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Destruction of the Fractional Quantum Hall Effect by Disorder

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I suggest that Hall steps in the fractional quantum Hall effect are physically similar to those in the ordinary quantum Hall effect. This proposition leads to a simple scaling diagram containing a new type of fixed point, which I identify with the destruction of the fractional states by disorder.

It can be argued that the remaining "cosmic" issue in the theory of the fractional quantum Hall effect is how the effect is destroyed by disorder. If all but a very few of the hierarchial quantum Hall states now believed to exist were not destroyed by disorder, then the effect would be impossible to observe because the range of magnetic field strengths over which the Hall conductance is constant would be vanishingly small. In thinking about this problem, it is important to realize that the type of rigor one is accustomed to seeing in consideration of localization phenomena in the absence of magnetic fields may not be possible in this case. The presence of a magnetic field almost certainly makes this problem harder than two-dimensional localization with coulomb interactions, an as-yet unsolved problem. Given the complexity of the system, it is appropriate, in my opinion, to make qualitative theories, based on guesswork if necessary, which can serve as a conceptual guide for formulating experiments. The work described in this paper is such a theory [1]. Its physical content is that all quantum Hall steps are equivalent and that fractional quantum Hall plateaus disappear with increasing disorder by narrowing continuously, in rough analogy with the closing of a superconducting gap with increasing temperature. In addition, the plateaus are nested, so that the complete destruction of any plateau implies the previous destruction of all those deriving from it hierarchically [2-4].

It is now well established that the ordinary quantum Hall effect, and by inference the fractional effect, does not occur in the absence of disorder [5-6]. The zero-resistance state and quantized Hall conductance are constant over a range of gate voltages or magnetic field strengths because electrons or holes added to the system are immobilized in localized states. Our present understanding of the role of localization in the fractional quantum Hall effect is by analogy with the ordinary effect [7]. Charge added or removed from the sytem we understand to be trapped as an excess of localized quasielectrons or quasiholes. This analogy is <u>quantitatively</u> consistent, in that the accuracy of the fractional quantization is so high. The gauge sum rule [5,7] says that the quantum of Hall conductance is related in a fundamental way to the charge of the "particles" localized at the Fermi surface, and there is good theoretical reason to expect the quasiparticle charge 1/3e, 2/5e,... to be as exact a quantum of nature as e. Our present understanding of the formation of the hierarchy is also by analogy. We believe we understand the formation of the 1/3 state by interacting electrons, and we also believe that the fractionally charged elementary excitations of the 1/3 state can do anything electrons can do, including condense into a second-generation 1/3 state. Furthermore there are experimental indications that the Hall conductances of the hierarchical states may also be "exact" quanta.

Since all available experimental evidence agrees quantitatively with these analogies, it is reasonable to guess that they may be correct and exact [1]. This is unproven, and it may be wrong, but it has nontrivial and experimentally verifiable ramifications.

The first of these is that the transition between the Quantum Hall states 0 → 1/3 should be the "same" as that between the states 0 → 1. There is now considerable evidence that in the absence of coulomb interactions, this transition is described by the renormalization group flows shown in Fig. 1 [8,9]. Several comments are in order. One interprets this diagram physically the same way one interprets the one-dimensional flows of the ordinary scaling theory of localization. When the sample is small, localization does not occur, and the conductivities take on their "mean field" values $\sigma_{xx}^{(o)}$ and $\sigma_{xy}^{(o)}$, given by

$$\sigma_{xx}^{(o)} = \frac{\rho \varepsilon e^2 \tau}{m} \frac{1}{H_o(\omega_c \tau)^2} \tag{1}$$

and

$$\sigma_{xy}^{(o)} = \frac{nec}{H_o} - \frac{\sigma_{xx}^{(o)}}{\omega_c \tau} \tag{2}$$

where ρ is the Fermi surface density of states, ε is the Fermi energy, n is the electron density, and τ is a suitable elastic collsion time. It is helpful to think of the mean field values in the low-disorder limit, when $\sigma_{xy}^{(o)}$ represents the electron density and $\sigma_{xx}^{(o)}$ represents the amount of disorder. As the sample is imagined to be made bigger, the conductivities evolve along the flow lines, and converge to the quantum Hall values for very large samples. The fixed point in the diagram corresponds physically to the phase transition in which the Hall conductance jumps discontinuously from 0 to 1 as the Fermi level ($\sigma_{xy}^{(o)}$) is raised through a narrow band of extended states. The β-function for this flow has actually been calculated only in the $\sigma_{xx} \rightarrow \infty$ limit and for a particular kind of disordering potential [8,9]. The most important aspects of the diagram, its topology and exponents, are guesses. The paper presented by D. C. Tsui [10] at this conference shows experimental "flows" that are nearly perfect replicas of Fig. 1. While one could argue that this agreement is an accident, since the coulomb interactions in Prof. Tsui's samples should have made the non-interacting theory irrelevant, I think the correct interpretation is that "typical" coulomb interaction strengths always give rise to flows which look like the noninteracting theory when projected onto the $\sigma_{xx} - \sigma_{xy}$ plane, and that the topology of the guessed noninteracting flows is correct. I also think that the agreement between locations of the fixed points is meaningful. There is reason to believe that the fixed point should be near the maximum of the self-consistent born conductivity of Ando [1,9,11] when the correlation length of the impurity potential is much shorter than the localization length. When this is not the case, the position of the fixed points should depend on the functional form of the impurity potential.

If the transitions $0 \rightarrow 1/3$, $1/3 \rightarrow 2/3$ and $2/3 \rightarrow 1$, are all equivalent to the transition $0 \rightarrow 1$, then we must assign to each a fixed point like than in Fig. 1. One guesses the σ_{xx} coordinate of these points to be 1/9 of the $0 \rightarrow 1$ value, since the self-consistent born conductivity scales as the square of the particle's charge. The second important ramification of the exactness of these analogies is that the $0 \rightarrow 1/3$, ... etc., fixed points are topologically incompatible with the $0 \rightarrow 1$ fixed point without the addition of at least one new, totally repulsive fixed point to separate them. Assuming that there is only one, we obtain the flow diagram for the fractional quantum Hall effect shown in Fig. 2.

The new fixed point, denoted by a box in Fig. 2, corresponds physically to the phase transition in which increasing disorder destroys the fractional states. That this is the case is shown in more detail in Fig. 3. The horizontal eigenflows out of the box, which may be idealized as flowing into the 0 → 1/3 and 2/3 → 1 circles, bound a basin of attraction for the 1/3 state. If the sample is sufficiently dirty that the starting value $\sigma_{xx}^{(o)}$ is above this line, the 1/3 state cannot be reached. If the starting point is below this line, the 1/3 state cannot be avoided. Immediately above the 0 → 1/3 circle, the line is vertical and is analogous physically to the vertical eigenflow line above the 0 → 1 circle. Crossing it by changing $\sigma_{xy}^{(o)}$ is tantamount to moving the Fermi level through a narrow extended-state band in the quasiparticle density of states. The curvature of this line at large values of $\sigma_{xx}^{(o)}$ and its disappearance into the box imply that the quasihole extended-state band floats upward in energy with increasing disorder, as does the extended state band in the center of a Landau level in the ordinary quantum Hall effect [12,13], and eventually collides with the quasielectron extended state band. The box is thus a gap-closing transition but for a mobility gap rather than a real gap.

The generalization of these ideas to describe the full hierarchy of fractional quantum Hall states is straightforward. In Fig. 2, I have included bifurcations down to the level of 1/5, 2/7, 2/5, 5/7 and 4/5. The 1/5 and 4/5 states, which have not been shown conclusively to exist, do not derive hierarchically from the 1/3 and 2/3 states, and thus need not be contingent upon them. I have shown them to be contingent because I believe this to be the case when the repulsive forces are coulombic.

The ideas presented here may be tested experimentally in a number of ways. A series of measurements of the energy gap of the 1/3 state by the activation energy method [14] for increasingly dirty samples would verify the prediction of a continuous mobility gap closing as would measurements of the plateau width. Conductivity measured at finite temperature or at nonzero frequency can test the effects of finite size. It is possible that the partial resurrection of the 1/3 plateau reported by McFadden et al. [15] is an effect of the box.

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Figure Captions

Fig. 1 Scaling diagram for the ordinary quantum Hall effect [8,9]. The units of conductivity are e^2/h.

Fig. 2 Scaling diagram for the fractional quantum Hall effect [1]. The units of conductivity are e^2/h.

Fig. 3 Illustration of the closing of the mobility gap of the 1/3 state with increasing disorder. A larger starting value for σ_{xx} implies more disorder, more broadening of the quasiparticle density of states (shown on left), and a smaller quasiparticle mobility gap.

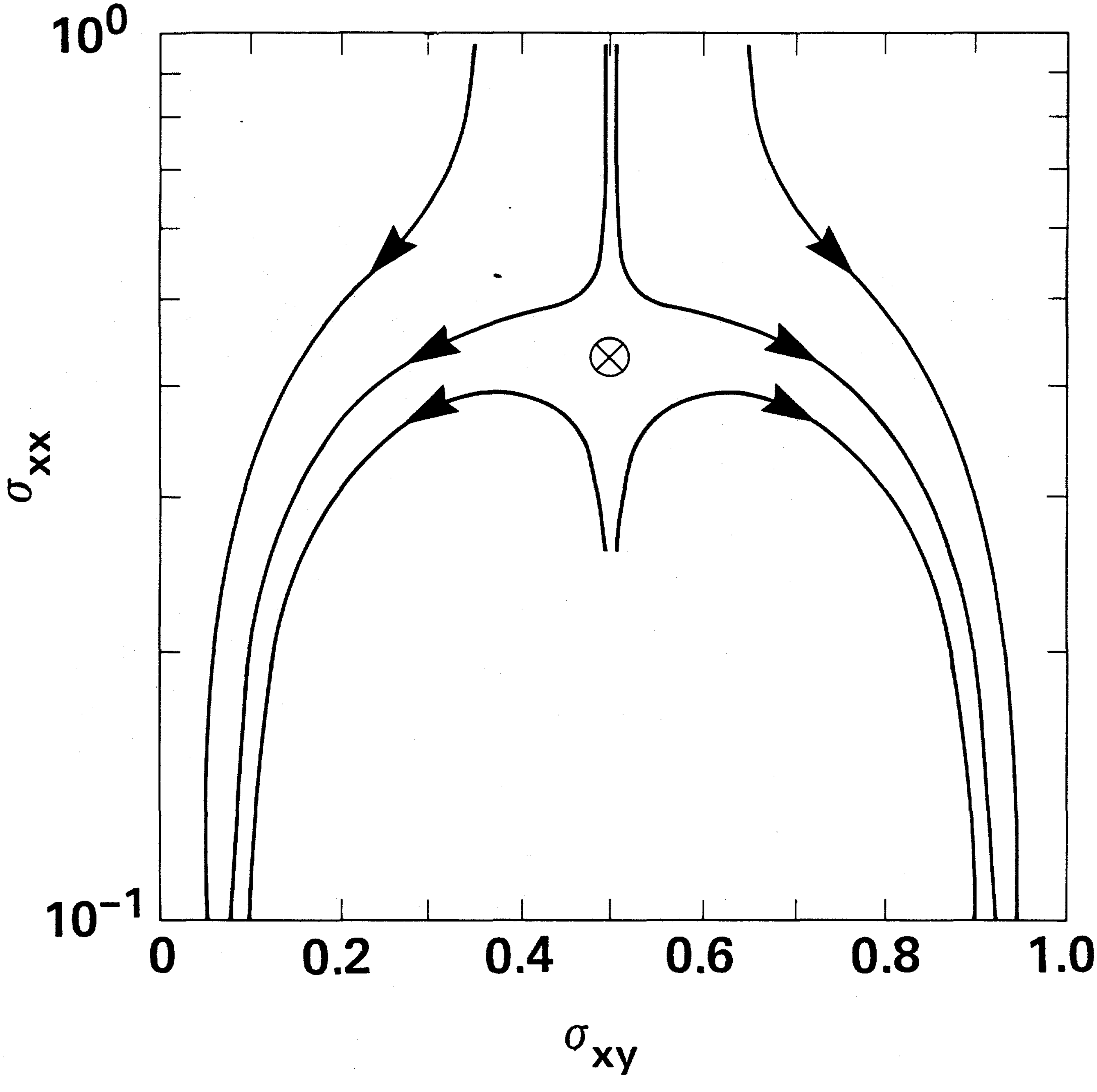

Figure 1

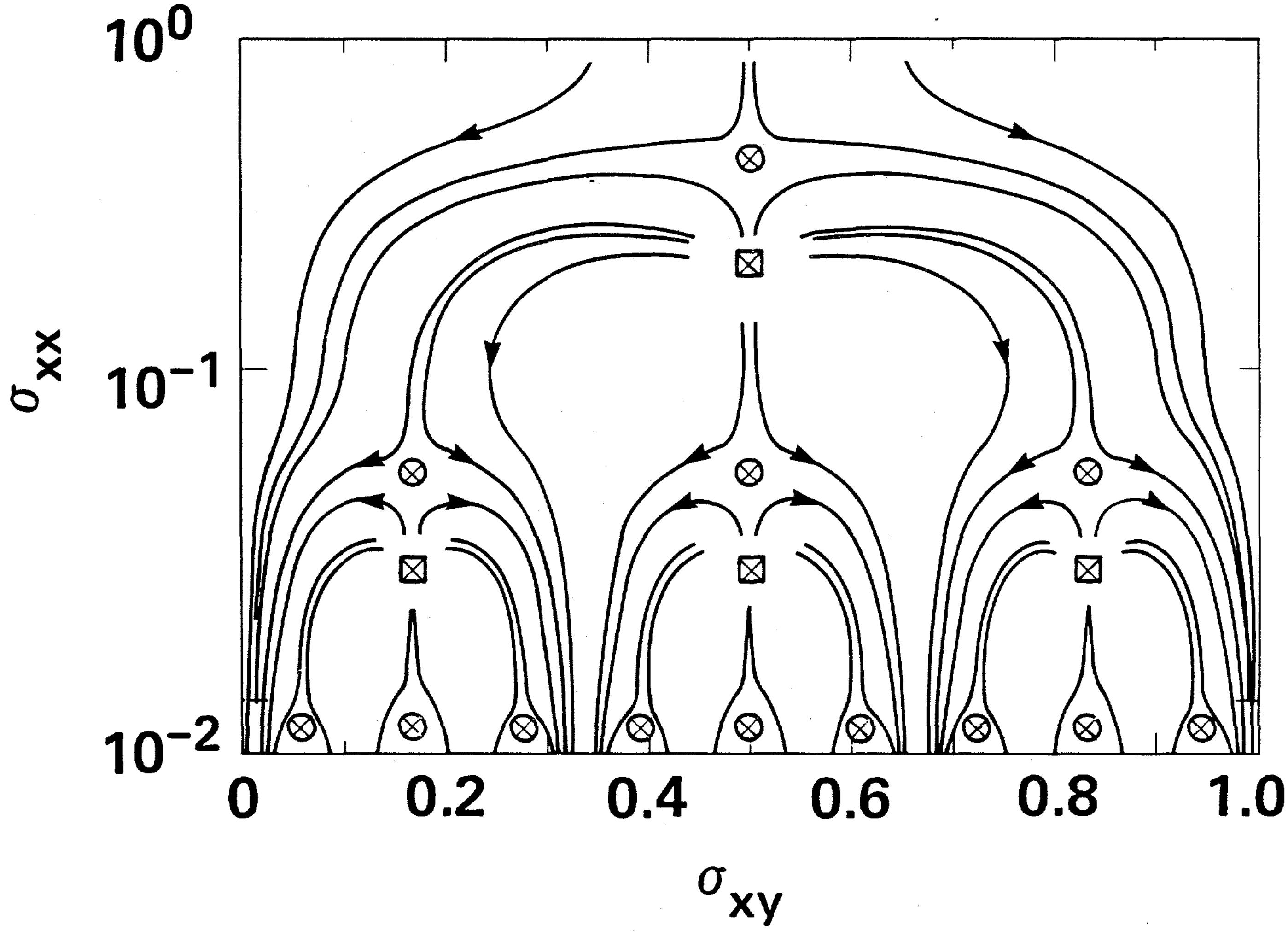

Figure 2

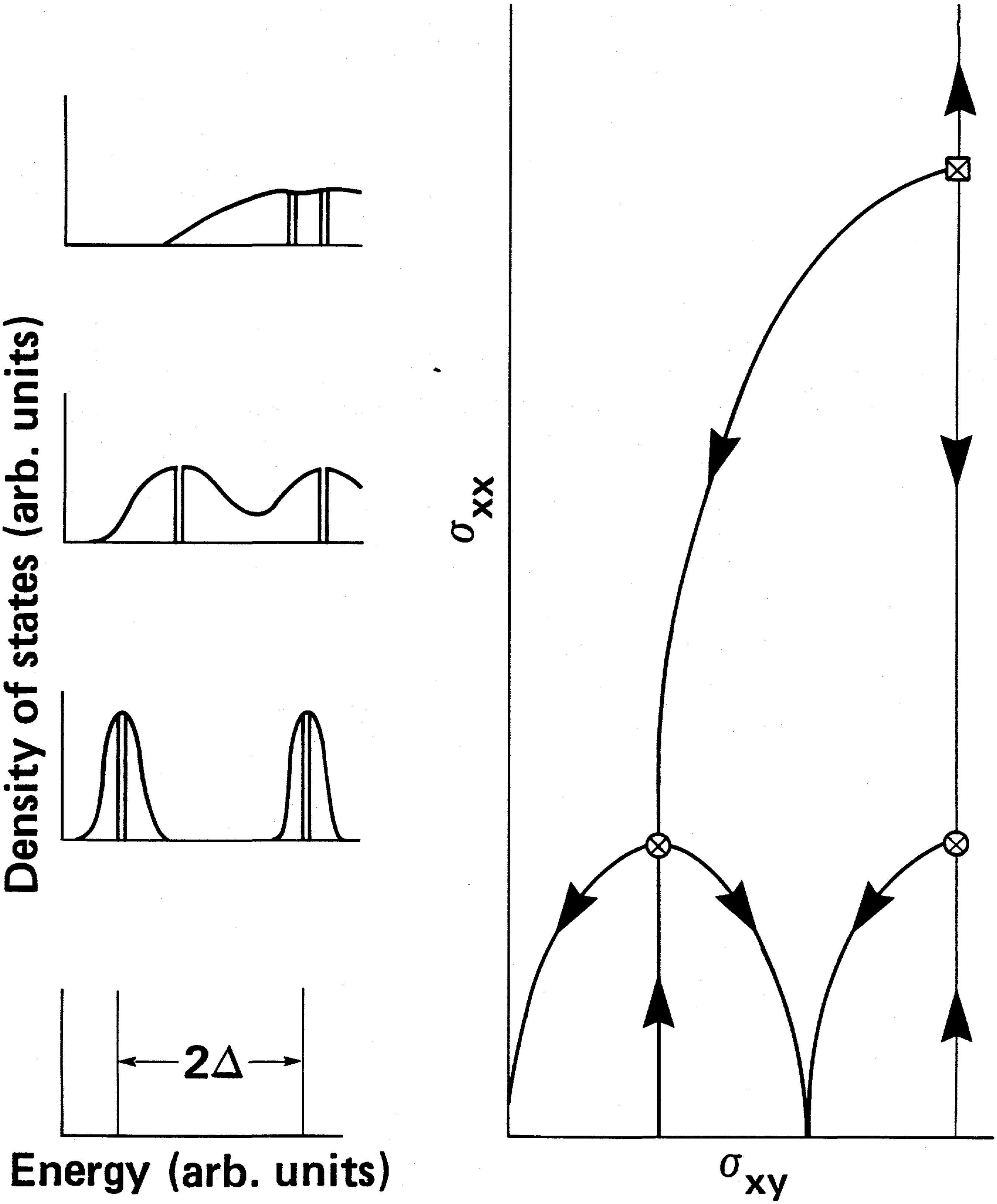

Figure 3