CONF-9010321--1

DE91 008316

TIME, DYNAMICS AND CHAOS*)

Integrating Poincaré's "Non-Integrable Systems"

Ilya Prigogine**)***)

I. Questioning Time

Time has always haunted man. Time is indeed our fundamental existential dimension. It has fascinated philosophers as well as scientists. It has often been stated that science has solved the problem of time. Is this really true? Indeed, a very fundamental property of the basic equations of physics, be it classical physics or quantum physics, is time reversibility. We may, in these equations, replace t by -t without changing the form of these equations. In contrast, in the macroscopic world, we deal with irreversible processes: +t and -t do not play the same role. There exists an "arrow of time." We come, therefore, to the strange conclusion that in the microscopic dynamic world, there would be no natural time ordering in contrast of what happens in the macroscopic world. For example, if we consider two positions of a pendulum, as represented in Figure 1, we cannot say which position comes earlier.

Figure 1

*) Presentation at Nobel Conference XXVI, Gustavus Adolphus College, Saint Peter, Minnesota, October 2 and 3, 1990.

**) The Solvay Institutes

***) Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin

With classical dynamics, time has lost its direction. Similarly, in quantum mechanics, we cannot speak about "older" or "younger" wave functions. But can this be the whole story? How can the time emerge from a time-reversible world? This conflict has become quite evident since the formulation by Clausius in 1865 of the well-known second law of thermodynamics. Clausius stated, "The entropy of the universe is increasing." This was the birth of evolutionary cosmology.

Figure 2

For every isolated system, entropy can only increase. Entropy expresses, as Eddington used to say, "the arrow of time." (The formulation of thermodynamics was the result of the work of engineers and physical chemists. The great mathematicians and physicists of this time considered it as the best as a useful practical tool however without any functional significance. The first to ask the question of the relation between entropy and the microscopic equations of motion was Boltzmann. Boltzmann was one of the main founders of kinetic theory. He tried to explain the increase of entropy as the result of molecular collisions leading to molecular disorder (the Maxwell velocity distribution law). Boltzmann's approach is still of great importance today as it leads for dilute gases to results that are in excellent agreement with experiment. Still, Boltzmann was defeated, as people were quick to point out to him that his results clashed with dynamic time reversibility (see i.e.[1]). Boltzmann was like a man in love with two women. He could not choose between his conviction that time

DISCLAIMER

irreversible evolution was an essential aspect of nature, and his confidence in the classical equations of motion which seem to prevent the existence of a privileged direction of time. I cannot go into details about this question, but let me stress that one of the aims of this lecture is to show that Boltzmann was right, but this involves quite recent results in which modern chaos theory plays an essential role. For Boltzmann's generation, as well as for the generations that followed, the conclusion of this debate was that the arrow of time was not in nature, but in our mind. Einstein's saying, time [as irreversibility] is an "illusion," is well known.[1]

I always found it curious that this conclusion did not trigger a crisis in science. How can we deny the existence of a privileged direction of time? As Popper wrote "this would brand uni-directional change as an illusion. This would make our world an illusion and with it, all our attempts to find more about our world." The ambition of classical science was to describe the behavior of nature in terms of universal, time-reversible laws. It is interesting to reflect on the relation between this ambition and the theological concepts that prevailed in the 17th century. For God, there is, of course, no distinction between past, present, and future. Is science not bringing us closer to God's like view of the universe?[2] This ambition of classical science was never realized. Often, science seemed close to this goal, and every time something failed. This gives a dramatic form to the history of western science. As you know, quantum mechanics is based on Schrödinger's equation, which is time reversible, but it had to introduce the measurement process and with it to attribute a fundamental role to the observer to obtain a consistent description. General relativity started as

a geometrical, "timeless" theory, to discover the need for some initial singularity or instability to obtain a consistent description of the cosmological evolution of our universe. To describe nature, we need both laws and events, and this, in turn, implies a temporal element, which was missing in the traditional presentation of dynamics including quantum theory and relativity. Last year's Nobel conference in Minnesota had the provocative title, "The End of Science?". I don't believe we can speak about the end of science, but indeed we come to the end of a certain form of rationality associated to the classical ideology of science. As I want to show here, in the building up of this new scientific rationality, non-equilibrium physics and "chaos" certainly will play an essential role.

II. The Time Paradox Builds Up

The 20th century is characterized by the discovery of quite unexpected features in which the arrow of time is essential. Examples are the discovery of unstable elementary particles and of evolutionary cosmology. I would like, however, first to emphasize in this lecture processes involving a macroscopic scale such as studied in non-equilibrium physics. A first remark: Contrary to what Boltzmann believed, irreversibility plays a constructive role. It not only is involved in processes leading to disorder, but also can lead to order This already appears in very simple examples such as presented in Figure 3. Consider two boxes containing two components, say hydrogen and nitrogen. If the boxes were at the same temperature, the proportion of these two components would be the same in the two compartments. If, on the contrary, we establish a temperature difference, we observe that the concentration of one of the components, say hydrogen, becomes larger in the compartment that is at the higher temperature.

Figure 3

The disorder associated with the flow of heat is used in this experiment to create "order". This is quite characteristic. Irreversibility leads both to order and disorder. A striking example is the case of chemical oscillations. Suppose we have a chemical reaction that may transform "red" molecules into "blue" ones and vice versa. It has been shown both theoretically and experimentally that, far from equilibrium, such a reaction may present a time-periodic behavior. The reaction vessel becomes in succession red, then blue, and so on. Let me emphasize how unexpected the appearance of chemical coherence is. We usually imagine chemical reactions as the result of random collisions between the molecules. Obviously, this cannot be the case far from equilibrium. We need long-range correlations to produce chemical oscillations. When we push such systems further away from equilibrium, the oscillations may become quite irregular in time. One then speaks about "dissipative chaos"; however, I shall not go into more details about this subject, which is treated adequately in many texts.[3][4] What is important is that irreversibility leads to new spacetime structures (which I have called "dissipative structures"), and which are essential for the understanding of the world around us. Therefore, irreversibility is "real". It cannot be in our mind, and we have to incorporate it in one

way or another in the frame of microscopic dynamics. Recently, there have been many monographs dealing with this problem, but I like to mention here the excellent introduction due to Peter Coveney and Roger Highfield, *The Arrow of Time*.[5] In this book, they called this problem, "time's greatest mystery."

But how to go beyond this paradox?

In the work done by my colleagues and me, we have followed the idea that the arrow of time must be associated with <u>dynamical instability</u>. Let me first present a very simple example of an unstable dynamic system, the so-called Baker transformation. See Figure 4. We consider a square. We squash it and put the right part on the top of the left as seen on Figure 4. This leads to a progressive fragmentation of the surface of the square. This is obviously an unstable dynamic system, as two points as close as one wants will finally show up in distant stripes. Such a system can be characterized by a Lyapounov exponent:

$$(\delta x)_t = (\delta x)_0 \exp(\lambda t)$$

Figure 4

The distance $(\delta x)_t$ between two neighboring trajectories increases exponentially with time. The coefficient, λ, is called the Lyapounov exponent, and is, in the case of the Baker transformation, equal to lg2. The existence of a positive Lyapounov exponent is characteristic of "chaotic" systems. There exists, then, a temporal horizon beyond which the concept of trajectory fails, and a probabilistic description is to be used. This is all well known. What I want to emphasize, however, is that in addition to the kinematic time t, we can introduce for such systems a "second, internal" time, T, which measures the number of shifts required to produce a given partition. For example, starting from the state (a) of Figure 4, we need two shifts to obtain state (c). As has been shown by our group, especially in the work of Prof. Misra,[6] this internal time is represented by an operator that leads to a non-commutative algebra very much like one we are using in quantum mechanics. Once you have the internal time, it is easy to construct an entropy and therefore to associate to the Baker transformation an arrow of time.

It is important to notice that the internal time refers to a global property as expressed by the partitions of the square. It is a "topological" property. It is only in considering the square as a whole that you can associate to it a given internal time or an "age." It is like when you look at some person. The age you will attribute to him does not depend on a specific detail of his body, but results from a global judgment. A detailed presentation of the Baker transformation and its relation to the arrow of time can be found in my book with Prof. Nicolis.[4] However, the Baker transformation corresponds to a highly idealized situation, and it is not clear on this basis why the arrow of time would be so prevalent in nature as testified by the universal validity of the second law of thermodynamics. That is the problem to which I want to turn now.

III. Poincaré's Theorem and the Science of Chaos--Large Poincaré Systems

In 1889, Poincaré asked a fundamental question.[7] [8] I should mention that his question was not formulated in these terms, but this is the formulation I shall use for the sake of the discussion. Poincaré asked if the physical universe is isomorphic to a system of non-interacting units. As it is well known, the energy (the "Hamiltonian" H) is generally formed by the sum of two terms, the kinetic energy of the units involved and the potential energy corresponding to their interactions. Therefore, Poincaré's question was, "Can we eliminate the interactions?"

This is indeed a very important question. If Poincaré's answer had been yes, there could be no coherence in the universe. There would be no life, and no Nobel Conferences. So it is very fortunate that he proved that you cannot, in general, eliminate interactions; moreover, he gave the reason for this result. The reason is the existence of resonances between the various units.

Figure 5

Everybody's familiar with the idea of resonance. This is the way the children learn to swing. Let's formulate more precisely Poincaré's question. We start with a Hamiltonian of the form,

$$H = H(p,q) \qquad (3.1)$$

where p,q are the momenta and the coordinates. We then ask the question if we can reduce it to the form

$$H = H(J) \qquad (3.2)$$

where J are the new momenta (the so-called action variables). In this form, the Hamiltonian depends only on the momenta. To perform the transformation from (3.1) to (3.2) Poincaré considered the class of transformations which conserve the structure of the Hamiltonian theory (so-called canonical or unitary transformations). More precisely, Poincaré considered Hamiltonians of the form

$$H = H_0(J) + \lambda V(J,\alpha) \qquad (3.3)$$

where λ is the coupling constant and V is the potential energy which depends both on the momenta J and the coordinates α (called the angle variables). For two degrees of freedom the potential can be expanded in a Fourier series

$$V(J_1,J_2,\alpha_1,\alpha_2) = \sum_{n_1,n_2} V_{n_1,n_2}(J_1,J_2)\, e^{i(n_1\alpha_1+n_2\alpha_2)} \qquad (3.4)$$

where n_1,n_2 are integers. The application of perturbation techniques leads then to expressions of the form

$$\frac{V_{n_1,n_2}}{n_1\omega_1+n_2\omega_2} \qquad (3.5)$$

with the frequencies ω_i defined as $\omega_i = \partial H_0/\partial J_i$. Here we see the dangerous role of resonances (or "small denominators")

$$n_1\omega_1 + n_2\omega_2 = 0 \qquad (3.6)$$

Obviously we expect difficulties when (3.6) vanishes while the numerator in (3.5) does not. This has been called by Poincaré[7] the "fundamental difficulty of dynamics". We come in this way to Poincaré's classification of dynamical systems.[7)8)10)] If there are "enough" resonances, the system is "non-integrable". A decisive progress in our understanding of the role of the resonances has been achieved in the 50's by Kolmogorov, Arnold, and Moser [the so-called KAM theory]. (See, i.e., [3].) They have shown that if the coupling constant in (3.3), λ, is small enough, [and also other conditions which I shall not discuss here are satisfied,] "most" trajectories remain periodic as in integrable systems. This is not astonishing. Formula (3.6) can be written as

$$\frac{\omega_1}{\omega_2} = -\frac{n_2}{n_1} \qquad \text{a rational number}$$

Now rationals are "rare" as compared to irrationals. However, whatever the value of the coupling constant λ, there appear now in addition, random trajectories characterized by a positive Lyapounov exponent and therefore by "chaos". This is indeed a fundamental result, since it is quite unexpected to find randomness at the heart of dynamics, which was always considered to be the stronghold of a deterministic description. However, it should be emphasized that the KAM theory has not solved the problem of the integration of Poincaré's non-integrable systems. The statement by Arnold that dynamical systems with even only two degrees of freedom, lie beyond our present mathematics has been widely quoted.

But there is a class of dynamical systems we call large Poincaré systems (LPS), for which we may indeed eliminate entirely Poincaré's divergences and therefore indeed "integrate" a class of Poincaré's "non-integrable systems. This result is the outcome of years of research with my colleagues in Brussels and Austin[9]; however, it is only recently that the problem of the integration of large Poincaré systems has been solved. I want to acknowledge from the start the fundamental contributions of Tomio Petrosky as well as of Hiroshi Hasegawa and Shuichi Tasaki.[10)-14)]

First, what is a large Poincaré system? It is a system with a "continuous" spectrum. For example, the Fourier series in formula (3.4) has now to be replaced by a Fourier integral. The resonance conditions takes then a new form. The resonance conditions for a small system with an arbitrary number of degrees of freedom are (see 3.4)

$$n_1\omega_1 + n_2\omega_2 + n_3\omega_3 + \dots = 0 \qquad (3.7)$$

where the n_i are integers. As mentioned, the resonance conditions express the existence of rational relations between frequencies. For large Poincaré systems, condition (3.4) has to be replaced by

$$k_1\omega_1 + k_2\omega_2 + k_3\omega_3 + \dots = 0 \qquad (3.8)$$

where the k_i are real numbers. Now resonances are "everywhere". The situation becomes similar to that in the Baker transformations, where also almost all motions are random motions. Moreover, large Poincaré systems are characterized by interactions involving integrations of resonances (examples follow). Before I shall consider examples, let me emphasize that the idea of large Poincaré systems remains meaningful in quantum mechanics. The frequencies ω_i become then energy levels. For small systems, the resonance condition (3.7) would correspond to accidental "degeneracies." But for large Poincaré systems, we have a continuous spectrum and the situation becomes quite similar to that in classical mechanics. (Large Poincaré systems have a surprising generality. We meet them everywhere both in classical and in quantum physics. Let me present two examples. The interaction between matter and electromagnetic fields leads to the emission of radiation (see Figure 6).

Figure 6

The life time of the excited states is given first approximation by what physicists call Fermi's golden rule which involves an integration over resonances

$$\int dk |V_k|^2 \delta(\omega_k - \omega_1) \qquad (3.9)$$

where ω_k are frequencies associated to the radiation and ω_1 is the energy level associated to the unstable state. This is an example of integration of resonances. All many-body systems involving "collisions" are LPS (see Fig. 7) as collisions also involve resonances (see Section V).

Figure 7

Large Poincaré systems are not integrable in the usual sense because of the Poincaré resonances, but what we want to point out in this lecture is that we can integrate them through new methods eliminating all Poincaré divergences. This leads to a new "global" formulation of dynamics (classical or quantum). As we deal here with chaotic systems, we may expect new features in this formulation of dynamics. Indeed, we shall find, as compared with the dynamics of integrable systems, an increased role of randomness, and above all a breaking of time symmetry and therefore the emergence of irreversibility at the heart of this new dynamics. We, in a sense, invert the usual formulation of the time paradox. The usual attempt was to try to deduce the arrow of time from a dynamics based on time reversible equations. In contrast, we now generalize dynamics to include irreversibility.

We may summarize the situation as follows.

Diagram I

On top we have the class of integrable systems (classical or quantum). This is the main field explored by dynamics. The basic structure of the dynamic integrable systems is expressed by celebrated laws (or

principles) such as the action principle which states that the trajectory is such that some functional (the action) is minimum. This structure has been the starting point both for quantum mechanics and general relativity.

But Poincaré's theorem limits the class of integrable systems. The basic question is then: what happens next? How nature solves the problem of the small denominators. Computer calculations do not lead to infinities!

As mentioned, a first step was the KAM theory. The physical effect of resonances is the appearance of random motion. For LPS almost all motions are random. The remarkable fact is that we then again can "integrate" the equations of motion. But now the structure of dynamics becomes radically different from that of integrable systems. It is fascinating that we now have to deviate from the structure inherent in the dynamic scheme as associated with the classical tradition.

IV. Poincaré's Theorem and the Quantum Mechanical Eigenvalue Problem

The first example I want to consider refers to quantum mechanics. As is well known, quantum mechanics has led to a kind of revolution in our thinking. In classical mechanics, "observables" are represented by numbers. The new point of view, taken by quantum mechanics, is to represent observables by operators. For example, the Hamiltonian H now becomes the Hamiltonian operator H_{op}. To this operator we associate eigenfunctions u_n and eigenvalues E_n

$$H|u_n\rangle = \epsilon_n|u_n\rangle \qquad (4.1)$$

The operator, H_{op}, acting on the eigen function, $|u_n\rangle$, reproduces this function multiplied by the eigenvalue, ϵ_n. The eigenvalues correspond to the numerical values of the physical quantity associated to the operator, H_{op}. Once we have a complete set of eigenfunctions and eigenvalues, we have the "spectral" representation (we drop the subscript "op") associated to H

$$H = \Sigma\, \epsilon_n |u_n\rangle\langle u_n| \qquad (4.2)$$

Finding the spectral representation (or solving the eigenvalue problem) is the central problem of quantum mechanics; however, this problem has only been solved in a few simple situations and most of the time we have to resort to perturbation techniques. We may start, as in the Poincaré theorem, with a Hamiltonian of the form

$$H = H_0 + \lambda V \qquad (4.3)$$

where we suppose that the eigenvalue problem can be solved for the "unperturbed" Hamiltonian H_0. We look then for eigenstates and eigenvalues of H, which we could expand in powers of the coupling constant λ. It is here that contact with Poincaré's classification can be made. For non-integrable Poincaré systems, the expansion of eigenfunctions and eigenvalues in powers of the coupling constant leads to the Poincaré catastrophe due to the divergence associated to the

small denominators. The relation between Poincaré's theorem and the quantum eigenvalue problem has been studied in a recent paper by T. T. Petrosky and the author[10].

Let us again consider the problem of quantum transitions (see Fig 6).* When we try to solve this problem by conventional perturbation theory, we come to Poincaré's divergence associated in the example to denominators of the form

$$\frac{1}{\omega_1 - \omega_k} \qquad (4.4)$$

where ω_1 is the energy of the excited states and ω_k the energy of a mode of the radiation corresponding to wave vector k. To avoid the divergence, we have to give a meaning to the denominator in (4.4).

Here enters the basic element which makes Poincaré non-integrable systems "integrable" in a new extended sense. We introduce a "natural time ordering" of the dynamic states. To make clear what we mean, consider a trivial example. A stone can fall into a water pond and produce outgoing waves. We may also have the inverse situation in which incoming waves would eject a stone. (See Fig. 8.)

Figure 8

In fact, only one of the situations is realized: the natural time ordering is the falling stone first, the outgoing waves next. Similarly, to give a

* We in fact consider the simple version called the Friedrichs model in which virtual processes are omitted [15]

meaning to Poincaré's denominators, we have to time-order the dynamical states -- the unstable atomic state first, the emission of radiation later. This corresponds to Bohr's picture in which the radiation emitted by the atom corresponds to a retarded wave. More precisely, to the transition $l \to k$ we associate the denominator

$$\frac{1}{\omega_l-\omega_k-i\epsilon} \qquad (4.5)$$

and to the transition $k \to l$ the denominator

$$\frac{1}{\omega_l-\omega_k+i\epsilon} \qquad (4.6)$$

As we have shown[11][12], this simple rule leads to the elimination of all Poincaré divergence when we integrate over the wave lengths of the radiation. It is a standard procedure in theoretical physics to express the difference between past and future through "analytic continuation". The general reader may just accept that we modify the Poincaré denominators differently according to the type of process to which they are associated. We obtain in this way complex solutions of Schrödinger's situation. The eigenvalues contain now an imaginary part corresponding to damping and the eigenstates have a broken-time symmetry. We have chosen this simple example because in this case there exists a standard solution which is, however, not analytic in the coupling constant (the particle disappears from the spectrum[15]).

We can therefore compare our results with the standard treatment and see if our approach makes sense. We indeed recover all

known results, but in addition, as we have states with broken time symmetry, we can introduce a functional which plays the role of Boltzman's $\mathcal{H}$ function and which decreases monotonously when the particle emits the radiation and decays to the ground state. (See Fig. 9.)

Figure 9

We can also, at least as a thought experiment, perform a time-inversion at time t_0 after the start of the decay. The result is represented schematically on Figure 10. At the time of the inversion t_0, the $\mathcal{H}$-quantity has a jump $\sim \exp(\gamma t_0)$, where $1/\gamma$ is the life-time of the unstable state. Then $\mathcal{H}$ starts to decrease again, at $2t_0$ we have $\mathcal{H}(2t_0) = \mathcal{H}(t=0)$ and the decrease of $\mathcal{H}$ continues until the particle has decayed.

Figure 10

As we can associate an $\mathcal{H}$-function to the particle decay the decay becomes an irreversible process.

We may summarize what we have done as follows: to avoid Poincaré's catastrophe we have enlarged the type of transformations which lead from the eigenfunctions of the unperturbed Hamiltonian H_0 (see 4.3) to the eigenvalues of the full Hamiltonian. In more technical terms, the situation is as follows: Poincaré considered only canonical (or unitary) transformations which among other properties keep the eigenvalues of H real. We introduce more general transformations leading to complex eigenvalues. The specific choice of these transformation follows from our time ordering of the dynamical states.

The result is already of great interest in the perspective of the epistomological problems which plague quantum mechanics. Let us first remind that the basic equation of quantum mechanics is the Schrödinger equation for the wave function Ψ

$$i\frac{\partial\Psi}{\partial t} = H_{op}\Psi \tag{4.7}$$

The equation as shown in all textbooks on quantum mechanics is time reversible and deterministic. (We exclude some "pathological" cases related to weak interactions.) The physical interpretation of Ψ is that it represents a probability "amplitude." In contrast, the probability proper is given by

$$\Psi\cdot\Psi^{cc} = |\Psi|^2 \tag{4.8}$$

We shall come back in the next section to this transition from probability amplitudes Ψ to probabilities proper $|\Psi|^2$.

Quantum mechanics is probably the most successful theory of physics, and still, the discussions about its conceptual foundation have never ceased. I recommend a recent book by J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics.*[16] Are they quantum jumps? This is quite a controversial problem. Schrodinger's equation (4.7) describes a smooth evolution. How then to include quantum jumps. In addition, (4.7) is time symmetric: If there is spontaneous emission, there should

be also spontaneous absorption. The conventional Copenhagen interpretation is that quantum jumps result from our measurement. This would be rather strange, since all of chemistry and life are the result of quantum jumps. How then could life be as a result of our measurements? Our method solves this problem as it associates to the quantum jump an irreversible event which can only occur in (our) future.

Before we come back to these fascinating problems underlying the close connection between the conceptual foundations of quantum mechanics and dynamical instability, let us make the following remark.

The example we have treated is a very simple one as we could introduce a natural time ordering in the frame of the usual quantum description (the so-called "Hilbert space"). But in general, this is impossible (think about scattering where all states play a symmetrical role). Then we shall see in the next section, we have to introduce a natural time ordering on the level of the statistical description. This leads to the integration of LPS in quite general situations and to a new form of dynamics, which breaks radically with the past.

V. Poincaré's Theorem and a Statistical Formulation of Dynamics

From the example studied in Section IV, it should be clear how we may avoid Poincaré's catastrophe: It is through introducing into the theory a time ordering of dynamical states which leads to well-defined "regularization" procedures for the small denominators (see 4.5-4.6). But how to introduce this time ordering? Here as we shall see now, we have to turn to the statistical description. Curiously our approach validates the way Boltzmann more than a century ago approached kinetic theory of gases. But Boltzmann could not guess the emergence of the chaos theory and did not know that he was studying "non-integrable Poincaré systems." (He, as well as Maxwell, placed therefore his hopes in ergodic theory which is indeed useful for the understanding of equilibrium, but not for dynamical purposes.)

In the early days of statistical mechanics, Gibbs introduced a quite fundamental concept, the "Gibbs ensembles." Instead of considering single dynamical systems, he considered a large number of dynamical systems evolving in the phase space associated to the coordinates $q_1 ... q_N$ and moments $p_1 ... p_N$ of the particles forming each dynamical system (see Fig. 11).

Figure 11

The description is then, in terms of the probability distribution ρ in phase space

$$\rho(q_1 ... q_N, p_1 ... p_N, t) \qquad (5.1)$$

This description remains also meaningful for quantum systems. The probability distribution ρ is then called the "density matrix." Once we know ρ we can calculate both the velocity distribution of the particles as well as the correlations existing between the particles.

How then does time enter into this description?

Let us consider a classical gas. Particles collide and these collisions give rise to correlations. See Figure 12. First we have binary

correlations, then ternary correlations, and time going on, correlations involving more and more particles.

Figure 12

The formation of correlations is somewhat reminiscent to that of a couple which has a conversation (This would correspond to a collision). Even when the partners go away, the memory of their conversation remains. The information associated to this conversation is time going on, spreading out to more and more participants.

Suppose we look at a glass of water. In this glass of water, there is an arrow of time that will, in fact, persist forever and corresponds to the creation of new correlations involving an ever-increasing number of particles. According to the correlations which exist between the molecules, we can distinguish "young" water from "old!" Computer experiments have been performed recently that show that binary correlations appear very rapidly. Ternary correlations involve longer time scales and so on. This time oriented flow of correlations breaks the symmetry involved in the classical description. Let us go from state A (of a many-body system) with no correlations at $t=0$ to a state B, a time t involving multiple correlations (see fig. 13).

Figure 13

Obviously, the transition from A to B involves quite different physical processes, then the inverse transition, from B to A.

The time ordering of correlation has to be introduced into dynamics to avoid Poincaré's catastrophes: Binary correlations come before ternary ones and so on. We have therefore to describe dynamics in terms of the time evolution of correlations.

This corresponds to a different point of view from that of classical dynamics: the question is no more to study the positions and moments of each particle time going on but to follow the evolution of the relations between the particles.[17] In this conceptual framework we can avoid Poincaré's catastrophe by treating transitions to higher correlations as "future oriented" and transitions to lower correlations as "past-oriented" exactly as we have done it in 4.5-4.6.

Gibbs' ensemble theory leads to an equation for the time evolution of the density matrix ρ

$$i\frac{\partial\rho}{\partial t} = L\rho \qquad (5.2)$$

which is formally quite similar to the Schrödinger equation (4.7). L is the so-called Liouville operator, which can be expressed in terms of the Hamiltonian both in classical and quantum mechanics. As we mentioned, Poincaré's theorem deals with Hamiltonians of the form

$$H = H_0 + \lambda V \qquad (5.3)$$

This corresponds to a decomposition of the Liouville operator

$$L = L_0 + \lambda L_V \qquad (5.4)$$

To solve Liouville's equation (5.2) we need as in the Schrödinger case to solve the eigenvalue problem (see 4.1)

$$L|f_n\rangle = l_n|f_n\rangle \qquad (5.5)$$

For integrable systems, there is no problem. The Liouville's equation (5.2) is then of no special interest as the problem reduces to the usual dynamical problems (finding trajectories or wave functions). However, the problem changes radically for non-integrable systems. Then the Liouville equation describes the emergence of chaos due to the destruction of the invariants of motion associated to the unperturbed system.

Again, Poincaré's theorem prevents us from finding solutions of (5.5) through unitary transformations (preserving the reality of l_n) which we could expand in powers of the coupling constant λ. As already mentioned, we solve this difficulty by introducing a supplementary element into the theory: the time ordering of the correlation. We then obtain a complex eigenvalue problem that can be solved and which leads to damping and to irreversibility through the occurrence of $\mathcal{H}$-functions (see Section IV). This new dynamics has some distinct features which present a basic departure from the features of the dynamics of integrable systems as described in classical or quantum theory.

To understand in qualitative terms what happens, let us analyze more closely what is involved in the idea of "collisions." In fact, a collision corresponds already to a complex process in which particles come close, exchange energy through resonance and depart. We can visualize a collision as a succession of states bound by resonance (see [8]). In a Hamiltonian system (the case of hard spheres as a limiting case which will not be considered here) a collision is not an instantaneous point-like event but has an extension both in space and time.

As has been shown recently by T. Petrosky and the author, the spectrum of the Liouville operator L is essentially determined by the dynamics of the collisions. This implies a radical deviation from the usual methods of dynamics valid for integrable systems where the evolution can be resolved into a succession of instantaneous space-time events (remember Feynman diagrams).* For this reason the dynamics of LPS can only be formulated on the statistical level, as we cannot reduce it nor to trajectories in the classical case nor to wave functions, as in the quantum case. This deviation from the great traditions of dynamics not so astonishing: we deal here with an aspect of dynamics that is totally absent in integrable systems. It is, however, already present in the KAM theory but there the behavior is so complex that it defies any quantitative description (we have to use qualitative criteria for the collapse of resonant tori as the result of the coalescence of resonances). It is precisely the main progress realized by the study of LPS to present a simple description of the physical processes due to resonances and which lead to Poincaré's non-integrability.

* For the reader familiar with kinetic theory, let me mention that the traditional kinetic equations (the Fokker-Planck equations) contain second derivatives, this is precisely due to the description of the collision as a two-stage process.

Our approach has been confirmed by numerical calculations performed on simple examples of LPS. We may start with a statistical distribution (as close as we want from a point in phase space). We see then the system going to various stages corresponding to the appearance of Lyapounov instability (see 2.1), folding in phase space and then diffusion as due to "collisions."

The main point I want to emphasize again is that instabilities destroy the very notion of trajectory (or of wave function in quantum mechanics) as the basic description is now in terms of statistical ensembles.

Let us now present some concluding remarks.

VI. Concluding Remarks

The integration of Poincaré's non-integrable dynamical systems leads for LPS to a new form of dynamics encompassing irreversibility (broken time symmetry) and exhibiting an increased role of probability both in classical and quantum mechanics. The time paradox we have described in Section 2 is in this way eliminated (see Fig. 14).

Figure 14

In the "old" situation, we had to bridge the microscopic time reversible level to the macroscopic level equipped with an arrow of time (Fig. 14). But how can time arise from no-time?

Now (Fig. 15) we have a new microscopic level with broken time symmetry out of which through averaging procedures emerge the macroscopic dissipative level. The "old" microscopic level has become unstable.

Figure 15

This leads to a better understanding of the role of chaos. In fact, there exist two quite different manifestations of chaos. When we study macroscopic equations which include dissipation, such as the reaction-diffusion equation, or the Stokes-Navier equation for fluids, we are already facing situations for which the basic microscopic description belongs to LPS. In other words, the very existence of such equations presupposes "dynamic chaos". This is not astonishing; indeed, properties such as friction or diffusion involve exchange of energy through collisions. These macroscopic equations may lead to chaos (chemical chaos as turbulence). This dissipative chaos lies "on top" of the dynamic chaos. As we mentioned, dissipative chaos is part of self-organization as it appears in nonequilibrium and non-linear systems. Examples of chemical coherence are oscillating chemical reactions. In short, therefore, macroscopic order as manifested in non-equilibrium is the outcome of dynamical chaos. Even the approach to equilibrium becomes the result of dynamical chaos. In all these cases, therefore, we have "Order out of Chaos" (see[1]).

Figure 16

Let us also mention that LPS are evolving systems. Once initial conditions are given, they go through various stages such as described by Lyapounov exponents, diffusional processes ... However, irreversibility is not related to Newtonian Time (or its Einsteinian generalization) but to an "internal" time as expressed in terms of the <u>relations</u> between the various units which form the system (such as the correlation between the particles). We cannot stop the flow of correlations, as we cannot prevent the decay of unstable atomic states.

Nabokov has written: What is real cannot be controlled, what can be controlled is not real. This is also true here. In addition to solving the time paradox, the dynamical laws obtained through the integration of LPS lead to a number of consequences which go far beyond our initial motivation. We have already mentioned some relations with the epistomological problems of quantum mechanics in Section 4. We can now go further. As is well known, the basic quantity in quantum mechanics is the probability amplitude Ψ which satisfies Schrodinger's equation (4.7) but we measure probabilities! Therefore, we need an additional mechanism to go from "potentialities" as described by the wave function to "actualities" as described by probabilities. In his introduction to "The New Physics," Paul Davies[9] wrote, "At the rock bottom, quantum mechanics provides a highly successful procedure for predicting the results of observations on microsystems, but when we ask what actually happens when an observation takes place, we get nonsense! Attempts to break out of this paradox range from the bizarre, such as the many universes interpretation of Hugh Everett, to the mystical ideas of John von Neumann and Eugene Wigner, who invoke the observer's consciousness. After half a century of argument, the quantum observation debate remains as lively as ever. The problems of the physics of the very small and the very large are formidable, but it may be that this frontier -- the interface of mind and matter -- will turn out to be the most challenging legacy of the New Physics." It is interesting that the solution to this fundamental problem may come from dynamical instability and chaos as in our new dynamical description we deal directly with probabilities. In this case, the breaking down of the superposition principle of quantum mechanics in LPS is due to dynamic instability without any appeal to esoteric considerations, such as the many-world theory or the existence of the new universal constant leading to a collapse of the wave function for macroscopic systems. We come to a realistic formulation of quantum mechanics eliminating the appeal to any observer situated outside physics.

This century has been dominated by two new conceptual frameworks: quantum mechanics and relativity. As has been often emphasized, (see, i.e., M. Sachs[20]) the intrusion of subjectivistic elements through the measurement process leads to difficulties when we want to combine quantum theory and relativity. However, non-integrable dynamical systems are likely also to alter relativity as the basic dynamical events (the collisions) do no more correspond to instantaneous and localized space-time events.

I believe that we are therefore indeed at the beginning of a "New Physics." Until now, our view of nature was dominated by the theory of integrable systems, both in classical and quantum mechanics. This corresponds to an undue simplification. The world around us involves instabilities and chaos, and this requires a drastic revision of some of the basic concepts of physics.

Let me conclude by expressing my conviction that, in the future, the non-integrability theorem of Poincaré will be considered as a turning point somewhat similar to the discovery that classical mechanics lead to divergences when applied to the black-body radiation. These divergences had to be cured by quantum theory. Similarly, Poincaré's divergences have to be cured by a new formulation of dynamics in the sense I have tried to describe in a qualitative way in this presentation.

__Acknowledgments__

This work is the outcome of many years of work of the Brussels-Austin group. It is impossible to quote all those who have contributed, but my special gratitude goes to Cl. George, F. Mayné and T. Petrosky.

We want also to thank The University of Texas at Austin, The Solvay Institutes, the Department of Energy (DOE grant DE-FG05-88ER13897), the Welch Foundation (grant WELCH FDN F-365), and the European Communities Commission (contract no. PSS*0143/B).

References

1) Ilya Prigogine and Isabelle Stengers, Order out of Chaos, Bantam Books, New York, 1984.

2) This question is discussed in Ilya Prigogine and Isabelle Stengers, Entre le Temps et Eternité, Fayard, Paris, 1988.

3) See the excellent introduction by M. Tabor, Chaos and Integrability in Nonlinear Dynamics, J. Wiley, New York, 1989.

4) G. Nicolis and Ilya Prigogine, Exploring Complexity, W.H. Freeman, New York, 1989.

5) P. Coveney and R. Highfield, The Arrow of Time, Allen, London, 1990.

6) For a summary, see ref.[4].

7) H. Poincaré, Methodes Nouvelles de la Mecanique Céleste, Gauthier Villars, Paris, 1892, reprinted by Dover, New York, 1957.

8) For a simple presentation, see Ilya Prigogine, Non Equilibrium Statistical Mechanics, Wiley Interscience, New York, 1962.

9) Let us only mention a few references in addition to [8]:
Ilya Prigogine, From Being to Becoming, W.H. Freeman, New York, 1980.
R. Balescu, Equilibrium and Non Equilibrium Statistical Mechanics, Wiley, New York, 1985.
Cl. George, F. Mayné and I. Prigogine, *Adv. in Chemical Physics*, 61(223), 1985.

10) Tomio Petrosky and Ilya Prigogine, *Physica* 147A (439), 1988.

11) Hiroshi Hasegawa, Tomio Petrosky, Ilya Prigogine, and Suichi Tasaki, to appear in *Foundations of Physics*, 1990.

12) Tomio Petrosky, Ilya Prigogine and Suichi Tasaki, to appear in *Physica A*, 1990.

13) Ilya Prigogine, Tomio Petrosky and Suichi Tasaki, to be published.

14) Tomio Petrosky, Ilya Prigogine and Suichi Tasaki, to appear in *Physica A*, 1990

15) See, e.g., G. Barton, Advanced Field Theory, Wiley Interscience, New York, 1963.

16) J.S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, 1987.

17) The idea of "dynamics of correlations" was introduced in ref.[8].

18) H. Hasegawa and B. Saphir, to be published.

19) "The New Physics," edited by Paul Davis, Cambridge University Press, 1989.

20) M. Sachs, Einstein versus Bohr, Open Court, Illinois, 1988.

Legends

Fig. 1: No time ordering in classical or quantum dynamics.

Fig. 2: Clausius formulation of the second law of thermodynamics

Fig. 3: Thermal diffusion experiment (see text)

Fig. 4: Baker transformation (see text)

Fig. 5: Resonance between coupled oscillators

Fig. 6: Quantum transition

Fig. 7: Collisions (see text)

Fig. 8: Example of temporal ordering

Fig. 9: H-function associated to the decay of an unstable particle (see text)

Fig. 10: Time inversion experiment (see text)

Fig. 11: Gibbs ensemble

Fig. 12: Flow of correlations

Fig. 13: Breaking of time symmetry (see text)

Fig. 14: The time paradox (see text)

Fig. 15: Elimination of the time paradox (see text)

Fig. 16: Chaos and Dissipation

Diagram 1

Classical Mechanics

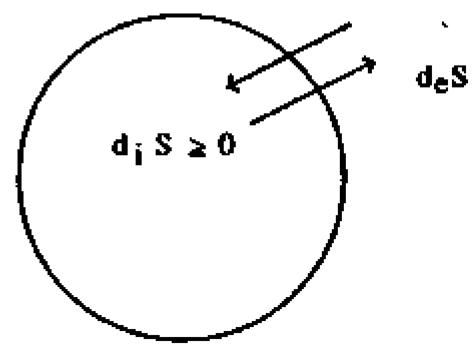
No time ordering

Quantum Mechanics

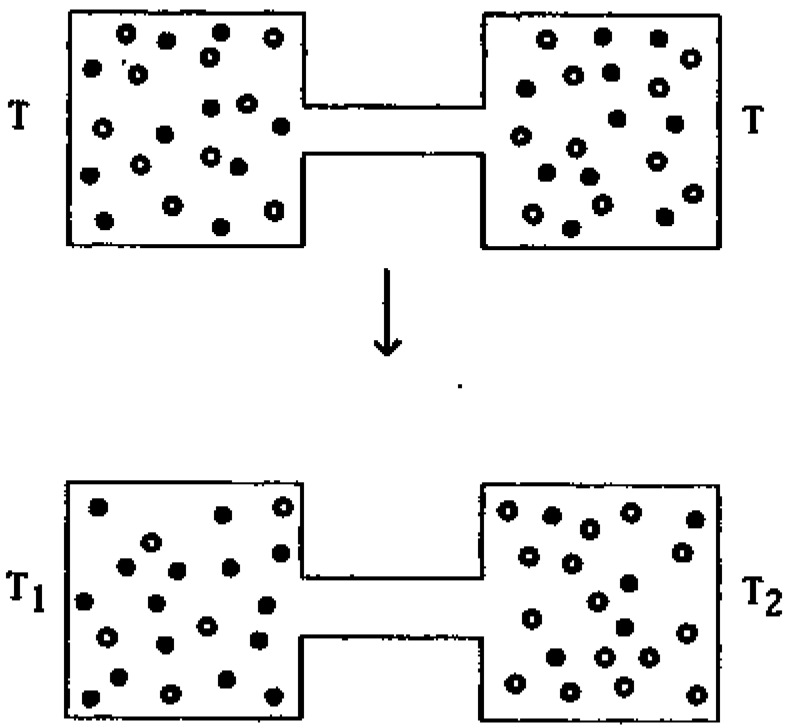
Ψ wave function $\begin{cases} \Psi(t_1) \\ \Psi(t_2) \end{cases}$

No time ordering

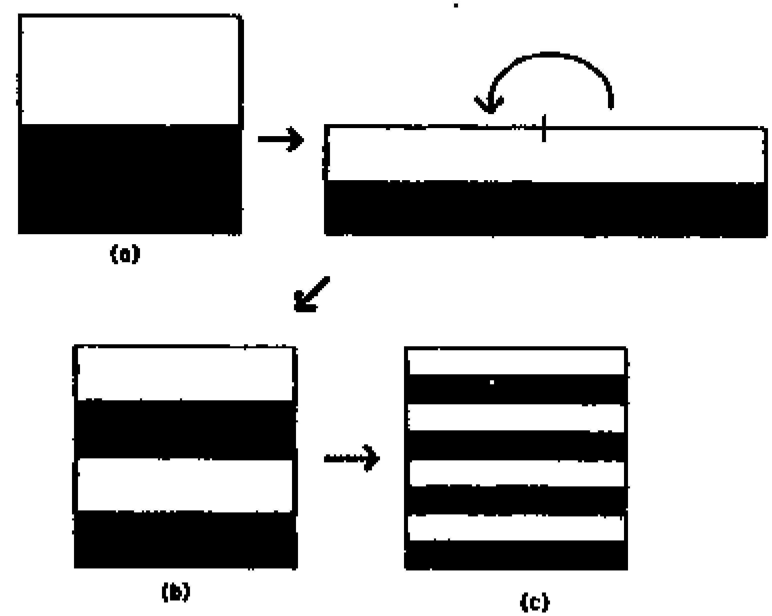
Clausius (1865)



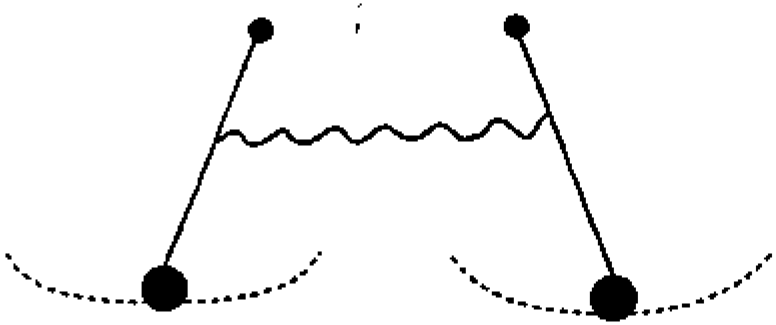
$$dS = d_eS + d_iS$$

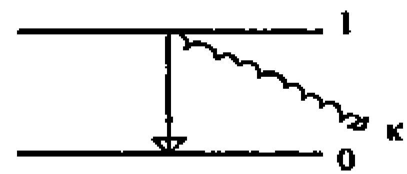


Baker Transformation



Resonances among coupled oscillators



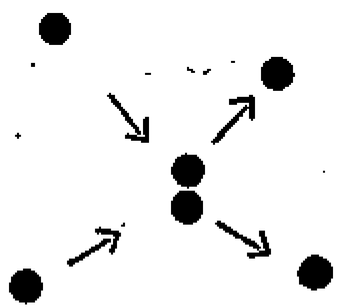


Quantum transition

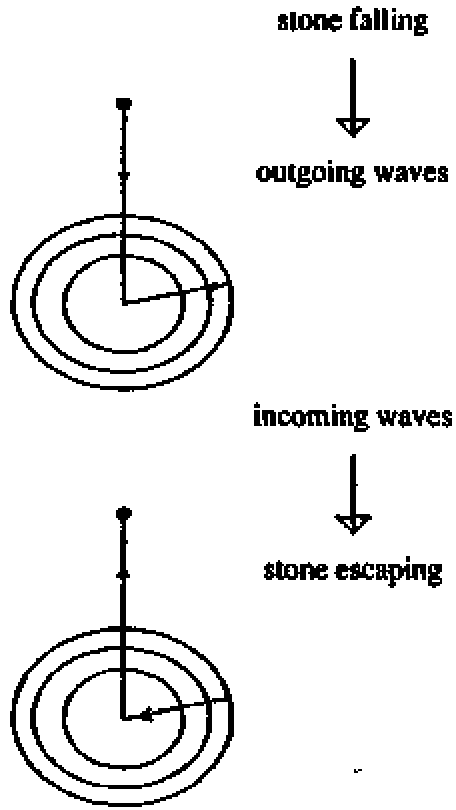
Integration over resonances

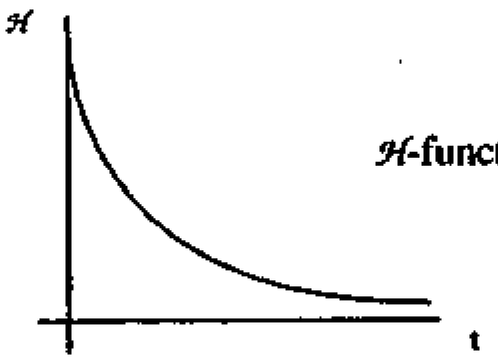
(– Fermi golden rule)

Collisions among gas molecules

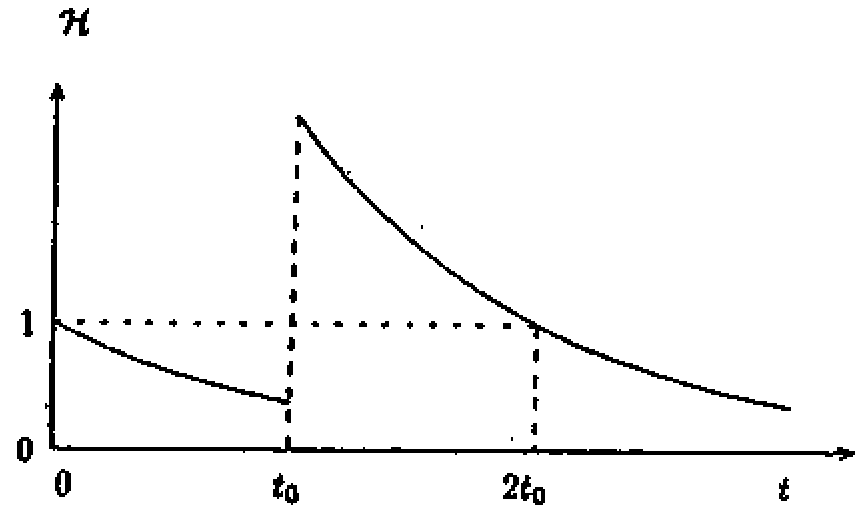


Collision – resonance



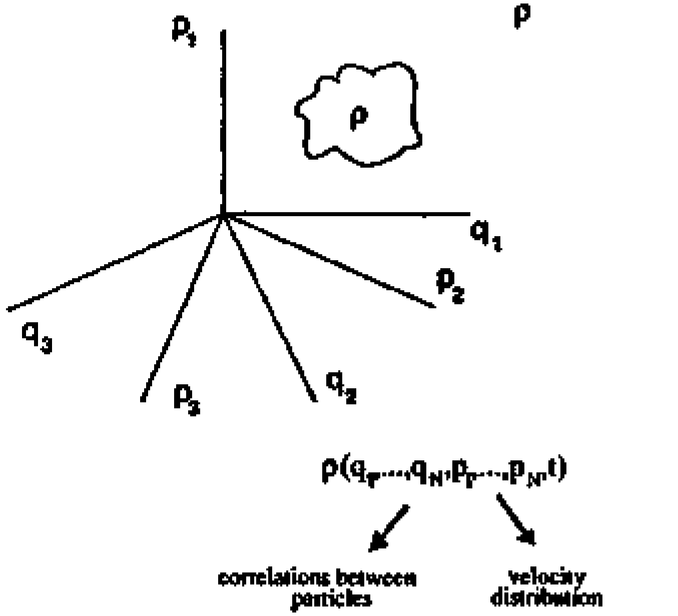


$\mathcal{H}$-function

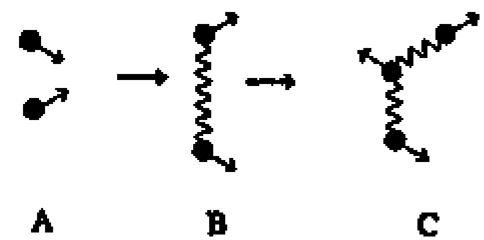


Statistical description
- Gibbs ensembles

description in terms of
probability distribution
ρ



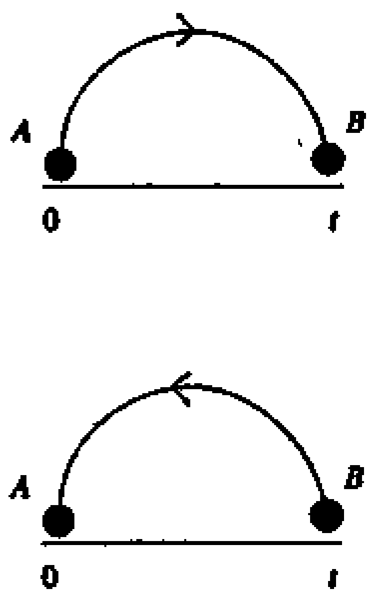
Flow of correlations



B : Binary correlations

C: Ternary correlations

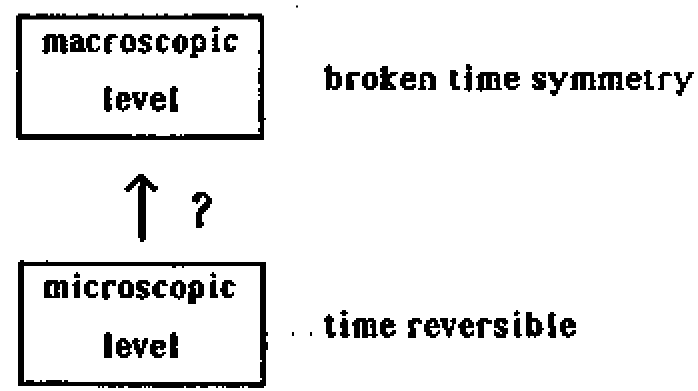
Trajectories and flow of correlations



A —> B : "Dispersion" of correlations

B —> A : "Reconcentration" of correlations

<u>Old Situation</u> (Integrable systems)



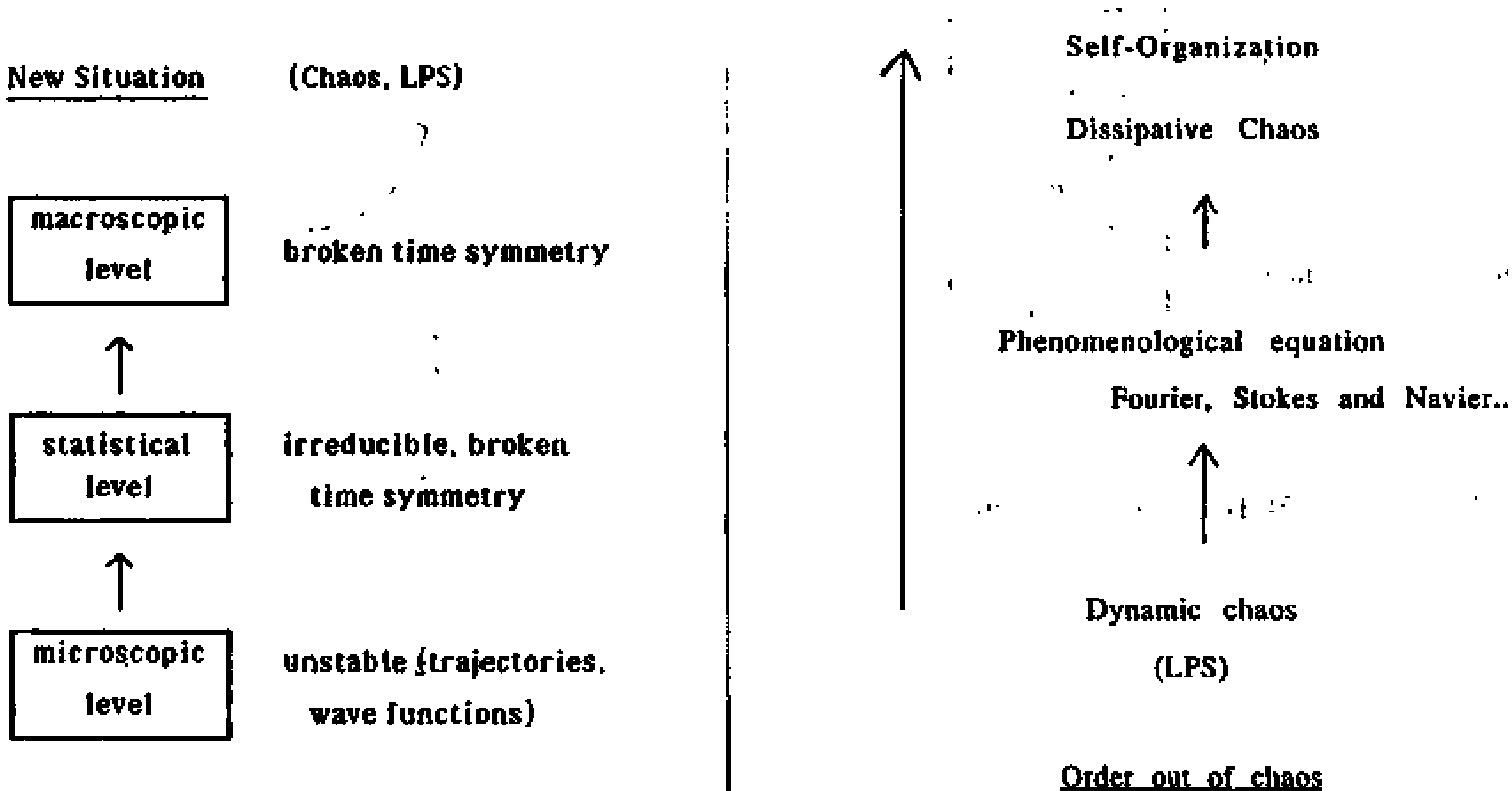
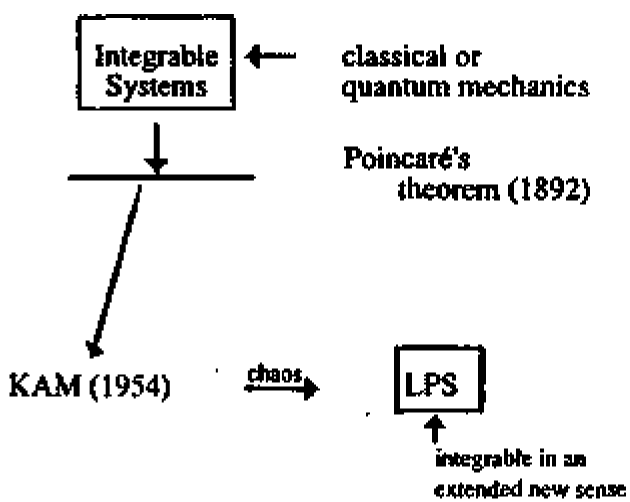

New Situation
(Chaos, LPS)
macroscopic level
broken time symmetry
statistical level
irreducible, broken time symmetry
microscopic level
unstable (trajectories, wave functions)
Self-Organization
Dissipative Chaos
Phenomenological equation
Fourier, Stokes and Navier...
Dynamic chaos
(LPS)
Order out of chaos

Diagram I


Integrable Systems
classical or quantum mechanics
Poincaré's theorem (1892)
KAM (1954)
chaos
LPS
integrable in an extended new sense