

ENHANCING THE MORRIS METHOD

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Abstract: The screening method proposed by Morris (1991) and recently improved by Campolongo et al. (2003) is very effective to screen a subset of few important input factors among a large number contained in a model. In this work the enhanced Morris method is first confronted with the variance based methods and then employed to assess the sensitivity of a financial model for option pricing.

Keywords: sensitivity analysis, screening designs, Morris method, variance based sensitivity indices, the Heston model, option pricing.

1. INTRODUCTION

A sensitivity analysis method widely used to screen factors in models of large dimensionality is the design proposed by Morris [1]. The Morris method deals efficiently with models containing hundreds of input factors without relying on strict assumptions about the model, such as for instance additivity or monotonicity of the model input-output relationship.

The Morris method is simple to understand and implement, and its results are easily interpreted. Furthermore it is economic in the sense that it requires a number of model evaluations that is linear in the number of model factors. The method can be regarded as global as the final measure is obtained by averaging a number of local measures (the elementary effects), computed at different points of the input space.

In very recent work [2] Campolongo and coworkers proposed an improved version of the Morris measure μ , denoted as μ^* , which is more effective in ranking factors in order of importance. Furthermore, they extended to the Morris measure a desirable property of the variance based methods: the capability to treat group of factors as if they were single factors.

Here we extend the study in [2] by testing the performance of μ and μ^* by groups on an analytical test function recently proposed by O'Hagan [3]. Results of the enhanced Morris method are also compared with those obtained by making use of the variance based sensitivity measures. The motivation for this comparison lies in the present trend that sees the variance based methods as particularly apt to sensitivity analysis, because of their desirable properties in terms of model independence, global nature, ease of interpretation and others [4]. At the same time the method of Morris is considerably cheaper than the variance based methods in terms of model evaluation, hence the interest in this comparison.

Results confirm that the Morris method, in its new version μ^* , is as efficient as the variance based techniques in identifying irrelevant factors, i.e. those factors that can be fixed at any given value within their range of uncertainty without significantly affecting the total output variance. Hence, it is recommendable as a valid alternative to the variance based when the problem is such that the cost of the variance based techniques is too high.

A theoretical link between the Morris and the variance based measures is also argued for by expressing the measure μ^* in terms of conditional variances.

Section 4 of this work is dedicated to an application of the enhanced Morris method to a real test case, a financial model here used to price a European call option. Results of the sensitivity analysis confirm the good quality of the model and encourage to extend its use to more delicate problems such those of pricing exotic options.

2. METHODOLOGY

2.1. The Morris method and its improved version

The experimental plan proposed by Morris is composed of individually randomized 'one-factor-at-a-time' experiments: the impact of changing one factor at a time is evaluated in turn. Each input factor may assume a discrete number of values, called levels, which are chosen within the factor range of variation.

The sensitivity measures proposed in the original work of Morris [1] are based on what is called an elementary effect. The elementary effect for the i th input is defined as follows. Let Δ be a predetermined multiple of $1/(p-1)$. For a given value of \mathbf{x} , the elementary effect of the i th input factor is defined as

$$EE_i(\mathbf{x}) = \frac{[y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(\mathbf{x})]}{\Delta}$$

where $\mathbf{x} = (x_1, x_2, \dots, x_k)$ is any selected value in Ω such that the transformed point $(\mathbf{x} + \mathbf{e}_i \Delta)$, where \mathbf{e}_i is a vector of zeros but with a unit as its i th component, is still in Ω for each index $i=1, \dots, k$. The finite distribution of elementary effects associated with the i th input factor, is obtained by randomly sampling different \mathbf{x} from Ω , and is denoted by F_i .

In Morris [1], two sensitivity measures were proposed for each factor: μ , an estimate of the mean of the distribution F_i , and σ , an estimate of the standard deviation of F_i . A high value of μ indicates an input factor with an important overall influence on the output. A high value of σ indicates a factor involved in interaction with other factors or whose effect is non-linear. Here we consider a third sensitivity measure, μ^* , which is an estimate of the mean of the distribution (here denoted as G_i) of the absolute values of the elementary effects [2].

We believe that μ^* is better than μ to rank factors in order of importance. The reason is that if the distribution F_i contains elements of opposite sign, which occurs when the model is non-monotonic, when computing its mean some effects may cancel each other out. Thus a

factor which is important but whose effect on the output has an oscillating sign may be erroneously considered as negligible, thus generating a mostly undesirable Type II error.

The performance of μ^* is tested on the analytical function presented in Section 3 and compared with that of the variance based methods described in the following subsection.

2.2. The variance based measures

Variance based methods choose as a measure of the main effect of a factor X_i on the output, an estimation of quantity $\frac{V_{X_i}(E_{\mathbf{X}_{-i}}(Y|X_i))}{V(Y)}$, which is known in the literature as the “first order effect” of X_i on Y , and denoted by S_i . Reasons for this choice are detailed in [4].

Another sensitivity measured based on the variance decomposition is the total sensitivity index, S_{T_i} . The total index is defined as the sum of all effects involving the factor X_i . S_{T_i} is

estimated by the quantity $\frac{E_{\mathbf{X}_{-i}}(V_{X_i}(Y|\mathbf{X}_{-i}))}{V(Y)}$.

The total index is the appropriate measure when the problem is that of Factors Fixing [4], i.e. that of identifying those factors that can be fixed to any given value within their range of variation because they are non-influent on the total output variance. A necessary and sufficient condition for factor X_i to be totally non-influent is that $S_{T_i} = 0$. In fact, if factor X_i is totally non-influent, then all the variance is due to \mathbf{X}_{-i} , and fixing this vector results in $V_{X_i}(Y|\mathbf{X}_{-i}) = 0$, as well as in $E_{\mathbf{X}_{-i}}(V_{X_i}(Y|\mathbf{X}_{-i})) = 0$. The reverse is also true: if $V_{X_i}(Y|\mathbf{X}_{-i}) = 0$ at all fixed points in the space of \mathbf{X}_{-i} , then X_i is non-influent, so that $S_{T_i} \equiv 0$.

Variance based techniques have several desirable properties. They are “model free”, in the sense of independent from assumptions about the model such as linearity, additivity and so on. They are global, i.e. they explore the entire interval of definition of each factor and the effect of each factor is taken as an average over the possible values of the other factors. They are usually quantitative, which is they can tell how much factor a is more important than factor b . They are able to treat grouped factors as if they were single factors, a property of synthesis that may be essential for the agility of the interpretation of the results.

The main drawback of the variance based measures is their computational cost, as they require a number of model evaluation such as $N \times (k + 2)$ where k is the number of input factors and N is of the order of $N = 500, 5000, \dots$, [4] which in some instances may result to be unaffordable. Note that this number can be lowered considerably if one desires to compute only the first order sensitivity indices, as shown by Ratto et al. [6]. Design based strategies to estimate sensitivity indices at low sample size are also proposed in [3].

In this work it is shown that the improved Morris measure represents a valid alternative to the variance based one when the aim of the analysis is that of screening few important factors among a large number, and the cost of applying variance based techniques would be excessive.

3. TESTING THE METHOD

In this Section we propose to test the performance of μ^* and that of the Morris strategy extended for groups on the analytical function recently proposed by Oakley and O'Hagan [3].

The test function is the following:

$$\mathbf{h}(\mathbf{x}) = \mathbf{a}_1^T \mathbf{x} + \mathbf{a}_2^T \cos(\mathbf{x}) + \mathbf{a}_3^T \sin(\mathbf{x}) + \mathbf{x}^T \mathbf{M} \mathbf{x}$$

where \mathbf{x} is a fifteen dimensional input vector while \mathbf{a}_1 , \mathbf{a}_2 , \mathbf{a}_3 and \mathbf{M} are respectively three (1×15) row vectors and a (15×15) matrix of parameters (Table 1). The unknown input factors are assumed to be independent and to follow a normal distribution $N(0,1)$. In [3] the emphasis is on computing first order sensitivity measures, and the test case is designed to have three groups of factors, with respectively high ($x_{11} - x_{15}$), medium ($x_6 - x_{10}$) and low ($x_1 - x_5$) values of S_i .

Table 1: Parameters of the analytical function proposed by O'Hagan.

a1	a2	a3	M														
0.01	0.43	0.10	-0.02	-0.19	0.13	0.37	0.17	0.14	-0.44	-0.08	0.71	-0.44	0.50	-0.02	-0.05	0.22	0.06
0.05	0.09	0.21	0.26	0.05	0.26	0.24	-0.59	-0.08	-0.29	0.42	0.50	0.08	-0.11	0.03	-0.14	-0.03	-0.22
0.23	0.05	0.08	-0.06	0.20	0.10	-0.29	-0.14	0.22	0.15	0.29	0.23	-0.32	-0.29	-0.21	0.43	0.02	0.04
0.04	0.32	0.27	0.66	0.43	0.30	-0.16	-0.31	-0.39	0.18	0.06	0.17	0.13	-0.35	0.25	-0.02	0.36	-0.33
0.12	0.15	0.13	-0.12	0.12	0.11	0.05	-0.22	0.19	-0.07	0.02	-0.10	0.19	0.33	0.31	-0.08	-0.25	0.37
0.39	1.04	0.75	-0.28	-0.33	-0.10	-0.22	-0.14	-0.14	-0.12	0.22	-0.03	-0.52	0.02	0.04	0.36	0.31	0.05
0.39	0.99	0.86	-0.08	0.004	0.89	-0.27	-0.08	-0.04	-0.19	-0.36	-0.17	0.09	0.40	-0.06	0.14	0.21	-0.01
0.61	0.97	1.03	-0.09	0.59	0.03	-0.03	-0.24	-0.10	0.03	0.10	-0.34	0.01	-0.61	0.08	0.89	0.14	0.15
0.62	0.90	0.84	-0.13	0.53	0.13	0.05	0.58	0.37	0.11	-0.29	-0.57	0.46	-0.09	0.14	-0.39	-0.45	-0.15
0.40	0.81	0.80	0.06	-0.32	0.09	0.07	-0.57	0.53	0.24	-0.01	0.07	0.08	-0.13	0.23	0.14	-0.45	-0.56
1.07	1.84	2.21	0.66	0.35	0.14	0.52	-0.28	-0.16	-0.07	-0.20	0.07	0.23	-0.04	-0.16	0.22	0.00	-0.09
1.15	2.47	2.04	0.32	-0.03	0.13	0.13	0.05	-0.17	0.18	0.06	-0.18	-0.31	-0.25	0.03	-0.43	-0.62	-0.03
0.79	2.39	2.40	-0.29	0.03	0.03	-0.12	0.03	-0.34	-0.41	0.05	-0.27	-0.03	0.41	0.27	0.16	-0.19	0.02
1.12	2.00	2.05	-0.24	-0.44	0.01	0.25	0.07	0.25	0.17	0.01	0.25	-0.15	-0.08	0.37	-0.30	0.11	-0.76
1.20	2.26	1.98	0.04	-0.26	0.46	-0.36	-0.95	-0.17	0.003	0.05	0.23	0.38	0.46	-0.19	0.01	0.17	0.16

The total variance of the output can be computed analytically and decomposed as the sum of first and second order effects:

$$V = \sum_i V_i + V^{2nd}$$

$$V_i = a_{1,i}^2 * E(x_i^2) + a_{2,i}^2 * E(\sin^2(x_i)) + a_{3,i}^2 * E(\cos^2(x_i)) + M_{i,i}^2 * E(x_i^4) + 2 * a_{1,i} * a_{2,i} * E(x_i \sin(x_i)) - [a_{3,i} * E(\cos(x_i)) + M_{i,i} * E(x_i^2)]^2$$

$$V^{2nd} = \sum_{i=1}^{15} \sum_{\substack{j=1 \\ j>i}}^{15} (\mathbf{M} + \mathbf{M}^T)_{i,j}^2$$

where $E(\bullet)$ indicates the mean operator. All the terms of higher order are zero.

Table 2 shows the rank of importance for the 15 input factors according to the revised Morris measure μ^* and to the Sobol' total effect index. The analytical values of the total effects are also reported. The total number of model evaluations needed to estimate each set of measures is reported in the first row. Results confirm that, with just 1024 model evaluations, the Morris revised measure is capable of identifying the subset of important factors ($x_{11} - x_{15}$). Note that when the total sensitivity indices are used, the factors end up partitioned in just two sets, that of the most influential factors ($x_{11} - x_{15}$), and that of the less influential ones (all others). Strictly speaking, none of the input factors of this test case can be fixed unless a rather high threshold is imposed. The least important factor's bottom marginal variance is in fact as high as 2.6 %. Four factors could be fixed if the threshold were 5%, while ten could be fixed if the threshold were 10%.

Table 2: Sensitivity analysis results for the test function in [3]. The analytical values of the total indices are reported together with the Sobol' estimates. The correspondent ranks are compared with that obtained through the Morris experiment.

Factor	ST(i) Analytics	ST(i) N=65563	Analytics Rank	ST(i) Rank N=65563	Morris Rank N=1024
X1	0.059	0.034	9	11	8
X2	0.063	0.032	8	12	9
X3	0.036	0.026	13	14	12
X4	0.055	0.035	11	10	10
X5	0.026	0.01	15	15	15
X6	0.041	0.038	12	9	13
X7	0.058	0.047	10	8	11
X8	0.082	0.067	7	7	7
X9	0.097	0.073	6	6	5
X10	0.036	0.027	14	13	14
X11	0.151	0.14	2	5	3
X12	0.148	0.172	3	2	2
X13	0.142	0.152	4	3	4
X14	0.141	0.143	5	4	6
X15	0.155	0.175	1	1	1

4. THE FINANCIAL PROBLEM

The problem is that of pricing a European call option. Different scenarios are assumed, corresponding to different possible strike prices and times to maturity. The dynamic of the underlying stock price is modeled according to the Heston Stochastic Volatility model (HEST, [5]), where the stock price follows the Black-Scholes stochastic differential equation SDE in which the volatility behaves stochastically over time:

$$\frac{dS_t}{dt} = (r - q)dt + \mathbf{s}_t dW_t \quad S_0 \geq 0.$$

The (squared) volatility follows the Cox-Ingersoll-Ross process:

$$d\mathbf{s}_t^2 = k(\mathbf{h} - \mathbf{s}_t^2)dt + \mathbf{q}\mathbf{s}_t d\tilde{W}_t \quad \mathbf{s}_0 \geq 0,$$

where $W = \{W_t, t \geq 0\}$ and $\tilde{W} = \{\tilde{W}_t, t \geq 0\}$ are two correlated standard Brownian motions such that $Cov[dW_t, d\tilde{W}_t] = \mathbf{r}dt$.

Here we also consider an extension of the HEST model that introduces jumps in the asset price [5]. Jumps here are assumed to occur as a Poisson process and the percentage jump-sizes are log-normally distributed.

In the Heston Stochastic Volatility model with jumps (HESJ), the SDE of the stock price process is extended to yield:

$$\frac{dS_t}{dt} = (r - q - \mathbf{I}\mathbf{m}_J)dt + \mathbf{s}_t dW_t + J_t dN_t \quad S_0 \geq 0,$$

where $N = \{N_t, t \geq 0\}$ is an independent Poisson process with intensity parameter $\mathbf{I} > 0$, i.e. $E[N_t] = \mathbf{I}t$. J_t is the percentage jump size (conditional on a jump occurring) that is assumed to be log-normally, identically and independently distributed over time, with unconditional mean \mathbf{m}_J . The standard deviation of $\log(1 + J_t)$ is \mathbf{s}_J :

$$\log(1 + J_t) \sim N\left(\log(1 + \mathbf{m}_J) - \frac{\mathbf{s}_J^2}{2}, \mathbf{s}_J^2\right).$$

The SDE of (squared) volatility process remains unchanged. J_t and N are assumed to be independent, as well as of W and of \tilde{W} .

Sensitivity analysis is performed first on the HEST model and then on its extended version with jumps HESJ. For HEST the input variables considered in the analysis are \mathbf{s}_0 , k , \mathbf{h} , \mathbf{q} and ρ . In the case where jumps are present \mathbf{I} , \mathbf{m}_J , \mathbf{s}_J are added. The initial condition for the underlying price S_0 is fixed at 100, while the interest rate r and the dividend yields q of the stock are respectively at 1.9% and 1.2%. The distributions chosen for the inputs are listed in Table 3. Both the Morris measure μ^* and the total sensitivity indices S_{T_i} are computed for the input factors in 42 different scenarios, a scenario being determined by a different value of the option strike price and of the time to maturity.

Table 3: Distributions for the inputs of the HEST and HESJ models.

Input	Distribution	Minimum	Maximum	Input	Distribution	Minimum	Maximum
\mathbf{s}_0	Uniform	0.04	0.09	ρ	Uniform	-1	0
κ	Uniform	0	1	λ	Uniform	0	2
η	Uniform	0.04	0.09	μ_j	Uniform	-0.1	0.1
θ	Uniform	0.2	0.5	σ_j	Uniform	0	0.2

Tables 4 and 5 show the ranking of the input factors obtained according to the two measures for the two versions of the model, HEST and HESJ, in some of the scenarios. For the variance based method, which is a quantitative method (as each index represent the fraction of the output variance due to the effect of that factor), we also reported values of the indices. The total number of model executions for the total sensitivity indices is 20480. For

the Morris experiment four levels are considered and 60 model executions performed to obtain the distribution of elementary effects for each input. The variance based method has also been repeated doubling the sample size to verify the convergence of the obtained sensitivity indices. Results confirm that the sample size 20480 can be considered sufficient for the estimation of the indices.

Table 4: Sensitivity analysis results of the HEST model for six selected scenarios

	ST(i) N=14336	ST(i) Rank N=14336	Morris Rank N=60	ST(i) N=14336	ST(i) Rank N=14336	Morris Rank N=60	ST(i) N=14336	ST(i) Rank N=14336	Morris Rank N=60
	Strike = 80			Strike = 100			Strike = 120		
	Time to maturity = 1y								
s0	0.742	1	1	0.821	1	1	0.411	2	2
k	0.026	4	5	0.050	4	4	0.033	5	5
h	0.055	3	3	0.075	3	3	0.045	4	4
q	0.013	5	4	0.084	2	2	0.110	3	3
r	0.194	2	2	0.009	5	5	0.448	1	1
	Time to maturity = 3y								
s0	0.493	1	1	0.348	1	1	0.201	3	2
k	0.158	3	3	0.273	2	2	0.231	2	3
h	0.362	2	2	0.267	3	3	0.182	4	4
q	0.065	4	4	0.165	4	4	0.149	5	5
r	0.042	5	5	0.049	5	5	0.324	1	1

Table 5: Sensitivity analysis results of the HESJ model for six selected scenarios.

	ST(i) N=2080	ST(i) Rank N=2080	Morris Rank N=90	ST(i) N=2080	ST(i) Rank N=2080	Morris Rank N=90	ST(i) N=2080	ST(i) Rank N=2080	Morris Rank N=90
	Strike = 80			Strike = 100			Strike = 120		
	Time to maturity = 1y								
s0	0.342	2	3	0.317	2	3	0.191	3	3
k	0.012	7	6	0.019	7	7	0.013	8	8
h	0.025	6	5	0.028	6	5	0.018	7	7
q	0.001	8	8	0.034	4	4	0.055	5	5
r	0.060	4	4	0.010	8	8	0.151	4	4
l	0.264	3	1	0.308	3	1	0.297	2	1
mj	0.040	5	7	0.030	5	6	0.044	6	6
sj	0.366	1	2	0.379	1	2	0.369	1	2
	Time to maturity = 3y								
s0	0.166	3	3	0.125	3	3	0.086	4	5
k	0.061	5	5	0.092	5	6	0.084	5	7
h	0.120	4	4	0.097	4	4	0.076	6	6

q	0.028	7	6	0.060	6	5	0.065	7	4
r	0.007	8	8	0.032	7	7	0.118	3	3
l	0.314	2	1	0.316	2	1	0.307	2	1
mj	0.031	6	7	0.030	8	8	0.032	8	8
sj	0.424	1	2	0.404	1	2	0.388	1	2

From both Tables it emerges that the rankings obtained with μ^* and with S_{T_i} are very similar in each of the scenarios, and in some cases even identical (especially for the HEST model), confirming reliability of the results.

The few cases where Morris inverts the ranking of two factors are those where their sensitivity indices values are very similar. In the worst case (in all 84 simulations we performed) the Morris design inverts 2 factors whose difference in the total indices represents nearly 18% of the total output variance. In general Morris can be considered successful in its goal of screening a subset of factors that can be fixed, as it never confounds groups of important and unimportant factors. If a factor is high ranked according to S_{T_i} it is also high ranked for Morris and vice versa.

The Morris method has the great advantage of a low computational cost. However, as a drawback it is not quantitative; the value of its measures can only be used to rank factors but cannot be interpreted as percentages of output variance. For this reason the S_{T_i} indices are used for analyzing the behavior of each input factors in different scenarios for example in the case of absence of jumps (Fig.1). In Figure 1 each dot refers to a scenario. The dot size highlights the importance of the factor in that scenario. The following conclusions can be drawn from the Figure:

- The three model parameters k, h, q are not very relevant at low times to maturity, but their importance increases with increasing the time horizon.
- The initial condition s_0 is the most important factor when the time to maturity is rather small and its importance decreases with time.
- The correlation ρ is also an influential parameter, especially when the option is not at-the-money.

When jumps are present the same conclusions can be drawn. Moreover the overall influence on the model outcomes of the three parameters related to jumps is relatively high, confirming the importance of the jumps inclusion. In particular, results show that λ and s_j are very much influential at all time horizons and strike prices (they are always among the three most important factors).

We also applied the Morris method to work with groups. Four groups were considered: the group of model parameters (k, h, q) relative to modeling the stochastic volatility; the initial condition s_0 , the group of model parameters (l, m_j, s_j) relative to jumps; and the correlation ρ . Results are plotted in Figure 2. In the plots the relative importance of the four groups is shown for all the considered scenarios. The total number of model evaluations for each scenario is $N=50$

The group of the jumps' parameters results to be always the most important, while the influence of s_0 and ρ depends upon the scenario characteristics. As expected, the group (k, h, q) is negligible at low times to maturity, but its importance increases with time horizon.

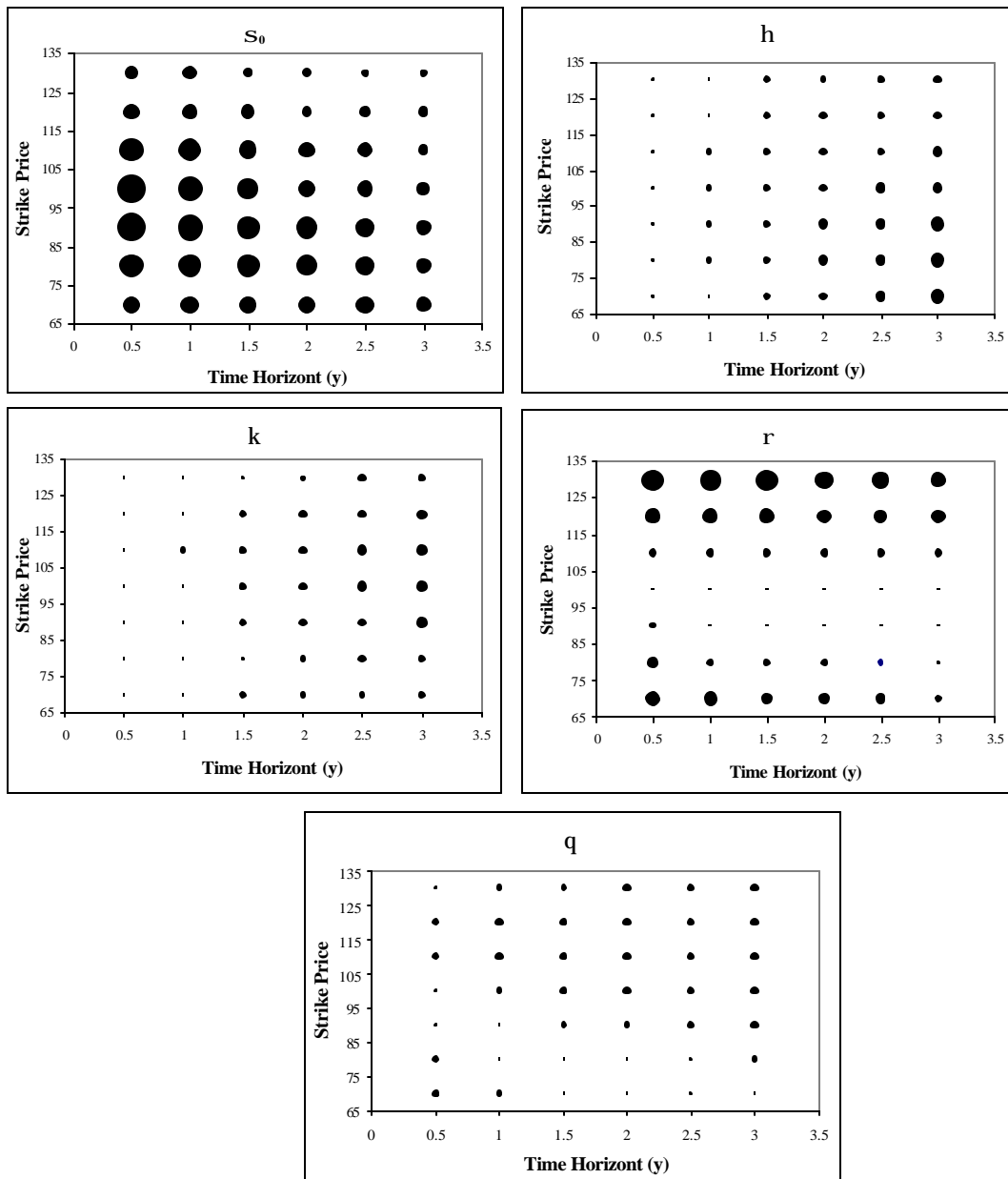
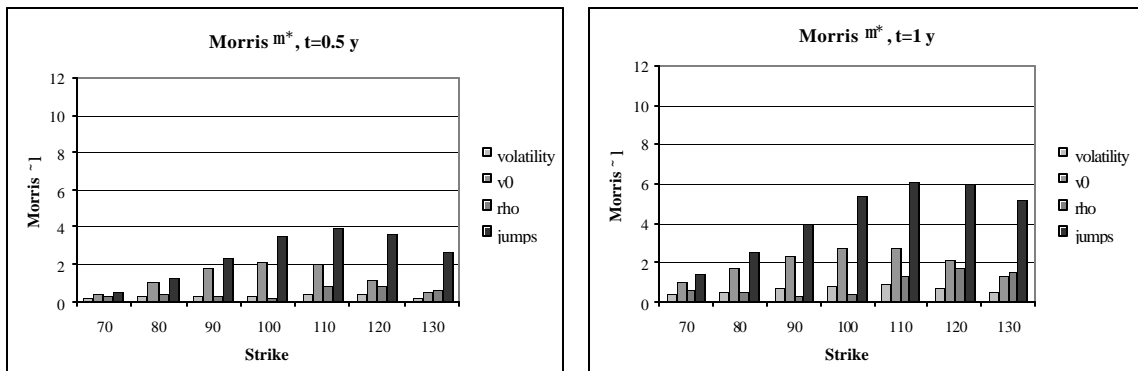


Figure 1: S_{T_i} results for the HEST model. The differences in the size of the dots represent the differences in the importance of the fixed input factors in all the considered scenarios.



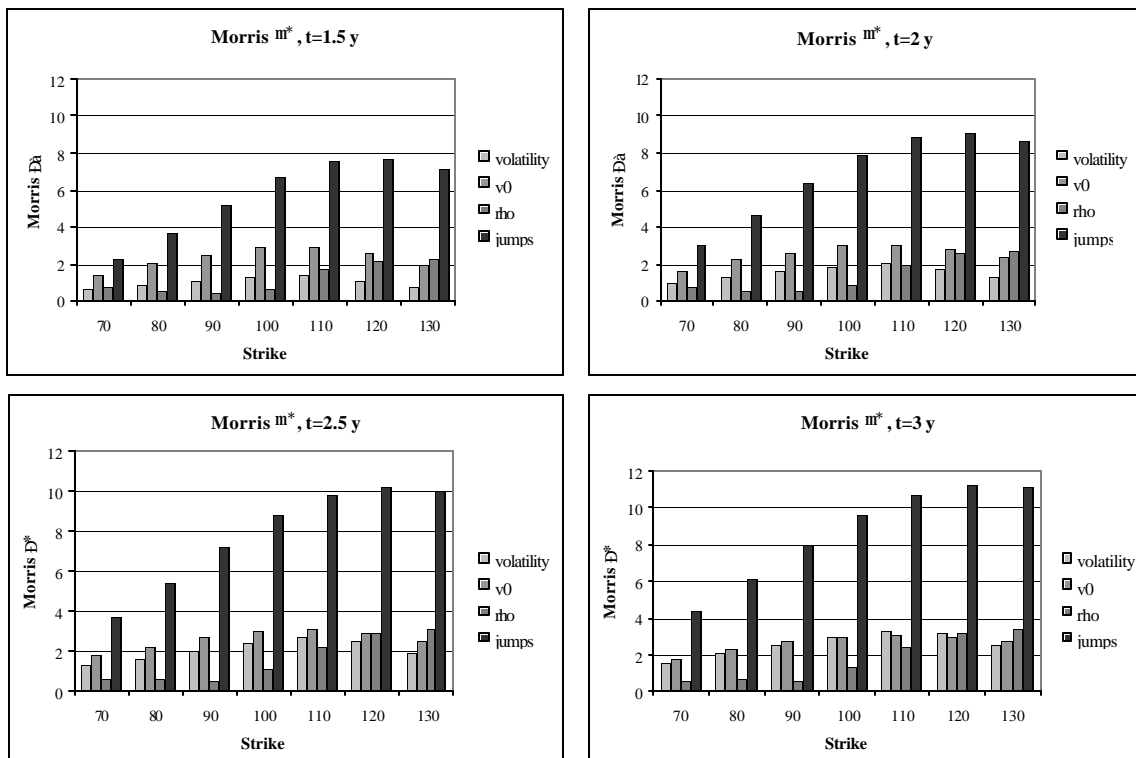


Figure 2: Screening results obtained through the Morris method for the HESJ model. 50 is total number of model evaluations for the Morris experiments. The bars plot the Morris revised μ^* , which can be used to screen the negligible factors in the model.

5. CONCLUSIONS

In this work we have confirmed the capability of the sensitivity measure μ^* , an improved version of the Morris measure introduced by Campolongo et al. [2], to distinguish between important and negligible model input factors at low computational cost. Also the updated measure has proved to be effective when factors are grouped.

Results of sensitivity analysis on the Heston model for pricing European option has allowed to concluding that jumps play a major role in determining the option price, thus stressing the need of including them in the model formulation. Furthermore results have underlined that, as expectable, at low time to maturity the initial condition for volatility needs to be accurately determined as the resulting option price is highly affected by its value. Its importance decreases as the time to maturity increases. Finally, it emerged that the correlation between the two Brownian motions needs to be carefully defined, especially when the option is not at the money, while the other model parameters are less important.

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