E911 PHASE 2 ACCURACY DEFINITION MODIFICATION PROPOSAL

MEAN RADIAL ERROR (MRE)

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Accuracy Proposals

- **RMS**
  - **Root Mean Square of all location attempt errors**
  - **Current FCC sanctioned accuracy definition**
  - **Assessment:**
    - › **Gives undue weight to outliers**
    - › **Includes all location attempts in accuracy calculation**
- **67%**
  - **Abscissa value at which the location error CDF crosses the 67% level**
  - **Assessment:**
    - › **Very insensitive to outliers**
    - › **Magnitude of largest 33% errors not counted in accuracy calculation**
- **90% RMS**
  - **Root Mean Square of location attempt errors with largest 10% excluded**
  - **Assessment:**
    - › **Insensitive to outliers**
    - › **Magnitude of largest 10% errors not counted in accuracy calculation**
- **New proposal: MRE (Mean Radial Error)**

Mean Radial Error (MRE) Proposal

- **Proposal development criteria**
  - all location attempts should be counted as part of the accuracy definition
  - a small fraction of location failures or "outliers" should not unduly skew the results of an otherwise excellent ALI technology
  - the original accuracy goals of the Commission should not be relaxed
- **MRE proposal**
  - Calculate accuracy as the Mean of the Radial Error
  - Comparison: RMS and MRE
    - RMS error equation (assumes actual location at origin)

$$A_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i^2 + y_i^2)}$$

Errors accumulated as magnitude squared

    - MRE error equation (same assumption)

$$A_{MRE} = \frac{1}{N}\sum_{i=1}^{N}\sqrt{x_i^2 + y_i^2}$$

Errors accumulated as magnitude

    - Errors accumulated by magnitude for MRE and magnitude squared for RMS
    - MRE includes all errors, but does not unduly magnify "outliers"
- **MRE provides for more equitable error weighting**

ACCURACY EQUIVALENCE

- **Probability theory**
  - **Gaussian probability density function (PDF)**

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \longrightarrow A_{RMS} = \sqrt{2\sigma^2}$$

  - **Transform this density by the function:** $r = \sqrt{x^2 + y^2}$
    - **Resulting random variable has a Rayleigh PDF**

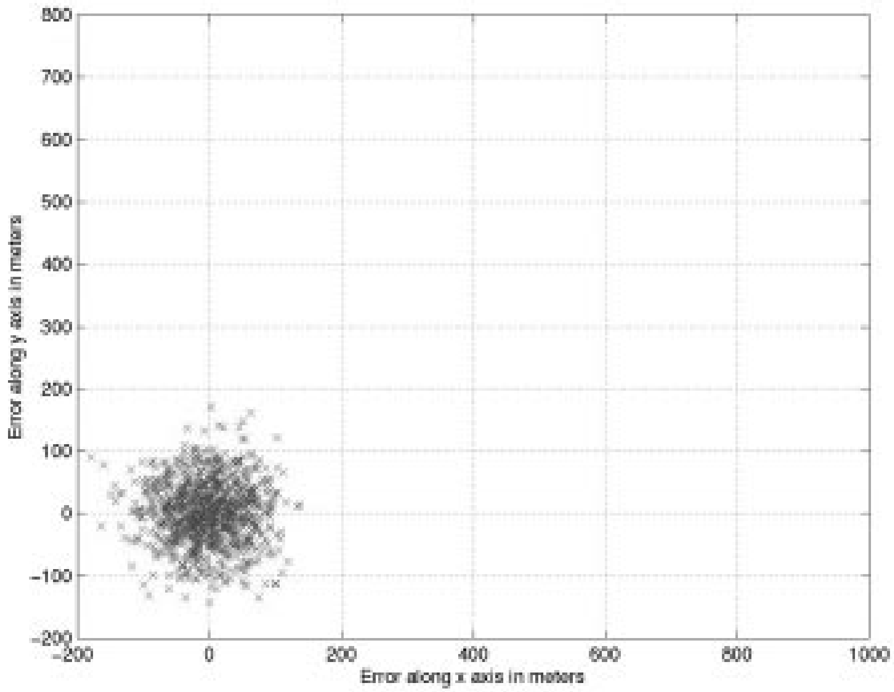
$$f_r(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} U(r)$$

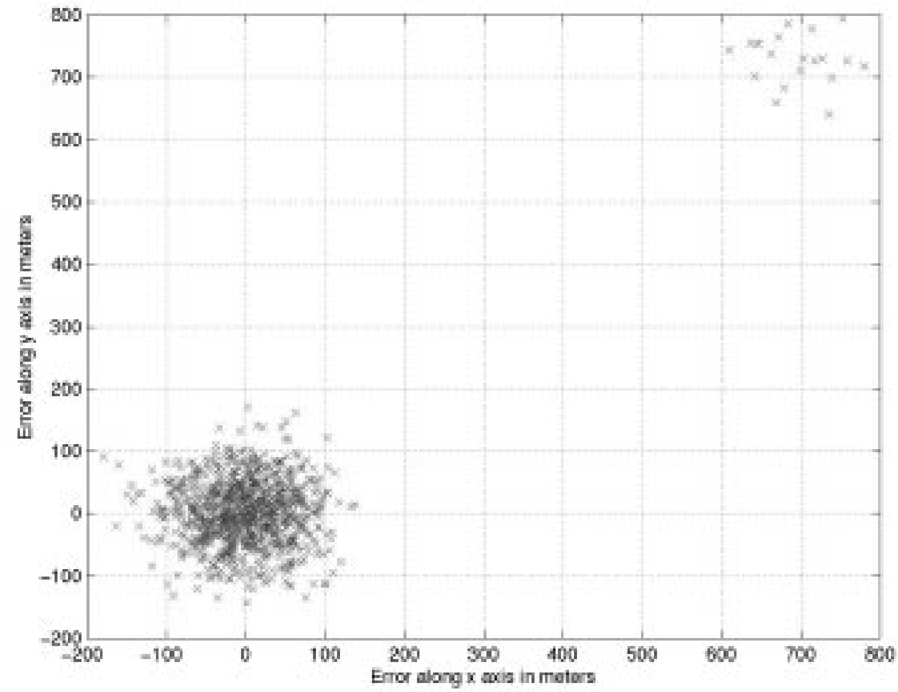
    - **The mean of this transformed Gaussian process is well known to be:**

$$A_{MRE} = \sigma\sqrt{\pi/2}$$

- **Accuracy equivalence**
  - **A_{RMS} = 125 m for σ = 88.39 m**
  - **For σ = 88.39 m, A_{MRE} = 110 m**
  - **Thus, setting a 110 meter MRE accuracy goal is equivalent to an RMS goal of 125 meters**
- **MRE accuracy goal of 110 meters is equivalent to 125 meter RMS goal**

ACCURACY DEFINITION CASE STUDIES

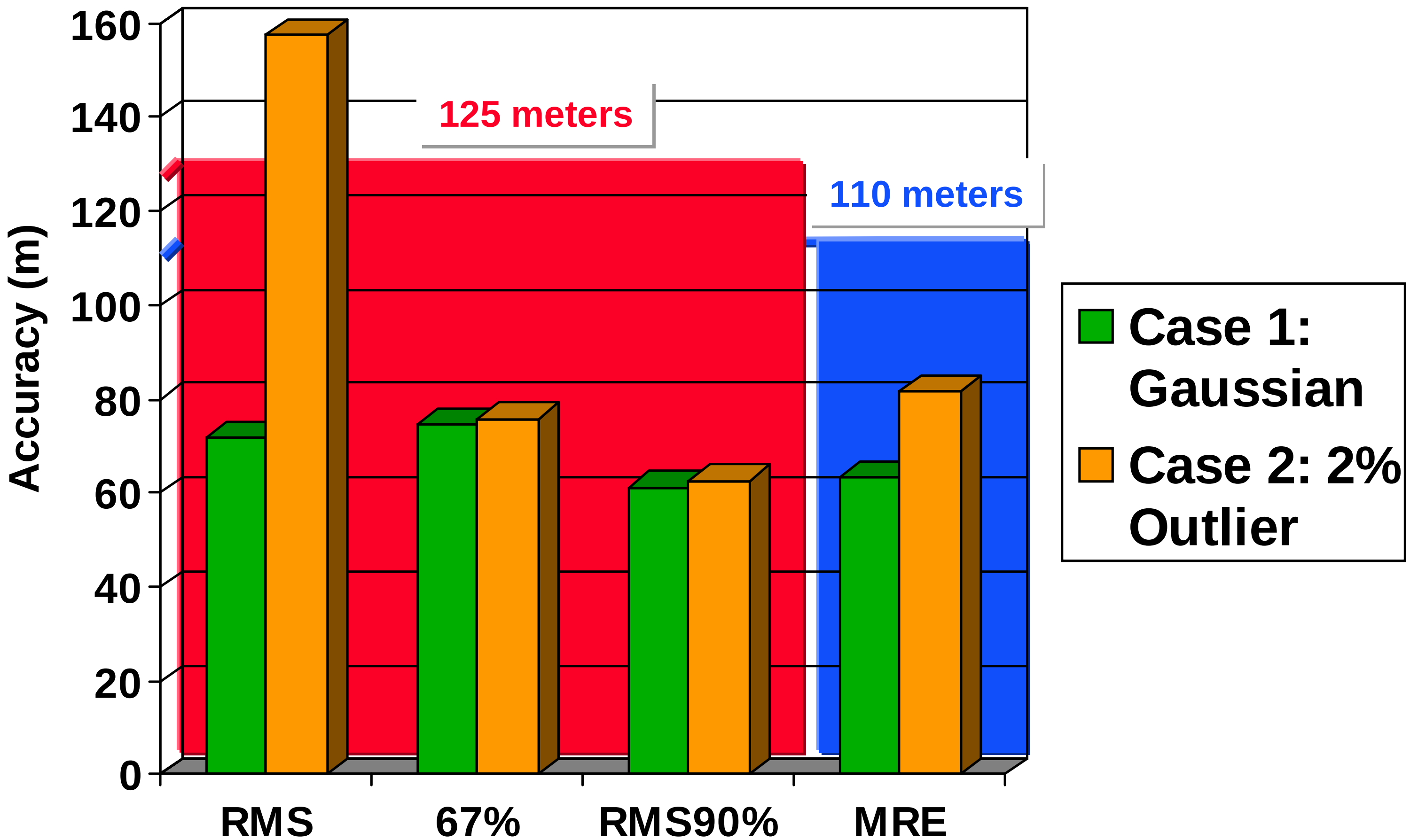




- **Case 1**
  - **1000 2-D Gaussian draws**
  - **σ = 50 m (both dimensions)**
  - **Radial error σ = 71 m**

- **Case 2**
  - **980 points identical to Case 1**
  - **20 "outliers" created**
    - **offset points by 1000 m**
    - **2% outlier probability**

ACCURACY RESULTS



- **Accuracy impact of 2% outlier as compared to Gaussian case**
  - **RMS accuracy degraded by 120%, non-compliant**
  - **RME accuracy degraded by 30%, compliant**
  - **67% and RMS90% accuracies degraded by less than 2%, compliant**

CONCLUSIONS

- The MRE definition:
  - Assures that all errors are counted in the accuracy calculation
  - Reduces sensitivity to outliers
  - Assures equivalent accuracy to the 125m/RMS goal (Gaussian assumption)
    - By setting MRE accuracy goal to 110 meters
- Win/Win for the interested parties
  - Public safety assured that all location errors counted in accuracy calculation
  - Carriers and vendors assured relief from excessive outlier impact on accuracy calculation
- We now have at least four accuracy definition proposals
  - RMS
  - 67%
  - 90% RMS
  - MRE
- Which one represents the best compromise between public safety requirements and wireless industry compliance difficulty?
- Let’s work together to expeditiously decide