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DISPERSION OF THE NEUTRON EMISSION IN U235 FISSION*

by

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DISCLAIMER

I. INTRODUCTION

During 1944 we made experiments and developed the theory for the neutron intensity fluctuations of a water boiler[1]. The fluctuations,

[1] Reported by F. de Hoffmann in Chapter 9 of "The Science and Engineering of Nuclear Power", Vol. II (Addison Wesley Press, Cambridge, Mass., 1949).

as measured by a counter depend on (a) the fluctuations in ν , the number of neutrons per fission, (b) the absolute criticality of the system (which is a measure of the average likelihood of starting and perpetuating a chain), (c) the efficiency of the counter and (d) the length of time over which counts are taken. Thus a measurement of the fluctuations together with a determination of (b), (c) and (d) will yield information on the fluctuations in the number of neutrons per fission.

Let $\bar{c}$ denote the average number of counts recorded in the counter per unit time. Then $\left[\overline{c^2} - (\bar{c})^2\right]/\bar{c}$ is a convenient measure of the fluctuations encountered; the quantity is unity when the fluctuations are of purely random origin, since then c has the form of a Poisson distribution. In the case of chain reactors we have greater fluctuations and we define the excess Y by:

$$\left[\overline{c^2} - (\bar{c})^2 / \bar{c} = 1 + Y\right] \qquad (1)$$

We have previously shown[2] that if t is the gate-width, defined

[2] We refer the reader to the derivation of Eq. (9-55) of Ref. 1; there is an obvious typographical error in the formula referred to--in particular ν^2 should be $\overline{\nu^2}$.

as being the time over which counts are taken, then in first approximation Y is given by

$$Y = \frac{\epsilon(\overline{\nu^2} - \overline{\nu})}{(\alpha\tau)^2}\left[1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right] \quad (2)$$

In (2), ϵ is the efficiency of the counter, i.e. the average number of counts per average fission[3] occurring in the water boiler. Furthermore,

[3]The fact that a neutron born at one place may not produce the same subsequent effect on the boiler as one born at another place has a small effect in our case. This is due to the symmetry of the boiler and also to its small size so that a neutron has a good chance of traversing the whole sphere during its lifetime. Calculations of Feynman (private communication 1944) have shown that this geometrical factor when taken account of amounts to about one percent and can really be neglected.

Incidentally, Feynman (private communication 1944) has further carried out the derivation of Eq. (2) with a continuum of neutron velocities and the result obtained is identical with that when only one group of neutron is assumed.

α is defined by the statement that one primary neutron introduced into the boiler means that $e^{-\alpha t}$ will be the expected number of neutrons present at time t due to the primary neutron. Finally, if one neutron is known to be present in the boiler then the probability of its producing fission in time dt is dt/τ, i.e. τ is the mean time between fission.

In Ref. 1 it is further shown that

$$\frac{\overline{\nu}}{\alpha \tau} = \frac{1}{1 - K'} \qquad (3)$$

where K' denotes the multiplication for an average neutron. Provided now that the gate-width t is short compared to the average delayed neutron period, the delayed neutrons do not significantly contribute to K' and we may write $K' = K_p$. We shall make the latter approximation from here on in. When the water boiler is run at delayed critical then[1]

$$1 - K_p = \gamma f \qquad (4)$$

where f is the fraction of fission neutrons which are delayed and γ their average effectiveness in the chain reaction as compared to prompt neutrons. Since all our fluctuation experiments were carried out at delayed critical, we therefore make the replacement (4). Denoting the brackets in (2) by the symbol B we thus find

$$Y = \frac{B \epsilon \, (\overline{\nu^2} - \overline{\nu})}{\overline{\nu} \, (\gamma f)^2} \qquad (5)$$

It will be noted that when the $\alpha t \gg 1$, B tends to unity and (5) becomes independent of the gate-width. Thus, a knowledge of Y, γf and ϵ enables one to determine $\overline{\nu^2}$.

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II. EXPERIMENTAL METHOD

The experiments were carried out on LOPO, the first water boiler in existence at Los Alamos[4]. Our experiment was designed to

[4] For details of LOPO the reader is referred to Rev. of Sci. Inst. 22, 489 (1951).

yield information concerning $\overline{\nu^2}$ and incidentally also to try to check out the validity of (2) as far as dependence on t is concerned. To this end two experimental methods were pursued. In the one case the counts as a function of time were recorded on film so that one and the same series of counts could be analyzed in terms of different gate-widths t. In the second case, a very prolonged series of counts were recorded electronically at a fixed gate-width which was chosen large enough so as to make B almost unity.

In each of the two methods used the neutron counter used was a BF_3 chamber arranged for maximum efficiency with equipment then existing in conjunction with the boiler. It was placed just outside the BeO tamper with blocks of paraffin all around it. The efficiency ϵ, found by comparing it with a U235 fission chamber of known efficiency placed in the center of the boiler was found to be $\epsilon = 3.51 \times 10^{-4}$ counts/fission. After the BF_3 chamber, there was a pre-amplifier, amplifier and discriminator. The rise time of this electronic system (designed by the Los Alamos electronics group under W. Higinbotham and M. Sands) was 5-6 microseconds.

Following the discriminator, the two methods were as follows:

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Film Method[5]

The output of the discriminator was fed into a scalar and

[5]We are indebted to Messrs. B. Brixner and J. Mack for the use of their equipment and for their kind help.

the scalar tapped off at the output of the first stage with the signal being fed to the vertical plates of a 5 inch oscillograph. On the horizontal plates a specially built linear sweep was imposed and adjusted to a frequency of 250 cycles. The screen was photographed by means of a continuously moving film, with the film moving vertically. The developed film showed a pattern like the sample shown in Fig. 1.

This pattern can be interpreted easily since the first stage of the scalar represents a flip-flop circuit, and consequently its output changes from a maximum to a minimum and back to a maximum with successive pulses. Thus a shift of the height of the line on the scope or film respectively will indicate that a pulse has registered[6].

[6]It is important that the sweep return very fast in order that pulses may not be lost between the end of one and the beginning of another sweep. Indeed, the sweep used had a return time of anly several microseconds.

The oscilloscope was photographed on 35 mm film at a film speed of approximately 100 feet per minute, by means of a General Radio Company Oscilloscope camera. The completed film was read on a micro-film reader by moving the film through the viewing apparatus, stopping at random and visually counting a pre-determined time interval on the film. The boiler itself was run at delayed critical and the intensity was adjusted so that the scalar recorded about 500 counts per second.

Electronic Method

This time the signal from the discriminator was fed through a gate circuit and then fed into another discriminator scalar unit. The gate circuit[7] was so designed as to let pulses pass through it starting from the time the switch was closed till a predetermined time later, in particular 283 milliseconds. Thus by closing the switch and reading the scalar, one obtained the number of counts for a 283 millisecond interval.

The boiler was run at critical and at an intensity of about 1000 counts/sec in the BF_3 chamber.

The true length of time of gate was determined by feeding 1000 cycles from a standard audio-oscillator into the input of the gate circuit and noting the number of counts recorded on the scalar. This also enabled one to make a check of constancy. Over a 12 hour run the gate reproduced to better than 2 percent.

III. EXPERIMENTAL DATA

Three 100 foot films were read for different gate-widths t. Since the average counting rate $\bar{c}$ was not the same on all three films

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[7] The gate circuit was kindly designed and built by C. P. Baker.

it was necessary to compute the Y + 1 for a particular gate-width from each film and then combine them properly so as not to get fictitious fluctuation due to variations in $\bar{c}$.

If on one film a number of gates m was counted for one particular gate-time it is shown in Appendix A that the computed Y + 1 from this film should be multiplied by the factor $m/(m - 1)$ to give the correct value of Y + 1. The date from several films were then combined by taking the mean correct Y + 1, weighted according to the respective m's. The probable error in the quantity obtained can be shown to be $\sqrt{2/e}$ where $e = \Sigma m$. Results of Y versus t are shown in Fig. 2 and the probable error indicated.

A single point at a high gate-time, namely, t = 283 milliseconds, was determined by the electronic method. 2200 individual gates were taken over a period of some 12 hours. These 2200 gates were then broken up into 76 small sets of from 10 to 50 gates in such a way that the mean $\bar{c}$ did not change violently during one set. Again this was done to avoid a fictitious increase of the quantity Y + 1 due to violent variations in $\bar{c}$. These sets were combined in the same fashion as described for combination of data from different films. The result was that for a gate-width of 283 milliseconds

$$Y = 4.17 \pm 0.16 \qquad (6)$$

This point is plotted on Fig. 2.

The mean deviation of the 76 individual Y's from their mean was computed and found to be in complete agreement with the deviation

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expected statistically. This instills confidence that the value of Y obtained does not contain fluctuations from other than statistical causes.

IV. INTERPRETATION OF DATA

We have pointed out that the electronic method used a gate time large enough so that $\alpha t \gg 1$. This we verify as follows: α is obtained from (3). Now the time between fissions always enters calculations as $1/\tau$ and therefore τ in (3) may be released by τ_p the average time between prompt fissions. The latter quantity has been determined[8] to be $\tau_p = 135 \pm 20$ microseconds for LOPO. Furthermore, the

[8] See Reference 4. It should be noted that the value of γf cited in this reference was based on a conversion factor between the amount of U235 in the LOPO solution and the resultant reactivity. This conversion factor was calculated in Reference 4 using the old 1944 cross-section for Boron and U235. A recomputation using a thermal absorption cross-section of 690 barns for U235 and 750 barns for Boron does not change the value of γf.

integral quantity γf for LOPO has been determined[8] as $8.5_5 \times 10^{-3}$. Making the replacement (4) in (3), and using $\overline{\nu} = 2.47$, we find

$$\alpha = 156 \pm \text{sec}^{-1} \tag{7}$$

Thus indeed, for the electronic method the quantity $\alpha t = 44$; i.e. very large compared to 1, making B = 0.98.

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Substituting Eq. (6), i.e. the value Y obtained from the electronic method into (5), we may now solve for $\overline{\nu^2}$ and find

$$\overline{\nu^2} = 7.8 \pm 0.6 \tag{8}$$

The film data as shown on Fig. 2 does confirm the theoretical prediction that the fluctuations should rise with increasing gate-widths. The dotted curve in Fig. 2 was constructed by assuming $\alpha = 156 \text{ sec}^{-1}$ is correctly given by (7) and normalizing the curve to the electronic method point at 283 milliseconds. It will be seen that the agreement of the dotted curve with the film method experiment is only qualitative. It would appear that a fit to experiment calls for a lower value of α than (7). In particular the solid curve which gives a reasonably good fit to the data was constructed with $\alpha = 115 \text{ sec}^{-1}$. This discrepancy does not reflect on the value of $\overline{\nu^2}$ deduced from (5) and (6) where α enters only very insensitively through B. Putting it another way, for the electronic method at 283 milliseconds we need only know the value of the integral quantity $\alpha\tau$ which is directly related to the measured γf, whereas for the variation of Y with t, an independent knowledge of both α and τ are called for.

V. DISCUSSION OF RESULT

Our experiment, performed in 1944, has determined the second moment in the distribution of the number of neutrons in the thermal fission of U235 as being 7.8 ± 0.6. To familiarize ourselves with the meaning of this number we might note that if ν were always 2.5, a physical impossibility, then $\overline{\nu^2} = 6.25$. If ν were to divide equally between 2 and 3, then $\overline{\nu^2} = 6.5$. It can be seen that if ν divided say between 2, 3 and 4 in such a way as to give $\overline{\nu} = 2.5$ the quantity $\overline{\nu^2}$ would vary between about 6.6 and 6.9. A Poisson distribution of $\overline{\nu}$ would lead to a value of $\overline{\nu^2} = 8.75$.

Since the work reported above was performed, the dispersion in the number of fissions from U235 has been determined directly[9] by

[9] Diven, B. C., et al, to be submitted to Phys. Rev.

Diven et al at Los Alamos. They do not determine the second moment, but rather obtained full information about the relative probability of the emission 1, 2, 3 --- neutrons per fission. They find that at 80 kev their data yields $\overline{\nu^2} = 7.32$. Furthermore, Leachman[10] has estimated on

[10] Leachman, R. B., to be submitted to Phys. Rev.

theoretical grounds that at thermal one would expect a value of $\overline{\nu^2} = 7.2$, which is allowed by our limit of error on (8).

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APPENDIX A. Derivation of the Correction Factor m/(m - 1)

In order to compute Y + 1 we have to compute the quantity $\overline{c^2} - (\bar{c})^2$, i.e., $(1/m) \sum_i (c_i - \bar{c})^2$.
Experimentally however, we measure the quantity

$$(1/m) \sum \left[c_i = (1/m) \sum_i c_i\right]^2 \tag{9}$$

Now

$$(1/m) \sum (c_i - \frac{1}{m} {}_i \; c_i)^2 = (1/m) \sum (c_i - \bar{c} + \bar{c} - \frac{1}{m} \sum_i c_i)^2$$

$$= (1/m) \sum_i (c_i - \bar{c})^2 - \left[(1/m) \sum (c_i - \bar{c})\right]^2 \tag{10}$$

Thus we have to evaluate the expected value of $(1/m \sum_i (c_i - c)$ to obtain the most probable difference between the two quantities. Now

$$\left[(1/m) \sum (c_i - \bar{c})\right]^2 = (1/m^2) \sum_i \sum_j (c_i - \bar{c})(c_j - \bar{c}) \tag{11}$$

In this sum the terms with $i \neq j$ are just as likely to be positive as negative and therefore on the average cancel out. This leaves only the terms with $i = j$ and hence the expected value of this quantity is:

$$(1/m^2) \sum_i (c_i - \bar{c})^2 \tag{12}$$

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Therefore

$$(1/m)\sum\left[c_i - (1/m)\sum_i c_i\right]^2 = \left[(1/m) - (1/m^2)\right]\sum_i (c_i - \bar{c})^2$$

$$= \left[(m - 1)/m\right](1/m)\sum_i (c_i - \bar{c})^2 \qquad (13)$$

or consequently

Expected true value of $(Y + 1) = \left[m/(m - 1)\right] \cdot \left[\text{measured value}\right]$.

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LEGEND FOR FIG. 2

The variation of Y as a function of gate-width t. All points shown were obtained by the film method except for the point at 283 milliseconds which was obtained by the electronic method. The dotted and solid curves were constructed as explained in section IV.

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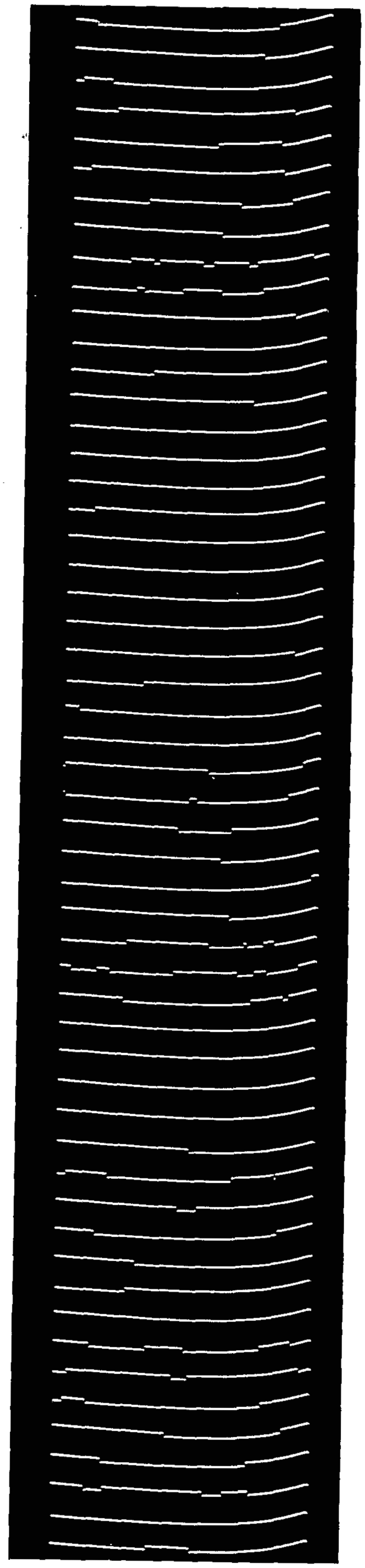

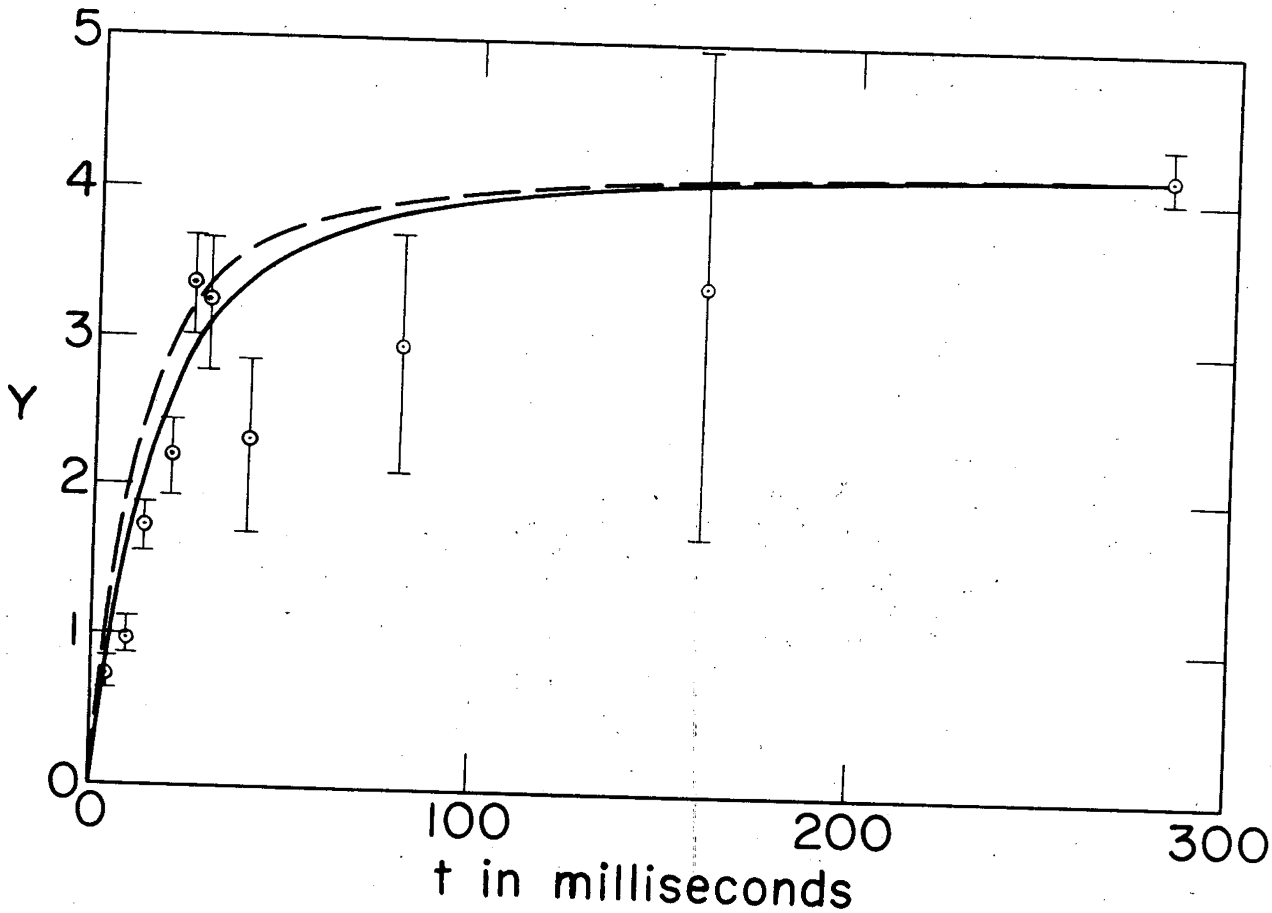

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