Report as of FY2006 for 2006OR76B: "Modeling Effects of Channel Complexity and Hyporheic Flow on Stream Temperatures"

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Report Follows

Modeling Effects of Channel Complexity and Hyporheic Flow on Stream Temperatures



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Modeling Effects of Channel Complexity and Hyporheic Flow on Stream Temperatures

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ABSTRACT

Stream temperatures are affected by multiple forcing functions, including surface heat exchange (including solar radiation, evaporation, conduction, and net long wave radiation) and hyporheic flows. Each of these forcing functions is directly influenced by the level of channel complexity in the stream channel and riparian shading. The interrelationship between channel complexity, hyporheic flow and stream temperature is highly complex, and efforts to manage for habitat diversity by managing channel complexity could result in unintended consequences on stream temperature. When planning modifications to stream channel complexity, consideration should be given to the effects such moderations could have on stream temperatures.

Urbanization has impacted many steams due to the construction of bank protections, levees, vegetation removal, etc. Such activities have eliminated side channels and reduced stream braiding, thereby reducing the overall channel complexity. Hulse et al. (2002) developed maps showing the channel configurations of the Willamette River in Oregon, USA in the years 1850 and 1995. These maps show a significant reduction in channel complexity in the intervening years. More complex stream channels provide greater habitat diversity and thus, are generally more desirable from a wildlife management perspective. Therefore, management of streams for increased channel complexity is gaining in popularity.

Knowing that stream channel complexity has diminished over time, an important question to consider is 'what were stream temperatures before we altered the natural channels?' This is an important issue in determining what natural conditions were and how we have strayed from these so-called 'natural' conditions as a result of channelization, dam building, and changes to the riparian vegetation and deforestation. Current Total Maximum Daily Load's (TMDL) rely on determining a 'natural' condition. In order to develop an understanding of what that is, a hydrodynamic and water quality computer simulation model has been applied to Oregon's Willamette River with several levels of channel complexity and varying rates of hyporheic flows. Adapting the model used to develop TMDL's for temperature in the Willamette River, the effects of present and past channel complexity on water temperatures was determined. The model used to develop the TMDL was the U. S. Army Corps of Engineers dynamic 2-D model CE-QUAL-W2, which consists of directly coupled hydrodynamic and water quality transport models

and simulates parameters such as temperature, algae concentration, dissolved oxygen concentration, pH, nutrient concentrations and residence time. The model also incorporates a dynamic shading algorithm for both vegetative and topographic shading on water bodies.

KEYWORDS

Temperature Modeling, Hyporheic Flow, CE-QUAL-W2, Willamette River

INTRODUCTION

The State of Oregon Department of Environmental Quality (DEQ) developed a river basin temperature model for the Willamette River basin. The study area included the Willamette River and all major tributaries. The model was used by DEQ to set temperature limits on point source dischargers and to evaluate the impact of management strategies on river temperatures to improve fish habitat. Stream temperatures directly influence habitat suitability for salmonids and other aquatic life by directly affecting metabolic rates, food requirements, growth rates, digestion rates, development rates, life-cycle timing, disease and parasite incidence, and predator-prey and competitor interactions (Lewis et al., 2000). The interrelationship between channel complexity, hyporheic flow and stream temperature is highly complex, and efforts to manage for habitat diversity by managing channel complexity could result in unintended consequences on stream temperature. When considering modifications to stream channel complexity, consideration should be given to the affects such moderations could have on stream temperatures.

Urbanization has impacted many streams due to the construction of bank protections, levees, vegetation removal, etc. Such activities have eliminated side channels and reduced stream braiding, thereby reducing the overall channel complexity. Hulse et al. (2002) developed maps showing the channel configurations of the Willamette River in Oregon, USA in the years 1850 and 1995. These maps (Figure 1 and Figure 2) show a significant reduction in channel complexity in the intervening years. More complex stream channels provide greater habitat diversity and thus, are generally more desirable from a wildlife management perspective. Therefore, management of streams for increased channel complexity is gaining in popularity.

The research goal is to investigate the extent which channel complexity and hyporheic flows can influence stream temperatures. Simulations will determine the relative difference observed in stream temperatures between the more- and less-complex channel systems with varying amounts of hyporheic flow and shade. Analysis will also evaluate critical densities and heights of streamside vegetation necessary to provide a net reduction in stream temperatures. From this work an assessment of 'natural' conditions for temperature in this section of the Willamette will be developed and compared to the 'natural' condition of the DEQ TMDL model.

MODEL DEVELOPMENT

Stream temperatures are influenced by processes that are external to the stream and by processes that occur within the stream system and the associated riparian zone. Most prominent of these forcing functions include incidence of solar radiation, topographic shade, vegetative shade, air temperature, relative humidity, wind speed and direction, precipitation, phreatic flows, and hyporheic flows (Poole & Berman 2000). Channel complexity is directly related to nearly all of these forcing functions. Broader streams have more surface area and thus have greater exposure to solar radiation. Deeply incised streams and narrow streams are likely to have more shading (on a percentage basis) from streamside vegetation. Stream channels located in deep, sharply cut or narrow valleys, as opposed to broad alluvial valleys, are

likely to experience more shading from surrounding topographic features. Streams located in deeply cut valleys are likely to have winds directed along the axis of the valley, thus greater wind exposure is possible, while broad alluvial valleys may experience less wind funneling, and thus have less exposure to winds. While riparian vegetation can provide shade, it can also trap cool or warm air in the stream corridor or provide shelter from prevailing winds.



Models developed to predict stream temperatures typically simulate the heat exchange functions given flow, meteorological, and stream channel configurations. CE-QUAL-W2 is a twodimensional, longitudinal/vertical, hydrodynamic and water quality computer simulation model developed by the US Army Corps of Engineers (Cole and Wells 2006). This model includes a compartmentalized heat exchange function based on the following:

 $H_n = H_s + H_a + H_e + H_c - (H_{sr} + H_{ar} + H_{br})$

where H_n = the net rate of heat exchange across the water surface; H_s = incident short wave solar radiation; H_a = incident long wave radiation; H_e = evaporative heat loss; H_c = heat conduction; H_{sr} = reflected short wave solar radiation; H_{ar} = reflected long wave solar radiation; and H_{br} = back radiation from the water surface. Each of the above compartments is solved individually to predict stream temperatures throughout the model's domain and over the time period of interest. CE-QUAL-W2 simulates the hydrodynamics of the system by simultaneous solution of the continuity and momentum equations. The results of the hydrodynamics are used in the solution of the energy continuity compartment. The hydrodynamic calculations affect the travel time and depth of flow through the river channel and thus can affect heat transfer processes significantly.

The governing equations in CE-QUAL-W2 include the x-momentum equation, the continuity equation, the free water surface equation, and the constituent transport equation. The six governing equations were derived from three-dimensional, turbulent and time averaged equations. A discussion of their derivation is supplied in Edinger and Buchak (1978) and Wells (1997). The six unknowns are pressure, \mathcal{L} ; horizontal velocity, \mathcal{L} ; vertical velocity, \mathcal{N} ; constituent concentration, \mathcal{Q} ; density, $\rho_{\mathcal{N}}$; and free water surface elevation, \mathcal{P} . If macrophytes are modeled, porosity $\boldsymbol{\zeta}$ is the ratio of plant volume in a model cell to total wetted cell volume. Conservation of mass is governed by the continuity equation:

$$\frac{\partial}{\partial x} (U\phi B) + \frac{\partial}{\partial z} (W\phi B) = q\phi B$$

where \mathbb{Z} is the channel width and $\underline{<}$ is the lateral inflow/outflow per unit volume. Assumptions implicit in the equation's derivation include a width-averaged channel and constant fluid density.

Conservation of fluid momentum in the horizontal direction is governed by the x-momentum equation:

$$\frac{\partial}{\partial t} (U\phi B) + \frac{\partial}{\partial x} (UU\phi B) + \frac{\partial}{\partial z} (WU\phi B) = -\frac{\phi B}{\rho_w} \frac{\partial p}{\partial x} + \frac{1}{\rho_w} \frac{\partial}{\partial x} (\phi B \tau_{xx}) + \frac{1}{\rho_w} \frac{\partial}{\partial z} (\phi B \tau_{xz})$$

 τ_x is the turbulent shear stress acting in the x-direction on the x-face of the control volume and τ_x is the turbulent shear stress acting in the x-direction on the z-face of the control volume.

The vertical momentum equation simplifies to the hydrostatic equation by assuming that vertical velocities are very low compared to horizontal velocities ($U \gg W$):

$$\frac{1}{\rho_w}\frac{\partial p}{\partial z} = g$$

The free water surface equation is obtained by integrating the continuity equation over depth:

$$\frac{\partial}{\partial t} \left(\phi B_{\eta} \eta \right) = \frac{\partial}{\partial x} \int_{\eta}^{h} U \phi B dz - \int_{\eta}^{h} q \phi B dz$$

where B_{i} is the surface width, $_{2}$ is the free water surface elevation and Z is the bottom elevation. In CE-QUAL-W2 the free water surface elevation is integrated over all the layers in a segment.

Constituent transport is governed by the constituent transport equation:

$$\frac{\partial}{\partial t}(\phi B \Phi) + \frac{\partial}{\partial x}(U\phi B \Phi) + \frac{\partial}{\partial z}(W\phi B \Phi) - \frac{\partial}{\partial x}\left(\phi B D_x \frac{\partial \Phi}{\partial x}\right) - \frac{\partial}{\partial z}\left(\phi B D_z \frac{\partial \Phi}{\partial z}\right) = q_{\phi}\phi B + S_K\phi B$$

where D_{x} and D_{z} the longitudinal and vertical temperature and constituent dispersion coefficients, respectively. q_{z} is the lateral inflow of constituent per unit volume and S_{z} is the kinetics source/sink term for constituent concentration.

Water density is governed by the equation of state and is a function of temperature T_{v} total dissolved solids concentration Φ_{TDS} and suspended solids concentration Φ_{s} :

$$\rho_w = f(T_w, \Phi_{TDS}, \Phi_{SS})$$

An algorithm which simulates hyporheic flow through the alluvial aquifer is being added to the CE-QUAL-W2 model code. The model will be able to capture the transient storage effects of hyporheic flow and the transfer of water across the river bed and banks. A conceptualized hyporheic flow zone is shown in Figure 3.



Figure 3. Illustration showing stream, semi-permeable stream bed, hyporheic zone, and the impermeable layer below the hyporheic zone.

Darcy's law is being used to estimate flow through the hyporheic zone. The head \leq [L] and hyporheic flow velocity \geq [L/T] are functions of x, y, and z such that

 $\phi = \phi(x, y, z)$

and

 $\vec{q} = \{q_x, q_y, q_z\}$

Applying Darcy's law and assuming the conductivity $k = k_x = k_y = k_z$ [L/T] is constant,

$$\vec{q} = -k\nabla\phi$$

or

$$\vec{q} = -k \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right)$$

Assuming $\frac{\partial \phi}{\partial y} = 0$ and that $q_z = -k \frac{\partial \phi}{\partial z} \approx 0$ within the hyporheic zone, then $\bar{q} = -k \frac{\partial \phi}{\partial x} \bar{i}$ and

$$q_x = -k \frac{\partial \phi}{\partial x}$$

The governing equation for hyporheic flow is derived using a control volume of length Δx_i , depth \mathscr{L} (thickness of hyporheic zone) and width Δy and assuming flow is only in the x-direction (Figure 4). The inflow is

$$Q_{\rm in} = -kB \frac{\partial \phi}{\partial x} \Delta y$$

Whereas flow rate out is



Figure 4. Control volume with length Δx , depth \mathcal{L} and width Δy .

A flow balance can be constructed giving

$$\underbrace{S \frac{\partial \phi}{\partial t} \Delta x \Delta y}_{\substack{\text{change in fluid volume} \\ \text{per unit time}}} = \underbrace{-kB \frac{\partial \phi}{\partial x} \Delta y}_{\text{flow in}} - \underbrace{\left(-kB \frac{\partial \phi}{\partial x} - \Delta x \frac{\partial}{\partial x} kB \frac{\partial \phi}{\partial x}\right) \Delta y}_{\text{flow out}} + \underbrace{N(x,t)}_{\substack{\text{sources/sinks}}}$$

where the dimensionless parameter \leq is the storativity and N(x,t) [L³/T] is the net flow rate of sources and sinks. Simplifying gives

$$S\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial x}kB\frac{\partial\phi}{\partial x} + \frac{N(x,t)}{\Delta x\Delta y}$$

Letting $\frac{N(x,t)}{\Delta x\Delta y} = \varepsilon(x,t)$
 $S\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial x}kB\frac{\partial\phi}{\partial x} + \varepsilon(x,t)$

The source/sink term $\mathcal{E}(x,t)$ [L/T] represents flow across the stream bed between the hyporheic zone and the stream:

$$\varepsilon(x,t) = \frac{k'}{b'} \left(\phi_{\circ} - \phi \right)$$

where $\phi_{=}$ water level in stream [L] k'=conductivity through stream bed [L/T] b'=thickness of streambed [L]

Substituting for $\mathcal{E}(x,t)$ gives the following governing equation:

$$S\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial x}kB\frac{\partial\phi}{\partial x} + \frac{k'}{b'}(\phi_{\circ} - \phi)$$

which can be solved to calculate the head \checkmark in the hyporheic zone.

The control volume approach is also used to derive the governing equations for constituent transport. It is assumed that flow and variation in concentration occur only in the x-direction. Given the mass dispersive flux $m_{_{\chi}}$ [M/L²-T] the rate of change in mass in the control volume can be expressed as:

$$\frac{\partial C}{\partial t} \Delta x \Delta y B = \underbrace{Cq_x B \Delta y}_{\text{Advected mass}} - \underbrace{\left(q_x C + \Delta x \frac{\partial Cq_x}{\partial x}\right)}_{\text{Advected flux out}} B \Delta y + \underbrace{m_x B \Delta y}_{\text{mass dispersive}} - \underbrace{\left(m_x C + \Delta x \frac{\partial m_x}{\partial x}\right)}_{\text{mass dispersive flux out}} B \Delta y + \underbrace{r \Delta x \Delta y B}_{\text{mass subsersive}} + \underbrace{m_x B \Delta y}_{\text{mass dispersive}} - \underbrace{\left(m_x C + \Delta x \frac{\partial m_x}{\partial x}\right)}_{\text{mass dispersive flux out}} B \Delta y + \underbrace{r \Delta x \Delta y B}_{\text{mass subsersive}} + \underbrace{m_x B \Delta y}_{\text{mass dispersive}} - \underbrace{\left(m_x C + \Delta x \frac{\partial m_x}{\partial x}\right)}_{\text{mass dispersive flux out}} + \underbrace{m_x B \Delta y}_{\text{mass dispersive}} + \underbrace{m_x B \Delta y}_{\text{m$$

The mass dispersive flux is:

$$m_x = -D_x \frac{\partial c}{\partial x}$$

where $D_{_{3}}$ [L²/T] is the coefficient of dispersion. Figure 5 shows the control volume for constituent transport.



Figure 5. Control volume for constituent transport.

Simplifying and substituting for m_1 gives

$$\frac{\partial C}{\partial t} + \frac{\partial Cq_x}{\partial x} = \frac{\partial}{\partial x} D_x \frac{\partial C}{\partial x} + r$$

Constituent transport between the hyporheic zone and the stream is modeled using the source/sink term r.

NUMERICAL SOLUTION SCHEME FOR CALCULATING HYPORHEIC HEAD

The head in the hyporheic zone was calculated using the governing equation

$$S\frac{\partial\phi}{\partial t} - \frac{\partial}{\partial x} \left(kB_T \frac{\partial\phi}{\partial x} \right) - \frac{k'}{b'} (\phi_w - \phi) = 0$$

Where
 $\boldsymbol{\zeta}$ =head [L]
 $\boldsymbol{\zeta}$ =storativity
 $\boldsymbol{\mathcal{L}}$ =width [L]
 $\boldsymbol{\mathcal{L}}$ = conductivity [L/T]
 ϕ_v =water level in stream [L]
 \boldsymbol{k} '=conductivity through stream bed [L/T]
 \boldsymbol{b} '=thickness of streambed [L]

Once the head \mathbf{q} is determined, the velocity \boldsymbol{q}_{2} can be estimated using

$$q_x = -k \frac{\partial \phi}{\partial x}$$

The head \mathbf{q} will be calculated at the center of a model cell. Figure 6 shows a sample grid.



Figure 6. Example grid used for hyporheic zone.

To determine the head in the hyporheic zone, an implicit finite difference scheme was applied where the time derivative was expressed as

$$\frac{\partial \phi}{\partial t} \approx \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t}$$

and the spatial derivatives were

$$\frac{\partial^2 \phi}{\partial x^2} \approx \theta \frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{\Delta x^2} + (1 - \theta) \frac{\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n}{\Delta x^2}$$

and $\frac{\partial \phi}{\partial x} \approx \theta \frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1}}{2\Delta x} + (1 - \theta) \frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x}$

where $\boldsymbol{\epsilon}$ is the time-weighting factor. A value of $\theta = 0$ indicates a fully implicit scheme, whereas a value of $\theta = 1$ is fully explicit. Substituting into the governing equation gives

$$\frac{\phi_{i}^{n+1} - \phi_{i}^{n}}{\Delta t} - \frac{\theta}{S_{i}\Delta x} \left(kB_{T} \Big|_{i+1/2} \frac{\phi_{i+1}^{n+1} - \phi_{i}^{n+1}}{\Delta x} - kB_{T} \Big|_{i-1/2} \frac{\phi_{i}^{n+1} - \phi_{i-1}^{n+1}}{\Delta x} \right) - \frac{(1 - \theta)}{S_{i}\Delta x} \left(kB_{T} \Big|_{i+1/2} \frac{\phi_{i}^{n} - \phi_{i}^{n}}{\Delta x} - kB_{T} \Big|_{i-1/2} \frac{\phi_{i}^{n} - \phi_{i-1}^{n}}{\Delta x} \right) + \theta \frac{k_{i}'}{S_{i}b_{i}'} \phi_{i}^{n+1} + (1 - \theta) \frac{k_{i}'}{S_{i}b_{i}'} \phi_{i}^{n} - \frac{k_{i}'}{S_{i}b_{i}'} \phi_{w} = 0$$

And rearranging results in

$$\begin{aligned} &\frac{\theta}{S_{i} \Delta x^{2}} \left(-kB_{T} \big|_{i-1/2} \right) \phi_{i-1}^{n+1} + \left(\frac{1}{\Delta t} + \theta \frac{kB_{T} \big|_{i-1/2}}{S_{i} \Delta x^{2}} + \theta \frac{kB_{T} \big|_{i+1/2}}{S_{i} \Delta x^{2}} + \theta \frac{k'_{i}}{S_{i} b'_{i}} \right) \phi_{i}^{n+1} \\ &+ \frac{\theta}{S_{i} \Delta x^{2}} \left(-kB_{T} \big|_{i+1/2} \right) \phi_{i+1}^{n+1} = \left(\frac{1-\theta}{S_{i} \Delta x^{2}} \right) kB_{T} \big|_{i-1/2} \phi_{i-1}^{n} \\ &+ \left(\frac{1}{\Delta t} - (1-\theta) \frac{k'_{i}}{S_{i} b'_{i}} - (1-\theta) \frac{1}{S_{i} \Delta x^{2}} \left(kB_{T} \big|_{i+1/2} + kB_{T} \big|_{i-1/2} \right) \right) \phi_{i}^{n} + \left(\frac{1-\theta}{S_{i} \Delta x^{2}} \right) kB_{T} \big|_{i+1/2} \phi_{i+1}^{n} + \frac{k'_{i}}{S_{i} b'_{i}} \phi_{w} \\ \text{or} \end{aligned}$$

$$-\theta \frac{kB_{T}|_{i-1/2}}{S_{i}\Delta x^{2}} \phi_{i-1}^{n+1} + \left(\frac{1}{\Delta t} + \theta \frac{kB_{T}|_{i-1/2}}{S_{i}\Delta x^{2}} + \theta \frac{kB_{T}|_{i+1/2}}{S_{i}\Delta x^{2}} + \theta \frac{k'_{i}}{S_{i}b'_{i}}\right) \phi_{i}^{n+1}$$

$$-\theta \frac{kB_{T}|_{i+1/2}}{S_{i}\Delta x^{2}} \phi_{i+1}^{n+1} = (1-\theta) \frac{kB_{T}|_{i-1/2}}{S_{i}\Delta x^{2}} \phi_{i-1}^{n} + \left(\frac{1}{\Delta t} - (1-\theta) \frac{k'_{i}}{S_{i}b'_{i}} - (1-\theta) \left(\frac{kB_{T}|_{i+1/2}}{S_{i}\Delta x^{2}} + \frac{kB_{T}|_{i-1/2}}{S_{i}\Delta x^{2}}\right)\right) \phi_{i}^{n} + \left((1-\theta) \frac{kB_{T}|_{i+1/2}}{S_{i}\Delta x^{2}} \phi_{i+1}^{n} + \frac{k'_{i}}{S_{i}b'_{i}} \phi_{w}\right)$$

This equation was solved using a tri-diagonal matrix solver pre-existing in the CE-QUAL-W2 source code to determine the head $_{\triangleleft}$ in the hyporheic flow zone.

STEADY STATE HEAD TEST

The hyporheic flow module was initially tested separately from CE-QUAL-W2 by simulating steady state conditions with fixed head boundary conditions and leakage between an aquifer and a overlying body of water (Figure 7). The governing equation for the steady state system is

$$\frac{\partial}{\partial x} \left(k B_T \frac{\partial \phi}{\partial x} \right) + \frac{k'}{b'} \left(\phi_w - \phi \right) = 0$$

since $\frac{\partial \phi}{\partial t} = 0$. If $kB_T = 0$ the governing equation simplifies to

$$kB_T \frac{\partial^2 \phi}{\partial x^2} + \frac{k'}{b'} (\phi_w - \phi) = 0$$

with fixed head boundary conditions $\phi(x=0) = \phi_{\circ}$ and $\phi(x=L) = \phi_{L}$ where \mathbb{Z} is the distance to the downstream boundary condition. To solve, the governing equation can be rewrote

$$kB_T \frac{\partial^2 (\phi - \phi_w)}{\partial x^2} - \frac{k'}{b'} (\phi - \phi_w) = 0$$

and letting $f(x) = \phi(x) - \phi_w$ such that

$$kB_T \frac{\partial^2 f}{\partial x^2} - \frac{k'}{b'} f = 0$$

where $f(x=0) = \phi_{\circ} - \phi_{w}$ and $\phi(x=L) = \phi_{L} - \phi_{w}$. If $\lambda = \sqrt{\frac{k'}{kB_{T}b'}}$ the governing equation can be

written

$$\frac{\partial^2 f}{\partial x^2} - \lambda f = 0$$

The solution for $\mathcal J$ has the form

$$f(x) = c_1 e^{-\lambda x} + c_2 e^{\lambda x}$$

where C_1 and C_2 are constants. At x = 0, the boundary condition is

$$f(0) = \phi_\circ - \phi_w = c_1 + c_2$$

giving $c_{\scriptscriptstyle 2} = \phi_{\scriptscriptstyle \circ} - \phi_{\scriptscriptstyle W} - c_{\scriptscriptstyle 1}$.

At x = L, the boundary condition is

$$f(L) = \phi_L - \phi_w = c_1 e^{-\lambda L} + c_2 e^{\lambda L} = c_1 e^{-\lambda L} + (\phi_\circ - \phi_w - c_1) e^{\lambda L}$$

resulting in

$$c_{1} = \frac{\phi_{L} + \phi_{w}(e^{\lambda L} - 1) - e^{\lambda L}\phi_{o}}{e^{-\lambda L} - e^{\lambda L}}$$

and
$$c_{2} = \phi_{o} - \phi_{w} - \frac{\phi_{L} + \phi_{w}(e^{\lambda L} - 1) - e^{\lambda L}\phi_{o}}{e^{-\lambda L} - e^{\lambda L}}$$

The solution for $\omega(\omega)$ is thus

The solution for $\phi(x)$ is thus

 $\phi(x) = c_1 e^{-\lambda x} + c_2 e^{\lambda x} + \phi_w$



Figure 7. Hyporheic flow test case where conditions are steady-state, the upstream and downstream head boundary conditions are fixed, and leakage occurs between aquifer and overlying water body.

Five simulations were conducted with varying parameter values.

Table 1 lists the coefficients used in the different test simulations. The model grid consisted of 12 model segments, each 10 m long ($\Delta x=10$ m). The storativity, hyporheic zone conductivity, stream bed conductivity, and stream bed thickness were assumed to be constant. The comparisons between the analytical solution and model predictions for head were shown in Figure 8. Error statistics, including mean error, absolute mean error, and root mean square error were listed in Table 2. The average absolute mean error for all the steady-state test cases was 0.003 m. Source code used for the steady state head test is shown in Appendix.

Test #	Upstream Head (m) ${oldsymbol{\phi}}_{\epsilon}$	Downstream Head (m) $oldsymbol{\phi}_{l}$	Overlying Head (m) ${oldsymbol{\phi}_{v}}$	Stora- tivity	Hyporheic zone cond. (m/s) Z	Hyorheic zone thick. (m) B 7	Stream bed cond. (m/s) k '	Stream bed thick. (m) b '
1	3.0	2.5	2.75	0.0001	0.004	10.0	0.00004	0.2
2	4.0	3.0	3.9	0.0002	0.001	1.0	0.00001	0.4
3	3.0	4.0	3.5	0.0001	0.004	5.0	0.00002	0.4
4	2.0	1.0	2.5	0.0001	0.006	5.0	0.0004	0.3
5	3.0	1.0	2.0	0.0001	0.008	10.0	0.00001	2.0

Table 1. Coefficient values used model test of steady-state conditions with leakage.

Figure 8. Comparison of model predictions with analytical solution for steady state test cases with leakage to hyporheic zone.

Table 2.	Error statistics of mode	l predictions with analy	tical solutions for st	teady state test case	es with leakage to
hyporhe	eic zone.				

Test #	Mean Error (m)	Absolute Mean	Error	Root Mean Square Error
		(m)		(m)
1	0.000	0.001		0.001
2	0.003	0.004		0.008
3	0.000	0.001		0.001
4	0.009	0.009		0.018
5	0.000	0.000		0.000
Average	0.002	0.003		0.006

CONSTITUENT TRANSPORT TEST

Another test case was used to compare model predictions of constituent transport in the hyporheic zone with an analytical solution. Model predictions were made using a CE-QUAL-W2 test code which included the hyporheic flow module. Constituent transport in the hyporheic zone is modeled using the following governing equation:

$$\frac{\partial C}{\partial t} + \frac{\partial Cq_x}{\partial x} = \frac{\partial}{\partial x} D_x \frac{\partial C}{\partial x} + r$$

The solution of to the constituent transport equation was determined using an advectivediffusion solution scheme pre-existing in CE-QUAL-W2. For the test case transport across the stream bed was assumed to be zero (r = 0). The horizontal velocity q_{2} and dispersion D_{3} were assumed to be constant giving

$$\frac{\partial C}{\partial t} + q_x \frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial^2 x}$$

The initial concentration in the hyporheic zone was set to zero and the concentration at the left hand boundary x = 0 was C_{c} . The initial condition and boundary conditions were thus

$$C(0,t) = C_{\circ} , \quad 0 < t < \infty$$
$$C(x,0) = 0 , \quad 0 < x < \infty$$

The analytical solution to this equation is

$$C(x,t) = \frac{C_{\circ}}{2} \left[erfc\left(\frac{x-q_{x}t}{\sqrt{4Dt}}\right) + erfc\left(\frac{x+q_{x}t}{\sqrt{4Dt}}\right) \exp\left(\frac{q_{x}x}{D}\right) \right]$$

The test case was diagrammed in Figure 9. With increasing time the constituent front travels to the right due to advection while also spreading out because of dispersion. The coefficient parameters used in the test cases were listed in Table 3. The concentration at the left hand boundary C_c was assumed to be 100 mg/l. The model grid consisted of 100 segments, each 10 m long ($\Delta x = 100$ m). Model predictions are compared with the analytical solution in Figure 10. The mean error, absolute mean error, and root mean square error of the test cases were listed in Table 4.



Figure 9. The constituent transport test case where a constituent of concentration C_c is released continuously at the location x = 0 starting at time t = 0

Test #	Upstream Head (m) ${oldsymbol{\phi}}_{c}$	Downstream Head (m) ${oldsymbol{\phi}}_{l}$	Dispersion (m^2/s) D_{λ}	Stora- tivity	Hyporheic zone cond. (m/s) Z	Hyorheic zone thick. (m) $oldsymbol{B}_{1}$	Stream bed cond. (m/s) K'	Stream bed thick. (m) b '
6	3.0	2.5	0.02	0.0001	0.020	10.0	0.0	0.2
7	3.0	2.5	0.001	0.0001	0.100	1.0	0.0	0.2
8	3.0	2.5	0.020	0.0001	0.020	5.0	0.0	0.2

Table 3. Coefficient values used model constituent transport test.

Tahle 4	Frror statistics of model	nredictions with analy	vtical solutions for	r constituent transr	nort test
		predictions with anal	y lical 301010113 101	constituent transp	5011 1051

Test #	Mean Error(mg/l)	Absolute	Mean	Root	Mean	Square
		Error(mg/l)		Error(n	ng/l)	
6	-2.0	2.0		3.3		
7	-0.2	0.2		0.6		
8	-1.0	1.0		1.8		
Average	-1.1	1.1		1.9		

Figure 10. Comparisons of model predictions with analytical solution for constituent transport test cases.

INTEGRATION WITH CE-QUAL-W2

A specialized input file was created to input hyporheic coefficients. Table 1 lists the coefficients in the input file "hyporheic.npt".

Variable Name	Equation Variable	Description
THETAH	E	Time weighting factor. $\theta = 0$ indicates a
		fully implicit scheme, whereas a value of
		heta=1 is fully explicit
THI	-	Initial temperature in hyporheic zone
		(Celsius)
UHH	-	Upstream branch boundary condition.
		UHH=0 for no-flux boundary, UHH=-1 for
		head boundary
DHH	-	Downstream branch boundary condition.
		DHH=0 for no-flux boundary, DHH=-1 for

Table 5.	List of	coefficients	used in	hyporheic.n	pt input file.
Lable 5.	LISC OI	coefficients	uscu m	nypornete.n	թե ութաէ ուշ

Variable Name	Equation Variable	Description
		head boundary
STOR	2	Storativity (-)
КС	7	Hyporheic zone conductivity (m/s)
ВТ	B_7	Hyorheic zone thickness (m)
WHP	Ду	Stream bed width (m)
КР	k'	Stream bed conductivity (m/s)
BP	<i>b</i> '	Stream bed thickness (m)
DXH	D_{λ}	Dispersion in groundwater (m ² /s)

An example file is shown below. The columns are eight spaces wide. This example file corresponds to a model consisting of a single branch, with 20 segments.

hyporheic input file: hyporheic.npt

	THETAH 0.55	THI 12.0					
br1	UHH -1	DHH 0					
SEG	STOR	KC	BT	WHP	KP	BP	DXH
1	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
2	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
3	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
4	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
5	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
б	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
7	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
8	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
9	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
10	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
11	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
12	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
13	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
14	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
15	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
16	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
17	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
18	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
19	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001
20	0.0001	0.500	2.0	15.0	0.1000	0.2	0.001

MODEL APPLICATION

Initially, the CE-QUAL-W2 model is being applied to an idealized riverine system consisting of a single main channel and then to the same idealized system, but with the addition of side channels. Assumptions used in the model development, for both systems, includes 15-meter tall dense streamside vegetation, diurnal air temperature fluctuations (7°C to 21°C) based on current meteorological data, constant flow rates and inflow stream temperatures, 44° north

latitude, no wind, a domain length of one mile, and with and without hyporheic flows. Also, changes in channel geometry is being explored in the main channel and the side channels. The results of the two models will be compared to evaluate differences in predicted temperature regimes between these two idealized systems.

The CE-QUAL-W2 model is being developed for the two Willamette River channel configurations shown in Figure 1 and Figure 2. The temperature regimes predicted for the two channel configurations are being compared. A sensitivity analysis is being performed to determine the dominant forcing functions affecting stream temperatures and evaluate critical levels for these forcing functions.

CONCLUSION

A model has been developed for simulating hyporheic flow in rivers. The hyporheic flow model is one-dimensional and based on Darcy's groundwater flow equation. Flow exchange between the stream and hyporheic zone is simulated across a semi-permeable stream bed. Constituent transport in the hyporheic zone is being modeled using the one-dimensional advective-diffusion equation. The hyporheic flow model has been coupled to the hydrodynamic and water quality model CE-QUAL-W2 and has been tested. The hyporheic flow model has been shown to reproduce analytical solutions. The combined impact of multiple stream channels and hyporheic flow will be evaluated. It will also be used to model to temperatures in the Willamette River, Oregon. Past and present channel configurations are being simulated in order to determine the impact of channelization on stream temperatures. Model predictions will be compared with data to validate the model's suitability for simulating present conditions.

When the project is complete a tool will be available that can model flow and constituent transport in the hyporheic zone of streams. This hyporheic flow feature will be part of future versions of CE-QUAL-W2. The prediction of pre-development or natural condition stream temperatures often necessary in TMDL studies will be made easier with a tool simulating the combined effect of hyporheic flow and channel complexity.

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Appendix

The code used in the CE-QUAL-W2 model is written in Fortran 90/95. This appendix contains the test codes used to verify the hyporehic flow algorithms in the CE-QUAL-W2 model.

```
! Head test program
 parameter(imx=12,kmx=5,nbr=1,nwb=1,tmend=86400.0)
 real thetah
 real head1(imx), stor(imx), kc(imx), bt(imx), whp(imx), kp(imx)
 real bp(imx),dxh(imx),kcb(imx),kpb(imx),porh(imx)
 real uhy(imx), qh(imx), qh(kmx), qb(imx), elws(imx), dlx(imx), volh(imx)
 real aa(imx),vv(imx),cc(imx),dd(imx)
  integer us(nbr),ds(nbr),bs(nwb),be(nwb),cus(nbr),uhh(nbr),dhh(nbr)
  logical uhyp_external(nbr),dhyp_external(nbr)
 double precision c1,c2,ush,dsh,head2(imx),lambda,headm(imx),dist(imx)
 open(1,file="uhout.dat",status='unknown')
 open(2,file="qout.dat",status='unknown')
 open(3,file="headout.dat",status='unknown')
  open(4,file="volhout.dat",status='unknown')
 us(1)=2;ds(1)=11;bs(1)=1;be(1)=1
! dlx = segment length, dlt=time step
 dlx=10.0
 dlt=10.0
 tconv=86400.0
! ush= upstream head, dsh=downstream head
 ush=3.0
 dsh=2.5
 delth=ush-dsh
  iu=us(1)
 id=ds(1)
! elws= water surface elevation of overlying water body
 elws(1)=ush
 elws(imx)=dsh
 do i=iu,id
     elws(i)=ush - delth*real(i-2)/real(imx-3)
!
    elws(i)=2.75
  end do
! read coefficients
 open (712, file='hyporheic.npt',status='old')
 read (712,'(//(8x,f8.0))')thetah
 read (712,'(//(8x,2i8))') (uhh(jb), dhh(jb), jb=1,nbr)
 read (712,'(/)')
 do i=1,imx
                                                             (712, '(8x, 8f8.0)')
   read
stor(i),kc(i),bt(i),whp(i),kp(i),bp(i),dxh(i),porh(i)
```

```
end do
 close(712)
! head conditions
 do jb=1,nbr
   uhyp_external(jb) = uhh(jb) == -1;dhyp_external(jb) = dhh(jb) == -1
 end do
! hyporheic geometry and constants
 do jw=1,nwb
   do jb=bs(jw),be(jw)
     iu=us(jb)
      id=ds(jb)
      stor(iu-1)=stor(iu)
      stor(id+1)=stor(id)
      kc(iu-1)=kc(iu)
      kc(id+1)=kc(id)
     bt(iu-1)=bt(iu)
     bt(id+1)=bt(id)
     kp(iu-1)=kp(iu)
     kp(id+1)=kp(id)
     bp(iu-1)=bp(iu)
     bp(id+1)=bp(id)
     do i=iu-1,id
        kcb(i)=(kc(i)*bt(i)+kc(i+1)*bt(i+1))/2.0
        kpb(i)=kp(i)/bp(i)
      end do
      kcb(id+1)=kcb(id)
      kpb(id+1)=kpb(id)
   end do
 end do
 do i=iu,id
   volh(i)=dlx(i)*whp(i)*bt(i)*porh(i)
 end do
 head1=elws
 head2=head1
 time=0.0
 do while (time<=tmend)</pre>
   time=time+dlt
   do jw=1,nwb
     do jb=bs(jw),be(jw)
        cus(jb)=us(jb)
        iu=cus(jb)
        id=ds(jb)
        aa = 0.0; cc = 0.0; vv = 0.0; dd = 0.0
        aa(iu)=0.0
        cc(iu)=-thetah*kcb(iu)/(stor(iu)*dlx(iu)**2)
        if(uhyp_external(jb))then
          vv(iu)=1/dlt+thetah*(kcb(iu-1)+kcb(iu))/(stor(iu)*dlx(iu)**2)+
          thetah*kpb(iu)/stor(iu)
                                          &
          dd(iu)=(1-thetah)*(kcb(iu-1)/(stor(iu)*dlx(iu)**2))*head2(iu-1)+
```

```
(1/dlt-(1.0-thetah)*kpb(iu)/stor(iu)-(1.0-
thetah)*(kcb(iu)+kcb(iu-1))/ &
               (stor(iu)*dlx(iu)**2))*head2(iu)+
                                                     &
               (1-thetah)*(kcb(iu)/(stor(iu)*dlx(iu)**2))*head2(iu+1) + &
               kpb(iu)/stor(iu)*elws(iu) +
                                                    &
               thetah*kcb(iu-1)/(stor(iu)*dlx(iu)**2) * head2(iu-1)
        else
vv(iu)=1/dlt+thetah*kcb(iu)/(stor(id)*dlx(id)**2)+thetah*kpb(iu)/stor(iu)
          dd(iu) = (1/dlt - (1.0 - thetah) * kpb(iu) / stor(iu) - (1.0 - thetah) *
                                                                        ራ
          kcb(iu)/(stor(iu)*dlx(iu)**2))*head2(iu)+
                                                        λ
               (1-thetah)*(kcb(iu)/(stor(iu)*dlx(iu)**2))*head2(iu+1)
&
                 kpb(iu)/stor(iu)*elws(iu)
        end if
        do i=iu+1,id-1
          aa(i)=-thetah*kcb(i-1)/(stor(i)*dlx(i)**2)
          vv(i)=1/dlt+thetah*(kcb(i-
1)+kcb(i))/(stor(i)*dlx(i)**2)+thetah*kpb(i)/stor(i)
          cc(i)=-thetah*kcb(i)/(stor(i)*dlx(i)**2)
          dd(i)=(1-thetah)*(kcb(i-1)/(stor(i)*dlx(i)**2))*head2(i-1)
                                                                               +
&
                 (1/dlt-(1.0-thetah)*kpb(i)/stor(i)-(1.0-thetah)*(kcb(i)+
&
                 kcb(i-1))/(stor(i)*dlx(i)**2))*head2(i) +
                                                              &
                 (1-\text{thetah})*(\text{kcb}(i)/(\text{stor}(i)*\text{dlx}(i)**2))*\text{head}2(i+1)
kpb(i)/stor(i)*elws(i)
        end do
        cc(id)=0.0
        aa(id)=-thetah*kcb(id-1)/(stor(id)*dlx(id)**2)
        if(dhyp_external(jb))then
          vv(id)=1/dlt+thetah*(kcb(id-
1)+kcb(id))/(stor(id)*dlx(id)**2)+thetah*kpb(id)/stor(id)
          dd(id)=(1-thetah)*(kcb(id-1)/(stor(id)*dlx(id)**2))*head2(id-1)
&
              (1/dlt-(1.0-thetah)*kpb(id)/stor(id)-(1.0-thetah)*(kcb(id-1)+
&
              kcb(id))/(stor(id)*dlx(id)**2))*head2(id) +
                                                              &
              (1-thetah)*(kcb(id)/(stor(id)*dlx(id)**2))*head2(id+1)
&
              kpb(id)/stor(id)*elws(id) + &
              thetah*kcb(id)/(stor(id)*dlx(id)**2) * head2(id+1)
        else
          vv(id)=1/dlt+thetah*kcb(id)/(stor(id)*dlx(id)**2)+thetah*kcb(id-1)/
&
          (stor(id)*dlx(id)**2)+thetah*kpb(id)/stor(id)
          dd(id)=(1-thetah)*(kcb(id-1)/(stor(id)*dlx(id)**2))*head2(id-1)
                                                                               +
&
            (1/dlt-(1.0-thetah)*kpb(id)/stor(id)-(1.0-thetah)*kcb(id-1)/
&
                    (stor(id)*dlx(id)**2))*head2(id) +
                                                         8
              (1-thetah)*(kcb(id)/(stor(id)*dlx(id)**2))*head2(id+1)
                                                                               +
kpb(id)/stor(id)*elws(id)
        end if
        call tridiag(aa,vv,cc,dd,iu,id,imx,head1)
! calculating hyporheic velocity and flow rate between cells - assuming no
flux boundaries at branch ends
```

```
do i=iu,id-1
          uhy(i)=kcb(i)*(head1(i)-head1(i+1))
          qh(i)=uhy(i)*(bt(i)*whp(i)+bt(i+1)*whp(i+1))/2.0
        end do
        if(uhyp_external(jb))then
           uhy(iu-1)=kcb(iu)*(head1(iu-1)-head1(iu))
           qh(iu-1)=uhy(iu-1)*bt(iu)*whp(iu)
        end if
        if(dhyp_external(jb))then
          uhy(id)=kcb(id)*(head1(id)-head1(id+1))
          qh(id)=uhy(id)*bt(id)*whp(id)
        end if
! correcting flows so that volume balances...
        do i=iu,id
         qb(i)=whp(i)*dlx(i)* kpb(i) * (elws(i)-headl(i))
         qh(i)=qh(i-1)+qb(i)
        end do
        head2=head1
      end do
    end do
    write(1,55)time/tconv,uhy
    write(2,55)time/tconv,qh
    write(3,55)time/tconv,head2
    write(4,55)time/tconv,volh
55 format(q10.4,<imx>(2x,f12.5))
  end do
  open(14,file='head_end.dat',status='unknown')
  write(14,'("
                     Х
                          model
                                    eqn")')
  dist(1)=0.0
  do i=2,imx
    dist(i)=dist(i-1)+dlx(i-1)/2.0+dlx(i)/2.0
  end do
! calculating analytical solution
! assuming constant kp, bt, kc and bp
  lambda=sqrt(kp(2)/(kc(2)*bt(2)*bp(2)))
    cl=(dsh+elws(2)*(exp(lambda*dist(imx))-1.0)-exp(lambda*dist(imx))*ush)/
&
       (exp(-lambda*dist(imx))-exp(lambda*dist(imx)))
  c2=ush-elws(2)-c1
  do i=1,imx
    headm(i)=c1*exp(-lambda*dist(i)) + c2*exp(lambda*dist(i)) + elws(2)
  end do
  do i=1,imx
    write(14,'(f8.2,2f8.3)')dist(i),head2(i),headm(i)
  end do
  stop
```

end

```
* * * * * * * * * * * * * * * * * * *
!*
                                          SUBROUTINE TRI
DIAG
* * * * * * * * * * * * * * * * * * *
SUBROUTINE TRIDIAG(A,V,C,D,S,E,N,U)
 INTEGER, PARAMETER :: I2=SELECTED_INT_KIND (3)
 INTEGER, PARAMETER :: R8=SELECTED_REAL_KIND(15)
 INTEGER,
                                 INTENT(IN) :: S, E, N
 REAL
                     DIMENSION(:), INTENT(IN) :: A(E), V(E), C(E), D(E)
      ,
 REAL,
                     DIMENSION(:), INTENT(OUT) :: U(N)
 REAL, ALLOCATABLE, DIMENSION(:)
                                        :: BTA, GMA
 ALLOCATE (BTA(N), GMA(N))
 BTA(S) = V(S)
 GMA(S) = D(S)
 DO I=S+1,E
   BTA(I) = V(I) - A(I) / BTA(I-1) * C(I-1)
   GMA(I) = D(I) - A(I) / BTA(I-1) * GMA(I-1)
 END DO
 U(E) = GMA(E)/BTA(E)
 DO I = E - 1, S, -1
   U(I) = (GMA(I)-C(I)*U(I+1))/BTA(I)
 END DO
 Deallocate (BTA, GMA)
END SUBROUTINE TRIDIAG
```