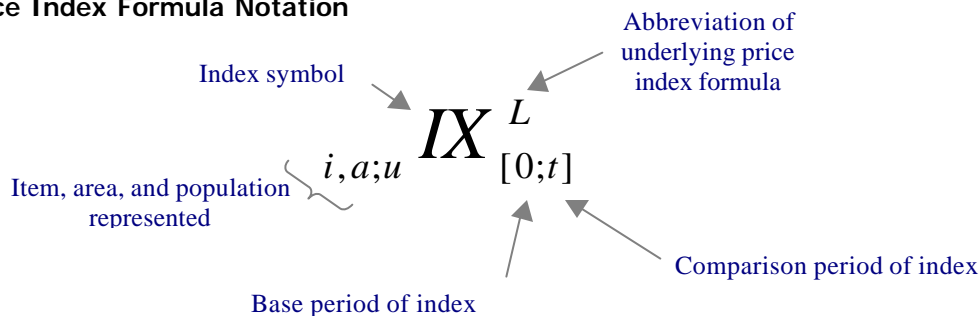


# An Introductory Look at the Chained Consumer Price Index

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## ESTIMATION METHODOLOGY

### 1. Price Index Formula Notation



### 2. The Cost-of-Living Index Concept

$${}_i IX_{[0;t]}^C = \frac{\min \sum_i P_t \times_i Q_t \Big|_{U=U_0}}{\sum_i P_0 \times_i Q_0}$$

- The minimum expenditure ( $P_t \cdot Q_t$ ) required in comparison period ( $t$ ) to attain the same level of satisfaction or utility ( $U_0$ ) achieved in base period ( $0$ ), divided by the actual expenditure ( $P_0 \cdot Q_0$ ) in base period ( $0$ ).

### 3. Price Index Formulas Commonly Used to Approximate a Cost-of-Living Index

#### FIRST ORDER APPROXIMATIONS:

LASPEYRES:	$IX_{[0;t]}^L = \sum_i s_0 \left( \frac{{}_i P_t}{{}_i P_0} \right)$
PAASCHE:	$IX_{[0;t]}^P = \left[ \sum_i s_t \left( \frac{{}_i P_0}{{}_i P_t} \right) \right]^{-1}$
GEOMETRIC MEAN:	$IX_{[0;t]}^G = \prod_i \left( \frac{{}_i P_t}{{}_i P_0} \right)^{s_0}$

#### KEY:

${}_i P_t$	= Price of item (i) in comparison period (t)
${}_i P_0$	= Price of item (i) in base period (0)
${}_i s_t$	= Expenditure on item (i) in comparison period (t), divided by expenditures on all items in comparison period (t)
${}_i s_0$	= Expenditure on item (i) in base period (0), divided by expenditures on all items in base period (0)

#### SECOND ORDER APPROXIMATIONS:

TORNQVIST:	$IX_{[0;t]}^T = \prod_i \left( \frac{{}_i P_t}{{}_i P_0} \right)^{\left( \frac{{}_i s_0 + {}_i s_t}{2} \right)}$
FISHER IDEAL:	$IX_{[0;t]}^F = \left( IX_{[0;t]}^L \times IX_{[0;t]}^P \right)^{1/2}$

#### 4. Estimation of Price Change in the Chained Consumer Price Index (C-CPI-U)

##### LOWER-LEVEL AGGREGATION:

$${}_{i,a}IX_{[0;t]}^L = \sum_{k \in i,a} k S_0 \left( \frac{{}_k P_t}{{}_k P_0} \right) \quad \text{or} \quad {}_{i,a}IX_{[0;t]}^G = \prod_{k \in i,a} \left( \frac{{}_k P_t}{{}_k P_0} \right)^k S_0$$

##### UPPER-LEVEL AGGREGATION:

	Long-term Price Change	Month-to-Month Price Change
Initial C-CPI-U	${}_{I,A}IX_{[z;y,t]}^{Gi} = {}_{I,A}IX_{[z;y-1,12]}^{Gr} \times \prod_{n=1}^t {}_{I,A}IX_{[n-1;n]}^{Gi}$	${}_{I,A}IX_{[t-1;t]}^{Gi} = I_y \prod_{i,a \in I,A} \left( \frac{{}_{i,a}IX_{[0;t]}^{LorG}}{{}_{i,a}IX_{[0;t-1]}^{LorG}} \right)^{i,a} S_{b_1}$
Interim C-CPI-U	${}_{I,A}IX_{[z;y,t]}^{Gr} = {}_{I,A}IX_{[z;y-1,12]}^T \times \prod_{n=1}^t {}_{I,A}IX_{[n-1;n]}^{Gr}$	${}_{I,A}IX_{[t-1;t]}^{Gr} = I_y \prod_{i,a \in I,A} \left( \frac{{}_{i,a}IX_{[0;t]}^{LorG}}{{}_{i,a}IX_{[0;t-1]}^{LorG}} \right)^{i,a} S_{b_2}$
Final C-CPI-U	${}_{I,A}IX_{[z;y,t]}^T = {}_{I,A}IX_{[z;t-1]}^T \times {}_{I,A}IX_{[t-1;t]}^T$	${}_{I,A}IX_{[t-1;t]}^T = \prod_{i,a \in I,A} \left( \frac{{}_{i,a}IX_{[0;t]}^{LorG}}{{}_{i,a}IX_{[0;t-1]}^{LorG}} \right)^{\frac{i,a S_{t-1} + i,a S_t}{2}}$

##### KEY:

k = unique good or service	$IX^L$ = Laspeyres elementary index
A = CPI aggregate area	$IX^G$ = Geometric Mean elementary index
a = CPI elementary area	$IX^{Gi}$ = Initial C-CPI-U index
I = CPI aggregate item	$IX^{Gr}$ = Interim C-CPI-U index
i = CPI elementary item	$IX^T$ = Final C-CPI-U index
0 = elementary index base period	${}_{i,a}S_{bi}$ = expenditure for elementary item (i) in area (a) in expenditure period (b), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b)
t = month	${}_{i,a}S_{br}$ = expenditure for elementary item (i) in area (a) in expenditure period (b), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b)
y = year	${}_{i,a}S_t$ = expenditure for elementary item (i) in area (a) in month (t), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t)
${}_k P_t$ = price of good (k) in month (t)	${}_{i,a}S_{t-1}$ = expenditure for elementary item (i) in area (a) in month (t-1), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t-1)
${}_k P_0$ = price of good (k) in base-period (0)	$\lambda_y$ = Adjustment factor used in year (y) to calculate Initial (y) and Interim (y-1) C-CPI-U indexes published in year (y); NOTE: $\lambda_y=1$ for C-CPI-U indexes published in 2002.
${}_k S_0$ = expenditure for good (k) in base period (0), divided by expenditure for all (k) goods in elementary item (i), area (a) in base period (0)	
z = December 1999 index base period	
$b_i$ = expenditure reference period of CPI-U index of year (y); NOTE: $b_i = 1999-2000$ for $y=2002$ and $y=2003$ .	
$b_r$ = expenditure reference period of CPI-U index of year (y+1). NOTE: $b_r = b_i$ for $y=2002$ and $b_r = 2001-2002$ for $y=2003$ .	