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# PHYSICAL LIMITATIONS TO DIRECTIONAL ANTENNA SYSTEMS IN THE STANDARD BROADCAST BAND

By

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# PHYSICAL LIMITATIONS TO DIRECTIONAL ANTENNA SYSTEMS IN THE STANDARD BROADCAST BAND

In adjusting and maintaining directional antenna systems there are bound to be some variations about the theoretical pattern, caused by changes in the adjustment, aging of the equipment, variation in ground losses, etc. It is proposed to study these variations on a statistical basis.

Let the theoretical pattern of the directional antenna system be given by

$$\mathcal{E}' = A e^{j\beta} = \sum_i \mathcal{E}'_i \tag{1}$$

where

$$\mathcal{E}'_i = A_i e^{j\beta_i} \tag{2}$$

In the above $\mathcal{E}'_i$ is the theoretical vector field from element i and has the magnitude $A_i$ and phase $B_i$. To each vector $\mathcal{E}'_i$ may be added a small "error" vector $\Delta_i$ to account for the variations from this theoretical field intensity. Thus, the true field from element i is given by

$$\mathcal{E}_i = \mathcal{E}'_i + \Delta_i \tag{3}$$

where

$$\Delta_i = \epsilon_i e^{j\delta_i} \tag{4}$$

$\epsilon_i$ being the magnitude and $\delta_i$ the phase of the "error" vector $\Delta_i$:

The resultant field intensity from the array becomes

$$\mathcal{E} = \mathcal{E}' + \Delta = \sum_i \mathcal{E}'_i + \sum_i \Delta_i \tag{5}$$

where

$$(6)\quad \Delta = \epsilon e^{j\delta} = \sum_i \Delta_i = \sum_i \epsilon_i e^{j\delta_i}$$

The relationship between $\varepsilon_i$, $\varepsilon_i'$ and $\Delta_i$ is illustrated in Figure 1.

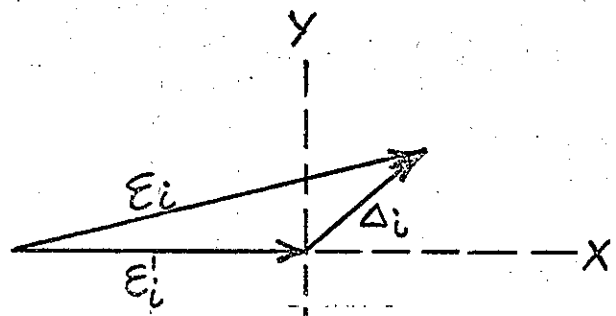

Figure 1

Let us assume that $\Delta$ is a Rayleigh vector - i.e. that the probability of the magnitude of $\Delta_i$ being between $\epsilon_i$ and $\epsilon_i + d\epsilon_i$ is given by

$$(7)\quad W(\epsilon_i)\,d\epsilon_i = \frac{2\epsilon_i}{\epsilon_{oi}^2} e^{-\frac{\epsilon_i^2}{\epsilon_{oi}^2}} d\epsilon_i$$

and all values of the phase $\delta_i$ have an equal probability of occurring. The r.m.s. value or standard deviation of $\epsilon_i$ is obtained from

$$(8)\quad \overline{\epsilon_i^2} = \int_0^\infty \frac{\epsilon_i^3}{\epsilon_{oi}^2} e^{-\frac{\epsilon_i^2}{\epsilon_{oi}^2}} d\epsilon_i = \epsilon_{oi}^2$$

the overhead bar indicating the averaging process. The assumption of a Rayleigh distribution is equivalent to saying that the orthogonal $x_i$ and $y_i$ components of $\Delta_i$ (see Figure 1) are each independently and normally distributed with mean zero and the same standard deviation $\epsilon_{oi}/\sqrt{2}$.

That the sum of any number of Rayleigh vectors is also a Reyleigh vector may easily be proven by considering the summation in terms of the two orthogonal components x and y. Since the sum of any number of normal variables is another normal variable with mean equal to the sum of the individual means and variance - i.e. standard deviation squared - equal to the sum of the individual variances, the resultant x and y components for the vector sum of a number of Rayleigh vectors must each be normal with means zero and variances equal to the sum of the individual component variances $\frac{\epsilon_{0i}^2}{2}$. Thus the vector sum of any number of Rayleigh vectors must also be a Rayleigh vector with an r.m.s. magnitude obtained from

$$(9) \quad \epsilon_0^2 = \epsilon_{0x}^2 + \epsilon_{0y}^2 = \sum_i \epsilon_{0i}^2 = \epsilon_{01}^2 + \epsilon_{02}^2 + \ldots$$

Vandivere[1] has derived the distribution for the magnitude $E$ of the sum of the constant vector $\mathcal{E}'$ plus a Rayleigh vector $\Delta$ as

$$(10) \quad P(E \leq E_r) = \frac{2}{\pi \epsilon_0^2} \int_0^{E_r} E e^{-\frac{E^2+A^2}{\epsilon_0^2}} I_0\left(\frac{2EA}{\epsilon_0^2}\right) dE$$

where $I_0(Z)$ is the modified Bessel function of zero degree. By an appropriate change of variable (10) may be put into more convenient form

$$(10a) \quad \begin{cases} P\left(\frac{E}{A} \leq \frac{E_r}{A}\right) = \frac{2}{\pi b^2} \int_0^{\frac{E_r}{A}} V dV e^{-\frac{V^2+1}{b^2}} I_0\left(\frac{2V}{b^2}\right) \\ b = \frac{\epsilon_0}{A} \end{cases}$$

Unfortunately, this integral can only be solved by either graphical means or by use of a slowly convergent series. A family of curves[2] for (10a) are plotted in Figure 2.

The moments of the distribution (10) are given by the series

$$(11) \quad \overline{E^q} = \frac{b^q A^q}{\pi} e^{-\frac{1}{b^2}} \sum_{j=0}^{\infty} \frac{(j+q)!}{j!\, b^{2j}}$$

By operating on the orthogonal components the average value of $E^2$ is found more readily. Thus, if we assume, without loss of generality, that the constant vector A lies along the X axis, then

$$(12) \quad \overline{E^2} = \overline{(A+X)^2 + y^2} = \overline{A^2} + \overline{X^2} + \overline{y^2}$$
$$= A^2 + \epsilon_0^2 = A^2 + \sum_i \epsilon_{0i}^2$$

For the special case of $\epsilon_0 = A$

$$(13) \quad \overline{E} = 0.637A$$
$$\overline{E^2} = 2A^2$$

As a special case, probably the most important one, let us assume that the r.m.s. error magnitudes are a constant percentage of the individual theoretical fields - i.e.,

(14) $$\epsilon_{oi} = a A_i$$

so that

(15) $$\begin{cases} \epsilon_o = \sqrt{\sum_i a^2 A_i^2} = a\sqrt{\sum_i A_i^2} = a A_{RSS} \\ A_{RSS}^2 = \sum_i A_i^2 \end{cases}$$

where $A_{rss}$ is the theoretical R.S.S. (root sum squared) field. We thus arrive at the very important conclusion that the magnitude of the resultant error vector is proportional to the theoretical R.S.S. field and not the R.M.S. (root mean squared) or the resultant field in the pertinent direction. This error vector should be substantially the same in all azimuthal directions, and should be combined with the theoretical field in every direction to give the expected field.

It should be pointed out that for the more unstable arrays the R.S.S. field of the antenna system may be three or more times as large as the R.M.S. field so that the use of the R.S.S. field as a base for limiting the nulls in directional arrays offers more protection factor than the use of the R.M.S. field as a base.

It is also interesting to note that in critical null directions the error field may be larger than the theoretical field so that these random variations will greatly affect the ability of the D.A. system to achieve very sharp nulls with stability. A good antenna system design will require a low ratio of R.S.S. to R.M.S. to yield stable nulls.

The relative effect of variations in amplitude versus variations in phase may be compared. Thus,

(16a) $$b_{i1} = \left(\frac{\epsilon_{oi}}{A_i}\right)_1$$

describes the effect of a change in amplitude and

(16b) $$b_{i2} = \left(\frac{\epsilon_{oi}}{A_i}\right)_2 = \tan\alpha_i \cong \alpha$$

gives the effect on the error vector of a change in phase (see Fig. 3)

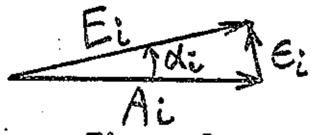


Figure 3

For an equivalent effect on the resultant field magnitude a phase tolerance of

$$\alpha_{degrees} = \frac{180}{\pi} b_{i1} \tag{17}$$

is required. For a permissible change equivalent to a 5% change in amplitude

$$\alpha = \frac{180}{\pi}(.05) = 2.86 \text{ degrees} \tag{18}$$

The assumption of a Rayleigh distribution for the error vector yields results which are in good agreement with field intensity measurements made by Robert M. Silliman[3], who made inverse distance field intensity measurements over a period of four months. This data is plotted in Figure 4. The solid curves are distributions of the type described by (10) with the indicated constants. Some work was done along these lines on antennas for broadside arrays in the microwave region by Ruze[4]. For this case the theoretical field A was negligible in comparison with the error fields but the assumption of a Rayleigh distribution for these error vectors was made and checked quite well with measurements. On the basis of these two independent sets of measurements the assumption of a Rayleigh distribution for these error vectors appears to be a good one.

The results of this study may be summarized as follows:

1. The random variations of the field intensity radiated from a directional antenna system may be described statistically by (10) as the sum of the constant theoretical field plus a randomly varying Rayleigh error field.

2. This error field is proportional to the theoretical R.S.S. field of the array.

It is expected that the theory developed in this report will hold for directional antenna arrays in all bands but for the moment our primary interest is in the standard broadcast band.

References:

1/ Report of Committee III in preparation for the Clear Channel Hearing, Docket 6741 (Appendix, "Some Notes on Probability Functions and Distributions")

2/ Redrawn from "Propagation in the FM Broadcast Band" by Kenneth A. Norton, Advances in Electronics, Volume I, pp. 406-408

3/ Before the F.C.C. in Docket 9515, Exhibit 39 for WXKW

4/ "Physical Limitations on Minimum Side Lobes in Broadside Arrays" by John Ruze of Air Force Cambridge Research Labs.

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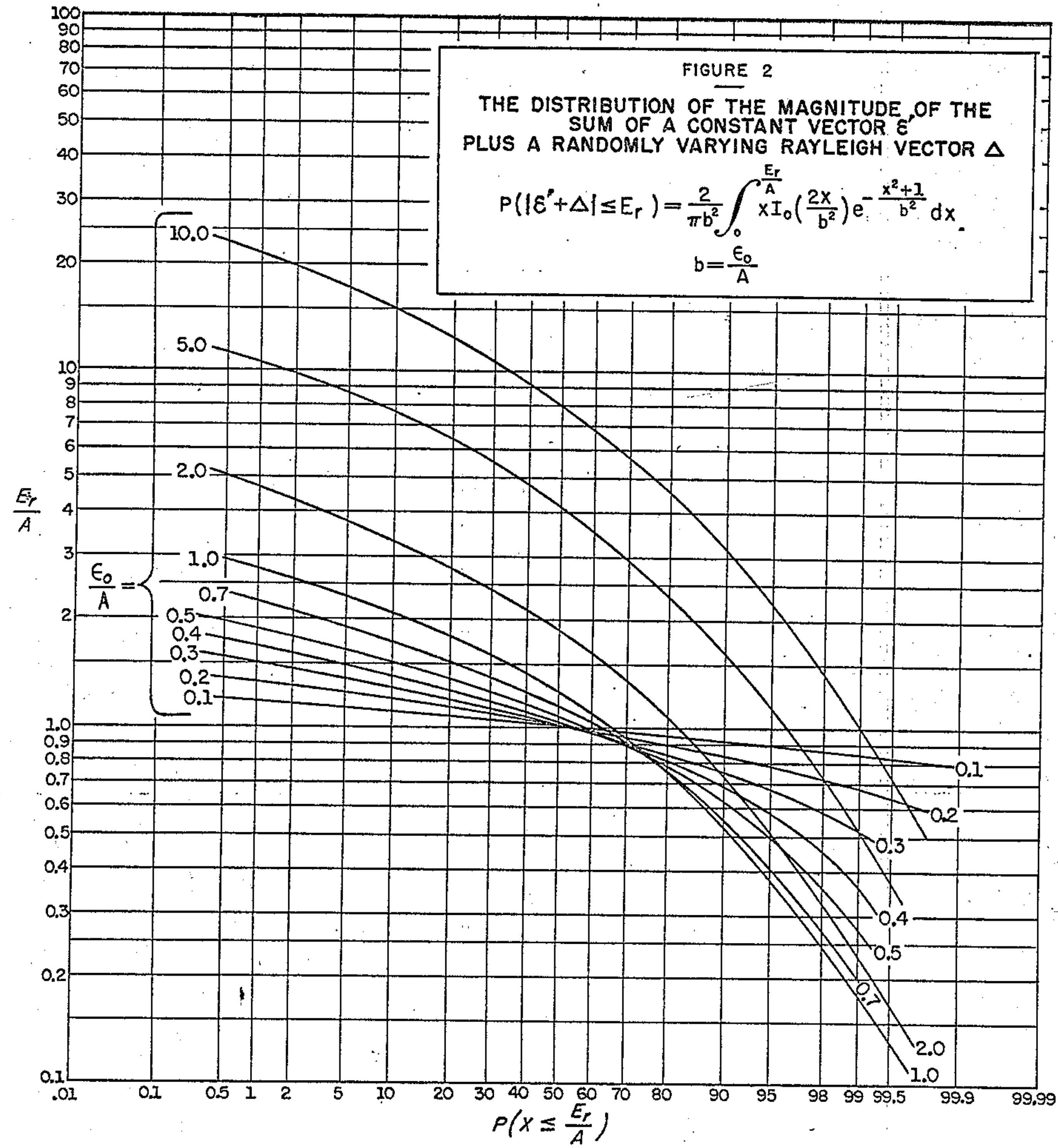


FIGURE 2

THE DISTRIBUTION OF THE MAGNITUDE OF THE SUM OF A CONSTANT VECTOR $\mathcal{E}'$ PLUS A RANDOMLY VARYING RAYLEIGH VECTOR $\Delta$

$$P(|\mathcal{E}' + \Delta| \leq E_r) = \frac{2}{\pi b^2} \int_0^{\frac{E_r}{A}} x I_o\left(\frac{2x}{b^2}\right) e^{-\frac{x^2+1}{b^2}} dx$$

$$b = \frac{\epsilon_o}{A}$$

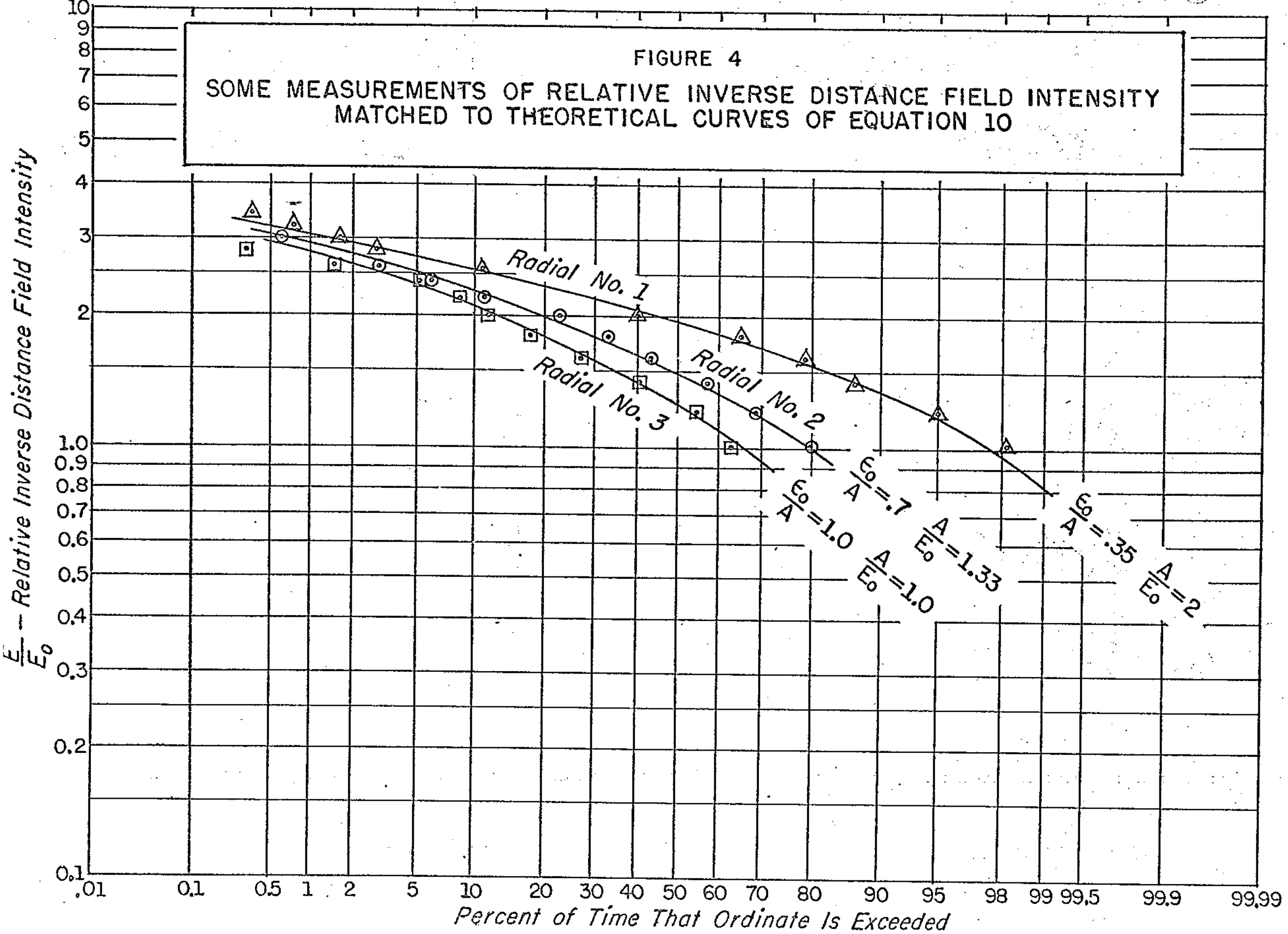

FIGURE 4
SOME MEASUREMENTS OF RELATIVE INVERSE DISTANCE FIELD INTENSITY MATCHED TO THEORETICAL CURVES OF EQUATION 10
Radial No. 1
Radial No. 2
Radial No. 3
$\frac{\epsilon_o}{A}=.35$ $\frac{A}{E_o}=2$
$\frac{\epsilon_o}{A}=.7$ $\frac{A}{E_o}=1.33$
$\frac{\epsilon_o}{A}=1.0$ $\frac{A}{E_o}=1.0$
$\frac{E}{E_o}$ – Relative Inverse Distance Field Intensity
Percent of Time That Ordinate Is Exceeded
10
9
8
7
6
5
4
3
2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
.01
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.5
99.9
99.99