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The Parity of the Neutral Pion and the Decay $\pi^{0} \rightarrow 2e^{+} + 2e^{-}$

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ABSTRACT

Two hundred and six electronic decays of the π^0, $\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$, have been observed in a hydrogen bubble chamber. The decay distributions of the electron pairs and the total rate for this process are shown to be in good agreement with theory. An examination of correlations of the e^+e^- pair decay planes on the basis of electrodynamic predictions is in agreement with the hypothesis that the π^0 is pseudoscalar, but disagrees for scalar pions by 3.6 standard deviations.

I. INTRODUCTION

The parity of the pion is, of course, important in all manifestations of the strong interaction. Perhaps the only previous direct experimental evidence on this incisive quantity is from the s wave capture of π^- mesons in deuterium.[1] It is essential in the interpretation of these experimental results to demonstrate that the capture proceeds indeed from the s state, and this has been done in a convincing way.[2]

It was first pointed out by Yang[3] that the decay of the neutral pion offers, at least in principle, a means for an unambiguous measurement of the parity: the plane polarizations of the two photons must be parallel for scalar pions, and perpendicular in the pseudoscalar case. Unfortunately there are no known techniques for the measurement of this polarization correlation. The chief reason for this difficulty is that although the plane of the pair produced by the photon is well correlated with the polarization, the angle between positron and electron is so small that the scattering in converters of practicable thickness destroys this correlation.

It is remarkable and fortunate that this angle is much larger ($\simeq \sqrt{mc^2/E}$ in place of mc^2/E) in the internal conversion of the γ-rays, so that the plane may be measured. The decay $\pi^o \rightarrow 2e^+ + 2e^-$ offers then a means of determining the parity of the π^o. However, the decay is rare; the rate is expected to be of the order of $(1/160)^2$, already on the basis of Dalitz's original work on these internally converted pairs.[4]

We report here an experimental study of this process, based on an observation of 8×10^6 π^o decays in some 836,000 bubble chamber pictures. Two hundred and six completely electronic decays were observed. A preliminary report based on one-half of the events has previously been published.[5]

II. EXPERIMENTAL DETAILS

π^- mesons from the Nevis Cyclotron are slowed down by polyethylene absorber and stopped in a hydrogen bubble chamber 12 in. in diameter, 6 in. in depth, placed in a 5.5 kgauss magnetic field. π^o's were produced in the reaction $\pi^- + p \rightarrow n + \pi^o$ which proceeds for 62% of the stopped pions. The pictures were obtained in a series of three separate exposures, the last of which consisted of 380,000 pictures accumulated at a chamber cycle rate of 90 pictures per minute. On the average there are 15 stopping pions per picture (Fig. 1). The events were measured on a digitized scanning machine, on three views, with a measurement accuracy of $\pm$ 2 microns on the film. From these measurements and the known magnetic field, the vector momentum of each track as well as other parameters necessary for the analysis were calculated on an IBM 650 computer.

Portions of the film were scanned by two independent persons. Comparison of these results indicate an efficiency of $\sim$ 90% for single Dalitz pairs and $\sim$ 95% for double Dalitz pairs. Due to the rare occurrence of the latter, no fiducial, dip, or depth restrictions were applied to the selection of events. It was, however, required that each pair have its origin within 1 mm of the stopping π^-.

Two hundred and six decays of the type $\pi^o \rightarrow e^+ + e^- + e^+ + e^-$ and $\sim$ 100,000 decays $\pi^o \rightarrow \gamma + e^+ + e^-$ were found. Fig. 1 is a reproduction of one of the former. Some of these events were rejected for the following reasons:

a) 18 events have their vertex near the chamber wall or window so that the momenta of two or more tracks are not measurable. Events in which at least three of the four momenta are not measurable were discarded.

b) 42 events have two tracks whose included angle is less than 3^o. Since the measurement accuracy of the direction of a track in space is of the order of 1^o-2^o, the plane of such e^+e^- pairs cannot be determined adequately.

All events except those in category (a) were measured a minimum of five times. The variations in the momenta in successive measurement gave the measurement error. The net error assigned to each track was a combination of measurement and multiple scattering error taken in quadrature.

The π^o's produced in the charge exchange reaction emerge with a unique energy of 137.8 Mev corresponding to a momentum of 27.5 Mev/c. In each event therefore the energy of each track was normalized so that the sum of the four electron energies was equal to 137.8 Mev, the weighting factor being inversely proportional to the square of the percentage error in the energy of each track. In the few cases where the energy (but not the direction) of one of the electrons was indeterminate, it was set equal to the difference between 137.8 Mev and the sum of the energies of the remaining three.

The normalized tracks were then transformed to the rest frame of the π^o and all further work is based on the normalized, transformed momenta.

III. THEORETICAL BACKGROUND AND SELECTION CRITERIA

The original argument of Yang is based entirely on conservation laws. The argument in the case of the direct decay into two pairs is however necessarily a two step argument; first, two virtual photons are produced, and these then convert internally. The overall process is calculated with the help of electrodynamics. Kroll and Wada[6] have given arguments to support the view that the resultant correlations in the planes of the two pairs follow directly from well demonstrated features of quantum electrodynamics and should be only insignificantly affected by the unknown mesonic form factor. In what follows, we present experimental results to substantiate this view also empirically. The main point is that although the arguments necessary for the interpretation of our experiment are not based exclusively on invariance but rest also on electrodynamics, we feel that this is more a formal than practical blemish. The theoretical arguments are on solid ground. In addition, they are supported by experiments in the following way: The parity argument rests on the correlation between the planes of the two pairs. The experimental support of the theoretical analysis comes from comparing <u>other</u> predicted features with experiment. These fall into two classes, the single and the double pair decay.

The single pair decay, $\pi^{o} \rightarrow \gamma + e^{+} + e^{-}$, has been discussed by Dalitz,[4] Kroll and Wada,[6] and by Joseph[7]. The decay can be described by two parameters; Kroll and Wada chose the quantities:

$$x^{2} = (E_{+} + E_{-})^{2} - \left|\vec{P}_{+} + \vec{P}_{-}\right|^{2} \quad \text{and} \quad y = \frac{\left|E_{+} - E_{-}\right|}{\left|\vec{P}_{+} + \vec{P}_{-}\right|} .$$

Extensive experimental results exist.[8] The results of Samios,[8] based on 4000 events found in these pictures and analyzed by the same methods we use here, are reproduced in Figs. 2 and 3. The experiment is seen to confirm the theory.

The theoretical analysis of the two pair decay, $\pi^{o} \rightarrow 2e^{+} + 2e^{-}$, is more involved. There are five non-trivial parameters needed to describe this decay. Furthermore, the analysis into pairs is ambiguous. If 1^{+}, 2^{+} denote the positrons, and 1^{-} and 2^{-} the electrons (Fig. 4), then it is possible to form the two pairs in two ways: $(1^{+},1^{-})(2^{+},2^{-})$ and $(1^{+},2^{-})(2^{+},1^{-})$. The Pauli principle demands that the decay probability has the form

$$P_{t} \simeq \left|M[(1^{+},1^{-})(2^{+},2^{-})]\right|^{2} + \left|M[(1^{+},2^{-})(2^{+},1^{-})]\right|^{2}$$

$$+ 2\mathrm{Re}(M^{x}[(1^{+},1^{-})(2^{+},2^{-})]\ M[(1^{+},2^{-})(2^{+},1^{-})])$$

where the two M's are matrix elements for a particular pairing. For most decays, one of the two pairings [in Fig. 4 the pairing $(1^{+},1^{-})(2^{+},2^{-})$] gives a much larger matrix element than the other. This is related to the fact that also in direct conversion, the probable angle between members of a pair is small. Kroll and Wada[6] have calculated the properties and especially the correlations with decay using only the pairing giving the larger matrix element, neglecting interference. This will be a good approximation in most but not all cases. Their results were prsented in the convenient form

$$P_t = A(x_1, y_1, x_2, y_2)[1 \pm \alpha(x_1\ y_1\ x_2, y_2 \cos 2\varphi] \qquad (1)$$

where the x's and y's are as previously defined and φ is the angle between the two planes.

$$A = \frac{1}{2x_1 x_2}\left\{1 + y_1^2\, y_2^2 + y_1^2 + y_2^2 + \frac{4m^2}{x_1^2}(1 + y_2^2) + \frac{4m^2}{x_2^2}(1 + y_1^2) + \frac{(4m^2)^2}{x_1^2 x_2^2}\right\} \qquad (2a)$$

$$\alpha = \frac{y_1^2 + y_2^2 - 1 - y_1^2\, y_2^2 + \frac{4m^2}{x_1^2}(1 - y_2^2) + \frac{4m^2}{x_2^2}(1 - y_1^2) - \frac{(4m^2)^2}{x_1^2 x_2^2}}{1 + y_1^2\, y_2^2 + y_1^2 + y_2^2 + \frac{4m^2}{x_1^2}(1 + y_2^2) + \frac{4m^2}{x_2^2}(1 + y_1^2) + \frac{(4m^2)^2}{x_1^2 x_2^2}} \qquad (2b)$$

The - and + signs are for the scalar and pseudoscalar cases, respectively. More recently Rockmore[9] has made unpublished results available to us in which no such approximation is made, in which therefore all decays are handled equally well. Unfortunately, the resulting expressions are formidable and we have not succeeded in handling them, especially in performing the necessary integrations. We are, therefore, constrained to use the approximate expression (1). In order not to exceed the limit of validity of (1) we use the arbitrary criterion

$$|M(1^+,1^-;2^+2^-)|^2 > 10\ |M(1^+2^-,2^+,1^-)|^2 + 2\mathrm{Re}\,[M^x(1^+2^-,2^+,1^-)\ M(1^+1^-,2^+2^-)] \qquad (3)$$

One is now in a position to compare the experimental decay distributions of the double Dalitz pairs with the theoretical predictions. The distributions in y and x/μ (μ = pion mass) are presented in Figs. 5 and 6. The plotted events include the above 146 events plus the additional events where one included angle is less then 3°. In the case of the 34 events which do not satisfy (3), the pairing which gives the larger value for the matrix element is used. Again, there is good agreement between theory and experiment.

The distribution in α furnishes another check on the theory. In the present experiment, the values of α varied from ~ 0 to 0.8. This distribution $g(\alpha)$ for the 112 events satisfying (2) is plotted in Fig. 7. The average value of α, $\langle\alpha\rangle$ for this experimental distribution is

$$\langle\alpha\rangle = 0.18 \pm 0.02$$

in agreement with the theoretical prediction, $\alpha = 0.18$.

The agreement between theoretical expectation and these several experimental observations lead us to believe that the theory is also competent to deal with the dependence of the asymmetry on the parity.

IV. RESULTS

A. Branching Ratio for Double Pair π^{o} Decay

The branching ratio $R = P_t(2e^+ + 2e^-)/P_t(2\gamma)$ was measured in two ways. The first involved a knowledge of the flux, the Panofsky ratio P and the total number of events. A flux count made of the pictures comprising the 3rd run gave

$(7.77 \pm 0.10) \times 10^6$ π^- stoppings, which using a Panofsky ratio $P = 1.62 \pm 0.06$ [8] yields $(4.80 \pm 0.19) \times 10^6$ π^o's. In this same group of pictures there appeared 146 double Dalitz pairs. The 1 mm cut-off criterion adopted would have introduced one background event in this sample due to the internal conversion of one pair and the external conversion of the second within 1 mm of the origin. Using a 95% scanning efficiency previously estimated, one obtains:

$$\left(\rho_{2\pi^o}\right)_1 = \frac{(146-1)}{0.95(4.8)\ 10^6} = (3.18 \pm 0.30)\ 10^{-5}$$

The second method utilized a knowledge of the internal conversion coefficient for single Dalitz pairs, the Panofsky P and the ratio of double to single Dalitz pairs. Single Dalitz pairs are produced in the two reactions:

$$\pi^- + p \rightarrow n + \pi^o \ ; \ \pi^o \rightarrow e^+ + e^- + \gamma \qquad \text{(a)}$$

and $$\pi^- + p \rightarrow n + e^+ + e^- \qquad \text{(b)}$$

and their respective theoretical conversion coefficient $\rho_a = 0.01196$ [7] and $\rho_b = 0.00710$ [7]. In this latter case, it is more profitable to use the total sample of 206 double Dalitz pairs and 104,573 single Dalitz pairs. The number of background events in these two categories are 2 and 180, respectively, therefore:

$$\left(\rho_{2\pi^o}\right)_2 = \frac{(206-2)/0.95}{(104,573-100)/0.90} \times \frac{(1.62\ \rho_a + \rho_b)}{1.62} = (3.02 \pm 0.32)10^{-5}$$

Both these values agree with the theoretically predicted value of 3.47×10^{-5} as calculated by Kroll-Wada. This latter agreement for $\rho_{2\pi^0}$ gives further confidence in the use of the theoretical distributions in determining the parity of the neutral pion.

B. Pion Parity

Of the 146 measurable events with no correlation angle smaller than 3°, 112 survived criterion (3). It is now possible to plot the number of these events against the angle between the two plane normals. This is shown in Fig. 8. The result can be fitted to the distribution $1 + \alpha \cos 2\varphi$, with $\alpha_{exp} = - 0.12 \pm 0.15$. This agrees very well with the theoretical expectation for the pseudoscalar pion $\alpha_{th}^{ps} = -0.18$, and misses the scalar theoretical expectation $\alpha_{th}^{s} = + 0.18$ by two standard deviations.

The statistical validity of this result can be considerably strengthened by making use of additional information. The theoretical asymmetry parameter is a sensitive function of the other measured variables, that is the x's and y's of each of the two pairs. In general, $\alpha_i(x_1y_1,x_2y_2)$ for event i is large if the angles between the members of the pairs are large and the energies divide up nearly equally. In any case, α_i^s and α_i^{ps} can be computed according to (2b) for each event for scalar and pseudoscalar pions, respectively, (the difference is one of sign only). The statistically best procedure is then to compute the likelihood functions $L^s = \prod_{i=1}^{112} (1 + \alpha_i^s \cos 2\varphi_i)$ and $L^{ps} = \prod_{i=1}^{112} (1 + \alpha_i^{ps} \cos 2\varphi_i)$.

We find then,

$$\frac{L^{ps}}{L_s} = \frac{1535}{1} .$$

This likelihood ratio corresponds to 3.3 standard deviations.

We present here yet another statistical analysis which seems to us more transparent than the likehood method, but is equivalent. The statistical value of a particular event in discriminating between the two signs in the asymmetry $1 \pm \alpha \cos 2\varphi$ is proportional to α^2. We have, therefore, weighted each event by α^2 and formed $\bar{\alpha}_{exp}$, $\bar{\alpha}_{exp} = \frac{2}{n} \sum_i^n \alpha_i^2 \cos 2\varphi_i$ where it should be kept in mind that α_i is the function (2b) evaluated for this event, and φ_i is the measured angle between the planes. $\bar{\alpha}_{exp}$ is the asymmetry parameter which will fit Fig. 9. The theoretically expected value is $\bar{\alpha}_{th} = \frac{\sum_i \alpha_i^3}{\sum_i \alpha_i^2}$.

The statistical error is $\frac{\sqrt{\sum \alpha_i^4}}{\sum \alpha_i^2}$ standard deviations.

In this way we find

$$\bar{\alpha}_{exp} = -0.41 \pm 0.24$$

$$\bar{\alpha}^{s}_{th} = +0.47$$

$$\bar{\alpha}^{ps}_{th} = -0.47$$

The experimental results are, therefore, in good agreement in the case of pseudoscalar pions, and fail in the scalar case by 3.6 standard deviations.

C. Time Reversal

It has recently been noted by Bernstein and Michel[10] that the internally converted γ-rays from π^o decay may offer a test of the invariance of the strong and electromagnetic couplings under time reversal. The predicted correlation, using normal electrodynamics but lifting the restrictions of time reversal, is of the form

$$P_t = A(x_1y_1x_2y_2)\left\{1 - \alpha(x_1x_2y_1y_2)\cos 2\varphi + C\alpha(x_1x_2y_1y_2)\sin 2\varphi\right\}$$

where A and α are as previously defined and C = 0 if and only if the interaction is invariant under time reversal. The last term vanishes if one folds the experimental distribution into the interval $0^\circ < \varphi < 90^\circ$, therefore the region examined was $0 < \varphi < 180^\circ$. The angle φ was defined as follows

$$\cos\varphi = (1^- \times 1^+)\cdot(2^- \times 2^+)$$

The result, $\langle C\rangle = -0.77 \pm 0.53$ is inconclusive due to the large error.

V. CONCLUSION

We have presented here measurements on the decay $\pi^0 \rightarrow 2e^+ + 2e^-$ which have been interpreted to yield a measurement of the pion parity. The analysis depends on a) invariance arguments due to Yang and originally presented for the two photon decay of the π^0; and b) on the validity of electrodynamics as applied to this problem. The validity of the invariance arguments need hardly be discussed; if they are not valid, then the parity is also without meaning. The electrodynamical theory is by now extensively verified. Nevertheless, we have presented here several additional experimental checks of the theory; namely the predictions of the theory for those features of the internal conversion of π^0 decay which are not directly used in the parity determination. These checks are:

1) The angular and energy distributions for the leptons in the decay $\pi^o \rightarrow \gamma + e^+ + e^-$.

2) The angular and energy distributions for the individual pairs in the decay $\pi^o \rightarrow 2e^+ + 2e^-$.

3) The distribution in $\alpha(x_1y_1, x_2y_2)$ for the decay $\pi^o \rightarrow 2e^+ + 2e^-$.

4) The absolute rates for the two internal conversion processes.

In all cases, agreement with the theory is good and we have a high level of confidence in the theory for predicted correlations.

On the basis of 112 analyzable events out of a total of 206, we find then a correlation which agrees very well (0.25 standard deviations) in the case of a pseudoscalar pion, and disagrees with the correlations predicted for the scalar case by 3.6 standard deviations. This reinforces the long standing result from the π^- capture in deuterium.[1]

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FIGURE CAPTIONS

Fig. 1 Photograph of a typical double internal conversion.

Fig. 2 Differential distribution in y for single Dalitz pairs.

Fig. 3 Differential distribution in x/μ for single Dalitz pairs.

Fig. 4 Pairing of electrons and positrons.

Fig. 5 Differential distribution in y for double Dalitz pairs.

Fig. 6 Differential distribution in x/μ for double Dalitz pairs.

Fig. 7 Experimental distribution in α for 112 double Dalitz pairs.

Fig. 8 Plot of the frequency distribution of the angle φ between planes of polarization. Included are two curves corresponding to $\alpha = \pm 0.18$.

Fig. 9 Plot of the weighted frequency distribution of the angle φ between planes of polarization.

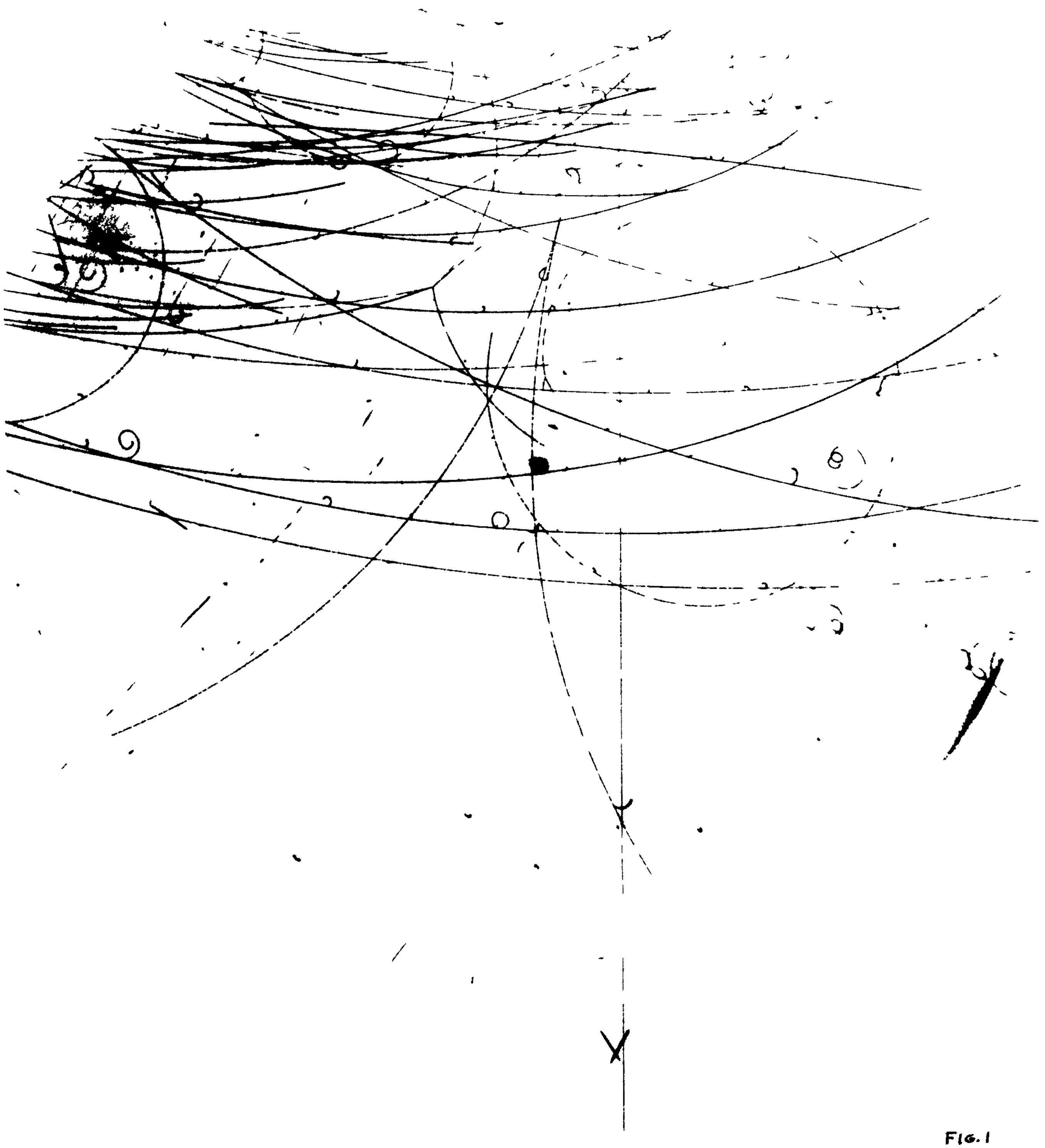

FIG. 1

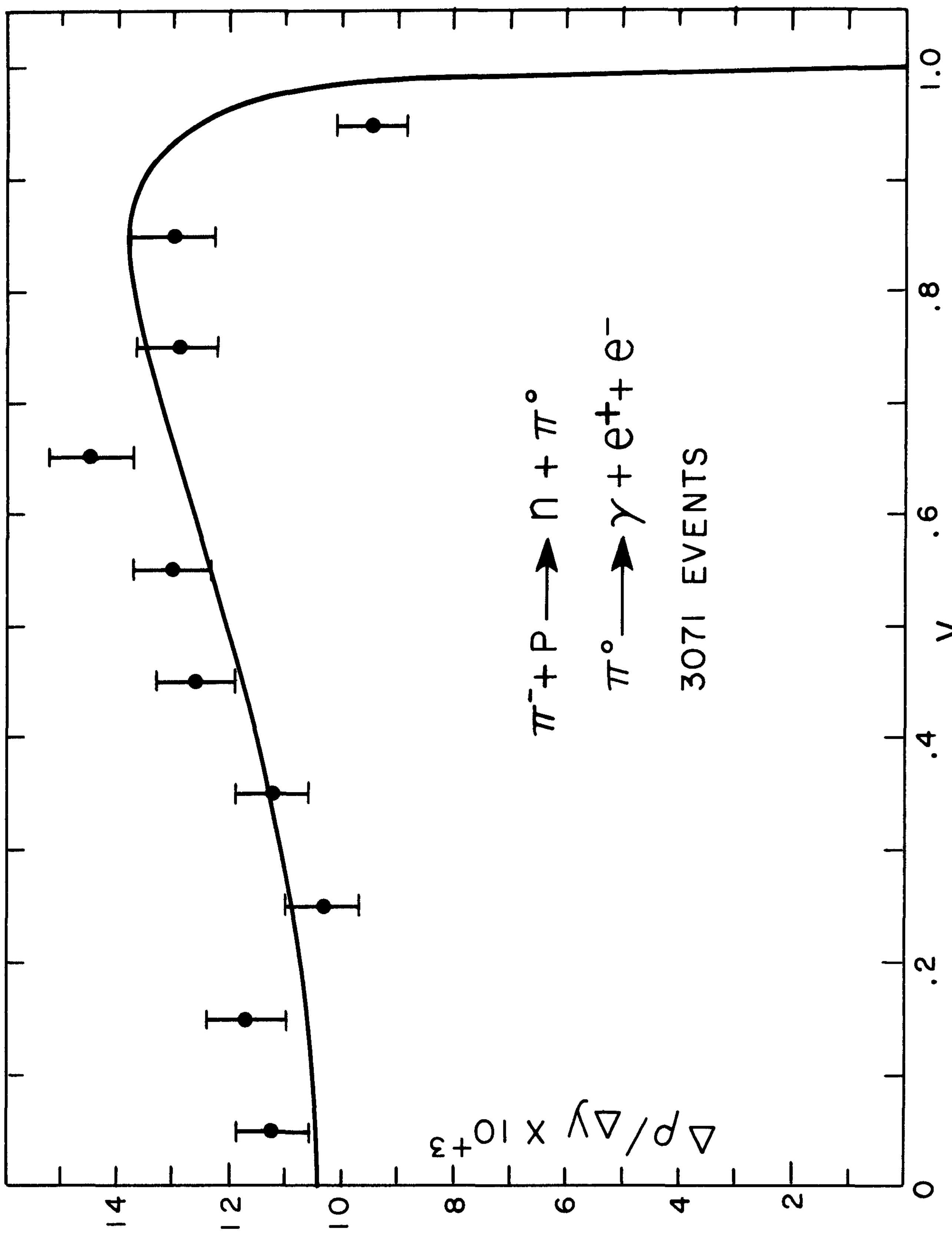

FIG 2

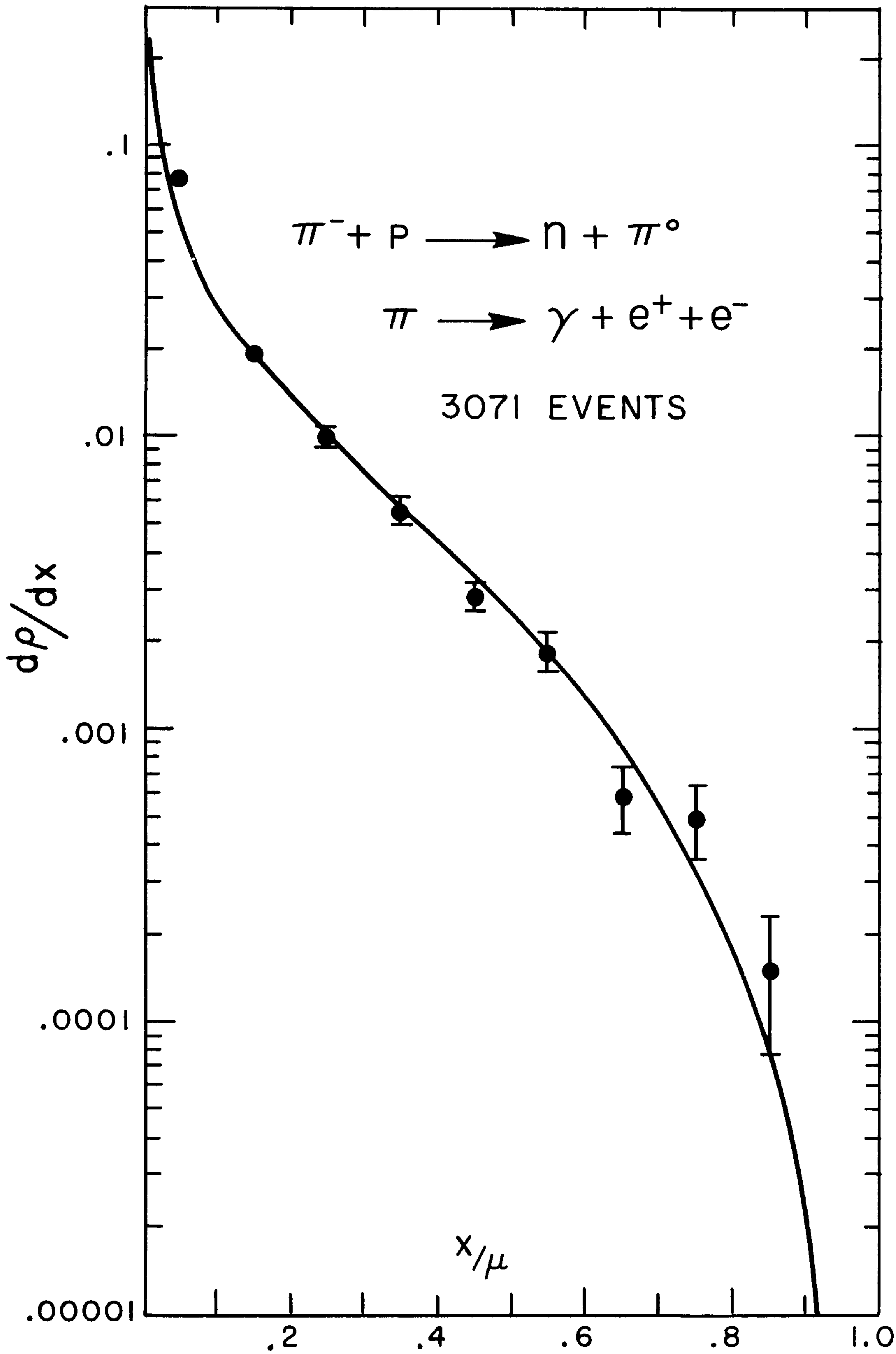

FIG 3

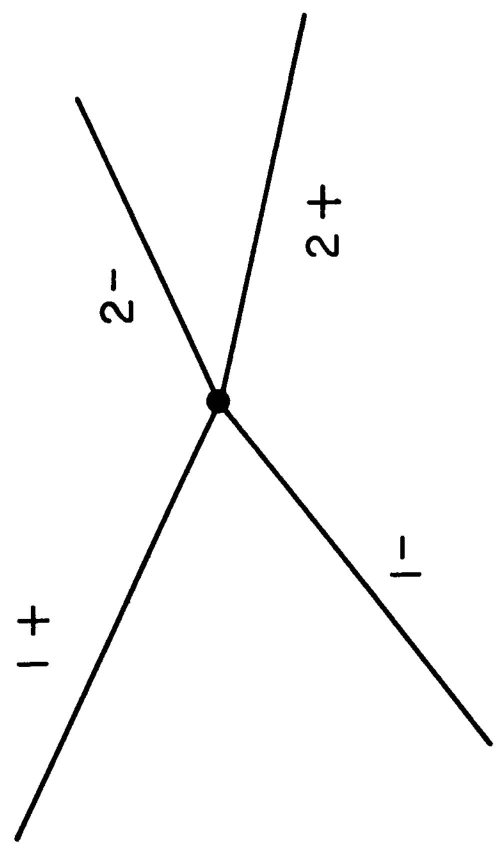

FIG 4

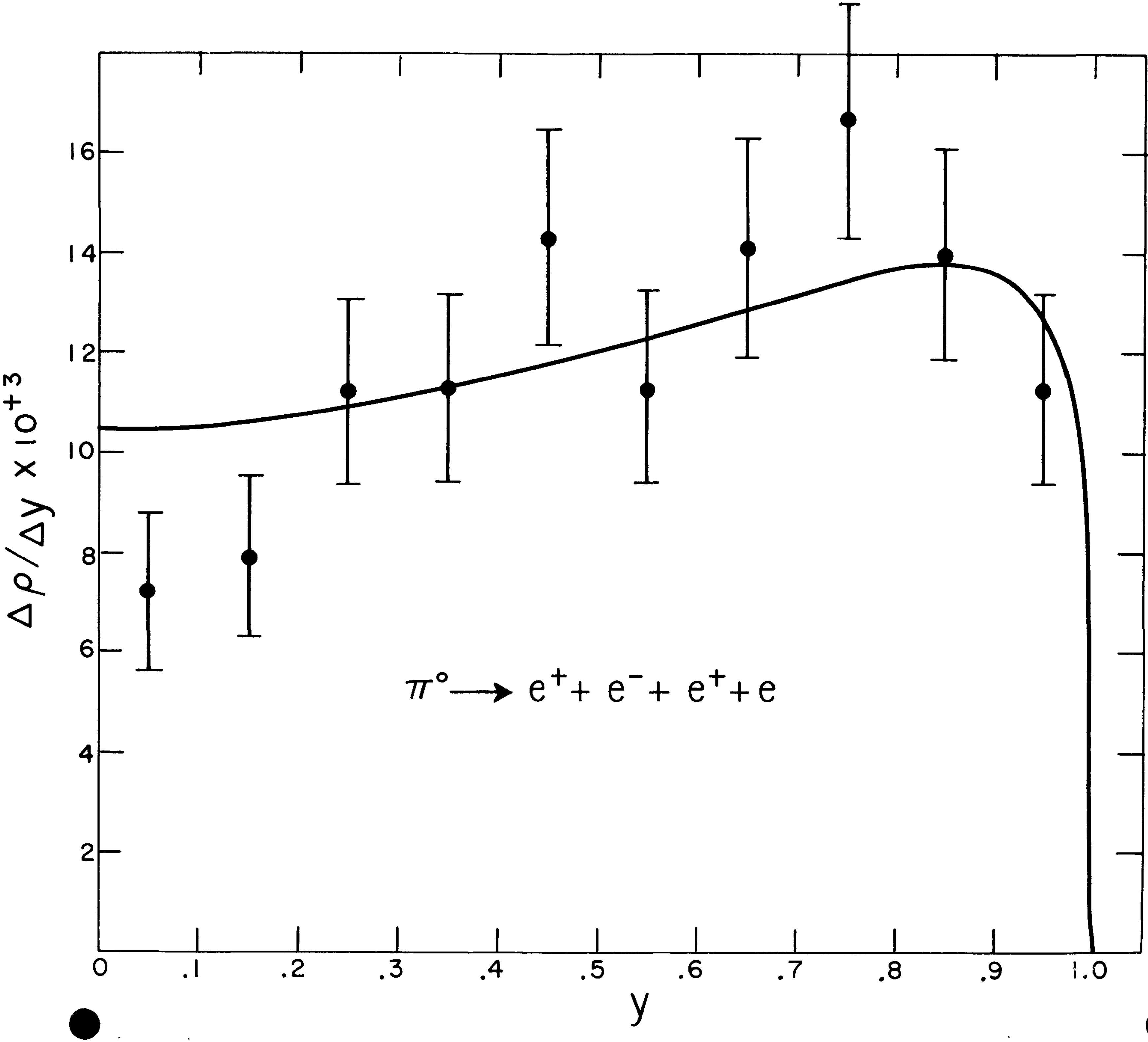

FIG 5

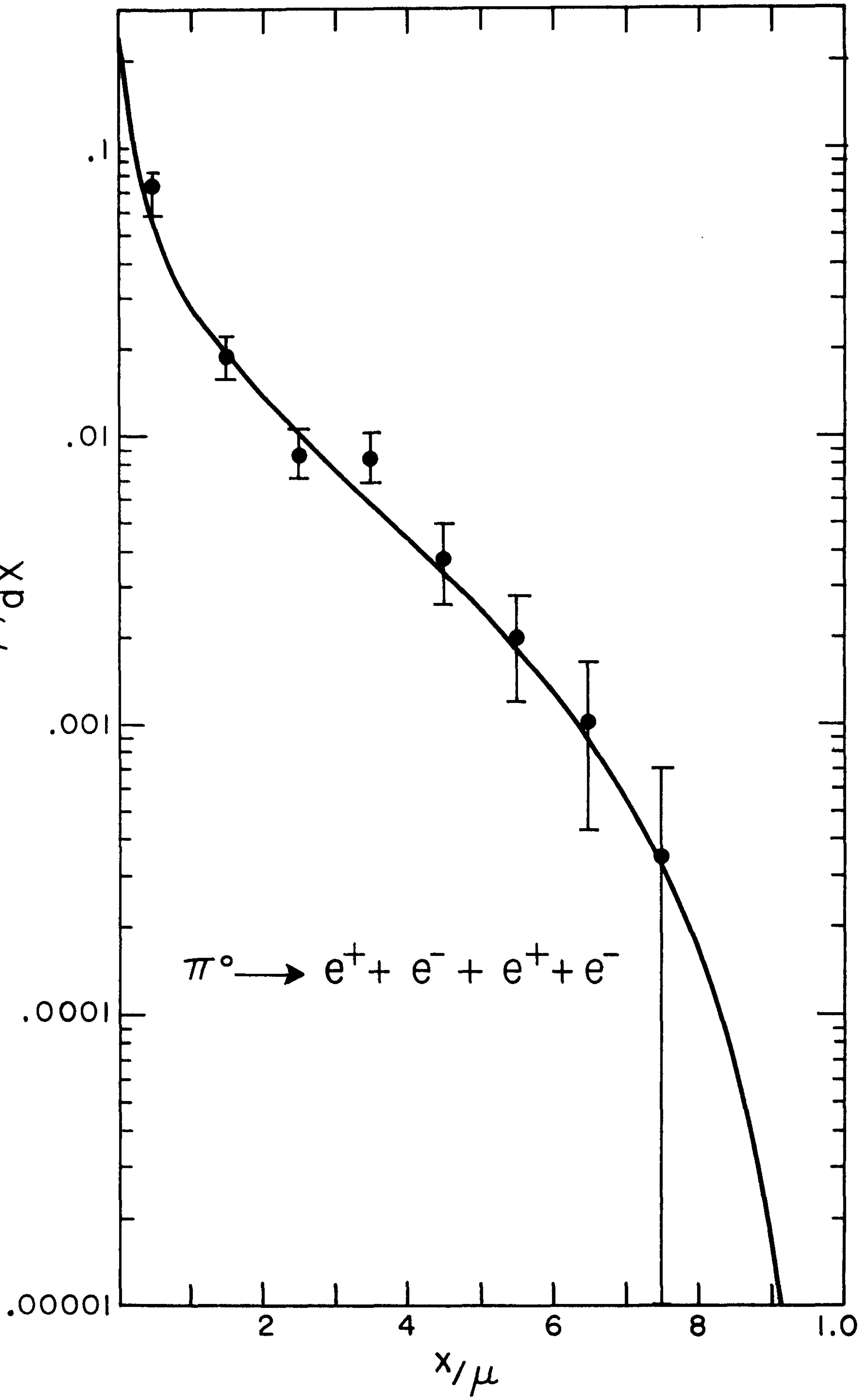

FIG 6

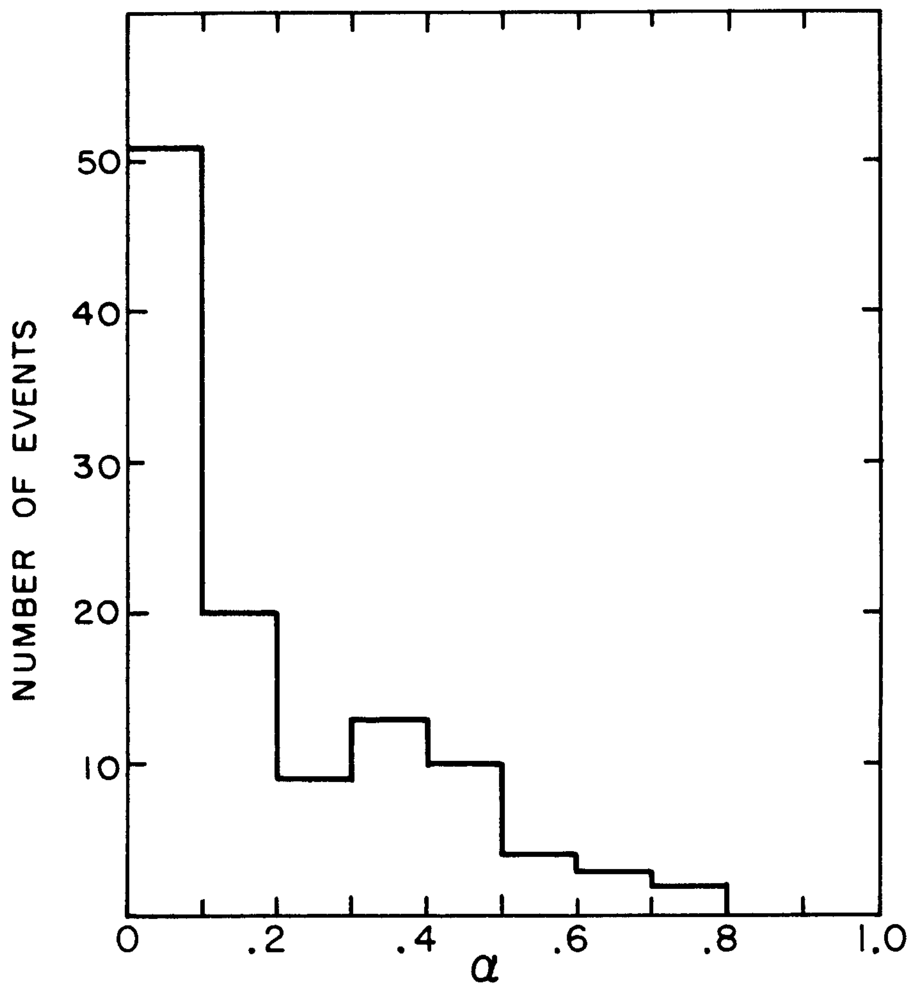

FIG 7

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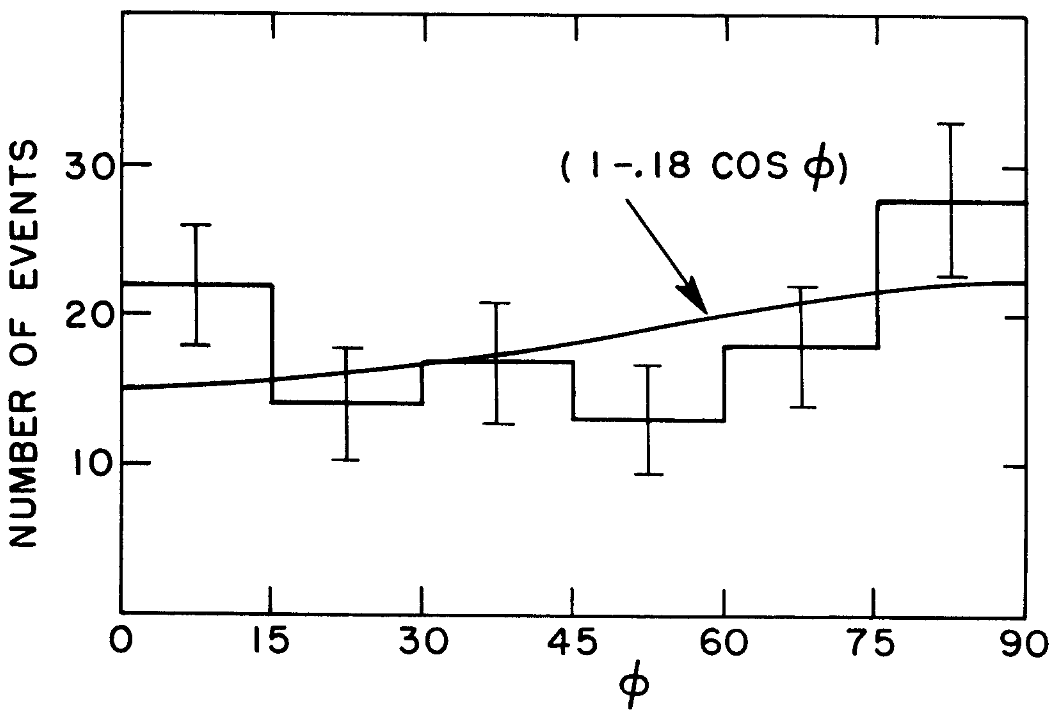

FIG 8

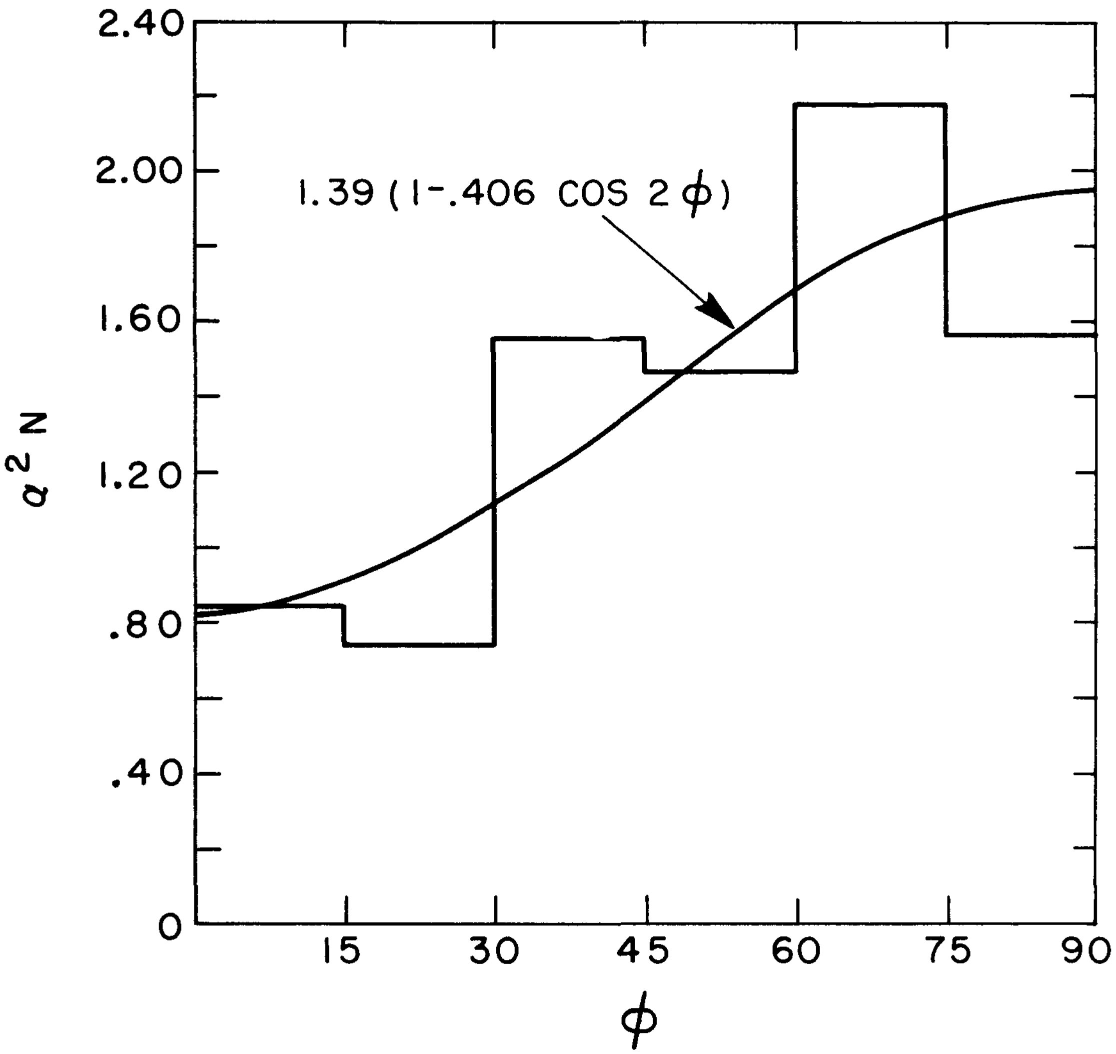

FIG 9