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SOME NOTES ON WIDEBAND FEEDBACK AMPLIFIERS

by

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SOME NOTES ON WIDEBAND FEEDBACK AMPLIFIERS

Introduction

The extension of the passband of wideband amplifiers is a highly important problem to the designer of electronic circuits. Throughout the electronics industry and in many research programs in physics and allied fields where extensive use is made of video amplifiers, the foremost requirement is a passband of maximum width. This is necessary if it is desired to achieve a more faithful reproduction of transient wave forms, a better time resolution in physical measurements, or perhaps just a wider band gain-frequency response to sine wave signals. The art of electronics is continually faced with this omnipresent amplifier problem. In particular, the instrumentation techniques of nuclear physics require amplifiers with short rise times, a high degree of gain stability, and a linear response to high signal levels.

While the distributed amplifier[1] may solve the problems of those seeking only a wide passband, the requirements of stability and linearity necessitate using feedback circuits. This paper considers feedback amplifiers from the standpoint of high-frequency performance. The circuit conditions for optimum steady-state (sinusoidal) and transient response are derived and practical circuits (both interstage and output) are presented which fulfill these conditions.

In general, the results obtained may be applied to the low-frequency end.

General

The fundamental limitation in feedback amplifiers arises from the over-all phase shift in the amplifier and in some cases, the feedback circuit as well. As the shift in phase approaches 180 degrees on either side of the mid-band, the feedback becomes positive, resulting in regeneration and possible oscillation. The relationships between attenuation and phase shift necessary for amplifier stability have been formulated and published[2].

It is the phase shift and its attendant difficulties that make feedback over more than three stages impractical for video amplifiers; and while three-tube feedback[3] is feasible on theoretical grounds, it is difficult to design practical circuits which use the feedback to the best advantage. Fig. 1 shows two of the most satisfactory three-tube feedback loops which have been used for video purposes.

The circuit in Fig. 1a accomplishes the feedback through the common plate impedance of T_1. The one serious shortcoming of this circuit is that the gain of T_1 is not stabilized. This tube merely serves to drive the feedback loop encompassing T_2, T_3, and T_4. For this reason the circuit is inadequate when used in those applications where the virtues of feedback; e.g., stability, linearity, are required.

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The circuit shown in Fig. 1b has been used extensively in recent years. It is completely stabilized; and, since there are a variety of outputs available, it is a highly useful circuit. Examination of this loop discloses that an output from point A would be stabilized with current feedback; from point B, with voltage feedback; and from point C, with voltage feedback. Outputs either in-phase or 180 degrees out-of-phase with the input may be obtained from points B and A respectively. However, this circuit has relatively poor high-frequency response for the following reasons: (a) phase shift occurs over three tubes while gain is realized from only two; and (b) the feedback resistor in the cathode circuit of T_1 is usually of such a magnitude as to reduce considerably the unfedback gain of T_1, which decreases the over-all stability and frequency response.

Cognizance of the shortcomings in the above circuits has led to the investigation of other possibilities for feedback over three tubes. However, there appears to be no completely satisfactory solution to the problem of feeding a portion of the output voltage of a three-stage amplifier back to the grid circuit of the first tube over a wide band of frequencies.

Other feedback circuits that have been used for video purposes are shown in Fig. 2. The design of the circuit in Fig. 2a for a wide passband calls for relatively small values of R_1 and R_2. If this condition is fulfilled, it is impossible

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to operate tube T_2 at appropriate d.c. potentials. The solution to this problem is the insertion of a capacitor in series with the feedback resistor sufficiently large to offer a very low impedance to the lowest frequency it is desired to amplify. Unfortunately, because of the necessarily large physical size of this capacitor, it will possess a large stray capacitance to ground seriously affecting the high-frequency response of the loop.

There are many modifications of the circuit in Fig. 2b, all of which are considered unsatisfactory because feedback occurs only around T_2, and the gain of T_1 is left unstabilized.

Two circuits which utilize two-tube feedback to the greatest degree are shown in Figs. 3 and 4. The first uses current feedback; and the second, voltage feedback. The first inverts the input signal, and the second does not. The second circuit may be modified to function as an output stage or as a unity-gain amplifier with a very low output impedance. These two circuit units are adaptable to nearly every situation where wide-band feedback amplifiers are needed.

Strictly speaking, the circuit in Fig. 3 is a three-tube loop. However, from the standpoint of frequency response, it can be analyzed as a two-tube loop inasmuch as R_k is so small (generally less than 10 ohms) that the cathode circuit of T_3 is a frequency independent feedback device over the band in which we are interested.

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It can be shown by conventional analysis that the cathode current of T_3 is given approximately by

$$i_3 \cong \frac{gm_3 K_{12} e_s}{1 + gm_3 R_k K_{12}} \cong \frac{e_s}{R_k}$$

where

$gm_3 = gm$ of T_3

$K_{12} =$ gain of T_1 and T_2

and if T_3 is connected as a triode, the gain becomes simply R_L/R_k. The mid-band gain of the loop to the grid of T_3 is

$$K_{FB} = \frac{K_{12}}{1 + gm_3 R_k K_{12}} \quad (1)$$

From elementary feedback theory, the gain of any amplifier with feedback is

$$K_{FB} = \frac{K}{1 - \beta K} \quad (2)$$

where K is generally a complex expression. It can be seen by comparing equation (1) with equation (2) that the term, gm_3R_k, is equivalent to $-\beta$, the feedback factor. Hence in this circuit the amount of the feedback is a function of gm_3 and R_k.

If the output of this loop is taken from the anode of T_3, it is important that the tube be connected as a triode for the reasons discussed below. The same considerations apply to the circuit in Fig. 1b.

The total cathode current of T_3 is stabilized by the feedback network. Unfortunately, if the tube is operated as

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a pentode and the cathode current is divided between screen and plate, it does not necessarily follow that the plate current will always be a fixed proportion of the cathode current. For this to be true, it is necessary that

$$\frac{gm_p}{gm_s} = \text{a constant}$$

where

gm_p = gm of plate with respect to control grid

gm_s = gm of screen grid with respect to the control grid

for all bias levels and that the constant be the same for all tubes. A plot of gm_p/gm_s versus grid bias for a particular tube, type 6AK5, is given in Fig. 5. The curve indicates that it is possible to pick a range of operation where the ratio of the plate and screen transconductances is constant. However, it is doubtful if other tubes exhibit precisely the same constant. This condition is necessary if the gain is to be independent of individual variations in tube characteristics.

In the amplifier of Fig. 3 the use of the triode is not a particular hardship. The anode load R_L is made sufficiently small so that the gain of the stage is less than 2, resulting in a tolerable input capacitance to the tube.

The circuit in Fig. 4 represents an attempt to eliminate the shortcomings of the circuit in Fig. 2a. Placing the tubes in cascade makes it possible to operate them at appropriate supply potentials, and the configuration results in a net decrease in the number of components. The gain of the loop is given approximately by

$$K_L \cong \frac{R_a + R_b}{R_a} \tag{3}$$

In most designs R_a is of such a size as to have little effect on the unfedback gain of T_1.

Since the circuits in Figs. 3 and 4 appear to utilize two-tube feedback advantageously, an analysis leading to design criteria is worthwhile. The analysis will first be made with the intention of obtaining a flat gain frequency characteristic. Then will be derived the circuit conditions necessary to obtain a monotomic rise* of the output pulse when the input is subjected to a unit step-function.

Steady-state Analysis

The equivalent high-frequency circuit of a two-stage feedback loop is shown in Fig. 6. In the figure the circuit block labelled β is the feedback network which in this case will be a frequency independent attenuator. Mathematically β carries the usual notation; i.e., it is the ratio of the voltage fed back to the output voltage. T_1 and T_2 are pentodes with a ratio of transconductance to input and output capacity. The unfedback gain of an amplifier such as this at any frequency f is

$$K = \frac{gm^2 R_1 R_2}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)} \tag{4}$$

$$= \frac{Km}{1 - \frac{f^2}{f_1 f_2} + jf\left(\frac{1}{f_1} + \frac{1}{f_2}\right)}$$

* A rise with absolutely no overshoot.

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where

$$f_1 = \frac{1}{2\pi R_1 C_1} \quad f_2 = \frac{1}{2\pi R_2 C_2} \quad K_m = gm^2 R_1 R_2 = \text{mid-band gain}$$

and where both tubes are assumed to be operating with the same gm. It is convenient, mathematically, if we let

$$\frac{f_1}{f_2} = n \quad \text{and} \quad \frac{f}{f_1} = m$$

then

$$K = \frac{K_m}{1 - nm^2 + jm\,(1+n)} \qquad (4a)$$

Combining equations (2) and (4) we obtain

$$\left|K_{FB}\right| = \frac{K_m}{1 - \beta K_m}\left[\frac{1}{\sqrt{\left(1 - \frac{nm^2}{1-\beta K_m}\right)^2 + \left(\frac{m(1+n)}{1-\beta K_m}\right)^2}}\right] \qquad (5)$$

This equation is the rationalized expression for the gain of the feedback amplifier. If the quantity in the bracket of equation (5) is plotted as a function of m for various values of n, the set of curves given in Fig. 7 results. The curves indicate that as n is increased, the peaking becomes successively less pronounced and the point of maximum gain moves to a lower frequency. It follows that if this point of maximum gain is made to occur at zero frequency, there will be no peaking whatsoever.

To obtain the value of m at which the gain is a maximum, the quantity under the radical sign in the denominator of equation (5) is differentiated with respect to m and the

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result equated to zero. This manipulation results in

$$m = \pm\sqrt{\frac{1-\beta K_m}{n} - \frac{(1+n)^2}{2n^2}} \qquad (6)$$

If then, m is set equal to zero, the relationship between n and $(1-\beta K)$ is determined which will produce a flat response; viz:

$$1 - \beta K_m = \frac{(1+n)^2}{2n} \qquad (7)$$

When $\beta K >> 1$ and $n >> 1$ this becomes (neglecting the negative sign)

$$n = 2\beta K_m \qquad (8)$$

Further analysis shows that the upper half-power frequency f_o of an amplifier designed with $n = 2\beta K_m$ will be

$$f_o \cong \frac{f_1}{\sqrt{2}} \quad \text{where } n >> 1 \qquad (9)$$

These relationships make it possible to formulate certain design criteria for amplifier circuits such as are illustrated in Figs. 3 and 4.

If f_o is specified, R_1 may be computed from the formula below

$$R_1 = \frac{1}{2\pi f_1 C_1} = \frac{1}{2\sqrt{2}\pi f_o C_1} \qquad (10)$$

Since

$$2\beta K_m = n = \frac{R_2 C_2}{R_1 C_1}$$

then

$$R_2 = 2\beta K_m R_1 \left(\frac{C_1}{C_2}\right) \qquad (11)$$

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The unfedback mid-band gain will be

$$K_m = gm^2 R_1 R_2 = gm^2 R_1^2 \, 2\beta K_m \left(\frac{C_1}{C_2}\right)$$

Therefore

$$\beta = \frac{1}{2(gmR_1)^2} \left(\frac{C_2}{C_1}\right) \qquad (12)$$

As an example of design, it is desired that an amplifier of the type shown in Fig. 3 have an upper half-power frequency of 15 megacycles and a stability factor βK of 20.

Using a type 6AK5 tube, the following constants may be obtained:

C_1 = 11 μμf

C_2 = 16 μμf (greater than C_1 since T_3 is connected as a triode)

gm = 5 ma/v

From equation (10)

$$R_1 = \frac{1}{2\sqrt{2} \times 3.14 \times 15 \times 10^6 \times 11 \times 10^{-12}} \cong 750 \text{ ohms}$$

Therefore

$$R_2 = 2 \times 20 \times 750 \times \left(\frac{11}{15}\right) \cong 20K$$

and

$$\beta = 0.05$$

In this circuit $\beta = gm_3 R_k$. Therefore if gm_3 equals 6 ma./volt, then

$$R_k = \frac{0.05}{6 \times 10^{-3}} \cong 9 \text{ ohms}$$

Any mid-band gain may be obtained by adjustment of R_L. However, the limitations imposed by the input capacitance

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of T_3 and by the frequency response of the anode circuit of T_3 necessitate the use of a load resistor less than 500 ohms. This latter qualification is, of course, dependent upon the amount of capacitive loading that there is on the plate. In this particular design a 270 ohm load is a good compromise and will give a gain of 30.

A circuit using the constants derived above is presented in Fig. 8, and its measured gain-frequency characteristic is given in Fig. 9. It is seen that the measured response has an upper half-power frequency of 15.5 megacycles, closely approximating the 15 megacycles for which the amplifier was designed.

The design for the circuit of Fig. 4 is somewhat different. In this case, the value established for β determines the ratio of $\frac{R_a}{R_a + R_b}$ and the mid-band gain of the amplifier will be simply the reciprocal of β (where $\beta K >> 1$).

In this case the gain obtainable is

$$K_L = \frac{1}{\beta} = 2(gmR_1)^2\left(\frac{C_1}{C_2}\right)$$

$$= \left(\frac{gm}{2\pi f_o C_1}\right)^2 \left(\frac{C_1}{C_2}\right) \qquad (13)$$

If f_o is the band-width of the amplifier, the gain-bandwidth product of the amplifier is

$$Kf_o = \left(\frac{gm}{2\pi C_1}\right)^2 \left(\frac{C_1}{C_2}\right)\left(\frac{1}{f_o}\right)$$

It is seen from this that the gain-bandwidth product is proportional to the square of the ratio $\left(\frac{gm}{C_1}\right)$. Hence it is

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exceedingly important that the tubes be operated with a maximum transconductance and that the stray wiring capacitance be minimized. For instance, type 6AK5 tubes usually are operated at a gm of 5 ma/volt. However, it is possible to obtain 6 ma/volt from them without exceeding the dissipation limits of the tube. If this change is made in the operating point, the gain-bandwidth product will be increased by $(6/5)^2$, or nearly 50 per cent. Furthermore, if the wiring capacitance is decreased by 1 $\mu\mu$f, or 10 per cent, the product of K_L and f_o will be increased by 20 per cent. Hence, the importance of the figure-of-merit is greatly emphasized in these circuits.

The Transient Analysis

The transient analysis of the system is most conveniently effected through the use of methods based on the Laplace Transformation[4,5]. This technique permits determination of the transient response directly from the steady-state analysis.

From equation (4) we obtain the following complex expression for gain

$$K = \frac{K_m}{(1 + j\omega T_1)\ (1 + j\omega T_2)}$$

where

$$T_1 = R_1 C_1 \qquad\qquad T_2 = R_2 C_2$$

Replacing $j\omega$ by the complex variable s, the system function is obtained.

$$K(s) = \frac{K_m}{(1 + sT_1)\ (1 + sT_2)} \tag{14}$$

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Combining equations (2) and (14) the gain with feedback becomes

$$K_{FB}(s) = \frac{K_m}{1 - \beta K_m} \left[\frac{1}{1+\left(\frac{T_1 + T_2}{1 - \beta K_m}\right) s + \left(\frac{T_1 T_2}{1 - \beta K_m}\right) s^2} \right] \quad (15)$$

In order that this amplifier may have a response corresponding to critical compensation; i.e., a transient response with minimum rise time but with no overshoot, the denominator of the bracketed expression must have two real and equal roots. Hence it is necessary that

$$\left(\frac{T_1 + T_2}{1 - \beta K_m}\right)^2 = \frac{4T_1 T_2}{1 - \beta K_m}$$

And therefore

$$\frac{T_1}{T_2} + 2 + \frac{T_2}{T_1} = 4(1 - \beta K_m)$$

Reverting to the notation used in the steady-state analysis

$$\frac{1}{n} + 2 + n = 4(1 - \beta K_m) \cong n$$

which is just twice the value for n required to produce a flat frequency-gain characteristic.

Substitution of $-\beta K = \frac{n}{4}$ into equation (5) and subsequent numerical solution results in the upper half-power frequency f_o being $f_{1/3}$. It can be shown[6] that an amplifier with an upper half-power frequency of f_1 will have a minimum rise time of

$$RT = \frac{1}{3f_o} = \frac{.33}{f_o}$$

Therefore, the rise time that can be expected from this amplifier is

$$RT_{(min)} = \frac{1}{f_1}$$

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As an illustration of design, if f_1 is made equal to 30 megacycles, the rise time will be 0.03 μseconds. Using the same tubes as before (6AK5), R_1 will be 500 ohms and R_2 is a function of the stability factor βK, as before. However, in this case,

$$R_1 C_1 \cong \frac{R_2 C_2}{4 \beta K}$$

Hence

$$R_2 = 4\beta K R_1 \left(\frac{C_1}{C_2}\right)$$

And

$$\beta = \frac{1}{4(gmR_1)^2}\left(\frac{C_2}{C_1}\right) = \left(\frac{3\pi f_o C_1}{gm}\right)^2 \left(\frac{C_2}{C_1}\right)$$

Therefore, the gain bandwidth product is

$$K_L f_o = \frac{f_o}{\beta} = \left(\frac{gm}{3\pi C_1}\right)^2 \left(\frac{C_2}{C_1}\right) \left(\frac{1}{f_o}\right)$$

Hence, it is seen that the $K_L f_o$ product is $\left(\frac{4}{9}\right)$ of the value obtained for the steady-state response.

It was seen in both steady-state and transient analysis that the high frequency performance is a function of f_1 which, in this analysis, is the upper half-power frequency of that stage, having the shortest time constant in the anode circuit. This being the case, considerable improvement can be obtained by compensating the f_1 stage with $L = R_1^2 C_{1/4}$. This increases f_1 by approximately 1.5, resulting in a like improvement in the response of the loop. If one stage is compensated, the time constant in the other stage must be adjusted accordingly.

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The Output Stage

In most amplifiers the bottleneck limiting the over-all frequency response is most likely to be the output stage where large voltage swings are required. The exact analysis of output stages is not convenient because the tubes cannot be considered as linear devices over the wide range of operating conditions. For this reason, the design is best done empirically.

Fig. 10 shows a modification of the circuit in Fig. 4 which has proved to be a successful output stage. The circuit will deliver 50 volt pulses with rise times of the order of 0.03 µseconds. The output impedance of the amplifier is given by the relation

$$Zo = \frac{1}{gm^2 \beta K_1}$$

where

$$gm_2 = gm \text{ of } T_2$$
$$K_1 = \text{gain of } T_1$$

As the frequency is increased, the gain of T_1 will be reduced. And it is evident from the above expression that as the gain of T_1 decreases, the output impedance increases at the same rate. Hence, the effectiveness of voltage feedback as an impedance transformer is reduced at the high frequencies where a low output impedance is most desirable. For this reason it is important to minimize the capacitive loading on the plate of T_2. If only positive pulses are to be amplified, this stage should be followed by a cathode follower. Critical compensation is achieved by the adjustment of the trimmer C and should be done under actual operating conditions with step signals applied to the input.

Unity-gain Amplifier

The circuit of Fig. 4 modified to function as a unity-gain amplifier is given in Fig. 11. Such a circuit is exceedingly useful as an output stage for pulsers, signal generators, etc., where an exceedingly low output impedance over a wide frequency range is desirable. The circuit is particularly adapted for driving coaxial cables and low impedance delay lines. The signal level that can be handled is determined by the permissible current change in T_2 and by the value of R_2. The permissible output voltage swing may be increased by making T_2 parallel combinations of similar tubes. R_2 may take the form of a terminated coaxial cable or delay line, as well as a resistance. The circuit has a mid-band output impedance of

$$Zo = \frac{1}{gm_2 K_1}$$

In the circuit shown this is approximately 1.5 ohms.

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5. H. S. Carslaw, J. C. Jaeger, Operational Methods in Applied Mathematics, (Oxford University Press, 1944)

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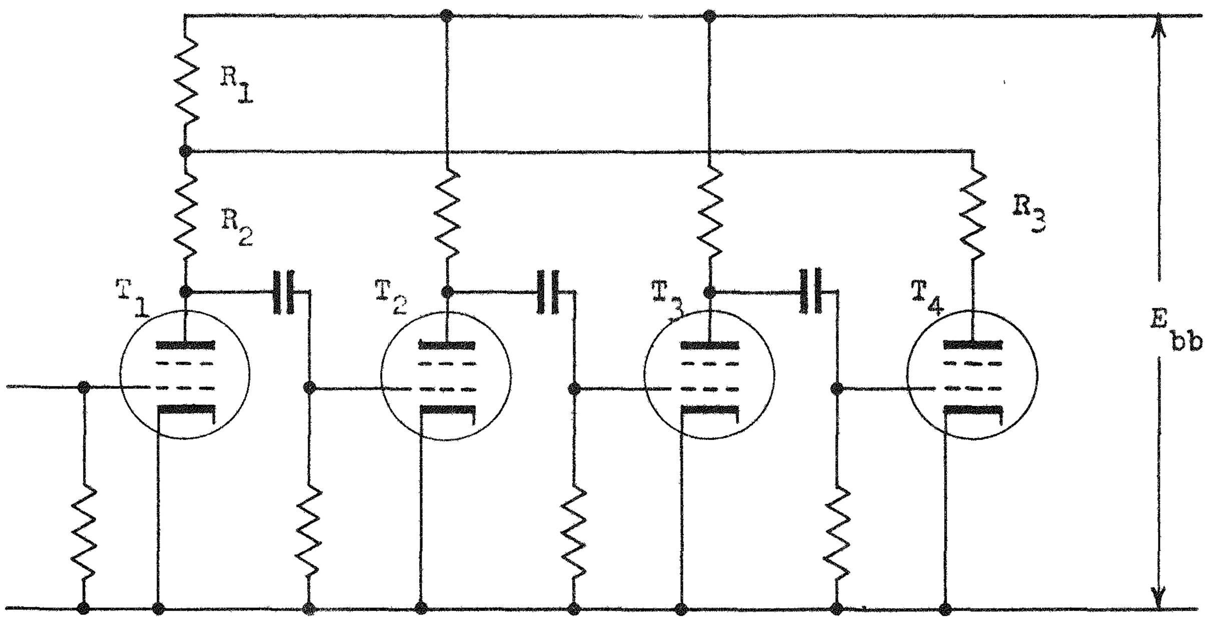

Fig. 1-a

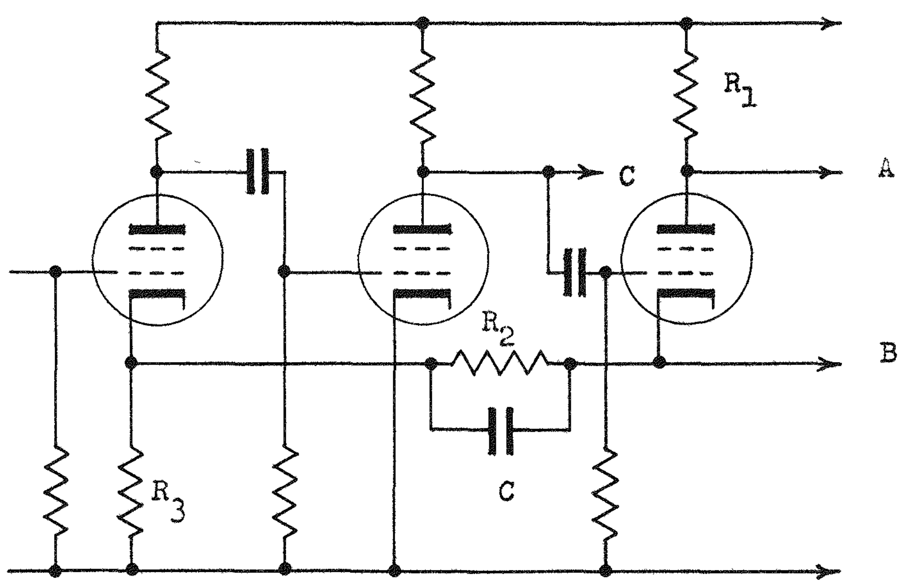

Fig. 1-b

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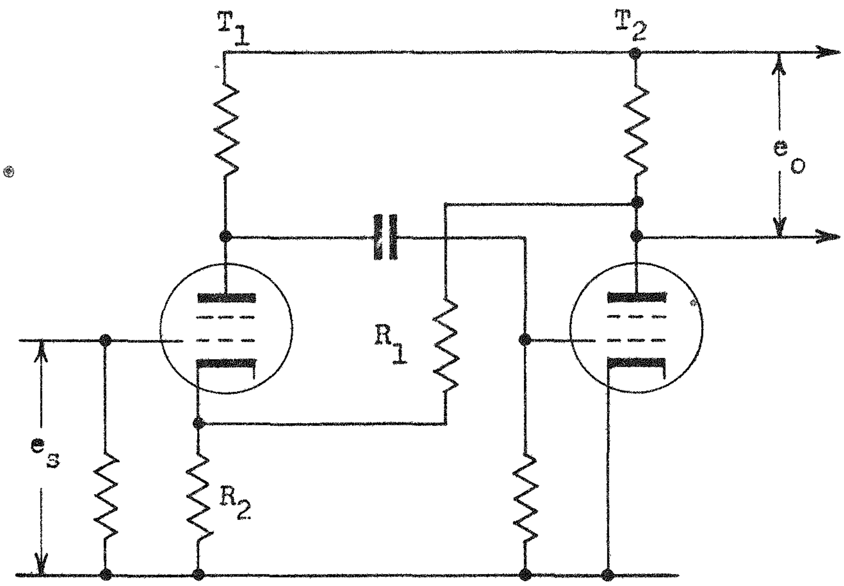

Fig. 2-a

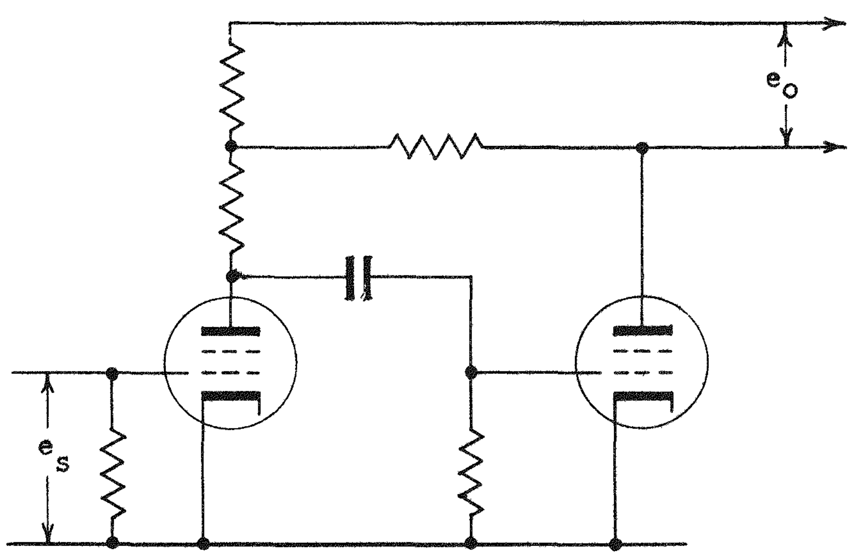

Fig. 2-b

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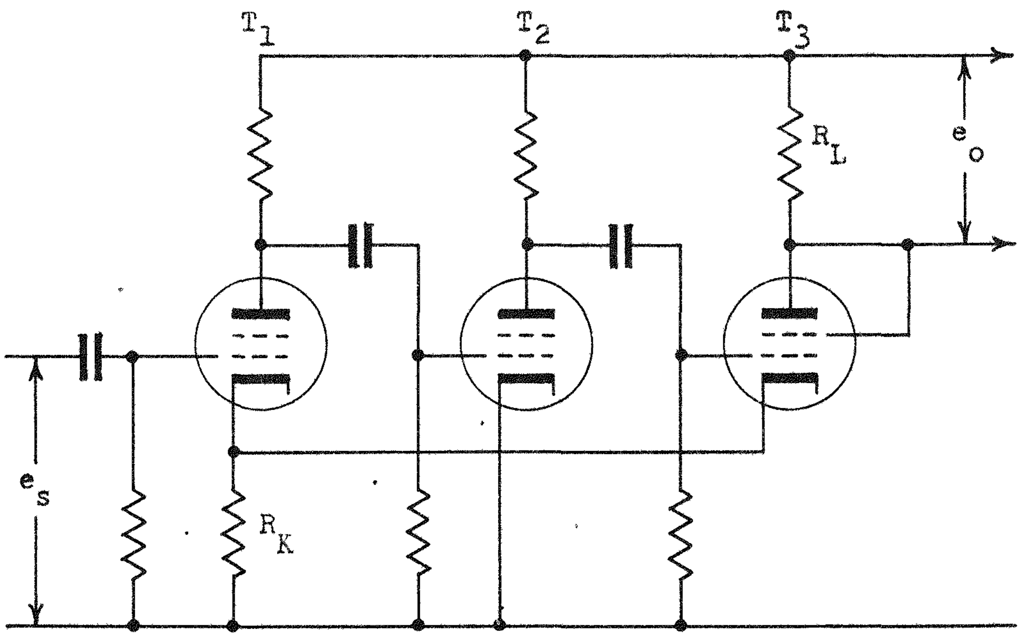

Fig. 3

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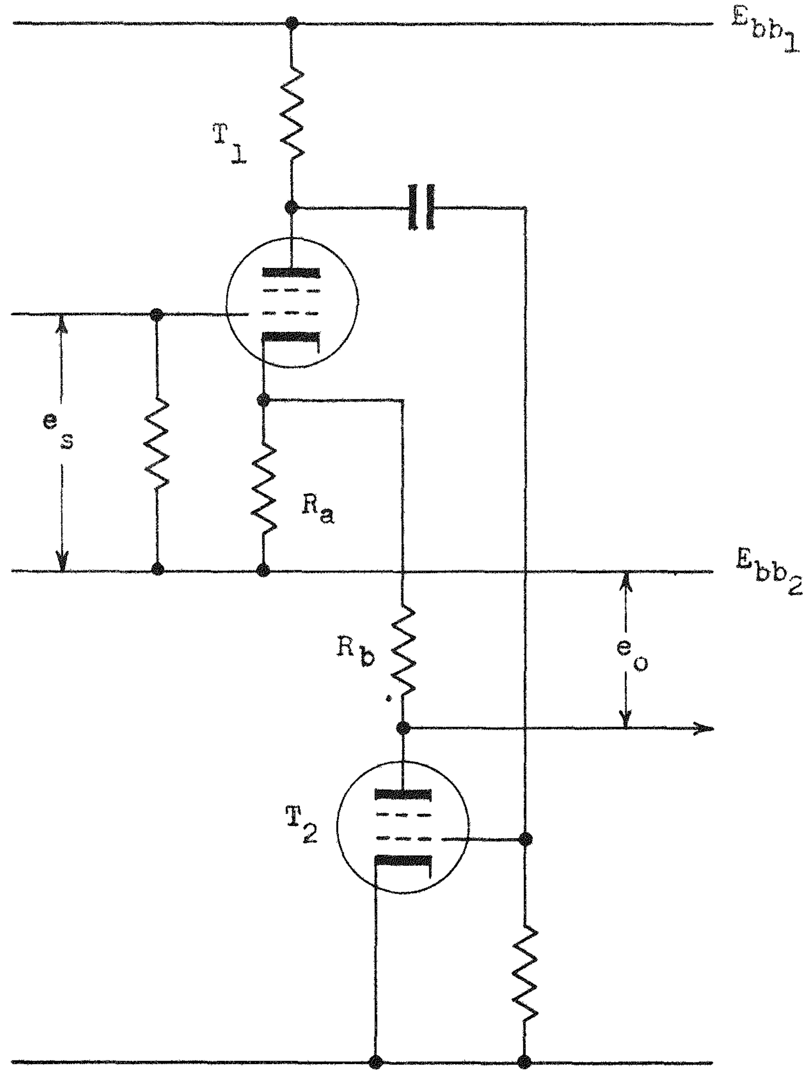

Fig. 4

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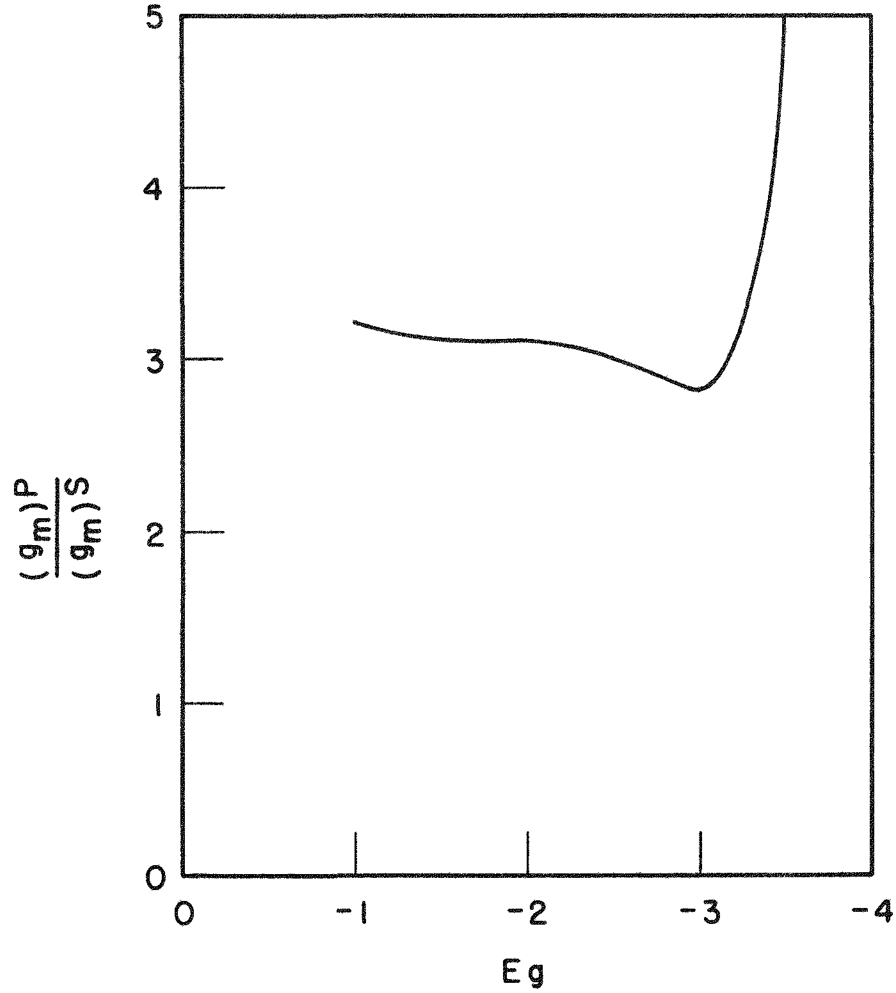
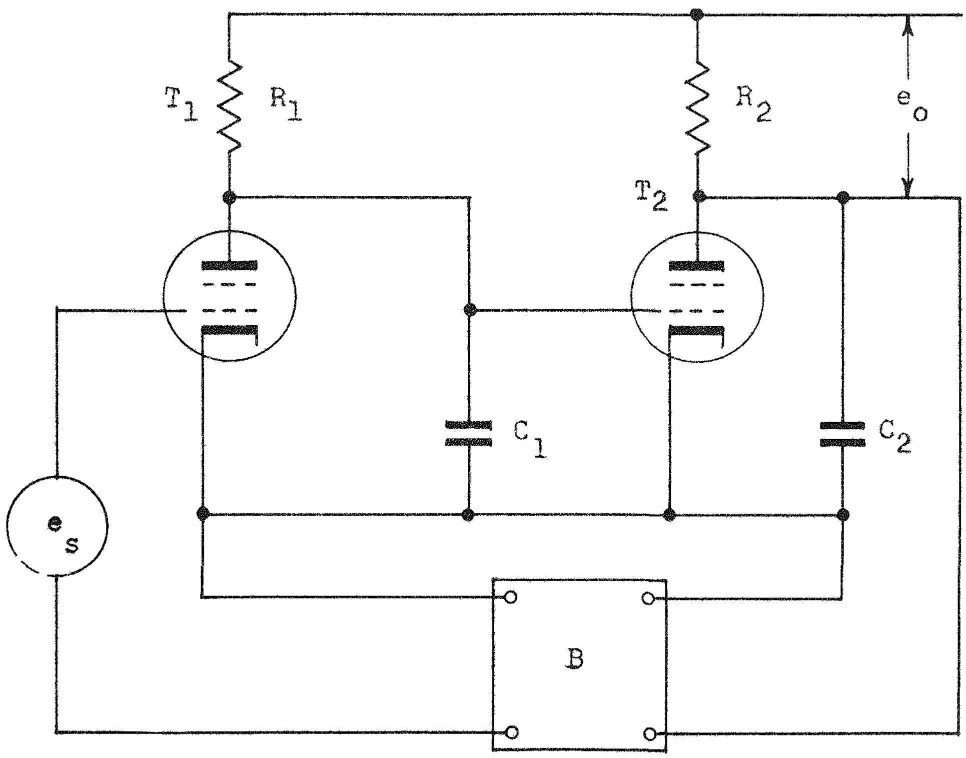


Fig. 5



Equivalent High Frequency Circuit

Fig. 6

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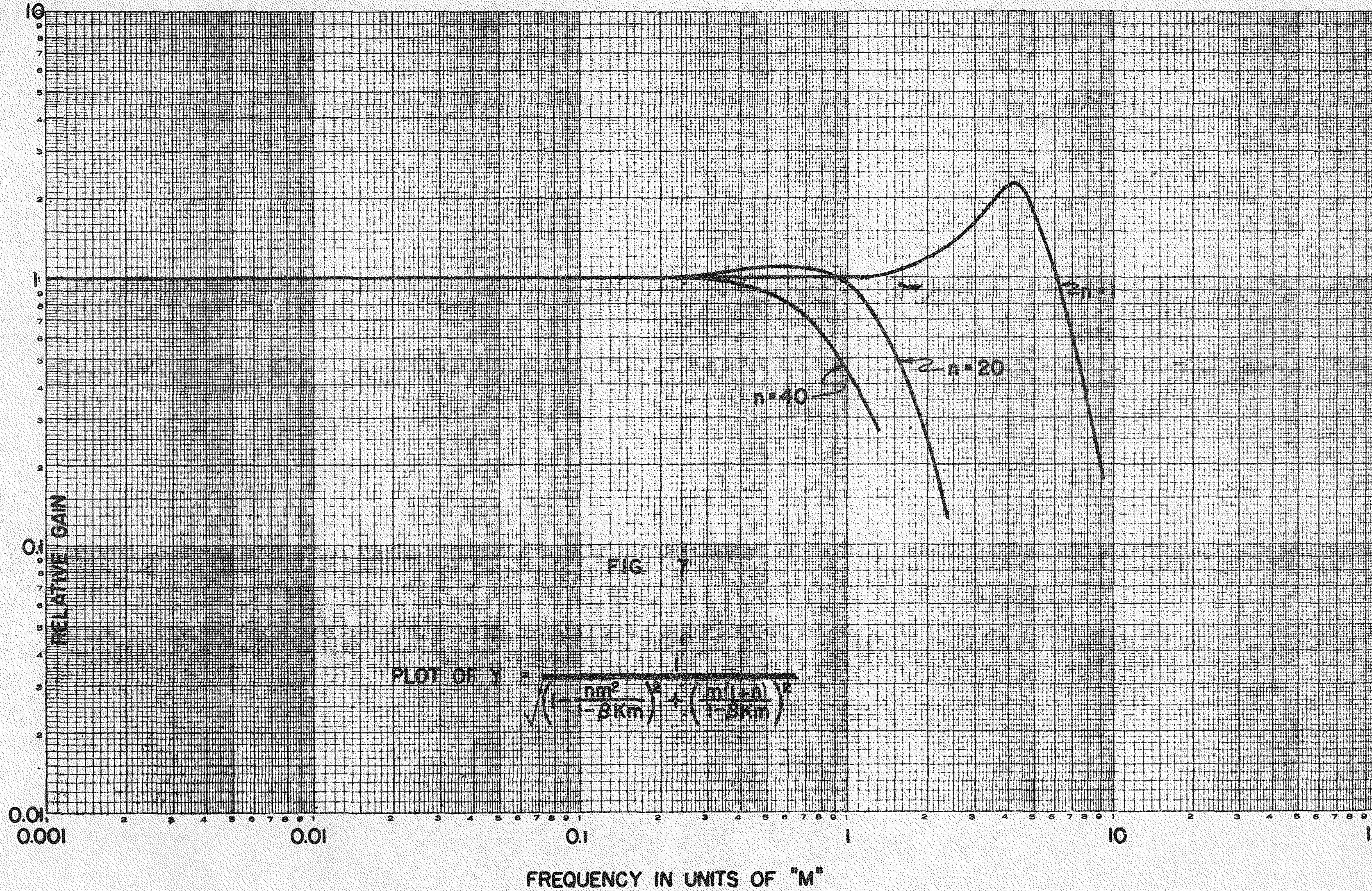

FIG. 7

PLOT OF $Y = \dfrac{1}{\sqrt{\left(1 - \dfrac{nm^2}{1-\beta Km}\right)^2 + \left(\dfrac{m(1+n)}{1-\beta Km}\right)^2}}$

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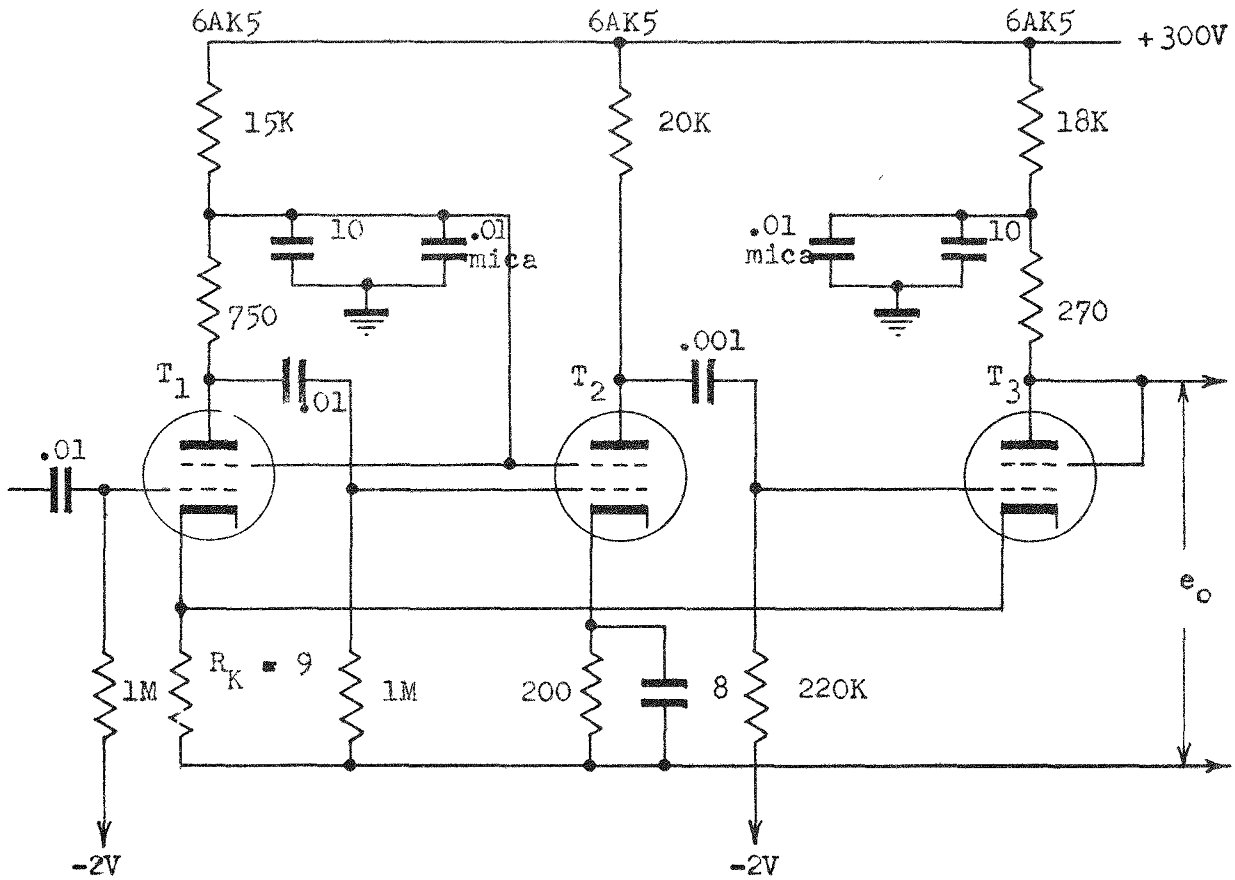
6AK5
6AK5
6AK5
+300V
15K
20K
18K
10
.01
mica
.01
mica
10
750
270
.001
T1
T2
T3
.01
.01
1M
RK = 9
1M
200
8
220K
eo
-2V
-2V

Fig. 8

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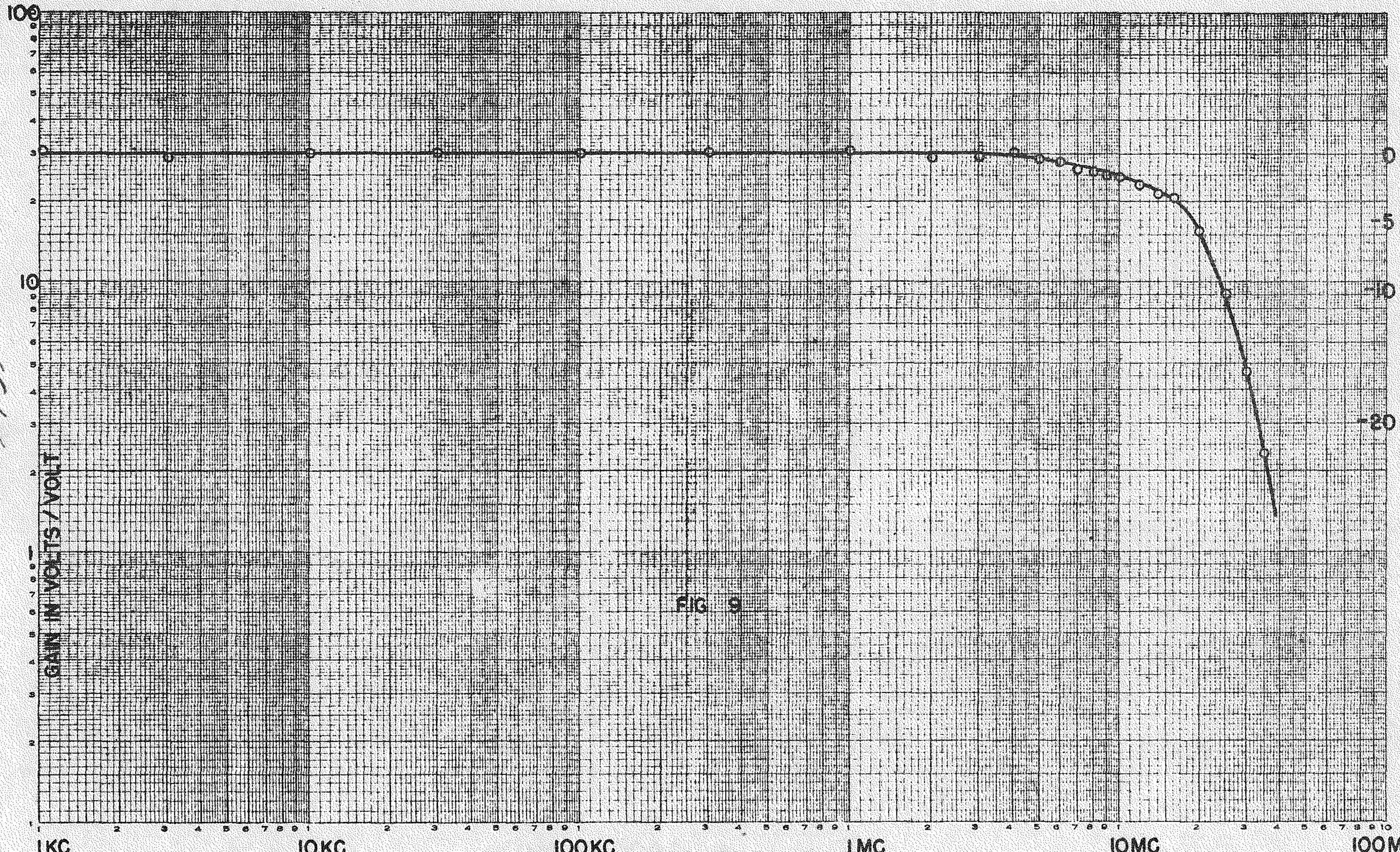

FIG 9

AMPLIFIER RESPONSE

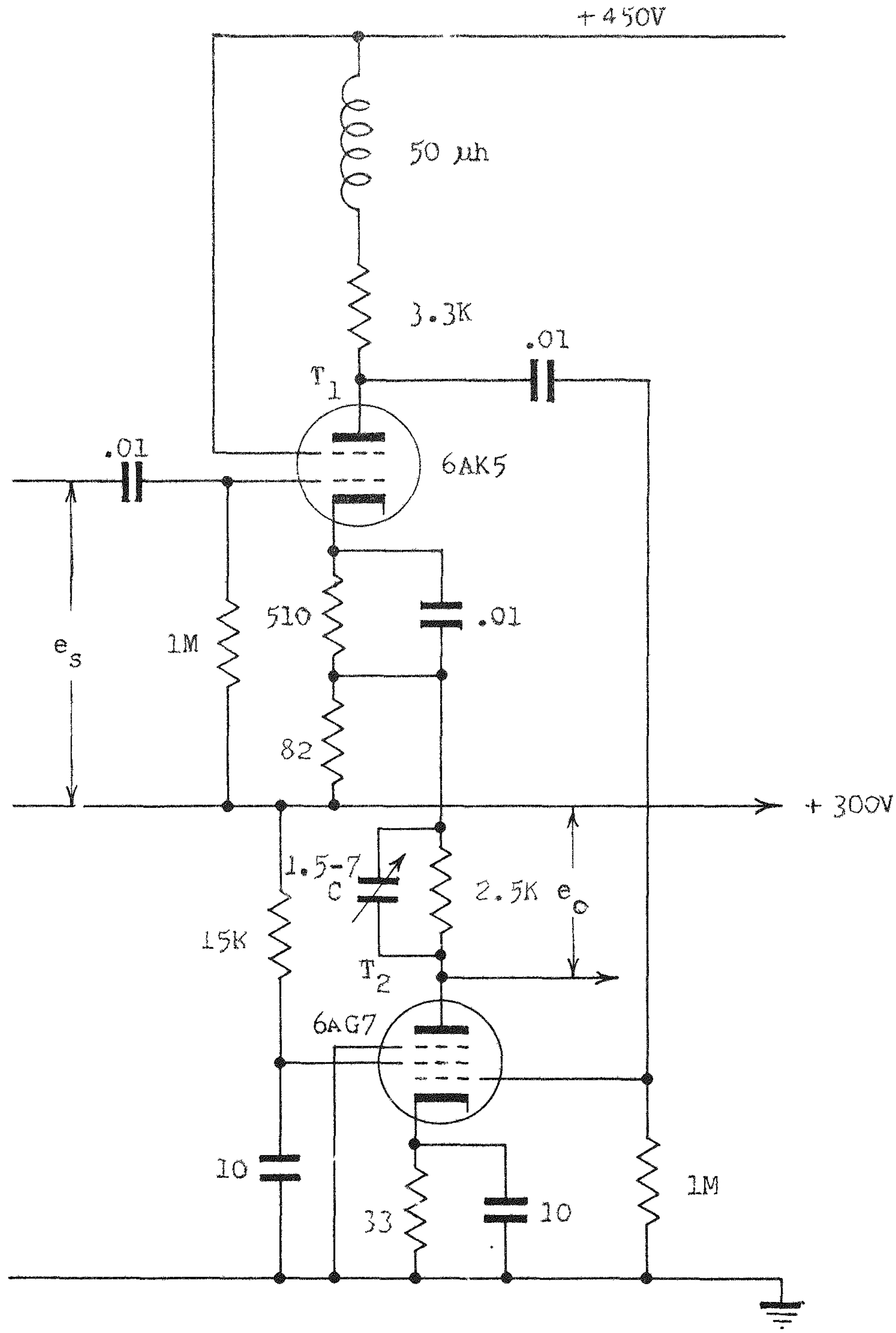
+450V
50 μh
3.3K
.01
T1
6AK5
.01
510
.01
es
1M
82
+300V
1.5-7
C
2.5K
ec
15K
T2
6AG7
10
33
10
1M

Fig. 10

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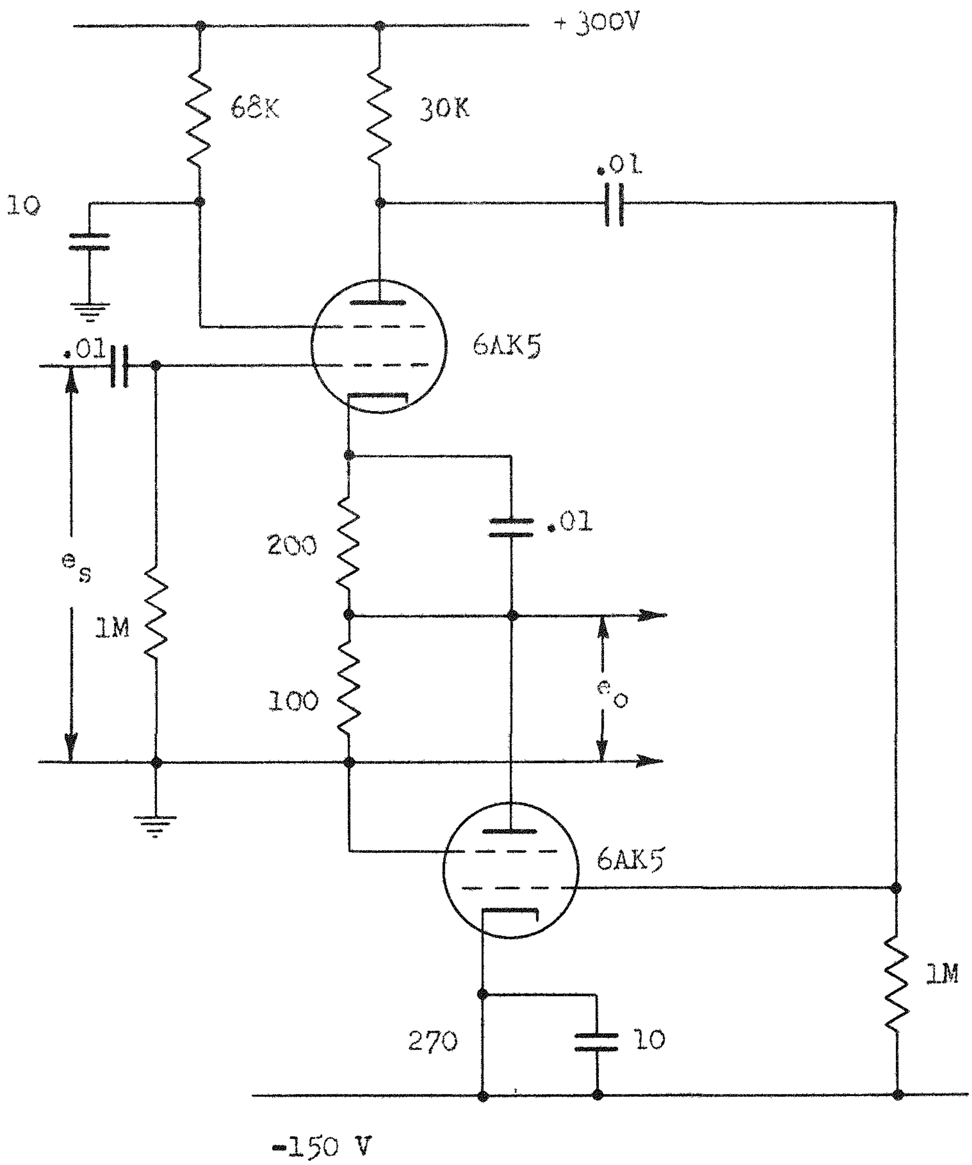

Fig. 11

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