MEASUREMENT OF THE BRANCHING RATIO $K_L \to 2\pi^0 / K_L \to 3\pi^0$ [*, †]

M. Banner,[††] J. W. Cronin, J. K. Liu,
and J. E. Pilcher

Palmer Physical Laboratory,
Princeton University,
Princeton, New Jersey

August 14, 1968

---

[*] Work supported by the U. S. Office of Naval Research, Contract No. N0014-67-A-0151-0001, and by the U. S. Atomic Energy Commission Contract No. AT(30-1)-2137.

[†] This work made use of computer facilities supported in part by a National Science Foundation grant, No. NSF-Gp 579.

[††] On leave from Département de Physique des Particules Elementaires, Centre d'Etudes Nucléaires, Saclay, France.

Accelerator
P.O. Box 682
Princeton, New Jersey 08540

ATTENTION: Document Office

AEC Contract
AT(30-1)-2137

DISCLAIMER

MEASUREMENT OF THE BRANCHING RATIO $K_L \rightarrow 2\pi^0/K_L \rightarrow 3\pi^0$ [*†]

M. Banner,[††] J. W. Cronin, J. K. Liu, and J. E. Pilcher

Palmer Physical Laboratory,
Princeton University,
Princeton, New Jersey

ABSTRACT

We have measured the branching ratio $(K_L \rightarrow 2\pi^0)/(K_L \rightarrow 3\pi^0)$ to be $(4.6 \pm 1.1) \times 10^{-3}$. Combined with a recently reported branching ratio $(K_L \rightarrow 3\pi^0)/(K_L \rightarrow$ all modes) = (0.209 ± 0.011), we find $(K_L \rightarrow 2\pi^0)/(K_L \rightarrow$ all modes) = $(0.96 \pm 0.23) \times 10^{-3}$, and $|\eta_{00}| = (2.3 \pm 0.3) \times 10^{-3}$.

The apparatus described in the previous letter(1) has been used to measure the branching ratio $(K_L \rightarrow 2\pi^0)/(K_L \rightarrow 3\pi^0)$. In the limited space available, we shall demonstrate that the $2\pi^0$ decay was observed and that the number of these decays was normalized in a reasonable way to the number of $3\pi^0$ γ-rays observed in the spectrometer. A paper giving a detailed description of this experiment is under preparation.

The spectrometer measurement allows us to isolate a group of potential $K_L \rightarrow 2\pi^0$ decays from the dominant mode $K_L \rightarrow 3\pi^0$ by the requirement that the transverse momentum, $P_\perp$, of a single γ-ray from K_L decay be greater than 165 MeV/c. The resolution in $P_\perp$ has a 4% standard deviation, plus a bremsstrahlung tail on the low momentum side which does not affect the separation. The recorded time of flight for each event permits a transformation to the c.m. system of the decaying particle. However, since the separation from the $3\pi^0$ mode depends only on $P_\perp$, it is independent of any error in the time of flight.

In the absence of background, all events with $P_\perp > 165$ MeV/c would be either $K_L \rightarrow 2\pi^0$ or $K_L \rightarrow \gamma\gamma$. This idea was used in a previous experiment(2) which reported a very large preliminary value for the branching ratio $(K_L \rightarrow 2\pi^0)/(K_L \rightarrow 3\pi^0)$. Subsequent analysis of that experiment and the present experiment have indicated a substantial background originating from γ-rays produced by interactions of neutrons and K_L's in the helium-filled beam volume and in the walls of the steel chambers which surrounded the beam. Of the total of 643 γ-rays that were measured to have a $P_\perp > 165$ MeV/c, approximately 150 were found to be from $K_L \rightarrow \gamma\gamma$.(1) Of the remaining

500, only about one in four was from $2\pi^0$ decay.

The γ-rays detected in the steel chambers in coincidence with the spectrometer γ-ray had energies ranging from 10-400 MeV. In order to maximize detection efficiency and thereby reduce the dependence on correction factors, a γ-ray criterion of two or more contiguous sparks was chosen. This modest requirement permitted a large accidental flux of sparks from neutron and K_L interactions to qualify as γ-rays. This accidental rate was about 25%. In addition, there was a 20% accidental rate for short, 2-gap recoil protons, which were classified as charged particles since their tracks originated in either the first or second plate in the aluminum section of the steel chambers.

The set of $K_L \rightarrow 2\pi^0$ events can be divided into three categories, depending on the number of γ-rays detected. No events were considered without a spectrometer γ-ray with $P_\perp > 165$ MeV/c. An event was classified as a type C if all three other γ-rays were detected in the steel chambers. When only two additional γ-rays were observed, the event was classified a type A if the missing γ-ray was associated with the π^0 of the spectrometer γ-ray, and a type B if it was associated with the other π^0. As we shall see below, the type A and type C events were relatively free of background from accidentals and interactions.

For the discussion that follows, the spectrometer γ-ray is labelled as 4 and its associated γ-ray as 3. The γ-rays associated with the other π^0 are designated 1 and 2. The direction of 4, which converted in the thin radiator, was known with a precision of 2.5°. The shower directions of 1, 2, and 3 have not been used in the analysis scheme which did, however, assume that the decay point lay on the trajectory of 4.

Consider an event of type C. If the decay point were known, there would be a 3-constraint fit to the hypothesis $K_L \rightarrow 2\pi^0$. However, since the decay point is only known to lie along a line, only a 2-constraint fit is possible. For a hypothetical decay point on the trajectory of 4, the directions of 1, 2, and 3 are assumed to lie along the lines drawn from this point to their conversion points. The time-of-flight information is then used to obtain the directions $\vec{u}_i$ of the four γ-rays in the c.m. system. Momentum conservation in the center-of-mass yields

$$p_1 \vec{u}_1 + p_2 \vec{u}_2 + p_3 \vec{u}_3 + p_4 \vec{u}_4 = 0 \quad ,$$

where p_i is the c.m. momentum of the i^{th} γ-ray. The solution to this set of 3 equations is

$$p_i = \lambda_i \, p_4 \quad , \quad i = 1,3.$$

Here λ_i depends only on the directions of the γ-rays and not on the measured momentum, p_4. The mass of the decaying particle, m^*, and the masses of the associated pairs of γ-rays, m_{12} and m_{34}, can then be calculated to be

$$m^* = p_4 \, (1 + \lambda_1 + \lambda_2 + \lambda_3) \quad ,$$
$$m_{12} = p_4 \, [2\lambda_1\lambda_2(1 - \vec{u}_1 \cdot \vec{u}_2)]^{1/2} \quad ,$$
$$m_{34} = p_4 \, [2\lambda_3(1 - \vec{u}_3 \cdot \vec{u}_4)]^{1/2} \quad .$$

If the decay point were known, these three quantitites could be used to check the hypothesis $K_L \rightarrow 2\pi^0$. To find the decay point, a χ^2 is formed using only the mass ratios (m_{12}/m^*) and (m_{34}/m^*), which are independent of p_4. This χ^2 is defined by

$$\chi^2 = [(m_{12}/m^* - m_\pi/m_k)^2 + (m_{34}/m^* - m_\pi/m_k)^2 \,]/(\sigma_\pi/m_k)^2 \quad ,$$

where m_π and m_k denote the π^o and K_L masses respectively, and $(\sigma_\pi/m_k) = 0.02$ is an estimate of the fitting precision. A minimum value for χ^2 is obtained by searching along the spectrometer γ-ray trajectory in 0.5 in. steps. For an acceptable fit, the best decay point may be no more than 2 in. outside the beam volume. All three possible pairings of the three observed γ-rays are tried and the minimum χ^2 is retained. In the case where an accidental track gave a fourth γ-ray in the steel chambers, that combination of three γ-rays giving the best overall χ^2 is retained. The events are limited to have at most one γ-ray converting in the first two gaps. The probability of having two such γ-rays is less than 1%. The measured spectrometer γ-ray energy, p_4, is also required to lie between 165 and 240 MeV.

A total of 74 type C candidates were found, and Fig. 1a shows the χ^2 distribution for 54 events with $\chi^2 < 50$. The distribution is essentially flat beyond $\chi^2 = 5$ and a cut was made at $\chi^2 = 9$. The m^* mass distribution of the 42 events with $\chi^2 \leq 9$ is shown in Fig. 1b. After a cut of $350 \leq m^* \leq 650$ MeV was imposed, 39 events remained, forming a peak centered around 498 MeV. This leads us to the conclusion that the $K_L \rightarrow 2\pi^o$ decay was indeed observed. We wish to emphasize that m^* was not constrained in the χ^2 search, but was calculated using p_4 only after a minimum χ^2 was obtained.

Figure 1c shows the resulting distribution for $(\vec{u}_1 \cdot \vec{u}_2)$, the cosine of the opening angle of the other π^o. In addition, we find the distance between the fit point and the vertex located by the γ-ray directions shows a peak about zero with a standard deviation of 3.5 in. Furthermore, the number of sparks observed for a γ-ray is well correlated with its energy as determined by the fit.

In the fit for a type A event, γ-rays 1 and 2 are assumed to be associated with a π^o from $K_L \rightarrow 2\pi^o$ decay. Knowledge of the directions of these γ-rays in the c.m. is sufficient to predict p_4. Two solutions are possible, and the one which gives better agreement with p_4 is kept. The predicted value of p_4 is required to agree within 8 MeV of the measured value, the latter having been increased by 4% to account for bremsstrahlung losses. The precision of 8 MeV was chosen to match the spectrometer resolution. The decay point was also adjusted to give the best fit. Since a type A event is more susceptible to an accidental fit than a C event, no fits were permitted with a track beginning in the first gap of a steel chamber. Events which failed the type C fit, and those with two particles in the steel chambers were tried for a type A fit. In all, a total of 26 type A events were found. The distribution for $(\vec{u}_1 \cdot \vec{u}_2)$, the cosine of the opening angle of the other π^o is shown in Fig. 1d.

The type B events allow only a check that a single γ-ray in the steel chambers can form a π^o with the spectrometer γ-ray. This check is too weak and is highly susceptible to contamination by accidentals. Hence, only type C and type A events are used to calculate a final branching ratio.

To study the background of events with $p_\perp > 165$ MeV/c in the spectrometer due to neutron and K_L interactions, a run was made with a heavy gas, SF_6, in the decay volume. The interaction rate in this gas has been estimated to be ~ 14 times that in helium.(3) Furthermore, the presence of SF_6 increased the accidental rate in the steel chambers significantly. The SF_6 running period was 1/8 that of the helium run, leading to an estimate of 1.7 times as many interactions in the beam volume. For the SF_6 run, 7 type C and 12 type A events were found where 5 and 3 decay events were expected respectively for a run of 1/8 duration. In addition, 5 events were found with $m^* > 650$ MeV. Hence, a background of 1 type C event and 6 type A events was predicted for the helium run. A total of (7 ± 4) background events was therefore subtracted from the category A + C.

We have considered the possibility that some of the events are in fact $3\pi^0$ events that spill past $P_\perp = 165$ MeV because of bad resolution. Using the 4% resolution found for $K_L \rightarrow \gamma\gamma$, no $3\pi^0$ events are expected with $P_\perp > 165$ MeV. The transverse momentum, $P_\perp$, has been remeasured for all the A and C events, and only one event was found in which a significant mistake had been made. Independent of either the spectrometer resolution or the possibility of a component of the K_L beam being off axis, we can estimate the maximum possible $3\pi^0$ background. Approximately 1000 experimental $3\pi^0$ events were analysed with the same $2\pi^0$ fitting scheme, except the measured c.m. energy, P_4, was replaced with a value chosen randomly between 165 and 230 MeV. This is the only background spectrum compatible with the observed E_{cm} distribution for the A and C events. The resulting distribution in m^* is shown in Fig. 2. If the 3 events in our experiment with $m^* > 650$ MeV are due to $3\pi^0$ background, then

3.5 type C events with $350 \leq m^* \leq 650$ MeV can be expected from such a source. Combined with a similar analysis of the $3\pi^0$ data for type A events, an upper limit of 7 A + C events are expected if the $3\pi^0$ background were truly present. However, since 3 events with $m^* > 650$ MeV are expected on the basis of 5 similar events found in the SF_6 run, we make no correction for a $3\pi^0$ background.

Given an observed γ-ray in the spectrometer, we have evaluated the detection efficiency for each event category. This efficiency calculation involved the geometry of the steel chambers, the γ-ray conversion probability by the processes of pair production and Compton scattering, and the probability of observing one electron traversing at least two gaps (~ 0.3 radiation lengths at normal incidence). This latter "seeability" factor was calculated as a function of γ-ray energy and angle of incidence, using known bremsstrahlung cross sections. It is essentially unity for γ-ray energies > 50 MeV. A summary of the detection efficiencies for 0, 1, 2, and 3 γ-rays detected in the steel chambers is shown in Table I. The efficiency for type A is larger than that for type B because the γ-ray associated with the spectrometer γ-ray is generally of low energy. While variations in "seeability" may affect the relative efficiencies for A and C, the efficiency of A + C remains rather stable.

The combined detection efficiency for A + C is thus 0.68. This value must be modified because some true A and C events fail to pass the analysis scheme, and some B events may fit as A events. The only way to compute these corrections is by identical analysis of Monte Carlo generated events. It must be emphasized that the analysis scheme used only (1) the direction of the spectrometer γ-ray, (2) the conversion points of the other γ-rays in the steel chambers, and (3) the observed momentum of the spectrometer γ-ray. The errors in each of these quantities can be calculated quite reliably and the Monte Carlo generated events are expected to be realistic. These fitting corrections are small and the overall efficiency for A + C events, after analysis, is 0.64. In order to estimate an error for this efficiency, the parameters which are involved in the calculations have been varied and the A + C efficiency has been found to be quite stable, having an extreme range of values (0.64 ± 0.06). It was noted that the addition of a random γ-ray can transform a type A to a type C in about 20% of the cases, but it cannot cause events to be lost from the combined category of A + C. To be absolutely safe, the efficiency for A + C has been taken as (0.64 ± 0.10) for subsequent calculations.

From the observed (65 ± 8) A + C events, (7 ± 4) are subtracted for background, leaving a total of (58 ± 9) events. Using (0.64 ± 0.10) for the detection efficiency, the number of $2\pi^0$ events observed in the spectrometer with $p_\perp > 165$ MeV/c is found to be (91 ± 21). The Monte Carlo calculations indicate that 0.570 of all $2\pi^0$ events detected in the spectrometer have $p_\perp > 165$ MeV/c. Hence a total of $N_{2\pi^0} = (160 \pm 37)$ $2\pi^0$ events were observed in the spectrometer as compared with $N_{3\pi} = (6077 \pm 85)$ 3π events.$^{(1)}$

The branching ratio is given by

$$\frac{K_L \to 2\pi^0}{K_L \to 3\pi^0} = \left(\frac{N_{2\pi^0}}{N_{3\pi}}\right)\left(\frac{6 + 2R'}{4}\right)\left(\frac{\bar{\epsilon}}{\epsilon_{2\pi^0}}\right) ,$$

where $R' = (0.60 \pm 0.03)$ is the branching ratio $(K_L \to \pi^+\pi^-\pi^0)/K_L \to 3\pi^0)$,$^{(4)}$

and $(\bar{\epsilon}/\epsilon_{2\pi^o}) = (0.0968 \pm 0.0072)$ is the relative efficiency of the spectrometer for $K_L \rightarrow 3\pi$ as compared with $K_L \rightarrow 2\pi^o$. The factor $(\bar{\epsilon}/\epsilon_{2\pi^o})$ is obtained in a manner entirely similar to $(\bar{\epsilon}/\epsilon_{\gamma\gamma})$ discussed in the previous letter.(1)

We find

$$\frac{K_L \rightarrow 2\pi^o}{K_L \rightarrow 3\pi^o} = (4.6 \pm 1.1) \times 10^{-3}.$$

This is in strong disagreement with the earlier result of $(18.9 \pm 3.1) \times 10^{-3}$.(2) Using the new value(4) for the branching ratio $(K_L \rightarrow 3\pi^o)/(K_L \rightarrow \text{all}) = 0.209 \pm 0.011$, we find

$$\frac{K_L \rightarrow 2\pi^o}{K_L \rightarrow \text{all}} = (0.96 \pm 0.23) \times 10^{-3},$$

and the CP violating parameter $|\eta_{oo}|$(5) to be

$$|\eta_{oo}| = (2.3 \pm 0.3) \times 10^{-3},$$

or

$$|\eta_{oo}|^2 = (5.1 \pm 1.2) \times 10^{-6}.$$

This is to be compared with the result of Bartlett et al.,(6) who find $|\eta_{oo}|^2 = (-2 \pm 7) \times 10^{-6}$, and with the fully analyzed experiment of Gaillard et al.,(7) who find $|\eta_{oo}|^2 = (13 \pm 4) \times 10^{-6}$.

On the basis of this experiment, we conclude that the previous result(2) obtained using only the pair spectrometer was incorrect. We have no evidence to conclude that $|\eta_{oo}|$ is different from $|\eta_{+-}|$, and hence a CP violation in $K_L \rightarrow 2\pi$ decay containing only $|\Delta I| = 1/2$ transitions is compatible with this experiment.

ACKNOWLEDGEMENTS

We wish to thank Professor M. G. White, Dr. Walter Wales, and all the PPA Operating crew for their wonderful cooperation during this experiment. We are also grateful to Mr. B. Knapp and Mr. M. Shochet for their help during the early stages of this experiment. Mr. A. Nathan's assistance in rescanning and remeasuring the film is gratefully acknowledged.

REFERENCES

* Work supported by the U.S. Office of Naval Research, Contract No. N0014-67-A-0151-0001, and by the U.S. Atomic Energy Commission Contract No. AT(30-1)-2137.

† This work made use of computer facilities supported in part by a National Science Foundation grant, No. NSF-Gp 579.

†† On leave from Département de Physique des Particules Elementaires, Centre d'Etudes Nucléaires, Saclay, France.

(1) M. Banner, J. W. Cronin, J. K. Liu, and J. E. Pilcher, preceding letter.

(2) J. W. Cronin, P. F. Kunz, P. C. Wheeler, and W. S. Risk, Phys. Rev. Letters, 18, 25 (1967).

(3) B. Margolis, CERN Report 67/1541/5-TH. 865, Geneva (1967). (Unpublished)

(4) I. A. Budagov, H. Burmeister, D. C. Cundy, W. Krenz, G. Myatt, F. A. Nezrick, H. Sletten, G. H. Trilling, W. Venus, H. Yoshiki, B. Aubert, P. Heusse, I. Le Dong, E. Nagy, C. Pascaud, L. Behr, P. Beillière, G. Boutang, J. van der Velde, Nuovo Cimento, to be published.

(5) See, for example, T. T. Wu and C. N. Yang, Phys. Rev. Letters, 13, 380, (1964).

(6) D. F. Bartlett, R. K. Carnegie, V. L. Fitch, K. Goulianos, D. P. Hutchinson, T. Kamae, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters, to be published.

(7) J. M. Gaillard, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. J Ratcliffe, H. Faissner, and H. Reithler, Rutherford Laboratory Preprint RPP/H/35, unpublished. A preliminary

report of this experiment was given by J. M. Gaillard, F. Krienan, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. Ratcliffe, P. Day, A. G. Parham, B. T. Payne, A. C. Sherwood, H. Faissner, and H. Reithler, Phys. Rev. Letters, 18, 20 (1967).

FIGURE CAPTIONS

Figure 1a Distribution of χ^2 for 54 type C candidates with $\chi^2 \leq 50$. Events with $\chi^2 > 9$ are cut. The solid curve is the Monte Carlo prediction.

1b Distribution of m^*, mass of the decaying particle, for 42 events with $\chi^2 \leq 9$. Events with $350 \leq m^* \leq 650$ MeV are kept. The solid curve is the Monte Carlo prediction.

1c Distribution of $\vec{u}_1 \cdot \vec{u}_2$ for 39 type C events with $\chi^2 \leq 9$ and $350 \leq m^* \leq 650$ MeV. The solid curve is the Monte Carlo prediction.

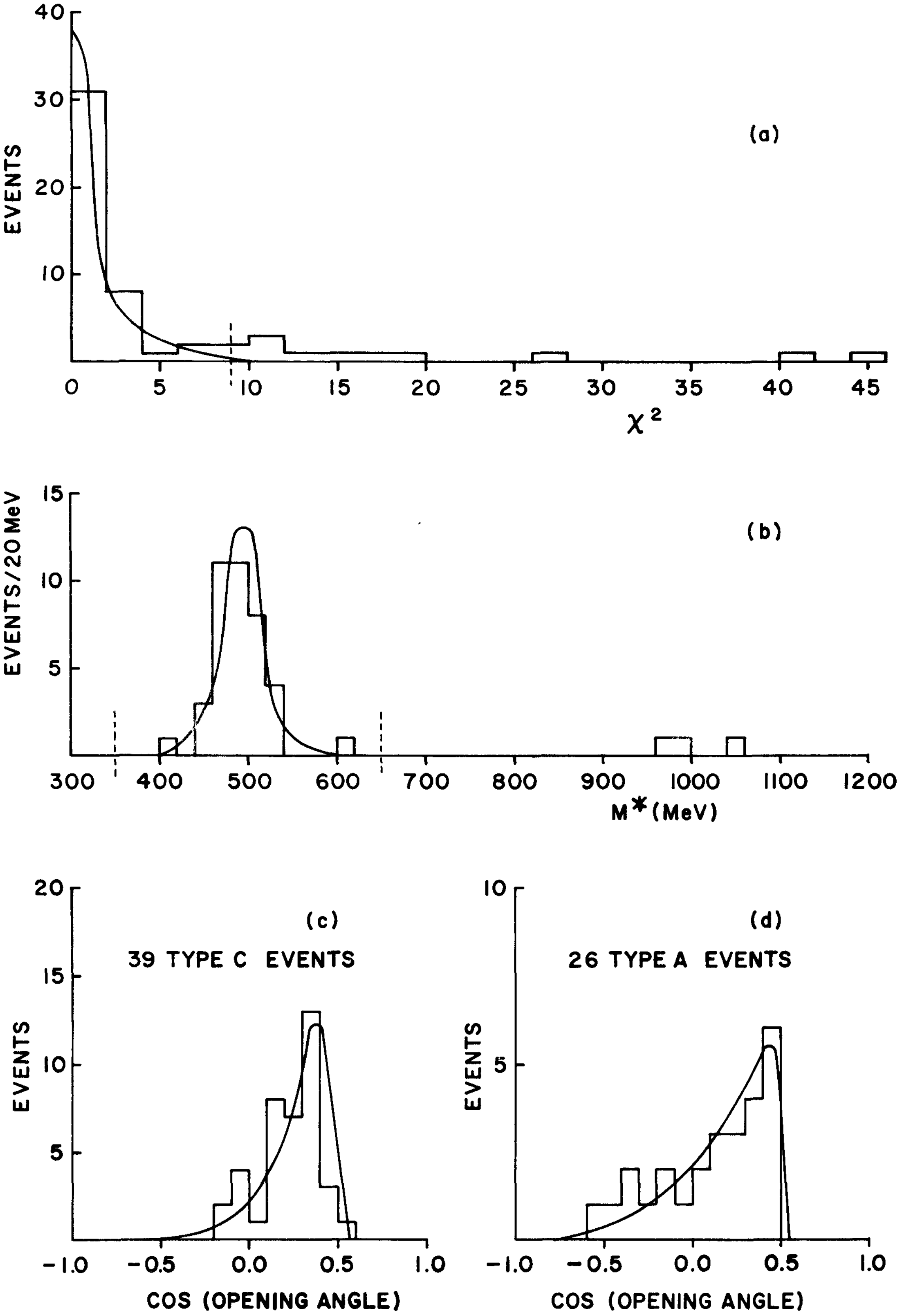
1d Distribution of $\vec{u}_1 \cdot \vec{u}_2$ for 26 type A events. The solid curve is the Monte Carlo prediction.

Figure 2 Distribution of m^* for experimental $3\pi^0$ events analysed as $2\pi^0$, but with E_{cm} chosen at random between 165 and 229 MeV. These events are normalized to the 39 observed type C events whose m^* distribution is also shown.

TABLE I

Detection Efficiences for γ-rays from $K_L \rightarrow 2\pi^0$

Condition	No. of γ-rays detected in steel chambers				
	0	1	2(type A)	2(type B)	3(type C)
(1) Geometry Alone	0.00	0.02	0.10	0.12	0.76
(2) Conversion probability included.	0.00	0.09	0.20	0.17	0.54
(3) Seeability and conversion probability included.	0.02	0.14	0.29	0.16	0.39



(a)
EVENTS
40
30
20
10
0
5
10
15
20
25
30
35
40
45
χ^2
(b)
EVENTS/20 MeV
15
10
5
300
400
500
600
700
800
900
1000
1100
1200
M*(MeV)
(c)
39 TYPE C EVENTS
EVENTS
20
15
10
5
−1.0
−0.5
0.0
0.5
1.0
COS (OPENING ANGLE)
(d)
26 TYPE A EVENTS
EVENTS
10
5
−1.0
−0.5
0.0
0.5
1.0
COS (OPENING ANGLE)

FIG. 1

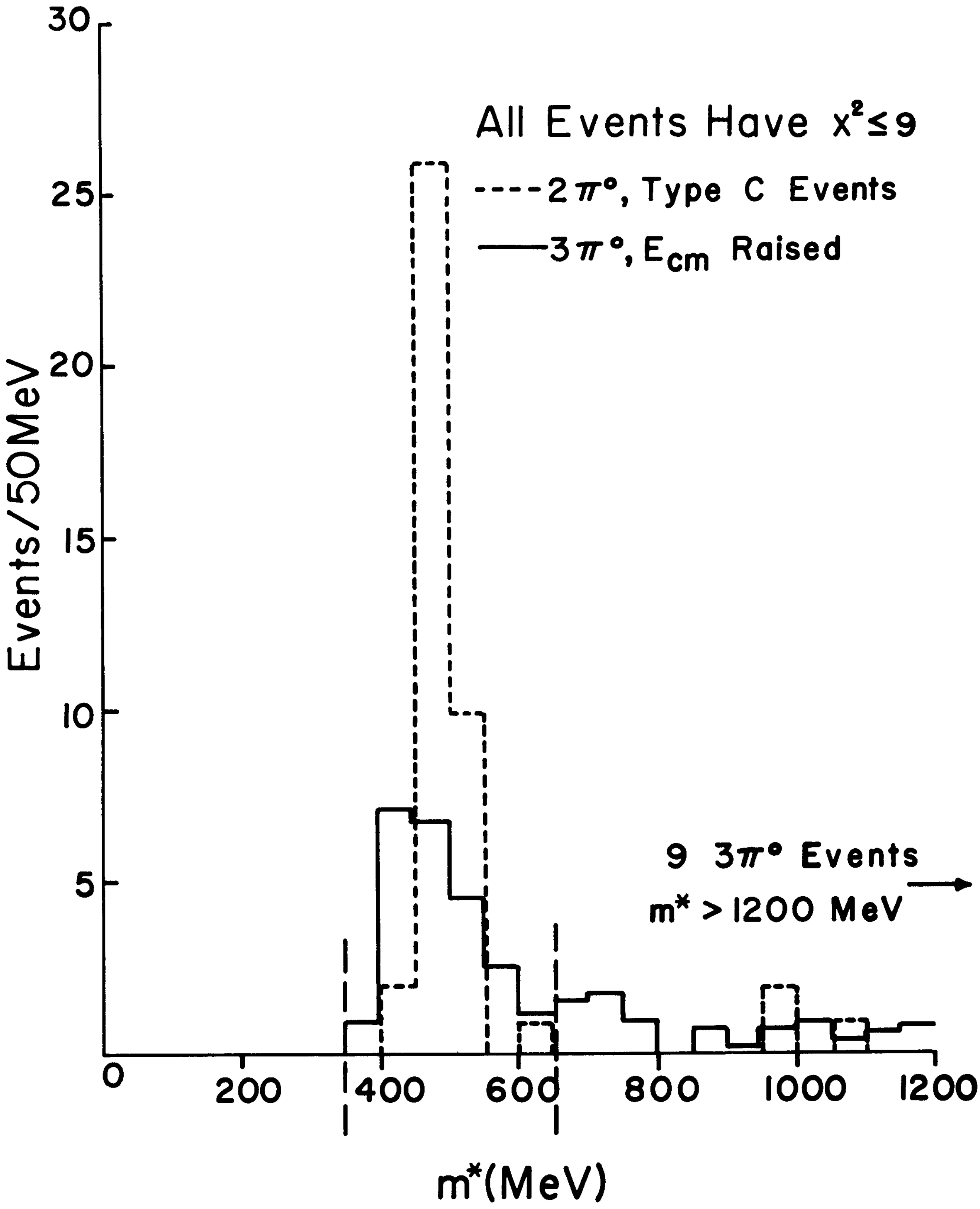

30
25
20
15
10
5
0
Events/50MeV
All Events Have $x^2 \leq 9$
----$2\pi^\circ$, Type C Events
——$3\pi^\circ$, E_{cm} Raised
9 $3\pi^\circ$ Events
$m^* > 1200$ MeV
200
400
600
800
1000
1200
m^*(MeV)

FIG. 2