	1.82	1.90 ± 0.25
$\frac{K_L^0 \rightarrow \pi^+\pi^-\pi^0}{K^+ \rightarrow \pi^+\pi^0\pi^0}$	2	1.96	1.75 ± 0.12

The $|\Delta I| = \frac{1}{2}$ rule also predicts relations in the decay of K^+ and K^0 between the energy distributions of the pions in the kaon centre of mass; however, we will not discuss this here. We only want to mention that within the experimental errors it seems that the predictions of the $|\Delta I| = \frac{1}{2}$ rule are verified in the kaon-three-pion decay, except for the last number of Table 6 where theory and experiment are separated by 1.5 σ.

However, I wish to point out that for the second number there was for years a strong discrepancy because of experimental errors, and the conclusions drawn by some theoreticians was that the admixture of $|\Delta I| = \frac{3}{2}$ necessary to explain the discrepancy was of the same order of magnitude as the one necessary to explain the $K^{\pm}$ decay into two pions. However, they had to put forward an hypothesis leading to predictions, later discarded by experiments. The relatively good agreement between the $|\Delta I| = \frac{1}{2}$ rule and experiments in K decays compares with the relatively good agreement also found in other types of strange particle decays: branching ratio $\Gamma(\Lambda \rightarrow p + \pi^-)/\Gamma(\Lambda \rightarrow n + \pi^0)$, relations between the asymmetry parameters of Σ^+ and Σ^-. This makes more striking the considerable violation of the $\Delta I = \frac{1}{2}$ rule in the branching ratio of $\Gamma(K_L^0 \rightarrow \pi^+\pi^-)$ with respect to $\Gamma(K_L^0 \rightarrow \pi^0\pi^0)$.

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APPENDIX D

CALCULATION OF THE EIGENVALUES OF THE MASS MATRIX IN MATTER

We go from the $(K^0\ \bar{K}^0)$ system to the $(K_L\ K_S)$ system by the transformation

$$C = \begin{pmatrix} p & -q \\ p & q \end{pmatrix} .$$

(See Chapter II, Professor Gourdin.)
The operator

$$\begin{pmatrix} f & 0 \\ 0 & \bar{f} \end{pmatrix}$$

becomes

$$C \begin{pmatrix} f & 0 \\ 0 & \bar{f} \end{pmatrix} C^{-1} = \frac{1}{2pq} \begin{pmatrix} p & -q \\ p & q \end{pmatrix} \begin{pmatrix} qf & qf \\ -p\bar{f} & q\bar{f} \end{pmatrix}$$

$$= \begin{vmatrix} \frac{f+\bar{f}}{2} & \frac{f-\bar{f}}{2} \\ \frac{f-\bar{f}}{2} & \frac{f+\bar{f}}{2} \end{vmatrix} .$$

Thus the additional term introduced by the matter becomes in the $(K_L\ K_S)$ system

$$- \frac{\pi N}{m} \begin{pmatrix} f+\bar{f} & f-\bar{f} \\ f-\bar{f} & f+\bar{f} \end{pmatrix} .$$

We want to diagonalize.

$$M = \begin{pmatrix} M_L - \frac{\pi N}{m}(f+\bar{f}) & -\frac{\pi N}{m}(f-\bar{f}) \\ -\frac{\pi N}{m}(f-\bar{f}) & M_S - \frac{\pi N}{m}(f+\bar{f}) \end{pmatrix} .$$

The new basis will be

$$\begin{aligned} K_L^{\mathrm{matt}} &= K_L' = K_L + \alpha\, K_S \\ K_S^{\mathrm{matt}} &= K_S' = K_S + \beta\, K_L \end{aligned} = \begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix} \begin{pmatrix} K_L \\ K_S \end{pmatrix}$$

$$\begin{pmatrix} M_L' & 0 \\ 0 & M_S' \end{pmatrix} = C^{-1}\, M C$$

$$C = \begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix} \qquad C^{-1} = \frac{1}{1-\alpha\beta}\begin{pmatrix} 1 & -\alpha \\ -\beta & 1 \end{pmatrix} .$$

This leads to the four relations

$$M_L' = \frac{1}{1-\alpha\beta}\left\{ M_L - \frac{\pi N}{m}(f+\bar f) + \beta\frac{\pi N}{m}(f-\bar f) - \alpha\frac{\pi N}{m}(f-\bar f) - \alpha\beta\left[M_S - \frac{\pi N}{m}(f+\bar f)\right]\right\}$$

$$M_S' = \frac{1}{1-\alpha\beta}\left\{ M_S - \frac{\pi N}{m}(f+\bar f) + \alpha\frac{\pi N}{m}(f-\bar f) - \beta\frac{\pi N}{m}(f-\bar f) - \alpha\beta\left[M_L - \frac{\pi N}{m}(f+\bar f)\right]\right\}$$

$$0 = -\alpha\left[M_L - \frac{\pi N}{m}(f+\bar f)\right] - \frac{\pi N}{m}(f-\bar f) + \alpha\left[M_S - \frac{\pi N}{m}(f+\bar f)\right] + \alpha^2\frac{\pi N}{m}(f-\bar f)$$

$$0 = \beta\left[M_L - \frac{\pi N}{m}(f+\bar f)\right] + \beta^2\frac{\pi N}{m}(f-\bar f) - \beta\left[M_S - \frac{\pi N}{m}(f+\bar f)\right] - \frac{\pi N}{m}(f-\bar f) .$$

The two last relations reduce to

$$\alpha\,(M_S - M_L) + (\alpha^2 - 1)\frac{\pi N}{m}(f-\bar f) = 0$$

$$\beta\,(M_L - M_S) + (\beta^2 - 1)\frac{\pi N}{m}(f-\bar f) = 0 .$$

These relations require $\alpha = -\beta$

$$M_L' = \frac{1}{1+\beta^2}\left\{ M_L - \frac{\pi N}{m}(f+\bar f) + 2\beta\frac{\pi N}{m}(f-\bar f) + \beta^2 M_S - \frac{\pi N}{m}(f+\bar f)\right\}$$

$$\boxed{\begin{aligned} M_L' &= M_L - \frac{\pi N}{m}(f+\bar f) - \frac{\beta^2}{1+\beta^2}(M_L - M_S) + \frac{2\beta}{1+\beta^2}\frac{\pi N}{m}(f-\bar f) \\ M_S &= M_S - \frac{\pi N}{m}(f+\bar f) + \frac{\beta^2}{1+\beta^2}(M_L - M_S) - \frac{2\beta}{1+\beta^2}\frac{\pi N}{m}(f-\bar f) .\end{aligned}}$$

If $\beta^2 \ll 1$ then $\beta = (\pi N/m)[(f-\bar f)/(M_L - M_S)]$ leading to the relation (I.4) and (I.5) in the text.

We know by experience, that $\alpha = -\beta$, the amplitude of the regenerated short-lived neutral kaon is at most a few per cent since the maximum of the intensity of regenerated kaons is 10^{-3} in any material. This justifies the approximation $\beta^2 \ll 1$.

* * *

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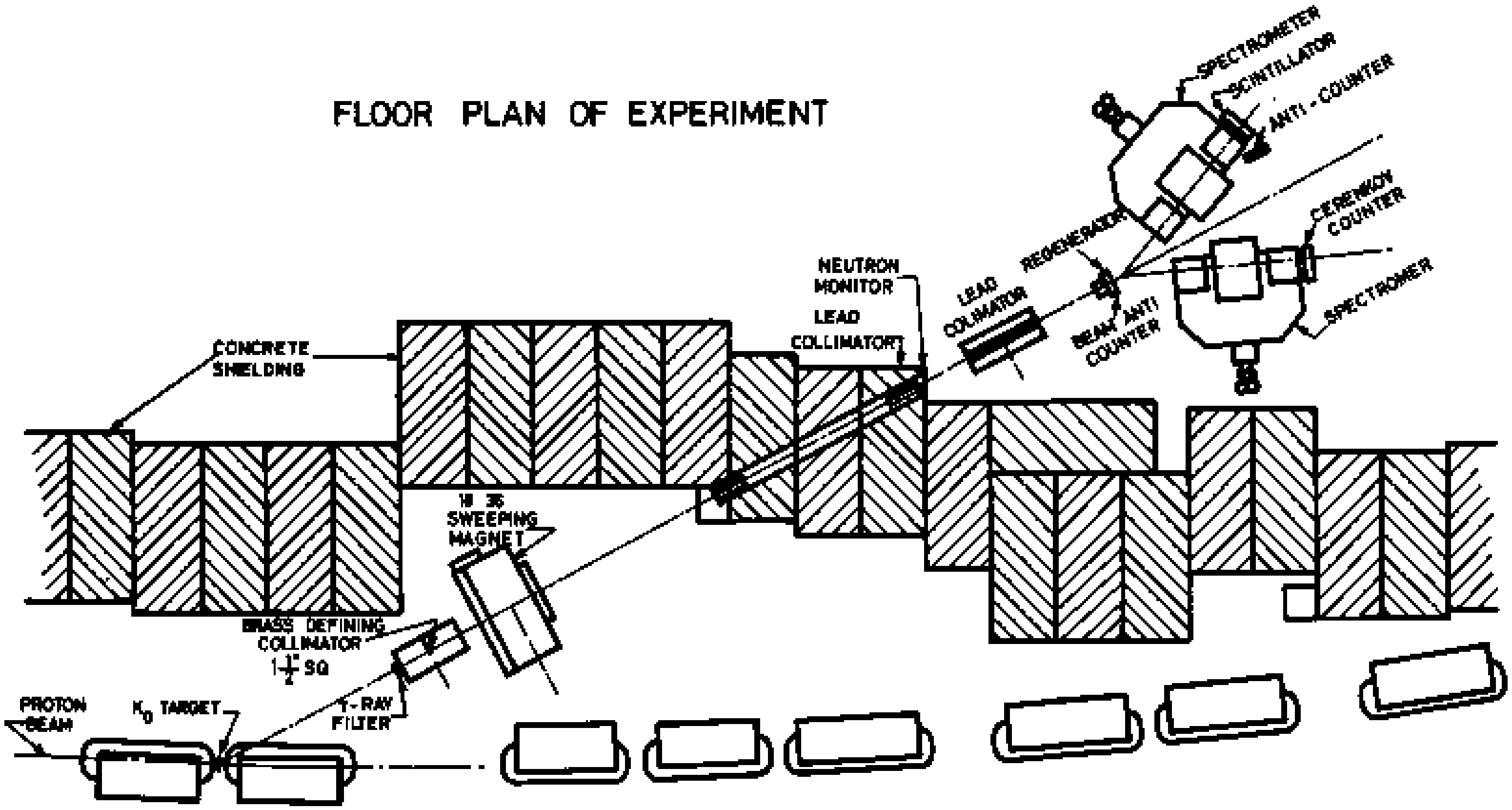


Fig. 1 : The experimental layout of Christenson et al.[12].

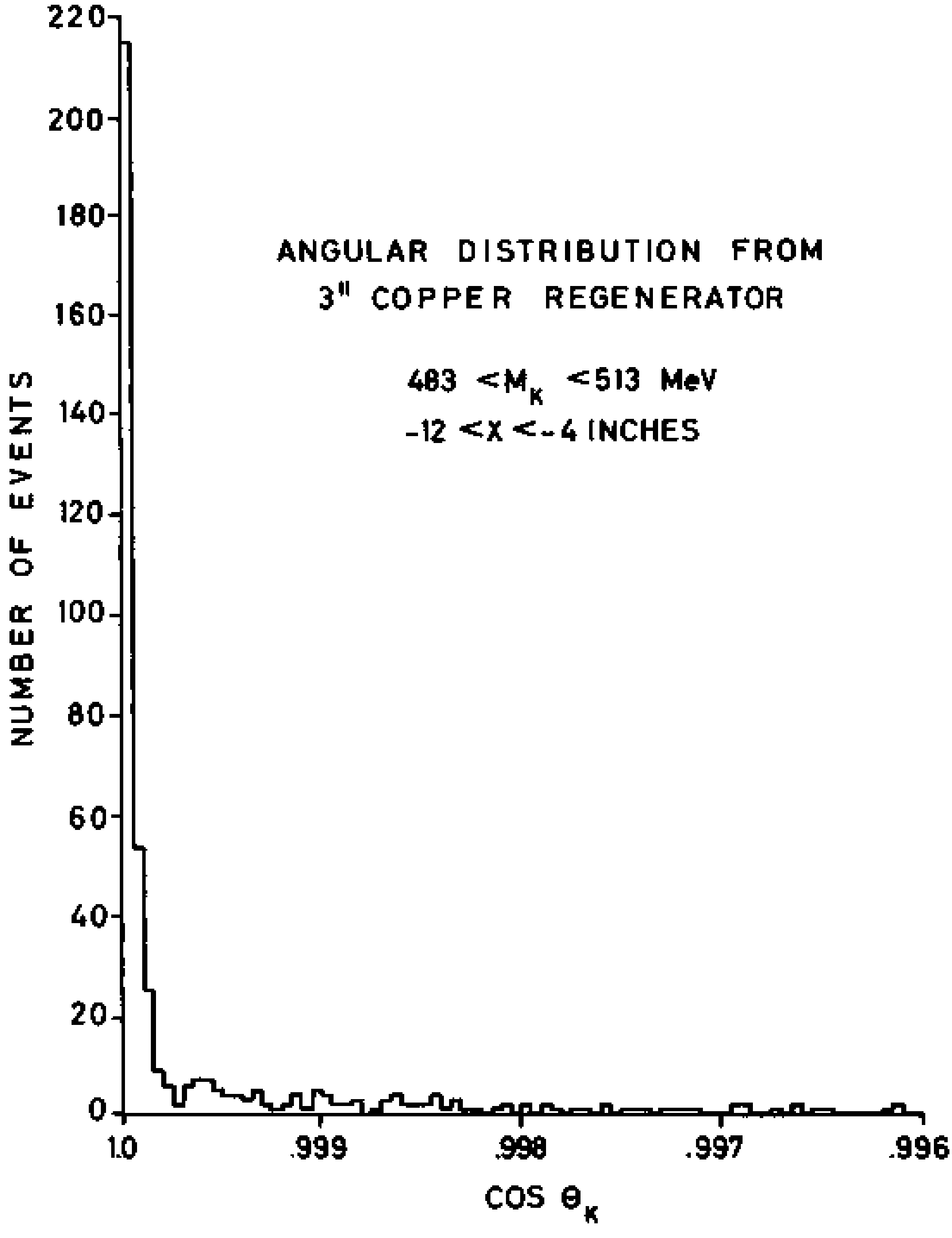


Fig. 2 : The angular distribution of the K_S regenerated by a thick regenerator. From Christenson's thesis[21].

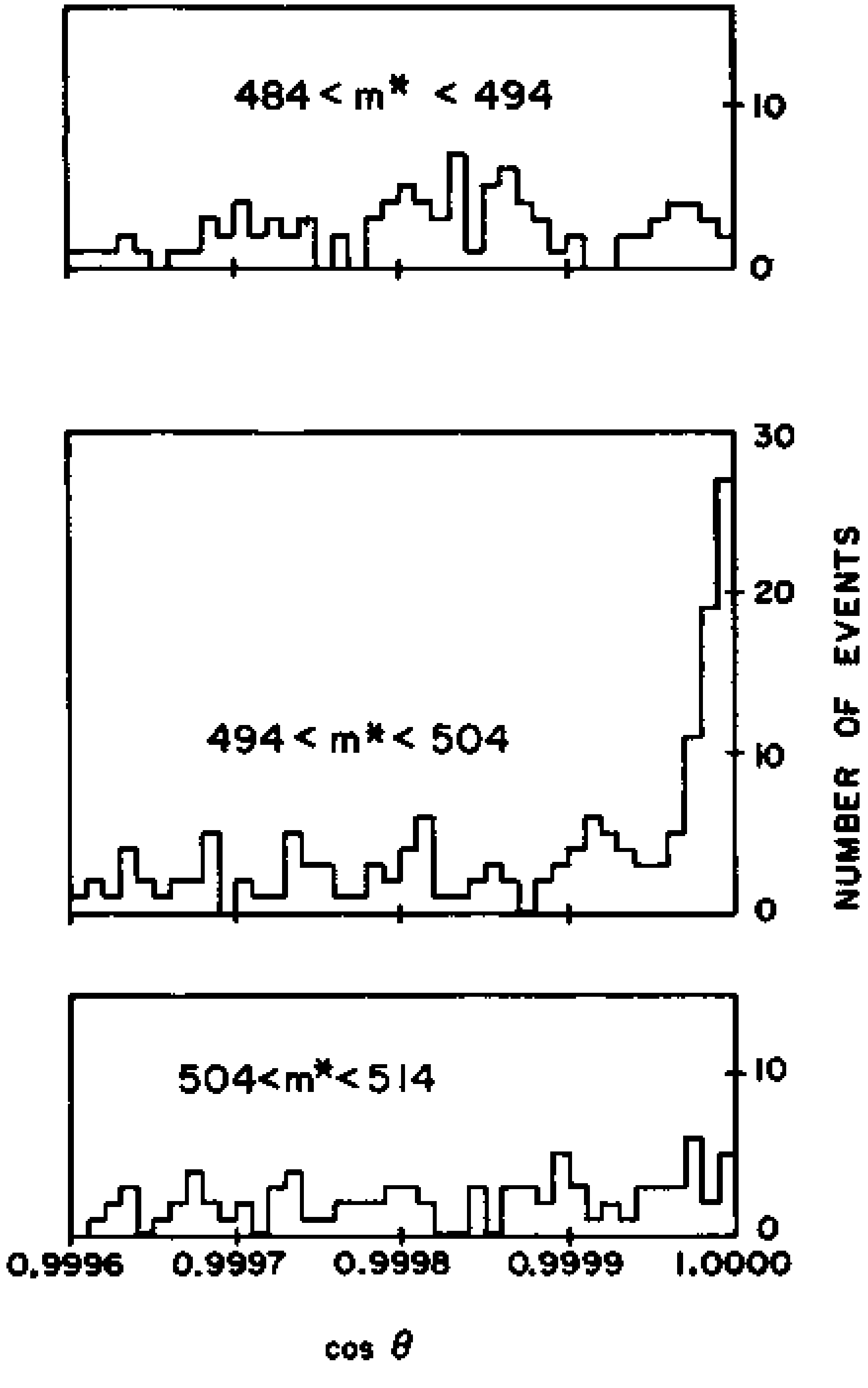


Fig. 3 : The angular distribution of the neutral kaon emitting two pions, with no regenerator, as a function of the effective mass. Events with cos θ > 0.9995. Christensen et al.[12].

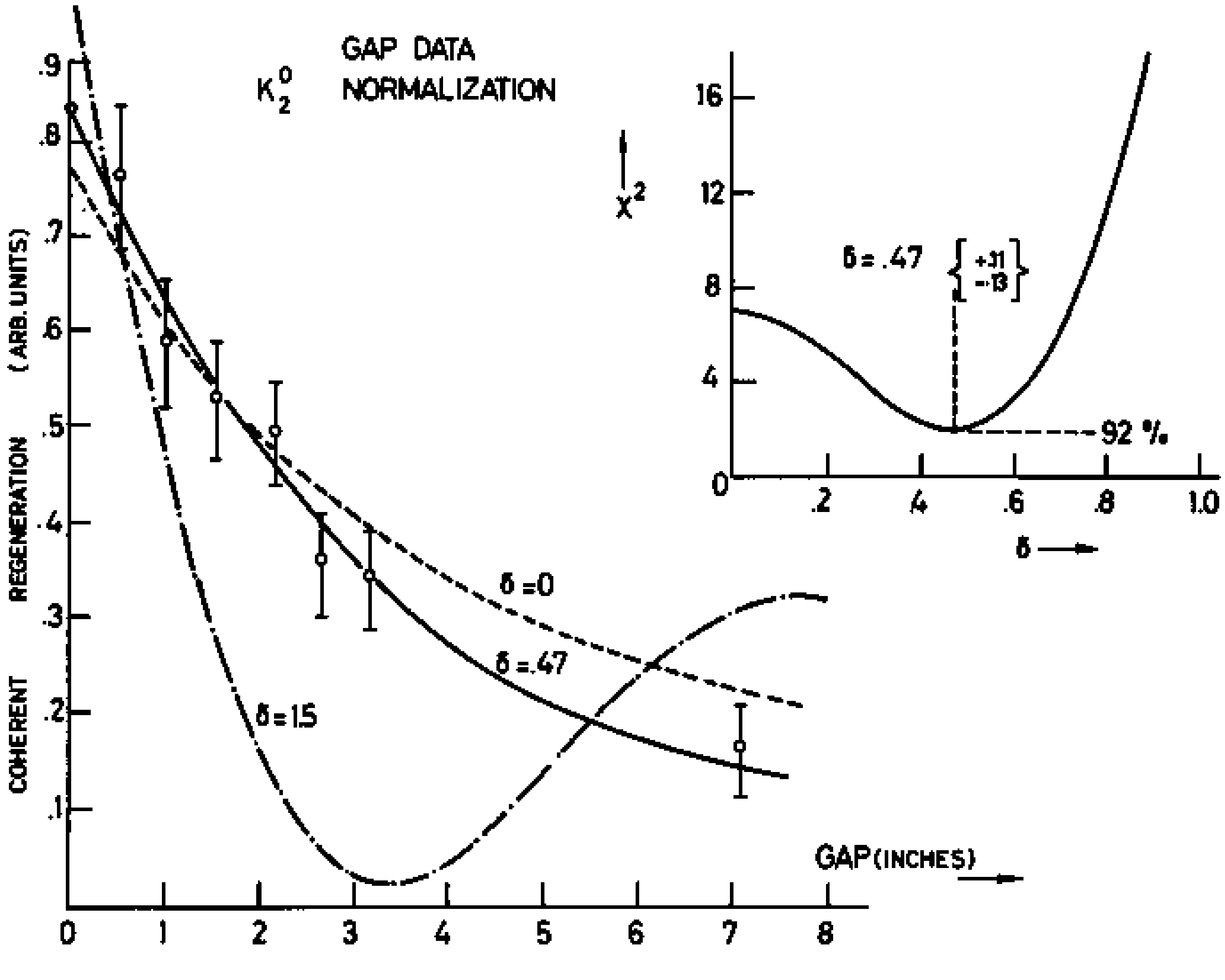


Fig. 4 : The distribution of K_S regenerated from two slabs separated by a gap. From Christenson's thesis[23].

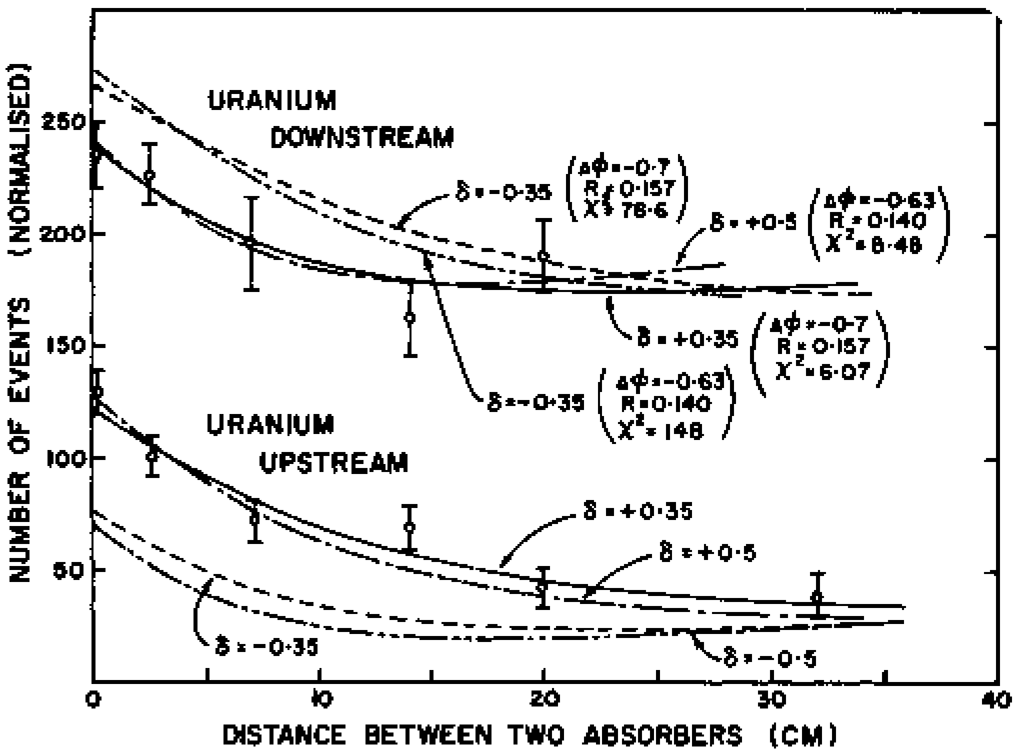
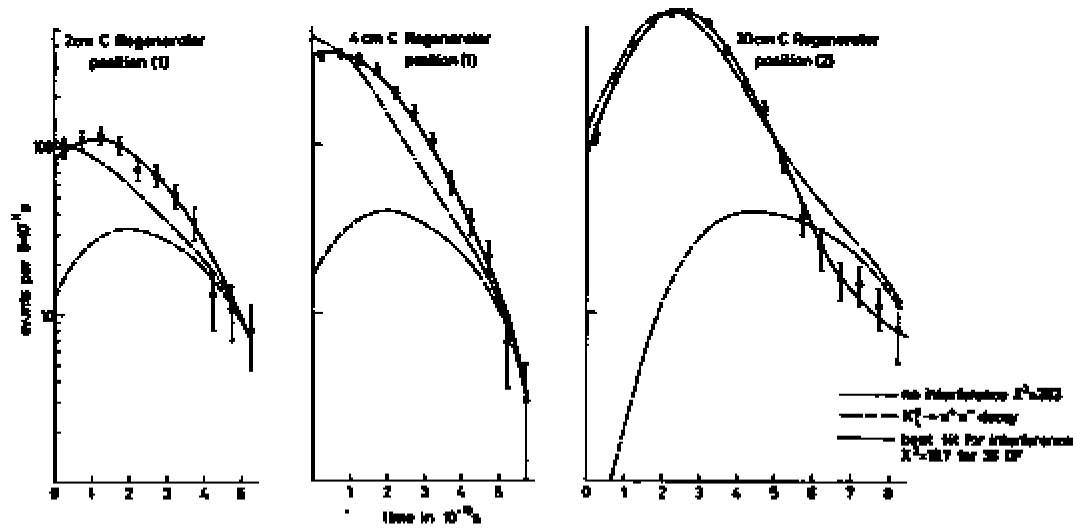
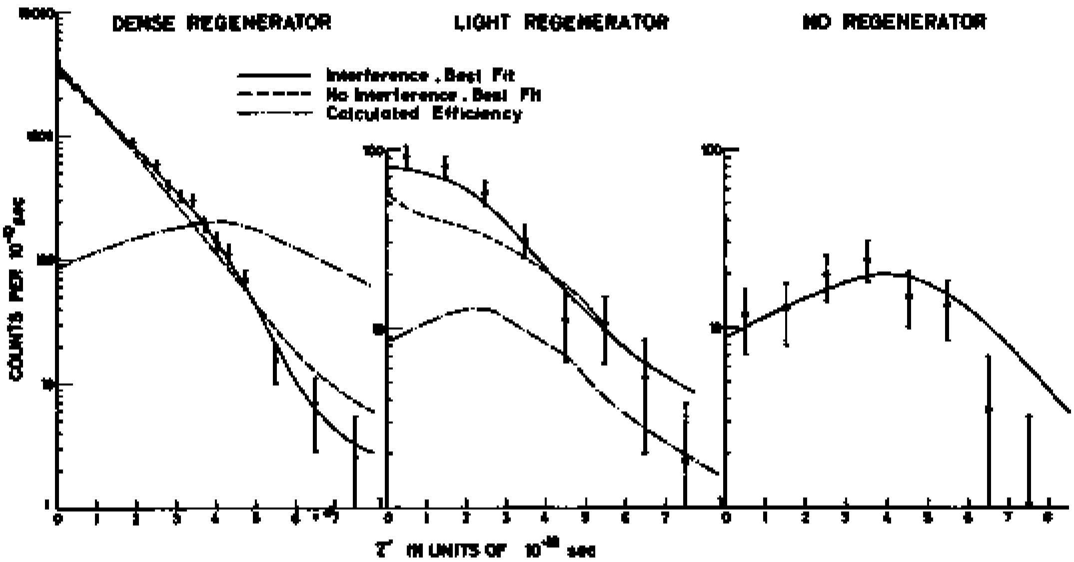


Fig. 5 : Computed regeneration curves and the experimental points for the case of an Okun'-Kobzarev experiment using a combination of uranium and carbon. Jovanovitch et al.[34].



a) Bott-Bodenhausen et al.



b) Alff-Steinberger et al.

Fig. 6 : Interference in the $\pi^+\pi^-$ decay between K_L and regenerated K_S.

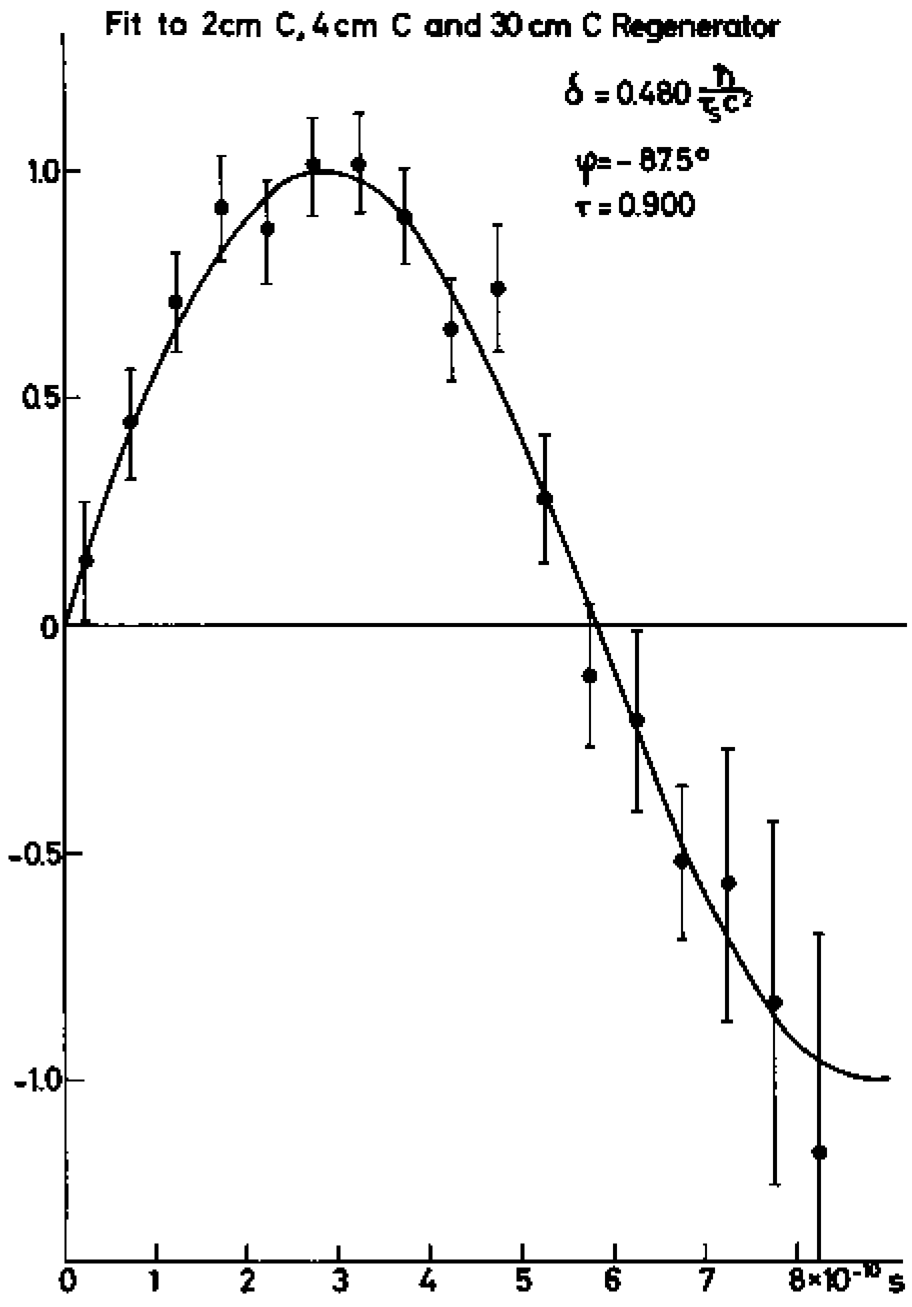


Fig. 7 : Experimental data treated in such a way as to isolate the interference term $\cos(\varphi + \Delta m\tau)$ Bott-Bodenhausen et al.[29].

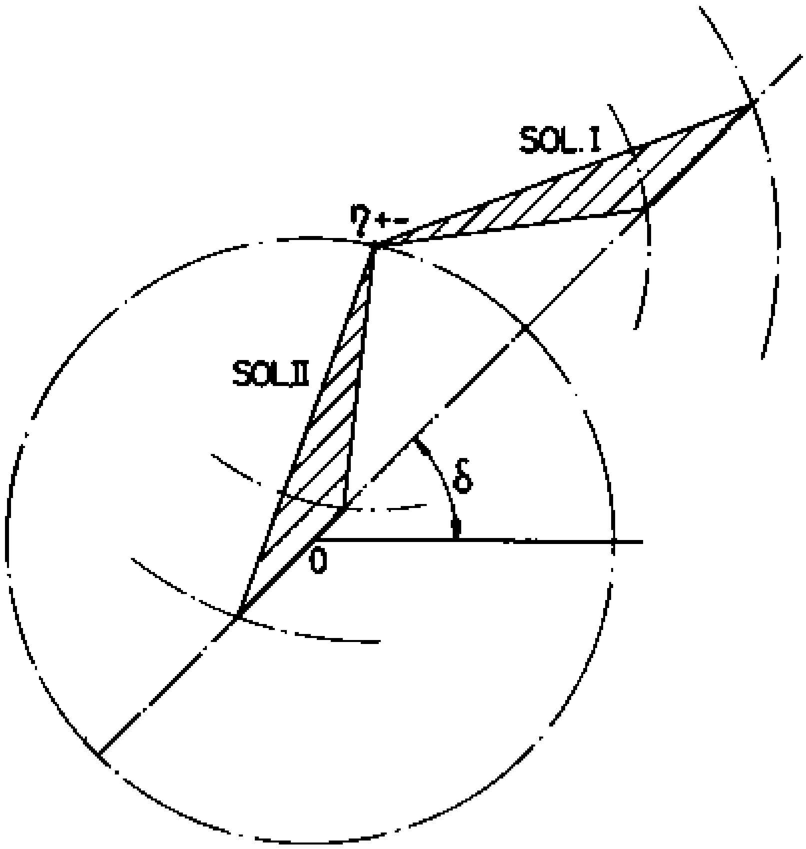


Fig. 8 : TCP invariance solutions for $\varphi_{+-} = 80°$; $\delta = 44°$; $r = 1.1 \pm 0.22$.

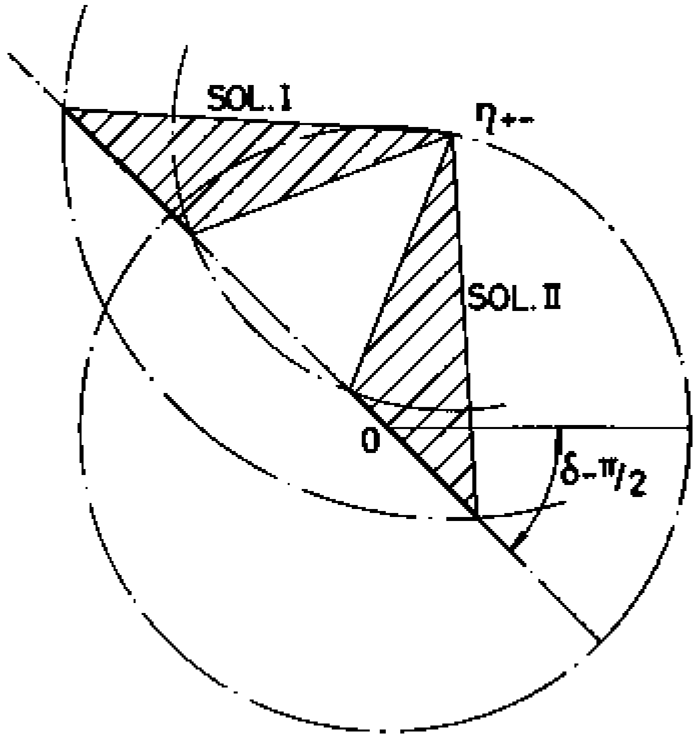


Fig. 9 : Time reversal invariance solutions for $\varphi_{+-} = 80°$; $\delta = 44°$; $r = 1.1 \pm 0.22$.

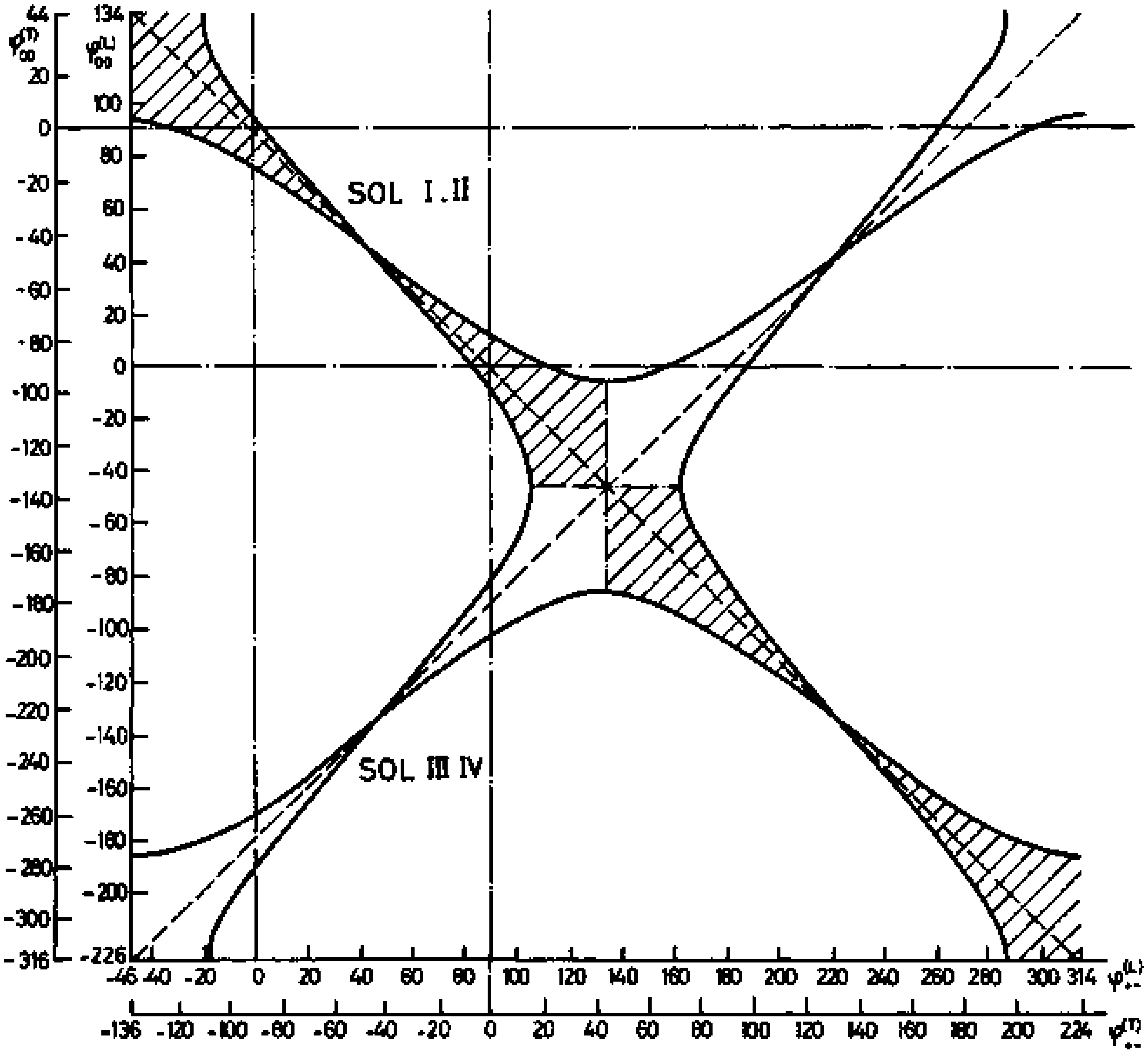


Fig. 10 : φ_{00} function of φ_{+-} for $r = 1.1 \pm 0.22$.

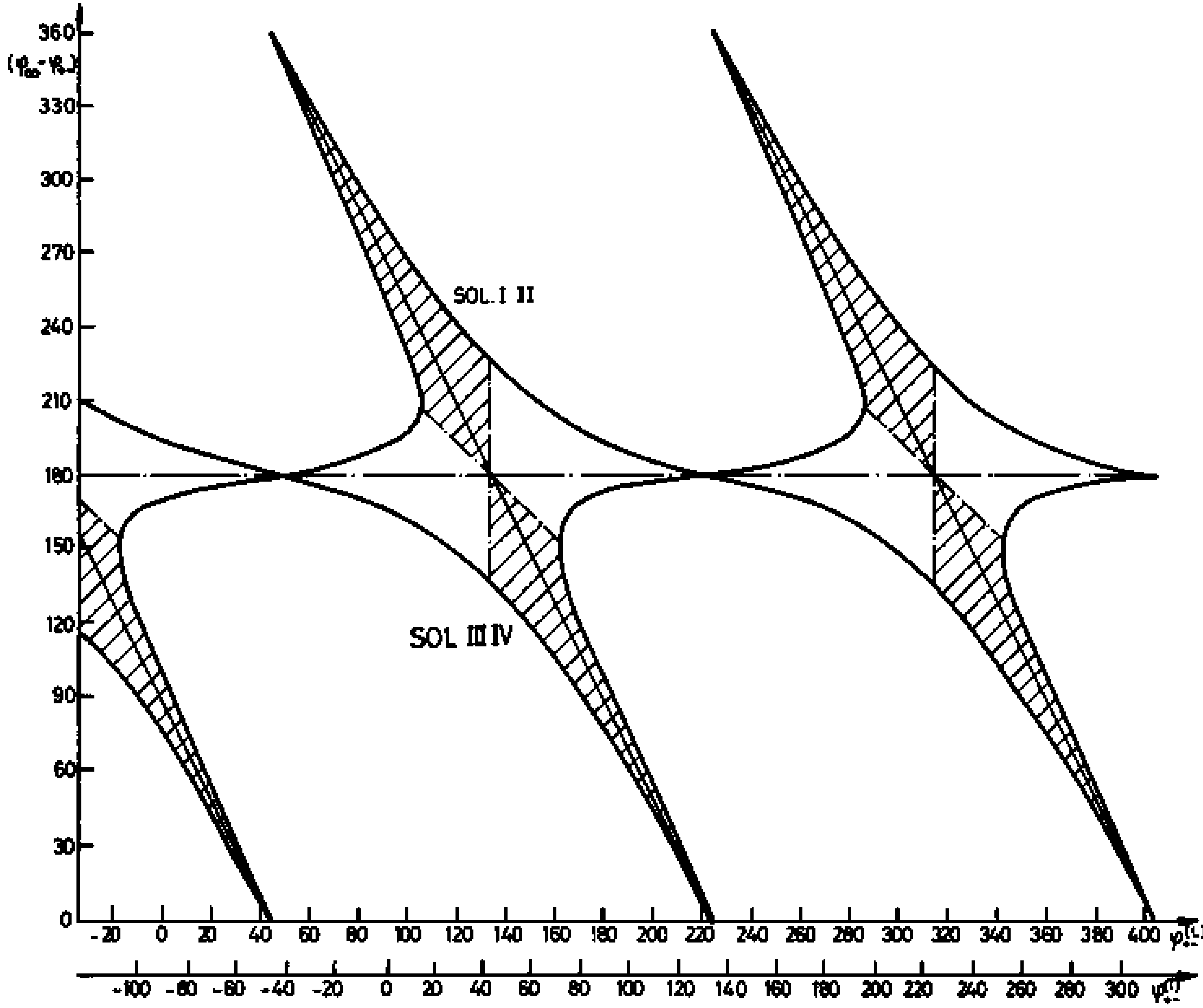


Fig. 11 : $\varphi_{oo} - \varphi_{+-}$ function of φ_{+-} for $r = 1.1 \pm 0.22$.

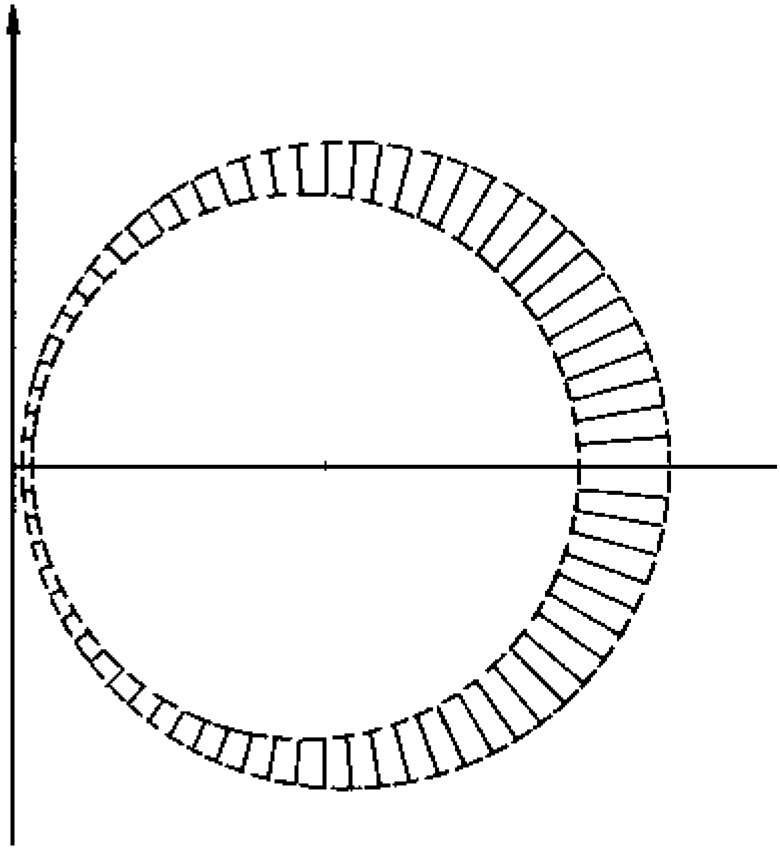


Fig. 12 : Allowed values of ω in the complex ω plane.

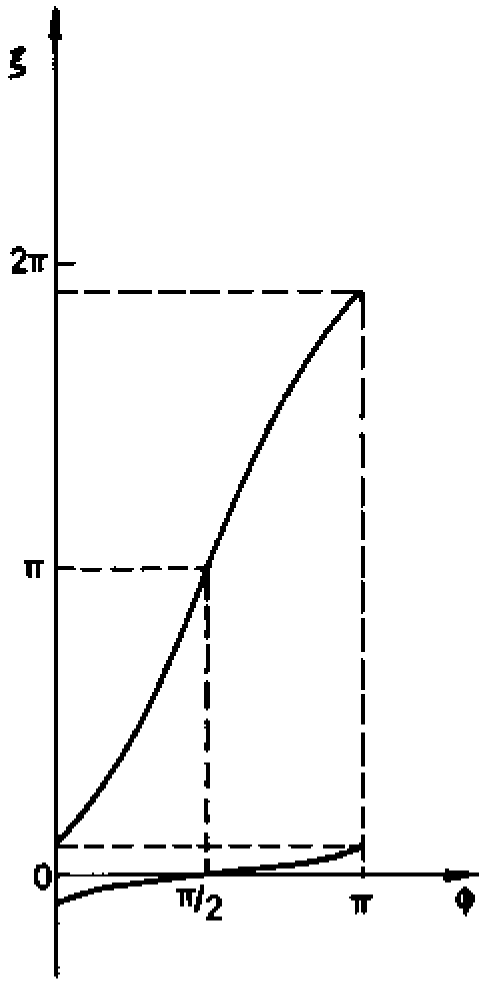


Fig. 13 : The angle ξ as a function of ϕ for TCP invariance.

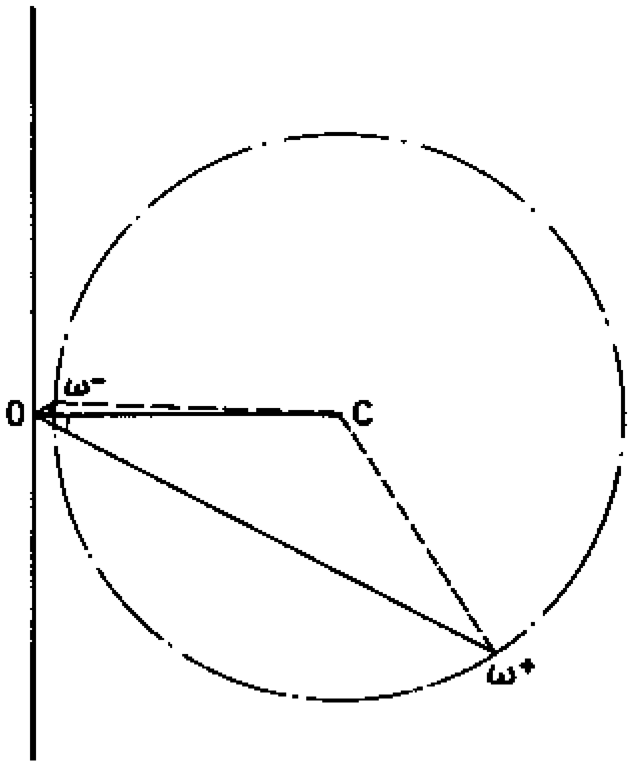


Fig. 14 : The two solutions ω^+ and ω^- for ω.

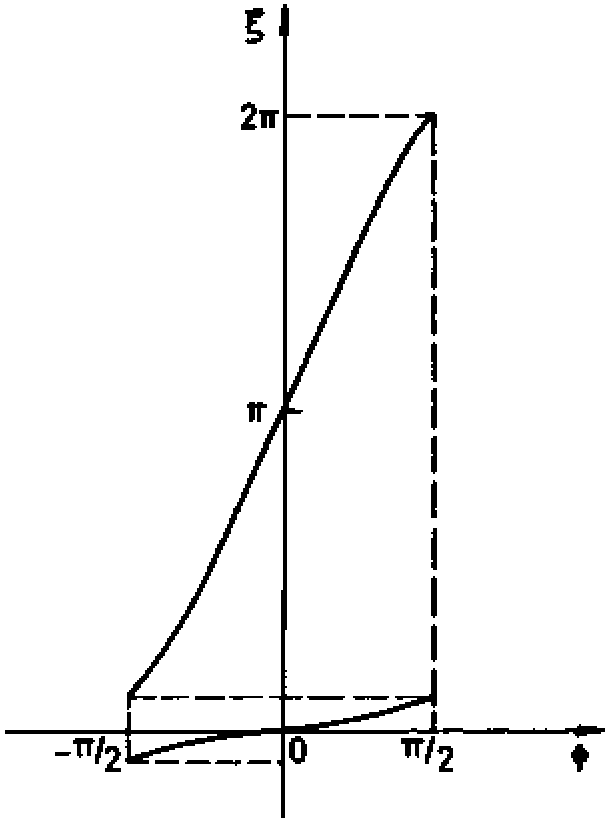


Fig. 15 : The angle ξ as a function of Φ for time reversal invariance.

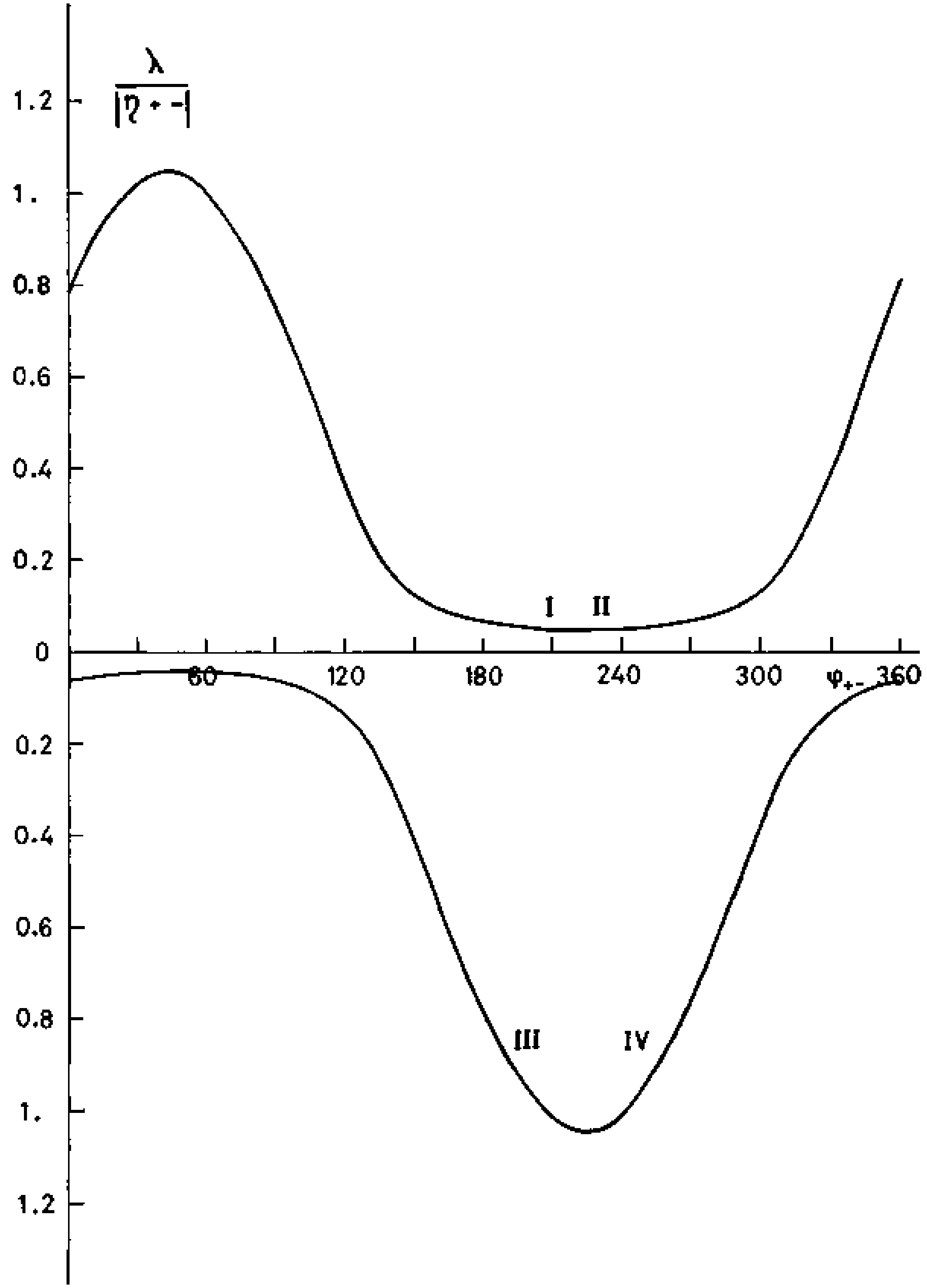


Fig. 16a : $\left|\frac{\lambda}{\eta_{+-}}\right|$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

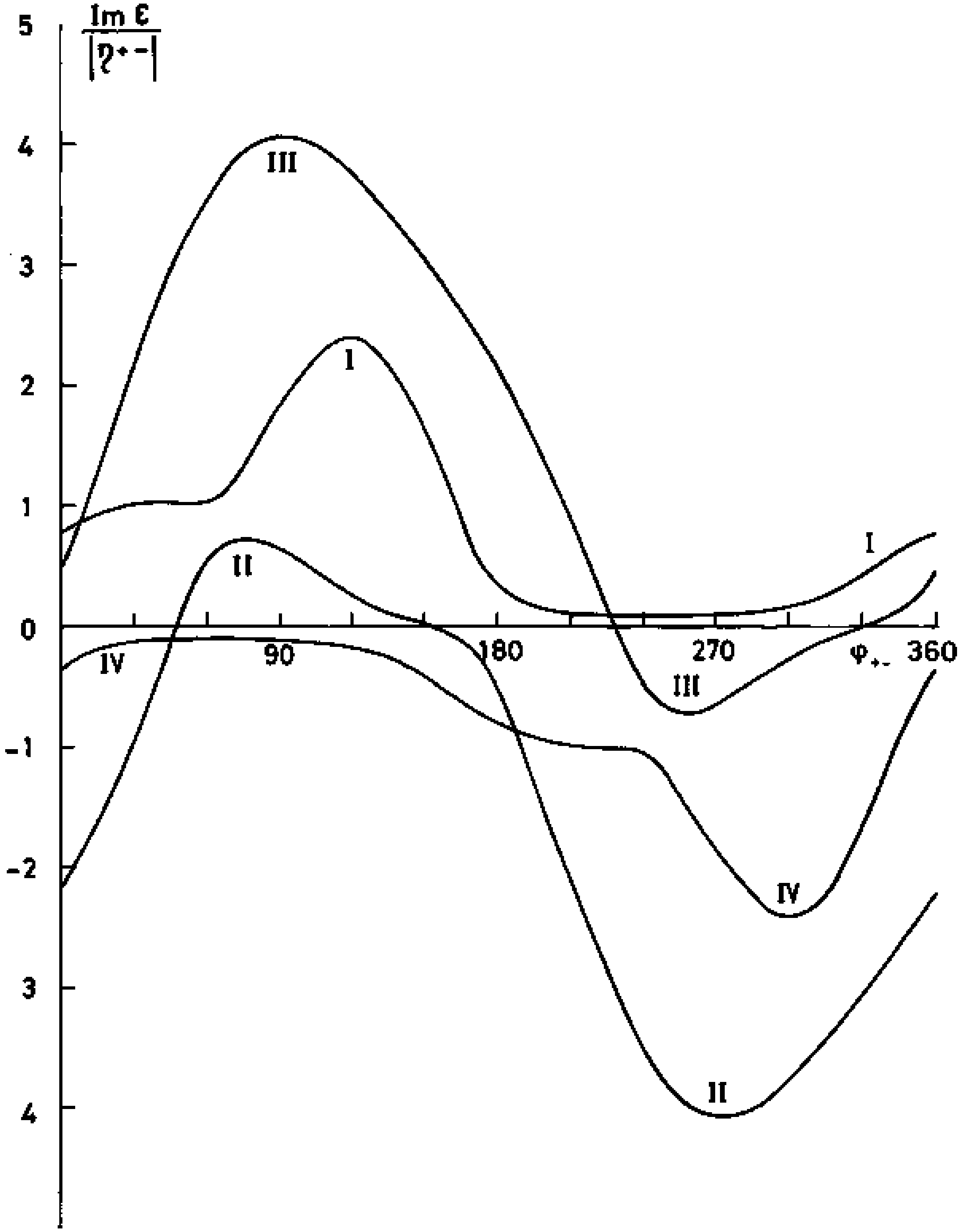


Fig. 16b : $\mathrm{Im}\,\epsilon = \theta_0$ for $R = 0.447$; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

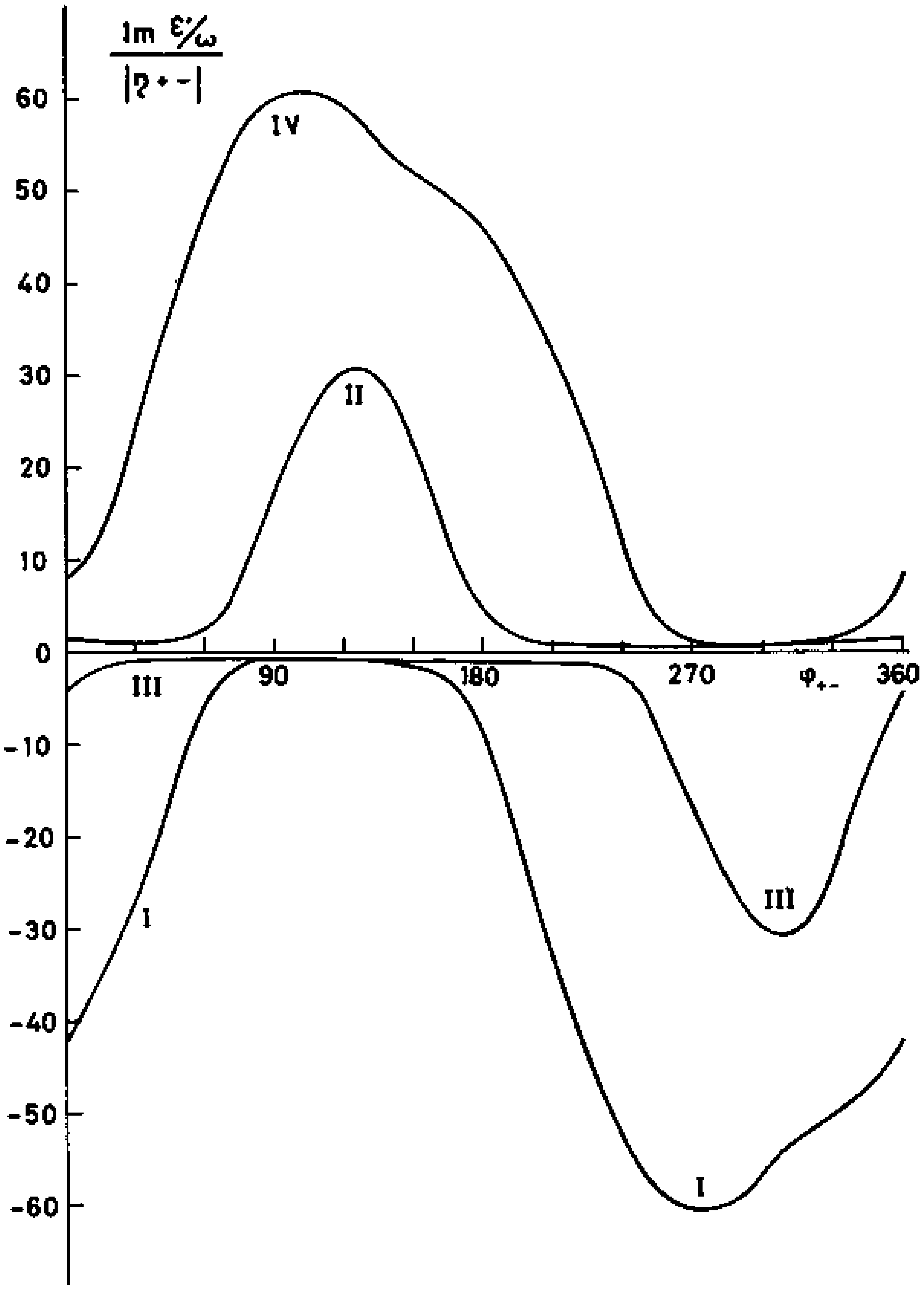


Fig. 16c : $\mathrm{Im}\,\frac{\varepsilon'}{\omega} = \tan\theta_2$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

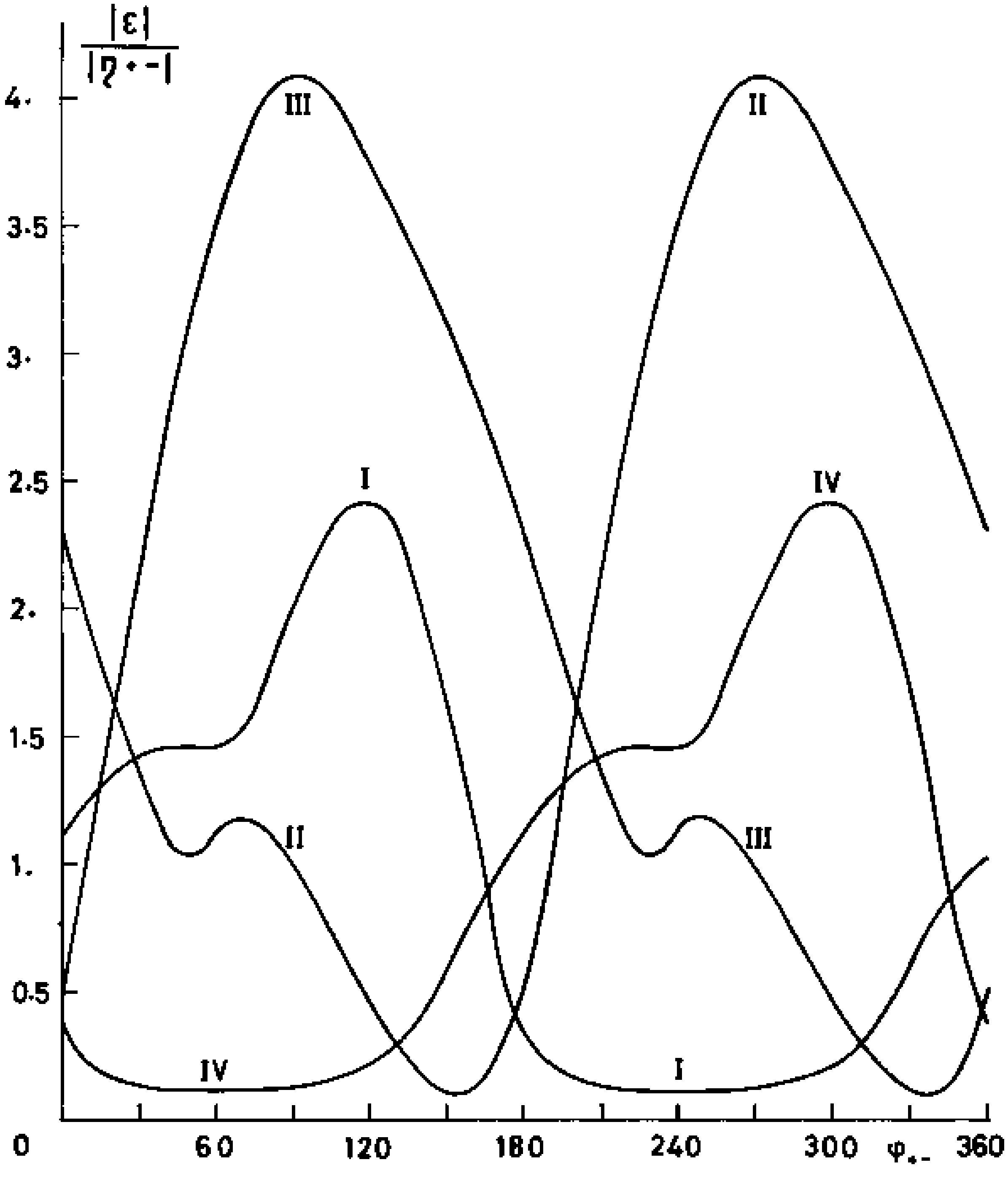


Fig. 16d : $\frac{|\varepsilon|}{|\eta_{+-}|}$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

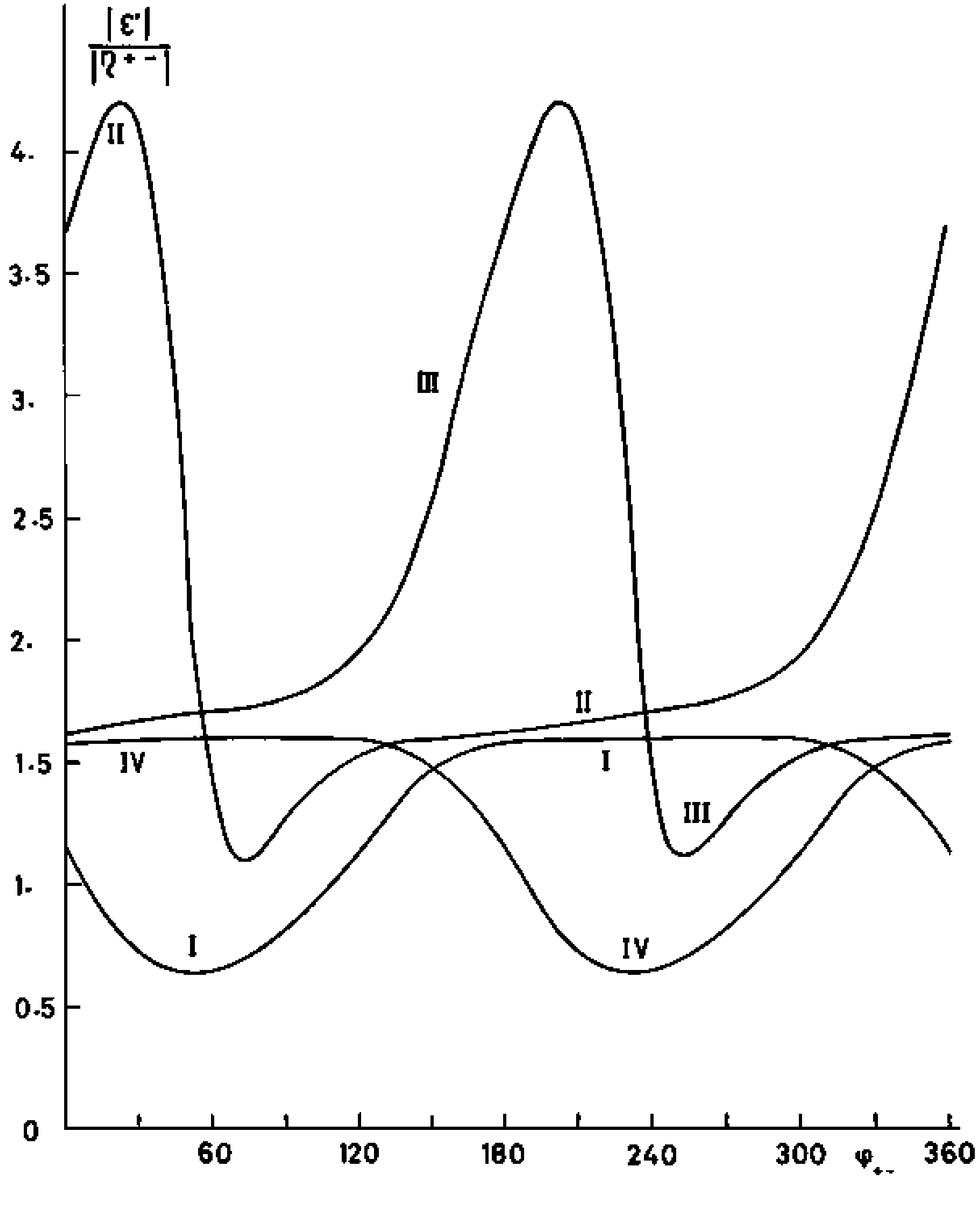


Fig. 16e ; $\frac{|\varepsilon'|}{|\eta_{+-}|}$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

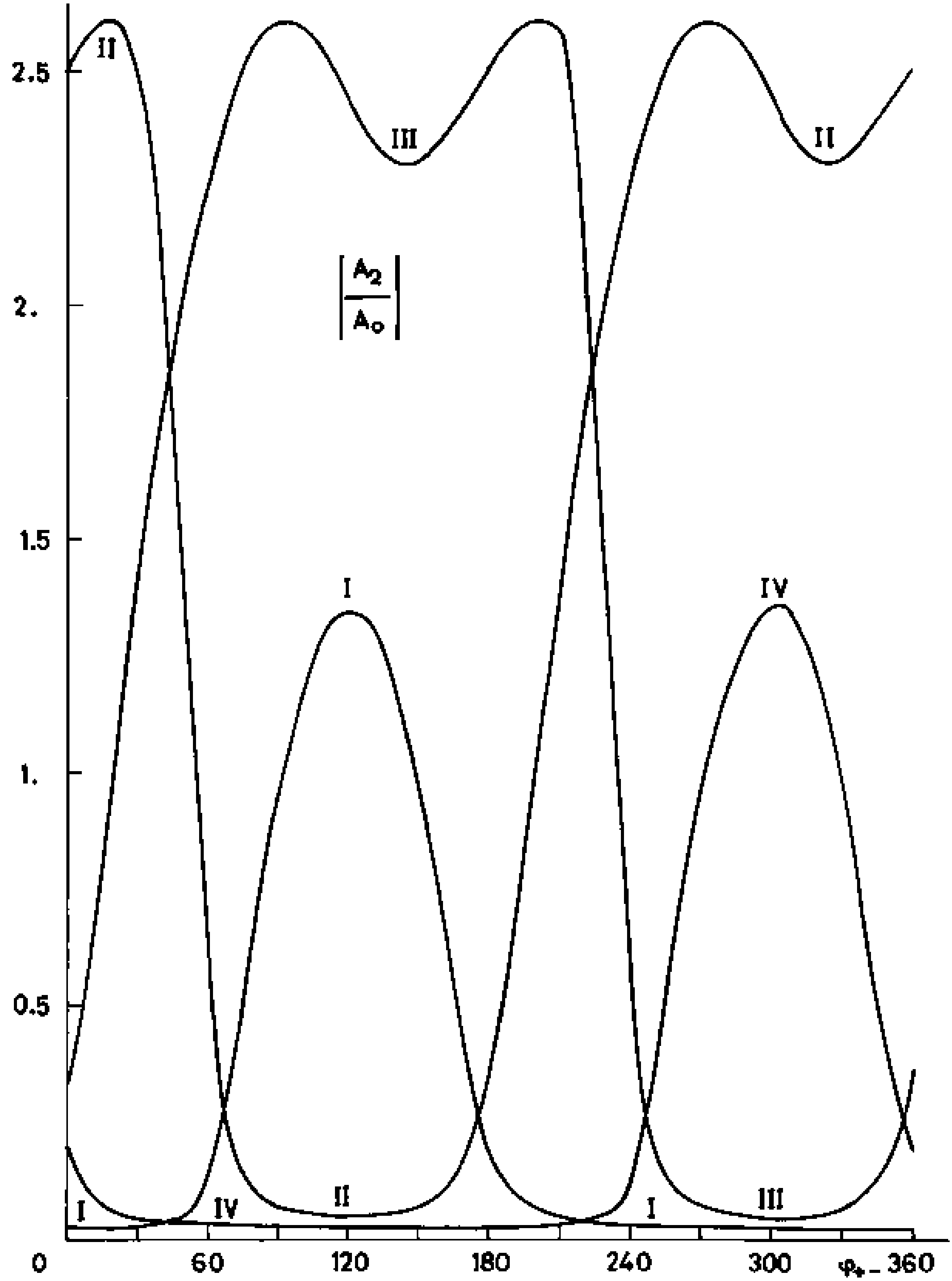


Fig. 16f $\left|\frac{A_2}{A_0}\right|$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right|$ = 2.45

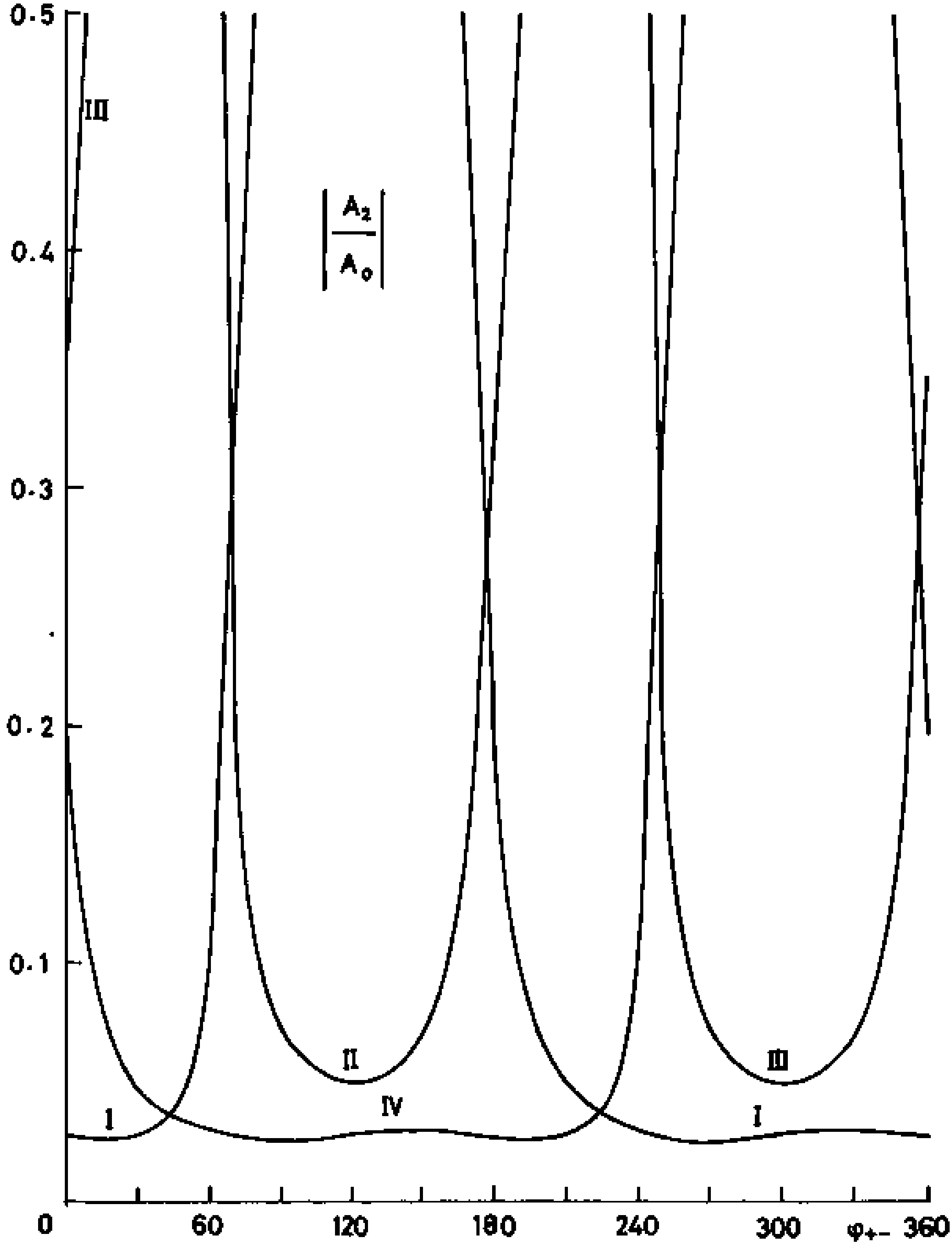


Fig. 16g : $\left|\frac{A_2}{A_0}\right|$ for the small $|s|$ solution.

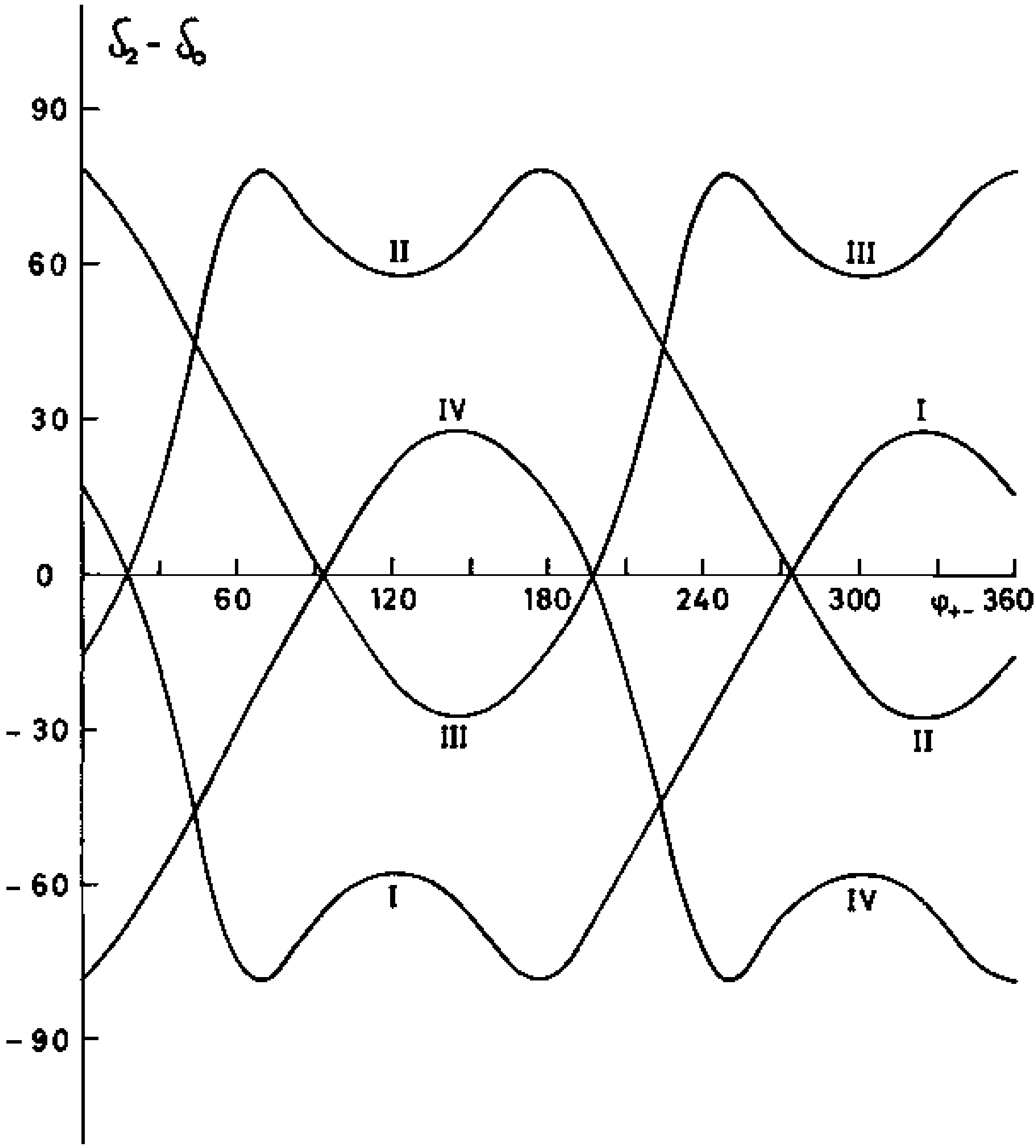


Fig. 16h : $\delta_2 - \delta_0$ for R = 0.447; $\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 2.45$

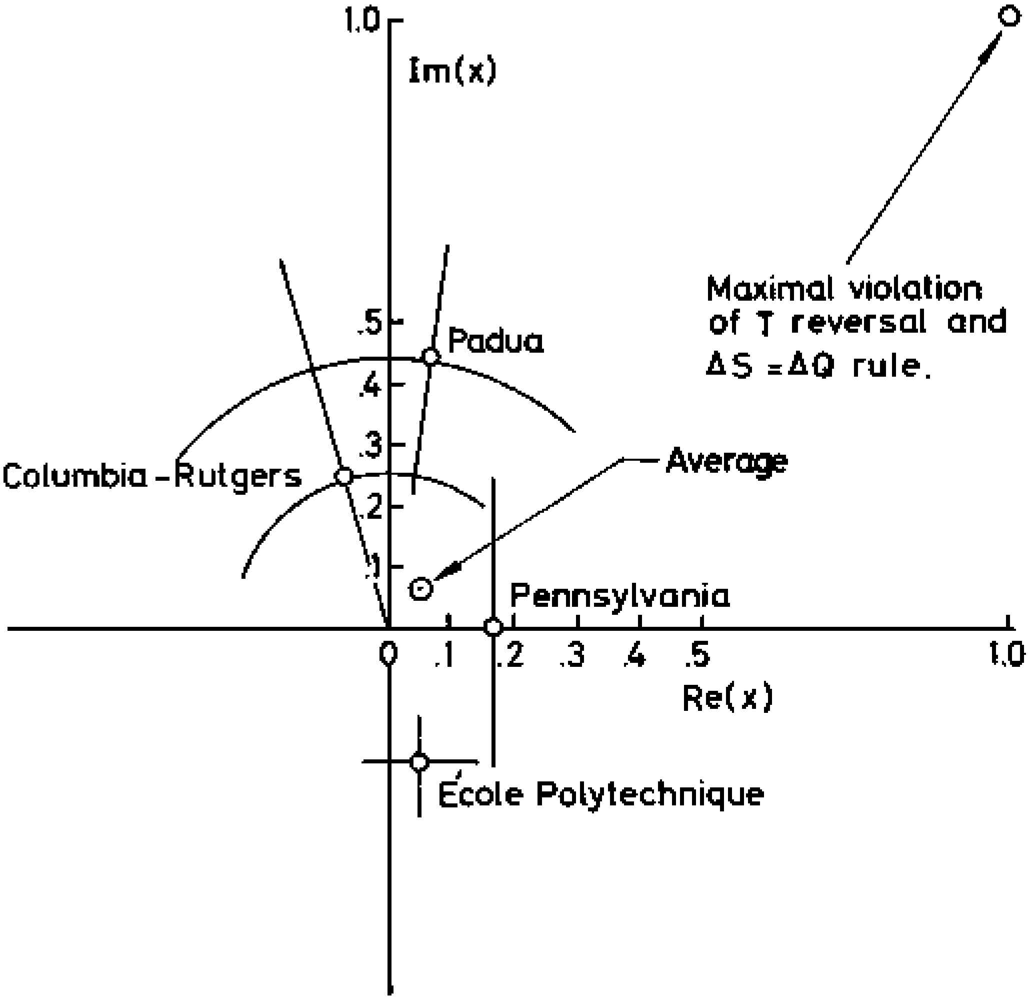


Fig. 17 : The measurements of the parameter expressing the violation of the $\Delta Q = \Delta S$ rule.