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THE EIGHTFOLD WAY:
A THEORY OF STRONG INTERACTION SYMMETRY

MURRAY GELL-MANN

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A THEORY OF STRONG INTERACTION SYMMETRY*

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CONTENTS

We attempt once more, as in the global symmetry scheme, to treat the eight known baryons as a supermultiplet, degenerate in the limit of a certain symmetry but split into isotopic spin multiplets by a symmetry-breaking term. Here we do not try to describe the symmetry violation in detail, but we ascribe it phenomenologically to the mass differences themselves, supposing that there is some analogy to the μ-e mass difference.

The symmetry is called unitary symmetry and corresponds to the "unitary group" in three dimensions in the same way that charge independence corresponds to the "unitary group" in two dimensions. The eight infinitesimal generators of the group form a simple Lie algebra, just like the three components of isotopic spin. In this important sense, unitary symmetry is the simplest generalization of charge independence.

The baryons then correspond naturally to an eight-dimensional irreducible representation of the group; when the mass differences are turned on, the familiar multiplets appear. The pion and K meson fit into a similar set of eight particles, along with a predicted pseudoscalar meson χ^0 having I = 0. The pattern of Yukawa couplings of π, K and χ is then nearly determined, in the limit of unitary symmetry.

The most attractive feature of the scheme is that it permits the description of eight vector mesons by a unified theory of the Yang-Mills type (with a mass term). Like Sakurai, we have a triplet

ρ of vector mesons coupled to the isotopic spin current and a singlet vector meson ω^o coupled to the hypercharge current. We also have a pair of doublets M and $\overline{M}$, strange vector mesons coupled to strangeness-changing currents that are conserved when the mass differences are turned off. There is only one coupling constant, in the symmetric limit, for the system of eight vector mesons. There is some experimental evidence for the existence of ω^o and M, while ρ is presumably the famous I = 1, J = 1, π-π resonance.

A ninth vector meson coupled to the baryon current can be accommodated naturally in the scheme.

The most important prediction is the qualitative one that the eight baryons should all have the same spin and parity and that the pseudoscalar and vector mesons should form "octets", with possible additional "singlets".

If the symmetry is not too badly broken in the case of the renormalized coupling constants of the eight vector mesons, then numerous detailed predictions can be made of experimental results.

The mathematics of the unitary group is described by considering three fictitious "leptons", ν, e^-, and μ^-, which may or may not have something to do with real leptons. If there is a connection, then it may throw light on the structure of the weak interactions.

I Introduction

It has seemed likely for many years that the strongly interacting particles, grouped as they are into isotopic multiplets, would show traces of a higher symmetry that is somehow broken. Under the higher symmetry, the eight familiar baryons would be degenerate and form a supermultiplet. As the higher symmetry is broken, the Ξ, Λ, Σ, and N would split apart, leaving inviolate only the conservation of isotopic spin, of strangeness, and of baryons. Of these three, the first is partially broken by electromagnetism and the second is broken by the weak interactions. Only the conservation of baryons and of electric charge are absolute.

An attempt[1,2] to incorporate these ideas in a concrete model was the scheme of "global symmetry", in which the higher symmetry was valid for the interactions of the π meson, but broken by those of the K. The mass differences of the baryons were thus attributed to the K couplings, the symmetry of which was unspecified, and the strength of which was supposed to be significantly less than that of the π couplings.

The theory of global symmetry has not had great success in predicting experimental results. Also, it has a number of defects. The peculiar distribution of isotopic multiplets among the observed mesons and baryons is left unexplained. The arbitrary K couplings (which are not really particularly weak) bring in several adjustable constants. Furthermore, as admitted in Reference 1 and reemphasized recently by Sakurai[3,4] in his remarkable articles predicting vector mesons, the global model makes no direct connection between physical couplings and the currents of the conserved symmetry operators.

In place of global symmetry, we introduce here a new model of the higher symmetry of elementary particles which has none of these faults and a number of virtues.

We note that the isotopic spin group is the same as the group of all unitary 2x2 matrices with unit determinant. Each of these matrices can be written as exp(iA), where A is a hermitian 2x2 matrix. Since there are three independent hermitian 2x2 matrices (say those of Pauli), there are three components of the isotopic spin.

Our higher symmetry group is the simplest generalization of isotopic spin, namely the group of all unitary 3x3 matrices with unit determinant. There are eight independent traceless 3x3 matrices and consequently the new "unitary spin" has eight components. The first three are just the components of the isotopic spin, the eighth is proportional to the hypercharge Y (which is +1 for N and K, -1 for Ξ and $\overline{K}$, 0 for Λ, Σ, π, etc.), and the remaining four are strangeness-changing operators.

Just as isotopic spin possesses a three-dimensional representation (spin 1), so the "unitary spin" group has an eight-dimensional irreducible representation, which we shall call simply $\underset{\sim}{8}$. In our theory, the baryon supermultiplet corresponds to this representation. When the symmetry is reduced, then $\underset{\sim}{I}$ and Y are still conserved but the four other components of unitary spin are not; the supermultiplet then breaks up into Ξ, Σ, Λ, and N. Thus the distribution of multiplets and the nature of strangeness or hypercharge are to some extent explained.

The pseudoscalar mesons are also assigned to the representation $\underset{\sim}{8}$. When the symmetry is reduced, they become the multiplets K, $\overline{K}$, π, and χ, where χ is a neutral isotopic singlet meson the existence of which we predict. Whether the PS mesons are regarded as fundamental or as bound states, their Yukawa couplings in the limit of "unitary" symmetry are describable in terms of only two coupling parameters.

The vector mesons are introduced in a very natural way, by an extension of the gauge principle of Yang and Mills[5)]. Here too we have a supermultiplet of eight mesons, corresponding to the representation $\underset{\sim}{8}$. In the limit of unitary symmetry and with the mass of these vector mesons "turned off", we have a completely gauge-invariant and minimal theory, just like electromagnetism. When the mass is turned on, the gauge invariance is reduced (the gauge function may no longer be space-time-dependent) but the conservation of unitary spin remains exact. The sources of the vector mesons are the conserved currents of the eight components of the unitary spin[6)].

When the symmetry is reduced, the eight vector mesons break up into a triplet ρ (coupled to the still-conserved isotopic spin current), a singlet ω (coupled to the still-conserved hypercharge current), and a pair of doublets M and $\overline{M}$ (coupled to a strangeness-changing current that is no longer conserved). The particles ρ and ω were both discussed by Sakurai. The ρ meson is presumably identical to the I = 1, J = 1, π-π resonance postulated by Frazer and Fulco[7)] in order to explain the isovector electromagnetic form factors of the nucleon. The ω meson is no doubt the same as the I = 1, J = 0 particle or 3π resonance predicted by Nambu[8)] and later by Chew[9)] and others in order to explain the isoscalar form factors of the nucleon. The strange meson M may be the same as the K^* particle observed by Alston et al.[10)].

Thus we predict that the eight baryons have the same spin and parity, that K is pseudoscalar and that χ exists, that ρ and ω exist with the properties assigned to them by Sakurai, and that M exists. But besides these qualitative predictions there are also the many symmetry rules associated with the unitary spin. All of these are broken, though, by whatever destroys the unitary symmetry, and it is a delicate matter to find ways in which these effects of a broken symmetry can be explored experimentally.

Besides the eight vector mesons coupled to the unitary spin, there can be a ninth, which is invariant under unitary spin and is thus not degenerate with the other eight, even in the limit of unitary symmetry. We call this meson B^o. Presumably it exists too and is coupled to the baryon current. It is the meson predicted by Teller[11)] and later by Sakurai[3)] and explains most of the hard-core repulsion between nucleons and the attraction between nucleons and antinucleons at short distances.

We begin our exposition of the "eightfold way" in the next Section by discussing unitary symmetry using fictitious "leptons" which may have nothing to do with real leptons but help to fix the physical ideas in a rather graphic way. If there is a parallel between these "leptons" and the real ones, that would throw some light on the weak interactions, as discussed briefly in Section VI.

Section III is devoted to the $\underset{\sim}{8}$ representation and the baryons and Section IV to the pseudoscalar mesons. In Section V we present the theory of the vector mesons.

The physical properties to be expected of the predicted mesons are discussed in Section VII, along with a number of experiments that bear on those properties.

In Section VIII we take up the vexed question of the broken symmetry, how badly it is broken, and how we might succeed in testing it.

II The "Leptons" as a Model for Unitary Symmetry

For the sake of a simple exposition, we begin our discussion of unitary symmetry with "leptons", although our theory really concerns the baryons and mesons and the strong interactions. The particles we consider here for mathematical purposes do not necessarily have anything to do with real leptons, but there are some suggestive parallels. We consider three leptons, ν, e^-, and μ^-, and their antiparticles. The neutrino is treated on the same footing as the other two, although experience suggests that if it is treated as a four-component Dirac field, only two of the components have physical interaction. (Furthermore, there may exist two neutrinos, one coupled to the electron and the other to the muon.)

As far as we know, the electrical and weak interactions are absolutely symmetrical between e^- and μ^-, which are distinguished, however from ν. The charged particles e^- and μ^- are separated by the mysterious difference in their masses. We shall not necessarily attribute this difference to any interaction, nor shall we explain it in any way. (If one insists on connecting it to an interaction, one might have to consider a coupling that becomes important only at exceedingly high energies and is, for the time being, only of academic interest.) We do, however, guess that the μ-e mass splitting is related to the equally mysterious mechanism that breaks the unitary symmetry of the baryons and mesons and splits the supermultiplets into isotopic multiplets. For practical purposes, we shall put all of these splittings into the mechanical masses of the particles involved.

It is well known that in present quantum electrodynamics, no one has succeeded in explaining the e-ν mass difference as an electromagnetic effect. Without prejudice to the question of its physical origin, we shall proceed with our discussion as if that mass difference were "turned on" along with the charge of the electron.

If we now "turn off" the μ-e mass difference, electromagnetism, and the weak interactions we are left with a physically vacuous theory of three exactly similar Dirac particles with no rest mass and no known couplings. This empty model is ideal for our mathematical purposes, however, and is physically motivated by the analogy with the strongly interacting particles, because it is at the corresponding stage of total unitary symmetry that we shall introduce the basic baryon mass and the strong interactions of baryons and mesons.

The symmetric model is, of course, invariant under all unitary transformations on the three states, ν, e^-, and μ^-.

Let us first suppose for simplicity that we had only two particles ν and e^-. We can factor each unitary transformation uniquely into one which multiplies both particles by the same phase factor and one (with determinant unity) which leaves invariant the product of the phase factors of ν and e^-. Invariance under the first kind of transformation corresponds to conservation of leptons ν and e^-. It may be considered separately from invariance under the class of transformations of the second kind (called by mathematicians the unitary unimodular group in two dimensions).

Each transformation of the first kind can be written as a matrix $e^{i\phi}1$, where 1 is the unit 2x2 matrix. The infinitesimal transformation is $1 + i(\delta\phi)1$ and so the unit matrix is the infinitesimal generator of these transformations. The transformations of the second kind are generated in the same way by the three independent traceless 2x2 matrices, which may be taken to be the three Pauli isotopic spin matrices τ_1, τ_2, τ_3. We thus have

$$1 + i \sum_{k=1}^{3} \delta\vartheta_k \frac{\tau_k}{2} \qquad (2.1)$$

as the general infinitesimal transformation of the second kind.

Symmetry under all the transformations of the second kind is the same as symmetry under τ_1, τ_2, τ_3, in other words charge independence or isotopic spin symmetry. The whole formalism of isotopic spin theory can then be constructed by considering the transformation properties of this doublet or spinor (ν, e^-) and of more complicated objects that transform like combinations of two or more such leptons.

The Pauli matrices τ_k are hermitial and obey the rules

$$\begin{aligned} \mathrm{Tr}\, \tau_i \tau_j &= 2\delta_{ij} \\ [\tau_i, \tau_j] &= 2ie_{ijk}\, \tau_k \\ \{\tau_i, \tau_j\} &= 2\delta_{ij}\, 1 \qquad . \end{aligned} \tag{2.2}$$

We now generalize the idea of isotopic spin by including the third object μ^-. Again we factor the unitary transformations on the leptons into those which are generated by the 3x3 unit matrix 1 (and which correspond to lepton conservation) and those that are generated by the eight independent traceless 3x3 matrices (and which form the "unitary unimodular group" in three dimensions). We may construct a typical set of eight such matrices by analogy with the 2x2 matrices of Pauli. We call them $\lambda_1 \ldots\ldots \lambda_8$ and list them in Table I. They are hermitian and have the properties

$$\begin{aligned} \mathrm{Tr}\, \lambda_i \lambda_j &= 2\delta_{ij} \\ [\lambda_i, \lambda_j] &= 2if_{ijk}\, \lambda_k \\ \{\lambda_i, \lambda_j\} &= \tfrac{4}{3}\, \delta_{ij}\, 1 + 2d_{ijk}\, \lambda_k \qquad , \end{aligned} \tag{2.3}$$

where the f_{ijk} are real and totally antisymmetric like the Kronecker symbols e_{ijk} of Eq. (2.2), while the d_{ijk} are real and totally symmetric. These properties follow from the equations

$$\mathrm{Tr}\, \lambda_k \left[\lambda_i, \lambda_j\right] = 4if_{ijk}$$

$$\mathrm{Tr}\, \lambda_k \left\{\lambda_i, \lambda_j\right\} = 4d_{ijk} \qquad (2.4)$$

derived from (2.3).

The non-zero elements of f_{ijk} and d_{ijk} are given in Table II for our choice of λ_i. Even and odd permutations of the listed indices correspond to multiplication of f_{ijk} by ± 1 respectively and of d_{ijk} by +1.

The general infinitesimal transformation of the second kind is, of course,

$$1 + i \sum_i \delta\vartheta_i \frac{\lambda_i}{2} \qquad (2.5)$$

by analogy with (2.1). Together with conservation of leptons, invariance under the eight λ_i corresponds to complete "unitary symmetry" of the three leptons.

It will be noticed that λ_1, λ_2, and λ_3 correspond to τ_1, τ_2, and τ_3 for ν and e^- and nothing for the muon. Thus, if we ignore symmetry between (ν, e^-) and the muon, we still have conservation of isotopic spin. We also have conservation of λ_8, which commutes with λ_1, λ_2, and λ_3 and is diagonal in our representation. We can diagonalize at most two λ's at the same time and we have chosen them to be λ_3 (the third component of the ordinary isotopic spin) and λ_8, which is like strangeness or hypercharge, since it distinguishes the isotopic singlet μ^- from the isotopic doublet (ν, e^-) and commutes with the isotopic spin.

Now the turning-on of the muon mass destroys the symmetry under λ_4, λ_5, λ_6, and λ_7 (i.e., under the "strangeness-changing" components of the "unitary spin") and leaves lepton number, "isotopic spin", and "strangeness" conserved. The electromagnetic interactions (along with the electron mass) then break the conservation of λ_1 and λ_2, leaving lepton number λ_3, and strangeness conserved. Finally the weak interactions allow the strangeness to be changed (in muon decay) but continue to conserve the lepton number n_ℓ and the electric charge

$$Q = \frac{e}{2}\left(\lambda_3 + \frac{\lambda_8}{\sqrt{3}} - \frac{4}{3} n_\ell\right) \qquad (2.6)$$

where n_ℓ is the number of leptons minus the number of antileptons and equals 1 for ν, e^-, and μ^- (i.e., the matrix 1).

We see that the situation is just what is needed for the baryons and mesons. We transfer the symmetry under unitary spin to them and assign them strong couplings and basic symmetrical masses. Then we turn on the mass splittings, and the symmetry under the 4th, 5th, 6th, and 7th components of the unitary spin is lifted, leaving baryon mumber, strangeness, and isotopic spin conserved. Electromagnetism destroys the symmetry under the 1st and 2nd components of the spin, and the weak interactions destroy strangeness conservation. Finally, only charge and baryon number are conserved.

III Mathematical Description of the Baryons

In the case of isotopic spin $\underset{\sim}{I}$, we know that the various possible charge multiplets correspond to "irreducible representations" of the simple 2x2 matrix algebra described above for (ν, e^-). Each multiplet has $2I + 1$ components, where the quantum number I distinguishes one representation from another and tells us the eigenvalue $I(I + 1)$ of the operator $\sum_{i=1}^{3} I_i^2$, which commutes with all the elements of the isotopic spin group and in particular with all the infinitesimal group elements $1 + i \sum_{i=1}^{3} \delta\vartheta_i I_i$. The operators I_i are represented, within the multiplet, by hermitian $(2I + 1) \times (2I + 1)$ matrices having the same commutation rules

$$[I_i, I_j] = ie_{ijk} I_k \qquad (3.1)$$

as the 2x2 matrices $\tau_i/2$. For the case of $I = 1/2$, we have just $I_i = \tau_i/2$ within the doublet.

If we start with the doublet representation, we can build up all the others by considering superpositions of particles that transform like the original doublet. Thus, the antiparticles e^+, $-\bar{\nu}$ also form a doublet. (Notice the minus sign on the antineutrino state or field.) Taking $\frac{e^+e^- + \bar{\nu}\nu}{\sqrt{2}}$, we obtain a singlet, that is, a one-dimensional representation for which all the I_i are zero. Calling the neutrino and electron e_α with $\alpha = 1, 2$, we can describe the singlet by $\frac{1}{\sqrt{2}} \bar{e}_\alpha e_\alpha$ or, more concisely, $\frac{1}{\sqrt{2}} \bar{e}e$. The three components of a triplet can be formed by taking $e^+\nu = \frac{1}{2} \bar{e}(\tau_1 - i\tau_2)e$, $\frac{e^+e^- - \bar{\nu}\nu}{\sqrt{2}} = \frac{1}{\sqrt{2}} \bar{e}\, \tau_3\, e$, and $\nu e^- = \frac{1}{2} \bar{e}(\tau_1 + i\tau_2)e$. Rearranging these, we have just $\frac{1}{\sqrt{2}} \bar{e} \tau_j e$ with $j = 1, 2, 3$. Among these three states, the 3x3 matrices I_i^{jk} of the three components of $\underset{\sim}{I}$ are given by

$$I_i^{jk} = - ie_{ijk} \qquad (3.2)$$

Now let us generalize these familiar results to the set of three states ν, e^-, and μ^-. Call them ℓ_α with $\alpha = 1, 2, 3$ and use $\bar{\ell}\ell$ to mean $\bar{\ell}_\alpha \ell_\alpha$, etc. For this system we define $F_i = \frac{\lambda i}{2}$ with $i = 1, 2, \ldots 8$, just as $I_i = \frac{\tau i}{2}$ for isotopic spin. The F_i are the 8 components of the unitary spin operator $\underset{\sim}{F}$ in this case and we shall use the same notation in all representations. The first three components of $\underset{\sim}{F}$ are identical with the three components of the isotopic spin $\underset{\sim}{I}$ in all cases, while F_8 will always be $\frac{\sqrt{3}}{2}$ times the hypercharge Y (linearly related to the strangeness). In all representations, then, the components of $\underset{\sim}{F}$ will have the same commutation rules

$$[F_i, F_j] = if_{ijk} F_k \qquad (3.3)$$

that they do in the simple lepton representation for which $F_i = \lambda_i/2$. (Compare the commutation rules in Eq. (2.3).) The trace properties and anticommutation properties will not be the same in all representations any more than they are for $\underset{\sim}{I}$. We see that the rules (3.1) are just a special case of (3.3) with indices 1, 2, 3, since the f's equal the e's for these values of the indices.

We must call attention at this point to an important difference between unitary or $\underset{\sim}{F}$ spin and isotopic or $\underset{\sim}{I}$ spin. Whereas, with a simple change of sign on $\bar{\nu}$, we were able to construct from $\bar{e}_\alpha$

a doublet transforming under $\mathbf{I}$ just like e_α, we are not able to do the same thing for the $\mathbf{F}$ spin when we consider the three antileptons $\bar{\ell}_\alpha$ compared to the three leptons ℓ_α. True, the antileptons do give a representation for $\mathbf{F}$, but it is, in mathematical language, <u>inequivalent</u> to the lepton representation, even though it also has three dimensions. The reason is easy to see: when we go from leptons to antileptons the eigenvalues of the electric charge, the third component of $\mathbf{I}$, and the lepton number all change sign, and thus the eigenvalues of F_8 change sign. But they were $\frac{1}{2\sqrt{3}}$, $\frac{1}{2\sqrt{3}}$, and $\frac{-1}{\sqrt{3}}$ for leptons and so they are a different set for antileptons and no similarity transformation can change one representation into the other. We shall refer to the lepton representation as $\mathbf{3}$ and the antilepton representation as $\bar{\mathbf{3}}$.

Now let us consider another set of "particles" L_α transforming exactly like the leptons ℓ_α under unitary spin and take their antiparticles $\bar{L}_\alpha$. We follow the same procedure used above for the isotopic spin and the doublet e. We first construct the state $\frac{1}{\sqrt{3}}\bar{L}_\alpha \ell_\alpha$ or $\frac{1}{\sqrt{3}}\bar{L}\ell$. Just as $\frac{\bar{e}e}{\sqrt{2}}$ gave a one-dimensional representation of $\mathbf{I}$ for which all the I_i were zero, so $\frac{\bar{L}\ell}{\sqrt{3}}$ gives a one-dimensional representation of $\mathbf{F}$ for which all the F_i are zero. Call this one-dimensional representation $\mathbf{1}$.

Now, by analogy with $\frac{\bar{e}\,\tau_i\, e}{\sqrt{2}}$ with i = 1, 2, 3, we form $\frac{\bar{L}\,\lambda_i\,\ell}{\sqrt{2}}$ with i = 1, 2,......8. These states transform under unitary spin $\mathbf{F}$ like an irreducible representation of dimension 8, which we shall call $\mathbf{8}$. In this representation, the 8x8 matrices F_i^{jk} of the eight components F_i of the unitary spin are given by the relation

$$F_i^{jk} = -if_{ijk} \quad , \qquad (3.4)$$

analogous to Eq. (3.2).

When we formed an isotopic triplet from two isotopic doublets, in the discussion preceding Eq. (3.2), we had to consider linear combinations of the $\frac{\bar{e}\,\tau_i\,e}{\sqrt{2}}$ in order to get simple states with definite electric charges, etc. We must do the same here. Using the symbol ~ for "transforms like", we define

$$\Sigma^+ \sim \tfrac{1}{2}\,\bar{L}(\lambda_1 - i\lambda_2)\ell \qquad \sim D^+\nu$$

$$\Sigma^- \sim \tfrac{1}{2}\,\bar{L}(\lambda_1 + i\lambda_2)\ell \qquad \sim D^o e^-$$

$$\Sigma^o \sim \tfrac{1}{\sqrt{2}}\,\bar{L}\,\lambda_3\,\ell \qquad \sim \frac{D^o\nu - D^+e^-}{\sqrt{2}}$$

$$p \sim \tfrac{1}{2}\,\bar{L}(\lambda_4 - i\lambda_5)\ell \qquad \sim S^+\nu$$

$$n \sim \tfrac{1}{2}\,\bar{L}(\lambda_6 - i\lambda_7)\ell \qquad \sim S^+e^-$$

$$\Xi^o \sim \tfrac{1}{2}\,\bar{L}(\lambda_6 + i\lambda_7)\ell \qquad \sim D^+\mu^-$$

$$\Xi^- \sim \tfrac{1}{2}\,\bar{L}(\lambda_4 + i\lambda_5)\ell \qquad \sim D^o\mu^-$$

$$\Lambda \sim \tfrac{1}{\sqrt{2}}\,\bar{L}\,\lambda_8\,\ell \qquad \sim (D^o\nu + D^+e^- - 2S^+\mu^-)/\sqrt{6} \qquad (3.5)$$

The most graphic description of what we are doing is given in the last column, where we have introduced the notation D^o, D^+, and S^+ for the $\bar{L}$ particles analogous to the $\bar{\ell}$ particles $\bar{\nu}$, e^+, and μ^+ respectively. D stands for doublet and S for singlet with respect to isotopic spin. Using the last column, it is easy to see that the isotopic spins, electric charges, and hypercharges of the multiplets are exactly as we are accustomed to think of them for the baryons listed.

We say, therefore, that the eight known baryons form one degenerate supermultiplet with respect to unitary spin. When we introduce a perturbation that transforms like the μ-e mass difference, the supermultiplet will break up into exactly the known multiplets. (Of course, D will split from S at the same time as e^-, ν from μ^-.)

Of course, another type of baryon is possible, namely a singlet neutral one that transforms like $\frac{1}{\sqrt{3}}\bar{L}\ell$. If such a particle exists, it may be very heavy and highly unstable. At the moment, there is no evidence for it.

We shall attach no physical significance to the ℓ and $\bar{L}$ "particles" out of which we have constructed the baryons. The discussion up to this point is really just a mathematical introduction to the properties of unitary spin.

IV Pseudoscalar Mesons

We have supposed that the baryon fields N_j transform like an octet $\underset{\sim}{8}$ under $\underset{\sim}{F}$, so that the matrices of $\underset{\sim}{F}$ for the baryon fields are given by Eq. (3.4). We now demand that all mesons transform under $\underset{\sim}{F}$ in such a way as to have $\underset{\sim}{F}$-invariant strong couplings. If the 8 mesons π_i are to have Yukawa couplings, they must be coupled to $\bar{N}\Theta_i N$ for some matrices Θ_i, and we must investigate how such bilinear forms transform under $\underset{\sim}{F}$.

In mathematical language, what we have done in Section III is to look at the direct product $\underset{\sim}{\bar{3}} \times \underset{\sim}{3}$ of the representations $\underset{\sim}{\bar{3}}$ and $\underset{\sim}{3}$ and to find that it reduces to the direct sum of $\underset{\sim}{8}$ and $\underset{\sim}{1}$. We identified $\underset{\sim}{8}$ with the baryons and, for the time being, dismissed $\underset{\sim}{1}$. What we must now do is to look at $\underset{\sim}{\bar{8}} \times \underset{\sim}{8}$. Now it is easy to show that actually $\underset{\sim}{\bar{8}}$ is equivalent to $\underset{\sim}{8}$; this is unlike the situation for $\underset{\sim}{\bar{3}}$ and $\underset{\sim}{3}$. (We note that the values of Y, I_3, Q, etc. are symmetrically disposed about zero in the $\underset{\sim}{8}$ representation.) So the antibaryons transform essentially like the baryons and we must reduce out the direct product $\underset{\sim}{8} \times \underset{\sim}{8}$. Standard group theory gives the result

$$\underset{\sim}{8} \times \underset{\sim}{8} = \underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{10} + \underset{\sim}{\overline{10}} + \underset{\sim}{27} \quad , \qquad (4.1)$$

where $\underset{\sim}{\overline{27}} = \underset{\sim}{27}$ (this can happen only when the dimension is the cube of an integer). The representation $\underset{\sim}{27}$ breaks up, when mass differences are turned on, into an isotopic singlet, triplet, and quintet with $Y = 0$, a doublet and a quartet with $Y = 1$, a doublet and a quartet with $Y = -1$, a triplet with $Y = 2$, and a triplet with $Y = -2$. The representation $\underset{\sim}{10}$ breaks up, under the same conditions, into a triplet with $Y = 0$, a doublet with $Y = -1$, a quartet with $Y = +1$, and a singlet with $Y = +2$. The conjugate representation $\underset{\sim}{\overline{10}}$ looks the same, of course, but with equal and opposite values of Y. None of these much resembles the pattern of the known mesons.

The $\underset{\sim}{8}$ representation, occurring twice, looks just the same for mesons as for baryons and is very suggestive of the known π, K, and $\bar{K}$ mesons plus one more neutral pseudoscalar meson with $I = 0$, Y = 0, which corresponds to Λ in the baryon case. Let us call this meson χ^o and suppose it exists, with a fairly low mass. Then we have identified the known pseusoscalar mesons with an octet under unitary symmetry, just like the baryons. The representations $\underset{\sim}{1}$, $\underset{\sim}{10}$, $\underset{\sim}{\overline{10}}$, and $\underset{\sim}{27}$ may also correspond to mesons, even pseudoscalar ones, but presumably they lie higher in mass, some or all of them perhaps so high as to be physically meaningless.

To describe the eight pseudoscalar mesons as belonging to $\underset{\sim}{8}$, we put (very much as in (3.5))

$$
\begin{aligned}
\chi^o &= \pi_8 \\
\pi^+ &= (\pi_1 - i\pi_2)/\sqrt{2} \\
\pi^- &= (\pi_1 + i\pi_2)/\sqrt{2} \\
\pi^o &= \pi_3 \\
K^+ &= (\pi_4 - i\pi_5)/\sqrt{2} \\
K^o &= (\pi_6 - i\pi_7)/\sqrt{2} \\
\overline{K^o} &= (\pi_6 + i\pi_7)/\sqrt{2} \\
K^- &= (\pi_4 + i\pi_5)/\sqrt{2}
\end{aligned}
\tag{4.2}
$$

and we know then that the matrices of $\underset{\sim}{F}$ connecting the π_j are just the same as those connecting the N_j, namely $F_i^{jk} = -if_{ijk}$.

To couple the 8 mesons invariantly to 8 baryons (say by γ_5), we must have a coupling

$$2ig_o\overline{N}\,\gamma_5\Theta_i N\pi_i \tag{4.3}$$

for which the relation

$$[F_i, \Theta_j] = if_{ijk}\,\Theta_k \qquad (4.4)$$

holds. Now the double occurrence of $\underset{\sim}{8}$ in Eq. (4.1) assures us that there are two independent sets of eight 8x8 matrices Θ_i obeying (4.4). One of these sets evidently consists of the F_i themselves. It is not hard to find the other set if we go back to the commutators and anticommutators of the λ matrices in the $\underset{\sim}{3}$ representation (Eq. (2.3)). Just as we formed $F_i^{jk} = -if_{ijk}$, we define

$$D_i^{jk} = d_{ijk} \qquad (4.5)$$

and it is easy to show that the D's also satisfy Eq. (4.4). We recall that where the F matrices are imaginary and antisymmetric with respect to the basis we have chosen, the D's are real and symmetric.

Now what is the physical difference between coupling the pseudoscalar mesons π_i by means of D_i and by means of F_i? It lies in the symmetry under the operation

$$R:\ p \leftrightarrow \Xi^-,\ n \leftrightarrow \Xi^0,\ \Sigma^+ \leftrightarrow \Sigma^-,\ \Sigma^0 \leftrightarrow \Sigma^0,\ \Lambda \leftrightarrow \Lambda$$

$$K^+ \leftrightarrow \pm K^-,\ K^0 \leftrightarrow \pm \overline{K^0},\ \pi^+ \leftrightarrow \pm\pi^-,\ \pi^0 \leftrightarrow \pm\pi^0,\ \chi^0 \leftrightarrow \pm\chi^0\ , \qquad (4.6)$$

which is not a member of the unitary group, but a kind of reflection. In the language of N_i, we may say that R changes the sign of the second, fifth, and seventh particles; we note that λ_2, λ_5, and λ_7 are imaginary while the others are real. From Table II we can see that under these sign changes f_{ijk} is odd and d_{ijk} even.

It may be that in the limit of unitary symmetry the coupling of the pseudoscalar mesons is invariant under R as well as the unitary group. In that case, we choose either the plus sign in (4.6) and the D coupling or else the minus sign and the F coupling. The two possible coupling patterns are listed in Table III.

If only one of the patterns is picked out (case of R-invariance) it is presumably the D coupling, since that gives a large $\Lambda\pi\Sigma$ interaction (while the F coupling gives none) and the $\Lambda\pi\Sigma$ interaction is the best way of explaining the binding of Λ particles in hypernuclei.

In general, we may write the Yukawa coupling (whether fundamental or phenomenological, depending on whether the π_i are elementary or not) in the form

$$L_{int} = 2ig_o \, \overline{N} \, \gamma_5 \left[\alpha D_i + (1 - \alpha) \, F_i\right] N \, \pi_i \quad . \qquad (4.7)$$

We note that in no case is it possible to make the couplings ΛKN and ΣKN both nuch smaller than the $N\pi N$ coupling. Since the evidence from photo-K production seems to indicate smaller effective coupling constants for ΛKN and ΣKN than for $N\pi N$ (indeed that was the basis of the global symmetry scheme) we must conclude that our symmetry is fairly badly broken. We shall return to that question in Section VII.

A simple way to read off the numerical factors in Table III, as well as those in Table IV for the vector mesons, is to refer to the chart in Table V, which gives the transformation properties of mesons and baryons in terms of the conceptual "leptons" and "L particles" of Section III.

An interesting remark about the baryon mass differences may be added at this point. If we assume that they transform like the μ-e mass difference, that is, like the 8th component of the unitary spin, then there are only two possible mass-difference matrices, F_8 and D_8. That gives rise to a sum rule for baryon masses:

$$1/2\ (m_N + m_\Xi) = 3/4\ m_\Lambda + 1/4\ m_\Sigma \quad , \qquad (4.8)$$

which is very well satisfied by the observed masses, much better than the corresponding sum rule for global symmetry.

There is no particular reason to believe, however, that the analogous sum rules for mesons are obeyed.

V Vector Mesons

The possible transformation properties of the vector mesons under $\underset{\sim}{F}$ are the same as those we have already examined in the pseudoscalar case. Again it seems that for low mass states we can safely ignore the representations $\underset{\sim}{27}$, $\underset{\sim}{10}$, and $\underset{\sim}{\overline{10}}$. We are left with $\underset{\sim}{1}$ and the two cases of $\underset{\sim}{8}$.

A vector meson transforming according to $\underset{\sim}{1}$ would have $Q = 0$, $I = 0$, $Y = 0$ and would be coupled to the total baryon current $i\overline{N}\gamma_\mu N$, which is exactly conserved. Such a meson may well exist and be of great importance. The possibility of its existence has been envisaged for a long time.

We recall that the conservation of baryons is associated with the invariance of the theory under infinitesimal transformations

$$N \longrightarrow (1 + i\ \varepsilon)N \quad , \qquad (5.1)$$

where ε is a constant. This is gauge-invariance of the first kind. We may, however, consider the possibility that there is also gauge invariance of the second kind, as discussed by Yang and Lee[12]. Then we could make $\mathcal{E}$ a function of space-time. In the free baryon Lagrangian

$$L_N = - \bar{N}(\gamma_\alpha \partial_\alpha + m_o)N \tag{5.2}$$

this would produce a new term

$$L_N \longrightarrow L_N - i\bar{N}\gamma_\alpha N \partial_\alpha \varepsilon \tag{5.3}$$

which can be cancelled only if there exists a neutral vector meson field B_α coupled to the current $\bar{N}\gamma_\alpha N$:

$$L_B = -1/4 \, (\partial_\alpha B_\beta - \partial_\beta B_\alpha)^2$$

$$L_{int} = if_o \bar{N}\gamma_\alpha N \, B_\alpha \tag{5.4}$$

and which undergoes the gauge transformation

$$B_\alpha \longrightarrow B_\alpha + 1/f_o \, \partial_\alpha \varepsilon \quad . \tag{5.5}$$

As Yang and Lee pointed out, such a vector meson is massless and if it existed with any appreciable coupling constant, it would simulate a kind of anti-gravity, for baryons but not leptons, that is contradicted by experiment.

We may, however, take the point of view that there are vector mesons associated with a gauge-invariant Lagrangian plus a mass term, which breaks the gauge invariance of the second kind while leaving inviolate the gauge invariance of the first kind and the conservation law. Such situations have been treated by Glashow[13], Salam and Ward[14], and others, but particularly in this connection by Sakurai[3].

The vector meson transforming according to **1** would then be of such a kind. Teller[11], Sakurai[3], and others have discussed the notion that such a meson may be quite heavy and very strongly coupled, binding baryons and anti-baryons together to make the pseudoscalar mesons according to the compound model of Fermi and Yang[15]. We shall leave this possibility open, but not consider it further here. If it is right, then the Yukawa couplings (4.7) must be treated as phenomenological rather than fundamental; from an immediate practical point of view, it may not make much difference.

We go on to consider the **8** representation. An octet of vector mesons would break up into an isotopic doublet with Y = 1, which we shall call M (by analogy with K -- the symbol L is already used to mean π or μ); the corresponding doublet $\overline{M}$ analogous to $\overline{K}$; a triplet ρ with Y = 0 analogous to π; and a singlet ω^o with Y = 0 analogous to χ^o.

We may tentatively identify M with the K^* reported by Alston et al.[10] at 884 Mev with a width $\Gamma \approx 15$ Mev for break-up into $\pi + K$. Such a narrow width certainly points to a vector rather than a scalar state. The vector meson ρ may be identified, as Sakurai has proposed, with the I = 1, J = 1, π-π resonance discussed by Frazer and Fulco[7] in connection with the electromagnetic structure of the nucleon. The existence of ω^o has been postulated for similar reasons by Nambu[8], Chew[9], and others.

In principle, we have a choice again between couplings of the $\underset{\sim}{D}$ and the $\underset{\sim}{F}$ type for the vector meson octet. But there is no question which is the more reasonable theory. The current $i\bar{N}F_j\gamma_\alpha N$ is the current of the F-spin for baryons and in the limit of unitary symmetry the total F spin current is exactly conserved. (The conservation of the strangeness-changing currents, those of F_4, F_5, F_6, and F_7, is broken by the mass differences, the conservation of F_2 and F_3 by electromagnetism, and that of F_3 and F_8 separately by the weak interactions. Of course, the current of the electric charge

$$Q = e(F_3 + \frac{F_8}{\sqrt{3}}) \qquad (5.6)$$

is exactly conserved.)

Sakurai has already suggested that ρ is coupled to the isotopic spin current and ω to the hypercharge current. We propose in addition that the strange vector mesons M are coupled to the strangeness-changing components of the F spin current and that the whole system is completely invariant under $\underset{\sim}{F}$ before the mass-differences have been turned on, so that the three coupling constants (suitably defined) are approximately equal even in the presence of the mass differences.

Now the vector mesons themselves carry F spin and therefore contribute to the current which is their source. The problem of constructing a nonlinear theory of this kind has been completely solved in the case of isotopic spin by Yang and Mills [5)] and by Shaw [5)]. We have only to generalize their result (for three vector mesons) to the case of F spin and eight vector mesons.

We may remark parenthetically that the Yang-Mills theory is irreducible, in the sense that all the 3 vector mesons are coupled to one another inextricably. We may always make a "reducible" theory by adjoining other, independent vector mesons like the field B_α discussed earlier in connection with the baryon current. It is an interesting mathematical problem to find the set of all irreducible Yang-Mills tricks. Glashow and the author[16)] have shown that the problem is the same as that of finding all the simple Lie algebras, one that was solved long ago by the mathematicians. The possible dimensions are 3, 8, 10, 14, 15, 21, and so forth. Our generalization of the Yang-Mills trick is the simplest one possible.

But let us "return to our sheep", in this case the 8 vector mesons. We first construct a completely gauge-invariant theory and then add a mass term for the mesons. Let us call the eight fields $\rho_{i\alpha}$, just as we denoted the eight pseudoscalar fields by π_i. We may think of the N_i, the π_i, and the $\rho_{i\alpha}$ as vectors in an 8-dimensional space. (The index α here refers to the four space-time components of a vector field.) We use our totally antisymmetric tensor f_{ijk} to define a cross product

$$(\underset{\sim}{A} \times \underset{\sim}{B})_i = f_{ijk} A_j B_k \quad . \tag{5.7}$$

The gauge transformation of the second kind analogous to Eqs. (5.1) and (5.5) is performed with an eight-component gauge function $\underset{\sim}{\phi}$:

$$\underset{\sim}{N} \rightarrow \underset{\sim}{N} + \underset{\sim}{\phi} \times \underset{\sim}{N}$$
$$\underset{\sim}{\rho}_\alpha \rightarrow \underset{\sim}{\rho}_\alpha + \underset{\sim}{\phi} \times \underset{\sim}{\rho}_\alpha - (2\gamma_o)^{-1} \partial_\alpha \underset{\sim}{\phi}$$
$$\underset{\sim}{\pi} \rightarrow \underset{\sim}{\pi} + \underset{\sim}{\phi} \times \underset{\sim}{\pi} \qquad (5.8)$$

We have included the pseudoscalar meson field for completeness, treating it as elementary. We shall not write the π-N and possible π-π couplings in what follows, since they are not relevant and may simply be added in at the end. The bare coupling parameter is γ_o.

We define gauge-covariant field strengths by the relation

$$\underset{\sim}{G}_{\alpha\beta} = \partial_\alpha \underset{\sim}{\rho}_\beta - \partial_\beta \underset{\sim}{\rho}_\alpha + 2\gamma_o \underset{\sim}{\rho}_\alpha \times \underset{\sim}{\rho}_\beta \qquad (5.9)$$

and the gauge-invariant Lagrangian (to which a common vector meson mass term is presumably added) is simply

$$L = -\frac{1}{4} \underset{\sim}{G}_{\alpha\beta} \cdot \underset{\sim}{G}_{\alpha\beta} - m_o \overline{\underset{\sim}{N}} \cdot \underset{\sim}{N} - \overline{\underset{\sim}{N}} \gamma_\alpha \cdot (\partial_\alpha \underset{\sim}{N} + 2\gamma_o \underset{\sim}{\rho}_\alpha \times \underset{\sim}{N})$$
$$- \frac{1}{2} \mu_o^2 \underset{\sim}{\pi} \cdot \underset{\sim}{\pi} - \frac{1}{2}(\partial_\alpha \underset{\sim}{\pi} + 2\gamma_o \underset{\sim}{\rho}_\alpha \times \underset{\sim}{\pi}) \cdot (\partial_\alpha \underset{\sim}{\pi} + 2\gamma_o \underset{\sim}{\rho}_\alpha \times \underset{\sim}{\pi}) \quad . \qquad (5.10)$$

There are trilinear and quadrilinear interactions amongst the vector mesons, as usual, and also trilinear and quadrilinear couplings with the pseudoscalar mesons. All these, along with the basic coupling of vector mesons to the baryons, are characterized in the limit of no mass differences by the single coupling parameter γ_o. The symmetrical couplings of $\underset{\sim}{\rho}_\alpha$ to the bilinear currents of baryons and pseudoscalar mesons are listed in Table IV. In Section VII, we shall use them to predict a number of approximate relations among experimental quantities relevant to the vector mesons.

As in the case of the pseudoscalar couplings, the various vector couplings will have somewhat different strengths when the mass differences are included, and some couplings which vanish in (5.10) will appear with small coefficients. Thus, in referring to experimental renormalized coupling constants (evaluated at the physical masses of the vector mesons) we shall use the notation $\gamma_{N\Lambda M}$, $\gamma_{NN\rho}$, etc. In the limit of unitary symmetry, all of these that do not vanish are equal.

VI Weak Interactions

So far the role of the leptons in unitary symmetry has been purely symbolic. Although we introduced a mathematical F spin for ν, e^-, and μ^-, that spin is not coupled to the eight vector mesons that take up the F spin gauge for baryons and mesons. If we take it seriously at all, we should probably regard it as a different spin, but one with the same mathematical properties.

Let us make another point, which may seem irrelevant but possibly is not. The photon and the charge operator to which it is coupled have not so far been explicitly included in our scheme. They must be put in as an afterthought, along with the corresponding gauge transformation, which was the model for the more peculiar gauge transformations we have treated. If the weak interactions are carried[17] by vector bosons X_α and generated by a gauge transformation[18,19] of their own, then these bosons and gauges have been ignored as well. Such considerations might cause us, if we are in a highly speculative frame of mind, to wonder about the possibility that each kind of interaction has its own type of gauge and its own set of vector particles and that the algebraic properties of these gauge transformations conflict with one another.

When we draw a parallel between the "F spin" of leptons and the F spin of baryons and mesons, and when we discuss the weak interactions at all, we are exploring phenomena that transcend the scheme we are using. Everything we say in this Section must be regarded as highly tentative and useful only in laying the groundwork for a possible future theory. The same is true of any physical interpretation of the mathematics in Sections II and III.

We shall restrict our discussion to charge - exchange weak currents and then only to the vector part. A complete discussion of the axial vector weak currents may involve more complicated concepts and even new mesons[20)] (scalar and/or axial vector) lying very high in energy.

The vector weak current of the leptons is just $\bar{\nu}\gamma_\alpha e + \bar{\nu}\gamma_\alpha \mu$. If we look at the abstract scheme for the baryons in Eq. (3.5), we see that a baryon current with the same transformation properties under F would consist of two parts: one, analogous to $\bar{\nu}\gamma_\alpha e$, would have $|\Delta \underset{\sim}{I}| = 1$ and $\Delta S = 0$, while the other, analogous to $\bar{\nu}\gamma_\alpha \mu$, would have $|\Delta \underset{\sim}{I}| = 1/2$ and $\Delta S/\Delta Q = +1$. These properties are exactly the ones we are accustomed to associate with the weak interactions of baryons and mesons.

Now the same kind of current we have taken for the leptons can be assigned to the conceptual bosons L of Section III. Suppose it to be of the same strength. Then, depending on the relative sign of the lepton and L weak currents, the matrices in the baryon system may be F's or D's.

Suppose, in the $\Delta S = 0$ case, the relative sign is such as to give F. Then the resulting current is just one component of the isotopic spin current; and the same result will hold for mesons. Thus we will have the conserved vector current that has been proposed[17)] to explain the lack of renormalization of the Fermi constant.

In the $\Delta S = 1$ case, by taking the same sign, we could get the almost-conserved strangeness-changing vector current, the current of $F_4 + iF_5$.

Further speculations along these lines might lead to a theory of the weak interactions[21)].

VII Properties of the New Mesons

The theory we have sketched is fairly solid only in the realm of the strong interactions, and we shall restrict our discussion of predictions to the interactions among baryons and mesons.

We predict the existence of 8 baryons with equal spin and parity following the pattern of N, Λ, Σ, and Ξ. Likewise, given the π and its coupling constant, we predict a pseudoscalar K and a new particle, the χ^0, both coupled (in the absence of mass differences) as in Eq. (4.7), and we predict pion couplings to hyperons as in the same equation.

Now in the limit of unitary symmetry an enormous number of selection and intensity rules apply. For example, for the reactions

PS meson + baryon $\longrightarrow$ PS meson + baryon, there are only 7 independent amplitudes. Likewise, baryon-baryon forces are highly symmetric. However, the apparent smallness of $g_1^2/4\pi$ for $NK\Lambda$ and $NK\Sigma$ compared to $N\pi N$ indicates that unitary symmetry is badly broken, assuming that it is valid at all. We must thus rely principally on qualitative predictions for tests of the theory; in Section VIII we take up the question of how quantitative testing may be possible.

The most clear-cut new prediction for the pseudoscalar mesons is the existence of χ^0, which should decay into 2γ like the π^0, unless it is heavy enough to yield $\pi^+ + \pi^- + \gamma$ with appreciable probability. (In the latter case, we must have $(\pi^+\pi^-)$ in an odd state.) $\chi^0 \rightarrow 3\pi$ is forbidden by conservation of I and C. For a sufficiently heavy χ^0, the decay $\chi^0 \rightarrow 4\pi$ is possible, but hampered by centrifugal barriers.

Now we turn to the vector mesons, with coupling pattern as given in Table IV. We predict, like Sakurai, the ρ meson, presumably identical with the resonance of Frazer and Fulco, and the ω meson, coupled to the hypercharge. In addition, we predict the strange vector meson M, which may be the same as the K^* of Alston et al.

Some of these are unstable with respect to the strong interactions and their physical coupling constants to the decay products are given by the decay widths. Thus, for $M \rightarrow K + \pi$, we have

$$\Gamma_M = 2 \frac{\gamma^2_{MK\pi}}{4\pi} \frac{k^3}{m_M^2} \quad , \tag{7.1}$$

where k is the momentum of one of the decay mesons. We expect, of course, a $\cos^2\vartheta$ angular distribution relative to the polarization of M and a charge ratio of 2:1 in favor of $K^0 + \pi^+$ or $K^+ + \pi^-$.

For the I = 1, J = 1, π-π resonance we have the decay $\rho \rightarrow 2\pi$ with width

$$\Gamma_\rho = \frac{8}{3} \frac{\gamma^2_{\rho\pi\pi}}{4\pi} \frac{k^3}{m_\rho^2} \qquad (7.2)$$

Using a value $m_\rho = 4.5\ m_\pi$, we would have $\Gamma \approx m_\pi \frac{\gamma^2}{4\pi}$ and agreement with the theory of Bowcock et al.[7] would require a value of $\frac{\gamma^2}{4\pi}$ of the order of 2/3. If, now, we assume that the mass of M is really around 880 Mev, then Eq. (7.1) yields $\Gamma_M \approx \frac{\gamma^2}{4\pi} \cdot 50$ Mev. If the width is around 15 Mev, then the two values of $\frac{\gamma^2}{4\pi}$ are certainly of the same order.

We can obtain information about vector coupling constants in several other ways. If we assume, with Sakurai and Dalitz, that the Y^* of Alston et al.[22] (at 1380 Mev with decay $Y^* \rightarrow \pi + \Lambda$) is a bound state of $\overline{K}$ and N in a potential associated with the exchange of ω and ρ, then with simple Schrödinger theory we can roughly estimate the relevant coupling strengths. In the Schrödinger approximation (which is fairly bad, of course) we have the potential

$$V(\text{triplet}) \approx -3 \frac{\gamma_{NN\omega}\ \gamma_{KK\omega}}{4\pi} \frac{e^{-m_\omega r}}{r} + \frac{\gamma_{NN\rho}\ \gamma_{KK\rho}}{4\pi} \frac{e^{-m_\rho r}}{r} \quad . \qquad (7.3)$$

If ω has a mass of around 400 Mev (as suggested by the isoscalar form factor of the nucleon) then the right binding results with both

$\frac{\gamma^2}{4\pi}$ of the order of 2/3.

A most important result follows if this analysis has any element of truth, since the singlet potential is

$$V(\text{singlet}) \approx -3 \frac{\gamma_{NN\omega}\gamma_{KK\omega}}{4\pi} \frac{e^{-m_\omega r}}{r} -3 \frac{\gamma_{NN\rho}\gamma_{KK\rho}}{4\pi} \frac{e^{-m_\rho r}}{r} \quad . \qquad (7.4)$$

A singlet version of Y^* should exist considerably below the energy of Y^* itself. Call it Y_s^*. If it is bound by more than 100 Mev or so, it is metastable and decays primarily into $\Lambda + \gamma$, since $\Lambda + \pi$ is forbidden by charge independence. Thus Y_s^* is a fake Σ^0, with $I = 0$ and different mass, and may have caused some difficulty in experiments involving the production of Σ^0 at high energy. If, because of level shifts due to absorption, Y_s^* is not very far below Y^*, then it should be detectable in the same way as Y^*; one should observe its decay into $\pi + \Sigma$.

Bound systems like Y^* and Y_s^* should occur not only for $\overline{K}N$ but also for $K\Xi$. (In the limit of unitary symmetry, these come to the same thing.)

The vector coupling constants occur also in several important poles. (For the unstable mesons, these are of course not true poles, unless we perform an analytic continuation of the scattering amplitude onto a second sheet, in which case they become poles at complex energies; they behave almost like true poles, however, when the widths of the vector meson states are small.) There is the pole at $q^2 = -m_M^2$ in the reactions $\pi^- + p \rightarrow \Lambda + K^0$ and $\pi^- + p \rightarrow \Sigma + K$; a peaking of K in the forward direction has already been observed in some of these reactions and should show up at high energies in all of them. Likewise the pole at $q^2 = -m_\pi^2$ in the reaction $K + N \rightarrow M + N$ should be observable at high energies and its strength can be predicted directly from the width of M. In the reactions $\pi + N \rightarrow \Lambda + M$ and $\pi + N \rightarrow \Sigma + M$, there is a pole at $q^2 = -m_K^2$ and measurement of its strength can determine the coupling constants $g_{NK\Lambda}^2/4\pi$ and $g_{NK\Sigma}^2/4\pi$ for the K meson.

In πN scattering, we can measure the pole due to exchange of the ρ meson. In KN and $\bar{K}N$ scattering, there are poles from the exchange of ρ and of ω; these can be separated since only the former occurs in the charge-exchange reaction. In NN scattering with charge-exchange, there is a ρ meson pole in addition to the familiar pion pole. Without charge exchange, the situation is terribly complicated, since there are poles from π, ρ, ω, χ, and B.

When the pole term includes a baryon vertex for the emission or absorption of a vector meson, we must remember that there is a "strong magnetic" term analogous to a Pauli moment as well as the renormalized vector meson coupling constant.

In a relatively short time, we should have a considerable body of information about the vector mesons.

VIII Violations of Unitary Symmetry

We have mentioned that within the unitary scheme there is no way that the coupling constants of K to both $N\Lambda$ and $N\Sigma$ can both be much smaller than 15, except through large violations of the symmetry. Yet experiments on photoproduction of K particles seem to point to such a situation. Even if unitary symmetry exists as an underlying pattern, whatever mechanism is responsible for the mass differences apparently produces a wide spread among the renormalized coupling constants as well. It is true that the binding of Λ particles in hypernuclei indicates a $\pi\Lambda\Sigma$ coupling of the same order of magnitude as the πNN coupling, but the anomalously small renormalized constants of the K meson indicate that a quantitative check of unitary symmetry will be very difficult.

What about the vector mesons? Let us discuss first the ρ and ω fields, which are coupled to conserved currents. For typical couplings of these fields, we have the relations

$$\gamma^2_{\rho\pi\pi} = \gamma^2_o Z_3(\rho) \left[V^\rho_\pi (0)\right]^{-2} , \qquad (8.1)$$

$$\gamma^2_{\rho NN} = \gamma^2_o Z_3(\rho) \left[V^\rho_1 (0)\right]^{-2} , \qquad (8.2)$$

$$\gamma^2_{\omega NN} = \gamma^2_o Z_3(\omega) \left[V^\omega_1 (0)\right]^{-2} , \qquad (8.3)$$

etc. Here each renormalized coupling constant is written as a product of the bare constant, a vacuum polarization renormalization factor, and a squared form factor evaluated at zero momentum transfer. The point is that at zero momentum transfer there is no vertex renormalization because the source currents are conserved. To check, for example, the hypothesis that ρ is really coupled to the isotopic spin current, we must check that γ_o^2 in (8.1) is the same as γ_o^2 in (8.2). We can measure (say by "pole experiments" and by the width of the π-π resonance) the renormalized constants on the left. The quantities V^2 are of the order unity in any case, and their ratios can be measured by studying electromagnetic form factors[23].

The experimental check of "universality" between (8.1) and (8.2) is thus possible, but that tests only the part of the theory already proposed by Sakurai, the coupling of ρ to the isotopic spin current. To test unitary symmetry, we must compare (8.2) and (8.3); but then the ratio $Z_3(\rho)/Z_3(\omega)$ comes in to plague us. We may hope, of course, that this ratio is sufficiently close to unity to make the agreement striking, but we would like a better way of testing unitary symmetry quantitatively.

When we consider the M meson, the situation is worse, since the source current of M is not conserved in the presence of the mass differences. For each coupling of M, there is a vertex renormalization factor that complicates the comparison of coupling strengths.

An interesting possibility arises if the vector charge - exchange weak current is really given in the $|\Delta S| = 1$ case by the current of $F_4 \pm iF_5$ just as it is thought to be given in the $\Delta S = 0$ case by that of $F_1 \pm iF_2$ (the conserved current) and if the

$\Delta S = 0$ and $|\Delta S| = 1$ currents are of equal strength, like the $e\nu$ and $\mu\nu$ currents. Then the leptonic $|\Delta S| = 1$ decays show renormalization factors that must be related to the vertex renormalization factors for the M meson, since the source currents are assumed to be the same. The experimental evidence on the decay $K \to \pi$ + leptons then indicates a renormalization factor, in the square of the amplitude, of the order of 1/20. In the decays $\Lambda \to p$ + leptons and $\Sigma^- \to n$ + leptons, both vector and axial vector currents appear to be renormalized by comparable factors.

The width for decay of M into $K + \pi$, if it is really about 15 Mev, indicates that the renormalized coupling constant $\gamma^2_{K\pi M}/4\pi$ is not much smaller than $\gamma^2_{\rho\pi\pi}/4\pi \approx 2/3$ and so there is at present no sign of these small factors in the coupling constants of M. It will be interesting, however, to see what the coupling constant $\gamma^2_{N\Lambda M}/4\pi$ comes out, as determined from the pole in $\pi^- + p \to \Lambda + K^0$.

We have seen that the prospect is rather gloomy for a quantitative test of unitary symmetry, or indeed of any proposed higher symmetry that is broken by mass differences or strong interactions. The best hope seems to lie in the possibility of direct study of the ratios of bare constants in experiments involving very high energies and momentum transfers, much larger than all masses[24)]. However, the theoretical work on this subject is restricted to renormalizable theories. At present, theories of the Yang-Mills type with a mass do not seem to be renormalizable[25)], and no one knows how to improve the situation.

It is in any case an important challenge to theoreticians to construct a satisfactory theory of vector mesons. It may be useful to remark that the difficulty in Yang-Mills theories is caused by the mass. It is also the mass which spoils the gauge invariance of the first kind. Likewise, as in the μ-e case, it may be the mass that produces the violation of symmetry. Similarly, the nucleon and pion masses break the conservation of any axial vector current in the theory of weak interactions. It may be that a new approach to the rest masses of elementary particles can solve many of our present theoretical problems.

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TABLE I.

A Set of Matrices λ_i.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

TABLE II.

Non-zero elements of f_{ijk} and d_{ijk}. The f_{ijk} are odd under permutations of any two indices while the d_{ijk} are even.

ijk	f_{ijk}	ijk	d_{ijk}
123	1	118	$1/\sqrt{3}$
147	1/2	146	1/2
156	-1/2	157	1/2
246	1/2	228	$1/\sqrt{3}$
257	1/2	247	-1/2
345	1/2	256	1/2
367	-1/2	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	1/2
678	$\sqrt{3}/2$	355	1/2
		366	-1/2
		377	-1/2
		448	$-1/(2\sqrt{3})$
		558	$-1/(2\sqrt{3})$
		668	$-1/(2\sqrt{3})$
		778	$-1/(2\sqrt{3})$
		888	$-1/\sqrt{3}$

TABLE III.

Yukawa interactions of pseudoscalar mesons with baryons, assuming pure coupling through D.

$$L_{int}/ig_o = \pi^0 \left\{ \bar{p}\gamma_5 p - \bar{n}\gamma_5 n + \frac{2}{\sqrt{3}} \bar{\Sigma}^0\gamma_5\Lambda + \frac{2}{\sqrt{3}} \bar{\Lambda}\gamma_5\Sigma^0 - \bar{\Xi}^0\gamma_5\Xi^0 + \bar{\Xi}^-\gamma_5\Xi^- \right\}$$

$$+ \pi^+ \left\{ \sqrt{2}\, \bar{p}\gamma_5 n + \frac{2}{\sqrt{3}} \bar{\Sigma}^+\gamma_5\Lambda + \frac{2}{\sqrt{3}} \bar{\Lambda}\gamma_5\Sigma^- - \sqrt{2}\, \bar{\Xi}^0\gamma_5\Xi^- \right\}$$

$$+ \text{h.c.}$$

$$+ K^+ \left\{ - \frac{1}{\sqrt{3}} \bar{p}\gamma_5\Lambda + \bar{p}\gamma_5\Sigma^0 + \sqrt{2}\, \bar{n}\gamma_5\Sigma^- - \frac{1}{\sqrt{3}} \bar{\Lambda}\gamma_5\Xi^- + \bar{\Sigma}^0\gamma_5\Xi^- + \sqrt{2}\, \bar{\Sigma}^+\gamma_5\Xi^0 \right\}$$

$$+ \text{h.c.}$$

$$+ K^0 \left\{ - \frac{1}{\sqrt{3}} \bar{n}\gamma_5\Lambda - \bar{n}\gamma_5\Sigma^0 + \sqrt{2}\, \bar{p}\gamma_5\Sigma^+ - \frac{1}{\sqrt{3}} \bar{\Lambda}\gamma_5\Xi^0 - \bar{\Sigma}^0\gamma_5\Xi^0 + \sqrt{2}\, \bar{\Sigma}^-\gamma_5\Xi^- \right\}$$

$$+ \text{h.c.}$$

$$+ \chi^0 \left\{ - \frac{1}{\sqrt{3}} \bar{p}\gamma_5 p - \frac{1}{\sqrt{3}} \bar{n}\gamma_5 n - \frac{2}{\sqrt{3}} \bar{\Lambda}\gamma_5\Lambda + \frac{2}{\sqrt{3}} \bar{\Sigma}^+\gamma_5\Sigma^+ + \frac{2}{\sqrt{3}} \bar{\Sigma}^0\gamma_5\Sigma^0 + \frac{2}{\sqrt{3}} \bar{\Sigma}^-\gamma_5\Sigma^- - \frac{1}{\sqrt{3}} \bar{\Xi}^0\gamma_5\Xi^0 - \frac{1}{\sqrt{3}} \bar{\Xi}^-\gamma_5\Xi^- \right\}$$

TABLE III (cont.)

Yukawa interactions of pseudoscalar mesons with baryons, assuming pure coupling through F.

$$L_{int}/ig_o = \pi^o\,(\bar{p}\gamma_5 p - \bar{n}\gamma_5 n + 2\,\overline{\Sigma^+}\gamma_5\Sigma^+ - 2\,\overline{\Sigma^-}\gamma_5\Sigma^- + \overline{\Xi^o}\gamma_5\Xi^o - \overline{\Xi^-}\gamma_5\Xi^-)$$

$$+ \pi^+\,(\sqrt{2}\,\bar{p}\gamma_5 n - \sqrt{2}\,\overline{\Xi^o}\gamma_5\Xi^- - 2\,\overline{\Sigma^+}\gamma_5\Sigma^o + 2\,\overline{\Sigma^o}\gamma_5\Sigma^-)$$

+ h.c.

$$+ K^+\,(-\sqrt{3}\,\bar{p}\gamma_5\Lambda + \sqrt{3}\,\bar{\Lambda}\gamma_5\Xi^- - \bar{p}\gamma_5\Sigma^o - \sqrt{2}\,\bar{n}\gamma_5\Sigma^- + \overline{\Sigma^o}\gamma_5\Xi^- + \sqrt{2}\,\overline{\Sigma^+}\gamma_5\Xi^o)$$

+ h.c.

$$+ K^o\,(-\sqrt{3}\,\bar{n}\gamma_5\Lambda + \sqrt{3}\,\bar{\Lambda}\gamma_5\Xi^o + \bar{n}\gamma_5\Sigma^o - \sqrt{2}\,\bar{p}\gamma_5\Sigma^+ - \overline{\Sigma^o}\gamma_5\Xi^o + \sqrt{2}\,\overline{\Sigma^-}\gamma_5\Xi^-)$$

+ h.c.

$$+ \chi^o\,(\sqrt{3}\,\bar{p}\gamma_5 p + \sqrt{3}\,\bar{n}\gamma_5 n - \sqrt{3}\,\overline{\Xi^o}\gamma_5\Xi^o - \sqrt{3}\,\overline{\Xi^-}\gamma_5\Xi^-)$$

TABLE IV.

Trilinear couplings of ρ's to π's and N's.

$$L_{int}/i\gamma_o = M_\alpha^+\Big\{-\sqrt{3}\,\bar{p}\gamma_\alpha\Lambda + \sqrt{3}\,\bar{\Lambda}\gamma_\alpha\Xi^- - \bar{p}\gamma_\alpha\Sigma^o - \sqrt{2}\,\bar{n}\gamma_\alpha\Sigma^- + \overline{\Sigma^o}\gamma_\alpha\Xi^-$$

$$+ \sqrt{2}\,\overline{\Sigma^+}\gamma_\alpha\Xi^o - \sqrt{3}\,K^-\partial_\alpha\chi^o + \sqrt{3}\,\chi^o\partial_\alpha K^- - K^-\partial_\alpha\pi^o$$

$$+ \pi^o\partial_\alpha K^- - \sqrt{2}\,\overline{K^o}\partial_\alpha\pi^- + \sqrt{2}\,\pi^-\partial_\alpha\overline{K^o}\Big\}$$

$+ h.c.$

$$+ M_\alpha^o\Big\{-\sqrt{3}\,\bar{n}\gamma_\alpha\Lambda + \sqrt{3}\,\bar{\Lambda}\gamma_\alpha\Xi^o + \bar{n}\gamma_\alpha\Sigma^o - \sqrt{2}\,\bar{p}\gamma_\alpha\Sigma^+ - \overline{\Sigma^o}\gamma_\alpha\Xi^o$$

$$+ \sqrt{2}\,\overline{\Sigma^-}\gamma_\alpha\Xi^- - \sqrt{3}\,\overline{K^o}\partial_\alpha\chi^o + \sqrt{3}\,\chi^o\partial_\alpha K^o + \overline{K^o}\partial_\alpha\pi^o$$

$$- \pi^o\partial_\alpha\overline{K^o} - \sqrt{2}\,K^-\partial_\alpha\pi^+ + \sqrt{2}\,\pi^+\partial_\alpha K^-\Big\}$$

$+ h.c.$

$$+ \rho_\alpha^+\Big\{\sqrt{2}\,\bar{p}\gamma_\alpha n - \sqrt{2}\,\overline{\Xi^o}\gamma_\alpha\Xi^- - 2\,\overline{\Sigma^+}\gamma_\alpha\Sigma^o + 2\,\overline{\Sigma^o}\gamma_\alpha\Sigma^- + \sqrt{2}\,K^-\partial_\alpha K^o$$

$$- \sqrt{2}\,K^o\partial_\alpha K^- - 2\,\pi^-\partial_\alpha\pi^o + 2\,\pi^o\partial_\alpha\pi^-\Big\}$$

$+ h.c.$

$$+ \rho_\alpha^o\Big\{\bar{p}\gamma_\alpha p - \bar{n}\gamma_\alpha n + 2\,\overline{\Sigma^+}\gamma_\alpha\Sigma^+ - 2\,\overline{\Sigma^-}\gamma_\alpha\Sigma^- + \overline{\Xi^o}\gamma_\alpha\Xi^o - \overline{\Xi^-}\gamma_\alpha\Xi^-$$

$$+ K^-\partial_\alpha K^+ - K^+\partial_\alpha K^- - \overline{K^o}\partial_\alpha K^o + K^o\partial_\alpha\overline{K^o} + 2\,\pi^-\partial_\alpha\pi^+$$

$$- 2\,\pi^+\partial_\alpha\pi^-\Big\}$$

$$+ \omega_\alpha^o\Big\{\sqrt{3}\,\bar{p}\gamma_\alpha p + \sqrt{3}\,\bar{n}\gamma_\alpha n - \sqrt{3}\,\overline{\Xi^o}\gamma_\alpha\Xi^o - \sqrt{3}\,\overline{\Xi^-}\gamma_\alpha\Xi^- + \sqrt{3}\,K^-\partial_\alpha K^+$$

$$- \sqrt{3}\,K^+\partial_\alpha K^- + \sqrt{3}\,\overline{K^o}\partial_\alpha K^o - \sqrt{3}\,K^o\partial_\alpha\overline{K^o}\Big\}$$

TABLE V.

Transformation properties of baryons and mesons, assuming pseudoscalar mesons coupled through D.

$$K^+ \sim \frac{\mu^+\nu + S^+\overline{D^0}}{\sqrt{2}}$$

$$K^0 \sim \frac{\mu^+e^- + S^+D^-}{\sqrt{2}}$$

$$\pi^+ \sim \frac{e^+\nu + D^+\overline{D^0}}{\sqrt{2}}$$

$$\pi^0 \sim \frac{\bar{\nu}\nu - e^+e^- + D^0\overline{D^0} - D^+D^-}{2}$$

$$\pi^- \sim \frac{\bar{\nu}e^- + D^0D^-}{\sqrt{2}}$$

$$\chi^0 \sim \frac{\bar{\nu}\nu + e^+e^- - 2\mu^+\mu^- + D^0\overline{D^0} + D^+D^- - 2S^+S^-}{\sqrt{12}}$$

$$\overline{K^0} \sim \frac{e^+\mu^- + D^+S^-}{\sqrt{2}}$$

$$K^- \sim \frac{\bar{\nu}\mu^- + D^0S^-}{\sqrt{2}}$$

$p \sim S^+\nu$

$n \sim S^+e^-$

$\Sigma^+ \sim D^+\nu$

$\Sigma^0 \sim \frac{D^0\nu - D^+e^-}{\sqrt{2}}$

$\Sigma^- \sim D^0e^-$

$\Lambda \sim \frac{D^0\nu + D^+e^- - 2S^+\mu^-}{\sqrt{6}}$

$\Xi^0 \sim D^+\mu^-$

$\Xi^- \sim D^0\mu^-$

TABLE V (cont.)

$$M^+ \sim \frac{\mu^+\nu - S^+\overline{D^0}}{\sqrt{2}}$$

$$M^0 \sim \frac{\mu^+e^- - S^+D^-}{\sqrt{2}}$$

$$\rho^+ \sim \frac{e^+\nu - D^+\overline{D^0}}{\sqrt{2}}$$

$$\rho^0 \sim \frac{\overline{\nu}\nu - e^+e^- - D^0\overline{D^0} + D^+D^-}{2}$$

$$\rho^- \sim \frac{\overline{\nu}e^- - D^0D^-}{\sqrt{2}}$$

$$\omega^0 \sim \frac{\overline{\nu}\nu + e^+e^- - 2\mu^+\mu^- - D^0\overline{D^0} - D^+D^- + 2S^+S^-}{\sqrt{12}}$$

$$\overline{M^0} \sim \frac{e^+\mu^- - D^+S^-}{\sqrt{2}}$$

$$M^- \sim \frac{\overline{\nu}\mu^- - D^0S^-}{\sqrt{2}}$$

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REFERENCES

1) M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

2) J. Schwinger, Annals of Physics 2, 407 (1957).

3) J. J. Sakurai, Annals of Physics 11, 1 (1960).

4) J. J. Sakurai, "Vector Theory of Strong Interactions", unpublished.

5) C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954). Also, R. Shaw, unpublished.

6) After the circulation of the preliminary version of this work (January, 1961) the author has learned of a similar theory put forward independently and simultaneously by Y. Ne'eman (Nuclear Physics, to be published). Earlier uses of the 3-dimensional unitary group in connection with the Sakata model are reported by Y. Ohnuki at the 1960 Rochester Conference on High Energy Physics. A. Salam and J. Ward (Nuovo Cimento, to be published) have considered related questions. The author would like to thank Dr. Ne'eman and Professor Salam for communicating their results to him.

7) W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960). See also J. Bowcock, W. N. Cottingham, and D. Lurie, Phys. Rev. Letters 5, 386 (1960).

8) Y. Nambu, Phys. Rev. 106, 1366 (1957).

9) G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

REFERENCES (cont.)

10) M. Alston et al., to be published.

11) E. Teller, Proceedings of the Rochester Conference, 1956.

12) C. N. Yang and T. D. Lee, Phys. Rev. 98, 1501 (1955).

13) S. L. Glashow, Nuclear Physics 10, 107 (1959).

14) A. Salam and J. C. Ward, Nuovo Cimento 11, 568 (1959).

15) E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1949).

16) S. L. Glashow and M. Gell-Mann, to be published.

17) R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

18) S. Bludman, Nuovo Cimento 9, 433 (1958).

19) M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

20) M. Gell-Mann, talk at Rochester Conference on High Energy Physics, 1960.

21) Earlier attempts to draw a parallel between leptons and baryons in the weak interactions have been made by A. Gamba, R. E. Marshak, and S. Okubo, Proc. Nat. Acad. Sci. 45, 881 (1959), and Y. Yamaguchi, unpublished. Dr. S. L. Glashow reports that Yamaguchi's scheme has much in common with the one discussed in this paper.

22) M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 5, 518, (1960).

23) M. Gell-Mann and F. Zachariasen, "Form Factors and Vector Mesons", to be published.

24) M. Gell-Mann and F. Zachariasen, "Broken Symmetries and Bare Coupling Constants", to be published.

25) Kamefuchi and Umezawa, to be published. Salam and Kumar, to be published.