POSSIBILITIES FOR AZ 4TH CREDIT MATHEMATICS COURSES

July 15, 2008

This document contains several suggestions for 4th credit courses with their correlations to the 2008 AZ Mathematics Standard. Descriptions of each course along with topics that are generally taught in the course are included along with specific correlations to the 2008 AZ College and Work Readiness Performance Objectives for Mathematics.

ARIZONA DEPARTMENT OF EDUCATION

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RESEARCH INTRODUCTION

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In December 2007, the Arizona State Board of Education adopted **R7-2-302.02** which increased the graduation requirements to include a 4th credit of mathematics beginning with the freshman class of 2013. On June 24th, 2008, the State Board of Education also approved a new Mathematics Standard K-12 that included performance objectives for grades 11 and 12 (College Work Readiness). This change will impact the education community in Arizona including students, educators, parents, and the larger community. Why is it important for a student to take four credits of mathematics? What does research say about placing all students into a four year mathematics program?

NCTM (2000) begins their text by posing the question, "Imagine a classroom or a school district where all students have access to high-quality, engaging, mathematics instruction (p. 2). It isn't sufficient to place students into a four year program just to satisfy state requirements. They must be engaged in high-quality learning. Marzano (2003) finds that effective teachers use more effective instructional strategies and also have more instructional strategies at their disposal. '

The National Mathematics Advisory Panel found that success in mathematics education is important to individual citizens because it opens options for college and career and it increases prospects for future income. Horn & Nunez (2000) and Horowitz (2005) found that the probability that a student will enroll in a four-year college is correlated to the completion of high school mathematics programs beyond the level of Algebra 2.

The NCTM Standards (2000) tell us that mathematical power consists of students who confidently engage in complex mathematical tasks, draw on knowledge from a wide variety of mathematical topics, approach problems from differing perspectives, are flexible and resourceful problem solvers, work productively and reflectively, communicate their ideas and results effectively, and place a value on mathematics. According to the National Research Council (1989), mathematics is involved in seeking solutions, not just memorizing procedures; exploring patterns, not just memorizing formulas; and formulating conjectures, not just doing exercises. It is important that the fourth year of mathematics encompass these ideas, the crux of what "significant mathematics" entails.

The National Research Council (2002) reminds us that the U.S. population is becoming increasingly diverse and education must mirror the diversity of the broader population. Therefore, there must be a variety of fourth year courses offered to students to help prepare them for the 21st century while honoring that diversity. Ma and Wilkins (2007) inform us that we need to explore how mathematics educators can use mathematics coursework to promote mathematics achievement for all students. Mathematics

courses must be available to all students and they must be given the opportunity to succeed through many paths including the diversity of the courses offered to them.

For many districts, the new 4th credit requirement for mathematics will pose a significant challenge. Not all students will be comfortable taking a Pre-Calculus course to satisfy this new requirement. New courses may need to be developed to help students achieve success in all four credits of high school mathematics. Teachers implementing any new curriculum and those who have used the same curriculum for a number of years need to be shown how to effectively implement the curriculum. Ball (2001) reminds us that new curriculum must be accompanied by ongoing opportunities for teachers to learn.

Marzano (2003) informs us that "a guaranteed and viable curriculum" has the most impact on student achievement. He says that this is both providing "opportunity to learn" and "time" (p. 22). Moses and Cobb (2001) saw mathematics literacy as a "civil right" that has been historically available to only an elite few. If we deny access to students to certain mathematical courses, we go beyond limiting their future educational and occupational opportunities. They argued it eventually limits a person's access to citizenship.

In this document, there are suggestions for six fourth credit mathematics courses including Pre-Calculus, Probability and Statistics, Trigonometry, Discrete Mathematics, Mathematical Models, and Financial Literacy. This is by no means an all-encompassing list. However, this document may help Districts to address the need for a diverse population of students to be successful in fourth credit courses that are both meaningful and "mathematically significant". This document includes a course description for each of these six courses, topics/outcomes, and correlations to the 2008 AZ College and Work Readiness Mathematics performance objectives. *Possibilities for AZ 4th Credit Mathematics Courses* should be used as a starting point to help to better equip our students for mathematics in the 21st century.

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ARIZONA FOURTH CREDIT GRADUATION REQUIREMENTS
Adopted 12/10/2007 by Arizona State Board of Education

c. Four credits of mathematics to minimally include the following:

i. Two credits containing course content covering the following areas in preparation for proficiency at the high school level on the AIMS test: Number Sense and Operations; Data Analysis, Probability and Discrete Mathematics; Patterns, Algebra and Functions; Geometry and Measurement; and Structure and Logic. These credits shall be taken consecutively beginning with the ninth grade unless a student meets these requirements prior to the ninth grade pursuant to subsection 1(c)(iv).

ii. One credit covering Algebra II or course content equivalent to Algebra II. Courses meeting this requirement may include, but are not limited to, career and technical education and vocational education, economics, science, and arts courses as determined by the local school district governing board or charter school.

iii. One credit that includes significant mathematics content as determined by the local school district governing board or charter school.

iv. Courses successfully completed prior to the ninth grade that meet the high school mathematics credit requirements may be applied toward satisfying those requirements.

v. The mathematics requirements may be modified for students using a personal curriculum pursuant to R7-2-302.03.

Strand 1: Numbers & Operations

Concept 1: Number Sense	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Solve problems and equations that require the number system to be extended from real to complex numbers.					x	
PO 2. Convert between radical and exponential forms of numerical expressions.					x	x

Concept 2: Numerical Operations						
PO 1. Explore different forms of complex numbers; determine if the properties of the real number system extend to complex numbers and matrices.						
PO 2. Perform computations with complex numbers.						
PO 3. Describe the relationship between real and complex numbers including plotting complex numbers as points in a plane.						
PO 4. Define polar coordinates; relate polar coordinates to Cartesian coordinates.	x		x			
PO 5. Convert complex numbers to trigonometric form and then multiply the results.	x		x			
PO 6. Apply DeMoivre's Theorem to calculate products, powers, and roots of complex numbers.	x		x			

Concept 3: Estimation						
PO 1. Recognize the limitations of estimations by assessing the amount of error resulting from estimation and determining whether the error is within acceptable tolerance limits.		x				x

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 1: Data Analysis (Statistics)						
PO 1. Solve problems by estimating and computing with one-variable and two-variable data.		x			x	x
PO 2. Compare data sets using graphs and summary statistics, including variance and standard deviation, with or without technology. (connects to SCHS-S1C4-02)		x			x	x
PO 3. Compute and explain summary statistics for distributions of data including measures of center and spread, including variance and standard deviation.		x			x	x
PO 4. Explain how sampling methods, bias, and the phrasing of questions asked during data collections impact the conclusions that can be drawn. (connects to SCHS-S1C3-04)		x			x	x
PO 5. Identify misleading uses of data and explain why they are misleading.		x			x	x
PO 6. Explain the differences between randomized experiments and observational studies; and determine the appropriateness of using each in given situations.		x			x	x
PO 7. Determine when arguments based on data mistake correlation for causation.		x			x	x
PO 8. Draw a line of best fit for scatter plot with or without technology; describe how the correlation coefficient relates to fit; and explain when it is appropriate to use the regression equation to make predictions. (connects to SCHS-S1C3-01)		x			x	
PO 9. Use matrices to organize and represent data.		x		x	x	x

Concept 2: Probability	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Apply probability concepts to calculate the probability of events and to make informed decisions in practical situations.		x			x	x
PO 2. Use the principal characteristics of the normal distribution to estimate probabilities.		x			x	x
PO 3. Estimate probabilities and predict outcomes using one- and two-variable data.		x			x	x
PO 4. Determine the conditional probability of an event given that another event occurs, decide if two events are dependent or independent, and determine the probability of an event given the probability of the complementary event.		x			x	

Concept 3: Discrete Mathematics – Systematic Listing and Counting

PO 1. Use the binomial theorem and Pascal's Triangle to solve problems.	x			x		
PO 2. Explain the connections between the binomial coefficients, entries of Pascal's triangle, and combinations.	x			x		

Concept 4: Vertex-Edge Graphs

PO 1. Study the following topics related to vertex-edge graphs: Euler circuits, Hamilton circuits, the Travelling Salesperson Problem (TSP), minimum weight spanning trees, shortest paths, vertex coloring, and adjacency matrices				x		
PO 2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.				x		
PO 3. Devise, analyze, and apply algorithms for solving vertex-edge graph problems.				x		
PO 4. Extend work with adjacency matrices for graphs, such as interpreting row sums and using the nth power of the adjacency matrix to count paths of length *n* in a graph. (connects to MCWR-S3C3-07)				x		

Strand 3: Patterns, Algebra, and Functions

Concept 1: Patterns

PO 1. Use models (i.e., algebraic formulas) to represent and analyze sequences and series: • ***explicit formulas for nth terms,*** • ***sums of finite arithmetic series, and*** • ***sums of finite geometric series.***	x			x	x	x
PO 2. Apply recursion equations involving arithmetic and geometric sequences to solve problems.	x			x	x	x
PO 3. Distinguish between explicit and recursive formulas; convert explicit formulas to recursive formulas and convert recursive formulas to explicit formulas making good choices about when to use which.				x	x	
PO 4. Solve problems involving recursion.				x	x	
PO 5. Use and interpret sigma notation to represent summation.				x	x	

Concept 2: Functions and Relationships	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.	x				x	x
PO 2. Use function notation flexibly: evaluate a function for a value represented by a variable expression.	x		x		x	x
PO 3. Graph absolute value, and step and other piecewise-defined functions identifying their key characteristics.	x				x	
PO 4. Graph exponential functions identifying their key characteristics. (connects to MCWR-S4C2-03)	x				x	x
PO 5. Sketch the graphs and determine the key characteristics of power functions in the form f(x) = ax^n , a≠ 0, for positive integral values of n.	x		x		x	
PO 6. Graph polynomial functions identifying their key characteristics.	x		x		x	x
PO 7. Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions.	x		x			x
PO 8. Find the domain, range, intercepts, and asymptotes of conic sections.	x					
PO 9. Find domain, range, intercepts, periods, amplitudes, and asymptotes of trigonometric functions.			x			
PO 10. Given a function: • ***find the inverse of the function,*** • ***determine whether the inverse is a function,*** • ***explain why the graph of a function and its inverse are reflections of each other over the line y = x.***	x		x			
PO 11. Find approximate solutions for polynomial equations with or without graphing technology.	x				x	x
PO 12. Use theorems of polynomial behavior (including but not limited to the Fundamental Theorem of Algebra, Remainder Theorem, the Rational Root Theorem, Descartes Rule of Signs, the Conjugate Root Theorem) to find the zeros of a polynomial function.	x					
PO 13. Relate logarithms and exponential functions as inverses; prove basic properties of a logarithm using properties of its inverse; and apply those properties to solve problems.	x					x
PO 14. Combine functions by composition, as well as by addition, subtraction, multiplication, and division including any necessary restrictions on the domain.						
PO 15. Determine if functions are even, odd, or neither both algebraically and graphically.						
PO 16. Identify the degree of a given polynomial function and write a polynomial function of a given degree.						
PO 17. Develop an informal notion of limits.	x					

Concept 3: Algebraic Representations	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Rewrite and describe the need for equivalent forms of algebraic expressions.		x	x		x	
PO 2. Apply the laws of exponents including rational and negative exponents to rewrite expressions in alternative forms.					x	
PO 3. Solve systems of three linear equations in three variables with or without technology.					x	
PO 4. Use matrices to represent everyday problems that involve systems of linear equations.					x	x
PO 5. Simplify radical expressions by performing operations on them.			x		x	x
PO 6. Divide a polynomial by a lower degree polynomial.					x	
PO 7. Find complex solutions for quadratic equations.	x				x	
PO 8. Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression with and without technology.	x		x		x	x
PO 9. Use matrix operations and the inverse of a matrix to solve problems. (connects to MCWR-S2C4-04)	x					
PO 10. Represent vectors as matrices.	x					
PO 11. Add, subtract, and compute the dot product of two-dimensional vectors; multiply a two-dimensional vector by a scalar.	x					

Concept 4: Analysis of Change						
PO 1. Analyze and describe how a change in an independent variable leads to a change in a dependent variable.	x		x		x	x
PO 2. Identify patterns in a function's rate of change identifying intervals of increase, decrease, constancy, and if possible, relate them to the function's verbal description or its graph.	x		x		x	x
PO 3. Analyze change in various contexts by modeling and solving word problems using functions and equations.	x		x		x	x
PO 4. Compare relative magnitudes of functions and their rates of change.	x		x		x	x
PO 5. Solve problems involving compound interest.						x
PO 6. Demonstrate the relationship between: • simple interest and linear growth • compound interest and exponential growth.					x	x
PO 7. Determine the total cost of purchasing consumer durables over time given different down payments, financing options, and fees.					x	x
PO 8. Apply a variety of strategies to use tax tables and determine, calculate, and complete yearly federal income tax.					x	x
PO 9. Develop a personal budget including debit, checking, and savings accounts by interpreting multiple personal budget examples.					x	x
PO 10. Determine an effective retirement savings plan to meet personal financial goals including IRA's, ROTH accounts, and annuities.					x	x
PO 11. Compare and contrast the role of insurance as a device to mitigate risk and calculate expenses of various options.					x	x

Strand 4: Geometry and Measurement

Concept 1: Geometric Properties	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Perform basic geometric constructions using a variety of methods (e.g., straightedge and compass, patty/tracing paper, or technology):						
• perpendicular bisector of a line segment,						
• bisector of an angle, and						
• perpendicular or parallel lines.						
PO 2. Explore geometries other than Euclidean geometry in which the parallel postulate is not true.						
PO 3. Apply the law of cosines and the law of sines to find missing sides and angles of triangles.	x		x			
PO 4. Use basic trigonometric identities including Pythagorean, reciprocal, half-angle and double-angle, and sum and difference formulas to solve equations and problems.	x		x			
PO 5. Examine the application of multivariable equations to multiple dimensions including surfaces, cross-sections, and *n*-dimensional objects.						

Concept 2: Transformation of Shapes						
PO 1. Describe the effect that changes in the parameters of a quadratic function have on the shape and position of its graph $(f(x) = a(x-h)^2+k)$.	x					x
PO 2. Describe the effect that changes in the parameters of an exponential function have on the shape and position of its graph $(f(x) = ab^x)$.	x					x
PO 3. Describe how changing the parameters of a trigonometric function affects the shape and position of its graph ($f(x) = A \sin B(x-C)+D$ or the other trigonometric functions).			x			

Concept 3: Coordinate Geometry						
PO 1. Graph the solution set of a system of two or three linear inequalities, and given an ordered pair determine whether it is a solution to the system.					x	x
PO 2. Determine an equation of a circle given its center and radius; given an equation of a circle, find its center and radius.					x	
PO 3. Graph equations of conic sections explaining the relationships between and among their algebraic form and key characteristics.	x					
PO 4. Graph all six trigonometric functions identifying their key characteristics.			x			
PO 5. Evaluate all six trigonometric functions of angles between (0 degrees and 360 degrees, 0 and 2π radians) using the unit circle in the coordinate plane.			x			
PO 6. Convert between rectangular and polar coordinates	x		x			
PO 7. Graph equations given in polar coordinates.	x		x			

Concept 4: Measurement						
PO 1. Explain, use, and convert between degree and radian measures for angles.	x		x			

Strand 5: Structure and Logic

Concept 1: Algorithms and Algorithmic Thinking	Pre-Calculus	Probability & Statistics	Trigonometry	Discrete Mathematics	Mathematical Modeling	Financial Literacy
PO 1. Use a variety of approaches (inductive and deductive reasoning, estimations, generalizations, formal and informal methods of proof) to analyze algorithms.	x	x	x		x	x

Concept 2: Logic, Reasoning, Problem Solving, and Proof						
PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.	x	x	x	x	x	x
PO 2. Solve problems by using theorems, formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).	x	x	x	x	x	x
PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.	x	x	x	x	x	x
PO 4. Generalize a solution strategy for a single problem to a class of related problems; explain the role of generalizations in inductive and deductive reasoning.	x	x	x		x	x
PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.	x	x	x		x	x
PO 6. Synthesize mathematical information from multiple sources to draw a conclusion, make inferences based on mathematical information, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.	x	x	x		x	x
PO 7. Analyze and explain the general properties and behavior of functions or relations using algebraic and graphing techniques.		x	x		x	x
PO 8. Use inductive and deductive reasoning to make, analyze, and validate or refute conjectures and/or proofs.		x				x
PO 9. Use mathematical models to represent and analyze personal and professional situations.	x	x			x	x
PO 10. Differentiate, interpret, apply, and develop concepts in the context of personal and professional situations.	x	x			x	x
PO 11. Determine under what conditions a given statement (algebraic, geometric) is true.	x	x				

PRE-CALCULUS

Course Description: In Pre-calculus, students continue to expand upon the K-8, Algebra I, Algebra II, and Geometry foundations as they increase their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts as well as the relationships among them. Students use functions, equations, and limits to express generalizations and to analyze and understand a wide range of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, and trigonometry to model physical situations. They are given a preview of calculus Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal) tools. They use technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

Topics/outcomes:

- Introduction to functions **CWR 3.2.1, 3.2.2, & 3.2.6**
- Quadratic functions **CWR 3.2.1, 3.2.6, 3.3.7, & 4.2.1**
- One-to-One functions and their inverses **CWR 3.2.10**
- Polynomial functions **CWR 3.2.1, 3.2.2, 3.2.3, 3.2.6, 3.2.7, & 3.4.4**
- Dividing polynomials and synthetic division **CWR 3.3.6**
- Real zeros of polynomials **CWR 3.2.11, 3.2.12**
- Complex numbers **CWR 1.1.1, 1.2.2, & 1.2.3**
- Complex zeros and the Fundamental Theorem of Algebra **CWR 3.2.12**
- Exponential functions **CWR 3.2.1, 3.2.2, 3.2.4, 3.2.13, & 4.2.2**
- Logarithmic functions **CWR 3.2.1, 3.2.2, & 3.2.13**
- Exponential growth and decay problems **CWR 3.2.1 & 3.2.2**
- Trigonometric functions **CWR 3.29 & 4.2.3**
- The unit circle **CWR 4.3.5 & 4.4.1**
- Trigonometric graphs **CWR 4.2.3 & 4.3.4**
- Trigonometric functions of angles **CWR 4.3.5**
- The Law of Sines and The Law of Cosines **CWR 4.1.3**

- Fundamental trigonometric identities **CWR 4.1.4**
- Double-angle, half-angle, and product-sum formulas **CWR 4.1.4**
- Inverse trigonometric functions **CWR 3.2.10**
- Trigonometric equations **CWR 3.2.9**
- Polar coordinates **CWR 4.3.6**
- Graphs of polar equations **CWR 4.3.7**
- Polar form of complex numbers, DeMoivre's Theorem **CWR 1.2.4, 1.2.5, & 1.2.6**
- Vectors **CWR 3.2.10**
- The dot product **CWR 3.2.11**
- Systems of equations **CWR 3.3.3**
- Systems of linear equations **CWR 3.3.4**
- The algebra of matrices **CWR 1.2.9**
- Inverses of matrices and matrix equations **CWR 3.3.9**
- Determinants and Cramer's Rule
- Partial fractions
- Systems of inequalities **CWR 4.3.1**
- Conic sections including parabolas, ellipses, hyperbolas, and shifted conics **CWR 3.2.8, 4.3.2, & 4.3.3**
- Sequences and summation notation **CWR 2.3.1, 2.3.2, & 2.3.5**
- Arithmetic sequences **CWR 2.3.1, 2.3.2, & 2.3.5**
- Geometric sequences **CWR 2.3.1, 2.3.2, & 2.3.5**
- Counting Principles
- Mathematical induction **CWR 5.1.1 & 5.1.8**
- Binomial Theorem and Pascal's triangle **CWR 2.3.1 & 2.3.2**
- Finding limits numerically and graphically **CWR 3.2.17**

PROBABILITY AND STATISTICS

Course Description: Probability and Statistics will encompass descriptive and inferential statistics: organization of data; measures of central tendency, dispersion, and association; linear regression and correlation. The concept of probability and its properties include random variables: expected value, variance, independence, standard discrete and continuous distributions, and normal approximation. The study of sampling and statistical inference include: estimating population parameters, interval estimation, and hypothesis testing.

Topics/outcomes:

- Descriptive and inferential statistics **CWR 2.2.3 & 5.2.2**
- Variables and types of data **CWR 2.1.1**
- Data collection and sampling techniques **CWR 2.1.4 - 2.1.6**
- Organizing data **CWR 2.1.9**
- Histograms, frequency polygons and ogives **CWR 2.3.1, 2.3.2 & 5.2.1**
- Other types of graphs
- Measures of central tendency **CWR 2.1.3**
- Measures of variation **CWR 2.1.3 & 2.2.1 - 2.2.4**
- Exploratory data analysis **CWR 2.1.1 - 2.1.8**
- Tree diagrams and the multiplication rule for counting **CWR 2.2.1 & 2.4.1**
- Permutations and combinations **CWR 2.2.1, 2.3.1 & 2.3.2**
- The addition rules for probability **CWR 2.2.1**
- The multiplication rules and conditional probability **CWR 2.2.1 & 2.2.4**
- Probability distributions **CWR 2.2.1 & 2.2.2**
- Mean, variance and expectation **CWR 2.1.1, 2.1.2, & 2.2.1 - 2.2.3**
- The binomial distribution **CWR 2.2.1**
- The standard normal distribution and applications **CWR 2.1.1 & 2.1.2**
- The central limit theorem

- Confidence intervals for means and for proportions
- Hypothesis testing for means and proportions **CWR 2.1.3 - 2.1.8 & 5.2.6**
- Correlation and regression **CWR 2.1.7**

TRIGONOMETRY

Course Description: Trigonometry is a study of measures of angles, properties of graphs of trigonometric functions, fundamental identities, addition and half-angle formulas, inverse trigonometric functions, solutions of trigonometric equations, complex numbers, and properties of triangle solution.

Topics/Outcomes:

- Angles, degrees, arcs
- Similar triangles
- Trigonometric ratios and right triangles
- Degrees and radians **CWR 4.4.1**
- Linear and angular velocity
- Trigonometric functions and applications **CWR 3.2.2, 3.2.9, 3.3.12, 3.3.8, 3.4.1, 3.4.2, 3.4.3, & 3.4.4**
- Special right triangles
- Circular functions **CWR 4.3.5**
- Graphs of sine, cosine, and tangent functions **CWR 3.2.9, 4.2.3, & 4.3.4**
- Graphs of secant, cosecant, and cotangent functions **CWR 3.2.9, 4.2.3, & 4.3.4**
- Fundamental trigonometric identities **CWR 4.1.4**
- Sum, difference, and co-function identities **CWR 4.1.4**
- Double angle, half angle, product-sum & sum-product identities **CWR 4.1.4**
- Inverse trigonometric functions **CWR 3.2.10**
- Trigonometric equations
- Law of Sines & Law of Cosines **CWR 4.1.3**
- Areas of triangles
- Vectors **CWR 3.3.10**

- The dot product **CWR 3.3.11**
- Polar coordinates **CWR 1.2.4, 4.3.6, & 4.3.7**
- Complex numbers **CWR 1.2.5, 4.3.6, & 4.3.7**
- DeMoivre's Theorem **CWR 1.2.6, 4.3.6, & 4.3.7**

DISCRETE MATHEMATICS

Course Description: Discrete Mathematics introduces the topics of mathematical application, matrices and their uses, graph theory and its applications, counting and finite probability, as well as the processes of optimization, existence, and algorithm construction. Additional topics included are set theory, mathematical mappings, trees, circuit analysis, symbolic logic, linear programming, and other algebraic systems, logic, relations and functions, mathematical induction and recursion, combinatorics, discrete probability, analysis of algorithms, algebraic structures, techniques of counting with combinatorics and permutations, graph theory, sequences and series, and Pascal type triangles.

Students will make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

Upon successful completion of this course the student should be able to:

1. Construct mathematical models using graphs, trees, difference equations, and matrices.
2. Determine Euler and Hamilton paths and circuits.
3. Utilize graph theory to find efficient solutions to routing, scheduling, and networking problems.
4. Analyze selection methods in light of fairness criteria.
5. Find terms of sequences and sums of sequences.
6. Utilize recursive techniques in applications.
7. Perform matrix operations.
8. Develop and solve linear programming problems.
9. Utilize sequences and/or matrices to analyze the behavior of processes.
10. Place discrete mathematical topics in their historical context.
11. Utilize technology (if available) to solve discrete problems.

Topics/outcomes:

- mathematical applications **CWR 5.1.1**
- matrices and their uses **CWR 2.2.9, 3.3.4, 3.3.9 & 2.4.4**
    i. Define matrix operations.
    ii. Utilize matrices to solve systems of equations.
    iii. Define a linear programming problem.

iv. Calculate solutions to linear programming problems using augmented matrices and an appropriate calculator.

- graph theory and its applications **CWR 2.4.1 - 2.4.4**
- counting and finite probability **CWR 2.2.3**
- optimization **CWR 2.2.1 & 5.2.1**
- analyzing and constructing algorithms **CWR 5.1.1**
- set theory **CWR 2.4.1 & 2.4.2**
- mathematical mappings **CWR 2.4.1 - 2.4.4**
- trees and networks **CWR 2.4.1 - 2.4.4**
    i. Define network.
    ii. Define spanning tree.
    iii. Determine optimal solutions to network problems.
- circuit analysis **CWR 2.4.1 - 2.4.4**
    i. Determine optimal solutions to routing problems.
    ii. Define and identify Euler and Hamilton paths and circuits.
- symbolic logic
- linear programming and other algebraic systems **CWR 3.2.1**
- logic **CWR 5.2.1**
- relations and functions **CWR 3.2.1**
- mathematical induction and recursion **CWR 5.2.2 - 5.2.6**
- summation **CWR 3.1.5**
- discrete probability **CWR 2.2.1**
- analysis of algorithms **CWR 5.1.1**
- algebraic structures **CWR 5.2.6**
- techniques of counting with combinatorics and permutations **CWR 2.3.1 & 2.3.2**
- graph theory **CWR 4.4.1 - 4.4.4**

    i. Discuss Euler's and Hamilton's contributions to graph theory

- sequences and series **CWR 3.1.1 - 3.1.4**
    i. Introduce sequences and functions in the context of recursion.
    ii. Define recursive growth problems.
    iii. Calculate recursive growth sequences.
- Pascal type triangles and the binomial theorem **CWR 2.3.1 & 2.3.2**
- Social Choice **CWR 2.2.1, 5.2.1 - 5.2.6**
    i. Introduce mathematically-oriented voting methods
    ii. Define ballot counting.
    iii. Define voting methods.
    iv. Define voting power.
    v. Calculate power distributions.
    vi. Define fair-division problems.
    vii. Calculate a solution to fair-division problems.
    viii. Define apportionment.
    ix. Calculate a solution to apportionment problems.
    x. Discuss the effect of apportionment methods in the U.S. Congress

MATHEMATICAL MODELS

Course Description: In Mathematical Models, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. They use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations including the concrete, pictorial, numerical, symbolic, graphical, and verbal. They use technology (including, but not limited to graphing calculators, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.

Students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). They also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

Topics/outcomes:

- Use a variety of strategies and approaches to solve both routine and non-routine problems. Compare and analyze various methods for solving a real-life problem. **CWR 5.2.1**

- Use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines. **CWR 5.2.1 & 5.2.2**

- Select a method to solve a problem, defend the method, and justify the reasonableness of the results. **CWR 5.2.3**

- Interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatter plots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data. **CWR 2.2.2**

- Analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences. **CWR 2.2.3**

- Analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments. **CWR 2.2.4 & 2.2.5**

- Use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information. **CWR 2.2.8**

- Develop and implement a plan for collecting and analyzing data in order to make decisions. Formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions. **CWR 2.1.1, 2.1.2, 2.1.3, 2.1.4, & 2.1.5**

- Communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation**. CWR 2.1.6**

- Determine the appropriateness of a model for making predictions from a given set of data. **CWR 2.1.8**

- Uses probability models to describe everyday situations. **CWR 2.2.1 & 2.2.2**

- Compare theoretical and empirical probability.

- Use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc. **CWR 2.2.3**

- Use functional relationships to solve problems related to personal income. Use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions. **CWR 3.2.1 & 3.2.2**

- Solve problems involving personal taxes. **CWR 3.4.8**

- Analyze data to make decisions about banking. **CWR 3.4.9**

- Use algebraic formulas, graphs, and amortization models to solve problems involving credit. Analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option. **CWR 3.4.7**

- Use amortization models to investigate home financing, compare buying and renting a home, to investigate automobile financing, and compare buying and leasing a vehicle. **CWR 3.4.7**

- Uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. **CWR 3.4.9 & 3.4.10**

- Analyze types of savings options involving simple and compound interest and compare relative advantages of these options. **CWR 3.4.10**

- Analyze and compare coverage options and rates in insurance. **CWR 3.4.11**

- Investigate and compare investment options including stocks, bonds, annuities, and retirement plans. **CWR 3.4.10**

- Use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology. **CWR 3.2.1 & 3.2.4**

- Use trigonometric ratios and functions available through technology to calculate distances and model periodic motion. **CWR 3.2.9 & 4.3.4**

- Use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

- Uses algebraic and geometric models to represent patterns and structures. Use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture. **CWR 3.1.1, 3.1.2, & 3.1.4**

- Use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music. **CWR 4.2.1, 4.2.2, & 4.2.3**

FINANCIAL LITERACY

Course Description: The General Financial Literacy Core is designed for junior and senior students and represents those standards of learning that are essential and necessary for all students. The implementation of the ideas, concepts, knowledge, and skills contained in the General Financial Literacy Core will enable students to implement those decision-making skills they must apply and use to become wise and knowledgeable consumers, savers, investors, users of credit, money managers, citizens, and members of a global workforce and society.

The Core should be taught with respect for differences in learning styles, learning rates, and individual capabilities without losing sight of the common goals. Using a "hands-on" instructional approach involving techniques such as problem solving, reasoning, simulation, and direct application of the concepts of this Core to the world in which students live will empower them to incorporate the concepts of the General Financial Literacy Core into their lives. The General Financial Literacy Core will incorporate concepts and skills from mathematics, language arts, social studies, applied technology, character education, and applied service learning.

Topics/outcomes:

- Use a rational decision-making process to set and implement financial goals.

- Explain the effect that goals, decision-making, and planning have on personal financial choices and behaviors including: home ownership, work ethic, charity, civic virtue, net worth statements, budgets, income and expense records, insurance plans, savings and investing plans, personal financial plans, and short-term and long-term financial plans.

- Analyze the role that cultural, social, and emotional influences have on financial behavior. Analyze the impact of marketing, advertising, and sales strategies/techniques on purchasing decisions (e.g., impulse buying, delayed payment).

- Determine the role that financial decisions play and their impact on personal and societal consequences (e.g., increased consumer costs, inflation, family instability). Describe the social and economic consequences of bankruptcy.

- Understand various sources of income and the relationship between income and career preparation.

- Identify various forms of income and analyze the factors that affect these types of income including wages, investments, self-employment, insurance, leave, retirement. Compare income to the cost-of-living in various geographical areas. Analyze how economic conditions affect income.

- Identify and understand required income withholdings including taxes and the reason for taxation as well as the uses of tax revenues, Social Security, Medicare, payroll, and personal state and federal income taxes. **CWR 3.4.8**

- Analyze criteria for selecting a career and identify the impact of career choices on both income and financial stability. Describe the correlation between income and a worker's skills and education, the value of the work to society, the condition of the economy, and the supply and demand for workers.

- Describe the role of planning and maintaining a balanced budget and develop one. Develop, monitor and evaluate a personal budget that includes debit, checking, savings accounts. Evaluate the impact of major purchases on budgeting (e.g. automobile, housing). Interpret multiple personal budget examples. **CWR 3.4.9**

- Understand credit uses and compare costs associated with them. Calculate and compare costs associated with the use of credit including: finance charges, interest, late fees, default rates, closing costs. Calculate loan repayments and the total costs when borrowers make minimum payments. **CWR 3.4.7**

- Describe and analyze the impact of credit on money management. Compare both the advantages and disadvantages of different payment methods. **CWR 3.4.7**

- Analyze the rights and responsibilities of buyers and sellers under consumer protection laws including but not limited to: "Identity Theft and fraud", financial contracts, payment penalties, method of interest calculation, disclosure information, repayment, financial scams and schemes designed to defraud consumers such as pyramid schemes. **CWR 3.4.7**

- Compare and contrast the role of insurance as a device to mitigate risk and to calculate expenses of various options (e.g., automobile, health, homeowners, renter's, life, long term disability). **CWR 3.4.11**

- Develop an effective retirement savings plan to meet personal financial goals including IRA's, ROTH accounts, and annuities. Explain the effects of inflation on savings and investments. **CWR 3.4.10**

- Describe the value of investing and types of investments in the financial planning process including: stocks, bonds, real estate, hard assets. Compare long-term and short-term investments.

- Compare savings and investment including the risk, return, liquidity, and costs for savings and investments. Solve problems involving compound interest. Demonstrate the relationship between simple interest and linear growth and compound interest and exponential growth**. CWR 3.4.5 & 3.4.6**

CONCLUSION

This document was designed to help school districts be able to navigate the new 2008 AZ College and Work Readiness Performance Objectives in the Mathematics Standard. These performance objectives were designed to serve as a menu from which one would be able to extrapolate various learning expectations to enable them to design a fourth year course that would satisfy the new 2007 high school graduation requirements for mathematics. The State Board of Education determined in 2007 that this must be “one credit that includes significant mathematics content as determined by the local school district governing board or charter school.”

This document in no way contains all the possibilities for fourth credit courses that would meet these requirements for high school graduation. However, it is the hope of the Arizona Department of Education (ADE) that this document will serve as a springboard from which local school districts and charter schools may design fourth credit courses that meet the requirement of “significant mathematics”. This document is designed to be a living document on the website of the Arizona Department of Education. The ADE encourages educators to send their comments to improve this document to the attention of Mary Knuck, NBCT, M.Ed., Arizona Department of Education, Deputy Associate Superintendent, Standards-Based Best Practices, (602), 364-2353, (602) 364-0902-Fax, email: Mary.Knuck@azed.gov.