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FOCUSING IN LINEAR ACCELERATORS

Edwin M. McMillan

August 24, 1950

Berkeley, California

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FOCUSING IN LINEAR ACCELERATORS

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It is well known that phase stability and first-order focusing are incompatible in a simple linear accelerator without foils or grids in the path of the particles.(1) However, various attempts have been made to design more complicated field shapes and time variations that will circumvent this limitation. A general proof that the limitation exists in all cases was made by the author in 1945 but was not published at that time. This proof is given below.

A particle of charge e moves parallel to the z-axis with velocity v and is acted on by fields $\underset{\sim}{E}$ and $\underset{\sim}{H}$, which are periodic in time and nearly periodic along the z-axis. The time period is T and the corresponding repeat length L is equal to vT. Changes in velocity and direction during one repeat length will be neglected; this is what we mean by the term "first order." The time corresponding to the position z is then given by $t_o + z/v$, where t_o is an arbitrary starting time.

The focusing effect depends on the x and y force components F_x and F_y, whose mean values are given by:

$$\left.\begin{aligned} \bar{F}_x &= (e/vT)\int_o^L \left[E_x - (v/c)H_y\right] dz, \\ \bar{F}_y &= (e/vT)\int_o^L \left[E_y + (v/c)H_x\right] dz, \end{aligned}\right\} \qquad (1)$$

the field components being evaluated at the position and corresponding time

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(1) See for example J. C. Slater, Rev. Mod. Phys. 20, 473 (1948)

of the moving particle. If the line of motion is chosen so that both these forces vanish, the restoring force constants toward this line are:

$$\left.\begin{aligned} k_x &= - d\bar{F}_x/dx, \\ k_y &= - d\bar{F}_y/dy \end{aligned}\right\} \qquad (2)$$

From (1) and (2) we get:

$$k_x + k_y = (e/vT)\int_0^L \Big[- (\partial E_x/\partial x + \partial E_y/\partial y) + (v/c)(\partial H_y/\partial x - \partial H_x/\partial y)\Big]\, dz. \qquad (3)$$

With the aid of the Maxwell equations div $\underset{\sim}{E} = 0$, curl $\underset{\sim}{H} = (1/c)\partial \underset{\sim}{E}/\partial t$, this becomes:

$$k_x + k_y = (e/vT) \int_0^L \left[\partial E_z/\partial z + (v/c^2)\partial E_z/\partial t\right] dz \qquad (4)$$

Finally, since $\partial/\partial z = d/dz - (1/v)\partial/\partial t$, we can write:

$$k_x + k_y = (e/vT) \left[E_z\right]_0^L - (e/v^2T)(1 - v^2/c^2) \int_0^L (\partial E_z/\partial t)\, dz. \qquad (5)$$

Next consider the energy gain ΔW during one period, given by:

$$\Delta W = e \int_0^L E_z\, dz. \qquad (6)$$

This depends on the starting time t_o, and its rate of change with t_o is:

$$d(\Delta W)/dt_o = e \int_0^L (\partial E_z/\partial t)\, dz. \qquad (7)$$

Combining (5) and (7), we obtain the relation:

$$k_x + k_y = (e/vT) \left[E_z\right]_0^L - (1/v^2T)(1 - v^2/c^2)d(\Delta W)/dt_o \qquad (8)$$

Now, in order to have phase stability, $d(\Delta W)/dt_o$ must be positive, since an increasing t_o means that the particle is too slow and therefore has an energy deficiency. In order to have focusing, both k_x and k_y must be positive; this is clearly incompatible with the above requirement unless some help is obtained from the first term on the right of (8). This term depends on the difference in E_z experienced by the particle on leaving and entering the repeat length, and is obviously zero in the absence of foils or grids. With a foil placed so that the field strength on one side is zero, it is determined by the field strength on the other side at the instant the particle enters the foil. (The repeat length must be taken as ending at the foil, since the field equations used are not valid inside a conductor.) In the case of a grid, the effect is essentially the same, even though there is no field discontinuity through a grid opening; the focusing force can then be considered as arising from the charge lying between the equilibrium path and the displaced path.

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