EVALUATION OF COVERAGE AND RESPONSE IN THE MANUFACTURERS' SHIPMENTS, INVENTORIES, AND ORDERS SURVEY

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A. Background

The U.S. Bureau of the Census conducts the Manufacturers' Shipments, Inventories, and Orders (M3) survey to measure monthly changes in the manufacturing sector of the economy. Data collected in the M3 are used as components of the GNP and the index of leading economic indicators; as input to national economic policy; and as an analytical tool within the business community.

Monthly data on shipments, new orders, unfilled orders, and inventories are collected for 74 broad industry categories. These categories are groupings of 449 4-digit industries as defined in the 1972 Standard Industrial Classification (SIC) Manual.

The M3 is a company based survey; each company is instructed to provide data on all its manufacturing activities. A company, active in more than one industry category, is requested to report data separately for each category. Each separate report constitutes a reporting unit.

The monthly records which companies maintain often do not permit the reporting of required data on a category basis. At the request of the Census Bureau, most large diversified companies file separate reports at their internal divisional levels if they are unable to break out data by industry category. In this case, the reporting unit is classified into a single industry category based on the major activity. This introduces nonsampling errors into the M3 estimates at the category level and has contributed to the decision not to publish separate estimates for certain categories. These categories are aggregated with other M3 industry categories for publication purposes.

Within each industry category, estimates of month-to-month percentage changes in shipments, inventories, and unfilled orders are calculated. Only those units which report in the current month are used to estimate the percentage change from the previous month. The monthly value of shipments and end-of-month total for inventories and unfilled orders are developed as ratio estimates. The estimate for a given month is obtained by multiplying the prior month's total by the estimated percentage change.

The annual totals for shipments and the end-of-year inventories estimated from the M3 are benchmarked to the annual shipments and end-of-year inventories published from the Census of Manufactures or, during intercensus years, the annual survey of manufactures (ASM). The benchmarked M3 figures are used to revise the monthly estimates. The adjustment procedure minimizes revisions to the month-to-month movements originally estimated from the M3 while correcting the annual M3 totals to equal the benchmark numbers.

The Census of Manufactures, conducted at 5 year intervals, provides a detailed statistical profile of manufacturing in the U.S. On an annual basis, the ASM provides information similar to that collected in the census. The ASM is a probability sample of establishments (single manufacturing locations) selected to meet reliability constraints on estimated annual

shipments within 4-digit industries. A new sample is drawn every 5 years using the most recent census as a sampling frame. In the current ASM panel, unweighted establishment data comprised approximately 65 percent of the total 1982 census value of shipments.

Benchmarking of the M3 has caused substantial revisions to the M3 annual totals at the industry category level. This has long been of concern to the Census Bureau and has led to questions regarding the adequacy of sample coverage and response in the M3.

Unlike the ASM, the M3 is not a probability sample. It is essentially a cut-off sample of large companies which account for the majority of manufactured shipments at the U.S. level. The current panel consists of approximately 4,500 reporting units.

The original panel, drawn in 1962, contained about 7,500 reporting units. All large companies with employment greater than 1,000 were included in the sample. Smaller companies were sampled on a probability basis proportional to employment size. It quickly became apparent that the very small firms had difficulty reporting on a regular monthly basis. Due to inadequate response, companies with less than 100 employees were dropped from the panel in 1963. Deterioration of response for companies with 100-999 employees prompted a supplemental sample of 1,000 companies which were added to the panel in 1978.

With budget constraints and the importance of large companies in manufacturing, the Census Bureau's efforts have been concentrated on maintaining and improving response from the largest manufacturing companies. Response to the M3 survey is volunatry, that is, it is not required by law.

Recently, 1978-1982, efforts at response improvement were channeled towards contacting large nonrespondent companies through telephone calls, letters, and visits. This was discontinued in 1982 due to lack of success which may have been caused by the unwillingness of companies to file voluntary surveys during the recession of 1982. In 1984, the response improvement program resumed with emphasis on visits to the largest nonrespondents and companies with reporting problems. Response is crucial, since only the respondents influence the M3 estimates. Nonresponse is another source of nonsampling error in the survey.

B. Purpose of Study

As part of a broader program to examine the quality of data used in the index of leading economic indicators, the Office of Management and Budget has directed the Census Bureau to identify sources of errors and large revisions in the M3 estimates and recommend appropriate survey improvements.

The research presented in this paper was designed, first, to quantify suspected problems with panel coverage and nonresponse, and then, to direct the Census Bureau's resources to specific industry categories which would benefit the most from improvements in these areas. This study is one aspect of a comprehensive effort now underway at the Census Bureau to improve M3 methodology and data quality.

The M3 is not a probability sample. The original sample, selected in 1962, has deteriorated over time. Limited additions have been made to the panel; however, there have been no systematic, continuous updates to the panel to reflect the changing universe of manufacturers. For these reasons, inadequate panel coverage is suspected as a source of error within some industry categories and possibly published aggregates of these categories. At these levels, the present panel, which consists mainly of large companies, may not be representative of movement over all manufacturers.

The Census Bureau is also concerned with the effect of poor response on the quality of M3 estimates. Participation in the survey is not required by law; refusals may seriously affect the reliability of the M3 within certain industries. Besides complete nonresponse, there are problems with late response, sporadic response, and the combined reporting of data across industry categories.

Motivated by these concerns, research was conducted to evaluate the adequacy of the M3 panel and the reporting portion of that panel in measuring economic change.

There are no available sources to make this evaluation on a monthly basis. However, estimates of yearly changes are available from the ASM. Because the ASM is a probability sample, these estimates are representative of the entire manufacturing universe and can therefore be used as a means of evaluating the M3 survey.

Establishment data collected annually in the ASM were used to examine coverage and response in the M3. It should be stressed that the study involved no M3 data. ASM establishments were used to provide estimates for companies in the M3 panel and those companies which report in the M3 panel. In this way consistent comparisons could be made between the universe, the M3 panel, and the reporting portion of the panel.

The analysis was limited to shipments and inventories; orders data are not collected in the ASM. For these two items, rates of change from 1982 to 1984 were calculated. 1984 was the first survey year of a sample newly drawn from the 1982 census and is not comparable to the 1983 ASM panel on an establishment level; therefore, a 1-year rate of change would not be meaningful.

Specifically, the evaluation study was designed to meet the following objectives at the industry category level.

1. Using the complete ASM panel, estimate rates of change in shipments and inventories from 1982 to 1984 for the entire manufacturing sector.

2. Using appropriate subsets of the ASM, estimate rates of change for the portions of the manufacturing sector represented by:

   a. Companies in the M3 panel, including those companies which do not report.

   b. Companies which report in the M3 survey.

3. Statistically compare change estimated for the M3 panel to estimated change in the manufacturing sector as a whole. Significant differences would indicate the need to improve panel coverage.

4. Statistically compare change estimated for the reporting portion of the M3 panel to estimated change in the manufacturing sector as a whole. Significant differences would indicate a need to improve response.

C. Design of Study

This section highlights the basic procedures followed during the implementation of the study.

Identification of ASM Establishments in the M3 Panel

The key element in this study is the use of ASM establishment data to examine coverage and response in the M3, a company based survey. To accomplish this, companies in the M3 panel were linked with their corresponding ASM establishments.

Unlike the ASM, updates to company identification in the M3 are not regularly maintained to reflect changes in company ownership. Therefore a preliminary review of the M3 was conducted to ensure consistency of identification numbers between the ASM and the M3.

Each ASM establishment was then identified as either in the M3 panel or not in the M3 panel corresponding to the status of it's company.

Identification of ASM Establishments in the Reporting Portion of the M3 Panel

Within a given industry category, a company in the M3 panel was defined as an M3 reporter if its shipments were reported, in that category, for 7 or more months during the 1984 M3 survey year.

Each ASM establishment of a company in the M3 panel was identified as either an M3 reporter or an M3 nonreporter as follows.

If a company was defined as a reporter within a given category, then all its corresponding ASM plants, classified in that specific category, were also defined as M3 reporters.

An establishment, in a given industry category, was identified as a nonreporter if one of the following conditions was met 1) the company, classified in that category, reported shipments in 6 or fewer months during 1984; or 2) the company was not classified in that category within the M3. The second case would include a company which provided data on the establishment's activity within a combined report classified in another industry category. Currently, in the M3, when a company is not able to provide the proper level of detail, but does provide data combined over several categories, all the data are tabulated in a single category as reported. Generally, no attempt is made to split out the data. For purposes of this study, the detailed data which were not supplied were considered unreported.

Preparation of Data

For the vast majority of establishments in the 1984 ASM panel, data on the total value of shipments and total ending inventories were obtained from both the 1984 ASM and the 1982 census. The remainder of establishments in the panel were classified as either out-of-business prior to 1984 or newly opened operations after 1982. These establishments are included in the study with zero data in the missing year.

Establishment data may have been imputed during the regular ASM/census processing to correct for misreported or missing values. Establishments with total value of shipments imputed in either 1982 or 1984 or both years are excluded from the calculations of shipments rates of change. Similarly, establishments with imputed inventories data are excluded from the inventories calculations. These exclusions were made to prevent possible biases of imputation in the ASM from affecting the analysis. To the extent that response in the ASM is not random, exclusion of these cases may also affect the analysis. However, it was felt that the risk of imputation bias was more serious. Of the 59,511 establishments in the ASM panel, 46,429 were included in the shipments rates of change. These establishments accounted for 91 percent of the 1984 weighted ASM shipments. 39,737 establishments were included in the inventories rates and accounted for 84 percent of 1984 weighted ASM inventories.

Estimation of Rates of Change

As described above, each establishment in the ASM panel was classified as either included in the M3 panel or not included in the M3 panel. Establishments identified as in the M3 panel were further broken down between M3 reporters and M3 nonreporters.

Within each of the 74 industry categories in the M3, estimated rates of change in total value of shipments (TVS) from 1982 to 1984 were developed separately for the complete ASM panel and each of the above subsets of the ASM using the following ratio estimator.

$$\hat{R} = \sum_{j=1}^{n} w_j Y_j / \sum_{j=1}^{n} w_j X_j$$

where

n = the number of ASM establishments in the estimation cell.

w_j = the sampling weight assigned to establishment j during the selection of the 1984 ASM panel.

Y_j = TVS reported by establishment j in the 1984 ASM.

X_j = TVS reported by establishment j in the 1982 census.

Rates of change were also estimated for the industry categories aggregated up to the 2-digit SIC level. Using the same methodology, rates of change in total ending inventories (TIE) were estimated for each of the industry categories and their aggregates.

The rates of change developed over the ASM panel and the subsets of the ASM representing the M3 panel and M3 reporters will be denoted by $\hat{R}_{ASM}$, $\hat{R}_{M3}$, and $\hat{R}_{M3R}$, respectively. $\hat{R}_{ASM}$ is the estimator of the rate of change for all establishments in the manufacturing universe; $\hat{R}_{M3}$ is the estimator of the rate of change for all establishments associated with companies in the M3 panel; and $\hat{R}_{M3R}$ is the estimator of the rate of change for all establishments associated with reported companies in the M3 panel.

Comparison of the Rates of Change

The statistical analysis of coverage and response in the M3 survey was designed to direct the Census Bureau's immediate action to those industry categories most in need of improved coverage or response.

Criteria were needed to identify specific categories on which to target our efforts. The M3 survey staff established the following criteria by which to evaluate each of the industry categories.

1. Panel coverage was considered to be inadequate if, over the 2-year period, shipments or inventories rates of change within the M3 panel differed by more than 0.05 in absolute value from the universe as a whole.

2. Response was considered to be inadequate if, over the 2-year period, shipments or inventories rates of change within the reported portion of the panel differed by more than 0.05 in absolute value from the universe as a whole.

Similar criteria were established to evaluate the industry categories aggregated to the 2-digit SIC level. An absolute difference of 0.02 at this level was considered inadequate.

Corresponding with these criteria, t-statistics were developed to test the null hypothesis of adequate panel coverage and the null hypothesis of adequate response for each of the 74 industry categories and their aggregates. The t-statistics were then applied in two-tailed t-tests designed to reject the null hypothesis at the 5 percent level of significance.

Within a given industry category, the null hypothesis of adequate panel coverage was rejected if

$$t_{M3} = \frac{|\hat{R}_{ASM} - \hat{R}_{M3}| - 0.05}{\hat{V}(\hat{R}_{ASM} - \hat{R}_{M3})} > t_{1-\alpha/2}, \text{ where } \alpha = 0.05$$

The null hypothesis of adequate response was rejected if

$$t_{M3R} = \frac{|\hat{R}_{ASM} - \hat{R}_{M3R}| - 0.05}{\hat{\sigma}(\hat{R}_{ASM} - \hat{R}_{M3R})} > t_{1-\alpha/2}, \text{ where } \alpha = 0.05$$

The estimated standard errors were based on the approximation derived in the technical appendix.

Similar tests were made at the 2-digit SIC level, using the appropriate t-statistics.

D. Results

Analysis of panel coverage and response within each of the 74 M3 industry categories identified 34 categories which need improvement in measuring rates of change for shipments or inventories.

Not all categories are included separately within the M3 publications. Only 9 of 34 problem categories are published separately. The others are included within aggregates of categories for the M3 publications.

For shipments or inventories, inadequate panel coverage was identified in 14 industry categories; 4 of these categories have problems in both shipments and inventories. Inadequate response was found in 22 industry categories; 9 of these categories have response problems in both items. A category was not considered as a response problem in a particular item, if inadequate panel coverage was also found for that item.

Tables 1 and 2, which follow, provide detailed category counts by category size. Size is determined based on weighted 1984 ASM data. Separate counts are given for categories within the durable goods and nondurable goods sectors. This is an important distinction in the M3 survey. The durable goods sector is followed very closely as an indicator of economic conditions. This sector includes all capital goods industries as well as all household durable goods. Estimates in the durable goods sector are therefore useful to analyze investment and consumer well-being.

The results of this study relating to shipments are summarized in Table 1.

Table 1: Counts of M3 Industry Categories with Inadequate Panel Coverage/ Inadequate Response1/ in Estimating Change in Shipments (TVS) from 1982 to 1984

	All Industry Categories		Inadequate Panel		Inadequate Response	
DURABLE GOODS	Collected	Published	Collected	Published	Collected	Published
By category size:2/						
$50(bil) < TVS	7	4	0	0	1	0
30 < TVS ≤ 50	8	2	1	0	3	2
15 < TVS ≤ 30	8	4	1	0	4	2
10 < TVS ≤ 15	9	3	3	1	1	0
TVS ≤ 10	14	3	3	0	5	1
Total	46	16	8	1	14	5
NONDURABLE GOODS						
By category size:						
$50(bil) < TVS	7	4	1	0	2	1
30 < TVS ≤ 50	7	1	0	0	0	0
15 < TVS ≤ 30	5	3	0	0	1	1
10 < TVS ≤ 15	3	0	0	0	0	0
TVS ≤ 10	6	0	0	0	1	0
Total	28	8	1	0	4	2
U.S., TOTAL						
By category size:						
$50(bil) < TVS	14	8	1	0	3	1
30 < TVS ≤ 50	15	3	1	0	3	2
15 < TVS ≤ 30	13	7	1	0	5	3
10 < TVS ≤ 15	12	3	3	1	1	0
TVS ≤ 10	20	3	3	0	6	1
Total	74	24	9	1	18	7

1/"Inadequate" was determined according to the criteria on page 6 and tested at the 5 percent level of significance.

2/Based on weighted 1984 shipments, reported or imputed in the ASM.

The U.S. totals in Table 1 show that for 9 of the 74 industry categories, M3 panel coverage of shipments was inadequate. Of these, however, only one is published separately.

In the durable goods sector, the categories with inadequate panel coverage are concentrated in the smaller size breakdowns. Of the 23 industry categories with shipments less than $15 billion, 6 have inadequate panel coverage. On the other hand, of the 23 industry categories with shipments greater than $15 billion, only 2 categories were identified with inadequate coverage. The M3 panel, which is basically a cut-off sample of large companies (employment greater than 1,000), provides better coverage in the industry categories with high shipments. These categories are generally comprised of large companies.

In the nondurable goods sector, there is only one industry category with inadequate coverage. This particular category is large, with 1984 shipments in excess of $50 billion.

Table 1 also shows that inadequate reporting is a more common problem than inadequate panel coverage; the U.S. totals show 18 categories have inadequate response compared to 9 categories with inadequate panel coverage.

In each of the size breakdowns within the durable goods sector, with the exception of the $10-15 billion range, inadequate response is more common than coverage problems. This is most noticeable in the larger size classifications. Unlike problems with panel coverage, poor reporting is not concentrated in the smaller size categories. Of the 23 durable goods categories with shipments greater than $15 billion; 8 have inadequate response. Of the 23 categories with shipments less than $15 billion, 6 have inadequate response. Examination of the size breakdowns for nondurable goods tells a similar story. Of the 4 categories with inadequate response, 2 are in the largest size breakdown.

Table 2, on the following page, summarizes the results based on rates of change in inventories.

Table 2: Counts of M3 Industry Categories with Inadequate Panel Coverage/ Inadequate Response1/ in Estimating Change in Total Inventories at End of Year (TIE) from 1982 to 1984

	All Industry Categories		Inadequate Panel		Inadequate Response	
DURABLE GOODS	Collected	Published	Collected	Published	Collected	Published
By category size:2/						
$10.0(bil) < TIE	5	3	0	0	0	0
7.5 < TIE ≤ 10.0	6	4	0	0	4	2
5.0 < TIE ≤ 7.5	5	0	2	0	1	0
1.5 < TIE ≤ 5.0	17	7	2	0	3	0
TIE ≤ 1.5	13	2	3	0	3	1
Total	46	16	7	0	11	3
NONDURABLE GOODS						
By category size:						
$10.0(bil) < TIE	2	1	0	0	0	0
7.5 < TIE ≤ 10.0	2	1	0	0	0	0
5.0 < TIE ≤ 7.5	4	2	0	0	0	0
1.5 < TIE ≤ 5.0	13	4	1	0	1	0
TIE ≤ 1.5	7	0	1	0	1	0
Total	28	8	2	0	2	0
U.S., TOTAL						
By category size:						
$10.0(bil) < TIE	7	4	0	0	0	0
7.5 < TIE ≤ 10.0	8	5	0	0	4	2
5.0 < TIE ≤ 7.5	9	2	2	0	1	0
1.5 < TIE ≤ 5.0	30	11	3	0	4	0
TIE ≤ 1.5	20	2	4	0	4	1
Total	74	24	9	0	13	3

1/"Inadequate" was determined according to the criteria on page 6 and tested at the 5 percent level of significance.

2/Based on weighted 1984 TIE, reported or imputed in the ASM.

As can be seen from Table 2, inadequate panel coverage for inventories is a problem with 9 categories. However, this was not a problem in the larger size classifications. No panel inadequacies were found for the durable goods categories with TIE greater than $7.5 billion or for the nondurable goods categories with TIE greater than $5.0 billion.

Of the categories with inadequate response, all but 2 are in the durable goods sector. A very interesting feature of Table 2 is nonresponse in durable goods categories with TIE in the range of $7.5-10.0 billion; 4 out of 6 categories have a response problem. Compare this to 7 out of 35 categories with inadequate response in the smaller size ranges of $7.5 billion or less. However, of the largest categories (TIE of $10.0 billion or more), none have inadequate response.

Based on Tables 1 and 2, the durable goods sector has a proportionately large share of the problem categories in comparison with the nondurable goods sector. It is unclear why this occurs; further investigation is necessary, particularly since the majority of published categories with problems are in the durable goods sector.

In addition to testing the 74 industry categories, groupings of these categories at the 2-digit SIC level were examined. A 2-digit SIC was identified as having inadequate panel coverage, if the rates of change in shipments or inventories for establishments corresponding to the M3 panel differed from the entire universe by more than 2 percentage points. The criterion of 2 percentage points was also applied to identify inadequate response. Within the industry categories, however, a 5 percent points criteria was applied. At this lower level, larger differences were considered acceptable.

It should be noted that the rates of change for the 2-digit groupings were simply derived by aggregating the tabulations developed at the industry category level.

Of the 10 2-digit SIC classifications within the durable goods sector, 5 were identified as having inadequate panel coverage or response problems in measuring rates of change for shipments. Six were identified with inadequacies for inventories. In some categories problems were found in both shipments and inventories. In all, 6 of the 10 2-digit classifications within durable goods are in need of improvement in some area.

In the nondurable goods sector, 4 out of 10 2-digit SICs are in need of improvement in either shipments or inventories, or both. For shipments, three categories were identified with inadequate coverage or response. For inventories, 4 categories were identified with inadequacies.

E. Detailed Analysis

Each of the industry categories with inadequacies in panel coverage or response were examined individually to identify sources of the problem. For this paper, two problem categories were chosen as illustrations of this detailed analysis. A third category for which no inadequacies were identified is also included for comparison. The category "building materials and wire products" was examined to discover causes for inadequate response in shipments. For "wood products, except containers, buildings, and mobile homes," M3 panel problems in shipments and inventories were analyzed. Finally, as an industry category for which no inadequacies were discovered, "products of pulp and paperboard mills, except building paper" is presented.

Wood Products, Except Containers, Buildings, and Mobile Homes

Of all 46 industry categories in the durable goods sector, the category "wood products, except containers, buildings and mobile homes" is the largest category with a panel inadequacy in measuring change in shipments. It is one of the two largest categories in the durable goods sector with inadequate panel coverage in measuring change in inventories. In level of shipments and level of inventories, it is the major category in the 2-digit SIC classification, "lumber and wood products."

The universe of establishments within this category contains a large proportion of very small establishments. Based on establishment counts from the 1982 census, establishments with less than 10 employees comprised almost 70 percent of the total number of establishments. The majority of these small establishments were classified within "logging camps and logging contractors," one of the 13 4-digit industries which comprise this category.

For wood products, except containers, buildings, and mobile homes, the t-tests indicate that the portion of the universe covered by the M3 panel understates change in the entire universe by more than 5 percent for both shipments and inventories.

Table 3, below, presents information to analyze the rates of change in shipments in further detail. Unweighted ASM establishment counts and their corresponding weighted 1984 shipments, as percentages of the ASM panel total are given. Note that this analysis involves only those establishments which reported ASM shipments in both 1982 and 1984.

Table 3: Wood Products, except Containers, Buildings, and Mobile Homes: Estimated Rate of Change in Value of Shipments (VS) from 1982 to 1984 Based on Weighted Data Reported by ASM Establishments in Both 1982 and 1984

	Unweighted ASM Establishment Count	Weighted 1984 VS as % of Weighted ASM Total	Estimated Rate of Change
In the ASM Panel	1810	100.0	1.331
In the M3 Panel	706	42.3	1.253
Not in the M3 Panel	1104	57.6	1.394

Based on all establishments which report in the ASM, a growth in shipments of 33 percent was estimated for the universe as a whole. However, for the portion of the universe covered by the M3 panel, growth was estimated at only 25 percent.

In value of shipments, the M3 does not have good coverage of the universe; only 42 percent as estimated from 1984 weighted ASM data.

Not only is coverage of the universe value low, it was also found that the M3 panel is skewed towards large establishments. The growth pattern for these establishments differed from the small establishments as shown in Table 4.

Table 4: Wood Products, except Containers, Building, and Mobile Homes: Comparison of Estimated Rates of Change in Value of Shipments (VS) Over Large1/ and Small2/ ASM Establishments

	Unweighted ASM Establishment Count	Weighted 1984 VS as % of Weighted ASM Total	Estimated Rate of Change
In the ASM Panel			
Large	875	52.0	1.253
Small	935	48.0	1.428
Total	1810	100.0	1.331
In the M3 Panel			
Large	641	73.3	1.231
Small	65	26.7	1.506
Total	706	100.0	1.253

1/Large refers to establishments which are self-representing in the ASM panel and generally have employment greater than 250.

2/Small refers to all other establishments in the ASM panel.

The counts in Table 4 indicate that the companies in the M3 panel tend to consist of large establishments. Over 90 percent of the ASM establishments in the M3 panel are large. Less than 50 percent of all the ASM establishments are large.

Growth estimated for the M3 panel is dominated by the large establishments which grew at about 23 percent. The small establishments of companies in the M3 grew by an estimated 51 percent. However, these establishments account for a relatively small percent of 1984 shipments estimated for the M3 panel.

Estimated shipments for the universe of all establishments is fairly evenly distributed between large and small establishments. The small establishments, however, grew at a much higher rate and therefore dominate estimated growth for the universe of manufacturers within this category.

The difference in growth between the universe and the M3 panel is due mainly to the inability of the current M3 panel to capture high growth in shipments for the portion of the universe consisting of smaller establishments.

Table 5, below, provides information to analyze inadequate panel coverage in measuring the rates of change in inventories. The ASM establishment counts do differ from Table 3. Table 5 includes only those establishments which reported total ending inventories in 1982 and 1984, whereas Table 3 includes those establishments which reported shipments in both years.

Table 5: Wood Products, except Containers, Building, and Mobile Homes: Estimated Rate of Change in Total Inventories at End of Year (TIE) from 1982 to 1984 Based on Weighted Data Reported by ASM Establishments in Both 1982 and 1984

	Unweighted ASM Establishment Count	Weighted 1984 TIE as % of Weighted ASM Total	Estimated Rate of Change
In the ASM Panel	1487	100.0	1.145
In the M3 Panel	657	39.2	1.034
Not in the M3 Panel	830	60.8	1.230

Estimated growth for inventories in the M3 panel is about 3 percent. This understates growth for the universe of all manufacturers, estimated at almost 15 percent. A substantial portion of the total universe inventories is not covered by the M3 panel. Establishments corresponding to companies in the M3 panel accounted for roughly 40 percent of the 1984 universe value, as estimated from the ASM.

Similar to shipments, small establishments which do not correspond with M3 panel companies, experienced high percentage growth in total ending inventories compared with the larger establishments covered in the M3 panel. This is shown in Table 6 on the following page.

Table 6: Wood Products, except Containers, Buildings, and Mobile Homes: Comparison of Estimated Rates of Change in Total Inventories at End of Year (TIE) Over Large1/ and Small2/ ASM Establishments

	Unweighted ASM Establishment Count	Weighted 1984 VS as % of Weighted ASM Total	Estimated Rate of Change
In the ASM Panel			
Large	863	50.2	1.062
Small	624	49.8	1.243
Total	1487	100.0	1.145
In the M3 Panel			
Large	596	86.7	1.026
Small	61	13.3	1.093
Total	657	100.0	1.034

1/Large refers to establishments which are self-representing in the ASM panel and generally have employment greater than 250.

2/Small refers to all other establishments in the ASM panel.

In summary, the analysis of shipments and inventories indicates that M3 panel coverage in this category would improve by better representation of the small companies. This could be accomplished by a probability sample of the small companies and/or a smaller cut-off for inclusion within the panel. The cut-off of company employment greater than 1,000 was applied across all M3 industry categories when the panel was selected. Even with an expanded M3 panel, the quality of the M3 estimates would be dependent on the ability and willingness of the smaller companies to provide regular and timely response.

Building Materials and Wire Products

The industry category, "building materials and wire products" comprises a major portion of the shipments data within the 2-digit SIC classification, "fabricated metal products." Over all 74 industry categories in the M3, building materials and wire products ranks in the top 25 in level of shipments, based on weighted 1984 ASM data.

This category is comprised of 13 4-digit industries. Shipments are dominated by 3 industries: "fabricated structural metal"; "fabricated plate work (boiler shops)"; and "sheet metal work."

The universe of all establishments classified in the 13 industries is predominantly comprised of fairly small establishments, with individual employment less than 50. The ASM panel for this category, over 2,000 establishments, is relatively large compared to other M3 industry categories.

Table 7, below, gives the rates of change in shipments, from 1982 to 1984, estimated over the ASM panel as well as for the various subsets of the ASM panel involved in the analysis.

Table 7: Building Materials and Wire Products: Estimated Rates of Change in Value of Shipments (VS) from 1982 to 1984 Based on Weighted Data Reported by ASM Establishments in Both 1982 and 1984

	Unweighted ASM Establishment Count		Weighted 1984 VS as % of Weighted ASM Total		Estimated Rate of Change
In the ASM Panel	2151		100.0		1.096
In the M3 Panel	729		42.4		1.061
M3 reporters		345		17.4	1.022
M3 nonreporters		384		25.0	1.090
Not in the M3 Panel	1422		57.6		1.123

For this category, M3 panel coverage was not found inadequate in measuring actual change in shipments. However, response in the M3 panel was found inadequate. Based on the t-test described earlier, it was concluded that the actual difference between all M3 reporters and the universe exceeds 5 percentage points in measuring the rate of change in shipments.

Estimated growth for establishments corresponding to the reported portion of the M3 panel is 2.2 percent. Change estimated for the universe of all establishments is 9.6 percent. The M3 reporters understated overall growth. Both the M3 nonreporters and those establishments that are not in the M3 panel grew at a higher rate than the M3 reporters.

A large portion of the universe value of shipments is not covered by the M3 panel. Coverage in 1984 was only 42 percent as estimated from the ASM. Even so, if all companies in the M3 panel had reported, no inadequacies would have been found for this category. However, this is not the case. More than one-half of the ASM establishments in the M3 panel are classified as nonreporters. These M3 nonreporters made up 25 percent of the total weighted ASM shipments compared with 17 percent which came from the M3 reporters.

To determine the possible cause for this reporting problem, M3 nonreporters were examined in further detail.

In the M3, there are really two different types of nonreporting. First, a company may provide data for building materials and wire products on an irregular basis or not at all. However, the company is classified within building materials and wire products for the M3. For this study, a distinction was made between this type of nonreporting and nonreporting which occurs when a company is not classified within the category. The second case would include a company which is unable to provide detailed data and therefore reports its activity related to building materials and wire

products with data classified in another category within the M3. This is referred to as combined reporting. The second type of nonreporter would also include a company which does not report in any of the categories in which it is classified. Combined reporting, however, is suspected as the more common problem within this category.

Inadequate response within building materials and wire products is due to the inability of the M3 reporters to capture large growth for the second type of nonreporters, companies which are in the M3 panel but are not classified in this category. Further investigation showed that these nonreporters with classification problems grew at an estimated 13.1 percent compared with 4.6 percent growth for the other nonreporters.

If all the companies not classified in building materials and wire products had actually provided regular reports for their activities in this category, then growth estimated for M3 reporters would have been 6.5 percent instead of 2.2 percent. This would have been very close to estimated growth for the M3 panel and would not have differed from the estimate for the entire universe by more than 5 percent.

This study was not able to determine why the establishments with classification problems grew so comparatively fast. This study was able to show that efforts should be taken to improve the classification of M3 data within this category. The first step in this direction is to determine why companies within this category are unable to provide more detailed data.

Those companies which responded in the category less than 7 months should not be neglected since they comprise roughly one-half of the portion of M3 panel with reporting problems, based on weighted 1984 ASM shipments.

Products of Pulp and Paperboard Mills, except Building Paper

The "products of pulp and paperboard mills, except building paper" is one of the industry categories in the nondurable sector for which no panel or response inadequacies were identified. Table 8, below, presents detailed information on rates of change estimated for inventories.

Table 8: Products of Pulp and Paperboard Mills, Except Building Paper: Estimated Rates of Change in Total Inventories at End of Year (TIE) from 1982 to 1984 Based on Data Reported by ASM Establishments in Both 1982 and 1984

	Unweighted ASM Establishment Count		Weighted 1984 TIE as % of Weighted ASM Total		Estimated Rate of Change
In the ASM Panel	359		100.0		1.097
In the M3 Panel		271		81.9	1.096
M3 reporters		177		64.4	1.084
Response Problem		48		7.8	1.111
Reporting Problem		46		9.6	1.165
Not in the M3 Panel		88		18.1	1.105

The rate of change estimated for the universe and the M3 panel are both approximately 10 percent.

The M3 panel covers a substantial portion of the universe total ending inventories, almost 82 percent as estimated from the ASM. Although only 177 out of the 271 ASM establishments in the M3 panel actually correspond to companies which report in the M3, these 177 establishments still comprise a major portion of the weighted inventories over the entire ASM, about 65 percent.

It is interesting to note that those establishments corresponding to companies with reporting problems, that is, companies not classified within this category for the M3, had the largest growth, estimated at 16.52 percent. However, this comprises such a small part of the universe estimate compared with M3 reporters, that the M3 reporters were not found to differ significantly from the universe.

F. Conclusion

Of the 74 industry categories in the M3, 14 have inadequate panel coverage to measure change in either shipments or inventories, or both. Within these categories, panel coverage may be deficient for several reasons.

First, the M3 panel has not been properly maintained. The panel, selected in 1962, would require updating to ensure that all companies with employment greater than 1,000 are included in the panel. This is the cut-off which was originally applied. Within some problem categories this may be adequate if accurately applied.

Secondly, the cut-off point of 1,000 employees may be too large for some of the problem categories. Panel coverage could possibly be improved by selecting cut-offs which vary by industry category. A cut-off based on shipments may also provide better coverage.

Within some industry categories, a cut-off sample alone may not provide adequate coverage. A probability sample of the smaller companies may be necessary if the panel is to be improved. Such a sample would have to be reselected on a regular basis to ensure proper coverage of the changing universe.

Each of the 14 industry categories with inadequate coverage must be examined individually to determine appropriate measures to improve the panel. Any attempt to expand the panels within industry categories has to take into consideration the level of response which can be realistically expected as well as resource limitations of the Census Bureau.

Problems with poor response are more common than inadequate panel coverage. Twenty-two industry categories were identified as having response which was inadequate to measure economic change. Unlike problems with panel coverage, poor response is not concentrated in the smaller industry categories.

Inadequate response may be due to several reasons. Within some industry categories the combined reporting of data across several manufacturing activities is the major factor affecting the quality of the M3 estimates. This study showed this is a more widespread occurence than originally believed. If the problem of combined reporting cannot be reduced by improved reporting at the category level, the Census Bureau may consider enacting procedures to automatically break-out data for certain companies. In other cases, poor response is due to companies which do not report in the M3 at all or do not provide data on a regular and timely basis.

Currently, efforts at improving response are directed towards large companies. In certain categories it may be worthwhile to direct efforts towards the mid-size to small companies.

The Census Bureau is very concerned with the impact of nonresponse in the M3 survey and is anxious to reduce it as much as possible. As part of an overall research program, the Census Bureau intends to construct a nonresponse profile for the M3 survey. A sample of M3 panel units would be selected from both reporters and nonreporters. Information would be requested on the timing availability of data, knowledge of the M3 publication, and corporate policy relating to voluntary surveys, among other questions. This nonresponse profile may be particularly helpful to investigate reasons behind the higher concentration of categories with response problems in the durable goods sector compared to the nondurable goods sector.

Of the 34 industry categories identified as problems in response or panel coverage, estimates for only 9 are included separately in the M3 publications. If appropriate actions cannot be taken to improve these categories, the Census Bureau would consider aggregating their data to higher levels for publication purposes.

APPENDIX

APPROXIMATE VARIANCE OF THE DIFFERENCE BETWEEN TWO RATIOS ESTIMATES FOR EVALUATION OF THE M3

A. Description

The universe of manufacturing establishments within a given category can be split into those establishments which correspond to companies in the M3 panel and those establishments which do not correspond to M3 companies. These groups of establishments will be referred to as the Class 1 universe and the Class 2 universe, respectively.

To evaluate M3 panel coverage of economic change, the rate of change in the Class 1 universe was compared to the rate of change in the entire universe within each of the 74 M3 industry categories. This comparison was performed on the rates of change for the shipments and inventories items from 1982 to 1984.

For each item, the rate of change in the entire universe was estimated from those establishments in the 1984 ASM panel which reported the item in both the 1982 census and the 1984 ASM. From these ASM reporters, the subset of ASM establishments which correspond to companies in the M3 was used to estimate the rate of change for the Class 1 universe. The difference between these two estimates was then used to evaluate the adequacy of the M3 panel, based on the t-test described in the paper. To apply this test, it is necessary to approximate the variance of difference between these two ratio estimators.

This appendix presents the derivation of this approximate variance and formulates the sample estimator of this variance obtained from the ASM.

Analysis of response in the M3 follows the same methodology, where the Class 1 universe includes all establishments corresponding to companies which report in the M3, and the Class 2 universe includes the remainder of the universe within a given industry category.

Within each individual M3 industry category, the methodology described below was applied in the evaluation of M3 panel coverage and response for both the shipments and inventories items.

B. Notation

1. The Universe

Complete	Class 1	Class 2
$Y = \sum_{j=1}^{N} y_j$	$Y_1 = \sum_{j=1}^{N1} y_j$	$Y_2 = \sum_{j=1}^{N2} y_j$
$X = \sum_{j=1}^{N} y_j$	$X_1 = \sum_{j=1}^{N1} x_j$	$X_2 = \sum_{j=1}^{N2} x_j$
$R = Y/X$	$R_1 = Y_1/X_1$	$R_2 = Y_2/X_2$

where

y_j = 1984 value of the item (shipments of inventories) for establishment j

x_j = 1982 value of the item for establishment j

N = the number of establishments in the universe

N1 = the number of establishments in the universe, in Class 1

N2 = the number of establishments in the universe, in Class 2

Note: N1 + N2 = N

2. The ASM Panel (ASM reporters only)

The establishments in the ASM panel which reported shipments in both the 1982 census and the 1984 ASM will be referred to as ASM reporters of the shipments item. The ASM establishments which reported inventories in the 1982 census and the 1984 ASM will be referred to as ASM reporters of the inventories item.

Complete	Class 1	Class 2
$\hat{Y} = \sum_{j=1}^{n} w_j y_j$	$\hat{Y}_1 = \sum_{j=1}^{n1} w_j y_j$	$\hat{Y}_2 = \sum_{j=1}^{n2} w_j y_j$
$\hat{X} = \sum_{j=1}^{n} w_j x_j$	$\hat{X}_1 = \sum_{j=1}^{n1} w_j x_j$	$\hat{X}_2 = \sum_{j=1}^{n2} w_j x_j$
$\hat{R} = \hat{Y}/\hat{X}$	$\hat{R}_1 = \hat{Y}_1/\hat{X}_1$	$\hat{R}_2 = \hat{Y}_2/\hat{X}_2$

where

w_j = the sampling weight assigned to establishment j during the selection of the 1984 ASM panel

n = the number of ASM reporters of the item

n1 = the number of ASM reporters of the item, in Class 1

n2 = the number of ASM reporters of the item, in Class 2

Note: $n1 + n2 = n$

C. Assumptions

It is assumed that $\hat{R}$ estimated over the ASM reporters is the same as the rate of change that would have been estimated over the entire ASM panel if all establishments had reported. This is equivalent to the assumption that the ASM reporters accounted for equal proportions of the total weighted value over the entire ASM panel in 1982 and 1984. This is written as follows:

$\hat{Y} = k\hat{Y}'$ and $\hat{X} = k\hat{X}'$ which implies $\frac{\hat{Y}}{\hat{X}} = \frac{\hat{Y}'}{\hat{X}'}$

where

k = constant between 0 and 1

$$\hat{Y}' = \sum_{j=1}^{n'} w_j y_j$$

$$\hat{X}' = \sum_{j=1}^{n'} w_j x_j$$

n' = number of establishments in the entire ASM panel

It was also assumed that $\hat{R}_1$ and $\hat{R}_2$ estimated over ASM reporters are the same as the corresponding rates of change which would have been estimated if all establishments had reported. This can be written as:

$$\hat{Y}_1 = k_1 \hat{Y}'_1; \quad \hat{X}_1 = k_1 \hat{X}'_1$$

and

$$\hat{Y}_2 = k_2 \hat{Y}'_2; \quad \hat{X}_2 = k_2 \hat{X}'_2$$

where

k_1 = constant between 0 and 1

k_2 = constant between 0 and 1

$$\hat{Y}'_1 = \sum_{j=1}^{n'_1} w_j y_j$$

$$\hat{X}'_1 = \sum_{j=1}^{n'_1} w_j x_j$$

$$\hat{Y}'_2 = \sum_{j=1}^{n'_2} w_j y_j$$

$$\hat{X}'_2 = \sum_{j=1}^{n'_2} w_j x_j$$

n'_1 = the number of ASM establishments in Class 1

n'_2 = the number of ASM establishments in Class 2

Furthermore, we assumed that the proportions of the sample in Class 1 and Class 2 which we observed for ASM reporters are the same as the proportions which we would have observed if all establishments had reported. This can be written as:

$$\frac{\hat{Y}_1}{\hat{Y}} = \frac{k_1\hat{Y}'_1}{k\hat{Y}'} = \frac{\hat{Y}'_1}{\hat{Y}'}; \quad \frac{\hat{Y}_2}{\hat{Y}} = \frac{k_2\hat{Y}'_2}{k\hat{Y}'} = \frac{\hat{Y}'_2}{\hat{Y}'}$$

The same applies for $\hat{X}_1$ and $\hat{X}_2$.

This implies that

$k_1 = k_2 = k$

D. The Variance of $(\hat{R} - \hat{R}_1)$ and an Approximation of its Components

The variance of $(\hat{R} - \hat{R}_1)$ is

$$V(\hat{R} - \hat{R}_1) = V(\hat{R}) + V(\hat{R}_1) - 2\,\mathrm{Cov}(\hat{R}, \hat{R}_1)$$

To approximate its variance and covariance components, consider the following:

$$\hat{R} - R = \frac{\hat{Y} - R\hat{X}}{\hat{X}} \doteq \frac{\hat{Y} - R\hat{X}}{kX}$$

Estimation procedures in the ASM are unbiased, therefore $E(\hat{X}') = X$ and $E(\hat{Y}') = Y$.

$E(\hat{X}') = X$ implies that $E(\hat{X}) = E(k\hat{X}') = kX$. If n is large, the approximation $\hat{X} \doteq kX$ can be made.

Therefore

$$E(\hat{R} - R) \doteq \frac{E(\hat{Y} - R\hat{X})}{kX} = \frac{kE(\hat{Y}' - R\hat{X}')}{kX} = \frac{k(Y - RX)}{kX} = 0$$

which implies

$E(\hat{R}) \doteq R$

Similarly,

$$\hat{R}_1 - R_1 \doteq \frac{\hat{Y}_1 - R_1\hat{X}_1}{kX_1} \quad \text{and} \quad E(\hat{R}_1) \doteq R_1.$$

As a result, we can approximate the following:

$$V(\hat{R}) = E[\hat{R} - E(\hat{R})]^2 \doteq E(\hat{R} - R)^2 \doteq \frac{E[\hat{Y} - R\hat{X}]^2}{(kX)^2}$$

$$V(\hat{R}_1) = E[\hat{R}_1 - E(\hat{R}_1)]^2 \doteq E(\hat{R}_1 - R_1)^2 \doteq \frac{E[\hat{Y}_1 - R_1\hat{X}_1]^2}{(kX_1)^2}$$

$$Cov(\hat{R},\hat{R}_1) = E[\hat{R} - E(\hat{R})][\hat{R}_1 - E(\hat{R}_1)] \doteq E(\hat{R} - R)(\hat{R}_1 - R_1) \doteq \frac{E(\hat{Y} - R\hat{X})(\hat{Y}_1 - R_1\hat{X}_1)}{k^2X(X_1)}$$

The variances of $\hat{R}$ and $\hat{R}_1$ can be written as follows, in a form which can be estimated from the sample.

$$V(\hat{R}) \doteq \frac{V(\hat{Y}) + R^2V(\hat{X}) - 2R\ Cov(\hat{Y},\hat{X})}{(kX)^2}$$

$$V(\hat{R}_1) \doteq \frac{V(\hat{Y}_1) + R_1^2V(\hat{X}_1) - 2R_1Cov(\hat{Y}_1,\hat{X}_1)}{(kX_1)^2}$$

An estimable form of the covariance is derived in the section which follows.

E. Approximate Covariance of R and R_1

In Section D, it was shown that

$$Cov(\hat{R},\hat{R}_1) \doteq \frac{E(\hat{Y} - R\hat{X})(\hat{Y}_1 - R_1\hat{X}_1)}{k^2X(X_1)}$$

In this section we derive a form of the covariance which can be estimated from the sample.

Since $\hat{Y} = \hat{Y}_1 + \hat{Y}_2$ and $\hat{X} = \hat{X}_1 + \hat{X}_2$, we can write

$$\hat{Y} - R\hat{X} = (\hat{Y}_1 - R\hat{X}_1) + (\hat{Y}_2 - R\hat{X}_2)$$

and

$$E(\hat{Y} - R\hat{X})(\hat{Y}_1 - R_1\hat{X}_1) = E(\hat{Y}_1 - R\hat{X}_1)(\hat{Y}_1 - R_1\hat{X}_1) + E(\hat{Y}_2 - R\hat{X}_2)(\hat{Y}_1 - R_1\hat{X}_1)$$

Each unit in the ASM sample is drawn independently and a sampled unit cannot be in Class 1 and 2; therefore

$$E(\hat{Y}_2 - R\hat{X}_2)(\hat{Y}_1 - R_1\hat{X}_1) = E(\hat{Y}_2 - R\hat{X}_2)E(\hat{Y}_1 - R_1\hat{X}_1)$$

This is proven in the attachment.

Since $E(\hat{Y}_1 - R_1\hat{X}_1) = kE(\hat{Y}'_1 - R_1\hat{X}'_1) = 0$, we have

$$Cov(\hat{R},\hat{R}_1) \doteq \frac{E(\hat{Y}_1 - R\hat{X}_1)(\hat{Y}_1 - R_1\hat{X}_1)}{k^2X(X_1)}$$

By substituting $R = R_1 + (R - R_1)$ we have

$$(\hat{Y}_1 - R\hat{X}_1)(\hat{Y}_1 - R_1\hat{X}_1) = [(\hat{Y}_1 - R_1\hat{X}_1) - (R - R_1)\hat{X}_1](\hat{Y}_1 - R_1\hat{X}_1)$$

$$= (\hat{Y}_1 - R_1\hat{X}_1)^2 - (R-R_1)(\hat{Y}_1 - R_1\hat{X}_1)\hat{X}_1$$

$$= (\hat{Y}_1 - R_1\hat{X}_1)^2 - (R - R_1)(\hat{Y}_1\hat{X}_1 - R_1\hat{X}_1^2)$$

Take the expected value of the above and make the following substitutions:

$$E(\hat{Y}_1 - R_1\hat{X}_1)^2 \doteq (kX_1)^2V(\hat{R}_1)$$

$$E(\hat{Y}_1\hat{X}_1) = Cov(\hat{Y}_1,\hat{X}_1) + E(\hat{Y}_1)E(\hat{X}_1) = Cov\ (\hat{Y}_1\hat{X}_1) + k^2Y_1X_1$$

$$E(R_1\hat{X}_1^2) = R_1E(\hat{X}_1^2) = R_1[V(\hat{X}_1) + (E(\hat{X}_1))^2] = R_1[V(\hat{X}_1) + (kX_1)^2] = R_1V(\hat{X}_1) + R_1(kX_1)^2$$

Therefore

$$E(\hat{Y}_1 - R\hat{X}_1)(\hat{Y}_1 - R_1\hat{X}_1) \doteq$$

$$= (kX_1)^2V(\hat{R}_1) - (R - R_1)[Cov(\hat{Y}_1,\hat{X}_1) + k^2Y_1X_1 - R_1V(\hat{X}_1) - R_1(kX_1)^2]$$

$$= (kX_1)^2V(\hat{R}_1) - (R - R_1)[Cov(\hat{Y}_1,\hat{X}_1) - R_1V(\hat{X}_1)]$$

$$= [V(\hat{Y}_1) + R_1^2V(\hat{X}_1) - 2R_1Cov(\hat{Y}_1,\hat{X}_1)] - (R - R_1)[Cov(\hat{Y}_1,\hat{X}_1) - R_1V(\hat{X}_1)]$$

$$= V(\hat{Y}_1) + (R_1^2 + RR_1 - R_1^2)V(\hat{X}_1) - (R + R_1)Cov(\hat{Y}_1,\hat{X}_1)$$

$$= V(\hat{Y}_1) + RR_1V(\hat{X}_1) - (R + R_1)Cov(\hat{Y}_1,\hat{X}_1)$$

Finally, we can write the approximate covariance as

$$Cov(\hat{R},\hat{R}_1) \doteq [V(\hat{Y}_1) + RR_1V(\hat{X}_1) - (R + R_1)Cov(\hat{Y}_1,\hat{X}_1)] \ / (k^2X_1X)$$

F. Approximate Variance of $(\hat{R} - \hat{R}_1)$

We have

$$V(\hat{R} - \hat{R}_1) = V(\hat{R}) + V(\hat{R}_1) - 2Cov(\hat{R},\hat{R}_1)$$

and

$$V(\hat{R}) \doteq [V(\hat{Y}) + R^2V(\hat{X}) - 2RCov(\hat{Y},\hat{X})] \ / (kX)^2$$

$$V(\hat{R}_1) \doteq [V(\hat{Y}_1) + R_1^2V(\hat{X}_1) - 2R_1Cov(\hat{Y}_1,\hat{X}_1)] \ / (kX_1)^2$$

$$Cov(\hat{R},\hat{R}_1) \doteq [V(\hat{Y}_1) + RR_1V(\hat{X}_1) - (R + R_1)Cov(\hat{Y}_1,\hat{X}_1)] \ / (k^2X_1X)$$

Make these substitutions to obtain

$$V(\hat{R} - \hat{R}_1) \doteq [V(\hat{Y}) + R^2V(\hat{X}) - 2RCov(\hat{Y},\hat{X})] / (kX)^2$$
$$+ [V(\hat{Y}_1) + R_1^2V(\hat{X}_1) - 2R_1Cov(\hat{Y}_1,\hat{X}_1)] / (kX_1)^2$$
$$- 2[V(\hat{Y}_1) + RR_1V(\hat{X}_1) - (R + R_1)Cov(\hat{Y}_1,\hat{X}_1)] / (k^2X_1X)$$

This can be estimated from the ASM reporters in the ASM panel as follows:

$$v(\hat{R} - \hat{R}_1) = [v(\hat{Y}) + \hat{R}^2v(\hat{X}) - 2\hat{R}\widehat{Cov}(\hat{Y},\hat{X})] / \hat{X}^2$$
$$+ [v(\hat{Y}_1) + \hat{R}_1^2v(\hat{X}_1) - 2\hat{R}_1\widehat{Cov}(\hat{Y}_1,\hat{X}_1)] / \hat{X}_1^2$$
$$- 2[v(\hat{Y}_1) + \hat{R}\hat{R}_1v(\hat{X}_1) - (\hat{R} + \hat{R}_1)\widehat{Cov}(\hat{Y}_1,\hat{X}_1)] / (\hat{X}_1\hat{X})$$

where

$$v(\hat{Y}) = \sum_{j=1}^{n} w_j(w_j - 1)y_j^2 \text{ with } E[v(\hat{Y})] = k^2V(\hat{Y}')$$

$$v(\hat{X}) = \sum_{j=1}^{n} w_j(w_j - 1)x_j^2 \text{ with } E[v(\hat{X})] = k^2V(\hat{X}')$$

$$Cov(\hat{Y},\hat{X}) = \sum_{j=1}^{n} w_j(w_j - 1)y_jx_j \text{ with } E[\widehat{Cov}(\hat{Y},\hat{X})] = k^2Cov(\hat{Y}',\hat{X}')$$

$$v(\hat{Y}_1) = \sum_{j=1}^{n1} w_j(w_j - 1)y_j^2 \text{ with } E[v(\hat{Y}_1)] = k^2V(\hat{Y}'_1)$$

$$v(\hat{X}_1) = \sum_{j=1}^{n1} w_j(w_j - 1)x_j^2 \text{ with } E[v(\hat{X}_1)] = k^2V(\hat{X}'_1)$$

and

$$Cov(\hat{Y}_1,\hat{X}_1) = \sum_{j=1}^{n1} w_j(w_j - 1)y_jx_j \text{ with } E[\widehat{Cov}(\hat{Y}_1,\hat{X}_1)] = k^2Cov(\hat{Y}'_1,\hat{X}'_1)$$

Note that the approximation $V(\hat{R} - \hat{R}_1)$ is equivalent to the approximation which would have been applied if all ASM establishments had reported.

$$V(\hat{R} - \hat{R}_1) \doteq k^2[V(\hat{Y}') + R^2V(\hat{X}') - 2RCov(\hat{Y}',\hat{X}')] / (kX)^2$$

$$+ k^2[V(\hat{Y}'_1) + R_1^2V(\hat{X}'_1) - 2R_1Cov(\hat{Y}'_1,\hat{X}'_1)] / (kX_1)^2$$

$$- 2k^2[V(\hat{Y}'_1) + RR_1V(\hat{X}'_1) - (R + R_1)Cov(\hat{Y}'_1,\hat{X}'_1)] / (k^2X_1X)$$

$$= [V(\hat{Y}') + R^2V(\hat{X}') - 2RCov(\hat{Y}', \hat{X}')] / X^2$$

$$+ [V(\hat{Y}'_1) + R_1^2V(\hat{X}'_1) - 2R_1Cov(\hat{Y}'_1,\hat{X}'_1)] / X_1^2$$

$$- 2[V(\hat{Y}'_1) + RR_1V(\hat{X}'_1) - (R + R_1)Cov(\hat{Y}'_1,\hat{X}'_1)] / (X_1X)$$

Recall that the universe (size N) consists of establishments classified in either Class 1 (size N1) or Class 2 (size N2).

Consider the following sample estimates U_1 and Z_2 for the Class 1 universe and Class 2 universe, respectively:

$$\hat{U}_1 = \sum_{j}^{n1} w_j u_j = \sum_{j}^{N1} w_j u_j t_j = \sum_{j}^{N} w_j u_j t_j m_j$$

$$\hat{Z}_2 = \sum_{j}^{n2} w_j z_j = \sum_{j}^{N2} w_j z_j t_j = \sum_{j}^{N} w_j z_j t_j (1 - m_j)$$

where

$t_j = 1$ if establishment j was selected for the ASM
$= 0$ otherwise

and

$m_j = 1$ if establishment j is in the Class 1 universe
$= 0$ otherwise

Note that

$1-m_j = 1$ if establishment j is in the Class 2 universe
$= 0$ otherwise

Now,

$$E(\hat{Z}_2\hat{U}_1) = E[\sum_{j}^{N} w_j z_j t_j (1 - m_j)][\sum_{j}^{N} w_j u_j t_j m_j]$$

$$= E\,[\sum_{j}^{N} w_j z_j t_j (1 - m_j) w_j u_j t_j m_j] + \sum_{i \neq j}^{N}\sum w_j z_j t_j (1 - m_j) w_i u_i t_i m_i]$$

Now,

$E(t_j) = p_j$, where p_j = probability that j was selected for the ASM

$E(t_j t_i) = E(t_j)E(t_i)$ since each unit has an independent chance of selection

$w_j = 1/p_j$

and

$m_j(1 - m_j) = 0$

Therefore,

$$E(\hat{Z}_2\hat{U}_1) = \sum_{i \neq j}^{N} \sum z_j(1 - m_j)u_i m_i = \sum_{j}^{N} z_j(1-m_j) \sum_{j}^{N} u_j m_j$$

$$= (\sum_{j}^{N2} z_j)(\sum_{j}^{N1} u_j) = E(\hat{Z}_2)E(\hat{U}_1)$$

Finally, substituting $y_j - Rx_j$ for z_j and

$y_j - R_1x_j$ for u_j,

we have

$$E(\hat{Y}_2 - R\hat{X}_2)(\hat{Y}_1 - R_1\hat{X}_1) = E(\hat{Y}_2 - R\hat{X}_2)E(\hat{Y}_1 - R_1\hat{X}_1)$$