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TASK 2

TIMELIKE MOMENTA

IN QUANTUM ELECTRODYNAMICS*

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In this note we discuss the possibility of studying the quantum electrodynamics of timelike photon propagators in muon or electron pair production by incident high energy μ or e beams from presently available[1] proton or electron accelerators.

The lowest order Feynman diagrams for these trident[2] processes are shown in Figure 1. The virtual photon with momentum t is spacelike in diagrams 1(c) and 1(d) but timelike in diagrams 1(a) and 1(b). The detailed calculation of the Bethe-Heitler graphs and complete numerical results which allow for the form factors and recoil of the nucleus, polarized leptons, and exchange terms for identical leptons will be reported elsewhere.[3] For simplicity, we discuss here the characteristic features of the triple coincidence cross section

$$d^5\sigma/d\Omega_1 d\Omega_2 d\Omega_3 dE_1 dE_2 \;,$$

assuming the nucleus acts once as a static potential,[4] the Compton graph can be ignored,[5] and all spins are summed over. For convenience, we consider a muon producing an electron pair,

$$\mu + Z \rightarrow \mu + Z + e^+ + e^-, \tag{1}$$

although most of our results will hold for all four variations.[6]

In this paper we try to determine configurations which will insure that the diagrams 1(a) and 1(b) give the dominant contribution to the cross section and at the same time give a production rate which is large enough so that experiments with present machines may be performed. We thus try to obtain maximum sensitivity to possible modifications of quantum electrodynamics for the timelike momentum region above 100 MeV/c.

We have found it convenient to first select configurations whereby (A), the electron-positron pair is detected symmetrically with respect to the muon scattering plane and (B), the total momentum of the three final leptons is in the incident μ direction. Since the nucleus is assumed to be a static potential, the total energy of the three leptons is equal to energy of the incident particle. Requirement A implies that the interference contribution between the spacelike diagrams 1(c) and 1(d) and the timelike diagrams 1(a) and 1(b) vanishes since it is antisymmetric under the interchange of the electron and positron momenta, whereas the cross section is invariant for mirror symmetry.[7] Requirement B insures that the momentum transfer to the nucleus is minimized for fixed lepton energies and polar angles:

$$-q^2 = -(P - P_1 - P_2 - P_3)^2 = q_z^2 \cong (E_1\theta_1^2/2 + E_2\theta_2^2)^2 ,$$

(assuming small angles and zero lepton mass). With an incident 10 BeV/c μ beam and $\theta_1 \cong \theta_2 \cong .1$ rad, we have $q_z \cong 50$ MeV/c. Thus even in a high energy experiment, the nucleus acts as a Coulomb source, requiring only small unambiguous recoil and form factor corrections, and the cross section is nearly proportional to Z^2. Feasible rates for a muon beam can thus be obtained from a high Z target if we choose events with $|\vec{q}| \leq 80$ MeV/c. For simplicity, we will give results for Z = 10, where the Born approximation should still be reliable.[4]

The triple coincidence cross section for the Bethe-Heitler diagrams can be written as[8]

$$\frac{d^5\sigma}{dE_1 dE_2 (d\Omega)^3} = \frac{P_1 P_2 P_3}{P} \cdot \frac{Z^2\alpha^4}{2\pi^4} \cdot \left(m_\mu m_e\right)^2 \sum_{\text{spin}} \left|M_s + M_t\right|^2 \cdot \frac{1}{q^4} \tag{2}$$

where

$$M_t = M_a + M_b = \bar{u}\,(P_1) J_t^\mu u(P)\bar{u}(P_2)\gamma_\mu v(P_3) \tag{3}$$

$$M_s = M_c + M_d = \bar{u}(P_1)\gamma_\mu u(P)\bar{u}(P_2) J_s^\mu v(P_3) \tag{4}$$

and $\bar{u}(P_1)J_t^\mu u(P)$, $\bar{u}(P_2)J_s^\mu v(P_3)$ are the time-like and space-like conserved currents:

$$\begin{aligned} J_t^\mu &= (-\gamma^\mu \not{q}\gamma_o\omega_3^{-1} + \gamma_o\not{q}\gamma^\mu\omega_4^{-1}) + 2\gamma^\mu(E\omega_3^{-1} + E_1\omega_4^{-1}) \\ J_s^\mu &= (\gamma_o\not{q}\gamma^\mu\omega_1^{-1} + \gamma^\mu\not{q}\gamma_o\omega_2^{-1}) + 2\gamma^\mu(E_2\omega_1^{-1} + E_3\omega_2^{-1}) \end{aligned} \tag{5}$$

with

$$\bar{u}(P_1)J_t^\mu u(P)\cdot(P_2 + P_3)_\mu = 0; \quad \bar{u}(P_2)J_s^\mu v(P_3)\cdot(P_1 - P)_\mu = 0. \tag{6}$$

The denominations are

$$\begin{aligned} \omega_1 &= (P_1 - P)^2(q^2 + 2P_2\cdot q) \\ \omega_2 &= -(P_1 - P)^2(q^2 + 2P_3\cdot q) \\ \omega_3 &= (P_2 + P_3)^2(q^2 - 2P\cdot q) \\ \omega_4 &= (P_2 + P_3)^2(q^2 + 2P_1\cdot q) \end{aligned} \tag{7}$$

For many purposes the above expression is the most practical form for the numerical calculation[9] of the cross section since in this form large cancellations of gauge-variant quantities do not occur.

However, if we apply the requirements A and B, the cross section is obtained in a relatively simple analytical form:

$$D_\sigma(T)+D_\sigma(S) \equiv \frac{d^5\sigma}{dE_1 dE_2 (d\phi)^3 (d\theta)^3} = \frac{Z^2\alpha^4}{2\pi^4} \frac{P_1 P_2 P_3}{P} \left[S+T\right] \frac{\sin\theta_1 \sin^2\theta_2}{q^4} \tag{8}$$

where

$$S \equiv \left(m_e m_\mu\right)^2 \sum_{\text{spin}} \left|M_s\right|^2 = -4(P_1-P)^2(P_{2x}^2 + P_{2y}^2) q_z^2 \omega_1^{-2} \tag{9}$$

$$T \equiv \left(m_e m_\mu\right)^2 \sum_{\text{spin}} \left|M_t\right|^2 = \tfrac{1}{2}(P_2+P_3)^2\left[(\alpha^2+\beta^2)(2P_1\cdot P)+4\alpha\beta\delta - 4m_\mu^2\beta^2 + 4m_\mu^2 q_z^2(\omega_3^{-2}+\omega_4^{-2})\right]$$
$$+ \tfrac{1}{2}(P_2-P_3)^2\left[(\alpha^2+\beta^2)(P_1\cdot P) + 2\alpha\beta\delta + m_\mu^2(\alpha^2-\beta^2)\right], \tag{10}$$

with

$$\begin{aligned} \alpha &= q_z(\omega_3^{-1}+\omega_4^{-1}) \\ \beta &= -2(E\omega_3^{-1}+E_1\omega_4^{-1}) \\ \delta &= E_1 P_z - EP_{1z}\,. \end{aligned} \tag{11}$$

If we ignore the lepton mass in the numerators of (9) and (10),

$$S = 8(P_{2x}^2 + P_{2y}^2)(P_1\cdot P) q_z^2 \omega_1^{-2} \tag{12}$$

$$T = (P_2\cdot P_3)(P_1\cdot P)(\alpha+\beta)^2\,. \tag{13}$$

The behavior of S and T can best be understood by considering the incident energy E, the pair angle θ_2, and the desired momentum transfer squared of the timelike photon, $v = (P_2 + P_3)^2$, to be parameters, letting the final electron energy, muon energy, or muon angle to be an adjustable variable to insure a large timelike to spacelike ratio T/S and large cross section. If we consider only small angles, then (12) and (13) give

$$\frac{d^8\sigma}{dE_1 dE_2 (d\phi)^3 (d\theta)^3} = \frac{Z^2\alpha^4}{2\pi^4} \frac{16(1+S/T)}{E^4\theta_2^5} \frac{\lambda^{3/2}(2+r)^2}{\bar{v}(2+\lambda r)^6} \tag{14}$$

with

$$S = \frac{1}{E_2^2 \ \lambda r(r+2)} \ , \ \frac{T}{S} = \frac{\lambda^2}{\bar{v}(2+\lambda r)^2} \tag{15}$$

where the two variables

$$\lambda \equiv \theta_1^2/\theta_2^2 \quad \text{and} \quad r \equiv E_1/E_2 \tag{16}$$

are constrained by

$$\bar{v} \equiv v/E^2\theta_2^2 = \frac{4-\lambda r^2}{(r+2)^2} \ . \tag{17}$$

It is readily seen from equations (14-17) that the optimal condition for large T/S is given by large θ_1 (and hence small E_1 to satisfy condition B). In Figure 2a we have shown $D_\sigma(T)$ (as calculated from equations 8 and 10) and R, the fraction of the total trident cross section due to the square of the timelike graphs, as functions of E_2 with θ_2 and v as parameters. Although the ratio R decreases slowly with decreasing E_2, the partial cross section $D_\sigma(T)$ increases rapidly.

It must be emphasized that the cross sections presented so far, although remarkably simple, are only valid when the symmetry conditions A and B are imposed. The requirement that $\vec{q}$ is in the incident direction in fact minimizes the rate. In Figure 2b it is shown that if the direction of $\vec{q}$ is perturbed by varying θ_1 by 1 mrad, the cross section increases by a factor of 200. This feature holds quite generally at every point of Figure 2a: the maxima obtained by slighly relaxing condition B are roughly proportional to the minima.

The same feature of the cross section is seen in Figures 2c, d, and e where conditions A and B are relaxed as the momentum of the positron is changed.

The trident cross section is suppressed for $\vec{q}$ in the incident direction because of a selection rule against the transverse polarization contributions of the virtual photon. If $\vec{q}$ is in the Z direction the matrix element for virtual transverse photons in the forward direction will vanish by angular momentum conservation since the muon helicity is conserved in a series of vector interactions at high energies. The pairs produced by this transverse photon are further suppressed in the forward direction since high energy vector interactions require the electron and positron to have opposite helicities.[11] The timelike and longitudinal polarization contributions are small since the photon is relatively close to the mass shell.

Requirement A does not imply a dip. If one destroys mirror symmetry while keeping $\vec{q}$ in the incident direction, the cross section still is slowly varying and stays close to the minimum.

We have also shown the variations of the cross section for θ_1 fixed at 5 and 10 mrad above the symmetry angle θ_1^*, thus giving configurations where the cross section is large but slowly varying. The cross section is nominally 10^{-32} $cm^2/(MeV)^2(ster)^3$ at $P = 10$ BeV/c, $Z = 10$, $v = (100 \text{ MeV/c})^2$ for ranges $\Delta\theta_1 \cong \Delta\theta_2 \cong \Delta\theta_3 \cong 20$ mrad, $\Delta\phi_2 \cong \Delta\phi_3 \cong 40^{\circ}$, $\Delta E_1 \cong \Delta E_2 \cong \Delta E_3 \cong 1$ BeV with the ratio R above 0.8. Therefore favorable rates for experiments sensitive to the timelike region are possible.

In summary, we note that a triple coincidence measurement of reaction (I) in the kinematic region described in this paper enables one to study the quantum electrodynamics of the photon propagator and the vertex function in the timelike region above 100 MeV/c. We further note that an important test of μ-e universality in the timelike region can also be easily performed by measuring three muons in the final state, taking into account mass differences and statistics.[3]

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2. J.D. Bjorken and S.D. Drell, Phys. Rev. 114, 1368 (1959), have discussed pair production by high energy electrons as a possible test of the electron or muon vertex and the photon propagator in the spacelike region. F. Ternovski, JETP 565 (1960); M. Chen, Phys. Rev. 127, 1844 (1962); E. Johnson, Phys. Rev. 140, 1005 (1965), and references therein considered various integrated trident cross sections in various situations.

3. S.J. Brodsky and S.C.C. Ting, to be published.

4. Corrections to Born approximation can be readily estimated from the work of L.C. Maximon and H.A. Bethe, Phys. Rev. 87, 156 (1952) on photopair production and bremsstrahlung. The corrections are in general of order $(Z\alpha)^2$ but become negligible if the momentum transfer to the nucleus is small compared to the lepton mass or the lepton momenta transverse to the photon direction. Radiative corrections for the trident process are expected to be of order 10%.

5. In contrast with the photoproduction of symmetric pairs, the Compton graph, Figure 1e, does interfere with the timelike Bethe-Heitler graphs but still the contribution is estimated to be less than 2%. The smallness of this interference term (as well

as higher order Born terms and radiative corrections involving three electromagnetic interactions of the lepton current) can be readily checked experimentally when the timelike graphs dominate by using both polarities of the incident beam since the interference term with the timelike graphs is proportional to the charge of the incident beam.

Following A. Krass, Phys. Rev. 138, B1268 (1965) one can suppose that the size of the Compton term is characterized by a peripheral graph, whereby the virtual photons interact with the nucleus via one pion exchange, and the leptonic pair come from the e.m. decay of a vector meson. However, in comparison with the Bethe-Heither rate, the peripheral contribution is reduced by the factor

$$\vec{q}^{\,2}/M_p^2 \lesssim 0.01$$

(from the pion-nucleon interaction) and by roughly a factor of Z. Furthermore, whereas the Bethe-Heitler trident cross section increases rapidly for small angles, the contributions from Compton graphs are comparatively angle-independent.

6. If an electron beam is used, the experimental analysis will be complicated by pairs produced by bremsstrahlung.

7. The proof of antisymmetry is similar to the proof of Furry's theorem since the trace required for this interference term also occurs in the matrix element evaluation of a closed electron loop with three e.m. interactions. Since spins are

not measured, the symmetry condition A implies the cross section is invariant under the interchange of the electron and positron momenta. The interference term thus gives no net contribution even for finite acceptance, if the events are collected symmetrically. A similar situation is described by J.D.Bjorken, S. D. Drell and S. C. Frautschi, Phys. Rev. **112**, 1409 (1958) concerning the interference of Compton and Bethe-Heitler graphs for photopair production.

8. The notation is given in Bjorken and Drell, *Relativistic Quantum Fields*, McGraw Hill, 1965.
9. We find that for computer calculations it is more efficient to multiply the Dirac matrices directly instead of using the usual reduction formulae.
10. This still gives the cross section within 10%.
11. J. D. Bjorken and S. D. Drell, ref. 2.

FIGURE CAPTIONS

Figure 1. Lowest order Feynman diagrams for the trident process of Equation (1). Figures 1(a) to 1(d) give the Bethe-Heitler (Born approximation) contribution. Figure 1(e) represents the general Compton contribution.

Figure 2. Properties of the cross section of process (1) in the timelike region for incident momentum of 10 BeV/c and nuclear charge, Z = 10.

(a): Behavior of the partial cross section $D_\sigma(T)$ due to timelike graphs 1(a) and 1(b) and the ratio R of this contribution to the total trident cross section under the restrictive symmetry requirements (A) and (B) of the text.

(b): Behavior of the cross section and the ratio R when requirement (B) is relaxed; i.e.: $\vec{q}$ not restricted to the incident direction ($\theta_1 \neq \theta_1^*$). The curves in 2(a) and (b) are shown for virtual photon timelike momenta squared $v = (100\ \mathrm{MeV/c})^2$ and $v = (200\ \mathrm{MeV/c})^2$.

(c),(d),(e): Behavior of the cross section for $v = (100\ \mathrm{MeV/c})^2$ when requirements (A) and (B) are both relaxed by varying the positron coordinates from the mirror symmetrical arrangement.

The three curves plotted in each figure correspond to the three fixed values of θ_1. For $\theta_1 = \theta_1^*$ the minima in these curves correspond to the cross section with exact symmetry conditions A and B. All points shown in Figure 2 have $|\vec{q}| < 80$ MeV/c.

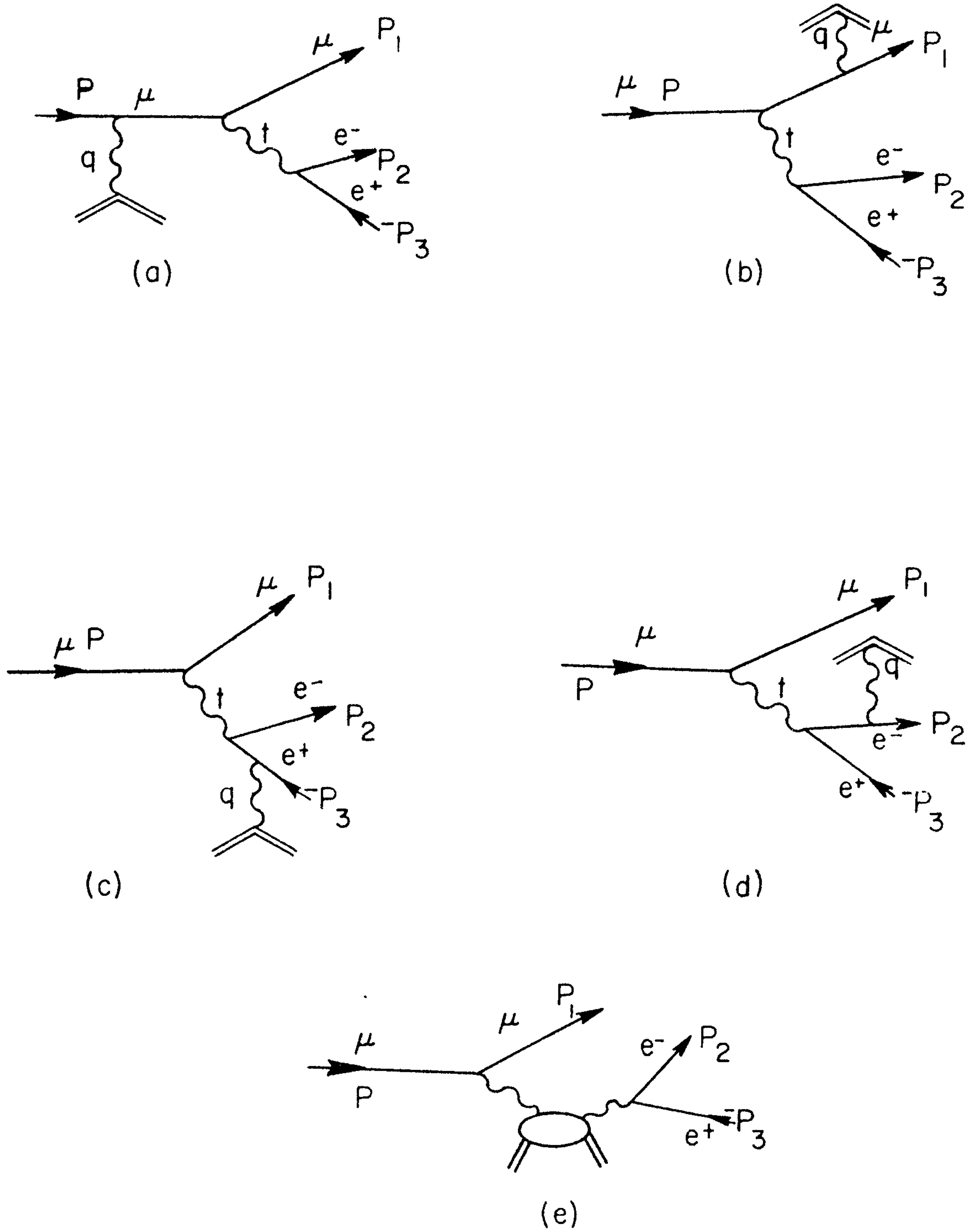
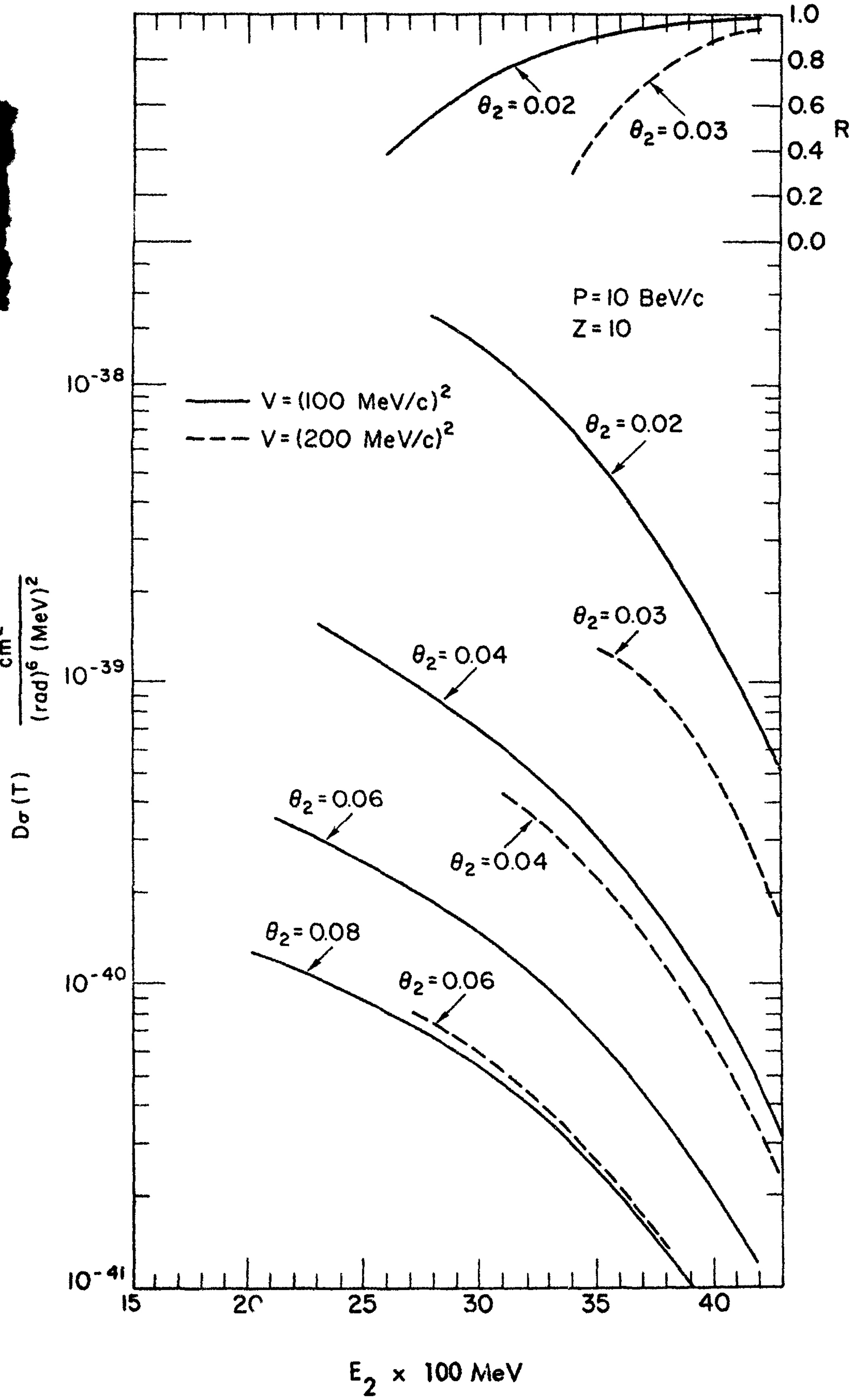


Figure 1



1.0
0.8
0.6
0.4
0.2
0.0
R
$\theta_2 = 0.02$
$\theta_2 = 0.03$
P = 10 BeV/c
Z = 10
$V = (100\ MeV/c)^2$
$V = (200\ MeV/c)^2$
$\theta_2 = 0.02$
$\theta_2 = 0.03$
$\theta_2 = 0.04$
$\theta_2 = 0.06$
$\theta_2 = 0.04$
$\theta_2 = 0.08$
$\theta_2 = 0.06$
10^{-38}
10^{-39}
10^{-40}
10^{-41}
$D\sigma(T)\ \frac{cm^2}{(rad)^6\ (MeV)^2}$
15
20
25
30
35
40
$E_2 \times 100$ MeV

Figure 2a

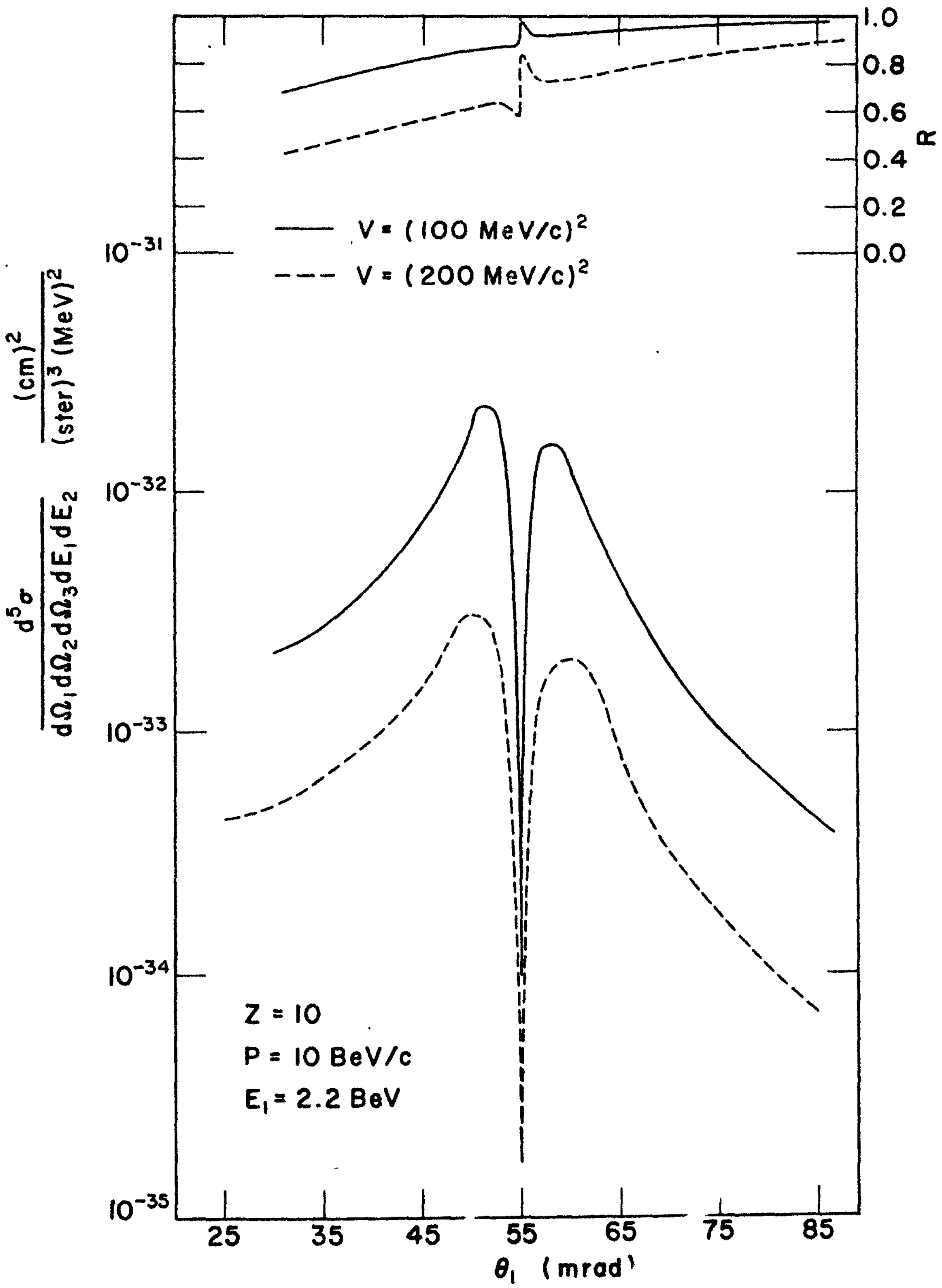

1.0
0.8
0.6
0.4
0.2
0.0
R
V = (100 MeV/c)^2
V = (200 MeV/c)^2
10^{-31}
10^{-32}
10^{-33}
10^{-34}
10^{-35}
$\frac{d^5\sigma}{d\Omega_1 d\Omega_2 d\Omega_3 dE_1 dE_2}$ $\frac{(cm)^2}{(ster)^3 (MeV)^2}$
Z = 10
P = 10 BeV/c
E_1 = 2.2 BeV
25
35
45
55
65
75
85
θ_1 (mrad)

Figure 2b