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How well do we need to know the beam properties at a neutrino factory ?

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Abstract

In principle, a neutrino factory can produce a beam with a well known ν_e and ν_μ flux. In practice, the uncertainties on the muon beam properties will introduce uncertainties into the calculated neutrino fluxes. We explore the relationship between the beam systematics and the systematic uncertainties on predicted event rates at a far site. The desired precision with which we must know the beam momentum, direction, divergence, momentum spread, and polarization are discussed.

1 Introduction

Neutrino factories, consisting of an intense muon–collider–like muon source [1] feeding a storage ring [2] with long straight sections [3], have caught the imagination of the neutrino physics community. Since the time that neutrino factories were first proposed [3] it has been claimed that a muon storage ring neutrino source can produce beams with well known flavor content and fluxes. In practice, uncertainties on the muon beam properties will introduce uncertainties in the calculated neutrino fluxes. In this note we explore the relationship between the beam systematics and the systematic uncertainties on predicted event rates at a far site. The desired precision with which we must know the beam momentum, direction, divergence, momentum spread, and polarization are discussed.

2 The calculation

To be explicit, we will consider a neutrino factory in which 30 GeV muons decay in a straight section pointing at a detector located at a distance $L = 2800$ km. To calculate event rates at the distant site we use the program described in Ref. [3].

In the muon rest-frame the distribution of muon antineutrinos (neutrinos) from the decay $\mu^\pm \to e^\pm + \nu_e\ (\overline{\nu}_e) + \overline{\nu}_\mu\ (\nu_\mu)$ is given by:

$$\frac{d^2N_{\nu_\mu}}{dxd\Omega} = \frac{2x^2}{4\pi}\left[(3-2x) \mp (1-2x)\cos\theta\right], \tag{1}$$

where $x \equiv 2E_\nu/m_\mu$, θ is the angle between the neutrino momentum vector and the muon spin direction, and m_μ is the muon rest mass. The corresponding expression describing the distribution of electron neutrinos (antineutrinos) is:

$$\frac{d^2N_{\nu_e}}{dxd\Omega} = \frac{12x^2}{4\pi}\left[(1-x) \mp (1-x)\cos\theta\right]. \tag{2}$$

Thus, the neutrino and antineutrino energy– and angular– distributions depend upon the parent muon energy, the decay angle, and the direction of the muon spin vector. For an ensemble of muons we must average over the polarization of the initial state muons, and the distributions become:

$$\frac{d^2N_{\nu_\mu}}{dxd\Omega} \propto \frac{2x^2}{4\pi}\left[(3-2x) \mp (1-2x)P_\mu\cos\theta\right], \tag{3}$$

and

$$\frac{d^2N_{\nu_e}}{dxd\Omega} \propto \frac{12x^2}{4\pi}\left[(1-x) \mp (1-x)P_\mu\cos\theta\right], \tag{4}$$

where P_μ is the average muon polarization along the chosen quantization axis, which in this case is the beam direction. These expressions enable the neutrino fluxes at a distant site to be calculated once the number of muon decays, the beam energy (and hence Lorentz boost), and the muon polarization are specified. In practice the muon beam will have a finite divergence and momentum spread, and the beam–forming straight section will have a finite length. For a precise calculation of the neutrino fluxes at a distant site these small effects are included in the calculation.

To calculate event rates we must integrate the neutrino fluxes at the far site over a finite angular interval. We will average the flux over a "spot" size with a radius of 100 m. The charged current (CC) interaction cross-sections in the detector increase approximately linearly with the neutrino energy:

$$\sigma_{\nu N} \sim 0.67 \times 10^{-38}\ cm^2 \times E_\nu(GeV) \tag{5}$$

and

$$\sigma_{\overline{\nu} N} \sim 0.34 \times 10^{-38}\ cm^2 \times E_{\overline{\nu}}(GeV). \tag{6}$$

Although these simple CC cross-section parameterizations neglect the evolution of parton distributions etc they are nevertheless adequate for the present study.

Finally, in the following we will present the variation of event rates with muon beam properties for ν_e and ν_μ CC interactions with no oscillations, and for a $\nu_e \to \nu_\mu$ oscillation signal. For the oscillations we will assume three active neutrino flavors, and a set of representative oscillation parameters: $\delta m^2_{32} = 3.5\times10^{-3}\ eV^2/c^4$, $\delta m^2_{12} = 5\times10^{-5}\ eV^2/c^4$, $s_{13} = 0.10$ ($sin^2\theta_{13} = 0.04$), $s_{23} = 0.71$ ($sin^2\theta_{23} = 1.0$), $s_{12} = 0.53$ ($sin^2\theta_{12} = 0.8$), and $\delta = 0$.

3 Results

We consider the variation of predicted event rates with beam momentum, direction, divergence, momentum spread and polarization.

3.1 Event rate versus beam energy

The CC event rates are shown as a function of the muon beam energy in Fig. 1 for ν_e and ν_μ interactions in the absence of oscillations, and for $\nu_e \to \nu_\mu$ oscillations corresponding to the three–flavor oscillation parameters described in the previous section. We find that, in the absence of oscillations, $dN/dE_\mu = 0.01$ GeV^{-1} for $E_\mu \sim 30$ GeV. Note that we would expect the number of CC events:

$$N \sim E_\mu^3/L^2 \ , \tag{7}$$

and therefore:

$$\Delta N/N = 3\ \Delta E_\mu/E_\mu \ . \tag{8}$$

A 1% uncertainty on the stored muon energy would result in an 3% uncertainty on the predicted CC event rate, consistent with the curves shown in Fig. 1. In the presence of oscillations the dependence of the signal rate with muon beam energy is modified (with respect to the unoscillated case) by the energy dependence of the oscillation probability. For the particular $\nu_e \to \nu_\mu$ example we consider, the rate dependence is given by $dN/dE_\mu = 0.07$ GeV^{-1}, corresponding to $\Delta N/N = 2\ \Delta E_\mu/E_\mu$.

3.2 Event rate versus beam direction

The CC event rates are shown as a function of the muon beam direction in Fig. 2 for ν_e and ν_μ interactions in the absence of oscillations, and for $\nu_e \to \nu_\mu$ oscillations corresponding to the three–flavor oscillation parameters described in the previous section. The calculation includes a Gaussian muon beam divergence of $\sigma_{\theta_x} = \sigma_{\theta_y} = 0.1/\gamma$ (where $\gamma = E_\mu/m_\mu$). For $E_\mu = 30$ GeV we have $\sigma_{\theta_x} = \sigma_{\theta_y} = 0.33$ mr. We would expect that beam direction offsets $\Delta\theta$ would result in negligible rate uncertainties provided $\Delta\theta << \sigma_\theta$. In Fig. 2 we see that $\Delta N/N \leq 0.01$ for $\Delta\theta < 2$ mr. For larger direction offsets there are substantial changes in the predicted event rates. For example, a 0.5 mr offset reduces the unoscillated event rates by $\sim 5\%$. We conclude that an offset $\Delta\theta < 0.6\ \sigma_\theta$ is desirable.

3.3 Event rate versus muon beam divergence

The CC event rates are shown as a function of the muon beam divergence in Fig. 3 for ν_e and ν_μ interactions in the absence of oscillations, and for $\nu_e \to \nu_\mu$ oscillations corresponding to the three–flavor oscillation parameters described in the previous section. The calculation assumes Gaussian divergences in the horizontal and vertical planes

$\sigma_{\theta_x} = \sigma_{\theta_y} = \sigma_\theta$. Note that once σ_θ exceeds $\sim 0.1/\gamma$ the dependence of the event rates on the muon beam divergence increases significantly. For $\sigma_\theta \sim 0.1/\gamma$ we find that $\Delta N/N \sim 0.03\ \Delta\sigma_\theta/\sigma_\theta$. Hence a 10% uncertainty on the beam divergence would result in a 0.3% uncertainty in predicted event rates at the far site.

3.4 Event rate versus muon beam momentum spread

The CC event rates are shown as a function of the muon beam momentum spread in Fig. 4 for ν_e and ν_μ interactions in the absence of oscillations, and for $\nu_e \rightarrow \nu_\mu$ oscillations corresponding to the three–flavor oscillation parameters described in the previous section. Although the statistical uncertainties on the calculation are substantial, the event rates can be seen to gradually increase with momentum spread (the high energy tail increases the rate faster than the low energy tail decreases the rate). We find that $\Delta N/N \sim 0.06\ \Delta\sigma_p/\sigma_p$. Hence a 10% uncertainty on the beam momentum spread would result in a 0.6% uncertainty in predicted event rates at the far site.

3.5 Event rate versus muon polarization

It was pointed out in Ref. [3] that if the muons in the storage ring were polarized, then the direction of the muon spins could be used to effectively switch on or off the ν_e flux at a distant detector. In principle this provides a powerful method to verify the nature of a given oscillation signal. The down-side of this "advantage" is that the muon polarization must be precisely known to avoid a significant systematic uncertainty on the fluxes at the far site.

The CC event rates are shown as a function of the muon beam polarization in Fig. 5 for ν_e and ν_μ interactions in the absence of oscillations, and for $\nu_e \rightarrow \nu_\mu$ oscillations corresponding to the three–flavor oscillation parameters described in the previous section. In the absence of oscillations, with increasing polarization (from -1 to +1) the ν_e flux increases and the ν_μ flux decreases. For example, if the muon polarization is believed to be zero, an uncertainty in the polarization ΔP will introduce an uncertainty in the unoscillated ν_μ rate $\Delta N_{\nu_\mu}/N_{\nu_\mu} = 0.4\ \Delta P$ and an uncertainty in the unoscillated ν_e rate $\Delta N_{\nu_e}/N_{\nu_e} = \Delta P$. Hence, it is desirable that the muon polarization is determined to better than 0.01 if the ν_e flux uncertainty is to be kept to O(1%). These results are for a beam with zero beam divergence. With a finite beam divergence we would expect correlations between the contributions from ΔP and $\Delta\sigma_\theta$. This deserves further study.

4 Summary

The results discussed in this note are summarized in Table 1. To be explicit, let us assume that we wish to keep the uncertainty on the neutrino fluxes to $< 1\%$. In this case, in addition to knowing the number of muon decays with a precision of better than 1%, we must also:

Table 1: Dependence of predicted CC event rates on muon beam properties at a neutrino factory. The last column lists the required precisions with which each property must be determined if the uncertainty on the neutrino flux at the far site is to be less than $\sim 1\%$.

Muon Beam property	Beam Type	Rate Dependence	Target Precision
Energy (E_μ)	ν (no osc)	$\Delta N/N = 3\ \Delta E_\mu/E_\mu$	$\sigma(E_\mu)/E_\mu < 0.003$
	$\nu_e \to \nu_\mu$	$\Delta N/N = 2\ \Delta E_\mu/E_\mu$	$\sigma(E_\mu)/E_\mu < 0.005$
Direction ($\Delta\theta$)	ν (no osc)	$\Delta N/N \leq 0.01$ (for $\Delta\theta < 0.6\ \sigma_\theta$)	$\Delta\theta < 0.6\ \sigma_\theta$
Divergence (σ_θ)	ν (no osc)	$\Delta N/N \sim 0.03\ \Delta\sigma_\theta/\sigma_\theta$ (for $\sigma_\theta \sim 0.1/\gamma$)	$\Delta\sigma_\theta/\sigma_\theta < 0.2$ (for $\sigma_\theta \sim 0.1/\gamma$)
Momentum spread (σ_p)	ν (no osc)	$\Delta N/N \sim 0.06\ \Delta\sigma_p/\sigma_p$	$\Delta\sigma_p/\sigma_p < 0.17$
Polarization (P_μ)	ν_e (no osc)	$\Delta N_{\nu_e}/N_{\nu_e} = \Delta P_\mu$	$\Delta P_\mu < 0.01$
	ν_μ (no osc)	$\Delta N_{\nu_\mu}/N_{\nu_\mu} = 0.4\ \Delta P_\mu$	$\Delta P_\mu < 0.004$

(i) Know the muon beam energy with a precision of $\leq 0.3\%$.

(ii) Have a muon beam divergence no larger than $\sim 0.1/\gamma$.

(iii) Know the muon beam divergence σ_θ with a precision of $\leq 20\%$. The requirements for non–Gaussian beam divergences deserves further study.

(iv) Know the neutrino beam direction with a precision of better than $0.6\ \sigma_\theta$.

(v) Know the muon momentum spread with a precision given by $\Delta\sigma_p/\sigma_p < 0.17$.

(vi) Know the average muon polarization with a precision $\Delta P_\mu < 0.01$.

The instrumentation that would be needed to achieve these goals deserves some consideration.

References

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[3] S. Geer, "Neutrino beams from muon storage rings: characteristics and physics potential", FERMILAB-PUB-97-389, 1997; Presented in the Workshop on Physics at the First Muon Collider and Front-End of a Muon Collider (Editors: S. Geer and R. Raja), November, 1997, and Published in S. Geer, Phys.Rev.D57:6989-6997,1998.

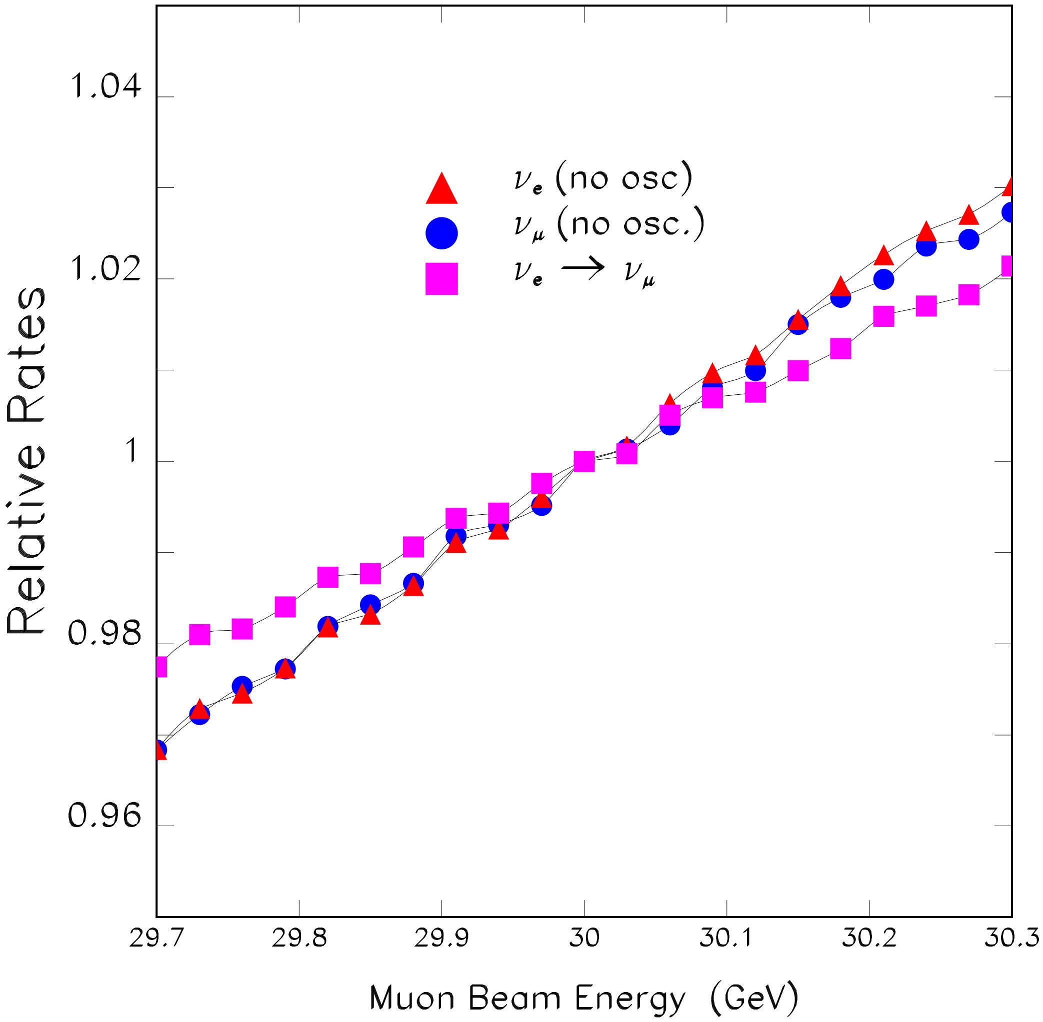

Figure 1: Dependence of CC neutrino interaction rates on muon beam energy for a detector at a far site located at $L = 2800$ km from the neutrino source. Rates are shown for ν_e (triangles) and ν_μ (circles) beams in the absence of oscillations, and for $\nu_e \rightarrow \nu_\mu$ oscillations (boxes) with the three–flavor oscillation parameters described in the text.

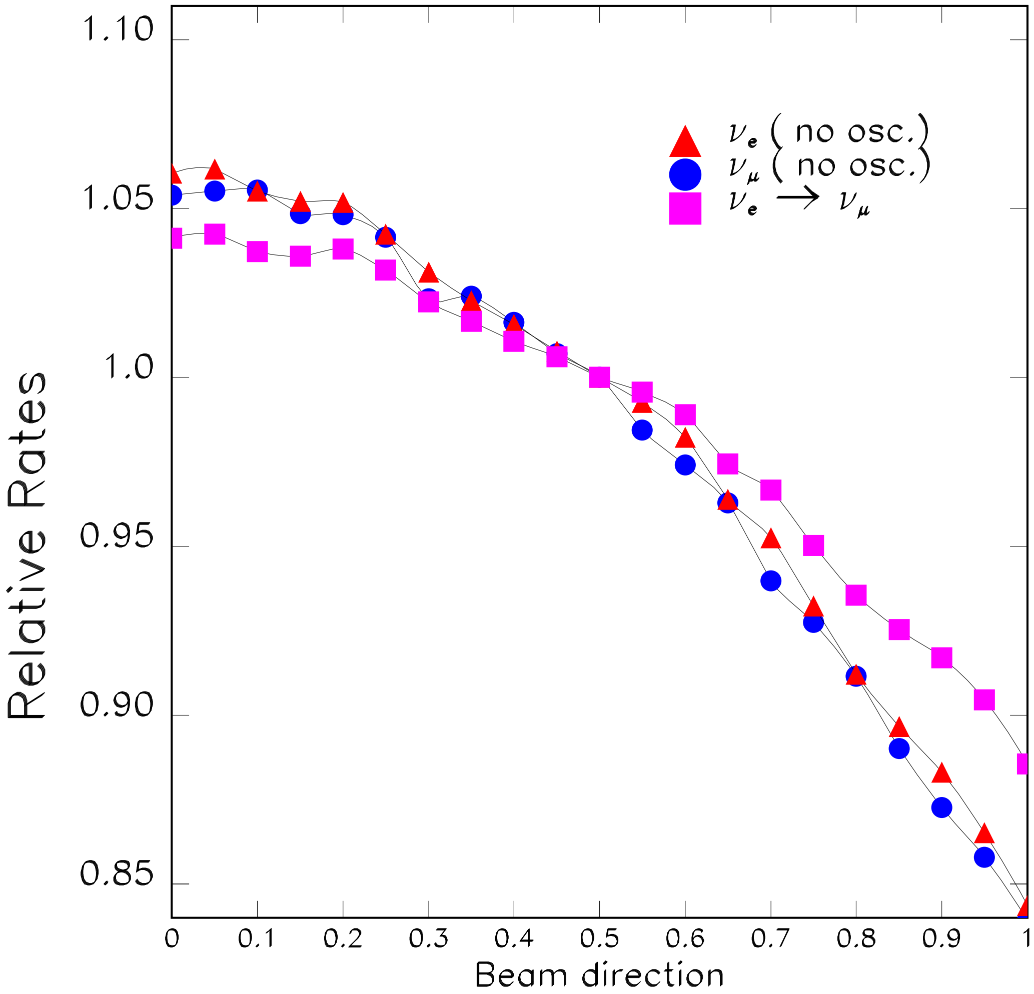

Figure 2: Dependence of CC neutrino interaction rates on the neutrino beam direction relative to a detector at a far site located at $L = 2800$ km from a muon storage ring containing 30 GeV unpolarized muons with a muon beam divergence of 0.33 mr. Rates are shown for ν_e (triangles) and ν_μ (circles) beams in the absence of oscillations, and for $\nu_e \to \nu_\mu$ oscillations (boxes) with the three–flavor oscillation parameters described in the text.

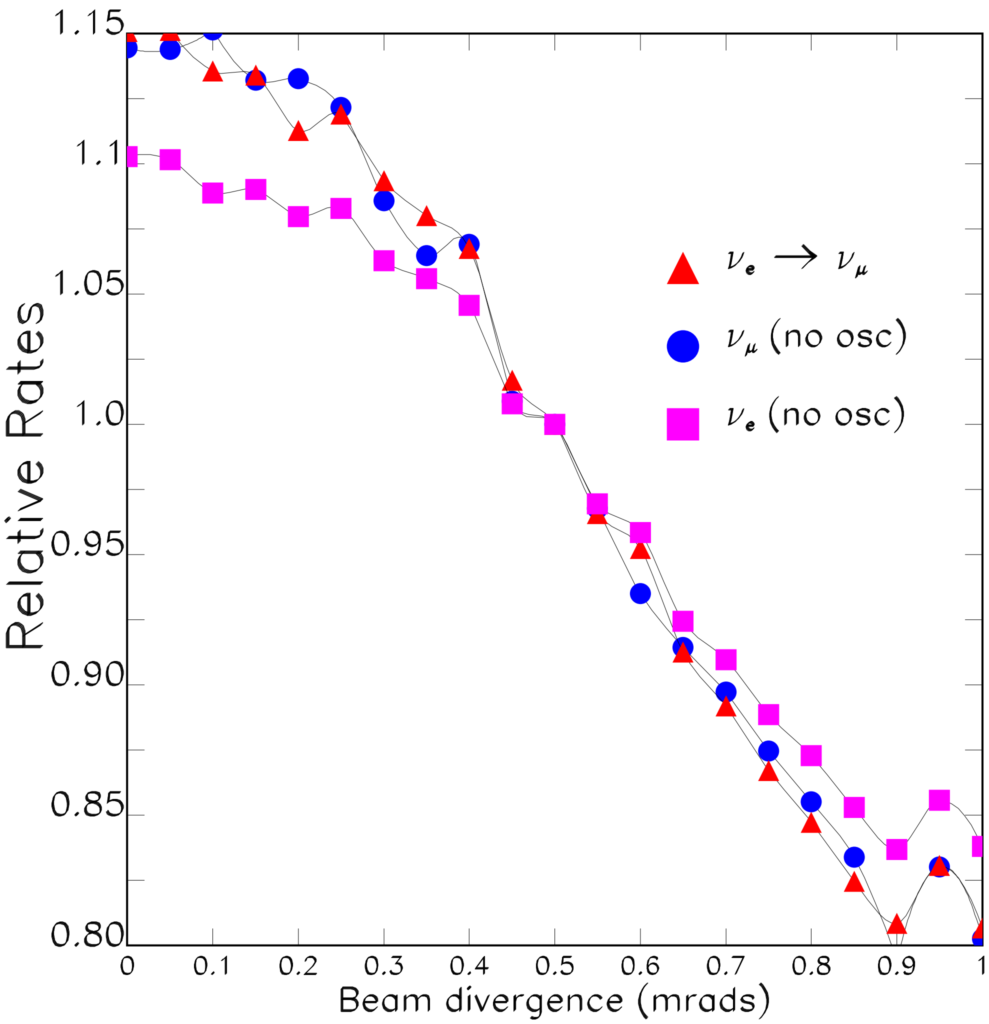

Figure 3: Dependence of CC neutrino interaction rates on the muon beam divergence for a detector at a far site located at $L = 2800$ km from a muon storage ring containing 30 GeV unpolarized muons. Rates are shown for ν_e (triangles) and ν_μ (circles) beams in the absence of oscillations, and for $\nu_e \rightarrow \nu_\mu$ oscillations (boxes) with the three–flavor oscillation parameters described in the text.

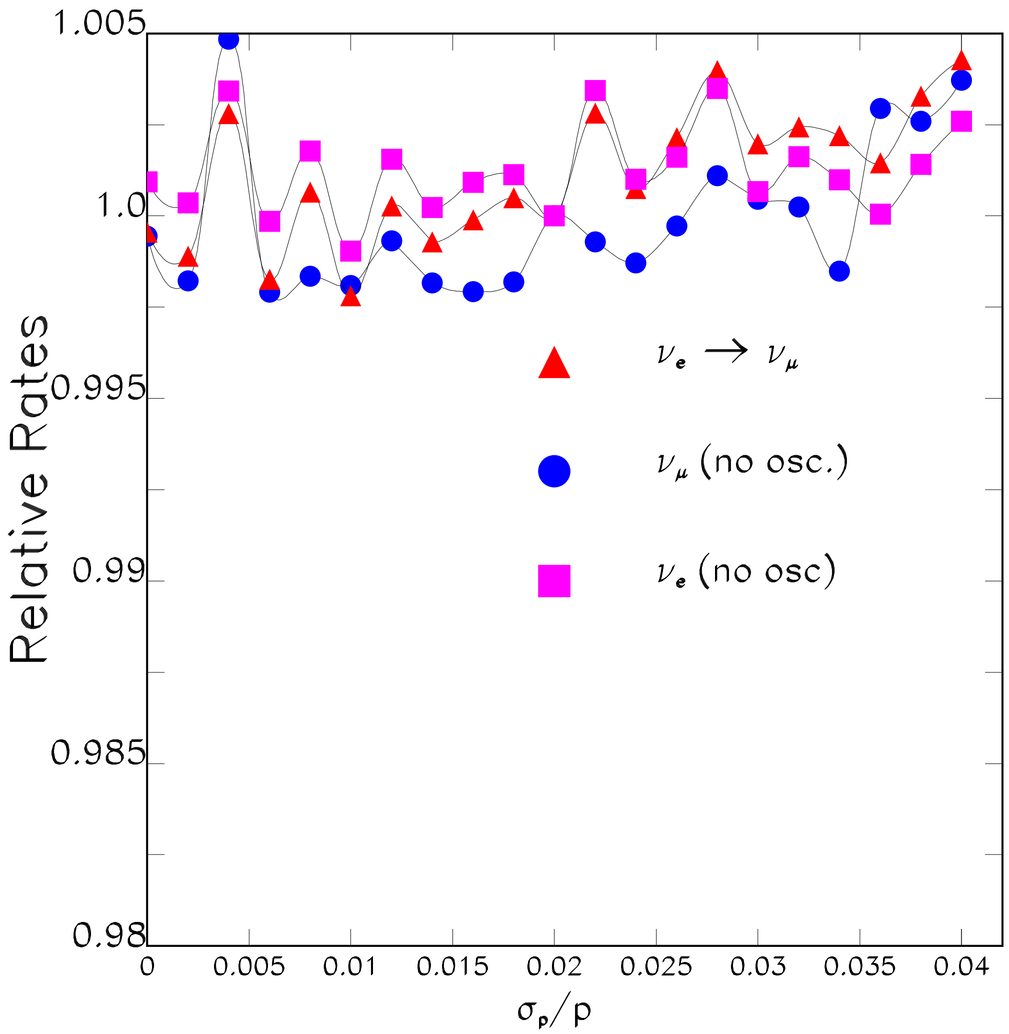

Figure 4: Dependence of CC neutrino interaction rates on the muon beam energy spread for a detector at a far site located at $L = 2800$ km from a muon storage ring containing 30 GeV unpolarized muons. Rates are shown for ν_e (triangles) and ν_μ (circles) beams in the absence of oscillations, and for $\nu_e \to \nu_\mu$ oscillations (boxes) with the three–flavor oscillation parameters described in the text.

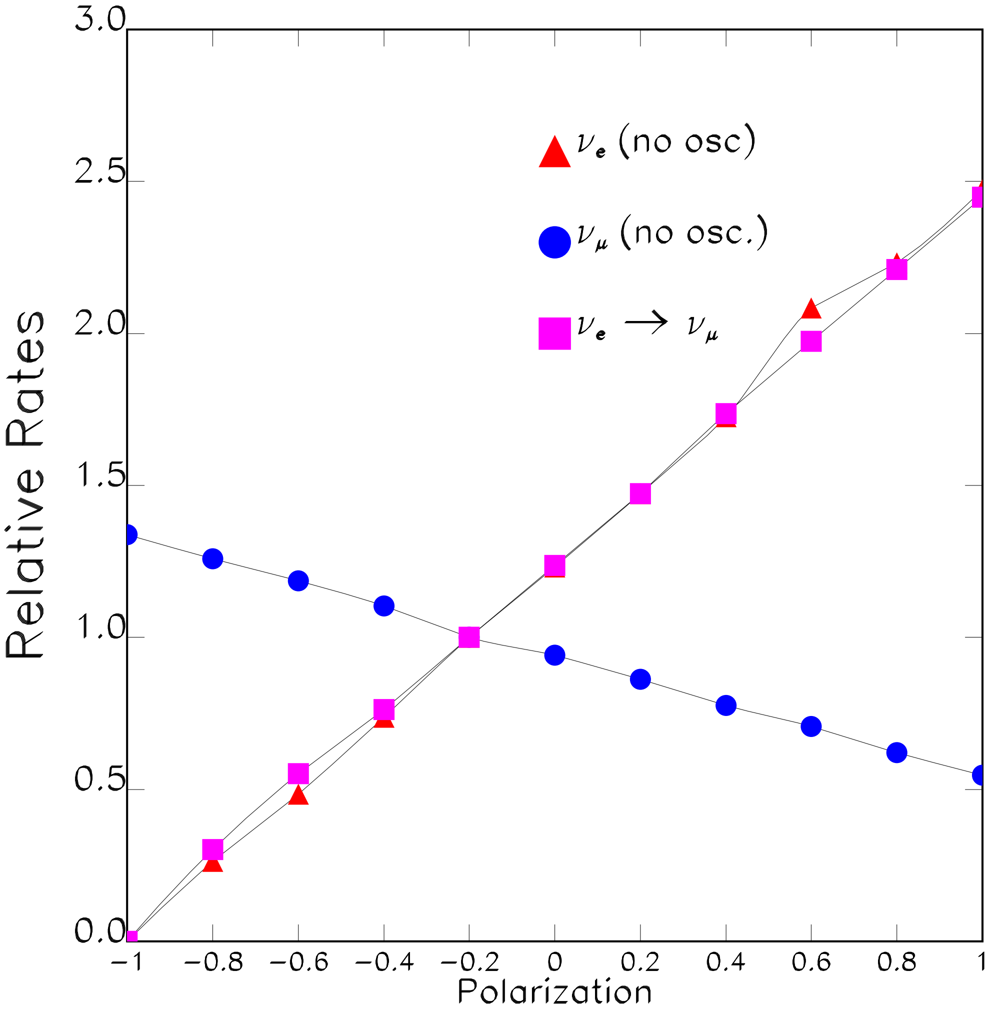

Figure 5: Dependence of CC neutrino interaction rates on the muon beam polarization for a detector at a far site located at $L = 2800$ km from a muon storage ring containing 30 GeV muons. Rates are shown for ν_e (triangles) and ν_μ (circles) beams in the absence of oscillations, and for $\nu_e \rightarrow \nu_\mu$ oscillations (boxes) with the three–flavor oscillation parameters described in the text.